

OBSERVATIONS ON MAKING RATES FOR EXCESS  
COMPENSATION INSURANCE

BY

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Workmen's compensation is based on the concept that losses due to industrial accidents, in an economic sense, should be regarded in the same way as losses resulting from depreciation or breakage of machinery and considered as a part of the cost of production. Under the Compensation laws the employer directly assumes the industrial accident losses and transmits them to the consumer through an increase in the price of the employer's products.

This situation leaves the employer in a state of uncertainty, for he may have no accidents and thus obtain a greater gain from the increased price of his products, he may have accidents so costly as to bring on insolvency, or he may have accidents of some intermediate status. In addition to his uncertainty as to the cost of accidents, the employer may be in need of special legal counsel, the aid of experienced adjusters, the advice of medical specialists, the cooperation of safety engineers to reduce the accidents to a minimum, or the service of a statistical department for issuing payments to the injured, keeping records, compiling reports and administering the necessary funds to assure payments for disabilities.

## FUNCTION OF COMPENSATION INSURANCE CARRIERS

It is the function of compensation insurance carriers to relieve the employer of the uncertainty as to the cost of accidents by assuming his legal responsibility for all accidents covered by the compensation law, and to furnish the employer the services that may be required to administer the law on the most efficient economic basis.

Employers are qualified in different degrees to assume the financial obligations placed directly upon them by the compensation acts and to provide the services necessary for administering the act most efficiently. Most employers desire and need to be fully protected against all liabilities incurred. Relatively few may desire protection only against accidents covered by

certain types of benefits, and still fewer desire protection only against extreme fluctuations in the annual losses. In the efforts of the carriers to supply the desires and needs of the employers in the several states different forms of coverage have been developed.

### CLASSIFICATION OF COVERAGES

For purposes of classification, with respect to the inclusiveness of compensation accidents, these coverages may be divided as follows:

I. *Full Coverage.* Under this coverage all obligations to the injured employees which the compensation law imposes upon the employer are assumed by the carrier.

II. *Partial Coverage.* The employer may chose to retain some of the obligations and to insure the others. For such employers these general forms of partial coverage are available.

1. *Coverage for a Fixed Percentage of Each Loss.* Under this form of coverage the assured retains a definitely fixed percentage of every loss and insures the rest. This form of coverage is known as *Co-Insurance*, and is mentioned merely for completeness of classification as it is of no real importance in compensation insurance.

2. *Coverage for Certain Types of Benefits.* The assured under this form of coverage retains the obligations for certain types of benefits and insures the obligations for the other types. The most common form of coverage under this division is *Ex-Medical Insurance* in which the assured assumes the medical losses and insures the indemnity losses.

3. *Coverage for Losses in Excess of a Fixed Limit per Accident.* This form of insurance is commonly known as *Excess Insurance* or *Deductible Average Insurance*. The assured under this coverage retains his obligations up to a fixed amount for each accident and insures all losses in excess of this fixed amount. If this fixed limit is high, *e. g.*, \$5,000 or more, this is known as *Excess Insurance*. If the limit is low, *e. g.*, \$500 or less, it is called *Deductible Average Insurance*. This form of insurance will be referred to hereafter as *Excess Insurance per Accident*.

4. *Coverage for Losses in Excess of a Fixed Percent of the Risk Premium.* Under this form of coverage the assured retains his obligations for all losses up to a fixed percentage of his premium at manual rates or merit-rated rates, and

insures all losses in excess of this fixed percentage. This form of excess insurance is rare and has not yet acquired a generally accepted name. It has been referred to as *Aggregate Stop Loss Ratio Insurance* and *Excess Loss Ratio Insurance*. In this discussion this form of excess insurance will be designated as *Excess Insurance per Loss Ratio*.

III. *No Coverage*. The assured may retain for himself all of his obligations to the injured employees. This case in which there is no coverage extended by a carrier is generally known as *Self-Insurance*. It is not included in the general topic of discussion and is mentioned for the sake of completeness of classification.

#### GRAPHICAL ILLUSTRATION

The types of coverage just classified may be illustrated by graphic forms. In Fig. 1 the accidents of a typical risk of larger size are represented. Each rectangular column represents a loss due to an accident of the type of benefit specified. The horizontal lines denote the fixed amounts indicated at the left. The larger rectangle at the right denotes the risk premium and the shaded portion of the rectangle denotes the aggregate risk losses. The numbers at the right of the rectangle indicate the loss ratio scale.

Under insurance for *Full Coverage* all of the losses represented by the individual rectangular columns are assumed by the carrier and under *No Coverage* or *Self-Insurance* the employer retains all the losses. In forms of insurance which extend partial coverage the losses are shared by the assured and the carrier. Under *Co-Insurance* each loss is split vertically and the same fixed percentage of every loss is retained by the assured, and the rest is assumed by the carrier. If *Ex-Medical* Insurance or any coverage by kind of injury is used the losses of specified types of benefits are retained by the assured and the losses of the other types of benefits are assumed by the carrier. If coverage is given for *Excess Insurance per Accident* the larger losses are split by the horizontal lines representing the fixed limit per accident. The portion above the line is assumed by the carrier and the lower portion and all small losses are retained by the assured.

The form of coverage designated as *Excess Insurance per Loss Ratio* may be represented by the large rectangle at the right of

the figure. Losses above the horizontal line representing the specific loss ratio used in the insurance contract are assumed by the carrier and those below the line are retained by the assured.

If the meaning of the rectangular columns is extended to denote the total loss due to the accident rather than the part covered by the compensation act then the graphic illustration of accidents—Fig. 1—may be used to indicate the share of the industrial hazard assumed by the employee. The risk of the employee involves the loss of his total wages from the date of accident until recovery and the cost of medical treatments. If his employment was not covered by the act, and he insured his industrial risk by taking out a policy similar to the compensation policy he would have *Co-Insurance*, for only 50% to 66⅔% of his wages are generally covered by compensation insurance. He would also have *Deductible Average Insurance* because he would bear his loss of wage during the waiting period. Generally he would carry his own *Excess Insurance per Accident* because the period during which he receives benefits or the total amount of benefits are usually limited in compensation acts. He would in some cases also have coverage by type of injury for in some of the states he is obliged to assume his own medical costs and is not covered for losses on account of death or industrial disease.

