

A SUGGESTED SYSTEM OF STANDARD NOTATION FOR
ACTUARIAL WORK IN WORKMEN'S COMPEN-
SATION INSURANCE.

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Actuarial science having kept pace with the rapid growth of workmen's compensation insurance, it seems opportune at the present time to attempt a standardization of the notation employed in rate making, experience rating, and the determination of loss reserves of this form of insurance. As a normal development of workmen's compensation insurance out of employer's liability insurance, certain terms such as premium, pure premium and manual rate, which have been included in insurance parlance for so long, were accepted without hesitation as part of the vocabulary of workmen's compensation insurance. The treatment of such terms might seem entirely superfluous at this time, until it is realized that in the development of rating procedure by many individual organizations different symbols have been assigned to the same terms, and in the various rating organizations a given symbol may represent different items. For the sake of completeness, then, this paper will attempt to cover all of the more common workmen's compensation insurance terms for which there seems to be a need of standardization.

In order that the problem may be attacked and developed with some logical sequence, the process of construction of workmen's compensation rates, according to the methods recently adopted by the National Council on Workmen's Compensation Insurance, will be used as a skeleton with the thought that, having covered the terms used in connection with the problems incidental thereto, this paper would have about completed its purpose.

The story of the rate revision itself has been told in detail by Mr. G. F. Michelbacher in his paper, "The Technique of Rate Making," which appeared in Volume VI, Part II, No. 14, of the *Proceedings* of this Society. References will be made from time to

time to portions of that paper. This will avoid lengthy explanation of specific operations in order that this paper may proceed more directly with its own problem—namely, that of establishing notation for these operations.

When the National Council on Workmen's Compensation Insurance undertook the task of establishing rates for the numerous classifications* for various states, there was available the experience of a number of representative carriers as reported to the state departments which require such data, or to central organizations, such as the National Workmen's Compensation Service Bureau and the National Association of Mutual Casualty Companies, part of whose function it has been to assemble such statistical information. The data was filed on the form known as Schedule "Z" and was subdivided by state, policy year and classification. For each classification there was reported the exposure in terms of payroll and premium. The payroll will be referred to by the symbol W , inasmuch as it represents the aggregate wages paid to all employees covered by the classification. It will also be necessary to indicate the state or policy years, and even the classification to which the payroll under consideration is related. The state and policy year will be indicated by a superscript or symbol appearing at the upper right corner of the symbol representing payroll. Thus if Yr represents the state of New York and 16 and 17 represent respectively the policy years 1916 and 1917, $W^{Yr::16:17}$ represents the entire payroll for the state of New York for the policy years 1916 and 1917. The classification may be conveniently indicated by the use as a subscript of the code number. Thus $W_{3632}^{Yr::16:17}$ would represent the payroll for Classification No. 3632 which was covered in New York during the policy years 1916 and 1917.

The following list of state abbreviations is submitted for use in connection with this method of indication with the full realization that, whereas they are not standard abbreviations, they possess the quality of brevity and are intended in part to bring to mind phonetically the states which they represent. Further, a two letter code has been established because of the fact that, although one letter would suffice in many instances, the same letters are suggested in other parts of this paper to represent different items. Where the system of subscripts, superscripts, presubscripts and

* See G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 205.

presuperscripts has been devised in such a way that the position of the modifying notation with respect to the main symbol will properly identify it, some confusion might be caused in the minds of those who had not thoroughly acquainted themselves with the system if there were any apparent duplication of symbols.

State	Symbol
Alabama	<i>Ab</i>
Alaska	<i>Al</i>
Arizona	<i>Ar</i>
Arkansas	<i>Ark</i>
California	<i>Ca</i>
Colorado	<i>Cl</i>
Connecticut	<i>Ct</i>
Delaware	<i>De</i>
Florida	<i>Fl</i>
Georgia	<i>Ga</i>
Idaho	<i>Id</i>
Illinois	<i>Il</i>
Indiana	<i>In</i>
Iowa	<i>Io</i>
Kansas	<i>Ks</i>
Kentucky	<i>Ky</i>
Louisiana	<i>La</i>
Maine	<i>Me</i>
Maryland	<i>Md</i>
Massachusetts	<i>Ms</i>
Michigan	<i>Mg</i>
Minnesota	<i>Mn</i>
Mississippi	<i>Mp</i>
Missouri	<i>Mo</i>
Montana	<i>Mt</i>
Nebraska	<i>Nb</i>
Nevada	<i>Nv</i>
New Hampshire	<i>Hm</i>
New Jersey	<i>Jr</i>
New Mexico	<i>Mx</i>
New York	<i>Yr</i>
North Carolina	<i>Cn</i>
North Dakota	<i>Dn</i>
Ohio	<i>Oh</i>
Oklahoma	<i>Ok</i>
Oregon	<i>Or</i>
Pennsylvania	<i>Pa</i>
Rhode Island	<i>Rd</i>
South Carolina	<i>Cs</i>
South Dakota	<i>Ds</i>

