ON VARIATIONS IN COMPENSATION LOSSES WITH CHANGES IN WAGE LEVELS

BY

PAUL DORWEILER

The typical compensation act provides that the injured or, in a case of fatality, his dependents, shall be paid a certain percentage of his average weekly wages but not more than a specified maximum amount nor less than a specified minimum amount, unless the wages of the injured are less than the minimum, in which case the actual wages shall be paid. This provision for indemnity benefits with variations in the percentage rates and the minimum and maximum amounts paid weekly is found in compensation acts generally.[1] A particular act may further provide several different percentage rates and sets of limits dependent on the type of injury or, in some cases, the number of dependents.[2] The minimum and maximum amounts paid weekly are sometimes not explicit but depend upon minimum and maximum weekly wages.

There are other factors affecting the weekly compensation, as the number of weeks per year used in calculating the annual earnings or the number of days per week used with the daily earnings in determining the average weekly wages.[3] These factors, which may be a part of the law or adopted as rules of administrative procedure, may be recognized through a corresponding adjustment in the percentage rate of weekly compensation. The effect of the limits which are imposed by some laws on the total amount paid for a single injury can be determined by methods not given here. Generally the effect of limits on the total amount paid is of minor importance.

It will be observed that the typical acts intend that the amount

---

(1) In these states fixed amounts independent of the wage of the injured are paid for the type of indemnity benefits specified:

Washington and Wyoming—all types of indemnity benefits.
Oregon—all types except temporary total disability.
Massachusetts and West Virginia—fatal cases, with widow and/or children dependents.

(2) See Table I. (3) See Table I, Column 8.

of the benefits shall depend, to some extent at least, on the wages paid the injured. The purpose of this paper is to examine, under conditions of changing wage levels, the relation of the compensation losses incurred to the exposure when expressed in payrolls and man-years, and to establish criteria for determining for which of these media there is greater responsiveness between losses and exposure.

LEGAL LIMIT FACTOR.

The legal limit factor may be defined as the ratio of the value of compensation benefits when evaluated with legal limits imposed to the value of the same benefits when evaluated without legal limits. The term may be used in reference to any one of the specific types of benefits or to a combination of them. When used without further qualification it will be assumed to apply to all of the indemnity benefits which are subject to legal limit restrictions. In this discussion "legal limit" will refer to the weekly limits only. A procedure for determining the legal limit factor is given in Appendix I.

EFFECT OF LEGAL LIMITS IN INDIVIDUAL CASES.

A graphical illustration of the effect of legal limits under a given law on the weekly compensation of individual cases is given in Chart I, in which the legal limit factor has been plotted against the weekly wage. It will be noted that if the wage is less than $\mathfrak{m}$, the minimum weekly compensation other than full wage, the factor is $1.00/r$; for wages between $\mathfrak{m}$ and w, which equals $\mathfrak{m}/r$, the factor follows the curve $F = \mathfrak{m}/rW$; for wages between w and $\overline{W}$, which is M/r, the factor is 1.00; and for wages in excess of $\overline{W}$, the factor follows the curve $F = M/rW$. If the law stipulates a fixed minimum compensation $\mathfrak{m}$ without the further condition "or actual wage", for wages less than $\mathfrak{m}$ the factor follows the broken part of the curve $F = \mathfrak{m}/rW$ above the solid line in the graph. In actual construction of the chart the general terms were given these specific values: $r = .66\frac{2}{3}$, $\mathfrak{m} = 8$, $w = 12$, $M = 20$, and $\overline{W} = 30$. As a matter of interest and for completeness of the graph, the values of the factor for the extreme wages are indicated even though these have no practical significance.

EFFECT OF LEGAL LIMITS ON AGGREGATES.

The legal limit factor for a state is made up from an aggregate of such individual cases as shown in Chart I. This aggregate has a definite average wage and a distinct frequency arrangement which is known as the wage distribution.

In some states there are different legal limits and different compensation percentage rates for various types of benefits. In Chart II the graphs of the legal limit factors for total disability, permanent partial disability, and death, as well as the combination of all three, are shown for New York for the whole range of average wages. The part of the chart of practical significance has been sectioned off in the rectangle between the lines $W = 17.5$ and $W = 40$, and $F = .65$ and $F = 1.05$.

The combined factor is the legal limit factor for the indemnity benefits of New York. This and similar factors for other states in Chart III will be used when discussing and comparing the effect of the legal limits on losses. It will be noted that the graphs of the legal limit factors for an aggregate of losses are smooth and do not have the sudden breaks found in Chart I.

The legal limit factors for indemnity losses corresponding to the combined factor (curve IV, Chart II) for New York, for wage distributions with average wages from $17.50 to $40, are plotted in Chart III for ten important compensation states. In the case of Massachusetts, where the benefits for fatal cases are different fixed weekly amounts dependent on widow and/or number of children but independent of the wage of the deceased, the factor applies to all indemnity losses except fatality. If it is desired to get a factor which when applied to all indemnity losses produces an equivalent effect, .84 times the values shown in the chart should be used.

The important part of this chart is comprised between the wage ordinates $W = 20$ and $W = 35$ as will be noted from Table II in which the average wages of all industries are given for the ten states under consideration. For individual industries the factors may and do fall to the extreme left and right and even beyond the limits of the chart.

