REMARKS ON COMPENSATION DIFFERENTIALS

BY

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To understand fully the cost relation of two compensation acts it is necessary to consider in detail the individual parts which appreciably affect the amount of compensation awarded. As the present day compensation laws have so many elements which enter to a perceptible degree into the determination of the cost of the act it becomes highly desirable to have for expressing their cost relationship some simple expression which is representative of the whole law. It is the function of the compensation differential to serve in this way as an approximate measure of the cost relation of two compensation acts.

Differentials as Index Numbers

When classified as a part of the subject matter of Statistical Theory compensation differentials belong to the division known as index numbers. These are numbers used to express the relative magnitude of statistical groups or of aggregates of variables when considered as of different times, or as of different places, or as of both different times and places. A typical compensation law has in its scale of benefits many cost items which apply in varying combinations to individual accidents. It is the problem in compensation differentials to determine indices for the cost of industrial accidents when considered for different times in the same state, or for the same time in different states, or for different times in different states.

Compensation Differentials Defined

The general meaning given to the term "differential" in the past has involved the comparison of the compensation costs or cost levels of two states. The term has also been used to designate more specifically the flat factor necessary to modify the pure premiums of one state so that when these modified pure premiums are applied to the payrolls of another state that state's aggregate losses will be reproduced exactly. The meaning attached to the word in these remarks is not in conflict with these usages.

Considering any one specific compensation law with reference to another compensation law which has been selected as the standard or basic law the compensation differential of the specific law relative to the other as the basic law shall be defined as the specific law's compensation cost per unit hazard-weighted payroll exposure when expressed in terms of the same cost unit of the basic law. The general object is to compare, for the two laws, the cost per unit payroll exposure when these payrolls refer to the same classification or to classifications of equal hazard. When, as in the general case, the classifications are not all of equal hazard it is proposed to weight the payrolls according to the hazard severity of the classifications.

For this purpose let it be assumed that there is available a true relative measure of the hazard in classifications. This measure may be conceived to consist of the loss in time and expense per unit time exposure which society suffers on account of accidents occurring within the classification. The relative numbers obtained by applying such a measure to each classification would be used in both the specific state and the basic state for weighting the payrolls which would then become "hazard-weighted payrolls."

No set of relative numbers based on the "time and expense" loss per unit time exposure is available. However, the cost of a representative compensation law is presumably approximately proportional to the loss in time and expense to society on account of compensation accidents. The payroll exposure for classifications may also be considered approximately proportional to the time exposure. In place of the unavailable relative numbers desired for weights there may now be substituted a new set of approximately the same relativity which express for each classification the compensation cost per unit payroll exposure, or the state pure premium provided that the law and payrolls refer to the same state and that $100 be made the payroll unit. If the basic law be selected to measure the relative loss to society by classifications and if the combined payrolls be chosen to serve as a relative measure of the time exposure then the basic pure premiums have approximately the same relativity as the set of relative numbers assumed at the outset and may be used for weighting the payrolls according to their hazard severity.

This roundabout way of defining compensation differentials might have been accomplished more directly if the differential had been defined as a comparison of the compensation cost of the specific state with the expected losses developed by the same payroll exposure under the conditions obtained by extending the basic act and the industrial conditions of the basic act to all the states. The longer method was chosen because it was desired to have the original definition which is based on a hazard-weighted payroll exposure free from the differential idea. The later use of the basic pure premium is only as an approximation. The expected losses as used in the shorter method directly involve basic pure premiums which result from a procedure involving the use of differentials.

Classification of Differentials

According to some of the more recent usages compensation differentials may be divided into law differentials and experience differentials.

> *Law differential* is the term used to express the relationship of the per unit hazard-weighted payroll cost of two compensation acts insofar as these costs are due to causes which are inherent in the act itself or which arise out of any definite interpretations of the act. The term *theoretical differential* is also applied to the law differential as just defined.
>
> *Experience differential* has been used more recently to express the cost relationship of the experience developed under two compensation acts or under two different periods of the same act.

Each of these two kinds of differentials may be divided again on the basis of including the cost of all accidents covered by the act, or of only certain types of these accidents.