Tennessee	<i>Tn</i>
Texas	<i>Tx</i>
Utah	<i>Ut</i>
Vermont	<i>Vt</i>
Virginia	<i>Va</i>
Washington	<i>Wa</i>
West Virginia	<i>Vw</i>
Wisconsin	<i>Ws</i>
Wyoming	<i>Wy</i>
* Eastern Group	<i>EG</i>
Central Group	<i>CG</i>
Western Group	<i>WG</i>
Southern Group	<i>SG</i>
United States	<i>US</i>

There are included in the above list all the states of the United States, although at the present time there are some in which workmen's compensation insurance is not written. There have also been added the four groups of states into which the National Council divided the workmen's compensation experience for territorial comparisons, as well as the United States, which in this instance refers to all of the states for which experience was available.

According premium the same treatment as was given payroll and allowing premium to be represented by *P*, the state or policy years and classification would be indicated by means of the same superscripts and subscripts. Thus, $P_{3632}^{Fr::16:17}$ would represent the premium earned from policies covering payrolls to the extent of $W_{3632}^{Fr::16:17}$ in New York for the policy years 1916 and 1917 for Classification No. 3632.

The losses were reported for each classification by nature of injury† and, although the division as to nature of injury was not made on exactly the same basis at all of the sources of reported information, nevertheless it was possible to segregate all of the loss payments into three general groups, namely, "Death and Permanent Total," "Medical," and "All Other." It has, however, been found advantageous to establish a universal form upon which such information should be reported and in the future it may be pos-

* See G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 218.

† See G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 215.

sible to subdivide the losses into more closely defined groups. The Universal Schedule "Z" form which has been adopted divides the losses into the following groups.

Death	Minor Permanent
Permanent Total	Indeterminate
Major Permanent	Temporary

Medical

It will therefore be necessary to refer to losses in a more detailed way than to payroll or premium. The nature of injury under which the losses were incurred will be indicated by a presubscript appearing at the upper left-hand corner of the symbol *L* which represents losses in general. Losses incurred for deaths in the State of New York during the policy years 1916 and 1917 for Classification No. 3632 would be represented by ${}^{DC}L_{3632}^{16:17}$. In order that there may be symbols for all kinds of injury for which reports may be received, the following symbols are suggested. For the same reason as obtained in the state symbolization a two letter code is used for nature of injury.

Nature of Injury	Symbol
Death	<i>DC</i>
Permanent Total	<i>PT</i>
Major Permanent	<i>MP</i>
Minor Permanent	<i>PM</i>
Indeterminate	<i>IC</i>
Temporary	<i>TT</i>
Temporary in Permanent Partial	<i>TP</i>
Medical	<i>MC</i>
Death and Permanent Total	<i>DP</i>
All Others	<i>AO</i>

The first operation which was performed on the experience of the various states was that of combining it on a basic level.* For this basic level the 1917 policy year of the state of New York was chosen. The *basic level* is the experience level as indicated by the workmen's compensation experience reported under policies written in the state of New York during the calendar year 1917 and expiring not more than one year later. New York will therefore be considered the *basic state*. A state other than the basic state whose experience is to be converted to the basic level is known as the *addi-*

* See G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 215.

tional state. The experience for the policy year 1916 for the state of New York must not, therefore, be referred to as basic experience but must be treated exactly in the same manner as the experience of an additional state.

In order to avoid an error which was inherent in the old method of law differentials, namely, that of ignoring the differences in accident distribution in the various states as regards nature of injury, the Manual was split into three groups called manual groups.* The first of these was made up of classifications representative of outdoor industries. It consisted of classifications under which were expected severe and long time temporary disabilities. This group was intended to reflect the effect of those portions of the compensation acts which dealt with temporary total benefits. The second group consisted of classifications under which dismembersments were expected. This group was intended to reflect the effect of the so-called specific schedules of the various compensation acts. The third group was comprised of the remaining classifications.