It will be observed that for Connecticut, which has a low compensation rate ($r = .50$), and a relatively high maximum weekly payment ($M = \$21$), and a very high effective maximum wage ($\overline{W} = \$42$), the limit factor diverges less from unity for the higher wage levels at the right of the chart than the other state factors. For Pennsylvania, where the compensation rate ($r = .65$) is relatively high, and the maximum weekly payment ($M = \$15$) and the effective maximum wage ($\overline{W} = \$23.08$) are relatively low, the legal limit factor diverges most widely from unity. In New Jersey, where the percentage rate of compensation in fatal cases is low ($r = .35$, etc.) and the fixed minimum weekly payment for all types of injury ($\mathfrak{m} = \$10$) is high, the result is a legal limit factor in excess of unity for the lower wage levels. Generally, for wage distributions where the average is very low, the minimum weekly compensation conditions are the more effective, while for distributions where the average is high the maximum weekly payments govern the limit factor. For the 1924-1930 wage levels (Table II), the effect of the minimum limits in most states is negligible.

VARIATION OF LOSSES WITH PAYROLL.

If compensation acts provided that indemnity benefits should be a fixed percentage of the weekly wages without any limitation as to weekly payments or as to the total amount to be paid, the payrolls would be an ideal medium to use for measuring exposure for indemnity losses. Medical benefits are not by law made responsive to wage levels, except by the rather vague and general provision in some of the acts that charges for industrial accidents should be no more than prevail for private treatment of such cases. There is, however, a long term responsiveness which correlates commodities in general with a price level and there is also a somewhat parallel variation of wages and medical costs between urban and rural communities. In this discussion this indirect and indefinite medical response to wage level will not be recognized. It will be assumed that there is no direct or immediate causal response of medical costs to variations in wage level. With this assumption it will be attempted to measure the degree of response which compensation losses, indemnity and medical combined, give to any change in wage level at different levels.

INDEX OF VARIATION.

A condition which might be termed "perfect variation", where under adequate exposure an $x\%$ increase or decrease in wage level will be accompanied by an $x\%$ increase or decrease in losses, will be represented by the index 1.00. "Perfect variation" may be considered as existing under a law which provides no medical benefits and in which the compensation rate is a definite $100r\%$ of the wages without any qualifications whatsoever. By an *index of variation y* will be meant that the losses change at the rate of $100y\%$ of the rate under a case of "perfect variation", which has an index of variation 1.00.

Under this definition, the legal limit factor for limits applying to all indemnity losses is the index of variation of the indemnity losses with the payrolls. The measure of the variation of all losses with the payrolls will depend on a combination of the variable indemnity losses with the medical losses, which, under the assumption, do not vary. Consider the case where the legal limit factor for indemnity is .90 and where the medical constitutes 30% of the total losses. Here the index of variation of the total losses is $.90 \times (1-.30)$ or .63. This means that for a given increase in the wage level the change in the actual losses caused by this increase in wages is .63 of what the change would be if all losses (indemnity and medical) increased in the same ratio as the wages. This relationship may be represented more generally by the equation:

$I = F(1 - R)$, where $I =$ index of variation
$F =$ legal limit factor
$R =$ medical ratio, medical losses to total losses

The index of variation thus far has been considered in a somewhat absolute sense: it has been considered in terms of an arbitrary condition representing "perfect variation". In actual operation the index of variation should be considered in a relative sense. Under a given compensation law the index of variation changes with the wage level and the medical ratio. The wage level and medical ratio underlying the basic rate are represented by an index which becomes incorporated in the pure premiums. Any change when considered with respect to its effect on the pure premiums must be taken relative to the wage level and medical ratio already in effect.

RESPONSIVENESS BETWEEN LOSSES AND EXPOSURE WHEN MEASURED IN PAYROLLS AND MAN-YEARS.

In Appendix II, consideration has been given to the loss ratios developed when exposure is méasured by payrolls and man-years under the same compensation act at various wage levels. A hypothetical set of rates for each exposure medium, which produces the expected loss ratio when operating at the basic wage level with index 1.00, is applied at another wage level with index $1 + x$ and the effect on the loss ratio is observed.

In the first part of the Appendix, formulas 14-18 have been developed for determining loss ratios under payroll and man-year exposure, and for determining the deviations of the actual loss ratios from the expected loss ratio, and also for determining the ratio of these deviations for the two exposure media. Formula 18 may be used to determine whether the deviation of the developed loss ratio from the expected loss ratio is greater under payroll or man-year exposure at a given wage level. It may be shown that the fraction in the formula is always negative and that under payroll exposure loss ratios are produced whose deviations from the expected are greater or less than those produced under the man-year exposure according as the value of the fraction is algebraically less or greater than -1.

In the second part of Appendix II, these formulas have been applied to calculate loss ratio indices for the same experience under payroll exposure and man-year exposure. The object of Table V is to show how loss ratios are affected by changes in wage level under payroll and man-year exposure media. In these states, for rates based on a $30 wage and with the medical assumed constant, the table shows greater responsiveness between losses and exposure when measured in man-years than when measured in payrolls. If the rates are based on lower wage levels, the greater responsiveness with man-year exposure decreases and then disappears, beginning with Connecticut at the $27.50 average wage level. Conversely, if rates are based on higher wage levels, there is an increase in the greater responsiveness of man-year exposure.

A casual survey of formulas 14-18 would indicate only four variables R_1, F_1, F_{1+x}, and x, and of these, the last three are interrelated. The first three are functions of two or more variables. The medical ratio R is a function of i and m, and the legal limit factors F are functions or r, $\mathfrak{m}$ and M (or w and $\overline{W}$), and W_n. The problem of determining the degree of responsiveness between compensation losses and exposure is an involved one requiring intensive study for its general solution.

