ABSTRACT OF THE DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

SOCIAL BUDGETING

W. R. WILLIAMSON

VOLUME XXIV, PAGE 17

WRITTEN DISCUSSION

MR. C. A. KULP:

Mr. Williamson presents a completely rational argument for a thorough re-analysis of American ways and means for meeting *all* of the average man's important risks. We suffer, he believes, from an over-emphasis on the purely private approach, and more particularly from the over-development of life as against other insurance and the related emphasis on the banking as against the insurance method of accumulating funds. As long as these were restricted in their effect to the top drawer of our population, to people generally quite able to care for themselves, there was no objection to them. The difficulties and the dangers—the latter word is mine not Williamson's though he will probably agree—which arise, are arising, because we assume that principles suitable and inevitable for a few of us will be suitable for all of us.

The Williamson arguments are essentially two. Social as against individual provision is better suited to the risks of the great masses; social provision is inevitable. The mere description of the average man's principal lifetime risks and their costs in terms of national income is a short and effective proof of the first argument. Of the five risks listed, two are clearly catastrophic. The cost even at minimum estimates runs to 16 per cent of income, and risks as important as these simply cannot be left to voluntary provision or private competition. The way things are, death gets more than its fair share, the banking more than the protective function, of the average man's income. I am grateful to Mr. Williamson for his candid treatment of more than one sacred insurance cow. As a people we are much too inclined to treat insurance, which we take to mean the private insurance

business, with reverence. It is and should be regarded as another economic and social institution. Of things that need saying I appreciate particularly his contrast of insurance as business and as cooperative enterprise, his comments on the reserve in Federal old age insurance.

But the implications of the Williamson statement of risks and their costs is deeper and more pervasive. It suggests, and I agree, that social provision is inevitable. Indeed in a country that to date insists that no man shall starve it is axiomatic. We are making social provision right now. The question is: what form shall this provision take? One form is social insurance. Conservatives may argue that they are opposed to social insurance; they can hardly argue against social provision because they are committed to social provision under the present arrangements. "When the bills are presented they must be paid."

The rub is that as a people we are not yet prepared for all of the consequences of the fact that the bills must be paid. We still most of us dwell in a happy land of political-economic juvenilia where we can annually demand more and more governmental services and costs, higher and higher standards, and yet believe that somehow, even if all the rest of the citizenry will have to pay, we somehow will be allowed to escape. It is a completely unrealistic attitude toward taxes, one that would seem strange in Britain or Germany at every level of income, but it is an American fact. We are all looking for a way to meet these costs out of other people's pockets. Social insurance is a shock because it itemizes and dramatizes a bill we thought hadn't existed. Most social insurance antis oppose it because they believe that it imposes a new cost.

One reason, then, for our distaste of social insurance is that it represents a new allocation of a tax. I cannot agree with Williamson on "the relative simplicity of universal contribution," except in perhaps the actuarial sense. Perhaps a more fundamental reason, one more rooted in our folkways, is our wistful nostalgia for a day and a country where such problems did not exist. It is easy to underestimate this force, even in a land that prides itself on having no past, only future. It explains our insistence that social insurance must not cramp or suppress the acquisitive or at least the independent instinct that we like to believe springs pure only

from American soil. On this, by the way, we are unduly fearful. Social insurance is generally, and in this country will inevitably be, insurance for the great mass of wage-earners, not for those few for whom individual and voluntary provision is suitable. Incidentally, even the British worker, insured for flat benefit rates in his social insurance schemes, is quite unable to understand the objections of Americans to social insurance. "The more you earn," he says, "the more you save and can add to flat insurance benefits": social insurance benefits, he considers "are just something to go on with for a while." Private saving, that is, and social provision, complement each other. The British Royal Commission on Health Insurance vetoed for the same reasons proposals to eliminate private and commercial health insurance bodies. These, said the Commission, are just as surely symbols of British self-help as any form of social insurance under state operation.

In brief, the social insurance institution must meet not one but two basic tests: it must be shaped to meet a particular risk; it must be shaped to suit as well as may be a particular people. On the first we are more likely to agree on details than on the second, but even here we have spilled a lot of ink arguing whether the unemployment risk is insurable. Probably not, by the standards of other social insurances; but whatever we call it, we are committed, as the whole world is committed, to some sort of orderly social provision for the unemployment risk. Social insurance institutions are traditional and social and political (in the broad sense) and not technical: this means in the nature of things they are empirical, flexible and thus (in the narrow sense) often illogical. Above everything else, to be able to do its work the social insurance institution or plan must have the confidence and support of its members and contributors. This is why the social insurance scheme in practice often includes characteristics entirely illogical or even improper on the point of pure theory. This explains why it is perhaps better to have an old age scheme complicated and limited by the banking element Williamson deplores (and I) rather than no old age scheme at all. State administration of unemployment insurance, which can be very bad, is perhaps better than waiting for the perfect way to handle unemployment, whatever that perfect way is. In short, here, as everywhere, you can't have perfection even if all parties could decide on

perfection. I am afraid that here, as everywhere, including I should say even Sweden, social insurance is also class legislation. How could it be otherwise?

Mr. Williamson does not expect apparently a completely rational treatment of what is admittedly a complex social, administrative, financial and tax-distribution problem. (His card-playing analogy, indeed, since it implies individual players and equality of skill, doesn't go nearly as far as his main argument.) But the rational content must and will grow. On such thoughtful discussion is laid the groundwork of an American social insurance institution more nearly fitted both to the risks and to the people that face and under any scheme will have to pay for them.

AUTHOR'S REVIEW OF DISCUSSION

MR. W. R. WILLIAMSON:

I appreciate Mr. Kulp's understanding comments. Whether the cooperative or social provision be called "insurance" or "security" or "services," we seem to be committed to a growing community of interest in these areas of need.

The paper was intended to carry certain challenges. I am glad to find Mr. Kulp questioning my "relative simplicity of universal contribution." The alternative, which we have so far adopted requires us to determine categories of coverage and other categories excluded from coverage. Definitions of coverage boundaries are necessarily complex, since the variety of human activities consistently defies the classifier. For example, the coverages of Titles II and III commonly exclude agricultural labor. A mass of decisions as to what constitutes "agriculture" is accumulating. A judge recently implied that some of the Federal decisions on this point seemed to him illogical. Of course they are. To decide that the horticultural part of plant care is non-agricultural, and that at some point in food-processing agriculture terminates forces variety of decision unless a single individual makes all the decisions, and probably even then. Since many members of the excluded categories are even more needy than the covered groups, alternative

provisions through either general or specialized relief, both commonly more demoralizing than "insurance" have to be devised. On providing for the welfare of our citizens our Federal government has long been committed to a policy of non-discriminatory benefits to the whole constituency. On caring for certain classes it is awkward to feel required to prove that no discrimination exists. It therefore seems probable that in a practical as well as an economic sense, simplicity lies with universality.

If uniformity, as well as universality, is accepted, the saving in record-keeping is tremendous. The broad sharing of social insurance can be much simpler than the meticulous accounting of the bank-book.

It is well to admit that there *is* a somewhat specious simplicity in leaving for later analysis most of the serious questions of detail. No apology is offered for this technique, since it seems necessary to limit the factors for consideration in any one discussion.

The real point of the card playing analogy is that we must have some inkling of the range and crude frequency distribution of the catastrophes against which insurance is provided. Is the average duration of life beyond 65 nearer 10 or 15 years? When jobs are lost are men out of work 1 week or 20? How much time is lost because of sickness in a year, 1 week per capita or 3 weeks? The pack of cards was supposed to follow in a labored fashion the simple aptness of a Biblical parable.

PURE PREMIUMS FOR COMPENSATION INSURANCE

ARTHUR G. SMITH

VOLUME XXIV, PAGE 35

WRITTEN DISCUSSION

MR. W. N. MAGOUN:

Mr. Smith has so skillfully diagnosed his "case" that no room is left for doubt as to what he considers to be the trouble. But whether the remedy he proposes will effect a satisfactory cure without causing other and equally distressing disturbances is open to possible doubt.

He finds the patient suffering from an over-dose of nationalism,

and I fear he would substitute an over-dose of sectionalism. May it not well be that a moderate dose of each in more suitable proportions than is now the case would be more effective?

In reviewing the workmen's compensation experience for a particular state, the classifications fall into three general groups. There may be argument as to the dividing line between these groups, but we will find that:

(1) The first group is comprised of those classifications with sufficient exposure within the state so that national experience may be entirely disregarded. No discussion of this group is here necessary. Each class within it is independent and in no way concerned with outside influences.