The losses within each manual group were then subdivided into three sub-groups. The first of these sub-groups consisted of "Death and Permanent Total" losses reported by number only. Inasmuch as death and permanent total cases were later treated on an average cost basis, it was only necessary to determine the accident rate in terms of number of accidents per unit of payroll, and these accident rates needed only to be applied to the average cost for death and permanent total cases established for a given classification to produce the death and permanent total partial pure premium for that classification in dollars and cents per unit of payroll. In establishing basic partial pure premiums for death and permanent total New York average values were used.† The second sub-group of losses was called the "All Other" group and consisted of all losses not death and permanent total or medical. The "Medical" losses made up the third sub-group.

Since "death and permanent total" losses were reported by number of cases it was only necessary to accumulate these additively to bring them to the basic level. In translating "all other" and "medical" losses to the basic level a method known as conversion

* *Ibid.*, page 217.

† See G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 221.

was employed. This method consisted of determining experience differentials known as conversion factors. A *conversion factor* is one such that, if it be applied to the losses of an additional state, the pure premiums developed by the combination of such modified losses with the actual losses of the basic state and the combined payrolls of the two states, when applied to the actual payrolls of the basic state, will exactly reproduce the losses experienced therein.

Mr. W. W. Greene in his original development of the formula,* which was adopted in a modified form by the Actuarial Committee of the National Council on Workmen's Compensation Insurance for the purpose of combining experience, assigned to his conversion factor the symbol E . The weakness of this symbol lies in the fact that it does not indicate in itself the particular state to the experience of which it is applicable nor the basis to which this experience is to be converted. Mr. A. H. Mowbray, in a memorandum to the Actuarial Committee of the National Council, suggested the use of a symbol C for conversion factor in general, with an appropriate superscript to indicate the additional state for which the experience was to be used and a subscript to represent the basic state. Thus, C_{Yr}^J would represent the factor for converting New Jersey experience to the New York basis. This might profitably be extended one step further by appending to the superscript the policy year or years to the experience of which the factor was applicable and by substituting for the subscript Yr the subscript B to represent the basic level. Thus, $C_B^{Jr:16:17}$ would represent the factor for converting New Jersey experience for the policy years 1916 and 1917 to the basic level. If, at a later time another state should be chosen as the base, a subscript such as $Jr:17$ could be used to indicate that the experience has been converted to the New Jersey 1917 level.

Since with but four exceptions the experience for the policy years 1916 and 1917 was used in the revision, a very natural question might be raised as to why the 16:17 need be included in the symbol for the conversion factors. The symbols are, however, suggested with the idea of general use, and it was thought advisable to abide by the fundamental principle of allowing the superscript to represent the particular policy year or years involved.

Conversion factors were established for the "all other" and

* See W. W. Greene, "Upon Combining Compensation Experience from Several States," *Proceedings*, Vol. VI, Part I, No. 13, pages 10-30.

"medical" loss groups and, in order to identify the conversion factor for a state and policy year with the proper loss group, the same device has been utilized as was employed in a preceding paragraph in associating losses with kind of injury. This was a superscript. In this way ${}^{AOC}_B^{::16:17}$ represents the conversion factor for converting the "all other" losses of New Jersey included in the experience of the 1916 and 1917 policy years to the basic level. Similarly, ${}^{MOC}_B^{::16:17}$ refers to the corresponding "medical" losses.

When the experience for all states had been converted to the basic level in the manner described, there was prepared an exhibit for each classification consisting of the converted experience for each state. The states were divided into four territorial groups called respectively Eastern, Central, Western and Southern. The experience for each of these groups of states, as well as for the whole country, appeared on the exhibit. The experience itself consisted of payroll, premium, number and cost of the "death and permanent total" cases, number of cases and amount of the "all other" losses, amount of the "medical" losses and the "total amount." All loss amounts were on a converted basis. In addition, partial pure premiums for "death and permanent total," "all other" and "medical" were presented together with the total pure premium. These partial pure premiums were obtained by dividing respectively the "death and permanent total," "all other," "medical" and "total" losses by the payroll in units of \$100.