TABLE I. Important Features of Compensation Laws Affecting Weekly Compensation Payments, Ten States

State	Type of Injury	Per Cent Rate r	Weekly Compensation Limits: Min. m	Weekly Compensation Limits: Max. M	Weekly Wage Limits: Min. w	Weekly Wage Limits: Max. W	Average Weekly Wage (d)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
California	All	65% (a)	$4.17	$25.00	$6.41	$38.46	$AE/52$
Connecticut	All	50	5.00	21.00	10.00	42.00	(26 wks. E) / (no. wks. worked)
Massachusetts	Fatal	Compensation is scheduled amounts, regardless of wage					
	Dism. & L of U	66⅔	4.00	10.00	6.00	15.00	$AE/(52-$ wks. lost if >2 wks.)
	Other	66⅔	9.00 (b)	18.00	13.50	27.00	
Michigan	All	66⅔	7.00	18.00	10.50	27.00	$DW \times 6$
Missouri	PTD over 300 wks.	25	6.00	20.00	24.00	80.00	No provision but
	Other	66⅔	6.00 (c)	20.00	9.00	30.00	$AE = 300 \times DW$
New Jersey	Fatal	35-60	10.00 (b)	20.00	Var.	Var.	$DW \times 5, 5\frac{1}{2}, 6,$
	Other	66⅔	10.00 (b)	20.00	15.00	30.00	$6\frac{1}{2}$ or 7
New York	Total Dis.	66⅔	8.00 (b)	25.00	12.00	37.50	
	Partial Dis.	66⅔	8.00 (b)	20.00	12.00	30.00	$(DW \times 300)/52$
	Fatal	15-66⅔	—	Var.	—	34.62	
Pennsylvania	Fatal	16-65	Various	Var.	12.00	24.00	$DW \times 5, 5\frac{1}{2}, 6, 6\frac{1}{2}$
	Other	65	7.00 (b)	15.00	10.77	23.08	or 7; or $AE/50$
Texas	All	60	7.00	20.00	11.67	33.33	$(DW \times 300)/52$
Wisconsin	Fatal	65	6.83	19.50	10.50	30.00	$(DW \times 300)/50$
	Other	70	7.35	21.00	10.50	30.00	

(a) In permanent disability cases involving life pensions becomes various after 240 weeks. (b) Or wage, if less than minimum compensation. (c) For temporary total, footnote (b) applies. (d) AE = annual earnings, DW = daily wage.

General Notes—In cases of partial disability, excluding scheduled specific dismemberments and loss of use (in Michigan dismemberment only), the rate r applies to the wage loss instead of total wage, except in New Jersey cases and in California and Missouri permanent partial cases, where it applies to total wage. The minimum limit does not apply in temporary partial disability except in New Jersey and New York, nor does it apply in permanent partial disability except in California, Missouri, New Jersey, and New York.

TABLE II. AVERAGE WEEKLY WAGES, ALL INDUSTRIES
FROM NATIONAL COUNCIL SEMI-ANNUAL WAGE CALL DATA, AND PENNSYLVANIA BUREAU POLICY YEAR DATA

STATE	CALENDAR YEAR						
	1924	1925	1926	1927	1928	1929	1930
California	$32.37	$32.57	$32.29	$31.90	$31.85	$31.86	$31.45
Connecticut	27.58	28.03	28.07	28.91	28.09	28.72	28.92
Massachusetts	27.11	27.35	27.37	27.09	27.85	27.51	28.00
Michigan	30.25	29.95	31.83	30.18	31.98	32.12	30.57
Missouri	—	—	—	27.04	27.26	26.90	26.47
New Jersey	29.89	31.82	31.84	32.37	33.26	32.30	32.58
New York	31.31	32.02	32.52	32.87	33.52	33.58	33.46
Pennsylvania	27.80	28.19	28.40	28.24	27.87	27.47	—
Texas	27.27	26.07	26.20	27.55	26.85	26.83	26.78
Wisconsin	25.81	26.92	27.61	28.02	28.55	28.80	28.23

TABLE III. PERCENTAGE DISTRIBUTION OF LOSSES IN CLASSIFICATION EXPERIENCE BY RATIO OF MEDICAL LOSSES TO TOTAL LOSSES; AND STATE AVERAGE MEDICAL RATIOS

Medical Ratio Group	Calif.	Conn.	Mass.	Mich.	Mo.	N. J.	N. Y.	Pa.	Texas	Wisc.
.10-.19	—	—	1%	1%	1%	28%	24%	9%	—	1%
.20-.29	1%	4%	35	17	55	54	55	37	55%	22
.30-.39	41	44	52	67	35	17	20	29	35	60
.40-.49	39	41	10	14	9	1	1	21	9	15
.50-.59	18	10	1	1	—	—	—	4	1	2
.60-.69	1	1	1	—	—	—	—	—	—	—
Average Medical Ratio	.42	.41	.32	.34	.32	.27	.26	.31	.29	.34

APPENDIX I.

DETERMINATION OF FACTORS FOR THE EFFECT OF LEGAL LIMITS ON WEEKLY COMPENSATION

1. FROM STANDARD WAGE DISTRIBUTION.

The wage distribution, Table IV, used as the standard, is the distribution given in Table I of Mowbray's paper on the effect of limits (Proceedings, Volume IX, p. 213). It has been graduated by Carver's method (Proceedings, Volume VI, pp. 52-72), projected to lower wage groups, and extended to 4,452 cases in order to make a total wage of $100,000 for convenience in using the table. Some arbitrary minor adjustments were necessary to bring the total to exactly $100,000. The column headings of Table IV are explained in its footnotes.

Let it be required to determine the legal limit factor under this wage distribution for a law compensating at the rate of 60% of wages subject to a maximum weekly payment of $18 and a *minimum of $6 or the actual wage if under $6.*

It will be observed that:

1. For weekly wages between $10, the effective minimum wage, and $30, the effective maximum wage, the weekly compensation is 60% of the wages.
2. For wages in excess of $30 the compensation is $18.
3. For wages between $10 and $6 the compensation is $6.
4. For wages under $6 the compensation is the actual wage.