#### PURE PREMIUM RELATIONSHIPS

It will be assumed in this discussion that for *Full Coverage* insurance the present rate making procedure, inclusive of the application of the merit rating plans, produces rates for the manual classifications and the individual risks which are substantially correct, not only as a whole but for each of the component parts. Under this assumption the classification pure premium for any one of the types of partial coverage enumerated may be considered to be fully determined when its relation to the full coverage pure premium has been definitely established. This ratio of the pure premium under partial coverage to the pure premium under full coverage will be known as the *Pure Premium Ratio*. As there are at present no adequate available data developed under partial coverage, it is necessary to determine the pure premiums for partial coverage from experience developed under full coverage. In this procedure there is a further tacit

assumption that experience developed under partial coverage is approximately the same as the analogous part of the experience developed under full coverage.

It is obvious that under *Co-Insurance* the pure premium for the portion insured by the carrier bears the same ratio to the full coverage pure premium that the portion of insured losses bears to the total losses. It is also evident that under a form of insurance covering certain types of benefits only, *e. g., Ex-Medical Insurance*, the pure premiums for the partial coverage may be determined from the relativity of the pure premiums of the component parts entering into the full coverage rates.

#### EXCESS INSURANCE PER ACCIDENT .

The Actuarial Committee of the National Bureau of Casualty and Surety Underwriters in 1920 had referred to it the problem of providing state rates for insuring compensation losses in excess of a specified limit per accident. At that time compensation experience for policy years 1916 and 1917 was the latest available. Carriers generally had not yet started keeping separate records of losses due to catastrophes. Rates based on data of compensation catastrophes were definitely out of consideration.

The Committee decided to build a frequency distribution of catastrophes based on reports of the United States Bureau of Mines for Pennsylvania anthracite coal mining. These reports give the total deaths and the deaths in every catastrophe involving five or more cases. The number of accidents involving 2, 3 or 4 deaths were supplied on a pro rata basis from Pennsylvania compensation reports. The data for the period 1880-1919 were tabulated for a catastrophe frequency distribution. In graduating this distribution the number of catastrophes and the number of fatal cases involved were preserved.

It was decided to establish excess rates for all classifications, except those which had a specific loading in the pure premiums for an inherent catastrophe hazard, on the basis of the relativity of the catastrophe deaths and the normal deaths in anthracite coal mining. It was assumed that the permanent total and fatal cases had the same relativity in catastrophes as in normal accidents. To provide for the cost of injuries other than death

and permanent total cases the variable loading factor given in column 5, Table I was added.

The classifications were divided, by schedules, into five groups on the basis of the ratio of the D & P T D partial pure premium to the total pure premium. This ratio which is designated by  $a$  was determined for each group of classifications in each state. There also was determined for each group, if possible, otherwise for the state as a whole, the average of the D & P T D losses per fatal case which is represented by the symbol  $A$ , and the ratio of the excess cost of individual injuries to the total D & P T D cost which is designated by  $b$ .

The pure premium ratios were determined for all limits in one tabular calculation for each of the five groups into which the industries of the states had been divided. The method for an individual case of the most general form is shown in the example following:

Let

$$a = \frac{\text{D \& PTD Losses}}{\text{Total Losses}}, \text{ for each classification group.}$$

$$A = \frac{\text{D \& PTD Losses}}{\text{No. of Fatals}}, \text{ for each group, or for state if group experience is not available or inadequate.}$$

$$l_1 = \text{the excess limit over which the losses are covered.}$$

$$b_{i_1} = \frac{\text{Losses on Individual Cases in excess of Limit } l_1}{\text{Total D \& PTD Losses}}, \text{ for whole state.}$$

$$R_{i_1} = \text{Pure Premium Ratio for Limit } l_1, \text{ for classification group.}$$

$$e = \text{Expected Loss Factor.}$$

$$f = \text{Portion of Manual rate, used as flat loading.}$$

$$g = \text{Portion of Excess rate, used as pro rata loading.}$$

Find  $n$  so that

$$(n - 1) A \leq l_1 < n \cdot A$$

Use this value of  $n$  to find  $N_n$ ,  $T_n$  and  $T'_n$  in Table I.

Substitute in the formula

$$R_{i_1} = a \cdot b_{i_1} \cdot \frac{T_n}{18976} + \left( \overset{1.0210}{x} - \frac{T'_n \cdot A + N_n \cdot l_1}{18976 A} \right) a$$

$$= a \left( \overset{1.0210}{x} + b_{i_1} \frac{T_n}{18976} - \frac{T'_n \cdot A + N_n \cdot l_1}{18976 A} \right)$$

$$\text{Percentage Rate} = 100 (e R_{i_1} + f) \div (1 - g)$$

## NEW YORK EXCESS RATES—1926

In 1926 the Actuarial Committee of the New York Compensation Inspection Rating Board and the Actuarial Committee of the National Council were asked to prepare rates for excess insurance. It was decided to divide the classifications of industries into groups on the basis of the catastrophe hazard and to make use of the catastrophe experience developed under compensation acts as far as possible. The Engineering Committee of the National Council prepared a grouping of the industrial classifications according to the inherent catastrophe hazard. On the basis of engineering judgment the classifications were assigned to these four groups.

- Group I —Serious inherent catastrophe hazard.
- Group II —Moderate inherent catastrophe hazard.
- Group III —Slight inherent catastrophe hazard.
- Group IV —No inherent catastrophe hazard.

The Committee had available an exhibit showing a compilation of a country-wide experience of accidents involving two or more serious cases for policy years 1922 and 1923. This exhibit gave, by classifications, the number of serious cases in excess of 1, 2, and 3 for any single accident, and for the classifications developing catastrophes, the total number of serious cases exclusive of catastrophes. The summary of this exhibit follows:

Catastrophe Group	Serious Cases Exclusive of Catastrophe	Total Number of Cases in Excess of $x$ on any single accident		
		$x = 1$	$x = 2$	$x = 3$
(1)	(2)	(3)	(4)	(5)
I	993	100	74	59
II and III	3,987	72	25	17
IV	3,213	58	23	6
Total	8,193	230	122	82

These data are admittedly too limited to have their indications accepted without modification by judgment. The Actuarial Committee of the New York Compensation Inspection Rating Board, on the basis of these data, supplemented by judgment, proceeded to determine a rate for covering losses in excess of \$10,000 per accident in New York. The judgment interjected represents the composite opinion of the Committee. It is prob-

able that no member agrees with every item of judgment entering into the calculation.

