ABSTRACT OF THE DISCUSSION OF THE PAPERS READ AT THE PREVIOUS MEETING.

NOTES OF POISSON'S EXPONENTIAL AND CHARLIER'S CURVE—A. H. MOWBRAY.

VOL. VI, PAGE 197.

WRITTEN DISCUSSION.

MR. H. C. CARVER:

Mr. Mowbray's "Notes on Poisson's Exponential and Charlier's Curves" are both interesting and significant: interesting because of the neat development of Poisson's Exponential Binomial Limit which is now available in the Proceedings for the students in this Society, and significant since we may infer that those who are closest to our fundamental actuarial problems desire a practical mathematical criterion to use in connection with "researches as to the stability of statistical series and the nature of disturbing forces which affect their value for rate making."

Numerous probability functions have been used in the treatment of statistical data. Among them are the following:

I. Pearson's Hypergeometric Series

$$F(x) = \frac{{}_pC_x \, {}_qC_{r-x}}{{}_nC_x}.$$

II. The Point Binomial

$$F(x) = {}_rC_x \, p^{r-x} q^x.$$

III. The Poisson Exponential Binomial Limit which Mr. Mowbray treats in the paper under discussion

$$F(x) = \frac{e^{-m} m^x}{\lfloor x}.$$

IV. The Normal Curve of Error

$$F(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x^2/2\sigma^2)}.$$

V. Charlier's Type A curve

$$F(x) = A_0 \phi_x + A_3 \phi_x^{\text{III}} + A_4 \phi_x^{\text{IV}} + \cdots,$$

where

$$\phi_x = \frac{1}{\pi}\int_0^\infty e^{-(\sigma^2\omega^2/2)}\cos x\omega d\omega.$$

VI. Charlier's Type B curve

$$F(x) = B_0\psi_{(x)} + B_1\Delta\psi_{(x)} + B_2\Delta^2\psi_x + \cdots,$$

where

$$\psi_{(x)} = \frac{e^{-\lambda}}{\pi}\int_0^\pi e^{\lambda\cos\omega}\cos[\lambda\sin\omega - x\omega]d\omega.$$

Of the first four, Pearson's Hypergeometric Series are the most powerful since the other three may be regarded as special cases.

Thus (denoting as usual the nth moment about the mean by ν_n) if

$$\nu_4 = 3\nu_2^2 + \frac{3\nu_3^2}{2\nu_2} - \frac{\nu_2}{2}.$$

I reduces to II.

Again, if $\nu_2 = \nu_3$, II reduces to III, while if $\nu_3 = 0$ and n be large, II approaches IV.

On the other hand, III, IV, V and VI are closely related, for if σ be large the expression for ψ_x in VI is represented approximately by the curve of error IV, while if λ be a positive integer or zero the value of $\psi_{(x)}$ in VI reduces to Poisson's Exponential Binomial Limit.

As Mr. Mowbray suggests the coefficients A_3, A_4 of V and the coefficients B_1, B_2, etc., may serve as a criterion of the perturbation of the basic probabilities: in other words we may be able to develop coefficients which will indicate to what extent and in what manner an observed frequency differs from the "normal" law or the "law of small numbers."

It occurs to me, however, that such coefficients would not be of the greatest value to us since our greatest interest centers around the function

$$nC_x p^{r-x}q^x,$$

which may be expressed, as Charlier has shown, by means of the series

$$F(x) = \phi_{(x)} + \frac{\sigma^2(p-q)}{L^3}\phi^{\mathrm{III}}_{(x)} + \frac{\sigma^2(1-6pq)}{L^4}\phi^{\mathrm{IV}}_{(x)} + \frac{\sigma^2(p-q)(1-12pq)}{L^5}\phi^{\mathrm{V}}_{(x)} + \cdots,$$

where, if σ be not small,

$$\phi_{(x)} = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x^2/2\sigma^2)}.$$

From the preceding it appears that if any law of distribution were Bernoullian, and not affected by any perturbations, the coefficients A_3, A_4 might still have significant values.

For this reason I believe that Charlier's Coefficient of Disturbancy or the criterion

$$\delta = \nu_4 - 3\nu_2^2 - \frac{3\nu_3^2}{2\nu_2} - \frac{\nu_2}{2}$$

might better serve the purpose.

It occurs to me, however, that interesting results might be obtained by writing the general law of error as

$$F(x) = B_0\psi_{(x)} + B_1\Delta\psi_{(x)} + B_2\Delta^2\psi_{(x)} + \cdots,$$

where $\psi_{(x)} = nC_x p^{r-x} q^x$, and then investigating the practicability of using the value of B_1, B_2, B_3, etc., as a criterion.

MR. R. HENDERSON:

Perhaps the best way in which I can discuss this paper is by giving the results of an investigation into which I was led by it and by the remark of Mr. Elderton in his book that Charlier's fitting of his Type B curve was arbitrary. The subject of the investigation was the systematic fitting of curves of the type $y = \psi(x)f(x)$, where $\psi(x)$ is any standard function of which the moments are known and $f(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots$.

Let s_n denote $\int x^n \phi(x)\,dx$

and let t_n denote $\int x^n \psi(x) f(x)\,dx$,

both integrals being taken over the complete range of $\psi(x)$. Then since $x^n\psi(x)f(x) = \psi(x)\,(b_0x^n + b_1x^{n+1} + b_2x^{n+2} + \cdots)$ we have

$$t_n = b_0s_n + b_1s_{n+1} + b_2s_{n+2} + \cdots.$$

If then we are given values of t_n equal in number to the arbitrary constants in $f(x)$ we have a series of simple equations to determine those constants.