From Table IV:

	NUMERICALLY	SYMBOLICALLY
1. Total weekly wages between $30 and $10. (Col. 7, line 28 – line 8.)	78176 – 961	$\Sigma C_{l_2} - \Sigma C_{l_1}$
Cost, 60% of wages.	.60 × 77215	$r(\Sigma C_{l_2} - \Sigma C_{l_1})$
2. No. of cases over $30. (Col. 6, line 29.)	620	$\Sigma N'_{l_2+1}$
Cost, $18 per case.	18 × 620	$M\Sigma N'_{l_2+1}$
3. No. of cases between $10 and $6 (Col. 5, line 8 – line 4.)	126 – 25	$\Sigma N_{l_1} - \Sigma N_{l_3}$
Cost, $6 per case.	6 × 101	$\mathfrak{m}(\Sigma N_{l_1} - \Sigma N_{l_3})$
4. Actual wages under $6. (Col. 7, line 4.)	111	ΣC_{l_3}
5. Total compensation cost, without limits.	.60 × 100,000	$r(100{,}000)$

If the compensation costs of the first four are added (in order 4, 3, 1, 2) and then divided by the cost in 5, the legal limit factor is obtained.

Numerically:

$$F = \frac{111 + 6 \times 101 + .60 \times 77215 + 18 \times 620}{.60 \times 100{,}000} = .9701$$

Symbolically:

$$F = \frac{\Sigma C_{l_3} + \mathfrak{m}\,(\Sigma N_{l_1} - \Sigma N_{l_3}) + r\,(\Sigma C_{l_2} - \Sigma C_{l_1}) + M\,\Sigma N'_{l_2+1}}{r\,(100{,}000)}, \text{ or}$$

$$\text{I. } 100{,}000\,F = \frac{1}{r}\Sigma C_{l_3} + w(\Sigma N_{l_1} - \Sigma N_{l_3}) + \Sigma C_{l_2} - \Sigma C_{l_1} + \overline{W}\,\Sigma N'_{l_2+1}$$

since $\mathfrak{m}/r = w$, and $M/r = \overline{W}$

If the weekly minimum $\mathfrak{m}$ applies also in cases where the wage is under $\mathfrak{m}$, the terms involving l_3 become irrelevant and are disregarded. The formula then becomes:

$$\text{II. } 100{,}000\,F = w\,\Sigma N_{l_1} + \Sigma C_{l_2} - \Sigma C_{l_1} + \overline{W}\,\Sigma N'_{l_2+1}$$

An analogous procedure may be followed to derive the formula in Case III when the law requires that the weekly wages shall not be taken in excess of $\overline{W}$ nor below w and in Case IV when the only restriction on wages is that they shall not be taken in excess of $\overline{W}$.

2. FROM ANY WAGE DISTRIBUTION.

If it is assumed that graduated wage distributions of like number of cases and the same average wage are substantially alike and that wage level changes may be represented approximately by percentage changes throughout the distributions affected, then a single standard wage distribution may be used to determine the legal weekly limit factor for any distribution having a known average wage.*

---

* These two assumptions are, in effect, equivalent to the assumptions regarding equal percentage departures made by Mowbray. See Mowbray—*Proceedings,* Volume IX, page 239, for tests as to accuracy of results produced. It should be recognized that the same degree of accuracy cannot be expected when the method is applied to distributions having very low or very high average wages.

In Chart IV, the wage distribution given in column 3, Table IV, is represented by the frequency curve D. The same distribution after all wages have been increased by a factor, $1/v$, is represented by the curve D'. The new curve is obtained by moving each of the ordinates of D to the right until its abscissa equals $1/v$ times the old. Conversely, D may be obtained from D' by compressing each abscissa of the latter to v times its former size. Under the assumptions stated the curve D' may be considered as representative of wage distributions whose average wage is $1/v$ times the average wage of D.

Let it be required to find the legal limit factor under a wage distribution D' for a law providing a compensation rate of $100r\%$ of wages with the weekly maximum compensation of M and a minimum of $\mathfrak{m}$. The effective maximum wage corresponding to M is M/r or $\overline{W}$, and the effective minimum corresponding to $\mathfrak{m}$ is $\mathfrak{m}/r$ or w. These wages w and $\overline{W}$ at which the limits become effective are fixed and are the same for all wage distributions. The ordinate on D' for the wage $\overline{W}$ cuts the curve at P'. The corresponding point on D is P, which has for its abscissa $v\overline{W}$. That is, an effective maximum wage $v\overline{W}$ in the D distribution, is relatively the same as the effective maximum wage $\overline{W}$ in the D' distribution.

Similarly it may be shown that the effective minimum wage w in the D' distribution is relatively the same as the effective minimum wage vw in the D distribution. If for any frequency distribution D' underlying a law, there be established for its w and $\overline{W}$ the corresponding effective minimum vw and effective maximum $v\overline{W}$ in the standard distribution D by means of the relation $v = W_s/W_n$, then the legal limit factor for D' may be found from D by entering Table IV with an effective minimum wage of vw and effective maximum wage of $v\overline{W}$.

