

1947 EXAMINATIONS OF THE SOCIETY

EXAMINATION COMMITTEE

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EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

April 9, 1947

Time 9:30 to 12:30 o'clock

1. (a) The ratio of the premium rate for risks of class A to the premium rate for risks of class B, is R . If the rate for risks of class A were to be increased x per cent and the rate for risks of class B increased y per cent, then the arithmetic mean of the two rates would be increased z per cent. But if the class A rate were increased y per cent and the class B rate increased x per cent, then the arithmetic mean would be increased kz per cent. Find the ratio of x to y in terms of k and R .
- (b) Find the sum of the first n terms of the series in which $(1 + a + a^2 + a^3 + \cdots + a^{k-1}) B^{k-1}$ is the k^{th} term.
2. (a) $1, \omega,$ and ω^2 are the three cube roots of unity. Evaluate $[2 + 2\omega + 5\omega^2]^6$
- (b) Using the binomial theorem, find $\frac{\sqrt[4]{401.2}}{\sqrt[3]{7.984}}$ to 4 significant figures.
- (c) If the number of combinations of 17 things taken r at a time plus the number of combinations of 17 things taken $(r - 1)$ at a time equals the number of combinations of 18 things taken $(r + 2)$ at a time, what is the number of combinations of r things taken 5 at a time?
3. (a) Solve the following equations for x and y :
 - (1) $y^2 = 5xy - 4x^2$
 - (2) $x^2 y^2 + 400 = 41xy$
- (b) If x is real, between what values must $\frac{x}{x^2 - 5x + 9}$ lie?

4. (a) Prove that if there be N sets containing m things, $2m$ things, $3m$ things . . . Nm things, respectively, then the number of ways in which a selection can be made consisting of exactly m things from each set, is

$$|Nm \div \{ |m \}^N$$

- (b) Given $\log_{10} 2 = .30$, solve the following equation:

$$\sqrt{5^a} + \sqrt{5^{-a}} = 5/2.$$

5. (a) What is the force of interest corresponding to an effective rate of 6%?
- (b) The amount of an ordinary annuity certain of \$100 per annum, payable annually, is \$6,667 and the present value is \$2,000. What is the effective rate of interest?
6. (a) A man leaves a perpetuity of \$1,000 annual rent to be equally divided between two hospitals. One is to receive the full annuity until it has received its share, after which the other will receive a perpetuity. Money is worth 4% effective. In what year must the payment be divided between the two hospitals?
- (b) A man borrows A dollars at $y\%$ interest, payable annually. He intends to build up a sinking fund to pay the debt off in n years by annual payments into a savings bank paying $x\%$ interest compounded semi-annually. Express in terms of A , x , y and n what the reduction in his total annual payment would be if the debt could be amortized (as to both principal and interest) by a series of n equal payments, the first to be made one year after the date of the loan.
7. A mortgage dated March 31, 1947 is to be retired by the payment of a fixed sum, S , at the end of each month for 20 years, the first payment to be made April 30, 1947. For each of the following questions, give an entirely numerical expression *indicating* the operations of addition, subtraction, multiplication, division, involution or evolution which would have to be performed if the answer to the question had to be determined without the use of any table whatsoever.
- (a) What should be the value of S if the mortgage secures a loan of \$10,000 bearing interest at the effective rate of 5% per annum?

7. (b) How much of the principal will remain unpaid on December 31, 1960, after the payment due on that date has been made?
8. (a) A \$1,000, 4 per cent bond, with dividends payable annually, is issued redeemable at par in 20 years. Fifteen years before this bond issue matures a man purchases one of these bonds at a price to yield him 3%. Seven years before maturity he purchases another of these bonds at a price to yield him 2%. What is the difference between the purchase prices of the two bonds?
- (b) A property costing \$10,000 has a scrap value of \$1,000 after 10 years of use. Compute the book value at the end of the seventh year, if depreciation charges are made according to the constant percentage of book value method.