(2) The second group is comprised of the borderline classifications, with sufficient state exposure to demand attention, yet without enough to be conclusive. In this group will be found the classes with such small credibility that they barely qualify for "review" under the present system, and those classes which, while just failing to qualify, nevertheless have almost the necessary credibility, and are in many instances of particular importance locally. It seems to me that this is the group shouting the loudest for attention.

(3) The third group is comprised of classifications with such a small state exposure that it is admittedly meaningless.

If I have interpreted Mr. Smith's paper correctly, he does not admit the existence of this third group, and in such case I cannot agree with him.

I will go along with him, however, in a willingness to see this third group kept as small as possible, which is the equivalent of enlarging the second group, and it is the treatment of this second group which I will briefly discuss.

With the use of a credibility formula I am in accord, but there is a need for more recognition of the individual state's experience by some means other than its mere inclusion in the national experience.

To illustrate, in the national experience for policy years 1930-1934, for the classification Sugar Refining, the Massachusetts payroll was $7,322,800 out of a total of $40,155,000, or 18%, and the Massachusetts "serious" losses were $48,625 out of a total of $194,881, or 25%. Yet when this class came up for review in Massachusetts, the formula produced no credibility whatever for Massachusetts "serious" losses, and the national pure premium

which is less than one-half the Massachusetts indication was proposed.

The reverse situation is difficult to justify. To cite an example of this kind, consider the classification Eyelet Manufacturing. In the national experience for policy years 1930-1934, the Massachusetts payroll was $7,578,600 out of a total of $19,118,500, or approximately 40%. It is true that in this national experience Massachusetts had one "serious" case with a national "serious" pure premium of .09 (attributable to the year 1930).

For the years 1931-1935 Massachusetts, with a payroll of $7,387,300, had no "serious" losses whatever. The formula produced no credibility, so that, except insofar as it comprised part of the total countrywide experience, the Massachusetts "serious" experience received no recognition, and the national "serious" pure premium of .29 (on Massachusetts basis .18) was proposed. The situation is further aggravated by the fact that the $19,118,500 of national experience actually produced a "serious" pure premium of only .12, and the national "serious" pure premium of .29 as proposed, is a "selected" pure premium.

I am not objecting to this selection, but merely point out that if Massachusetts over a period of five years had no "serious" losses, with 40% of the exposure, some small reduction from the national basis is indicated.

Though we should be cautious about placing too much reliability on small exposures, if a fine record is established, some definite recognition thereof creates a good feeling and tends to avoid criticism on the part of assureds. I do not advocate sacrificing sound principles or accuracy in order to cater to public opinion, but if it is possible to make some slight modification in established practices which will produce a result more satisfactory to local supervising authorities and assureds, and still maintain equally sound principles and no less degree of accuracy, I am in favor of the change.

In the two cases above mentioned, instead of including the Massachusetts experience as a part of the countrywide experience, and then saying that the Massachusetts "serious" pure premium should be determined solely therefrom, why should we not determine the countrywide experience, exclusive of any Massachusetts experience, and then compare the national and state.

giving recognition to the state, at least according to its proportion of the combined exposure and probably somewhat greater recognition through a weighting process.

Again, take the case under the present system, where to some extent the state's later and more valuable experience may be offset by its own earlier and hence less valuable experience.

Under the Pyroxylin Manufacturing classification, the national experience for policy years 1930-1934 shows a payroll of $5,606,700 of which Massachusetts produced about 90%. Surely the Massachusetts later experience covering policy years 1931-1935, representing 90% of all the experience available, is a better indication than the countrywide experience for 1930-1934 which includes one year prior to that used in the current local revision, and as the national experience becomes older, will include two or more years of prior experience, which further emphasizes the point. Under the formula treatment the Massachusetts credibility for the "serious" pure premium was 25%, for the "non-serious" 25%, and for the "medical" 50%.

Several of the Boards and Bureaus have followed the practice in recent rate revisions of tabulating and presenting for Committee consideration, the experience for some of the lesser classifications. In other words, the national pure premiums have not been blindly adopted for classifications having a particular local interest, even though the exposure was small.

I believe that a modest expansion of this procedure, taken in conjunction with some improved method of applying formula credibilities, to place somewhat more emphasis on the smaller state exposures, offers a solution of Mr. Smith's problem, which I understand to be primarily the placing of less reliance on the national pure premiums.

Several methods of treating the credibility formula, such as reducing the qualifying volume of expected losses for the several credibility groups, establishing more of such groups, or providing separate credibility criteria for each of the recognized "industry groups," might be considered; but that subject is important enough to warrant presentation in a paper devoted exclusively thereto, and I do not propose injecting it into this discussion.

For such classifications as have almost no state experience, I would still adhere to the national pure premiums, bearing in mind

that upon production of sufficient evidence that the local risks are substantially different in hazard from that contemplated by the national experience, it is always possible to recognize the local condition by reducing or increasing the national pure premiums on the basis of such facts.

To attempt to make rates for classifications with an extremely limited exposure, on the state experience alone, would seem to lead to that situation, admittedly undesirable, under which one or two bad losses would cause a wide fluctuation in the rates from year to year.

Abnormal fluctuations would be apt to result not only in unwarranted changes in rate relativity within the particular state, but also in the impairment of the reasonably steady rate relativity which exists between states resulting from the use of a common national base.

If reasonable stability of the rate making structure as a whole has been achieved, I would dislike to see that stability jeopardized by any treatment of the smaller and less important classes which might by reason of constant changes in their alignment, react unfavorably on the whole system of classification and rate relativity.

AUTHOR'S REPLY TO DISCUSSION

MR. ARTHUR G. SMITH:

I do not, as Mr. Magoun thinks, precisely deny the existence of a group of classifications with such limited state exposure as to be meaningless. Of course, there are such classifications. I will go farther and say that there are some where the entire national experience is meaningless but where, nevertheless, the scheme of selecting national pure premiums provides a minimum of 10% credibility. It hardly seems reasonable that such a limited volume is more worthy of consideration when it appears in an exhibit of national experience than in a state exhibit, especially in view of the fact that the national exhibit is quite likely to be a mixture of dissimilar exposure not representative of any given state, and

may furthermore be distorted by the application of the conversion factors. Volume for volume it is safe to say that state experience is more reliable than national and therefore if the national method is sound it should be still more sound when applied to a single state, at least down to a point corresponding to the smallest volume which is in fact used for national pure premium determination. It is true that there are some classifications which cannot be rated on state experience alone. I deliberately omitted mention of these because I was merely outlining the system proposed and did not think it necessary to mention that, as in the case of all of the present systems of pure premium selection of which I know, some exceptions to the general procedure will be required. Obviously exceptional treatment should be accorded those classifications showing no exposure whatever or only a few thousand dollars payroll a year. A number of such classes would probably be dropped entirely if the state experience were reviewed instead of the national pure premium accepted automatically, and there would be nothing to prevent the responsible committees from selecting pure premiums departing from the formula as they have done in every rate revision in New York since there has been a formula. In any event I see no virtue in blindly taking the national pure premiums even on this group of classifications without even having an opportunity of comparing them with whatever the state experience has to show.

Mr. Magoun is entirely correct in stating that the second group he mentions is the one needing most attention. Examples of the type cited for Massachusetts can readily be found for New York and probably for every other sizable state. A very definite step in the right direction was taken in New York in connection with the general revision effective July 1, 1938, where the formula was extended to permit 20, 15 or 10% credibility for corresponding expected losses. While this has been an improvement it does not take care of the classifications with medium credibility which continue to show a definite differential from national year after year. For such classifications the national pure premiums are not a proper base and their use as such prevents either reasonableness or adequacy of rate as the case may be.

Except on but the very smallest classes, where I admit some variation from the general proposal is necessary, I do not think

that applying a formula against present state pure premiums will produce any less stability than present methods. National pure premiums are not revised annually as are state pure premiums, and consequently, even though the larger volume may tend toward more stability this is offset by the fact that two or possibly three years are dropped and the same number added. In state revisions four-fifths of the experience is from policy years which were used in the previous revision. It is quite possible for one or two bad losses to cause a wide fluctuation in national indicated pure premiums, and if a minimum credibility of 10% is too high to achieve the desired degree of stability (either in national or in state revisions) it would not be impossible to adopt a smaller figure.

THE DISTRIBUTION OF CASUALTY ADMINISTRATION EXPENSE BY LINES OF INSURANCE

THOMAS F. TARBELL AND HARRY V. WAITE

VOLUME XXIV, PAGE 45

WRITTEN DISCUSSION

MR. PAUL DORWEILER:

The expenses incurred in operating the casualty insurance business are about one-half of the total cost. They constitute that part of the cost which has aroused greater resistance among the insurance buyers and has produced greater criticism by state administrative authorities. Every effort to allocate and measure the incidence of expenses more accurately and to introduce the distribution of expenses into the rating procedure more equitably should be of interest to insurance carriers.