TABLE IV. Legal Limit Factor Table — $W_s = 22.46$

l	W_l	N_l	C_l	ΣN_l	$\Sigma N'_l$	ΣC_l
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	2.50	3	7	3	4,452	7
2	3.50	5	18	8	4,449	25
3	4.50	7	31	15	4,444	56
4	5.50	10	55	25	4,437	111
5	6.50	14	91	39	4,427	202
6	7.50	20	150	59	4,413	352
7	8.50	28	238	87	4,393	590
8	9.50	39	371	126	4,365	961
9	10.50	54	567	180	4,326	1,528
10	11.50	72	828	252	4,272	2,356
11	12.50	94	1,175	346	4,200	3,531
12	13.50	119	1,606	465	4,106	5,137
13	14.50	146	2,117	611	3,987	7,254
14	15.50	175	2,713	786	3,841	9,967
15	16.50	203	3,349	989	3,666	13,316
16	17.50	229	4,008	1,218	3,463	17,324
17	18.50	249	4,606	1,467	3,234	21,930
18	19.50	263	5,129	1,730	2,985	27,059
19	20.50	269	5,514	1,999	2,722	32,573
20	21.50	269	5,784	2,268	2,453	38,357
21	22.50	260	5,850	2,528	2,184	44,207
22	23.50	247	5,804	2,775	1,924	50,011
23	24.50	230	5,635	3,005	1,677	55,646
24	25.50	208	5,304	3,213	1,447	60,950
25	26.50	187	4,956	3,400	1,239	65,906
26	27.50	165	4,537	3,565	1,052	70,443
27	28.50	144	4,104	3,709	887	74,547
28	29.50	123	3,629	3,832	743	78,176
29	30.50	105	3,202	3,937	620	81,378
30	31.50	88	2,772	4,025	515	84,150
31	32.50	74	2,405	4,099	427	86,555
32	33.50	62	2,077	4,161	353	88,632
33	34.50	52	1,794	4,213	291	90,426
34	35.50	42	1,491	4,255	239	91,917
35	36.50	35	1,278	4,290	197	93,195
36	37.50	29	1,087	4,319	162	94,282
37	38.50	24	924	4,343	133	95,206
38	39.50	20	790	4,363	109	95,996
39	40.50	16	648	4,379	89	96,644
40	41.50	13	540	4,392	73	97,184
41	42.50	11	467	4,403	60	97,651
42	43.50	9	392	4,412	49	98,043
43	44.50	7	311	4,419	40	98,354
44	45.50	6	273	4,425	33	98,627
45	46.50	5	233	4,430	27	98,860
46	47.50	4	190	4,434	22	99,050
47	48.50	3	145	4,437	18	99,195
48	49.50	3	149	4,440	15	99,344
49	50.50	2	101	4,442	12	99,445
50	51.50	2	103	4,444	10	99,548
51	52.50	2	105	4,446	8	99,653
52	53.50	1	53	4,447	6	99,706
53	54.50	1	55	4,448	5	99,761
54	55.50	1	56	4,449	4	99,817
55	56.50	1	57	4,450	3	99,874
56	60.50	1	60	4,451	2	99,934
57	66.50	1	66	4,452	1	100,000

Note: The symbols in the column heads of Table IV denote the following:

$l =$ line number of group.
$W_l =$ average wage of group.
$N_l =$ number of cases in group.
$C_l =$ total wage of group.
$\Sigma N_l =$ number of cases cumulated downward.
$\Sigma N'_l =$ number of cases cumulated upward.
$\Sigma C_l =$ total wages cumulated downward.

Formulas:

Case I. Compensation rate $= r$, minimum weekly compensation $= \mathfrak{m}$ or wages, maximum weekly compensation $= M$.

$$100{,}000\,F = \frac{1}{r}\,\Sigma C_{l_3} + w(\Sigma N_{l_1} - \Sigma N_{l_3}) + \Sigma C_{l_2} - \Sigma C_{l_1} + \overline{W}\,\Sigma N'_{l_2+1}$$

Case II. Compensation rate $= r$, minimum weekly compensation $= \mathfrak{m}$, maximum weekly compensation $= M$.

$$100{,}000\,F = w\,\Sigma N_{l_1} + \Sigma C_{l_2} - \Sigma C_{l_1} + \overline{W}\,\Sigma N'_{l_2+1}$$

Case III. Compensation rate $= r$, minimum weekly wage $= w$, maximum weekly wage $= \overline{W}$.

$$100{,}000\,F = w\,\Sigma N_{l_1} + \Sigma C_{l_2} - \Sigma C_{l_1} + \overline{W}\,\Sigma N'_{l_2+1}$$

Case IV. Compensation rate $= r$, maximum weekly wage $= \overline{W}$.

$$100{,}000\,F = \Sigma C_{l_2} + \overline{W}\,\Sigma N'_{l_2+1}$$

Where

$W_s =$ average wage, standard distribution, or 22.46.
$W_n =$ average wage, new distribution.
$v = W_s \div W_n$.
$r =$ compensation rate expressed in decimals.
$u = v \div r$.
$\mathfrak{m} =$ minimum weekly compensation.
$M =$ maximum weekly compensation.
$w = u\,\mathfrak{m}$, effective minimum wage.
$\overline{W} = u\,M$, effective maximum wage.
$F =$ legal limit factor.
l_1 is that value of l for which W_l is equal to or next lower than w.
l_2 is that value of l for which W_l is equal to or next lower than $\overline{W}$.
l_3 is that value of l for which W_l is equal to or next lower than $v\mathfrak{m}$.

APPENDIX II.

LOSS RATIOS UNDER PAYROLL AND MAN-YEAR EXPOSURES

1. DERIVATION OF FORMULAS.

Consider two industrial conditions alike in every respect except the underlying wage level, and consider the same compensation act as effective in each. Let one industrial condition be denoted by "I" and its wage level index be 1.00. Let the other industrial condition be denoted by "II" and its wage level index be $1 + x$.