Note: The following values will be useful in solving problems 5 to 8 inclusive:

$\log_{10} 1.06$	=	.0253
$\log_{10} e$	=	.434
$\log_{10} 1.995$	=	.3000
$(1.04)^{17}$	=	1.95
$(1.04)^{18}$	=	2.03
$a_{\overline{15} }$ (at 3%)	=	11.938
$a_{\overline{7} }$ (at 2%)	=	6.472

PART II

April 9, 1947

Time 1:30 to 4:30 o'clock

1. Determine the coördinates of the maximum, minimum and inflectional points of the curve

$$y = xe^{-x^2}$$

and draw a rough sketch of the curve.

2. (a) Given $\cos^2 U = \frac{1}{2} + \frac{1}{2} \cos 2U$, find the area bounded by the circle $\rho = a \cos \theta$ and the lines $\theta = 0$ and $\theta = 60^\circ$.
- (b) Expand $\log \left(\frac{1+x}{1-x} \right)$ in powers of x to three terms and determine the interval of convergence for this expansion.

3. (a) Find a function, $f(x)$, whose first derivative is

$$\frac{8x + 2}{x^3 - x^2 - 2x}$$

How many such functions are there and in what respect do they differ from each other?

(b) Find $\int_0^\pi (\sin^2 \phi + \cos \phi)^2 d\phi$.

4. (a) The strength of a rectangular beam varies as the product of the breadth and the square of the depth. Find the dimensions of the strongest beam that can be cut from a log whose cross section is an ellipse of semi-axes a and b .

(b) Find the limit of $(\sin x)^{\tan x}$ as x approaches $\frac{\pi}{2}$.

5. (a) Prove the following identity by the method of separation of symbols:

$$U_0 + x_{(1)} \Delta U_1 + x_{(2)} \Delta^2 U_2 + x_{(3)} \Delta^3 U_3 + \dots \\ = U_x + x_{(1)} \Delta^2 U_{x-1} + x_{(2)} \Delta^4 U_{x-2} + \dots$$

- (b) Given $f(3)=100$; $f(7)=448$; $f(11)=652$; $f(15)=776$; and $f(19)=884$; interpolate values of $f(4)$, $f(5)$ and $f(6)$ and state the assumption upon which your interpolation is based.

6. (a) Prove that $\Delta_{\frac{1}{\sqrt{x}}}^2 x^3 = x + y + z$.

- (b) Apply Lagrange's formula to find U_5 , given that

$$U_1 = 2, U_2 = 4, U_3 = 8 \text{ and } U_0 = 64$$

and explain why the results differ from those obtained by the series of powers of 2.

7. (a) From Newton's divided difference formula, derive the Gauss "forward" formula for the case in which $U_{-2}, U_{-1}, U_0, U_{+1}$ and U_{+2} are known.

- (b) Prove that the sum to infinity of the series

$$\frac{1^2}{2} + \frac{2^2}{3} + \frac{3^2}{4} + \dots \text{ is } e - 1,$$

8. (a) Prove that $[5] U_x = (5 + 5\delta^2 + \delta^4) U_x$.

$$\text{Where } \delta U_x = U_{x+\frac{1}{2}} - U_{x-\frac{1}{2}}.$$

- (b) Derive Simpson's rule for approximate integration from $U_x = (1 + \Delta)^x U_0$.

PART III

April 10, 1947

Time 9:30 to 12:30 o'clock

1. The value of a certain continuous variate, X , has been observed in each of 100 cases drawn at random from an indefinitely large class of cases in every one of which X has some one determinable value. The observed values are distributed as shown in the following table:

		<i>Number of Cases</i>	
12.5	$\leq X <$	17.5	0
17.5		22.5	1
22.5		27.5	4
27.5		32.5	12
32.5		37.5	20
37.5		42.5	25
42.5		47.5	21
47.5		52.5	11
52.5		57.5	5
57.5		62.5	1

- (a) Compute the arithmetic mean, the standard deviation and the skewness, α_3 , of the observations. Final operations of involution and division in the computation of α_3 , may be indicated without being performed.
- (b) State the probable error corresponding to each of the three results required in (a).
- (c) What is the meaning (in terms of the class from which the observed cases were drawn) of each of the statements of probable error required in (b)?
2. Suppose that you have available a table of values of

$$\phi(t) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}}, \text{ and of } \int_0^u \phi(t) dt,$$

from $t = 0$ to $t = \varepsilon$, and that on the basis of the data in Question 1, you have been asked to estimate for *each* of the intervals 12.5 to 17.5, 17.5 to 22.5, etc., the number of cases in which the value of X will be found to lie within that interval, out of any 1,000 cases drawn at random from the class from which the observed cases in Question 1 were drawn. Outline the procedure you would follow; and state the assumptions upon which your procedure would be based.