For example if $f(x)$ is assumed to be of the nth degree in x the $n+1$ arbitrary constants may be determined if we are given the values of t_r for values of r from 0 to n inclusive. The equations which determine $f(x)$ are

$$\begin{aligned}
f(x) &= b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 + \cdots,\\
t_0 &= b_0s_0 + b_1s_1 + b_2s_2 + b_3s_3 + b_4s_4 + \cdots,\\
t_1 &= b_0s_1 + b_1s_2 + b_2s_3 + b_3s_4 + b_4s_5 + \cdots,\\
t_2 &= b_0s_2 + b_1s_3 + b_2s_4 + b_3s_5 + b_4s_6 + \cdots,\\
t_3 &= b_0s_3 + b_1s_4 + b_2s_5 + b_3s_6 + b_4s_7 + \cdots,\\
t_4 &= b_0s_4 + b_1s_5 + b_2s_6 + b_3s_7 + b_4s_8 + \cdots.
\end{aligned}$$

Those familiar with the theory of determinants will see that the result of eliminating the arbitrary constants from this set of equations may be expressed compactly by equating to zero a determinant the form of which is quite evident. This furnishes the most direct formal solution of the general problem and it may be applied to any particular case of $\psi(x)$, by inserting the corresponding values of s_n.

For certain purposes however a different method of stating the solution, which shows the effect of taking into account successively the higher moments, is more useful and instructive. For this purpose we will designate by $b_{m:n}$ the addition made to b_m when the nth moment is taken into account so that we have

$$b_m = b_{m:m} + b_{m:m+1} + b_{m:m+2} + \cdots.$$

Also let $f(nx)$ be the addition to the value of $f(x)$ when the nth moment is taken in so that

$$f(x) = f(0x) + f(1x) + f(2x) + \cdots.$$

Then we have

$$t_0 = b_{0:0}s_0 \quad \text{or} \quad b_{0:0} = t_0/s_0,$$

$$f(0x) = b_{0:0} = t_0/s_0.$$

Also

$$f(1x) = b_{0:1} + b_{1:1}x, \quad 0 = b_{0:1}s_0 + b_{1:1}s_1,$$

$$t_1 - \frac{t_0 s_1}{s_0} = \frac{\begin{vmatrix} s_0, & s_1 \\ t_0, & t_1 \end{vmatrix}}{s_0} = b_{0:1}s_1 + b_{1:1}s_2.$$

The solution of this is

$$b_{0:1} = -\frac{s_1}{s_0}b_{1:1}; \quad b_{1:1} = \begin{vmatrix} s_0, & s_1 \\ t_0, & t_1 \end{vmatrix} \div \begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix},$$

$$f(1x) = \frac{\begin{vmatrix} s_0, & s_1 \\ t_0, & t_1 \end{vmatrix}}{\begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix}}\left(x - \frac{s_1}{s_0}\right) = \frac{\begin{vmatrix} s_0, & s_1 \\ t_0, & t_1 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1 \\ 1, & x \end{vmatrix}}{s_0 \cdot \begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix}}.$$

Again

$$f(2x) = b_{0:2} + b_{1:2}x + b_{2:2}x^2,$$

$$0 = b_{0:2}s_0 + b_{1:2}s_1 + b_{2:2}s_2,$$

$$0 = b_{0:2}s_1 + b_{1:2}s_2 + b_{2:2}s_3,$$

$$\begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ t_0, & t_1, & t_2 \end{vmatrix} \div \begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix} = b_{0:2} + b_{1:2}s_3 + b_{2:2}s_4.$$

The result of eliminating the constants from these equations is

$$f(2x) = \frac{\begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ t_0, & t_1, & t_2 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ 1, & x, & x^2 \end{vmatrix}}{\begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ s_2, & s_3, & s_4 \end{vmatrix}}.$$

We have now reached the point where it is possible to detect the law of formation of the series, the final result being

$$f(x) = \frac{t_0}{s_0} + \frac{\begin{vmatrix} s_0, & s_1 \\ t_0, & t_1 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1 \\ 1, & x \end{vmatrix}}{s_0 \cdot \begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix}} + \frac{\begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ t_0, & t_1, & t_2 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ 1, & x, & x^2 \end{vmatrix}}{\begin{vmatrix} s_0, & s_1 \\ s_1, & s_2 \end{vmatrix} \times \begin{vmatrix} s_0, & s_1, & s_2 \\ s_1, & s_2, & s_3 \\ s_2, & s_3, & s_4 \end{vmatrix}} + \cdots.$$

Let us now apply this method of reasoning to Charlier's Type A curve. It is customary to consider the total area or number of cases involved as a separate factor. We may therefore put $s_0 = t_0 = 1$. It is also customary to make the mean value of $\psi(x)$ coincide with the mean value of the data and to take that value as origin so that we have $s_1 = t_1 = 0$. It is also customary to make the second moment in $\psi(x)$ for this type agree with the final value so that $t_2 = s_2 = \sigma^2$, where

$$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x^2/2\sigma^2)}.$$

Also since $\psi(x)$ is symmetrical we have

$s_n = 0$ if n is an odd number. We have also

$$s_4 = 3\sigma^4,$$
$$s_6 = 15\sigma^6,$$
$$s_8 = 105\sigma^8.$$

We might substitute these values in the general expression already arrived at and thus obtain the solution but it is in this case easy to go back to the original equations, make the substitution there and apply the method to the resulting equations. If we do so and put μ_n for t_n since the mean value has been taken as origin we obtain

$$f(x) = 1 + \frac{1}{6} \cdot \frac{\mu_3}{\sigma^3}\left(\frac{x^3}{\sigma^3} - \frac{3x}{\sigma}\right) + \frac{1}{24}\left(\frac{\mu_4}{\sigma^4} - 3\right)\left(\frac{x^4}{\sigma^4} - \frac{6x^2}{\sigma^2} + 3\right).$$

This is the familiar expression for the Type A curve.