It was decided to reduce the number of catastrophe groups from four to three by combining Group II and Group III. The determination of the excess rate for a \$10,000 limit is shown somewhat in detail in Table II. Items of the service and expense loading factor were split on a judgment basis into a flat loading or fixed percentage of the manual rate, which represents the part proportional to the number of risks, and a pro rata loading which represents the part proportional to the excess rate, as follows:

Flat Loading	Pro Rata Loading
Inspection.....2.5%	Acquisition.....17.5%
Audit.....2.0	Taxes.....2.5
Administration.....3.75	Administration.....3.75
	Claims.....8.00
<u>8.25%</u>	<u>31.75%</u>

To obtain rates for losses in excess of \$25,000 per accident the Committee used the data of the anthracite coal mine catastrophe distribution. These data when subjected to the procedure followed in Table II indicated, for the three groups combined, an average rate of 14% for the \$25,000 limit. It was the belief that the adverse selection factor of 1.20—line 4, Table II,—was not needed for this high limit and that there should be a reduction in expense factors. The committee adopted an average rate of 10% for the combined groups. Judgment differential rates of 11%, 10% and 9% were assigned to the respective groups. The rates for excess limits of \$15,000 and \$20,000 per accident were then interpolated.

The percentages adopted for the \$10,000 excess limit in Table II are averages based on the experience of the whole group. It was the Committee's decision that the actual classification ratio of the serious pure premium to the total pure premium should be used in calculating the \$10,000 excess limit rates for each individual classification, instead of the group average shown in line 5, Table II. The rates for the higher excess limits were then obtained for each classification, by maintaining the relativity of the average group rates. In the final exhibit of rates recommended, each classification is given an excess rate which is expressed as a percentage of the full coverage manual rate.



## WISCONSIN EXCESS RATES—1926

The National Council Staff and Committees cooperated with the Wisconsin Compensation Rating and Inspection Bureau in developing rates and rules for excess insurance in Wisconsin. The maximum amount of compensation possible for a single injury under the Wisconsin act is limited so that single injury accidents could have no effect on excess cost over a \$20,000 limit per accident. The cost of a single injury in Wisconsin also is practically negligible on excess costs for limits of \$15,000 and \$10,000 per accident. This condition simplified the calculation of excess rates in Wisconsin.

The National Council using compensation catastrophe data and following the general procedure shown in Table II, but eliminating the effect of single injuries and using a different expense loading, determined these average rates for all classifications combined.

Excess Limit per Accident	Percentage Rate
\$10,000	10.0%
15,000	8.5
20,000	7.5

The items of the service and expense loading used for Wisconsin were divided as follows:

	Flat % of Manual Rate	Pro Rata % of Excess Rate
Inspection.....	1.0%	1.0%
Payroll Audit.....	1.0	1.0
Home Office.....	1.5	4.5
Claim.....	..	7.0
Taxes.....	..	3.5
Acquisition.....	..	17.5
	<u>3.5%</u>	<u>34.5%</u>

The classifications were divided, on the basis of engineering and underwriting judgment, into five groups according to catastrophe hazard. Judgment differentials for the five groups were established and minimum group rates for the \$10,000 limit were determined so that when applied to all classifications, except clerical office, premiums equal to 10% of the manual premium were produced. The minimum group rates for the \$15,000 limit and the \$20,000 limit were then determined by taking

respectively 85% and 75% of the minimum group rates for the \$10,000 limit. A flat rate of 2c. was selected for classifications covering clerical office force. The Wisconsin minimum rates which are expressed in monetary units and which are applied directly to the payroll are:

	Excess Rate for Limits of		
	\$10,000	\$15,000	\$20,000
Group 1.....	\$1.00	\$.85	\$.75
Group 2.....	.78	.66	.59
Group 3.....	.56	.48	.42
Group 4.....	.34	.29	.26
Group 5.....	.12	.10	.09

It is proposed to add to these minimum rates certain charges for unusual catastrophe hazards. These charges are to be added to the minimum rates irrespective of the classification grouping. The charges which necessarily are based on judgment are:

	Flat Charges for Excess Limits of		
	\$10,00	\$15,000	\$20,000
First Unusual Hazard.....	\$.20	\$.16	\$.12
Second " ".....	.10	.08	.06
Third " ".....	.03	.02	.01
Fourth " ".....	.03	.02	.01
Fifth " ".....	.03	.02	.01
Sixth " ".....	.03	.02	.01

EXCESS INSURANCE PER LOSS RATIO

This type of Excess Insurance has had a very limited usage. Under this coverage the carrier agrees to assume all losses sustained on a risk in excess of a definite percentage of the premium based on either manual rates or rates adjusted for the particular risk by the application of merit rating. This form of excess coverage which insures the stability of the loss ratio resulting from the aggregate risk losses introduces the size of the risk as a new factor to be considered in the determination of pure premium ratios. It is evident that in the very small risks there will be most likely a clear experience. If there happen to be any losses the loss ratio will probably be high, e. g., 100% or more. At the other extreme, for large risks, there will be few, if any, risks with-

out losses and few, if any, with loss ratios exceeding 100%. From purely theoretical considerations it may be demonstrated that there will be definite changes in the frequency distribution of risk loss ratios when these are grouped according to the size of the risk premium.

In Table III a tabulation of the distribution of loss ratios of risks by premium groups has been made for 24,838 compensation risks for New York state, for policy years 1924 and 1925. This tabulation excludes minimum premium risks and risks whose governing classifications have a per capita premium basis. The figures in the body of Table III denote the number of risks belonging to the premium group indicated in the left hand column and to the loss ratio group indicated by the column heading.

