1943 EXAMINATIONS OF THE SOCIETY

APRIL 7 AND 8, 1943

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

1. (a) Using the binomial theorem, evaluate $\sqrt[3]{528}$ to four places of decimals.

   (b) Prove the equality of the following binomial expansion:

$$1-\frac{1}{2}\cdot\frac{1}{3}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{3^2}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{1}{3^3}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\cdot\frac{1}{3^4}-\cdots\cdots$$

$$=1-\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2^2}-\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{1}{2^3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{1}{2^4}-\cdots\cdots$$

2. (a) If the arithmetic mean between a and b is twice as great as the geometric mean, show that

$$a:b=(2+\sqrt{3}):(2-\sqrt{3})$$

   (b) Prove by induction:

$$1+3+5+\cdots\cdots+(2n-1)=n^2$$

3. (a) Solve:

$$\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$$

   (b) For what values of m will the following equation have roots which are equal in magnitude and opposite in sign?

$$\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$$

4. (a) How many numbers of six digits each can be formed if neither 0 nor 1 is to occupy either end position and no repetitions of digits are allowed?

   (b) Find the number of combinations that can be made by taking four letters from the word "combinations".

(c) Find the number of permutations that can be made by taking four letters from the word "permutations".

5. (a) A paving assessment of \$845.19 is to be paid, principal and interest at 5%, in 10 equal annual installments. What is the indebtedness to the city at the time the sixth payment is due but not paid?

(b) What is the purchase price of a \$10,000 bond bearing half-yearly coupons of \$300 each and repayable in 25 years at a price of 110, when the investor is to realize 5% convertible semi-annually?

6. (a) A man buys a team of horses for \$500 and a wagon and harness for \$150. The team will have to be replaced in 10 years and the wagon and harness in 20 years. Feed and blacksmith bills cost \$200 a year, taxes and insurance \$20, and wages of driver \$600. What total sum will be required to purchase the equipment, to provide for its periodic replacements, and to assure payment of the annual expenses, assuming the costs remain constant and that money is worth 5 per cent?

(b) Derive a relationship between successive values of $s_{\overline{n}|}$ by which a table of $s_{\overline{n}|}$ could be constructed.

7. (a)* A house priced \$5,000 is sold on a basis of equal monthly payments to extend through 7 years, the first payment to be made at the end of the first month. What is the monthly payment if money is worth 5% effective?

(b)* What is the monthly payment for problem 7(a) if money is worth 5% compounded semi-annually?

8. (a)* A man buying a piece of property is offered the three following methods of payment:

(i) An annuity of \$1,900 for four years.

(ii) A four payment annuity of \$2,200 deferred for four years.

(iii) Four payments of \$2,100 at the end of two, four, six and eight years.

What is the present value of each method, money being worth 5 per cent?

(b)* In each case of problem 8(a), what additional payment at the end of the second year would extinguish the unpaid portion of the debt?

*NOTE: Questions marked with asterisk (*) — the answer may be stated in the form of an expression in which all numerical values are entered but not multiplied.

Given Values:

$j_{(12)}$ (at 5%) $= 0.0489$
v^2 (at 5%) $= 0.9070$
v^4 (at 5%) $= 0.8227$
v^6 (at 5%) $= 0.7462$
v^8 (at 5%) $= 0.6768$
v^{50} (at 2½%) $= 0.2909$
$(1.050625)^{\frac{1}{2}} = 1.0041$
$(1.050625)^{-7} = 0.7077$
$\frac{1}{a_{\overline{7}|}}$ (at 5%) $= 0.1728$
$\frac{1}{a_{\overline{10}|}}$ (at 5%) $= 0.1295$
$\frac{1}{a_{\overline{20}|}}$ (at 5%) $= 0.0802$

PART II

1. (a) Differentiate:

$$y = x^{e^x} \text{ and express in terms of } x.$$

(b) Given $\log_e 2 = 0.693$, find $\int_0^1 \frac{x^3\,dx}{1+x^4}$

(c) By the trapezoidal rule, evaluate approximately $\int_0^3 \frac{x^2\,dx}{1+x}$

2. (a) Evaluate $\int x^n \log x\,dx$.

(b) The altitude of a circular cone is 100 inches, and decreases at the rate of 10 inches per second; and the radius of the base is 50 inches, and increases at the rate of 5 inches per second. At what rate is the volume of the cone changing?

$$(V = \tfrac{1}{3}\pi r^2 h)$$

3. (a) Expand e^x by Maclaurin's series and determine for what values of x the series is convergent.