Turning now to the Type B curve where instead of $\psi(x)$ we use

$$\psi(\alpha) = \frac{e^m m^x}{\lfloor x}$$

or transferring to the mean value as origin

$$\psi(x) = \frac{e^{-m} m^{x+m}}{\lfloor x+m},$$

we have here $s_0 = 1$ and $s_1 = 0$ as before, also

$$s_2 = m,$$
$$s_3 = m,$$
$$s_4 = m + 3m^2,$$
$$s_5 = m + 10m^2,$$
$$s_6 = m + 25m^2 + 15m^3,$$
$$s_7 = m + 56m^2 + 105m^3,$$
$$s^8 = m + 119m^2 + 490m^3 + 105m^4.$$

Substituting these values in the equations and solving as before we have

$$f(x) = 1 + \frac{1}{2m^2}(\mu_2 - m)(x^2 - x - m)$$
$$+ \frac{1}{6m^3}(\mu_3 - 3\mu_2 + 2m)\{x^3 - 3x^2 - (3m-2)x + 2m\}$$
$$+ \frac{1}{24m^4}\{\mu_4 - 6\mu_3 - (6m-11)\mu_2 + 3m^2 - 6m\}\{x^4 - 6x^3$$
$$- (6m-11)x^2 + (14m-6)x + 3m^2 - 6m\} + \cdots,$$

AUTHOR'S REVIEW OF DISCUSSIONS.

MR. A. H. MOWBRAY:

As its title implies it was my expectation that these notes when presented would appear under the caption "Actuarial Notes," rather than with the full dignity of a formal paper. It did not seem to me these brief observations warranted the importance of such a form of presentation. However, I am very glad they have been so published as otherwise the very able discussions presented, especially that by Mr. Henderson, would probably not have found a place in our proceedings.

Both the critics of the paper are so much abler mathematicians than I that I hardly feel I should attempt any reply in detail to the points they have raised.

THE TECHNIQUE OF RATE MAKING AS ILLUSTRATED BY THE 1920 NATIONAL REVISION OF WORKMEN'S COMPENSATION INSURANCE RATES—G. F. MICHELBACHER.

VOL. VI, PAGE 201.

WRITTEN DISCUSSION.

MR. A. H. MOWBRAY:

Mr. Michelbacher's paper and my own are so closely related and we worked together so much in preparing them that it is rather difficult for me to prepare much of a discussion of this paper. There are, however, a few points, comment upon which may assist students in understanding the paper.

On page 2, Mr. Michelbacher says, "The procedure of revision is typical and differs only in detail from that which might be adopted for any revision of rates." I am a little doubtful whether this is not a bit too strong a statement. The peculiarities of Workmen's Compensation business have required the development of a very elaborate technique and I doubt whether in many other lines the technique is so highly developed.

In the same paragraph he refers to the methods used in the 1920 revision as "those which have been *established* after years of experimentation." Here again I think it is possible to take this statement as stronger than Mr. Michelbacher really intended. The methods have been established for present use, but I doubt whether they could be considered established indefinitely for the future.

On page 205 discussing the problem of classification Mr. Michelbacher says, "There should be as few classifications as possible." In this statement I heartily agree with him, if due consideration is given to the limitation "as possible." It is true that a substantial volume of experience is necessary to form a statistical basis of rate making but it is equally important that the experience be homogeneous, if it is to yield a dependable indication. Not only that but if the public is to be satisfied there must be no patent inclusion within the classification of risks which might reasonably otherwise be separately classified and which are distinctly better or distinctly worse than the general average of the classification. For example, if an attempt were made to reduce the number of classifications, probably one of the first steps would be to include all of the classifications in Group 461 (Exhibit V) in the same classification as the present classification 3632 (Exhibit IV). Yet a comparison of these two exhibits will show that the pure premiums for

the average of Group 461 are two or three cents less in practically every state, than the experience of classification 3632, which of itself contributes about 90 per cent. of the experience in Group 461, indicating that the classifications forming the other 10 per cent. are distinctly better than classification 3632.

On page 212, Mr. Michelbacher refers to Schedule "W" as "an underwriting 'gain and loss' exhibit" and implies rather than directly states that the sole value of Schedule "W" is the determination of expense loading in matters of this nature. The standard Schedule "W" now also contains a loss analysis feature that was very useful in connection with the projection theory. (See my paper—page 273).

Describing the classification groups on page 217, Mr. Michelbacher says, "The nature of the operations performed is the basis of classification rather than the relative proportion of permanent partial disability and of temporary total disability losses." This is true as to the method of procedure in selecting the classifications which should be included in the group, but as appears earlier in the paper and is more fully dealt with in my own, the reason for making this grouping was because of an expected difference in this respect between the classifications falling into the first group and those of either of the other two. In connection with the foot note on this page attention may be called to the discussion of the same matter in my paper on pages 258 and 259.

On the same page Mr. Michelbacher refers to the Committee having decided to group the experience in accordance with "Manual Classification Code." In my paper I have not referred to this decision primarily because the principle had been in vogue for so long and the actual grouping was not developed by the Actuarial Committee.

On page 218, Mr. Michelbacher refers to the decision to present the material by states and regional sections as well as by the country as a whole. I find that I did not present the reasons for this in my paper; this was an oversight. At previous rate revisions there have been contentions for differentiation between states and regions on the basis of a difference in accident frequency, but no attempt was made to recognize this because statistical data was not available to either prove or disprove this contention. Such data being presently available it was decided to present it in such form as to bring this feature out.

On page 228, Mr. Michelbacher says, as to basic pure premiums, "These pure premiums have no significance in themselves." It is true they have not, yet they are the key to the basic manual and are the "basic set of key rates" referred to on page 251 of my paper. This is well known to Mr. Michelbacher, and most of us closely in touch with the work, but the significance may not always be grasped by the readers not so thoroughly familiar with the way the work has been done.

At the time this paper was prepared as well as my own, the rate revision had not been completed and naturally things that came up after the last meeting of the society could not be fully covered. Probably the two most important were:

1. That in certain states we could not get enough data to use the projection method. It therefore became necessary to use theoretical law differentials as heretofor, but we used them not as flat differentials upon aggregate premiums, but as partial differentials upon partial premiums. We further introduced a certain element of projection by valuing limits of the New York law to which the basic pure premium referred, by the use of a wage distribution from New York state experience on the issues of 1917 and valuing the limits of the law of the other state upon the basis of a wage distribution of the year 1919, in that or a neighboring state.