#### CONTOUR MAP

Consider a rectangular field whose ordinates are divided on a percentage scale and whose abscissas are divided into premium groups on a scale proportional to the total losses within the group. From the data in Table III, for a given loss ratio, plot points on the median line of each group so as to divide the line in the ratio that the number of risks having loss ratios equal to or less than the given loss ratio bears to the total number of risks in the group. Plot similar points for several such arbitrarily selected loss ratios. If curves are fitted freely to the points of the different premium groups representing equal loss ratios there will result a plat such as is roughly sketched in Fig. 2.

The sketch in Fig. 2 may be considered a sort of limiting form approached by lines joining the plotted points, when the number of premium groups is indefinitely increased and the volume of experience is indefinitely large. The sketch is really a contour map of risks arranged first into ascending premium groups from left to right, and then arrayed according to size of loss ratio within each group.

#### THREE-DIMENSIONAL MODEL

To visualize a three-dimensional model constructed from this contour map imagine the part above each curve in the map as extended perpendicularly to the plane of the map on a scale of

one inch for each 100% of loss ratio. This process would form a model with sharp ridges. To overcome this defect suppose the number of contour lines on the map were indefinitely increased then there would result a model with a smooth curved surface somewhat as sketched in Fig. 3.

A model formed as just described would be constructed according to scale and from it many of the pure premium relations could be visualized and roughly determined. The part of its volume above the horizontal plane  $a$  inches above the base would represent to scale the losses in excess of a loss ratio of 100  $a$ %. The ratio of the volume above the plane to the total volume of the model would represent the pure premium ratio. From the model it can be visually ascertained that the losses in excess of the larger loss ratios are due entirely to the small premium group risks. To obtain relations which are representative of a whole group of risks it is necessary to divide the model by planes parallel to the  $Y O Z$ -plane. Within each of these sections of the model which now represent the premium group of risks the ratio of the volume above the horizontal plane to the volume of the whole section would represent the pure premium ratio of the group for the particular loss ratio corresponding to the horizontal plane.

#### CALCULATION OF PURE PREMIUM RATIOS

In determining the *Pure Premium Ratio* which will be defined as the ratio of the risk losses in excess of a specified loss ratio to the total risk losses, it will be necessary to make certain assumptions. It will be assumed that the ratio of the aggregate of the risks of a premium group approximately represents the ratio for the risks of average premium within the group, or with a lesser degree of accuracy it may be assumed that the ratio for the group represents the ratio for each risk within the group.

The pure premium ratios for *Excess Insurance per Loss Ratio* for each of any number of specified loss ratios may be determined in one tabular calculation for each premium group by the procedure followed in Table IV. The data in columns 1 and 2 of this table are taken from Table III. The rest of the process with the aid of the column headings is believed to be self-explanatory.

A general test as to accuracy may be applied to the calculation

in Table V. The sum of the individual loss ratios in column 3 may be applied to the average risk premium of the group. The product should equal approximately the total losses of the group. The close agreement found in this particular case is not to be expected generally. In passing judgment on the adequacy of these data it should be noted that while there are 2202 risks in the group, 1649 of these have no losses whatever and the losses of the last seven risks listed in the table account for one-third of the total. The very high loss ratio of the last risk group is larger than is to be expected in this volume of experience, and in part accounts for the lack of greater uniformity in the trend of the pure premium ratios in Table V.

If similar calculations of pure premium ratios are made for each of the other premium groups and if the ratios of column 10 of Table IV and the corresponding ratios of other groups are tabulated according to the premium group and the selected risk loss ratio there will result the tabulation which has been designated Table V.

From the different proportions in which the various types of injuries, each of which has probably a definite accident frequency distribution, enter into the classifications it would seem natural to infer that the different classifications or industries would produce different pure premium ratios for *Excess Insurance per Loss Ratio*. To test the validity of such an inference the risks for the Manufacturing Industry (schedule 5-25), the Contracting Industry (schedules 26-27), the Commercial Industry (schedule 34), and the Care, Custody and Maintenance of Buildings (schedule 36) were segregated. Pure premium ratios for each of these industries were then calculated. Only the lower premium groups were used as the data when divided by industries are entirely too limited for the upper groups. The results of these calculations are shown in Table V-a.

One would expect to derive less stable loss ratios and correspondingly higher pure premium ratios from the individual risk experience of those industries in which the serious accidents are relatively more frequent. In connection with the different parts of Table V-a it is interesting to consider the percentage distribution of losses within the different industries as shown in the following table which has been compiled from the country-wide

experience of policy years 1918-1922 brought to the basis of the 1925 New York Law.

Kind of Loss	Percentage Distribution of Losses by Industries				
	Manu- facturing	Contract- ing	Com- mercial	Care and Custody	All
Serious . . . . .	34%	44%	34%	38%	38%
Non-Serious . . . . .	41	37	40	38	39
Medical . . . . .	25	19	26	24	23
Total . . . . .	100%	100%	100%	100%	100%

The parts of Table V-a do not have the same degree of smoothness that is found in Table V, particularly among the higher selected loss ratios. Considering the limited volume of experience this is to be expected. A similarity appears when comparing the level and trend of the pure premium ratios for the manufacturing industry with the commercial, and those for the contracting industry with the care, custody and maintenance of buildings. It would be premature to derive from these limited data the definite conclusion that the indicated differences in the pure premium ratios in Table V-a are due to causes inherent in the industry.

SERVICE AND EXPENSE LOADING

The loading factor to be applied to this form of insurance depends largely on the service the assured desires or needs and on the limits specified in the policies. If the assured desires full claim, legal and safety engineering service there should be the same service and expense loading as provided under the full coverage. Generally the assured desires excess insurance because of his belief in the inherent merit of his risk and in the efficiency of his organization in rendering the necessary services. For its own protection against adverse selection of risks it is necessary that the carrier render some service the extent of which would depend on the nature of the risk and the size of the excess limits.

There is general agreement on the principle that a part of some of these loading items should be charged flat and the rest of the same items should be charged on a pro rata basis. There is no general agreement however, as yet, as to how much of the split items should be charged on the flat and how much on the pro

rata basis. Actual instances of a division of these items for *Excess Insurance per Accident* occur in the recommendations of the Actuarial Committees and are shown in the New York rates in Table II and in the Wisconsin excess rates. The question also arises, particularly in *Excess Insurance per Loss Ratio*, whether there should be any variation in the expense loading with different excess limits. No general agreement has yet been reached on this question.