(b) A third degree curve has a maximum at $-1, 4$ and a minimum at $1, 0$. Find the equation of the curve, and the point of inflection.

4. (a) Find the volume of the oblate spheroid generated by revolving the area bounded by the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ about the y axis.

(b) Using Taylor's series, expand $\tan x$ in powers of $\left(x - \frac{\pi}{4}\right)$ to three terms.

5. (a) Find the sixth term of the series 1, 37, 61, 77, ····· and prove by extending the table of differences.

(b) From the following data estimate the missing values by a method of finite differences:

Time—	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.	7 P.M.	8 P.M.
Temperature—	53	54	—	49	47	—	40	35

6. Sum, to twenty terms, $5 + 10 + 17 + 28 + 47 + 82 + \cdots\cdots$ (The answer may be left as a numerical expression without evaluating powers.)

7. Evaluate to four decimal places the root of the equation

$$x^3 + x - 3 = 0$$

which lies between 1.20 and 1.24, using Stirling's formula and two approximations; given that

$$1.20^3 = 1.728 \qquad 1.22^3 = 1.816$$
$$1.21^3 = 1.772 \qquad 1.23^3 = 1.861$$
$$1.24^3 = 1.907$$

8. (a) Use Newton's divided difference formula to find the form of the function $y = U_x$; given that

$$U_0 = 8 \qquad U_4 = 68$$
$$U_1 = 11 \qquad U_5 = 123$$

(b) Using Lagrange's formula, find U_4; given that

$$U_1 = 7$$
$$U_3 = 5$$
$$U_6 = 1$$

PART III

1. (a) Criticize the presentation of the data in the following table:

Amount of Rent Paid for Each Apartment in a Certain City

Rent (in dollars)	Number of Apartments (00 omitted)
20. to 30.	2
30. to 40.	14
40. to 50.	21
50. to 60.	8
60. to 70.	5
Total	50

(b) From the data of the table shown in problem 1(a) above (disregarding your criticism) compute the mode and the median, and give a formula for the geometric mean in terms of logarithms.

(c) From the data of the table shown in problem 1(a) above (disregarding your criticism) compute the standard deviation and the mean deviation.

2. Find the standard error of estimate in terms of the standard deviation and the Bravais-Pearson Coefficient of Correlation; given

$$\Sigma \rho^2 = m^2 \Sigma x^2 - 2m \Sigma xy + \Sigma y^2$$

$$\text{where } m = \frac{\Sigma xy}{\Sigma x^2}$$

3. Assuming that a second degree parabola is to be fitted to a mass of observed data, use the method of moments to derive equations which may be used to evaluate the constants in the equation of the curve.

4. (a) Fit a second degree parabola to the following data:

x	1	2	3	4	5
y	0	2	5	8	11

(b) Two types of electric bulbs were observed as to the length of life, and the following data were secured:

Type 1	Type 2
$N_1 = 100$	$N_2 = 100$
$M_1 = 1224$ hours	$M_2 = 1036$ hours
$\sigma_1 = 36$ hours	$\sigma_2 = 40$ hours

Determine the probable error of the difference between the means. Is the difference in the two means sufficient to warrant the conclusion that type 1 is superior to type 2? State your reasons.

5. (a) The trial balance of the ledger accounts of the X Company is given below as it appeared on December 31, 1941, prior to adjustment and closing. Adjustment items are given below. Show all adjusting and closing journal entries in connection with Operating Wages and Salaries, Depreciation, Bad Debts, and Rent.

Trial Balance—12-31-41

Cash	250,000	
Accounts Receivable	130,000	
Inventory—Finished Goods (1-1-41)	50,000	
Inventory—Goods in Process (1-1-41)	25,000	
Inventory—Raw Materials (1-1-41)	40,000	
Machinery and Equipment	1,040,000	
Reserve for Depreciation		200,000
Accounts Payable		58,000
Bonded Indebtedness		400,000
Surplus		500,000
Capital Stock		250,000
Sales		1,605,000
Sales Returns	100,000	
Discount on Sales	5,000	
Purchases	506,000	
Freight in on Purchases	4,000	
Discount on Purchases		3,000
Purchase Returns		7,000