2. We obtained evidence that medical costs were continuing to increase beyond the latest date to which our experience, even using the projection factor, could bring them. For several states therefore a percentage loading was applied to the medical pure premium to represent this further increase not yet represented in our experience used in the projection data. The amount of this loading was ascertained by comparing the realized increase in the state in question and in New York, on the theory that the increase of New York was representative of the country and that its not being realized in another state was due to slower spread of the movement among the doctors elsewhere.

MR. E. S. COGSWELL:

Mr. Michelbacher has performed a valuable service for the Society in giving us a complete description of the methods followed by the National Council in the rate revision which is still in process. As he states in his paper, this is a very large task, and it is still going on. The work of the revision began in November, 1919, and yet the General Rating Committee still has four or five more states to pass upon before the work will be completed.

In attending many of the sessions of the General Rating Committee it came forcibly to my attention that in spite of the fact that the Council had before it in its work, experience representing nearly Twelve Billion Dollars of payroll and over One Hundred and Sixteen Million Dollars of incurred losses, yet there are many classifications where the rate is based either on insufficient experience or determined largely by judgment. The Council has performed a valuable service in eliminating a number of classifications from the Manual so that the Manual now contains only 953 classifications in place of the 1319 classifications in the former Manual. In this revision only two years experience was used, namely, policy years 1916 and 1917. I think the Council acted wisely in excluding the experience of earlier years as conditions were different from

those prevailing in 1916 and 1917, and considerably different from those prevailing today. If conditions remain stable, however, the experience of three or more years should be used in future revisions as two years experience for a number of classifications does not give a wide enough spread of exposure and a better average is obtained by taking the experience of three or more years.

In attending sessions of the committee I was greatly impressed with the desire on the part of the members of the committee to make their decisions upon the basis of the statistics presented to the committee, rather than on the basis of judgment. The committee on several occasions spent several hours trying to find a statistical basis upon which to make a decision, although any one of the members could have decided the matter by underwriting judgment within a short period of time.

One of the new steps in rate making was the use of the projection method and the comparison of the 1916–1917 loss ratio with the 1919 loss ratio. Mr. Mowbray performed a valuable service by bringing this method to the committee's attention. It is very desirable, however, that careful check be made in the future to see if the actual state loss ratios of the year 1919, after the policies have been audited and the incurred losses determined with a reasonable degree of certainty, are within measurable distance of the loss ratios used by the committee. It is necessary in this connection not only to check the final loss ratios of the combined experience of the companies which furnished preliminary statistics to the committee, but also to determine the final loss ratios of the combined experience of all companies. It is necessary to see if the companies which reported preliminary experience are representative of the business as a whole.

Mr. Michelbacher is correct in saying that the problem of rate making is now exceedingly complex and there is danger of its being over-balanced on that account. The problem of making rates for a state is far more complicated than it was in the old Manual. As the basic pure premium is now divided into three parts it is necessary to translate these three parts to state sectional pure premiums. The translation of the D. & P. T. D. pure premiums requires reference to the Schedule in which the classification is placed as a different factor is used for each Schedule for each state. It is necessary to refer to one of three conversion groups in order to obtain the proper factor to translate the basic D. & P. T. D., All Other, and Medical pure premiums, to state pure premiums. After the state pure premiums are determined, a separate multiplier must be applied to each of the three sections of the pure premium and care must be taken to use the correct set of multipliers for one set is to be used for regular classifications, and another set for classifications subject to schedule rating. Then the sectional rates must be totaled and one cent added for catastrophe to obtain the final manual rate.

Under the old method of procedure where the pure premium was

not divided into sections, one multiplication and the addition of the one cent for catastrophe was all that was necessary to transform a basic pure premium into a state rate. Under the present method of rate making, six multiplications are necessary and the totaling of four figures. This applies to states where rates are based in large measure upon state experience. The problem is a little simpler for a state which has only a small volume of experience and whose rates are made on a theoretical basis.

The Council was very fortunate in having the services of Mr. Michelbacher throughout the work of revision, and the Society is fortunate in having him present his paper which gives a complete history of the methods used.

MR. A. L. KIRKPATRICK:

Until recently, the student of Workmens' Compensation Insurance has had very little available literature with which to study beyond the range of his own daily experience. There are a number of books available which treat largely with workmen's compensation as a social development rather than from the standpoint of the insurance business. The Proceedings of this Society contain numerous papers on various problems of compensation insurance but none has heretofore covered the problem of rate making in its entirety. The printed proceedings of the 1915 conference give little insight into the problem. The proceedings of the Augmented Standing Committee in 1917 as published by the National Workmen's Compensation Service Bureau are a little more complete, but to a person not in immediate touch with such work leave numerous questions unanswered. In Mr. Michelbacher's paper, anyone can follow the recent revision from start to finish and can find an explanation for all of his questions. Further than that it provides a permanent record of the present stage of development of rating procedure and rate making methods.

It is not difficult to understand the vast amount of work involved in such a revision and to appreciate the many problems which arise. Dr. Downey has criticized the method of reducing state experience to the New York level and proposes the revaluation of individual claims on the basis of New York benefits. Perhaps the results would have been more accurate than under the method used. Certainly the work involved would have been increased several fold. As the volume of compensation experience increases, an ever increasing amount of data becomes available and more and more refinement is permitted in the methods used. The question may naturally be raised as to how far these refinements may go before the amount of work and the expense involved become more than the business will permit.

There are two main problems involved in rate making. The first

is to establish the proper relativity between rates for classifications of different hazard so that each industry bears its fair share of the cost. The second is to obtain the correct level of rates in each state thus determining the aggregate premium income from all industries in the state. The necessity for a revision arises out of the fact that both of these elements are constantly changing.