There is also the broader question as to the effect of excess insurance on the general expense loading for full coverage. There is a certain overhead cost which is in part fixed and which is now spread over the relatively large premiums derived from full coverage rates. It is conceivable that the writing of excess insurance might reduce the total premium volume to such an extent that it would be necessary to increase the present percentage loading for full coverage in order to provide for this fixed overhead.

#### TEST OF CORRECT RELATIVITY OF EXCESS INSURANCE RATES

The generally accepted aim of rate making is to produce such rates for individual risks that all risks become equally desirable when judged solely on the basis of realizing the expected loss ratio. The fact that underwriters when considering only the loss experience prefer one risk or set of risks over others would indicate that in their judgment the proper rate had not yet been determined for the individual risks. It is the problem of the rate makers to determine such rates for excess insurance that when considered entirely with reference to realizing the expected loss ratio there can be no adverse selection of risks on account of differences in any of these conditions:

1. The excess limit used in the policies
2. The industry to which the risk belongs
3. The loss experience of the individual risk
4. The size of the risk.

#### EXCESS INSURANCE IN COMPENSATION AND LIABILITY LINES

The problem of rate making for excess insurance under compensation coverage is simpler than the rate making problem for excess insurance of other liability lines involving personal injury.

The cost of compensation for specific injuries within a state has become practically standardized through the compensation law. Compensation laws have no restrictions on the total amount an accident may cost aside from the prescribed cost for individual injuries. The variable necessary to consider in *Excess Insurance per Accident* is the accident cost frequency distribution and the variable necessary to consider in *Excess Insurance per Loss Ratio* is the loss ratio frequency distribution.

In Liability insurance involving personal injury there is little, if any, standardization of cost for specific accidents for a state. Verdicts secured for specific accidents which are almost identical in character and which occurred under very similar conditions will show wide variations in cost. Under these conditions the costs of accidents in liability insurance have a much larger spread. This results in frequency distributions of greater standard deviations for both the individual accidents which are used in *Excess Insurance per Accident* and for the aggregate accidents of a risk which are used in *Excess Insurance per Loss Ratio*. This characteristic of the frequency distribution indicates definitely that the pure premium ratios of either type of excess insurance, for limits which have the same relativity to the average cost, are greater under liability insurance than under compensation insurance.



TABLE I  
 SHOWING DISTRIBUTION OF FATAL CASES FOR  
 PENNSYLVANIA ANTHRACITE FOR YEARS 1880-1919, AND  
 AUXILIARY COLUMNS FOR DETERMINING PURE  
 PREMIUM RATIOS

FATALS per Accident $n$	No. of Accidents		Total FATALS	Factor for P. P. & T. T.	Cost (4) $\times$ (5)	$\Sigma$ (3) up $N_n$	$\Sigma$ (4) down $T_n$	$\Sigma$ (6) down $T'_n$
	Actual	Revised						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1			16,445	1.00	16,445			
2	(600)	600	1,200	1.20	1,440	836	16,445	16,445
3	(120)	120	360	1.18	425	236	17,645	17,885
4	(40)	40	160	1.16	186	116	18,005	18,310
5	19	20	100	1.14	114	76	18,165	18,496
6	15	15	90	1.12	101	56	18,265	18,610
7	12	11	77	1.10	85	41	18,355	18,711
8	3	7	56	1.09	61	30	18,432	18,796
9	6	5	45	1.08	49	23	18,488	18,857
10	8	4	40	1.07	43	18	18,533	18,906
11	..	3	33	1.06	35	14	18,573	18,949
12	3	2	24	1.05	25	11	18,606	18,984
13	4	1	13	1.05	14	9	18,630	19,009
15	1	1	15	1.05	16	8	18,643	19,023
17	..	1	17	1.05	18	7	18,658	19,039
19	1	..	..	..	..	6	18,675	19,057
20	..	1	20	1.05	21	6	18,675	19,057
25	..	1	25	1.05	26	5	18,695	19,078
28	2	..	..	..	..	4	18,720	19,104
34	..	1	34	1.05	36	4	18,720	19,104
58	1	1	58	1.05	61	3	18,754	19,140
72	1	1	72	1.05	76	2	18,812	19,201
92	1	1	92	1.05	97	1	18,884	19,277
Total	836	836	18,976				18,976	19,374

TABLE II  
SHOWING DETERMINATION OF EXCESS RATES FOR LOSSES  
IN EXCESS OF \$10,000 PER ACCIDENT

Line	Item	Classification Hazard Group		
		I Serious	II Moderate	III All Other
(1)	(2)	(3)	(4)	(5)
1.	Ratio Excess Cost on Single Injuries to Cost of Serious Injuries.....	.150	.153	.108
2.	Ratio Excess Cost on Catastrophes to cost of Serious Injuries	.085	.062	.005
3.	Ratio Excess Cost to Cost of Serious Injuries. Lines (1) + (2).....	.235	.184	.111
4.	Loading for Adverse Selection and Lack of Control. $1.20 \times (3)$ .....	.282	.221	.133
5.	Ratio Serious Pure Premium to Total Pure Premium.....	.459	.379	.342
6.	Ratio Excess Cost to Total Pure Premium. $(4) \times (5)$ .....	.129	.084	.045
7.	Ratio Excess Cost to Manual Rate. $.60 \times (6)$ .....	.078	.050	.027
8.	Add Flat Expense Loading of .0825 $(7) + .0825$ .....	.1605	.1325	.1095
9.	Loading for Pro Rata Expense $(8) \div (1 - .3175)$ .....	.235	.194	.160
10.	Per Cent of New York Manual Rate selected as average rate for excess of \$10,000 per accident.....	24%	19%	16%
11.	Percentage Rate selected for \$15,000 Excess Limit.....	17	15	13
12.	Percentage Rate selected for \$20,000 Excess Limit.....	13	12	11
13.	Percentage Rate selected for \$25,000 Excess Limit.....	11	10	9



**TABLE V**  
**PURE PREMIUM RATIOS FOR EXCESS INSURANCE PER LOSS RATIO.**  
 Table showing the Pure Premium Ratio—Ratio of Excess Cost to Total Cost—for Selected Risk Loss Ratios, by Premium Groups, for All Industries Combined.