Operating Wages and Salaries.....	237,000	
Rent	11,000	
Heat, Light and Power.....	15,000	
General Manufacturing Expense.....	30,000	
Advertising	50,000	
Selling Expense.....	250,000	
Administrative Expense.....	200,000	
Interest on Bonded Debt.....	20,000	
Dividends	60,000	
	3,023,000	3,023,000
Adjustment data:		
Inventories 12-31-41:		
Finished Goods.....		60,000
Goods in Process.....		40,000
Raw Materials.....		50,000
Accrued Operating Wages and Salaries (Unpaid on 12-31-41).....		3,000
Depreciation during 1941.....		40,000
Estimated Loss on Bad Debts.....		30,000
Estimated Federal Income Tax Payable		50,000
Rent Prepaid.....		1,000

(b) Set up sample ledger accounts (T accounts), and post the adjusting and closing entries, for the following accounts of the X Company:

Operating Wages and Salaries
Reserve for Depreciation
Estimated Loss on Bad Debts
Rent

6. & 7. Prepare a Profit and Loss Statement for the X Company for the year ending December 31, 1941.

8. (a) Prepare a Final Balance Sheet for the X Company as of December 31, 1941.

(b) A "book audit" is concerned only with the accuracy of the books of account, and requires no investigation of the basis for determining the amounts which are entered in those accounts which require the exercise of judgment. If you, as a stockholder, were told that the Balance Sheet and the Profit and Loss Statement of the X Company had been subjected to a book audit, what additional information would you want to have in order to judge the Company's actual financial condition?

PART IV

1. (a) Six persons throw for a stake of $100, which is to be won by the player who first throws heads with a penny. If they are to throw in succession, find the expectation of each player.

(b) In five throws with a single die, what is the chance of throwing the following:

(i) three aces exactly
(ii) three aces at least

2. (a) Three cards are drawn from an ordinary pack of 52 cards. What are the respective probabilities of the following results if the cards are drawn one at a time and replaced:
   (i) A spade each time
   (ii) At least one spade
   (iii) Three cards of the same suit
   (iv) Three cards of different suits
   (v) Two cards of same color and the other different.

   (b) If a coin is tossed 1000 times, how many times can you expect to get exactly five heads in succession?

3. From a bag containing n balls, all either white or black, all numbers of each being equally likely, a ball is drawn which turns out to be white; this is replaced, and another ball is drawn, which also turns out to be white. If this ball is replaced, prove that the chance of the next draw giving a black ball is $\frac{1}{2}(n-1)(2n+1)^{-1}$; given that

$$\sum_1^n r = \tfrac{1}{2} \cdot n(n+1)$$

$$\sum_1^n r^2 = \tfrac{1}{6} \cdot n(n+1)(2n+1)$$

$$\sum_1^n r^3 = \tfrac{1}{4} \cdot n^2(n+1)^2$$

4. (a) A speaks the truth 2 out of 3 times, B 4 times out of 5; they agree in the assertion that from a bag containing 6 balls of different colors a red ball has been drawn. Find the probability that the statement is true.

   (b) In a lottery all the tickets are blanks but one; each person draws a ticket and retains it; show that each person has an equal chance of drawing the prize.

5. (a) The present value per \$100 of annual wages for future payments to each of the three youngest children of a surviving widow, given in Table IV of Special Bulletin No. 207 of the N. Y. Department of Labor, is computed from the following formula:

$$15\ {}_1\bar{a}_{\overline{18-y}|} - 5\ {}_1\bar{a}_{x'} : \overline{{}_{18-y}|}$$

   If x is the widow's age and y the child's age, explain briefly the meaning of each term of the formula, and tell why this table can be used for only the three youngest children.

(b) Show that

$$1 + e_x = q_x + p_x (1 + q_{x+1}) + {}_2p_x (1 + q_{x+2}) + \cdots\cdots$$

6. (a) (i) Show algebraically that $a_{x:\overline{n}|} < a_n$

(ii) Express the annual premium for a 20 year term insurance entirely in terms of temporary life annuity symbols and the rate of interest.

(iii) Interpret in words the symbol ${}_nE_{xy}|_z$ and express its value in terms of pure endowments.