It may be a known fact that one industry is twice as hazardous as another in 1920 and accordingly given double the pure premium, but processes and methods of manufacturing are constantly changing so that in 1922 the relation of 2 to 1 may be entirely incorrect. These changes are much slower, however, than those affecting the levels of loss cost. There are numerous known factors and perhaps others that are not known. Movement in the business circle is constantly changing wage levels, accident frequency and speed with which injured laborers return to work. Changes in the compensation benefits and changes in the procedure of administrative bodies are frequent occurrences. All have a direct bearing on the loss cost and require revision of rate level.

Excepting changes in the compensation law, none of these variables can be measured, until the experience of the carriers is available. Schedule "Z" experience is two years old before it can be used. Loss Ratio experience is not stable for about the same period. In the recent revision, pure premiums were established on the level of 1917 policy year experience. The Actuarial Committee developed an ultimate loss ratio for 1919 policy year using the experience of that year developed to December 31, 1919, and later to June 30, 1920. By this means the gap from 1917 to 1919 policy years was bridged. But that experience is already nearly a half a year old and the rates have been put out in only a few states. Obviously there is need for a method of keeping rates on the current level.

Any policy year experience furnished by the companies is valueless until at least twelve months of the year have passed in order that all deposit premiums and some payroll audits may be recorded and a fairly dependable volume of paid losses accumulated. But it is desirable to get closer to the present state of affairs if that is possible. A calendar year experience is of too doubtful accuracy to be depended upon for rate making purposes. There is no dependable index of loss cost or of premium income except on a policy year basis, and that lags considerably behind present-day conditions.

There are certain indices of general business conditions such as gross railroad earnings, bank clearings, etc. It has been suggested that perhaps there are also indices in the compensation business which could be combined into an index number to measure current cost and income. This line of thought has not been developed very far but it may be pointed out that this method is used only as a qualitative indication and not as an exact measure such as rate making requires.

There have been methods proposed of reporting experience by the companies so that the experience may disclose at once just what is happening in the compensation business. The Actuarial Committee of the National Council is at present working on the problem and it is quite possible that the next general revision of rates will see a more refined method of keeping up with current cost than has been used up to the present time.

"THE ACTUARIAL PROBLEMS OF THE 1920 NATIONAL REVISION OF WORKMEN'S COMPENSATION INSURANCE RATES AND THE SOLUTIONS DEVELOPED BY THE ACTUARIAL COMMITTEE OF THE NATIONAL COUNCIL"—A. H. MOWBRAY.

VOL. VI, PAGE 250.

WRITTEN DISCUSSION.

MR. S. B. PERKINS:

When a participant in committee work attempts to discuss a record of the problems that it has had to solve, such as the record which Mr. Mowbray has submitted of the work of the Actuarial Committee, and particularly where the practice has been to handle each problem in a manner which would meet with unanimous approval, it might be possible to approach the task in an unfriendly and adversely critical way, but it certainly would not be normal. The few comments which follow will accordingly be more in the nature of suggestions of certain elaborations which, in the opinion of the writer, would lend additional value to Mr. Mowbray's paper, with an occasional recommendation as to the form of presentation.

As one of the objects to be attained in revising the manual, Mr. Mowbray has pointed to the general agreement between the Actuarial and General Rating Committees that there should be: "Right rates in each state—rates accurately measuring the compensation insurance cost of its industries under its law and industrial conditions, or in other words conforming to the closest reasonable degree with its own experience." Surely there can be no criticism of the effort *per se* to establish right rates in each state. But what are right rates? If one should define them as rates which would over the entire period of workmen's compensation insurance return to the carriers an amount of premium equivalent to the losses and legitimate expenses incurred, this would seem to be a fair statement. Very probably this is the fundamental idea in the minds of all who attempt to define right rates, but different points of view produce widely divergent results in the form of definitions.

It has been suggested that a period of years, immediately preceding the date as of which rates are to be established, be accepted as the basis to rate-making, with the idea that the comparison of losses and expenses incurred, with the payroll exposed, would produce proper rates. On the assumption that: (1) the relation of losses to exposure depends upon the particular position of current business in a trade cycle; (2) trade cycles are of equal duration

and the correspondent departures from normal are of equal violence; (3) the period over which experience is to be collected, corresponds exactly with the length of a trade cycle, and that, (4) all employers carry insurance continuously, rates established on this basis would undoubtedly comply with the requirements set forth in the original definition. It does not require a second thought, however, to realize the weakness of some of the assumptions.

In the first place there are many trade cycles in operation simultaneously, each the result of a separate set of economic forces. Since 1871 there have been thirteen complete upward and as many downward swings of prices. These have been modified by seasonal fluctuations and in turn have themselves modified cycles of more extended duration which have a little oftener than once a decade resulted in panicky conditions, with resultant complete financial readjustments. We are told that these again are but playing their part in cycles with a period of approximately forty years. It is evident therefore that trade conditions of any particular time are influenced by its relative position in all of these cycles.

Assuming for the moment that for the purpose of rate calculation no cycle of more than a five year period be considered as affecting the immediate problem, on the basis that the addition of a unit of a year's experience, whenever it became available with a corresponding rejection of the experience of the oldest year, would correct with sufficient promptitude the change of rate levels necessitated by the more slowly progressing cycles, there are certain requirements to be demanded of those of shorter periods.

Since compensation is based upon weekly earnings and since each Workman's Compensation Act specifies certain maximum benefits which may be allowed for each kind of injury, one requisite is that at a given phase of each cycle wages shall have returned to the same level. This, however, has not been the case in the past and there are no indications that it will be realized in the future.

For the sake of completing the thought, however, let us suppose that all of the conditions set forth as necessary did exist, and that rates had been established on this basis. We can imagine such a possibility, but can we imagine the state of mind of the large purchaser of insurance who just following a substantial wage increase, is informed that he is entitled to no rate adjustment because it is anticipated that within four or five years the matter will be adjusted by a practical application of economics. Little good will it do to assure him that when conditions were reversed the insurance carriers were accepting their loss with the same promise of future relief. Such might have been the case—would have been under this system of rate-making—but there is grave doubt in the mind of the writer as to how thoroughly the purchaser would be convinced, and further—to the practical point as to how long he would remain a purchaser.