Using certain assumptions and definitions which have been designated by "a", expressions for pure premium with weekly limits, expected losses, premium, and loss ratio are developed for condition I, first for payroll exposure, then for man-year exposure. The corresponding expressions are then developed for condition II, on the assumption that the rates effective in condition I have been retained intact. These developments are shown in the tabular form following:

ITEMS	INDUSTRIAL CONDITION			
		I		II
For Payroll Exposure				
1. Wage Level Index	a	1.	a	$1+x$
2. Payroll	a	P		$(1+x)P$
3. Limit Factor	a	F_1	a	F_{1+x}
4. Pure Premium, no limits	a	$i+m$		$i+m/(1+x)$
5. Pure Premium, with limits		F_1i+m		$F_{1+x}i+m/(1+x)$
6. Expected Losses, (2) × (5)		$P[F_1i+m]$		$(1+x)P[F_{1+x}i+m/(1+x)]$
7. Premium, (6) ÷ E		$P[F_1i+m]/E$		$(1+x)P[F_1i+m]/E$
8. Loss Ratio (6) ÷ (7)		E		$E[F_{1+x}i+m/(1+x)]/[F_1i+m]$
9. Deviation of Loss Ratio from Expected, E - (8)		0		$E[F_1i-F_{1+x}i+mx/(1+x)]/[F_1i+m$
For Man-Year Exposure				
10. Premium, Item 7 I		$P[F_1i+m]/E$		$P[F_1i+m]/E$
11. Expected Losses, Item 6		$P[F_1i+m]$		$(1+x)P[F_{1+x}i+m/(1+x)]$
12. Loss Ratio, (11) ÷ (10)		E		$(1+x)E[F_{1+x}i+m/(1+x)]/[F_1i+m$
13. Deviation of Loss Ratio from Expected, E - (12)		0		$E[F_1i-(1+x)F_{1+x}i]/[F_1i+m]$

; Ratio and Deviation Formulas Simplified *

"Payroll" Loss Ratio, II Item 8 II $= E\left[(1-R_1)\frac{F_{1+x}}{F_1} + \frac{R_1}{1+x}\right]$

"Man-Year" Loss Ratio, II Item 12 II $= E\left[(1-R_1)(1+x)\frac{F_{1+x}}{F_1} + R_1\right]$

"Payroll" Loss Ratio Deviation, II Item 9 II $= E\left[1 - \frac{R_1}{1+x} - (1-R_1)\frac{F_{1+x}}{F_1}\right]$

"Man-Year" Loss Ratio Deviation, II Item 13 II $= E(1-R_1)\left[1-(1+x)\frac{F_{1+x}}{F_1}\right]$

Ratio of Loss Ratio Deviations, II
(17) ÷ (16)

$$\frac{\text{"Man-Year" Deviation}}{\text{"Payroll" Deviation}} = \frac{1-(1+x)\frac{F_{1+x}}{F_1}}{1-\frac{F_{1+x}}{F_1}+\frac{R_1}{1-R_1}\cdot\frac{x}{1+x}}$$

Where P = payroll.
F_1 = legal limit factor at wage level 1.00.
F_{1+x} = legal limit factor at wage level $1+x$.
i = indemnity pure premium without legal limits, Condition I.
m = medical pure premium, Condition I.
E = expected loss ratio.
R_1 = medical ratio, Condition I.

* Simplification of Formulas.

14. "Payroll" Loss Ratio (Item 8 II)

$$= E\left[F_{1+x}\,i + \frac{m}{1+x}\right] \Big/ \left[F_1\,i + m\right]$$

$$= E\left[F_{1+x} + \frac{F_1 m}{F_1 i(1+x)}\right] \Big/ F_1\left[1 + \frac{m}{F_1 i}\right]$$

$$= E\left[\frac{F_{1+x}}{F_1} + \frac{R_1}{(1-R_1)(1+x)}\right] \Big/ \frac{1}{1-R_1}$$

$$= E\left[(1-R_1)\frac{F_{1+x}}{F_1} + \frac{R_1}{1+x}\right]$$

since $F_1\,i$ = indemnity pure premium under Condition I and $\frac{m}{F_1 i} = \frac{R_1}{1-R_1}$

By similar procedures formulas 15, 16, and 17 may be derived.

2. CALCULATION OF LOSS RATIO INDICES.

In Table V, loss ratio indices have been calculated for the ten states under consideration, on the assumption that the rates have been so keyed to a wage level of $30 average weekly wage that the permissible loss ratio would be produced if the conditions remained unchanged. It is assumed further that the medical ratio *R* given in Table III applies at this wage level and that the only factor which varies from those in the rate calculation is the wage level, for which the percentage change applies everywhere so that the relativity of classification payroll distribution is preserved. There is a lag between the wages used in determining the injured's weekly compensation and those underlying the premiums because past periods are used in determining average weekly wages (see Table I, Column 8). No allowance is made for this lag. This is equivalent to an assumption that the wage level has been in effect for a sufficient period to overcome the lag.

The loss ratio indices were calculated using formulas 14 and 15 for seven different wage levels which are shown in the column headings of the Table. Directly underneath each average wage is shown the wage level index based on 1.000 for the $30 average wage. In the body of the Table are shown loss ratio indices based on 1.000 for the expected loss ratio underlying the rate level. Two sets of indices are given for each state. On the first line marked "P" the indices are for a set of classification rates based on payroll exposure, and on the second line marked "M" the indices are for a set of rates based on man-year exposure. It is assumed that in each case the rates produce the permissible loss ratio at the $30 average wage level. For each exposure basis its own set of rates is retained for all seven wage levels. The indices in Table V may be viewed as applying to the state as a whole, with the relativity of classification payroll distribution preserved, or to a particular classification or group of classifications within the state.