In their paper "The Distribution of Casualty Administration Expense by Lines of Insurance," Messrs. Tarbell and Waite have made a creditable addition to the growing list of papers on expense analysis in the *Proceedings*. A reference to the two Indexes to the first twenty volumes of the *Proceedings* and the later individual numbers reveals an even dozen papers that have dealt with ex-

penses in casualty insurance. A casual survey of the list shows the scope of the field covered.

Author	*Date*	*Volume*	*Pages*	*Subject*
Woodward	1917	III	140-8	Provision for Expenses in Workmen's Compensation Premiums.
Kirkpatrick	1922	VIII	340-3	Current Notes on Allocation of Administration Expense by Lines.
Hull	1922	IX	38-50	Allocation of Administration Expense by Lines.
Michelbacher	1923	IX	242-65	Incidence of Acquisition Expenses under the New Rules of the Acquisition Cost Conference.
Craig	1923	X	9-16	Allocation of Expenses.
Tarbell	1924	X	107-18	Determination of Acquisition and Field Supervision Cost by Lines.
Bailey	1928	XIV	233-42	Allocation of Adjusting Expenses to Lines.
Van Tuyl	1929	XVI	121-130	Use of Hollerith Cards in Expense Analysis.
Perryman	1930	XVII	22-41	Theory of Distribution of Expenses.
Barber	1934	XXI	65-80	Compensation Expenses Per Policy.
Waite, H. V.	1935	XXII	15-31	Distribution of Inspection Cost by Line.
Tarbell & Waite	1937	XXIV	45-59	Distribution of Casualty Administration Expense by Line.

In these papers are revealed methods for allocating expenses to carriers, to lines, and to size of risk. There is still left to be treated however, the problem of allocating claim expense by kind of injury and by time periods. It would be interesting and possibly useful to know to what extent there is a variation in the cost of adjusting non-compensable, temporary total, minor, major, permanent total, and fatal workmen's compensation claims. There should be a study undertaken along this line. It would also be desirable to study the incidence of claim adjustment expense according to the period of time elapsed since the date of the accident. Knowledge gained from such a study would be useful in setting up reserves for claim adjustment expense and would serve to check the percentage now used in Schedule P to distribute calendar year unallocated claim expenses to policy years.

On rereading the papers on expenses in the *Proceedings* one may trace the development of a process which in some aspects has attained its highest stage in the paper under discussion. Under this process the carriers' internal operations are divided into departments which incur the same kind of expense: acquisition, inspection, claim, administration, and payroll audit. Each department is then subdivided into homogeneous units of employees doing the same work. These units are studied and the salaries are allocated: to lines directly, if the whole unit is devoted to one line; and on the basis of time studies, number of items treated, amount of paid losses, or amount of premiums, if two or more lines are involved. The distribution of salaries has come to be used as the basic distribution. Any other item of expense which cannot be allocated specifically or which it is not practicable to allocate specifically is then associated with and pro-rated on the basis of salaries.

In the paper, the authors have developed this procedure and applied it to an analysis of the administration expenses of the Travelers Insurance Company. The procedure is briefly outlined and explained with a statement regarding fundamentals and principles. I do not take exception to the fundamentals or principles enunciated; as, for example, that:

> All items which can be should be charged to specific lines or combination of lines.
>
> In setting up a system reasonableness should govern between the theoretical and practical.

It should be pointed out, however, that differences of opinion arise, not regarding the acceptance of these principles, but rather regarding their interpretation when applied in specific instances. There probably would be differences as to what constitutes "all items which can be" or what is "reasonableness." This is not meant as a criticism of the procedure, but rather to call attention to inherent difficulties.

The authors have gone into such detail, far beyond anything heretofore, that it may seem out of place to call for further explanation. However, there is a question regarding the meaning of "judgment" in assignment on basis of judgment. Does this mean an estimate of the time of employees without resorting to a time study, number of items, losses, or premiums as a basis for this esti-

mate—Page 50, Division A (2)? Apparently where some such basis is used for the separation between two lines, the procedure is not referred to as an assignment on basis of judgment—Page 50, Division B (2).

The method used by Messrs. Tarbell and Waite produces results that deviate from arbitrary judgment to the extent that the salaries in the ultimate units can be assigned to lines directly, or on some proper formula basis. That portion not so assigned, called the residue, must be allocated on an arbitrary basis. The accuracy of the procedure depends inversely upon the relative portion left in the residue. In a large organization having a high division of labor it follows that the residue is relatively small and the accuracy correspondingly high. In a smaller organization where an individual employee may have not only several lines to deal with but may even be associated with two or more kinds of expense there would result a residue that is relatively large. It follows that the method will not produce the accuracy in allocation of expense for such a small organization, that it will produce for the larger one. In other words, by this method the expense can be more accurately allocated in large than in small organizations.

The appraisal of the results produced under different methods of expense allocation should be on a relative basis. The test should not be the absolute degree to which the residue that is to be arbitrarily assigned has been minimized but a comparison of the residue under one method with the residues under other methods duly weighing the practicability of each method.

AUTHOR'S REVIEW OF DISCUSSION

MR. THOMAS F. TARBELL:

Mr. Dorweiler's discussion is of particular interest for the suggestions as respects the proposed study of claim expense by kind of injury and duration of disability, and for its contribution of a bibliography on papers on expense distribution appearing in the *Proceedings* over the past twenty years. There are many phases of expense distribution that will well repay further study and it is hoped that the *Proceedings* will contain a larger rather than a

smaller number of papers in the future on both the general subject and the more specialized divisions of the subject.

Mr. Dorweiler's comments regarding the interpretation of certain principles set out by the authors are well taken, and unquestionably certain rather concise or unelaborated statements could well have been enlarged upon. The general statement that "all items of expense which can be charged to a specific line . . . should be so charged" did not contemplate in the minds of the authors that every small item of expense should be examined to determine if it is subject to such treatment, but rather that all items of consequential amount should be so charged. Admittedly, some latitude is contemplated and the element of judgment or opinion will have some influence. The same general idea was in the minds of the authors in using the word "reasonableness." Conceivably, a system might be set up which would be ideal from a theoretical standpoint but would be unreasonable from a practical standpoint, particularly in view of the expense of maintaining the system. "Judgment" as used in the paper has the meaning assumed by Mr. Dorweiler—an estimate of the time of employees without resorting to a time study or other basis. This basis is used where the tasks performed by the individual vary rather materially by kind or nature but not so extensively by line. In such instances the individual is usually able to estimate rather closely the time spent on the average on the various lines.

It is undoubtedly correct, as Mr. Dorweiler points out, that greater accuracy of distribution is attainable in a large organization than in a small organization because of a greater degree of division of labor and concentration of work involving a single line, or two or three lines, in the larger organization. However, careful expense analysis, within practical limitations, is worthwhile regardless of size of company, because of its benefits in executive guidance, particularly since profit margins are narrow in some lines and underwriting results are subject to periodic fluctuation.

DISCUSSION OF EXPERIENCE RATING PLAN CREDIBILITIES

FRANCIS S. PERRYMAN

VOLUME XXIV, PAGE 60

WRITTEN DISCUSSION

MR. ROBERT V. SINNOTT:

Mr. Perryman's three *a priori* conditions must be accepted as entirely reasonable. His progress from axioms to ultimate conclusions are logically flawless in so far as a careful study of his paper reveals. His conclusions are acceptable from both scientific and practical viewpoints. This contribution to the business of rate-making is invaluable.

An Experience Rating Plan is a determination of the degree in which a risk differs from the average risk in its loss-producing potentialities and in the manifestation of this degree as a departure from the Manual Rate. In determining this degree of departure, we examine the risk's past experience and accept, as being significant, certain elements of the risk's history and reject others. We consider the size of the risk, the age of the experience, and the size of the individual losses. We recognize that the experience of a large risk is more significant than that of a small one; we recognize that more recent experience, provided it is reasonably developed, is more significant than older experience; and we also recognize that each successive dollar spent in the settlement of any claim is less significant than its predecessor.

Some of us use other language in assessing a risk's potentialities of loss and give consideration to the elements in the experience for which the assured is responsible. Many factors contributing to the occurrence of an accident exist through the deliberate choice of the assured. Promptness in obtaining medical aid by the assured, and the efficiency of such first aid treatments operate to minimize the severity of a loss. In general, we hold that the assured is more responsible for the occurrence of the accident than for its ultimate cost or that the assured's responsibility for loss arising out of any one accident diminishes with each succeeding dollar of loss expenditure.