Lower Limit Premium Group*	Number of Risks	Selected Risk Loss Ratios												
		00%	30%	50%	60%	70%	80%	100%	200%	500%	1000%	2000%	5000%	10000%
\$10	3,682	1.000	.978	.963	.957	.951	.945	.934	.890	.797	.709	.610	.471	.354
25	5,999	1.000	.947	.919	.907	.895	.884	.863	.779	.625	.482	.329	.144	.025
50	3,490	1.000	.919	.881	.864	.849	.834	.807	.698	.505	.353	.191	.037	
75	2,202	1.000	.929	.895	.880	.866	.852	.828	.737	.587	.451	.302	.184	.079
100	2,672	1.000	.911	.870	.851	.836	.821	.794	.698	.529	.377	.222	.073	
150	1,471	1.000	.874	.820	.797	.775	.756	.722	.598	.387	.234	.097	.012	
200	1,657	1.000	.842	.779	.752	.727	.704	.664	.529	.317	.158	.042		
300	940	1.000	.852	.790	.765	.742	.720	.682	.544	.321	.145	.051		
400	539	1.000	.861	.805	.783	.762	.752	.705	.572	.370	.220	.101		
500	629	1.000	.804	.732	.703	.677	.652	.608	.459	.252	.108	.016		
700	479	1.000	.734	.637	.600	.568	.541	.495	.341	.111	.026			
1,000	348	1.000	.705	.643	.605	.570	.540	.492	.341	.161	.036			
1,500	313	1.000	.667	.550	.506	.469	.436	.376	.196	.053	.003			
2,500	180	1.000	.666	.538	.490	.450	.414	.356	.200	.013				
4,000	141	1.000	.556	.386	.319	.266	.223	.163	.045					
8,000	61	1.000	.510	.318	.248	.204	.169	.132	.030					
16,000 and over	35	1.000	.460	.215	.137	.066	.028							

\*Each group extends from the given limit to the next.

TABLE IV  
SHOWING CALCULATION OF PURE PREMIUM RATIOS FOR PREMIUM GROUP \$75—\$99.  
ALL INDUSTRIES COMBINED, NEW YORK DATA FOR POLICY YEARS 1924 AND 1925, AS IN TABLE III

Average L. R. of Group	No. Risks in Group	Total Group L. R. (1) X (2)	Cost $\Sigma$ (3) down	Excess Cases $\Sigma$ (2) up	Excess L. R.	Cost Excess Cases (5) X (6)	Cost to Assureds (4) + (7)	(8) $\div$ (3a)	Pure Premium Ratio 1.00 - (9)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
90%	1,649	00	00	553	00	000	000	.0000	1.0000
05	100	500	500	453	10	4,530	50,03	.0282	.9718
15	75	1,125	1,625	378	20	7,560	9,185	.0514	.9486
25	50	1,250	2,875	328	30	9,840	12,715	.0712	.9288
35	26	910	3,785	302	40	12,080	15,865	.0888	.9112
45	26	1,170	4,955	276	50	13,800	18,755	.1050	.8950
55	20	1,100	6,055	256	60	15,360	21,415	.1199	.8801
65	12	780	6,835	244	70	17,080	23,915	.1339	.8661
75	12	900	7,735	232	80	18,560	26,295	.1472	.8528
85	13	1,105	8,840	219	90	19,710	28,550	.1598	.8402
95	11	1,045	9,885	208	100	20,800	30,685	.1718	.8282
105	14	1,470	11,355	194	110	21,340	32,695	.1830	.8170
115	10	1,150	12,505	184	120	22,080	34,585	.1936	.8064
125	6	750	13,255	178	130	23,140	36,395	.2037	.7963
135	13	1,755	15,010	165	140	23,100	38,110	.2133	.7867
145	6	870	15,880	159	150	23,850	39,730	.2224	.7776
160	14	2,240	18,120	145	175	25,375	43,495	.2435	.7565
185	9	1,665	19,785	136	200	27,200	46,985	.2630	.7370
250	44	11,000	30,785	92	300	27,600	58,385	.3268	.6732
350	15	5,250	36,035	77	400	30,800	66,835	.3741	.6259

TABLE IV—Continued

Average L. R. of Group	No. Risks in Group	Total Group L. R. (1) × (2)	Cost Σ (3) down	Excess Cases Σ (2) up	Excess L. R.	Cost Excess Cases (5) × (6)	Cost to Assureds (4) + (7)	(8) ÷ (3a)	Pure Premium Ratio 1.00 - (9)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
450	16	7,200	43,235	61	500	30,500	73,735	.4127	.5873
550	6	3,300	46,535	55	600	33,000	79,535	.4452	.5548
650	4	2,600	49,135	51	700	35,700	84,835	.4749	.5251
750	5	3,750	52,885	46	800	36,800	89,685	.5020	.4980
850	5	4,250	57,135	41	900	36,900	94,035	.5264	.4736
950	2	1,900	59,035	39	1,000	39,000	98,035	.5488	.4512
1,120	8	8,960	67,995	31	1,300	40,300	108,295	.6062	.3938
1,390	4	5,560	73,555	27	1,500	40,500	114,055	.6384	.3616
1,760	12	21,120	94,675	15	2,000	30,000	124,675	.6979	.3021
2,350	8	18,800	113,475	7	3,000	21,000	134,475	.7527	.2473
3,620	2	7,240	120,715	5	5,000	25,000	145,715	.8157	.1843
5,690	1	5,690	126,405	4	6,000	24,000	150,405	.8419	.1581
9,080	2	18,160	144,565	2	10,000	20,000	164,565	.9212	.0788
17,040	2	34,080	178,645				178,645	1.0000	.0000
Total	2,202	(a) 178,645	1,453,810	..	..	776,505	2,230,315	..	..

Check— { Average Risk Premium = \$86.10  
 Total Losses of Group = \$153,748  
 Test—1,786.45 × 86.10 = 153,813

TABLE V-a  
 PURE PREMIUM RATIOS FOR EXCESS INSURANCE PER LOSS RATIO.  
 Table showing the Pure Premium Ratio—Ratio of Excess Cost to Total Cost—for Selected Loss Ratios, by Premium Group  
 and by Industries.