The cost of the "death and permanent total" cases was calculated on an average value basis. Since the policy year 1917 for the state of New York was adopted as the basic level, the average values assigned to "death and permanent total" cases were determined from the 1917 Schedule "Z" for the state of New York. The manual classifications were segregated into schedules and for each schedule an actual average value was calculated. The average values were found to fall naturally into five groups, with the exception of a very few instances where there was not sufficient experience within a schedule to satisfactorily indicate an average. Such schedules were combined with others which, in the opinion of the Actuarial Committee of the National Council, were analogous with regard to those conditions upon which an average "death and permanent total" value depends.

In referring to basic pure premiums the symbol π will be em-

ployed while p will indicate the pure premium of an additional state. A partial pure premium will be indicated by the addition of a presuperscript referring to the division of the pure premium to which it applies. The state and policy year will be denoted by superscripts while a subscript will indicate the classification. Thus, the "death and permanent total" basic partial pure premium for Classification No. 3632 would be written $DP_{\pi_{3632}}$ and similarly $DP_{3632}^{Jr:17}$ would refer to the New Jersey 1917 "death and permanent total" partial pure premium for Classification No. 3632.

With the data submitted in the above described form it devolved upon the General Rating Committee of the National Council at this point to establish basic pure premiums for all of the manual classifications. In order to furnish a measure of the deviation of classification experience from the average experience of certain other more or less homogeneous classifications, the whole manual was divided into groups of classifications and the experience for these groups was combined and submitted to the Committee in addition to the individual classification experience. Before adopting partial pure premiums for a given classification, the Committee studied the data by states, territorial groups, and country as a whole, and compared the total classification experience with the group experience. Deaths or permanent total cases which had occurred as the result of catastrophes—a catastrophe being defined as an accident involving losses equal to or exceeding the cost of five deaths—were in general excluded on the theory that they represented a class of losses which should be covered by the one cent catastrophe loading which is to be applied to every rate. The Committee then proceeded to adopt partial and total pure premiums for the various classifications, the selections being governed principally by the experience. Where the experience was abnormal for one or more states and of enough volume to be indicative, and where possibly that abnormality could be specifically assigned to local conditions, an exception was created for that particular state or group of states. Generally, such experience was excluded from the total in determining basic pure premiums. Certain classifications were eliminated entirely, the experience reported for them being combined with the experience of the classifications under which would be written the policies which had furnished the particular data for the classifications which were eliminated. Other classifications were combined in the belief that it would be impossible for a payroll auditor to obtain a correct

division of the payroll, and with the knowledge that any differentiation in the hazards of the classifications on the basis of the limited experience available would be fallacious.

Having decided upon basic pure premiums on the basic level of New York, 1917, three problems presented themselves. The first problem was that of translating the basic pure premiums to the state level for which workmen's compensation rates were to be promulgated; the second, that of modifying the individual state pure premiums in such a way as to reflect the differences in all conditions which would effect pure premium level as between the period over which the experience was collected and that for which the rates would apply; and the third, the problem of measuring the effect which amendments to existing compensation acts, effective since 1917, should have had on the pure premium level.

The first problem was a comparatively simple one, the method having really been determined in the creation of conversion factors. It is obvious that any method which correctly translates the experience of a given state to the basic level may just as correctly be used to convert basic experience to the level of any additional state. This process was called translation and the factors were called translation factors. A *translation factor* is one which, when applied to an established partial basic pure premium, will produce a partial pure premium for an additional state.

The foregoing definition of a translation factor differs slightly from the definition originally adopted by the Actuarial Committee of the National Council on Workmen's Compensation Insurance in that "partial pure premium for an additional state" has been substituted for "partial gross rate for an additional state." The justification of this change lies in the fact that there are many distinct problems involved in creating rates for individual states from combined basic experience which may be solved independently. Were a translation factor defined as that factor which would translate basic pure premiums to state partial gross rates, the factor in itself could not be determined until every problem incidental to the creation of rates was settled, and such a factor in itself would be meaningless unless analyzed.

Since translation is therefore a particular application of conversion, the same general symbol as applies to conversion might also well apply to translation. For instance, $C_{r:17}^B$ would represent the factor for the translation of the basic experience to the level of

the 1917 policy year for the state of New Jersey. The fact that a translation factor applies to basic partial pure premiums and that a conversion factor applies to losses is immaterial, for since partial pure premiums are merely the ratio of losses to payroll, it is evident that a factor applied to the losses would produce the same effect upon the result as if it were applied to the pure premium obtained by the use of the unmodified losses. Nature of injury or loss groups and classifications should be indicated in the manner already described.