The principle that the larger the risk, the greater the degree of control the assured can exercise over the incidence and severity

of the accidents, by making use of physical and moral means of accident control and prevention, is universally accepted.

The professors of the responsibility doctrine also hold that with changing time comes changing custom. To hold an assured equally responsible, in his future compensation rate, for an accident which happened some time in the dim past and for an accident which happened only yesterday, is illogical.

It is important to note, however, that there is no reason to believe that this significance, or responsibility, grows greater or less in discrete jumps. So far as we know, the quantum theory is not yet involved in Experience Rating. The algebraic functions which we choose to represent it should proceed as smoothly and as continuously as is possible with due regard to the practical aspect. We must not overwhelm the rating bodies with a multiplicity of intricate calculations. In other words, in our procedure, we must measure as closely as possible the individuality of the risk and at the same time, produce results economically.

Our present rating plan is a compromise of this sort. The measure of significance of the past experience is the Modification; and each of the three principles stated above enters into its calculation. The Modification is a weighted average between the assured's own experience and the experience of the average risk. The weights, or credibilities, have been carefully calculated for various sizes of risks and proceed smoothly from 0 to 1, giving little or no credence to the experience of a small risk and full credence to the experience of a large one. It is this calculation to which greatest attention has been given in the past and which has, in Mr. Perryman's paper, reached what is probably its ultimate refinement, at least in form. I say in form because the initial point of qualification for rating, and the point of full self-rating, are chosen not with an eye to the actual significance of past experience of the risk but on the basis of expediency alone. In contrast to the meticulous calculation of credibility, the age of the experience is recognized by giving arbitrary, uniformly increasing weights to each successive year of experience. In the plan now in use, recognition is given to the inverse significance of each successive dollar of any one loss in a crude and curious way. We have set an arbitrary limit of say "a" dollars and the first "a" dollars of any one loss is called the "normal loss." The remainder of the loss is

called "excess loss." We hold that "normal loss" is highly characteristic of the risk; "excess loss" is not.

If the normal split is $1,000, then the $1,001st dollar is presumed to have far less significance than the $1,000th dollar but the $999th dollar is held of equal significance with the first dollar. We hold that the first $1,000 of loss is just as indicative in the case of a Clothing Manufacturer, with carefully guarded machines, as it is in the case of a conscienceless Contractor, who sends men into an unshored tunnel. We regard the normal split as immutable as the laws of the Medes and Persians, or the hitching post in front of the First National Bank, never changing in the face of circumstances. This device has limped along, posing as the truth far too long. We have held to it through thick and thin until we, ourselves have begun to believe it, although I have never heard of an explanation of the "normal split" adequate to satisfy a curious and unsympathetic assured. The multi-split principle of loss evaluation is a long step in the right direction toward the true method. A smoothly falling curve is substituted for the two horizontal lines of the present plan.

Any rating formula takes the significant losses from the risk's experience and compares them with losses from the average risk, chosen in the same way. An analysis of the formula, now in use, as well as the formulae cited in Mr. Perryman's paper, will indicate that this is done through the three devices described above. If these three ways of doing essentially the same thing could be reduced to two, or even to one, without loss in refinement, or an increase in complexity, the simplification should be welcome. I have attempted to do this with Mr. Perryman's Formula 31:

$$M = \frac{A_d + K_E + W(A_c - K_E)}{E_d + K_E + W(E_E - K_E)}$$

A_d is the portion of the actual loss selected by the Multi-Split Formula as being significant for risks below the Q point (i.e. small risks). A_c is the portion of the actual loss discarded by that Formula. As the size of the risk W become greater than 0 and a part of these discarded losses is reintroduced into the rating formula; *but* it is important to note that the discarded loss is not reintroduced in the same way in which it was discarded. In being reintroduced, the discarded portion of the last dollar to be expended carries the same weight as the discarded portion of the

first dollar. For example, if we have a loss of two dollars and the Multi-Split Formula takes ½ the first dollar, and rejects the other ½; accepts ¼ of the second dollar, and rejects ¾; then if W is .5, ¼ of the first dollar and ⅜ of the second dollar is brought back into the formula again. This may seem academic until we see that if we have a series of small losses of which we discard a certain aggregate amount; and a single large loss of which the same amount is discarded; and then these losses are reintroduced into the formula, the amount of the large loss, so reintroduced, will equal in amount the amount brought in for the smaller losses. The additional amount for the smaller losses should be the greater.

The Q point is therefore a critical point in Formula 31, due to the necessarily artificial method of treatment of losses. It would be an improvement if, instead of taking out these losses and then putting them back in again, this end could be accomplished in a single operation. With this thought in mind, I have examined the Multi-Split Discount Formula to see if it could be adapted to this requirement. This Formula is:

$$\text{Discounted Loss} = \text{Maximum Ratable Loss}\left(1 - r\,\frac{\text{Actual Loss}}{\text{Constant}}\right)$$

$$\text{or more briefly} D = M\left(1 - r\frac{L}{c}\right)$$

where $0 < r < 1$ and where the loss used in the rating is D if L is greater than c or L, if L is less than c. If M, r, or c instead of being constants were parameters which varied by size of risk, D could be made to vary. The practical difficulties of varying c, or r, seem too great to be overcome and it would be possible to get a rating value in excess of an actual loss by varying M; so, in the brief experimenting which I have done, I have explored the possibility of adding a parameter to the right-hand side of the formula, thus:

$$D = N + M\left(1 - r\frac{N - A}{c}\right)$$

where "N" increases from 0 at the Q point to infinity at the S point and where the portion of the loss used in the rating is D if the actual loss is greater than $N + c$, and the full actual loss, itself, if it is less than $N + c$. Thus, the A_d of Mr. Perryman's

Formula becomes variable by size of risk. Suppose for purposes of distinction, we call it A_v. Then to preserve the result of Mr. Perryman's deductions

$$A_v \text{ must equal } A_d + W A_e.$$

I had at hand a tabulation of W as a function of E for Georgia, where $r = 2/3$, $c = 300$, and by using a statewide, all risk, distribution of losses, indemnity and medical combined, and by using the suggested Discount Formula above, the following relationships between N and E were found to exist:

N	E	Losses Rated at Actual Value if Less Than $N + c$
0	4,000 Q point	300
50	5,744	350
100	7,332	400
200	9,996	500
300	12,132	600
400	14,412	700
500	16,292	800
600	18,004	900
750	20,620	1,050
900	22,848	1,200
1,000	24,248	1,300
1,250	27,584	1,550
1,500	30,568	1,800
1,750	32,900	2,050
2,000	34,485	2,300
2,500	35,692	2,800
3,000	36,700	3,300
4,000	38,328	4,300
5,000	38,968	5,300
	40,000 S point	Actual value of all losses used

The relationship between the two functions follows a curve of the exponential type, asymptotic to $E = S$ and passing through $c = 0$, $E = Q$.

I have made no effort to determine whether any direct mathematical relationship connects N and E. Such a relationship would necessarily be an empirical one since the relationship between number of losses and size of loss is fortuitous. Such an empirical relationship might be found which would fit the circumstances in most cases.

The Rating Formula would then be:

$$M = \frac{A_v + K_E (1 - W)}{E_d + K_E + W (E_E - K_E)}$$

with the practical advantage that the actual losses are treated only once and in a uniform manner for all risks. Moreover, there would be a definite limit to the amount at which any loss, regardless of its size, would be included in the rating plan $(N + M)$ up to the point of self-rating.

MR. HARMON T. BARBER:

Too infrequently there appears in the *Proceedings* a paper of the type which must be carefully studied in order to be fully appreciated. Mr. Perryman's paper is one, and a review of it is bound to arouse one's admiration of the skillful way in which the author has treated this difficult phase of rating theory. That the paper constitutes an exhaustive and logical exposition of the restrictions which should be imposed on credibility and of methods for incorporating these limitations in various types of experience rating plans, has been confirmed by the writer after stumbling through an abundance of mathematical entanglements. An acknowledgment is due to the author for the service which he has rendered by this comprehensive study but it is difficult to express it appropriately. In simple words Mr. Perryman's paper represents a good job well done and the writer feels that this view will be shared by all including those who will have occasion to consult it as a reference in the years to come.