3. By the Method of Least Squares, determine the values of the constants in the equation of a curve fitted to the following data. State the reason for your choice of the form of the equation.

t :	0	1	2	3	4	5	6
u :	2	6	15	28	46	68	95

4. (a) Prove that $r_{xy} = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2 \sigma_x \sigma_y}$

- (b) If A is the simple arithmetic mean and H is the simple harmonic mean of a group of relatives, show that the geometric mean of these two means fulfills the time reversal test.

5. From the following data prepare a profit and loss statement for the A B C Company for the period beginning April 1, 1946 and ending June 30, 1946.

Inventory 4/1/46	\$15,000.	Office Salaries	\$1,200.
Sales Allowances	100.	Advertising	500.
Purchases	10,000.	Rent	500.
Sales	20,000.	Sales Salaries	1,000.
In Freight	1,000.	Sales Travel	500.
Inventory 6/30/46	14,000.	Bad Debts	800.
Purchase Returns	700.	Purchase Discounts	400.
Sales Discounts	700.	Interest Expense	300.
Office Expense	300.	Shipping Expense	1,000.
Depreciation	200.	Interest Income	100.

6. Suppose the following additional data were available for the A B C Company as of June 30, 1946 prior to the making of any adjusting or closing entries :

Capital Stock	\$20,000.
Surplus	1,900.
Cash	7,000.
Accounts Payable	4,000.
Accounts Receivable	5,000.
Equipment	3,000.
Accrued Salaries	100.
Unexpired Insurance	150.
Interest Accrued on Investments	150.
Interest Expense Accrued	200.

- (a) Would the addition of the above data make any difference in the net operating profit and if so, how much?
- (b) Prepare a balance sheet for the A B C Company as of June 30, 1946 from the information contained above and in problem 5.
7. (a) A partnership maintains a Purchase Journal and a Cash Journal. How should purchases for cash be handled so that duplicate entries are not posted to the Ledger?
- (b) The X Company has been authorized to issue 3000 shares of \$100 par value stock. It has issued 2000 shares of this stock at a 5% discount. At the end of the first year the surplus of \$15,000 is considered too small for the discount to be charged off. Indicate how the net worth section of the balance sheet will appear.
8. Two proprietors, A and B, owning similar businesses decide to merge and form a corporation. Their balance sheets are as follows:

Assets:	<i>A</i>	<i>B</i>
Cash	\$ 20,000	\$ 50,000
Accounts Receivable	100,000	50,000
Merchandise	30,000	20,000
Plant	50,000	80,000
Total	<u>\$200,000</u>	<u>\$200,000</u>
Liabilities:		
Accounts Payable	\$ 10,000	\$ 25,000
Mortgage	90,000	95,000
Total	<u>\$100,000</u>	<u>\$120,000</u>
Proprietor	100,000	80,000

The authorized capitalization is \$200,000. A and B each subscribe for one half the stock. The corporation purchases the assets of the two proprietorships paying therefor with stock. B buys the rest of his stock for cash. Make the opening entries for the new corporation and close the books of A's business.