The second problem, that of bringing the state partial pure premiums to the period of rate application, was a complicated one because of the many elements which would directly influence the result. Among these there may be mentioned as prominent (a) change in accident rate—*accident rate* is defined as the number of accidents occurring per 1,000 employee years; (b) change in average accident severity—*accident severity* is the measure of the loss of earning capacity resulting from any one industrial accident; (c) change in ratio of compensation to wages; (d) change in administration of the compensation acts; (e) effect of industrial fatigue; (f) amendments to compensation acts, effective subsequent to 1917. Present day conditions of unemployment, labor turnover and industrial unrest, although much less tangible, have also a distinct influence on the pure premium level.

After a careful study of the situation, the Actuarial Committee came to the conclusion that a single factor would measure collectively the effect of not only the other elements hereinbefore listed exclusive of amendments, but any others that might exist. This factor consisted of the ratio of the loss ratios for the policy years 1919 and 1916–1917 on a modified basis* for the state under consideration. Provided the loss ratio for the policy years 1916–1917 could be obtained on the basis of the 1919 manual rates, such a factor would indicate the necessary adjustment in the pure premiums of the 1916–1917 level to produce adequate but not unreasonable pure premiums for 1919.

A *loss ratio* symbolized by ρ for the experience of any policy year is defined as the ratio between the incurred losses and the ultimate

* See A. H. Mowbray, "Actuarial Problems of 1920 Revision . . .," *Proceedings*, Vol. VI, Part II, pages 270–277; also see G. F. Michelbacher, "The Technique of Rate Making . . .," *Proceedings*, Vol. VI, Part II, No. 14, page 244.

earned premium. In this manner $\rho^{Yr:19}$ represents the loss ratio for the state of New York for the policy year 1919. By *incurred losses* for any given year is meant the aggregate amount of money which will have been paid to injured employees in the form of compensation benefits or for medical and surgical treatment for the benefit of injured employees when every case assignable to that particular policy year has been finally closed. Such losses are indicated by L_i . As of any given date the incurred losses may be divided into *paid losses* and *unpaid or outstanding losses*. These would be respectively L_p and L_o . In the event that a particular classification were referred to, the subscript representing incurred, paid or outstanding should appear first, and should be separated from the classification by a colon. Thus, $L_{i:3632}$ would represent the incurred losses for Classification No. 3632.

In dealing with loss ratios, unconverted incurred losses should be used. The reason for this is obvious, since, in projecting the basic experience to the individual state levels, the experience for the additional states have established their own levels; and since the factors which are to be hereafter developed apply to the state partial pure premiums, unconverted state experience must be used in connection therewith.

Ultimate earned premiums refer to the final adjusted premiums collected under policies assigned to a given policy year. Ultimate earned premiums differ from *written premiums* in that written premiums are based upon advance estimates of annual payrolls. The ultimate earned premiums may be divided as of any given date into *earned premiums* and *unearned premiums*. Earned premiums represent the premium obtained by applying the final adjusted rate to the payrolls actually expended at the time as of which earned premiums are being calculated. The difference between ultimate earned premium and earned premium is the unearned premium. Written, ultimate earned, earned and unearned premiums will be referred to respectively as P_w , P_u , P_e and P_o .

The accuracy of a loss ratio depends to a large extent upon the period between the beginning of the policy year under consideration and the date as of which the loss ratio is determined. In any case the results depend upon the accuracy with which the losses outstanding are estimated. The date, therefore, as of which a loss ratio is estimated, is sometimes as significant in interpreting the loss ratio as the ratio itself, and it might be well to make some

provision for this date in the symbol. The presubscript space is available for this time indication and a presubscript indicating the number of months after the beginning of the policy year under consideration should be used. To represent a loss ratio for the 1917 policy year for the state of New York estimated as of December 31, 1919, the symbol ${}_{36\rho}{}^{Yr:17}$ is recommended. The ratio between any two loss ratios would be symbolized as β in which case this relation exists.

$$\frac{{}_{12\rho}{}^{Yr:19}}{{}_{36\rho}{}^{Yr:17}} = \beta_{Yr:19/17}.$$

The factor β is termed *projection factor*. It was realized that such a factor would not apply equally to the various loss groups, and it was therefore split into three factors, which should be applied to the "Death and Permanent Total," "All Other" and "Medical" partial state pure premium respectively. These three factors shall be referred to as ${}^{DP}\beta$, ${}^{AO}\beta$, and ${}^{MC}\beta$ with the proper indication for state and policy years. The factor $\beta^{Tr:19/17}$ provides only for the adjustment of the pure premiums to the level of the 1919 policy year. If, however, it is eventually decided to attempt to forecast the conditions of 1920, 1921 or 1922 as affecting loss ratios it would not affect the symbols or procedure as previously set forth. The only changes necessary would be the alteration of the superscript from 19/17 to 20/17 or 21/17, depending upon whether the loss ratio for 1920 or for 1921 was estimated.