Essentially, the effect of experience rating on risk premium is to produce a cost to the assured which has many of the characteristics of the cost of a combination of deductible and coinsurance coverages. The prospective nature of the experience modification and the fact that it is predicated on the experience of several policies dims this comparison but does not alter the fundamental similarity. Thus under the original or elementary no-split experience rating plan which involved a comparison of risk loss ratio with the expected loss ratio of manual rates there existed a situation directly comparable with co-insurance. The credibility element in this plan as in other plans defined the degree or percentage of co-insurance which should prevail. As the premium size of the risk increased the loss provision in the adjusted premium approached the loss cost of self-insurance until it actually

attained this status at the self-rating point, the rate of approach being controlled by the behavior of credibility.

With the introduction of the split type of experience rating plan, first based on kind of injury and later involving the normal-excess type of split, certain desirable features analogous to deductible coverage were incorporated in the plan and were subjected to the coinsurance principle by means of dual credibilities. The multi-split principle represents a refined application of the deductible principle and should allow a higher degree of coinsurance or credibility than was practical under previous plans. In fact one advantage of the split type of plan is the greater opportunity which it offers to expand credibility or, as it might be expressed, to increase the assured's participation in the coinsurance relationship.

The intent of these comments is to direct attention to the point that credibility and the treatment of losses in experience rating are interdependent. Particularly is this so in the case of the multi-split plan which employs a loss valuation table from which is obtained the discounted loss (A_n) corresponding to each actual incurred claim cost. This may be illustrated by a transformation of formula (12) of the paper under discussion and which relates to a split plan similar to the present experience rating plan.

$$\text{Mod.} = 1 - Z_n \cdot \frac{E_n}{E} + Z_n \cdot \frac{A_n}{E} - Z_e \cdot \frac{E_e}{E} + Z_e \cdot \frac{A_e}{E} \quad (12)$$

$$= 1 - Z_n \frac{(E_n + E_e \cdot Z_e/Z_n)}{(E)} + Z_n \frac{(A_n + A_e \cdot Z_e/Z_n)}{(E)} \quad (12a)$$

Compare (12a) with formula (2) of the paper, the latter relating to the no-split type of plan with its recognized advantage of simplicity.

$$\text{Mod.} = 1 - Z + Z \cdot \frac{A}{E} \quad (2)$$

If by some contrivance the expression $A_n + A_e \cdot Z_e/Z_n$ of formula (12a) could be transformed to a single term corresponding to A of formula (2), some of the simplicity of the no-split plan might be captured for a large range of rated risks. Since varia-

tions in the ratio Z_e/Z_n are not important in their effect on the modifications of small rated risks it was a short step beyond assuming this ratio to be constant to test the principle that it should decrease in accordance with some definite plan as the size of the loss increases. In this way the underwriting tendency to regard the occurrence of exceptionally large losses as largely fortuitous as to the resulting cost might be recognized and standardized. This explains the genesis of the multi-split principle. The loss valuation table is entered with A and a rating value A_n is obtained which is equivalent to the sum of the terms of a geometrical progression with $r < 1$. Applying similar treatment to E in the aggregate gives E_n and thus a rating formula of the following type can be used for the great proportion of rated risks with premium of less than Q.

$$\text{Mod.} = \frac{A_n + K}{E_n + K}.$$

It is apparent that the loss valuation method should be regarded as an integral part of the credibility structure and should be subjected to the minute scrutiny which Mr. Perryman has applied to other elements. In his paper there is a promise to discuss the treatment of losses under the multi-split plan at greater length and it may be anticipated that this second chapter will prove to be a valuable supplement to the current study.

Throughout the paper frequent reference is made to the three cardinal conditions to which Z must be subject. With these as with other principles enunciated in the paper, the present writer has no quarrel but there is another restriction which is not susceptible to mathematical expression which should be added and preferably superimposed on the original three. This is the requirement that credibility create as little complication of the rating procedure as is possible and in those instances where a simplified approximation will produce results of the same general character as a more exact application of the theory, the choice should rest with the former.

There are several points where such a choice may be exercised and in its deliberations on the credibility elements of the multi-split plan the National Council Actuarial Committee has evidenced a disposition to follow a trend toward simplicity. For

example in experimenting with formula (31) sentiment has been expressed in favor of employing "straight-line" values of W and K_e, i.e. values which are simple functions of premium rather than to use the more refined "continuous" values resulting from an exact application of the principles developed in the paper. When these two series of values are calculated using the same value of S, the "straight-line" values appear at a disadvantage as respects the degree of effective credibility assigned to certain premium sizes. This disparity can be and should be corrected by a readjustment of S so that the "straight-line" values will produce the same general credibility effect as do the values which they are designed to replace. It is claimed for the "straight-line" values that they may be more readily explained and will be more easily understood by an inquisitive layman. Whether this advantage is material or not, the simplification of any part of the plan has a very definite appeal to the writer providing we do not stray too far from the path marked out by theory.

As a matter of fact the attractiveness of formula (30) in this respect is so strong that the writer dislikes to see it summarily abandoned. This formula achieves the maximum in simplicity by employing a constant K for all premium sizes in place of a variable K_e for risks above Q as in formula (31). Mr. Perryman properly points out that in some cases formula (30) violates the basic condition that Z should not exceed unity, or to state it in more practical terms, under formula (30) an assured may pay more than \$1.66 of adjusted premium for \$1.00 of actual incurred loss. In spite of the theoretical validity of the criticism there are several observations which weaken its effectiveness with the writer. In the first place the experience modification is not the ultimate result of experience rating but is only a step toward establishing an adjusted premium for the risk for the ensuing year. This adjusted premium is also effected by some highly variable conditions such as payroll exposure. If the extent of a risks operations should change materially, many of the theoretical niceties of the perfect rating plan are nullified as respects their practical effect on premium. Secondly the full effect on adjusted premium of any loss is not attained until its particular policy year has completed its full term of service in the experience period. Thus the premium effect of one loss cannot be readily

disassociated from the effect of other losses of the same year and of the several other policy years represented in the experience period, unless one goes to great lengths in analyzing several annual ratings. Then if the result is actually found to be, say, $1.50 premium effect per dollar of loss under formula (31) and $1.70 under formula (30) will any reasonable judgment praise one formula and condemn the other? In the consideration of experience rating credibility it is necessary to continually bear in mind the limitations of the statistical evidence on which the rating is predicated in order not to be inordinately exact in the treatment of a single element out of the many which affect premium cost.

There is an opportunity to apply the doctrine of simplicity to good advantage in dealing with the rating of risks at the lower end of the range of risks by premium size. The credibility curve illustrated in Figure II of the paper requires a slight modification in order for it to be truly representative because of the eligibility requirements which are a part of every experience rating plan. To be typical of actual conditions the curve should run along the E axis until the qualification size is reached and then rise to an appreciable value and follow the curve as depicted thereafter. If the process of stream-lining the credibility curve were to be carried out completely the curve would start with a reverse hook similar to the curve of Figure III for the same reasons as are set forth in the discussion which precedes it in the paper. However, it is not necessary to be precise in rating risks in this size-zone because the practical effect of the rating on the adjusted premium is nominal. Obviously as the size of premium diminishes a point is eventually reached where the administration expense of experience rating is no longer warranted—hence, eligibility requirements. This suggests the possibility of a still further simplified procedure to use in rating risks which are too small to meet the present qualification standard. A rule might be established reading somewhat as follows (with values to be adjusted to conform with the experience rating plan):

> For risks which fail to meet the premium qualification of the experience rating plan but which have produced total earned premiums of $500 or more during the last three completed policy periods prior to the current policy, the percentage

> credit or charge to apply to manual rates for the renewal policy shall be the difference between (a) a credit of 7% for each $1,000 of earned premium and (b) a charge of 14% for each $1,000 of discounted actual losses.

This rule departs from the procedure of the multi-split plan in several particulars. Actual earned premiums and an average excess ratio which does not vary by classification are used in determining expected losses. Credibility is proportional to premium and the credibility curve becomes a straight line which joins the credibility curve of the rating plan at the point of eligibility.