Rate-making for the life actuary must be a pleasure with the mortality experience running along in the even tenor of its way with only the occasional epidemic to furnish the necessary uncertainty. In compensation the "epidemic" is the rule—the "even tenor" non-existent. The whole complexion of compensation business changes with extreme rapidity. A wage increase or decrease effective on a certain date influences the adequacy of the rates in force from the very instant the change is made effective, to the monetary advantage of the carrier or assured depending upon whether the change is upward or downward. In neither case is complete justice done. On the other hand if for any reason a rate is changed, unless it be made effective on all business in force, the effect is not felt in its entirety until every policy then in force shall have expired or shall have had its rates adjusted.

The National Council has recognized this condition and the General Rating Committee has taken a corrective step by adopting a resolution reading in part as follows:

"*Resolved,* That the Rating Committee approve in its entirety the plan for the projection of rates from basic pure premiums with the following provisions: 1st—that the Actuarial Committee be instructed to develop the method by which the current level of rates may be promptly modified in the event of changes or other conditions affecting such rates; . . . "

Workmen's compensation rate-making has passed from the period of flat differentials to partial differentials, from the basis of comparative rate levels to that of individual state pure premiums determined from experience and from the era of conjectural factors to one of statistical analysis.

It is not impossible that a Compensation Cost Barometer may be established, set ahead to forecast conditions during which a given set of rates are to be operative. Very probably indications might differ from industry to industry; possibly even from classification to classification. This is for the future to determine. Certainly if such a Barometer could be successfully established, not only would the conditions of the basic definition be realized, but the burden of the premium would be so distributed for the employer as to more nearly meet temporary economic demands.

Under the caption "The Defect of Former Differential Methods" Mr. Mowbray has outlined the greatest weakness of the "flat differential," and under the caption "The Remedy Proposed" he has apparently presented very briefly the method by which the Actuarial Committee decided to surmount the difficulty, namely, by the use of "partial pure premiums, as for death benefits, for permanent disability compensation, for temporary disability compensation, and for medical and hospital service." This is apt to fix in the mind of the reader the impression that partial pure premiums were actually established for each of these subdivisions, whereas actually only three partial pure premiums were used. If

it did seem necessary to introduce the defects of former methods at this particular point in the paper, the next section might well have been captioned "The Remedy" or, possibly, the whole presentation of the "Subdivision of the Pure Premium" could have been dealt with once and for all.

In closing his presentation of the "Subdivision of the Pure Premium," Mr. Mowbray has quoted from a resolution of the Actuarial Committee in which it states its reasons for treating permanent partial disability losses and temporary losses as a single element of the pure premium. It is significant, however, that in drafting a uniform Schedule "Z" blank for future reportings there was inserted a line for permanent partial (major) and one for permanent partial (minor). The permanent partial (major) has been defined for the purpose of such reporting as

(*a*) Every permanent injury, not constituting permanent total disability, which involves the loss of sight of an eye or the loss of a hand, foot, arm, or leg;

(*b*) Every permanent injury involving the impairment to the extent of 50 per cent. or more of a hand, foot, arm, or leg;

(*c*) Any permanent injury, whether enumerated above or not, which is compensated on the basis of 25 per cent. or more of permanent total disability (or 25 per cent. or more of the full benefit for permanent total disability allowed under the Act applicable thereto).

There is little question but that by far the greater number of permanent partial losses can always be combined with the temporary losses and a conversion factor calculated for the resulting group with a satisfactory degree of accuracy. It seems equally clear that there are a number of permanent partial or even temporary cases of extended duration which, by virtue of the fact that they are individually costly, distort the present "all other" partial pure premium for certain classifications. The remedy would seem to be to accord such cases treatment similar to that accorded death and permanent total cases, namely, in principle, to spread their cost over groups or schedules of more or less homogeneous classifications.

The same criticism may be offered of the section "Conversion by Groups Considered Advisable" as was offered in regard to "The Remedy Proposed," namely, that emphasis has seemingly been laid upon the unmatured judgment of the Committee. In glancing at the section referred to attention is drawn to the four groups listed numerically, while in the following paragraph there appears a sentence—"It was found, however, that the differences between the factors produced for the commercial and light manufacturing were so slight as to warrant combining these two groups, and this was done reducing the number from four to three." The tentative four groups might have been presented in a less imposing manner and the final three groups given the prominence.

The length of this discussion will not permit a comprehensive review of Mr. Mowbray's treatment of Mr. Greene's formula but, in the estimation of the writer, the technical aspects of the problem might well have been presented as one of two appendices to the paper, the second of which will be referred to later. This would have permitted a descriptive rather than an analytic method of presentation. In the text of the paper might have been included a statement of the underlying assumptions, the limits of usefulness of the formula, the dangers of its misuse, together with some simple examples showing the effect which payrolls and losses independently have on the relation between R and E and the actual values of each.

It is obvious that the two errors which appear in Formula (5) on page 266 were typographical where Formula (5) appeared as

$$C_j^m = 1 - \frac{D\dfrac{\Sigma L_j}{\Sigma L_m}}{1+D} \quad \text{when} \quad (1+D) = \frac{\Sigma\left(J\dfrac{L_j+L_m}{j+M}\right)}{\Sigma L_j}. \quad (5)$$

It should have been written ts

$$C_j^m = \frac{1 - D\dfrac{\Sigma L_j}{\Sigma L_m}}{1+D} \quad \text{when} \quad (1+D) = \frac{\Sigma\left(J\dfrac{L_j+L_m}{J+M}\right)}{\Sigma L_j}. \quad (5)$$

The second appendix, mentioned earlier in the discussion, might have been presented in connection with the subject of Projection of Losses. Every carrier is interested in determining as soon as possible, and with the greatest possible accuracy, the loss ratio for the current policy year. As Mr. Mowbray has suggested in the closing paragraph of this section, it would make the paper too voluminous to go into all of the detail involved in arriving at a projected loss ratio for the current year; but, as has been suggested, an appendix outlining the whole procedure, including the method of correction for amendments and rate changes on the losses and premiums reported together with the method of subdividing the projection factor, might be of considerable value to some carriers to whom the process might come as an innovation.