TABLE V

LOSS RATIO INDICES—PAYROLL AND MAN-YEAR EXPOSURE
Showing Loss Ratio Indices based on rate level keyed to a $30 average weekly wage level and a medical Ratio R_1 taken from Table III.

Index for expected loss ratio = 1.000
Payroll exposure loss ratio index on line P
Man-year exposure loss ratio index on line M

STATE	Exposure	AVERAGE WAGE OF DISTRIBUTION Wage Level Index, $1+x$						
		$20.00	$22.50	$25.00	$27.50	$30.00	$32.50	$35.00
		.667	.750	.833	.917	1.000	1.083	1.167
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
California	P	1.233	1.161	1.101	1.048	1.000	.955	.912
$R_1 = .42$	M	.822	.870	.917	.961	1.000	1.034	1.064
Connecticut	P	1.221	1.151	1.093	1.043	1.000	.960	.921
$R_1 = .41$	M	.814	.863	.911	.957	1.000	1.039	1.075
Massachusetts	P	1.314	1.220	1.141	1.066	1.000	.941	.886
$R'_1 = .43$*	M	.876	.915	.951	.978	1.000	1.020	1.034
Michigan	P	1.293	1.213	1.138	1.067	1.000	.938	.881
$R_1 = .34$	M	.863	.910	.948	.978	1.000	1.016	1.028
Missouri	P	1.247	1.180	1.117	1.058	1.000	.944	.891
$R_1 = .32$	M	.832	.885	.930	.970	1.000	1.023	1.040
New Jersey	P	1.257	1.181	1.116	1.055	1.000	.949	.900
$R_1 = .27$	M	.838	.886	.930	.968	1.000	1.028	1.051
New York	P	1.198	1.143	1.094	1.047	1.000	.954	.909
$R_1 = .26$	M	.799	.857	.912	.960	1.000	1.033	1.060
Pennsylvania	P	1.347	1.250	1.160	1.076	1.000	.932	.871
$R_1 = .31$	M	.899	.938	.966	.987	1.000	1.009	1.016
Texas	P	1.210	1.151	1.098	1.049	1.000	.953	.906
$R_1 = .29$	M	.807	.863	.915	.962	1.000	1.032	1.057
Wisconsin	P	1.257	1.186	1.121	1.059	1.000	.944	.891
$R_1 = .34$	M	.839	.889	.934	.971	1.000	1.022	1.040

* .43 is composed of Medical .32, Fatal .11

CHART II.

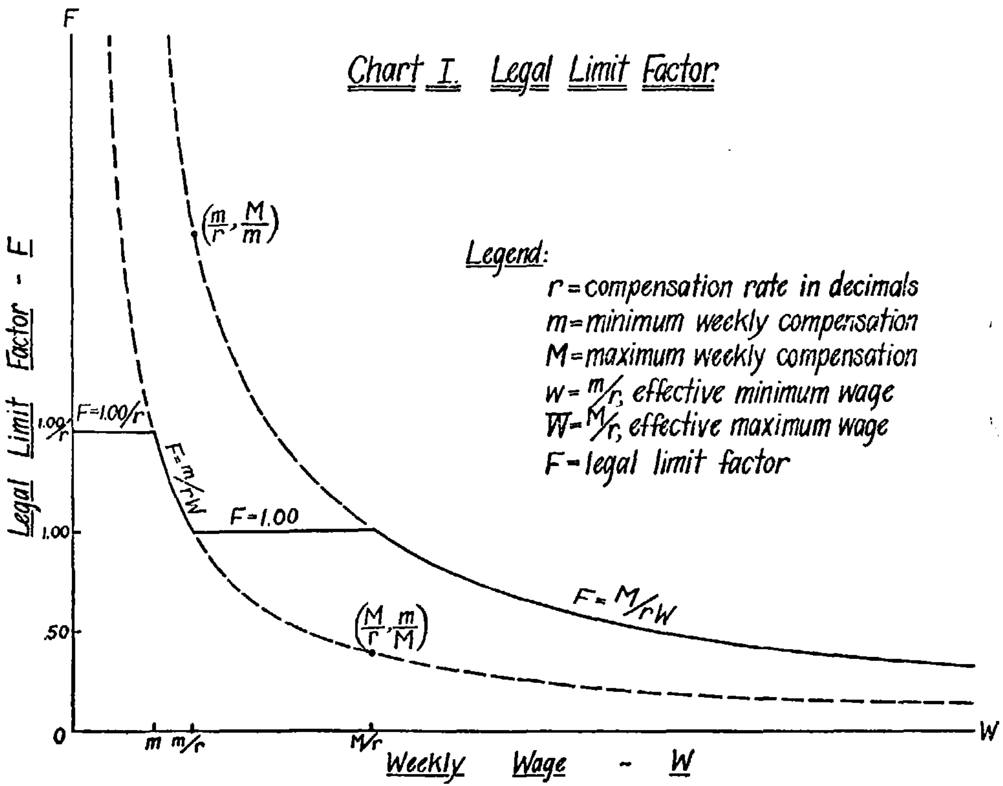
Chart I. Legal Limit Factor
F
(m/r, M/m)
Legend:
r = compensation rate in decimals
m = minimum weekly compensation
M = maximum weekly compensation
w = m/r, effective minimum wage
W̄ = M/r, effective maximum wage
F = legal limit factor
F=1.00/r
F=m/rW
F=1.00
F=M/rW
(M/r, m/M)
1.00/r
1.00
.50
0
m
m/r
M/r
W
Weekly Wage - W
Legal Limit Factor - F