The algebraic expression of the rule is:

$$\text{Credit} = .07\ (.001\ P) - .14\ (.001\ A_n).$$

Its derivation may be traced from formula (2) of the paper:

$$\text{Modification} = 1 - Z + Z\frac{A}{E} \text{ or } \text{Credit} = Z - Z\frac{A}{E}.$$

Adapting this to the conditions of formula (31) we have:

$$\text{Credit} = Z_n - Z_n\frac{A_n}{E_n}.$$

Now the credibility curve of formula (31) for premiums of less than the eligibility requirement is transformed to a straight line by making Z_n proportional to premium. The risk which is just eligible produces $1,500 premium over three years and is entitled to a credibility of .10. Hence $Z_n = .10$ per $1,500 premium or roughly .07 per $1,000 premium. Expressing the other symbols in the preceding formula in $1,000 units produces:

$$\text{Credit} = .07\ (.001\ P) - .07\ (.001\ P)\frac{(.001\ A_n)}{(.001\ E_n)}$$

$$\text{Assume } E_n = .83\ E = .83 \times .60\ P = .50\ P$$

$$\text{Then, Credit} = .07\ (.001\ P) - .14\ (.001\ A_n).$$

There are many advantages to be derived from a method of rating small risks by a rule of this nature. It answers the demand of the average employer that individual recognition be given to his risk in the determination of rates. It should obviate the necessity for lowering the eligibility requirements of the standard plan, as has been done recently in some states, with the increased burden of administration expense which follows such a move. The

rule can be applied readily and economically to experience as reported under the Unit Statistical plan. The modifications which result should tie in closely with those of the experience rating plan so that it can be asserted that the method really represents an approximate application of the standard plan.

Mr. Perryman's paper, although limited by title to credibilities of experience rating, tends to incite one's imagination as to possible variations in rating methods. For instance, the benefits resulting from an increased incentive toward accident prevention activities afforded by the imposition of moderate financial penalties for the occurrence of each accident are recognized as substantial. It is interesting to speculate on how effective would be an insurance plan which imposed an experience rating plan of the multi-split type on the premium cost of $50 deductible compensation coverage. This would in effect provide full credibility on the first $50 of every loss of every risk with diminishing degrees of credibility applying to successive increments in the size of each loss, the effective credibility on the excess over the deductible amount being dependent on risk size. Such a plan would offer to an assured all the required protection against occasional severe losses and at a cost which would be largely commensurate with his ability to suppress accidents. Some day we may be seriously considering a combination of this kind even though it seems highly visionary at present.

MR. MARK KORMES:

In this latest contribution to the theory underlying credibilities used in rating plans Mr. Perryman as usually displays his mastery of the subject and presents a complete analysis of the various approaches to this difficult and delicate problem. Beginning with an analysis of the experience modification formula underlying a no-split plan, that is where each loss is used with an equal weight, he develops the necessary conditions which must be met in order to have satisfactory credibility values and demonstrates the superiority (theoretical at least) of the use of a tangential parabola instead of a tangent straight line for reaching the self-rating point. Next he analyzes the modification formula of the present

plan which splits losses into "Normal" and "Excess" portions, an arbitrary value being considered as "Normal" and then proceeds with the development of several formulæ for the so-called "Multi-split" plan.

It is not within the frame of this discussion to comment on the desirability of a multi-split plan and therefore I shall proceed with the review of the several formulæ for experience modification.

I.

Mr. Perryman gives an extended analysis of three such formulæ although he explains (see paragraph 22) that many more could be devised. Before going further into the subject, it should be emphasized that in the multi-split plans similarly as in the no-split plan, the modification formulæ considered have only one self-rating point. This, as Mr. Perryman points out, was done advisedly for the sake of simplification. The first formula considered (Formula (14) in the paper) is:

$$\text{Modification} = \frac{A_n + E_e + K - W\ (A_e - E_e - K)}{E + K\ (1 - W)} \qquad (1)$$

where $W = 0$ for $E \leqq Q$ and $0 < W \leqq 1$ for $Q < E \leqq S$. The value of Q up to which the excess portion of actual losses would be disregarded and the value of S or the self-rating point must be selected on the basis of judgment and I find myself in full agreement with Mr. Perryman's suggestions as to the method of their selection (see paragraph 25). The values of W being zero up to $E = Q$ the modification for risks falling in that range (and, which is important, the vast majority will fall in this group!) is very simple, namely

$$\frac{A_n + E_n + K}{E + K} \qquad (1a)$$

The difficulty arises for risks where $E > Q$. The values of W must be determined as a function of E in such a manner as to satisfy certain fundamental conditions. The modification formula (1) allows the following credibility values for normal and excess portions

$$Z_n = \frac{E}{E + K\ (1 - W)},\ Z_e = W\, Z_n \qquad (2)$$

and in order to obtain satisfactory credibilities Mr. Perryman introduces a new function:

$$\zeta = Z_n + a\, Z_e = \frac{E\,(1 + a\, W)}{E + K\,(1 - W)} \tag{3}$$

which must obey certain terminal conditions for $E = Q$ and $E = S$ and for $Q < E \leqq S$ behave in a certain fashion (see pages 77-78) and where a is the maximum ratio of excess loss to normal loss. The conditions imposed upon ζ require the solution of a differential equation with given terminal values. It can be shown easily that there is an infinite variety of solutions and therefore it would be only of interest to find a formula as simple as possible. This Mr. Perryman does by an ingenious device, namely, by putting

$$\zeta = \frac{E}{Y} \tag{4}$$

and by subjecting Y to such conditions as to obtain satisfactory values of ζ he constructs Y as a sum of the ordinates of two hyperbolas and obtains finally for Y the expression (see Appendix I)

$$Y = \frac{C_3 + B_3\, E - A_1\, E^2}{(C_1 + E)\,(C_2 - E)} \tag{5}$$

where the A's, B's and C's are constants suitably determined. The substitution of (5) in (4) demonstrates that ζ is cubic.

Now an examination of Y discloses that in the interval considered the mean curvature of Y is very small and therefore it could be approximated by a straight line:

$$Y = A\, E + B \tag{6}$$

where $A = \dfrac{S - (Q + K)(1 + a)}{(S - Q)(1 + a)}$ and $B = \dfrac{S\,[(Q + K)(1 + a) - Q]}{(S - Q)(1 + a)}$

It should be noted that S, Q, K and a being constant values which are to be determined in advance, the calculation of A and B presents no difficulty. This approximation results in reducing the degree of the function ζ from 3 to 2 and it can be readily recognized that ζ will be a hyperbola. The calculation of Y for any given value of E is very simple and since

$$W = \frac{E + K - Y}{Y + K} \tag{7}$$

the calculation of W values presents no further difficulties. Table I shows the values of W, Z_n and Z_e calculated on basis of Y from formula (6) and gives a comparison with the values obtained by Mr. Perryman using the more exact formula (5).

It will be seen that the values obtained differ but slightly and formula (6) provides, therefore, a satisfactory approximation.

II.

The second formula for modification (see formula 31) analyzed is:

$$\text{Modification} = \frac{A_n + K_E + W\,(A_e - K)}{E_n + K_E + W\,(E_e - K)} \tag{8}$$

where, as before $W = 0$ for $E \leqq Q$ and $0 < W \leqq 1$ for $Q < E \leqq S$ and where K_E is also a function of E such that it has a constant value K, for $E \leqq Q$ and then increases in a suitable manner. This modification formula while simpler in form contains two functions which must be determined, W and K_E and Mr. Perryman demonstrates that W depends upon K_E and both depend upon the excess ratio r. Thus he uses

$$\zeta = \frac{E}{Y\,(1-r)} \tag{9}$$

and determines Y in a similar manner as for formula (3). It appears that also in this case a straight line approximation of Y will produce satisfactory results. It can be found without difficulty that

$$A = \frac{S - Q\,(a+1)(1-r) - K\,(a+1)}{(a+1)(1-r)(S-Q)}$$

$$\text{and } B = \frac{S\,[Q\,(a+1)(1-r) + K\,(a+1) - Q]}{(a+1)(1-r)(S-Q)}. \tag{10}$$

As before the above values can be readily determined and once they are determined for a given set of values the process of calculation of W depends only on the determination of K_E. This Mr. Perryman accomplishes by means of the hyperbola

$$K_E = \frac{(K - Q\,g)^2}{g\,E + (K - 2\,Q\,g)} + g\,E \tag{11}$$

where g represents the maximum value of r. Again the straight

line approximation appears to be in order, the mean curvature in the interval being very small and we obtain for K_E the straight line:

$$K_E = K + \frac{(g\,S - K)(E - Q)}{S - Q}. \tag{12}$$

This was recognized by the Actuarial Committee of the National Council who even proposes to go much further and to approximate W by means of the straight line

$$W = \frac{E - Q}{S - Q} \tag{13}$$

It seems to the reviewer, that the last suggestion goes a little too far in that it depresses credibilities for the larger sizes of risks as will be seen from a study of Table II.* The writer has calculated in Table III some values of W, K_E, Z_n and Z_e on basis of formulæ (10) and (12) and a comparison of these values with those obtained by use of the exact formulæ of Mr. Perryman discloses that the approximation suggested show very small departures. In this connection it should be also borne in mind that the break in continuity by using the straight line approximations will be obliterated in actual practice anyway on account of the use of finite intervals. Tabular values are shown in intervals of one thousandth and therefore could be represented by a graph resembling a staircase. This practical consideration further justifies the use of approximation formulæ.