It is somewhat difficult to determine upon reading the two captions "Projection to Present Conditions" and "Projection to Current Level" exactly what phase of the general problem is to be treated in each section. The "Projection to Current Level" might have been extended, using the subject matter under "Projection to Present Conditions" in a modified form as an introduction and with the "Change in Wage Levels," the "Increase in the Medical Service Costs" and "Other Conditions Produced Changed Cost" as subdivisions of the projection problem.

Two outstanding features of the Committee's work were in Mr.

Mr. Mowbray's opinion listed in his concluding paragraphs. In addition to these, the decision to allow each state to determine its own rate level on the basis of its own experience is in my estimation one of the most important features of the whole revision. Mr. Mowbray may have considered this a decision of the General Rating Committee or it may not have appeared to him to be properly listed under the Committee's work. Very probably, it could be considered as being covered in his first item—"The Change in Method of Combining Experience and Translating the Selected Basic Pure Premiums into State Pure Premiums," but it seems of sufficient importance to have been given prominence of "honorable mention."

Reference to the American Accident Table calls to mind that the recent National Council Rate Revision has been the first general rate revision which has been made entirely on the basis of American experience. Another milestone has been passed in the development of workmen's compensation insurance and rating procedure.

AUTHOR'S REVIEW OF THE DISCUSSION.

A. H. MOWBRAY:

The first part of Mr. Perkin's discussion of my paper is a defense of the present methods of rate making and probably calls for no reply from me. It strikes me that he has put quite clearly the difficulties of long term rate making, desirable as such rate making is in many ways.

Mr. Perkins criticizes the method of presentation of one or two items as tending to mislead the reader as to just what was done. Should this result it would indeed be unfortunate and very far from my intention. The paper, however, was written as a companion paper to that of Mr. Michelbacher's which precedes it in the same number of the *Proceedings* and it was intended to be read jointly with his. Both papers in a measure tell *what* was done in the revision. Mr. Michelbacher's paper was intended to tell, and I think does tell *how* it was done; my paper was intended to tell and I think does tell *why* it was done.

From this point of view it is more important, it seems to me, to give the full theoretical reasoning than the exact process in the few cases where a modification was used as a means of adapting the theory to limited material or short cut the work where trials with the full theoretical process had indicated that differences were insignificant.

Mr. Perkins makes the suggestion that the details of the projection method be more fully dealt with and the suggestion seems good. At the time the paper was written the theory was not sufficiently developed so that all of the important details of application had been brought out. It was therefore deemed best at that time to do no more than outline the theory for the simpler cases.

In preparing the factor for New York we had relatively clear sailing. Aside from sporadic changes of individual classification rates, the rates in New York had remained the same since January 1, 1918. Also the law had remained unchanged during the same period or substantially so. To get the projection loss ratio on 1919 issues, therefore, it was only necessary to make reasonably sure of the ratio of paid to incurred by a study of the record of earlier years and then to select in the light of record of earlier years a ratio of premiums written as recorded at the end of the calendar year of issue to the ultimate earned premiums. This done we could apply the factors directly to the figures as of December 31, 1919, on the issues of 1919 without modification and obtain a reasonable estimate of the ultimate incurred losses and ultimate incurred premiums giving the ultimate loss ratio.

In many states, however, the case was not so simple, the laws had been amended during the calendar year of 1919, affecting the losses under the unexpired portions of policies issued in 1918 and of policies issued in 1919 prior to the date of the amendment and affecting the cost throughout under 1919 policies issued subsequent to the day of amendment. The affect of the amendment therefore had to be allowed for (1) in getting the ratio of paid to incurred losses for the issues of 1918 in order that they might be compared with the earlier years (2) in adjusting the 1919 losses paid to a proper basis for comparison.

As the amendments did not uniformly affect all parts of the pure premium but bore more heavily on some parts than on others it was the judgment of the Committee that the projection factor, which cannot readily be analyzed, should be so determined as to exclude any of the affect of amendment and that the full amendment factor should be used in all states, in other words that the basis for modifying the 1919 paid losses should be to bring them to the basis of a uniform condition throughout the year on the level of the law used at the beginning rather than the end of the year.

In some cases also the level of rates had been changed during the period under review for determination of the projection factor and it is necessary to ascertain what the change in rate level had been and modify the earned or written premiums as the case might be to make them the equivalent of the December 31, 1919, manual rates. The reasons for, and ways of making this adjustment are perhaps sufficiently obvious and require no further comment. The methods of making an adjustment for amendments are more complicated and require a more elaborate statement.

As all losses occurring after the effective date of the amendment are compensated under the new law the first step is to determine the proportion of losses incurred after that date. As the basis for this determination the Committee made two assumptions:

(1) That the business written was uniformly distributed over the calendar year.
(2) That the losses incurred were always proportional to the exposure, that is to the volume of business in force during the time these losses are incurred.

To determine the ratio of paid to incurred for the 1918 issues when there was an amendment to the law in 1919 we first found the proportion of the exposure of 1918 issues after the date the amendment became effective and we considered this portion of the losses to be compensated at the higher rates. If we consider the level of the act in effect in 1918 to be represented by unity, the average value of the amendment as "a" and the portion of 1918 exposed after it came into effect as "m," then the incurred losses as reported are $(1+am)$ times what they would have been had the law remained unchanged. Therefore we divided the 1918 incurred losses as reported by $(1+am)$ to get them to a basis comparable with the payments in 1918. That is under the old law it will be apparent that in working out the ratio of paid to incurred we must always have the same law in mind in dealing both with the paid and the incurred.