> *Flat differential* is the term applied to a differential if the costs of accidents of all parts of the law are included. The terms *single differential* and *general differential* have also been used in the past to refer to the flat differential as here defined.
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> *Partial differential* is the term applied to a differential which refers to only some of the parts into which the cost of a compensation law may be divided. The partial differential is a differential of analogous parts of two laws. Other terms used for this differential are *differential by parts* and *group differential.* The latter term has more recently been used to refer to the differential—flat or partial—of a group of classifications as distinguished from the differential of all classifications.

Law Differentials

In the development of compensation differentials the law differentials, using the term as previously defined, by their very nature preceded experience differentials which had to await the maturing of a state's experience. The law differential came in with the introduction of the compensation basic manual in 1914 and 1915. The varying benefit scales which appeared in the laws of the different states required some index like the differential to express their relative measure.

Stated in a general way the law differential* is calculated by evaluating the cost of accidents in some typical or standard† distribution in accordance with the scale of benefits in both the act for which the differential is being determined and the act used as standard or basic. It is assumed that the distribution of accidents is developed under equal time exposures in each of the two acts. As the accident distribution and time exposures used in the evaluations are identical they represent equal severity exposures.

The process of evaluation may be represented symbolically by formula 1 where the primed letters refer to the basic state, and the other letters refer to the state whose differential is being determined. The letter L refers to losses or costs of accidents in units of weekly wages for the nature of injury indicated in the subscripts: DC, fatal; PT, permanent total; MP, major permanent partial; PM, minor permanent partial; TT, temporary total; MC, medical. The letter w refers to average weekly wages, P to payroll, k to the relative weights used which are the same for both states, and D denotes the differential.

$$(1)\quad D = \frac{(L_{DC} + L_{PT} + L_{MP} + L_{PM} + L_{TT} + L_{MC})\, w \div Pk}{(L'_{DC} + L'_{PT} + L'_{MP} + L'_{PM} + L'_{TT} + L'_{MC})\, w' \div P'k}$$

$$(1a)\quad = \frac{L_{DC} + L_{PT} + L_{MP} + L_{PM} + L_{TT} + L_{MC}}{L'_{DC} + L'_{PT} + L'_{MP} + L'_{PM} + L'_{TT} + L'_{MC}},$$

$$\text{since } \frac{P}{w} = \frac{P'}{w'} = \text{time exposure.}$$

The types of accidents underlying the L terms of a state, in general, are differently affected by the various cost items in the

*Refer Rubinow, *Proceedings*, Vol. IV, pp. 8-44.

†Refer Rubinow, *Proceedings*, Vol. IV, pp. 19-24. Outwater, *Proceedings*, Vol VII, pp. 57-77.

scale of benefits given in the acts. Any particular accident distribution gives a definite relative weighting to these terms in both numerator and denominator which results in a definite differential. Should another accident distribution be used a new differential would result for the same law. This variation due to the accident distribution used restricts the use which can be made of a flat differential. It is only in a general way representative of the state as a whole and does not generally express the flat differentials for individual classifications. For this reason the flat differential has not been used in converting experience in general rate making since the 1917 Revision.

Partial law differentials are calculated by the same general methods as flat law differentials with the exception that only the losses of the particular type of injury for which the differential is desired, are used. In the following formula for a partial law differential L refers to the losses of the particular nature of injury covered by the differential. The other characters have the same significance as in the formula 1 for the flat differential.

$$D = \frac{L\,w \div Pk}{L'\,w' \div P'k} = \frac{L}{L'} \tag{2}$$

To a smaller degree the partial differential is still dependent on the distribution of the accidents falling within its particular nature of injury. Insofar as the distribution of accidents varies by nature of injury between classifications the partial differential is not truly representative. A larger number of loss divisions according to type of injury would tend to overcome this but it would also necessitate the use of many partial differentials in comparing laws and would materially add to the detailed work in conversion of experience.

There is one factor—the state wage scale—not inherent in the law which since 1917 has been introduced in calculating law differentials. Prior to 1917 the Massachusetts wage distribution was generally used in all states. This action was prompted by necessity as this wage distribution was the only one available at the time.

Experience Differentials

In the actual operation of a compensation law there are several factors forming no part of the law which affect the cost. The accident frequency, the administration of the compensation law, the state safety laws and their enforcement, the personnel of the employers and employees, all these with still others are factors which affect the cost of compensation. It is the experience differential which serves to determine the relative cost of the compensation act in actual operation regardless of the source from which any part of the cost may come.