PART IV

April 10, 1947

Time 1:30 to 4:30 o'clock

1. (a) A party of n persons is seated at random at a round table. Find the probability that 3 specified persons will not be seated in adjacent chairs.
(b) Players of equal skill, A and B, were playing a set of games in which the first player to win 10 games wins the set. A had won 8 games and B had won 6 when they were obliged to stop playing. How should the stakes be divided?
2. (a) Two of a pack of 52 cards have been lost. From the remainder of the pack two cards are drawn and are found to be spades. What is the probability that neither of the two missing cards is a spade?
(b) A bag contains $\frac{n(n+1)}{2}$ counters, one of which is marked 1, two of which are marked 4, three of which are marked 9, etc. A person draws out a counter at random and is to receive as many dollars as the number marked upon it. What is his expectation?
3. (a) On a straight line of length L , two points are selected at random. Find the probability that the distance between them is greater than M , where M is less than L .
(b) From a box containing n tickets numbered 2, 4, 6, 8, \dots , $2n$, respectively, k tickets are to be drawn one at a time, each one being replaced before another is withdrawn. What is the probability that the sum of the numbers on the k tickets is exactly $2(n+k)$?
4. A wallet contains 2 five dollar bills and 3 one dollar bills; a second wallet contains 3 five dollar bills and 2 one dollar bills. A bill is drawn from the first and placed in the second and then a bill is drawn from the second and placed in the first. In the drawings, each available bill has an equal chance of being selected.
 - (a) What is the most probable value of the bills in the first wallet after the second operation?
 - (b) What is the expectation of a person who is to receive the second wallet after the second operation?

5. (a) Find in commutation symbols an expression for a_{30} where the interest rate is to be 4% for the first ten years and 5% thereafter.
- (b) Prove that $C_x = v \cdot D_x - D_{x+1}$ and from this equation deduce the relation between A_x , a_x and the rate of discount.
6. (a) A life annuity contract, issued at age 30, provides for the payment of \$1,000 a year, first payment at age 60. The annuity is to be purchased by annual premiums payable for 30 years. In event of the death of the annuitant prior to age 60, the net premiums already paid are to be returned to his estate, without interest. Express the annual net premium in commutation symbols.
- (b) Develop by the retrospective method an expression in commutation symbols for the fifth terminal reserve on a twenty payment, thirty year endowment policy issued at age 35.
7. (a) An insurance policy issued to a man aged 20 pays \$10,000 if death occurs during the first year, \$9,000 if death occurs during the second year, etc., with coverage ceasing at the end of the 10th year. What is the net single premium for this policy?

7. (b) Evaluate $a_{\overline{25}|40}$, given

$\mu_{25} = .00804$	$N_{35,35} = 313072.$
$\mu_{35} = .00888$	$N_{36,36} = 292975.$
$\mu_{36} = .00902$	$D_{25} = 37674.$
$\mu_{40} = .00977$	$D_{40} = 19727.$
$N_{25} = 770113.$	$D_{35,35} = 20097.$
$N_{40} = 344167.$	$D_{36,36} = 19073.$

Note: The answer may be expressed in terms of the numerical values of the commutation symbols without performing the final arithmetical operations.

8. (a) State in words the meaning of the symbol ${}_nE_x|y\bar{z}$ and derive an expression of its value in terms of pure endowments.
- (b) Write out the formula for the share of z in the case of an annuity $a_{x:\overline{y}|z}$, assuming y and z share the annuity equally throughout their joint lives after the death of x .

Note: The following values will be useful in solving problems 5 to 8 inclusive:

$$M_{20} = 13267$$

$$D_{20} = 46556$$

$$R_{21} = 384016$$

$$R_{31} = 268760$$

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

April 9, 1947

Time 1:30 to 4:30 o'clock

1. Name the various types of United States Government bonds and discuss the factors which determine the investment quality of these bonds.
2. (a) What are the various factors that should be considered before investing in a particular issue of municipal bonds?
(b) Discuss the relative desirability of equipment trust certificates and railroad first mortgage bonds in the portfolio of a casualty insurance company.
3. Do you believe the entry of the Federal government into the field of electrical power seriously impairs the value of investments in utilities of that nature on the part of casualty insurance companies? Discuss.
4. (a) What are the essentials of an insurance contract?
(b) Distinguish between breach of warranty and misrepresentation in an insurance contract. What are the possible effects of each in the event of loss?
5. (a) What are the four qualifications given in the New York Insurance Law which must be met by rating organizations in the making of rates?
(b) Outline briefly the steps which are being taken in the various states to meet the requirements of Public Law 15.
6. Outline briefly the provisions of the New York Insurance Law relating to the establishment of loss and loss expense reserves by casualty insurance and surety companies.