In case of amendment within the calendar year with whose issues we are dealing we have

Total losses paid out in calendar year = Losses paid on the basis of the old act—payments on account of losses subsequent to the date of amendment, that is the payments as affected by the amendment.

We may write symbolically

$$Pt = Pt_1 + Pt_2(1+a_2),$$

where

Pt = Total payments within the calendar year on account of losses issued in such year.
Pt_1 = Payments on account of accidents prior to the effective date of amendment.
Pt_2 = Payments on the basis of the old act on account of accidents subsequent to the effective date of amendment.
a_2 = Average effective value of amendment for period of its effective date to the end of calendar year.

The amendments which had to be considered generally increased the limits on the rate of compensation. An amendment which increased the term of compensation would increase the ultimate incurred loss but not immediately reflect itself in the payments and this would be a difficult matter to adjust. Fortunately we had no such case to deal with and this greatly simplified the work since we would not have to answer the question of the effective value of

the amendment for the period to which it applied, but could take quite readily the actual amendment factor and write our formula

$$Pt = Pt_1 + Pt_2(1 + a).$$

If we now divide through by the payments for the year on the basis of the old act we have

$$\frac{\text{Actual payments}}{\text{Payments basis old act}} = \frac{Pt_1}{Pt_1 + Pt_2} + \frac{Pt_2}{Pt_1 + Pt_2}(1 + a) = W_1 + W_2(1 + a),$$

where W_1 and W_2 are weights proportional to the payments in the two periods on the basis of the old act and $W_1 + W_2 = 1$. Hence we get

$$\text{Payments on the basis of the old act} = \frac{\text{Actual Payments}}{W_1 + W_2(1 + a)}.$$

It was found by study and test that under a uniform law in effect through the entire period the distribution of payments throughout the year is independent of the precise terms of the law within the limits of variation found in American practice.

This may seem startling at first, but upon reflection the phenomenon seems reasonable. The variations in American Compensation laws are sometimes in the rate of compensation, that is 50, 60, 65 or 66⅔ per cent. of wages but so long as it is uniformly either 50, 60 or some other percentage, the distribution of payments throughout the year is not affected by the percentage rate of compensation. The maximum and minimum limits of actual compensation are another feature as to which the laws vary considerably from state to state but so long as the limits remain the same throughout the year these variations will not affect the distribution of payments throughout the year. Our laws also vary with respect to the length of time compensation is extended for serious disabilities. This will affect in the second and subsequent years the distribution of paid losses to ultimate incurred but all laws compensate total disability for more than fifty-two weeks and rarely if at all does the specific compensation for permanent partial disability so vary that it will reflect itself within the first calendar year after the date of the accident and this is the maximum term which needs to be considered when we are considering the question of the proportion of payments on the basis of a uniform act in the calendar year of issue due to accidents occurring before or after a given date therein. Therefore for practical purposes this ratio is independent of the terms of the compensation act, and it will be readily apparent that this fact facilitates the work.

In general the Committee did not find it necessary or desirable to break up the projection factor. But in certain states where the

evidence of increase in cost of medical services collected by the Committee indicated that this increase had not been so rapid as the increase in payrolls, if the projection factor were unity or greater than unity, then the Committee felt it was desirable and necessary to break it up. In these cases the Committee used as the projection factor, applicable to the medical pure premium, the ratio of the respective percentages which the 1919 medical cost and wages bore to the 1917. The projection factor for each of the other elements was calculated by using the weights of the elements in the pure premium so as to bring the average projection factor to the value calculated as outlined above.

In such states a loading on the medical pure premiums was also recommended in anticipation of medical costs rising as they had in other states at least in the same proportion as wages in general. The loading recommended was based upon a comparison of the increase in medical cost in the local state shown by the Committee's investigation and the increase in New York and other states.

In certain states the data was too scanty to furnish a dependable basis for projection factor calculation and in a few others there was a peculiar condition about the experience which made the Committee mistrust the projection factor as calculated. In these states no projection factor was used but it was decided to use a law differential method, calculating, however, separate differentials for each of the partial pure premium divisions, and using a New York wage distribution corresponding to 1917 issues for valuing the limits under the New York law as it stood in 1917, and a wage distribution based upon local conditions of the year of issue 1919 for valuing the limits under the local law. In this way the wage change, at least as well as the statutory differences were brought under consideration. Because they embodied both the elements of translation from New York statute to state statute and at least the wage change part of the projection to present conditions we have referred to these different factors as "Projection-Translation Factors."

The Committee voted to put no loading into the rates for the effect of experience rating because the experience rating plan theoretically should be made to balance and the insertion of a loading to correct for lack of balance in the plan places a charge upon the risks which are not subject to experience rating. In this connection the Committee reviewed the experience rating plan and decided that the premiums be split for application of the plan in accordance with the way the rates were actually made, rather than in general rate groups as heretofore and that the modification factors should be developed on the same principles as employed in the rate making. The Committee felt that this procedure would tend to produce a balanced plan or at least one more nearly so than has been the case heretofore. Such reports as we have received up to the present date of the operation of the plan in New York, seems to indicate this is so.

Mr. Perkins points out a certain confusion in the headings "Projection to Current Level" on page 272, and "Projection to Present Condition" on page 270. The choice was unfortunate; it was in my mind that the head "Projection to Present Conditions" was a general heading for all that followed and I should have seen to it that it was so set up in type as to convey this impression.

Mr. Perkins refers to allowing each state to determine its own level as one of the outstanding features of the present work which he thinks I have not sufficiently emphasized. It has been my impression that notwithstanding the defects of flat law differential system it has been the custom, by means of loss ratios more or less to true up the level of rates for a given state with its own experience, but in the present revision we have gone beyond this and allowed the states own experience to determine the relativity between classifications at least so far as that experience was adequate to do so. This was the result of "The change in method of combining experience and translating the selected basic pure premiums into state pure premiums" which I characterized as the first of the outstanding features of the present revision.