Qualifications Desired in Experience Differentials

There are several qualifications of different degrees of importance which, if possible, it is desired to have in experience differentials. The value of the method of calculation may be rated according to the degree in which the resulting differentials possess these qualifications.

1. To reproduce experience in the aggregate.
2. To use all experience typical of territory.
3. To form a set of consistent differentials.
4. To be reasonably simple in application.
5. To weight exposure proportionally to hazard.

The first of these is the most important. No differential which would not reasonably reproduce the experience in the aggregate could be given a final qualifying rating. If only two states are concerned the aggregate experience may be exactly reproduced. In a general revision, however, where many states are involved there can only be approximate reproductions of actual experience. Assuming the necessary condition of approximately reproducing experience in the aggregate as applying to each of two methods which are equal otherwise except in the proportion of experience used then the method which makes use of all typical experience of a state is to be preferred.

In addition to the conversion of experience, differentials are used for the general comparison of laws. It is desirable that from a set of differentials on a given basic act the relation of any two of the set may be determined. In order that the results may be consistent the differentials between two states produced by the direct and inverse methods should be reciprocals. The fourth qualification is obvious and the fifth follows from the definition.

Bases of Experience Differentials

The assumption underlying the use of all compensation differentials is that the partial pure premiums of classifications when properly subdivided according to nature of injury have approximately a fixed relativity within states and that such variations as do occur are accidental fluctuations due largely to inadequate exposure. If there were ideal states where these assumed conditions were exactly realized then the ratio of the corresponding partial pure premiums of a classification in the two states would be constant and equal to the partial differential. Denoting the differential by D, the partial pure premium by p, and using primes to indicate the basic state and subscripts to refer to the classifications, the relation may be stated in the form of an equation.

$$(3) \qquad D = \frac{p_1}{p'_1} = \frac{p_2}{p'_2} = \frac{p_3}{p'_3} = \ldots \frac{p_n}{p'_n}$$

or

$$D = \frac{p_1 + p_2 + p_3 + \ldots p_n}{p'_1 + p'_2 + p'_3 + \ldots p'_n}$$

$$(4) \qquad D = \frac{\frac{1}{n}\,(p_1 + p_2 + p_3 + \ldots p_n)}{\frac{1}{n}\,(p'_1 + p'_2 + p'_3 + \ldots p'_n)}$$

$$(5) \qquad D = \frac{1}{n}\left(\frac{p_1}{p'_1} + \frac{p_2}{p'_2} + \frac{p_3}{p'_3} + \ldots \frac{p_n}{p'_n}\right)$$

In the development of compensation differentials many bases for experience differentials have been suggested. A smaller number have been discussed and tested and a still smaller number have been actually used. It is proposed to consider briefly only those which have been used either for general comparison of losses or for conversion purposes in rate revisions. The formulas given may be considered as applying either to flat differentials or partial differentials. In the case of flat differentials the characters should be considered as applying to all the experience and in the case of partial differentials as applying to only the experience under the nature of injury covered by the partial differential.

In the system of symbols used in the formulas the primed characters refer to the basic state, those not primed refer to the state for which the differential is being determined, L is used to denote the losses, P denotes the payrolls, p denotes state pure premiums, and Σ stands for the summation process.

1. Ratio of Average Values of Accidents.

$$(6) \qquad D = \frac{1}{n}\,\Sigma L \div \frac{1}{n'}\,\Sigma L', \text{ where } n = \text{number injured.}$$

The use of this basis for a differential whether applied to the losses of a single nature of injury or to those of a grouping of two or more of them assumes that the general accident frequency is the same in the two states. It is now generally recognized that at a time when compensation costs and pure premiums are increasing the average cost per injured may be decreasing due to the relatively greater increase in the accident frequency of the less severe accidents.

The use of this method would produce differentials that could hardly be expected to reproduce the losses in the aggregate nor would the experience used be given weight according to the hazard involved. Under this method all the typical experience of a state would be used, its application would be reasonably simple and the differentials produced would form a consistent set.

The use of average values and the number of cases in converting experience is not using an experience differential but rather a modified form of evaluating individual accidents of one state in accordance with the conditions in another state.