7. (a) Why are the following two cases decided by the United States Supreme Court in 1946 important to the insurance industry?
- a. Prudential *vs.* Benjamin
 - b. Robertson *vs.* California
- (b) Discuss the reward for risk taking.
8. The Comprehensive Rating Plan for National Defense Projects provides for combining workmen's compensation, automobile liability and property damage and other liability and property damage insurance for the rating of individual risks. From the standpoint of the economic theory of risk, do you think this rating method should be carried over to regular risks? Do you think that it would be desirable to include other lines of insurance? Discuss.

PART II

April 10, 1947

Time 9:30 to 12:30 o'clock

1. Explain the function of Schedule P and outline briefly how it operates. Limit your answer to Parts 1, 2, 5 and 5A.
2. (a) In the case of those lines of casualty insurance to which the claim or notice-average basis is applicable for the determination of known loss reserves, outline a method that might be used for determining a reserve for incurred but not reported losses.

(b) Describe and illustrate four situations that may cause a casualty insurance company to establish a voluntary underwriting reserve.
3. (a) Automobile policies are ordinarily written for a one year term and sometimes provide for the payment of premium on a quarterly installment basis. What, in your opinion, is the best method of handling the premium installments as respects the determination of unearned premium reserve? Give reasons.

(b) Outline the essential features of the Automobile Statistical Plan of the National Bureau of Casualty and Surety Underwriters.
4. Give the advantages of the information obtained from the

reporting of workmen's compensation experience under the Unit Statistical Plan as opposed to the Schedule Z method whereby each carrier submits its own experience by classification.

5. Explain why the rate changes that have taken place during the past year or so in automobile insurance invalidate the use of calendar period loss ratios for the evaluation of current underwriting results. What adjustment should be made in such data to render it useful for a proper evaluation of current results?
6. Outline briefly a method that might be used by a casualty company for the compilation of classification statistics on liability lines other than automobile that must be filed annually with external agencies for ratemaking purposes. Include a design of the premium and loss punch cards.
7. Outline briefly a method of distributing casualty administration expense by line of insurance for the following departments:
 - a. Actuarial and Statistical
 - b. Accounting
 - c. Compensation and Liability Underwriting
8. (a) What items are known as Non-Ledger Assets in the asset and liability pages of the annual statement blank of the N. A. I. C.? What items are considered Non-Admitted Assets?
 - (b) Outline briefly a method that might be used by a casualty company to determine its administration and payroll audit expense by size of risk for workmen's compensation insurance.

EXAMINATION FOR ENROLLMENT AS FELLOW

PART III

April 10, 1947

Time 1:30 to 4:30 o'clock

1. (a) Outline the burglary experience rating plan, giving the principal features.
 - (b) What purpose is served by the "B" and "W" values in the National Council experience rating plan for workmen's compensation risks? Give the modification formula and explain the effect of the "B" and "W" values.

2. Outline a program for interstate prospective experience rating for workmen's compensation insurance.
3. Discuss the merits of making the Retrospective Rating Plan available to assureds for a term of three or five years as an additional alternative to the present one-year basis.
4. Some of the cash sickness fund plans being considered for presentation to state legislatures contain provisions permitting the use of at least part of the large unemployment reserves built up during the war. Do you think these reserves should be used for cash sickness funds? Discuss.
5. Extensive consideration is being given currently to the problem of providing hospital and medical care, and accident and health coverage for the majority of the people. Are all of these areas in which casualty insurance companies can function effectively, or do you think some of them should be the responsibility of state or federal government? Discuss.
6. (a) Discuss the growing need for Product Public Liability insurance.
(b) Discuss the effect of business cycles upon the Casualty Insurance business.
7. In view of current conditions, outline an underwriting and production program for the balance of this year which you would recommend to a company writing all lines of casualty insurance.
8. A company not wishing to increase its relative volume of automobile business is faced with the problem of dealing with new business being offered as a result of the passage of financial responsibility laws in additional states. How would you advise this company to handle the situation thus created without seriously impairing its reputation for providing the public with adequate insurance facilities?