Industry	Prem. Group*	No. of Risks	Selected Risk Loss Ratios											
			30%	50%	60%	70%	80%	100%	200%	500%	1000%	2000%	5000%	10000%
Manufacturing	\$10	1,045	.9511	.9204	.9061	.8929	.8803	.8567	.7588	.5558	.3707	.1997	.0034	.0157
	25	1,357	.9213	.8811	.8635	.8471	.8315	.8027	.6959	.5003	.3354	.1942	.1034	
	50	648	.9108	.8700	.8532	.8383	.8246	.7987	.6915	.4940	.3317	.1510		
	75	414	.8783	.8221	.7981	.7757	.7547	.7174	.5936	.3766	.1660	.0340		
	100	512	.8756	.8225	.8006	.7818	.7647	.7331	.6150	.4326	.2679	.0797		
	150	240	.8883	.8382	.8169	.7971	.7779	.7437	.6374	.4480	.2501	.0607		
	200	356	.7840	.6962	.6594	.6261	.5957	.5433	.3794	.1969	.0753			
	300	223	.8540	.7951	.7711	.7505	.7313	.6969	.5845	.3852	.2459	.1201		
	400	142	.7590	.6267	.5907	.5571	.5261	.4720	.3168	.1491	.0062			
	500	196	.8110	.7348	.7033	.6754	.6495	.6026	.4538	.2189	.0671			
	700	138	.5943	.4682	.4228	.3824	.3471	.2876	.1514	.0202				
1,000	..	..	..	..	..	..	..	..	..	..				
Contracting	\$10	424	.9908	.9852	.9826	.9799	.9772	.9719	.9514	.9056	.8694	.8185	.6940	.5667
	25	1,468	.9631	.9418	.9323	.9232	.9144	.8973	.8265	.6817	.5341	.3724	.1723	.0276
	50	1,028	.9352	.9022	.8880	.8747	.8618	.8369	.7331	.5707	.4522	.3085	.1166	
	75	675	.9400	.9105	.8974	.8853	.8736	.8513	.7625	.6214	.4833	.3012	.1785	.1034
	100	772	.9341	.9013	.8868	.8738	.8620	.8412	.7644	.6219	.4845	.3105	.1185	
	150	535	.8935	.8476	.8289	.8123	.7977	.7723	.6841	.5183	.3639	.2196	.0572	
	200	579	.8604	.7996	.7729	.7479	.7245	.6818	.5370	.2793	.1153	.0266		
	300	358	.8948	.8492	.8294	.8111	.7940	.7621	.6293	.3804	.1554	.0495		
	400	203	.9168	.8789	.8645	.8515	.8395	.8175	.7255	.5625	.3888	.1931		
	500	223	.7882	.7132	.6820	.6527	.6260	.5786	.4110	.2304	.0994			
	700	182	.7855	.6985	.6649	.6356	.6114	.5742	.4489	.2129	.0661			
1,000	..	..	..	..	..	..	..	..	..	..				

\*Each group extends from the given limit to the next.

TABLE V-a—Continued

Industry	Prem. Group*	No. of Risks	Selected Risk Loss Ratios											
			30%	50%	60%	70%	80%	100%	200%	500%	1000%	2000%	5000%	10000%
Commercial	\$10	996	.9685	.9507	.9426	.9349	.9280	.9153	.8648	.7566	.6612	.5483	.3967	.2273
	25	1412	.9456	.9174	.9049	.8931	.8819	.8613	.7808	.6473	.5360	.4097	.2231	.0557
	50	769	.8959	.8476	.8261	.8065	.7886	.7554	.6340	.4080	.2629	.1345	.0140	
	75	495	.9371	.9077	.8945	.8821	.8703	.8488	.7672	.6011	.4598	.3089	.1760	
	100	599	.8904	.8409	.8193	.7998	.7817	.7474	.6328	.4619	.3341	.2212	.1149	
	150	296	.7686	.6826	.6454	.6122	.5825	.5302	.3571	.1358	.0151			
	200	309	.7818	.7080	.6782	.6523	.6291	.5859	.4498	.2535	.0965			
	300	157	.6575	.5310	.4799	.4347	.3953	.3308	.1173					
	400	80	.8008	.7290	.7007	.6723	.6449	.5921	.3966	.1039				
	500	79	.6063	.5213	.4899	.4586	.4273	.3669	.1790					
	700	59	.6029	.5048	.4761	.4569	.4402	.4115	.2871	.0239				
1,000	..	..	..	..	..	..	..	..	..	..				
Care, Custody	\$10	515	.9856	.9775	.9737	.9700	.9664	.9592	.9317	.8664	.8066	.7415	.6130	.4680
	25	738	.9402	.9076	.8931	.8793	.8663	.8421	.7431	.5624	.3797	.2038	.0395	
	50	433	.9127	.8709	.8520	.8341	.8176	.7879	.6763	.4675	.3024	.1074		
	75	261	.9033	.8557	.8352	.8160	.7980	.7655	.6534	.5036	.3831	.2528	.0450	
	100	264	.8972	.8495	.8294	.8103	.7922	.7595	.6389	.3926	.2016	.0543		
	150	143	.8284	.7591	.7277	.6997	.6757	.6328	.4703	.2970	.1980	.0330		
	200	164	.8135	.7438	.7158	.6910	.6701	.6405	.5476	.3715	.2034	.0272		
	300	73	.7801	.7113	.6826	.6558	.6291	.5832	.4474	.2753	.0841			
	400	51	.8512	.7934	.7645	.7355	.7066	.6517	.4480	.0867				
	500	45	.8311	.7554	.7257	.7014	.6797	.6405	.5378	.3622	.1595			
	700	36	.7371	.6341	.5908	.5528	.5149	.4472	.1897					
1,000	..	..	..	..	..	..	..	..	..	..				

\*Each group extends from the given limit to the next.