2. Ratio of Average State Pure Premiums.

$$(7) \qquad D = \frac{\Sigma L_x}{\Sigma P_x} \div \frac{\Sigma L'_x}{\Sigma P'_x}, \text{ where } x \text{ refers to classifications.}$$

On this basis the same relative distribution of industry in the two states is assumed. This generally is not true. This method might serve for determining the differentials between two periods of a particular state if it is known that no change in the general distribution of business has occurred in the interval. Under this method reasonably approximate reproductions of the total experience could hardly be expected in the general case. The experience also would not be weighted in proportion to the hazard. The method is reasonably simple in application, produces a consistent set of differentials and makes use of all the typical experience of the state.

3. Ratio of Means of Classification State Pure Premium.

$$D = \frac{1}{n} \Sigma p_x \div \frac{1}{n'} \Sigma p'_x \tag{8}$$

Formula **8** follows directly from the assumption underlying differentials—see formula **4.** The formula given is for the arithmetic mean. Each classification is weighted according to its pure premium regardless of the actual volume of exposure. Violent chance fluctuations due to limited exposures are unduly weighted. To overcome this objection the classes used might be restricted to those having large exposures in each of the two states. A variation of this method would consist in taking the mode or median instead of the arithmetic mean.

This method considered with respect to the arithmetic mean of a limited number of classifications would not reproduce the experience in the aggregate, would not weight the experience proportionally to the hazard nor would all the typical experience of the state be used. When restricted to the same classifications the direct and inverse processes would give differentials which are reciprocals and produce a consistent set. In case the pure premiums were already available, the method would be extremely simple in application. Even if these pure premiums had to be determined this method would be considered as qualifying with respect to simplicity in application.

4. Mean of Classification State Pure Premium Ratios.

$$D = \frac{1}{n} \Sigma \frac{p_x}{p'_x} \tag{9}$$

Formula **9** also follows directly from the assumption underlying differentials—see formula **5.** The formula is again for the arithmetic mean. Under this formula all classification having equal ratios are given equal weight regardless of volume of exposure. The extreme chance fluctuations which are probable in classifications of small exposure have undue influence. This objection might be overcome by restricting the classifications used to those having adequate exposure in each of the two states. The method might also be varied by taking the mode or median of the pure premium ratios in place of the arithmetic mean.

In general the aggregate experience would not be reproduced nor would all experience typical of the state be used. The differentials derived would not form a consistent set and the experience would not be weighted proportionally to the hazard. The only qualification reasonably met would be simplicity in application.

5. Comparison of Expected and Actual Losses.

$$D = \frac{1}{2}\left\{\frac{\Sigma P_x p'_x}{\Sigma L_x} + \frac{\Sigma L'_x}{\Sigma P'_x p_x}\right\} \qquad (10)$$

In formula **10** the mean of the differentials obtained by using each state's payroll exposure is used. This basis depending on state pure premiums like some of the preceding bases, gives undue influence to chance fluctuations in classifications of small exposure. To avoid these errors classifications of small exposure in either of the states might be omitted.

The differentials produced by this method have been considered as reasonably reproducing the experience and the method might be considered as qualifying regarding simplicity in application. On the modified basis all experience typical of the state would not be used. The differentials derived would not form a consistent set and the weighting would not be proportional to the hazard.

6. Greene's Formula.

$$D = \Sigma\left(\frac{P_x P'_x}{P_x + P'_x} p_x\right) \div \Sigma\left(\frac{P_x P'_x}{P_x + P'_x} p'_x\right) \qquad (11)$$

Formula **11** is designed to exactly reproduce the losses in the aggregate* between any two states. As given the formula is in its most general form. Under certain conditions imposed on formula **11** the preceding formulas **2, 3** and **5** become special cases†. This method is a comparison of state pure premiums when they are weighted by a function of the payrolls of both states. It is subject to chance fluctuations occurring in classifications of inadequate exposure.

In its actual use it has been modified and applied to a limited number of representative classifications of adequate exposure in each of the two states. Under these conditions when the work is organized its application becomes simpler than a cursory inspection would indicate. In results this method reasonably

*Refer Greene, *Proceedings*, Vol. VI, pp. 10-30.

