NOTE ON EXPERIENCE RATING CREDIBILITY

BY MARK KORMES

Mr. Perryman's extensive paper on "Experience Rating Credibilities"* forms the theoretical basis of the structure of the present multi-split experience rating plan used in Workmen's Compensation insurance. In my written discussion I have pointed out that the determination of Z and therefore W leads to a differential equation but Mr. Perryman ingeniously reduced the problem to a cubic equation for Z.

Recently I have had an occasion to design a rating plan where credibility would begin with a certain size of risk and would reach self-rating for another size both lower and upper limits selected by judgment. To make the formula as simple as possible I have selected the form

$$Z = \frac{E + fK}{E + K} \tag{1}$$

where E represents the size of risk, measured either by premium or expected losses or corresponding exposure and f is a function of E which varies from 0 when $E = Q$ to 1 when $E = S$. One can readily recognize that formula (1) corresponds to Mr. Perryman's formula (14B) and f corresponds to his $W = Z_s$.

The conditions for f are:

(a) $\frac{df}{dE} = 0$ for $E = Q$ and $E = S$

(b) $\frac{dZ}{dE}$ is positive

(c) $\frac{d(Z/E)}{dE}$ is negative

But condition (a) leads to a Bernoulian differential equation

$$\frac{df}{dE} = Af^2 + Bf \tag{2}$$

The solution of this equation is the well known logistic curve

$$f = \frac{C}{1 + e^{a + bf}} \tag{3}$$

*Proceedings Vol. XXIV

Since the constant C is usually very close to unity, $0 \leqq f \leqq 1$ and the constant b is always negative, simple differentiations show that conditions (a), (b) and (c) are satisfied. The actual determination of the constants also presents no difficulty. If f_1, f_2 and f_3 are three suitably selected equidistant values of f we have the following relations

$$C = \frac{2f_1 f_2 f_3 - f_2^2 (f_1 + f_3)}{f_1 f_3 - f_2^2} \tag{4.1}$$

$$a = ln \frac{(C - f_1)}{f_1} \tag{4.2}$$

$$b = \frac{l}{n} ln \frac{f_1(C - f_2)}{f_2(C - f_1)} \tag{4.3}$$

where ln is the natural logarithm and n represents the number of units on the E axis. It is quite clear that for different selection of the three values f_1, f_2 and f_3 a differently shaped curve will be obtained but such a selection will be always determined by practical desiderata. Thus, for example, in the case of Wisconsin taking $W_1 = .05$, $W_2 = .48$ and $W_3 = .91$ (the corresponding values of E are 15,000; 90,000 and 165,000; the interval is 75 units) we obtain:

$$W = \frac{1.0781}{1 + e^{3.0233 - .0229E}} \tag{5}$$

Several test calculations show that formula (5) gives the results which are very close to those given in the table of W for Wisconsin.