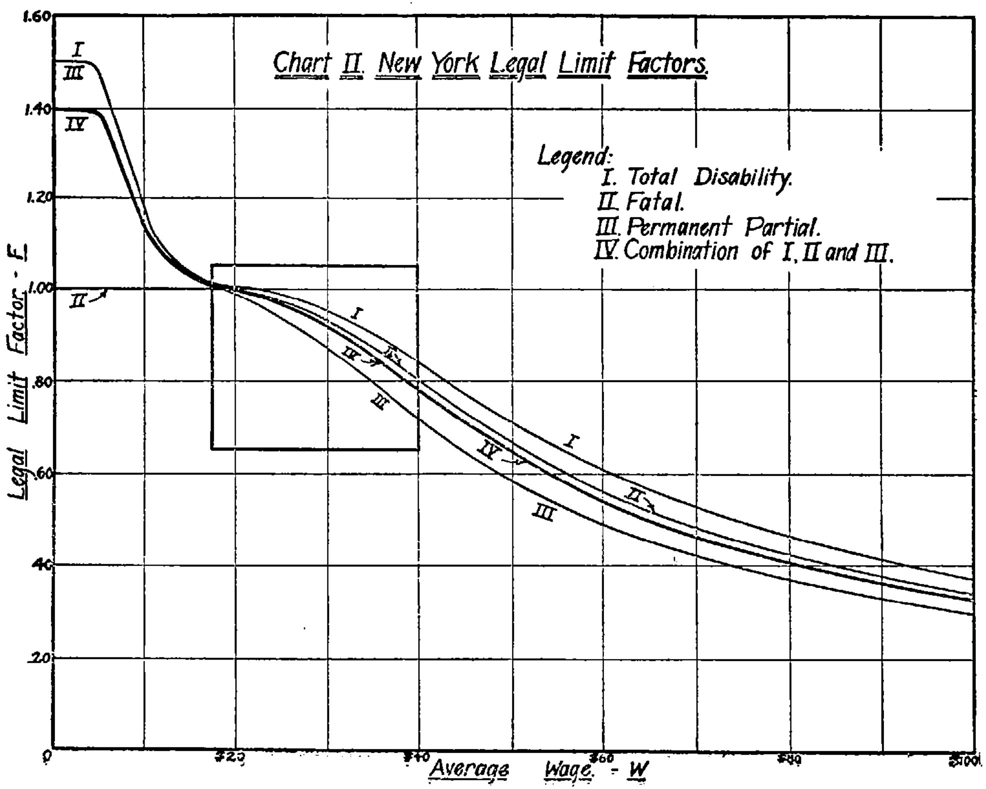
Chart II. New York Legal Limit Factors.
Legend:
I. Total Disability.
II. Fatal.
III. Permanent Partial.
IV. Combination of I, II and III.
Legal Limit Factor - F
1.60
1.40
1.20
1.00
.80
.60
.40
.20
0
$20
$40
$60
$80
$100
Average Wage - W
I
II
III
IV

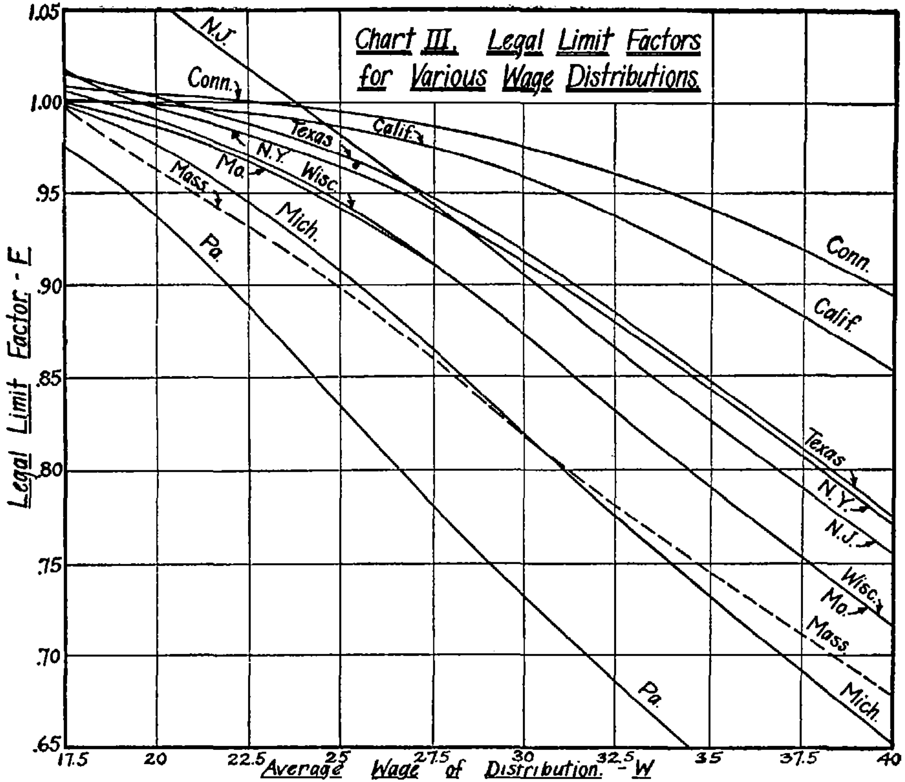
Chart III. Legal Limit Factors for Various Wage Distributions.
Legal Limit Factor - E
Average Wage of Distribution - W
1.05
1.00
.95
.90
.85
.80
.75
.70
.65
17.5
20
22.5
25
27.5
30
32.5
35
37.5
40
N.J.
Conn.
Texas
Calif.
N.Y.
Wisc.
Mo.
Mass.
Mich.
Pa.

CHART III.

Chart IV. Wage Distribution.