The third formula given in the paper becomes incidental to the first formula by replacing K with $K\,(1 - r)$. Since this formula presents no new problems it does not appear necessary to make any remarks.

III.

As regards the selection of a formula for practical application, I am inclined to agree with Mr. Perryman that the first formula is preferable. It requires only the determination of one variable and therefore the construction of one set of tabular values for any

* Based on a memorandum dated April 18, 1938 by Mr. Perryman to the Actuarial Committee of the National Council on Compensation Insurance. This defect can be corrected by changing the point *og* self-rating and the point Q.

given state. Some feel that the second formula is easier to explain. This, perhaps, is true as regards the formal appearance but difficulties will arise in explaining the two variable elements W and K_E. As explained above, most of these difficulties can be eliminated by the use of a straight line formula for both W and K_E but it is questionable whether the resulting lower credibility would produce satisfactory ratings on large size risks which have not yet reached the point of self-rating.

IV.

The actuarial fraternity owes a debt of gratitude to Mr. Perryman for this contribution to the theory of credibility as it has paved a way to new approaches to the problem of experience rating and undoubtedly will find application in more than one line of insurance. May I be permitted to close my remarks with a hint to the Educational Committee that it is very unfortunate that the Syllabus does not require the elements of trigonometry and plane analytic geometry. How can a candidate for Fellowship, who has passed successfully his Associateship examinations, be required to study and understand this paper? I have no doubt that it will be included among the standard requirements for Fellowship examinations.

TABLE I

COMPARISON OF RESULTS OF USING STRAIGHT LINE FORMULA FOR Y WITH RESULTS OBTAINED BY THE USE OF THE EXACT FORMULA

	Straight Line Y Basis				Quadratic Y Basis*		
	E	W	Z_n	Z_e	W	Z_n	Z_e
New York							
S = 140,000	14,000	.000	.670	.000	.000	.670	.000
Q = 14,000	20,300	.065	.759	.050	.040	.754	.030
K = 6,900	26,600	.127	.815	.104	.095	.810	.077
= 4	39,200	.247	.883	.219	.211	.878	.185
	51,800	.360	.921	.332	.328	.918	.301
	77,000	.568	.963	.547	.558	.962	.536
	102,200	.754	.984	.742	.776	.985	.763
	127,400	.922	.996	.919	.958	.998	.956
	140,000	1.000	1.000	1.000	1.000	1.000	1.000
Massachusetts							
S = 90,000	9,000	.000	.620	.000	.000	.620	.000
Q = 9,000	13,050	.060	.716	.044	.041	.711	.029
K = 5,500	17,100	.119	.779	.093	.095	.774	.074
= 4	25,200	.233	.856	.199	.208	.852	.177
	33,300	.343	.902	.310	.320	.899	.288
	49,500	.549	.952	.522	.543	.952	.517
	65,700	.740	.979	.723	.760	.980	.745
	81,900	.916	.994	.911	.950	.997	.947
	90,000	1.000	1.000	1.000	1.000	1.000	1.000
Georgia							
S = 42,000	4,200	.000	.503	.000	.000	.504	.000
Q = 4,200	6,090	.050	.608	.031	.050	.607	.030
K = 4,140	7,980	.101	.690	.070	.100	.682	.068
= 4	11,760	.201	.781	.157	.200	.780	.158
	15,540	.301	.843	.254	.301	.843	.253
	23,100	.502	.918	.460	.502	.918	.460
	30,660	.701	.961	.675	.703	.961	.675
	38,220	.901	.989	.890	.904	.990	.894
	42,000	1.000	1.000	1.000	1.000	1.000	1.000

* Taken from Table I of the paper.

TABLE II*

COMPARISON OF STRAIGHT LINE W AND K_E VALUES WITH TANGENTIAL CURVE VALUES FOR SECOND FORMULA

New York $S = 135{,}000$ $Q = 13{,}500$ $K = 6{,}900$ $g = .4$

E	$r = .4$				$r = .25$				$r = .1$			
	Straight Line		Curve		Straight Line		Curve		Straight Line		Curve	
	Z_n	Z_e	Z_n	Z_e	Z_n	Z_e	Z_n	Z_e	Z_n	Z_e	Z_n	Z_e
13,500	.900	.000	.900	.000	.795	.000	.710	.000	.710	.000	.710	.000
14,850	.910	.010	.935	.005	.800	.010	.730	.005	.715	.010	.730	.005
20,250	.940	.050	.975	.060	.830	.045	.765	.050	.740	.040	.765	.050
27,000	.960	.105	.990	.135	.850	.095	.785	.120	.765	.085	.785	.105
40,500	.980	.215	.998	.275	.880	.195	.820	.250	.795	.175	.820	.225
74,250	.995	.485	.999	.615	.905	.450	.895	.580	.845	.425	.895	.550
108,000	.999	.780	.999	.895	.970	.755	.970	.880	.935	.730	.970	.870
121,500	.999	.890	.999	.970	.985	.875	.990	.965	.970	.865	.990	.960
128,250	.999	.945	.999	.990	.990	.935	.997	.990	.985	.930	.997	.985
135,000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

* Based on Mr. Perryman's memorandum to the Actuarial Committee of the National Council dated April 18, 1938.

TABLE III

COMPARISON OF VALUE BASED ON STRAIGHT LINE Y AND K_E WITH THE VALUES BASED ON HYPERBOLIC Y AND K_E

(*New York*, $S = 140{,}000$, $Q = 14{,}000$, $K = 7{,}000$, $r = .333$, $g = .333$)

E	Straight Line Y and K_E			Hyperbolic Y and K_E*		
	W	Z_n	Z_e	W	Z_n	Z_e
14,000	.000	.857	.000	.000	.857	.000
20,300	.082	.909	.073	.034	.946	.032
26,600	.154	.937	.145	.092	.972	.089
39,200	.292	.967	.292	.218	.990	.216
51,800	.414	.982	.414	.346	.995	.344
77,000	.623	.994	.619	.594	.999	.593
102,200	.793	.999	.792	.813	1.000	.813
127,400	.933	1.000	.936	.971	1.000	.971
140,000	1.000	1.000	1.000	1.000	1.000	1.000

* Based on Table II of the paper.

AUTHOR'S REVIEW OF DISCUSSIONS

MR. FRANCIS S. PERRYMAN:

The three discussions by Messrs. Barber, Kormes and Sinnott are most helpful and sympathetic. In reviewing these discussions I think it will be best for me to make a few comments on each of them in turn and then conclude with some general remarks.

Taking Mr. Barber's comments first; he is quite evidently in favor of simplicity. He advocates "straight lines" instead of "curves" and with that I have no quarrel, as will be seen later. Mr. Barber goes even further and advocates Formula (30) instead of Formula (31) on the grounds of further simplicity and despite the very serious theoretical objections. This, of course, does not take me by surprise and as usual Mr. Barber has expressed his arguments very forcibly and clearly and, indeed, from the practical point of view he has a good deal on his side, although I am not prepared to concede his point. I believe we should be simple but not unsound. Mr. Barber proceeds (again not for the first time) to put forward the idea of a "tabular" plan for smaller risks. The plan he proposes is ingenious but, of course, the great objection to using such a plan is in the great difficulty of securing a smooth join between such a plan for smaller risks and the regular Experience Rating Plan for larger risks. As a matter of fact, it was the necessity for careful treatment of a similar join between the multi-split plan for medium-size risks and a self-rating plan for very large risks that caused the writing of the paper under discussion. In any experience rating plan we have at least two or three such joins to deal with, namely, one at the qualification point (this is usually not a serious problem); another one at what is called in the paper the "Q" point (namely, where we start to proceed toward self-rating); and another join at the self-rating point (this again is usually not a serious problem). Mr. Barber's "tabular" plan would introduce another join at what might be called the "T" point (namely, where the tabular plan ceases to be used). Mr. Barber's final suggestion as to the possibility of further progress in the rating of individual risks, involving the use of small deductibles, may possibly be worked out in the future—perhaps coupled up with Mr. Sinnott's idea of a variable

multi-split commencing point. However, such ideas do not seem to be within the range of practical consideration at present.