†Refer Mowbray, *Proceedings*, Vol. VI, pp. 260-267.

reproduces the experience in the aggregate and might be considered reasonable as to simplicity in application. In its modified form it would not make use of all experience typical of the state, the direct and inverse process would not produce exact reciprocals and the experience would not be weighted according to hazard.

7. Comparison of State Ratios of Actual Losses to Expected Basic Losses.

(12)
$$D = \frac{\Sigma_1^n \Sigma_1^t L_x}{\Sigma_1^n \Sigma_1^t P_x \pi_x} \div \frac{\Sigma_1^{n'} \Sigma_1^{t} L'_x}{\Sigma_1^{n'} \Sigma_1^{t} P'_x \pi_x}$$

Where π refers to basic pure premium, Σ_1^t denotes the summation for all classifications (t = total) for policy year, Σ_1^n denotes the summation of policy years. This is the formula in its most general form. If n and n' each become unity the formula refers to one year only and assumes the form **12a** in which it is generally used when applied to a single policy year or to a broad aggregate of experience brought to the basis of a single policy year.

(12a)
$$D = \frac{\Sigma_1^t L_x}{\Sigma_1^t P_x \pi_x} \div \frac{\Sigma_1^t L_x'}{\Sigma_1^t P_x' \pi_x}$$

If in (**12**) the primes are dropped and the constant factor C is introduced in the denominators of the fractions in the second member of the equation, and if C be chosen so that $C\pi_x$ will be R_x, the manual rate, and if n be taken as 3 and n' as 5 then the expression becomes the experience differential known as the projection factor.

(12b)
$$D = \frac{\Sigma_1^3 \Sigma_1^t L_x}{\Sigma_1^3 \Sigma_1^t P_x R_x} \div \frac{\Sigma_1^5 \Sigma_1^t L_x}{\Sigma_1^5 \Sigma_1^t P_x R_x}$$

In this form the formula is a comparison of the three-year and five-year manual rate loss ratios or the present rate level projection factor.

This method weights the classifications relatively to the time exposure times the basic pure premiums which is a constant times the hazard severity. The direct and inverse processes produce differentials which are reciprocals. This produces a system of consistent differentials which have a fixed relationship. The division of the differential of one state by the differential of another state will give the differential of the first state with

ERRATA

Formulae on page 278, *Proceedings* No. 26, in paper entitled "Remarks on Compensation Differentials" by Paul Dorweiler should read as follows:

(12) $$D = \frac{\Sigma_1^n \, \Sigma_1^t \, L_x}{\Sigma_1^n \, \Sigma_1^t \, P_x \, \pi_x} \div \frac{\Sigma_1^{n'} \, \Sigma_1^{t'} \, L'_x}{\Sigma_1^{n'} \, \Sigma_1^{t'} \, P'_x \, \pi_x}$$

(12a) $$D = \frac{\Sigma_1^t \; L_x}{\Sigma_1^t \; P_x \, \pi_x} \div \frac{\Sigma_1^{t'} \, L'_x}{\Sigma_1^{t'} \, P'_x \, \pi_x}$$

(12b) $$D = \frac{\Sigma_1^3 \, \Sigma_1^t \; L_x}{\Sigma_1^3 \; \Sigma_1^t \; P_x \, R_x} \div \frac{\Sigma_1^5 \; \Sigma_1^t \; L_x}{\Sigma_1^5 \, \Sigma_1^t \; P_x \, R_x}$$

reference to the second as the basic state. Thus from a set of differentials based on a given basic state the differentials based on any one of the given states may be produced. All the experience except special state classifications, (a) rated classifications, and unassigned discontinued classifications, all of which have no basic pure premiums, may be used thus making the experience quite representative of the state.

The formula itself would indicate that very much preliminary work is required in order to calculate the set of differentials. This would be the situation if everything in connection with it had to be worked out anew. The present rate revisions have been so organized, however, that much of a preceding rate revision may be salvaged in calculating the differentials to be used in the following. In case it should be desired to calculate partial differentials for parts which are not separately shown in the basic pure premium it would be necessary to work out the new basic pure premiums for the part desired and the work would be very much increased.

Comparison of Law and Experience Differentials

The law differential calculation requires a very close analysis of the compensation act and its interpretation in each of the many detailed items affecting the cost. The experience calculation requires no knowledge of the compensation act.