(b) Compute the present value of a temporary life annuity due on the life of an individual now aged 30 providing for a sequence of payments of \$500 a year for fifteen years followed by a sequence of ten payments of \$1000 each year; given that

$$N_{30} = 596800 \qquad N_{55} = 125000$$
$$N_{45} = 253700 \qquad D_{30} = 30400$$

7. Find the equal age w by means of the Gompertz-Makeham law of mortality, and prove the Law of Uniform Seniority.

8. (a) A special 20 year endowment policy issued at age x provides, in event of the death of the insured during the twenty year period, for a benefit of \$1.00 and the return, without interest, of all net premiums paid. If the insured survives the twenty year period the policy matures, the insured receiving only the face amount of \$1.00. Show that the net annual premium may be written in the form

$$\frac{M_x - M_{x+20} + D_{x+20}}{N_x - N_{x+20} - R_x + R_{x+20} + 20\,M_{x+20}}$$

(b) Show that Fackler's Accumulation Formula can be written in the form

$${}_{t+1}V_x = ({}_tV_x + P_x - c_{x+t})\,u_{x+t}$$

given $c_x = \dfrac{C_x}{D_x}$

PART V

1. (a) Outline the provisions for determining the excess limits premium for Occupational Disease (Paragraph 1 b) Coverage which are contained in the Compensation Manual of the National Council on Compensation Insurance.

(b) Explain the purpose and principles underlying the Workmen's Compensation and Employers' Liability minimum premiums and the rules for determining them when several classifications and states are involved.

2. (a) Under what conditions would a steam boiler insurance policy cover a fire loss resulting from a boiler explosion or from a machinery breakdown?

   (b) Under what conditions may additional insureds be included under a fidelity schedule, fidelity blanket or forgery bond, and what are the special premium charges, if any?

3. (a) A certain company is issuing both can and non-can accident and health policies. In accordance with current practice and existing laws, what steps can it take to protect itself against moral hazard resulting from an economic depression?

   (b) What additional safeguards do you think it advisable to adopt?

4. (a) Describe the New York Workmen's Compensation Premium Discount Plan.

   (b) What coverage is contemplated under a Manufacturers' and Contractors' policy for "after completion of operations"?

   (c) What is the difference in the application of the N.P.D. rules between (1) mercantile, mining, construction or erection operations, and (2) other operations?

5. (a) What was the result anticipated when the contingency loading program was introduced in Workmen's Compensation rate-making?
   What have been the actual results?

   (b) Discuss briefly some of the problems of "account current" methods of establishing rate levels.

6. Discuss the feasibility of making Workmen's Compensation rates on a governing classification basis.

7. (a) What is meant by saying that a rating classification is homogeneous? Compare the following lines of insurance by degree of homogeneity of classification:

      (a) Automobile, (b) Workmen's Compensation, (c) Manufacturers' and Contractors' Liability, (d) Burglary, (e) Fidelity.

   (b) Under present circumstances, what would you consider the causes of an observed indicated increase in average settlement costs for automobile liability insurance?

8. What is the purpose of a rate level factor? To what extent is it a trend factor? Does its function vary in any way in the different lines of insurance? Under what circumstances could it cause an unwarranted increase in the rate for a given classification; (a) in Automobile Insurance, (b) in Workmen's Compensation Insurance?

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

1. (a) The Treasury Department in recent financings has indicated a desire to place as many of its bonds as possible in the hands of individuals, institutions, banks other than those accepting demand deposits, and insurance companies.
   (1) What effect will this have upon inflationary trends?
   (2) Upon insurance company portfolios?

   (b) Discuss the effect upon low coupon long term bonds if interest rates should reach higher levels. How would this affect the balance sheets of casualty insurance companies?

2. Under what circumstances do you think bonds held by an insurance company should be amortized. Suggest rules for determining the propriety of amortizing individual bonds considering market value, financial ratings, and interest rates.

3. (a) Outline the analysis on income, disbursements, assets, and liabilities which should be made to determine the optimal amount of the assets of a carrier writing only Workmen's Compensation Insurance which should be held in cash or short maturities if it is the policy of the carrier to seek the best available yield on its assets.

   (b) Do you consider it advantageous for a company writing only Workmen's Compensation Insurance to hold all of its assets in cash or short maturities?

4. (a) Give the legal meanings of "waiver", "estoppel", and "election".
   (b) An insured has a Paymaster Robbery Policy. A holdup occurred at which time the money was in possession of a guard accompanying the custodian. What rights for recovery has the insured under the Policy? Give reasons.

5. Outline the provisions of the New York Code for capital and surplus of a casualty insurance company. Do you believe these requirements to be proper for the different lines in all cases?