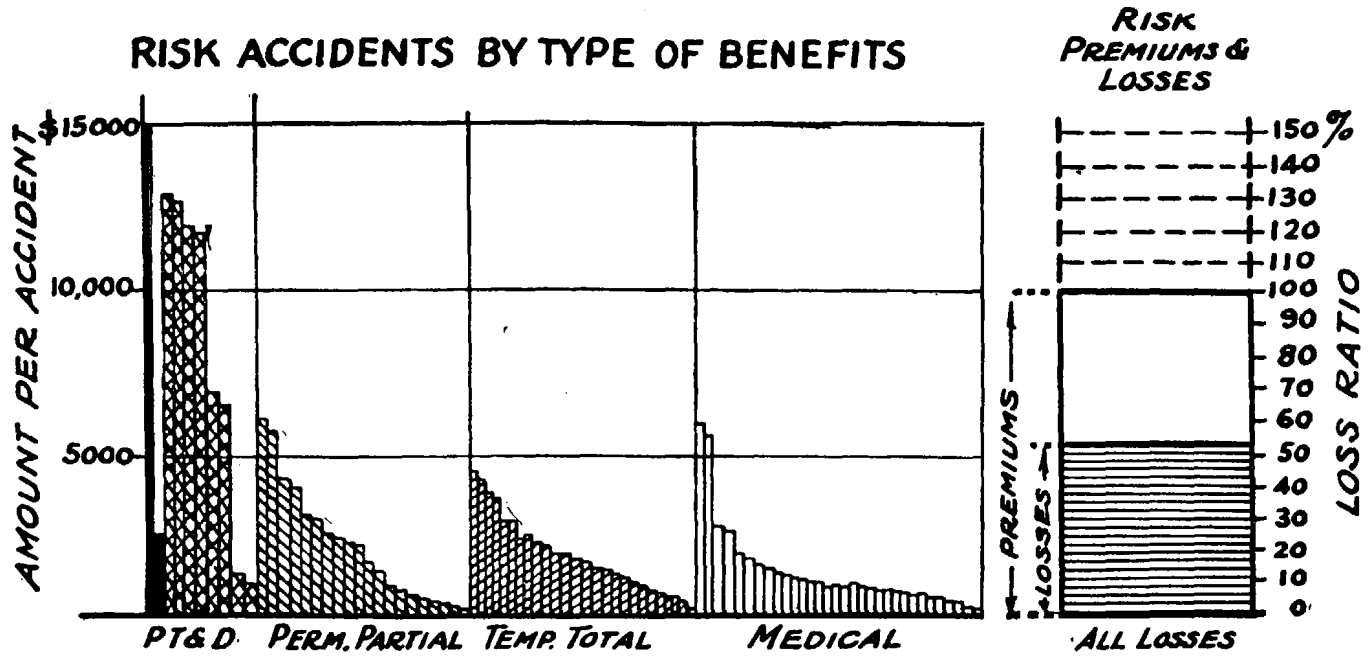


FIG. 1

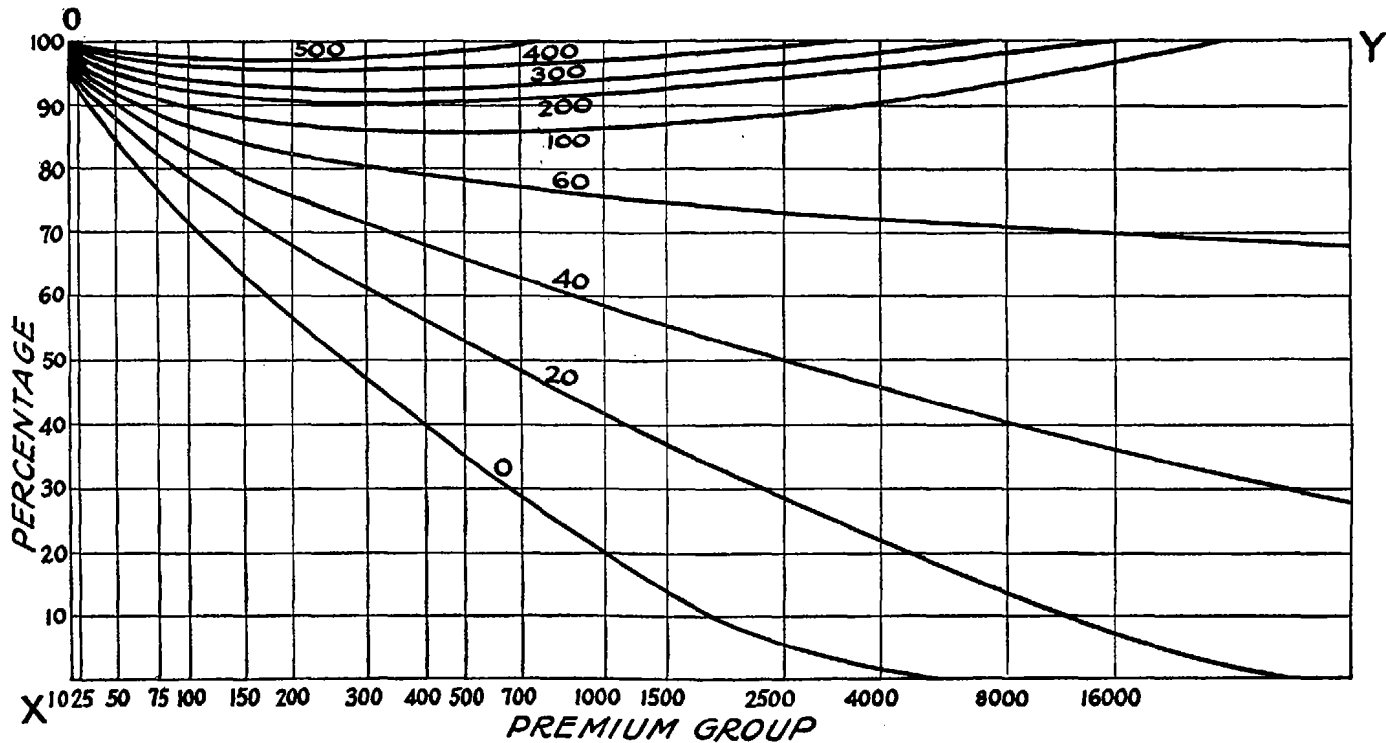


FIG. 2