The law differential calculation can be made at any time and without hindrance as to the location of the calculator. On this account it may be used to determine the costs of new laws and amendments by anyone without limitations as to place or time. The experience differential can only be made after the experience has fully developed and at such central sources as have available all the experience for the law and for the period desired. It is not adapted to calculations for new laws or amendments nor can it be used by individuals who do not have the experience of the central sources available.

By use of the law differential method the effect of any particular cost item of the law can be determined as *e. g.*, the waiting period or the weekly limits. It is also possible to determine the relative cost for sub-divisions of any nature of injury. The experience differential is of no value to determine the effect of any individual cost item and can only be used to determine differentials by nature of injury when the experience for this nature of injury is separated in the records.

The law differential can only measure the effect of the scale of benefits and wages. The many other factors entering to an important degree in the final cost of the law cannot be determined. The experience differential does measure the full effect of the cost regardless of its source and this is of greatest importance in general rate revisions.

History of Use of Differentials in Rate Revisions

In presenting in tabular form the compensation differentials used in the general rate revisions, the compensation law used as the basic act, the accident distributions, the wage distributions, and the experience period used for the basic pure premiums, it has been found convenient to divide the time since the introduction of compensation into four periods each corresponding approximately with the time elapsed between the beginning of the preparatory work for two consecutive rate revisions.

I. 1917 Revision Period, 1914-1918
- Basic Act, 1912 Massachusetts Law
- Experience Period, Policy Years 1914 and 1915
- Flat law differential based on Standard Accident Table and Massachusetts Wage Distribution was used both in conversion of experience and reversion of pure premiums.
- During latter part of this period in the 1918 Pennsylvania Revision these experience differentials were used in conversion for a few states.
  1. All other Indemnity — Formula **10**.
  2. Medical — Formula **10**.

II. 1920 Revision Period, 1918-1921
- Basic Act, 1920 New York Law
- Experience Period, Policy Years 1916 and 1917
- Conversion of Experience
  - Partial Experience Differentials—A set for each of the three groups into which industry was divided.
    1. All Other Indemnity—Based on Greene's Formula
    2. Medical—Based on Greene's Formula
  - Flat Experience Differentials—Used as pure premium conversion factor in few states.

Reversion* of basic pure premiums.
Partial Experience Differentials—The same as in conversion above.
1. D. & P. T. D.—Based on Average Values. Formula **6**
2. All Other—Same as in conversion
3. Medical—Same as in conversion
Partial Law Differentials—American Table and Actual State Wage, used for states having inadequate data.
1. D. & P. T. D.
2. All other
3. Medical
Projection Factor
Flat Experience Differential } Usage Varied in States
Partial Experience Differential }

III. 1923 Revision Period, 1921-1924.
Basic Act, 1920 New York Law
Experience Period, Policy Years 1918-1920
Conversion of Experience
Partial Law Differentials—American Accident Table and Actual State Wage
1. Fatal
2. Permanent Total
3. Major Permanent Partial
4. Minor Permanent Partial
5. Temporary Total
Partial Experience Differentials
1. Medical—Average values and judgment
2. Payroll—Based on comparison of Average Wages
Reversion of basic pure premiums to latest State Law Basis.
Partial Law Differentials
1. Serious
2. Non Serious
3. Medical
Partial Experience Differential—pure premium correction factor for each division
Projection Factor
Flat Experience Differential—Formula **12b**

IV. 1925 Revision Period, Since 1924.
Basic Act, New York Law Effective 1-1-1926
Experience Period, Policy Years 1918-1922

*The term "translation" was used in place of "reversion."

Conversion of Experience
- State Policy Year Experience to State Present Law Basis if Amendments Intervene.
- Partial Law Differential—American Table, State Wage for Policy Year
  1. Fatal
  2. Permanent Total
  3. Major Permanent Partial
  4. Minor Permanent Partial
  5. Temporary Total
- Partial Experience Differentials—Formula **12a**
  1. Serious
  2. Non Serious
  3. Medical

Reversion of basic pure premium to Present Law Basis
- Partial Experience Differentials—Formula **12a**—Weights given classification experience receiving state credit.
  1. Serious
  2. Non Serious
  3. Medical
- Partial Experience Differentials—Correction Factors—Formula **12b**

Projection
- Flat Experience Differentials—Weighted to apply to indemnity only.