6. (a) What types of governmental bodies levy taxes on casualty insurance premiums? What are the purposes and justification of these taxes?

   (b) Do you think that the two year carry-over provision of the excess profit section of the 1942 Federal Revenue Act would operate, under normal conditions, with equal fairness to a multiple line Casualty Insurance Company and a large manufacturing concern?

7. Discuss the desirability of establishing uniform supervised rates for (a) automobile liability insurance, (b) fidelity bonds, and (c) accident and health insurance.

8. Discuss the feasibility of private insurance carriers assuming the hazards of war damage in the United States.

PART II

1. (a) Discuss the desirability of carrying a special loss reserve for undisclosed occupational disease claims. What method or methods would you suggest for accumulating such a reserve?

   (b) Would you favor accumulating a special reserve from the catastrophe loading collected in Workmen's Compensation rates? Give reasons.

2. (a) Under Schedule "P", unallocated loss expenses are required to be allocated to policy year on the basis of specified percentages. What effect has this requirement on the "case basis" policy year incurred loss and loss expense ratios of (1) a rapidly growing company and (2) a rapidly retrogressing company?

   (b) Outline a method that might be used to test the adequacy of the aggregate loss and loss expense reserves established by a carrier in its financial statement of two years ago on its liability and compensation business assuming that only the information reported in the carrier's last three annual statements is available.

3. (a) What changes would you consider necessary and practical to be incorporated into the unit statistical plan for alleviating the manpower problem of the rate-making bureaus and of the insurance carriers?

   (b) What changes have been made in the statistical plans covering automobile casualty insurance for the purpose of alleviating the manpower problem in the rate-making bureaus and the offices of the carriers?

4. Discuss the advantages and disadvantages of converting the Convention Form of Annual Statement Blank for miscellaneous companies to an earned-incurred basis.

5. What changes would you make in the assets and liabilities of a casualty company as shown on the Convention Form of Annual Statement Blank for miscellaneous companies in order that the capital and surplus accounts would represent the true worth of the carrier as a going concern?

6. (a) What statistical and accounting records of subrogation should be maintained in automobile liability insurance?

   (b) Give two methods by which reinsurance recoverable on paid losses may be shown in the annual statement of miscellaneous companies.

7. (a) Discuss the validity and utility of experience statistics currently being collected on casualty insurance lines.

   (b) What reserves of a casualty insurance company should be given special attention because of current conditions? Why?

8. Design a punch card suitable for continuous accumulation of the "case basis" reserves on reported claims for a large multiple line casualty company, and describe the treatment that would be accorded the various types of transactions entering into the net reserves.

PART III

1. Outline in detail how you would estimate the underwriting loss to a carrier covering a large volume of Workmen's Compensation Insurance on the Retrospective Rating Plan if we assume that the ratio of losses to standard premiums on this business will be 80%. Assume that the items of expense of the carrier including claims expense and miscellaneous taxes approximate the allowances for company expenses in the Retrospective Rating Plan.

2. (a) What modifications are made in the formula for Retrospective Rating for ex-medical risks?

   (b) Outline the principal features of the automobile experience rating plan used in New York.

3. What new principles were introduced into the writing and rating of casualty insurance by the War Department Comprehensive Rating Plan? Discuss the desirability of applying these principles to all large casualty risks.

4. (a) Under what circumstances and to what extent do you consider that compulsory methods are justified in distributing any social cost? Discuss the application of your opinions to

   (a) automobile injury
   (b) unemployment
   (c) disability

   (b) In some industries the principle underlying the price structure is to charge what the traffic will bear. Discuss the application of this principle to casualty insurance rates.

5. Present in broad outline a social program to meet the minimum needs of the average man and discuss the probable effect of such a program on the economic structure of this country.

6. (a) What are the three guiding principles underlying the recommendations in the Beveridge Report?

   (b) What would be the effect on private casualty insurance if such a plan as the Beveridge Plan should be adopted?

7. A stock casualty insurance carrier writing Workmen's Compensation and Automobile Liability Insurance is offered for sale. Outline the calculations which the seller should make in determining a fair value for the net worth. What other factors should be considered in the actual purchase price?

8. Certain causes have contributed to the creation of the present problem of the rating of interstate risks. What are the causes and how have they created the problem? What solutions have you to offer keeping in mind the demands of the buyers of insurance for such interstate risks?