Mr. Sinnott's general remarks, like those of the other discussers, are very helpful in elucidating the ideas and aims of the paper. The paper itself was so long that I hesitated to enlarge my remarks on the basic objectives and the discussers have helped me out by stating these objectives clearly in different language. I like Mr. Sinnott's remarks on the absence of a quantum theory in experience rating but must resist the temptation to enlarge mathematically on this aspect of the subject. He is, of course, perfectly right that from the point of view of theory improvements could be introduced by having variable points at which to commence to discount losses and perhaps variable discount ratios, but I am afraid that the multi-split plan is complicated enough without making more of the constants variable. Mr. Sinnott's suggestion for the introduction of a further parameter N is ingenious but I believe impractical. In any case, does not the plan as set forth in the paper accomplish something very similar to what Mr. Sinnott wants? As the size of the risk goes to the self-rating point, the maximum value for any loss goes from what Mr. Sinnott calls "M" to infinity in theory, but in practice, to a limited value since presumably average death and permanent total values would be used with suitable maxima for catastrophes. One further point must be borne in mind and that is that the treatment of the actual losses must always be paralleled by a similar treatment of expected losses. The two are always dealt with in the same way, and this must not be forgotten. I would hesitate to put forward a plan in which E_v has to be calculated in a manner corresponding to the A_v of Mr. Sinnott's suggestion. Any plan to be used in practice must be even less complicated than the multi-split plan dealt with in the paper and, as will be seen later, it is suggestions directed toward simplification that we want rather than further complications.

I am grateful to Mr. Kormes for his sympathetic treatment of the mathematics involved in my paper. His suggestions regarding a straight line Y are very sound, although, as indicated below, I would rather go further, directly to a straight line W, correcting, if necessary, for the rather lower credibilities which this entails by suitably adjusting the self-rating point. Mr. Kormes' straight

line Y and the straight line W, mentioned below, both involve, theoretically, discontinuities at the Q point but I do not believe this is an insuperable obstacle. I am very much in accord with Mr. Kormes' remarks regarding the absence from the Society's syllabus of branches of mathematics like trigonometry and plane geometry. Other actuarial societies have seen the wisdom of having their students grounded in such subjects. Trigonometrical functions are not solely surveyors' measuring functions but are very useful in analysis and crop up there continuously. For instance, they found their way into a paper of mine on rate levels in Volume XX of *Proceedings*. They should certainly be understood by actuaries. I am rather reminded of an anecdote of Professor de Morgan, the eminent British mathematician and actuary of a century ago. He was explaining to a friend some actuarial problem involving probability and in the formula he set down the symbol π appeared. The friend asked what that stood for; de Morgan answered that it was a constant number that frequently occurred in mathematics and said that the simplest illustration he could give was that it was the ratio of the circumference of a circle to the diameter. The friend replied, "Now, I know you are talking nonsense because what has a circle to do with this actuarial problem?" Plane geometry is also useful, for graphs and geometrical illustrations often give an easy visual picture of a function, as, for example, a graph of the simple credibility function $Z = E/(E + K)$ (see Fig. I).

It seems advisable in closing these remarks to mention briefly some developments regarding the subject matter of the paper that have taken place since the paper was written. Except where otherwise indicated these remarks apply principally to Formula (31) as this is the formula that the Actuarial Committee of the National Council has had chiefly under discussion. Last November I calculated full values of K_E and W for a particular state (Georgia), for certain determinate values of K, Q and S. The Committee felt that the computations involved were rather complicated so in December I investigated the shape of the W curves for various states with varying methods of determining K and found that the following formula was satisfactory.

$$W = 1 - \frac{(x-1)^2 \{L + (2L + M)x\}}{L + Mx - x^2}$$

where $x = (E - Q)/(S - Q)$ as in Formula (21) and L, M are constants. We were assuming that S would be taken as ten times Q and found that constants of $L = .0161538$ and $M = 1.4523077$ (which were chosen so as to give $W = .375$ at $x = .3$ and $W = .825$ at $x = .7$) gave good results for all states. Note that in the above formula, when $x = 0$ W always $= 0$ and $W^1 = 0$, while when $x = 1$, $W = 1$ and $W^1 = 0$.

Later on, in order to obtain a formula on which could be based a standard table of W and ballasts $B_E = K_E(1 - W)$, so that values could be readily got out for any particular state, once values of Q, S and K had been fixed, I proposed to modify the K Formula given in equation (38) of the paper to

$$K_E = K + (g S - K)\frac{(1 + f)\, x^2}{x + f}$$

where f is a constant, which must be less than

$$\frac{K - Q g}{g S + Q g - 2 K}.$$

I found that in practice, if as usual, $S = 10 \cdot Q$, the value of .03 was satisfactory for f. Then B_E can be written as

$$\frac{(1 + f)\, g\, x^2\, (1 - W)}{x + f} S + \left\{1 - \frac{(1 + f)\, x^2}{x + f}\right\}(1 - W)\, K$$

or say $B_1 S + B_2 K$. I gave a table of W, B_1 and B_2 in terms of E/S from which, for any state, the necessary table of W and B_E for various values of E could be readily computed. The specimen two lines below will serve as illustration.

E/S	W	B	B
.386315	.400	.072360	.419101
.389947	.405	.072739	.413153

Suppose now for example that for a particular state $S = 85{,}000$ and $K = 5{,}500$ then multiplying the E/S column by 85,000 and calculating $B_E = B_1 S + B_2 K$ we get

E	W	B_E
32837	.400	8456
33145	.405	8455

Then for E between 32837 and 33145 we use .400 for W and 8456 for B_E. This represents perhaps the high-water mark of complexity of treatment of multi-split plan credibilities. The next steps

were directed toward simplifying the procedure. It should be emphasized here, however, that such simplification is only in the calculation of the tables which must necessarily accompany a multi-split experience rating plan. In the actual rating of a risk the procedure is just the same, for instance, whether the tables are based on curved W's or on straight line W's. Simplicity, however, is extremely helpful to the rating organizations in preparing the necessary tables and also in explaining such tables to the supervising authorities and to assureds.

The Actuarial Committee of the National Council, as mentioned by Mr. Kormes, decided to test "straight line" W and K_E values and accordingly in April I gave the results of my investigations along these lines. By "straight line" W is meant putting $W=(E-Q)/(S-Q)$ i.e. x of the paper and $K_E=K+(g\,S-K)\,x$ i.e. a straight line is drawn in Fig. IV from g to the point $(S_1\ g\ S)$ on the line $K_E = g\,E$. Mr. Kormes has given an illustration of the values thus produced in comparison with curved values (these curved values were calculated from the table just mentioned). Mr. Kormes thinks straight line credibilities are on the low side but I don't think this is necessarily so and in any case suitable adjustment can easily be made by changing the S values. The straight line values will fill all the requirements laid down in the paper, except that of giving a smooth join at Q and at S. The break in continuity at S is scarcely noticeable and that of Q is hidden by the other break in continuity caused by the use in actual practice of tables of W and B_E proceeding by discreet intervals. In my memorandum to the Actuarial Committee I illustrated this graphically. My conclusion was that straight line values were satisfactory.

For comparison I give straight line values corresponding to the curved ones given above for $S = 85{,}000$ and $K = 5{,}500$.

E	W	B_E
32789	.320	9289
33171	.325	9307

The table is, of course, in exactly the same form as the previous one and either can be used with equal facility. It is the lower W and higher B_E values (for the same values of E) that produce lower credibilities.

There have been numerous discussions of many other phases of the multi-split plan but I think the above completes the account of the developments regarding credibilities, the subject of the paper.

In conclusion, let me say that a multi-split plan, as set up with "straight line" W and ballast values is probably easier to explain, although I believe that curved W values could be almost as easily justified. If a formula such as (31) is used and not what I still believe is theoretically preferable (namely 14) then even with straight line values there remains, it seems, some difficulty in explaining the reason for an increase in the ballast above Q. Mr. Barber would obviate this by not increasing the ballast, but I believe it necessary to have the increase. It can be explained as follows: below Q we are using A_n and E_n with a ballast of K. Above Q we bring in A_e and E_e proportionately, i.e., $W A_e$ and $W E_e$, and to balance these we must add apportionate amount of an "excess" ballast K_e, i.e., we must add $W K_e$ to K. Now in order to reach self-rating at S we must start to take the balance out proportionately as E is increased so that it is entirely out at S. That is we take $(1 - W)$ only of the total ballast. The final ballast is thus $B_E = (1 - W)(K + W K_e)$. Now if we take $g S$ for K_e we get the straight line ballast as above. For convenience of course we calculate the final ballast B_E ahead of time and show it in the rating table.

Here, as in the paper itself, I have wandered perhaps somewhat from the discussion of credibilities to consideration of other aspects of the multi-split plan. It is hard to avoid doing this, for after all in any experience rating plan the various component parts are quite closely interconnected.