The solution of the third problem, namely that of evaluating amendments effective subsequent to 1917, is dependent in its accuracy upon the stability of the distribution of accidents by nature of injury throughout the various compensation states. The National Council on Workmen's Compensation Insurance has, on the basis of the entire compensation experience reported, compiled what is known as the American Accident Table. This table is a distribution of industrial accidents by nature of injury and presents the number of "Fatal," "Permanent Total," "Permanent Partial" and "Temporary Total" accidents occurring per 100,000 compensable industrial accidents. The fatal cases are distributed by number, relationship and age of dependents. The average age of those permanently and totally injured is indicated. Permanent partial cases are distributed by nature of injury. In this distribution

nature of injury has been interpreted as reflecting the surgical effect of an industrial accident.

Temporary total cases and temporary total cases resulting in permanent partial disability have been distributed by duration of total disability. To denote the number of temporary total cases in the American Accident Table with a duration of disability of more than thirty weeks the symbol ${}_{30}TT$ is used. The number of temporary total cases resulting in permanent partial disability of which the duration of total disability is less than thirty weeks is represented by ${}_{30}TP$. Use of the symbol TT without modification denotes the total number of temporary total cases in the American Accident Table. Similarly, PP represents the total number of permanent partial cases; MP major permanent partial cases; PM minor permanent; PT permanent total; and DC death cases. Permanent partial cases may be further subdivided into dismemberment cases PP^d and non-dismemberment cases PP^n .

The benefits payable for the number of accidents of any type that appears in the American Accident Table should be indicated by the addition of the presuperscript c to the symbols just described. Thus ${}^cTT^{Jr:17}$ would represent the cost of "Temporary Total" accidents according to the American Accident Table and under the 1917 New Jersey Compensation Act.

In valuing annuities under the various compensation acts the regular life insurance notation should be continued. Thus, ${}_n a_x^{(m)}$ would represent the present value of a temporary life annuity payable m times a year for n years at age x the first payment at the end of the first period. A deferred annuity should be denoted by ${}_n | a_x^{(m)}$ which would represent the present value of an annuity, the first payment to be made at the end of the first period after n years payable m times a year at the present age x and payable until the end of life. Again, $a_x^{(m)}$ would represent the present value of an ordinary life annuity payable m times a year, the first payment to be at the end of the first period and to continue to the end of life. There have been various symbols used with reference to the present value of an annuity payable to a widow until death or remarriage. It is recommended that a_x^w represent the present value of an annuity payable to a widow until death or remarriage.

Factors which reflect the effect of amendments to compensation acts are known as *amendment factors*. Amendment factors have

been calculated for the three leading groups and may be indicated by $DP\alpha$, $AO\alpha$ and $MC\alpha$, which refer respectively to the "death and permanent total," "all other" and "medical" loss groups.

The product of the projection and amendment factors and the corresponding state partial pure premiums would produce state partial pure premiums on the basis of the period of rate application. The state partial pure premiums here referred to of course are obtained by multiplying the basic partial pure premiums by the translation factor applying to the state under consideration. The sum of the three state partial pure premiums on this basis would produce the state total pure premiums for any desired classification. There remains the determination of the expense, tax and catastrophe loadings to have the complete data from which manual rates may be determined. Symbolizing the expense factor by ϵ , the tax by τ , the catastrophe loading by γ , and the manual rate by R , any manual rate could be determined by performing the following algebraic calculation:

$$R = \frac{DP_p DP_\alpha DP_\beta + AO_p AO_\alpha AO_\beta + MC_p MC_\alpha MC_\beta}{1 - (\epsilon + \tau)} + \gamma.$$

R in this formula represents the manual rate for a nonschedule rate classification. If, however, the classification be a schedule rated one and if σ be assigned as the symbol for the schedule rating factor then

$$R = \frac{(DP_p DP_\alpha DP_\beta + AO_p AO_\alpha AO_\beta + MC_p MC_\alpha MC_\beta)\sigma}{1 - (\epsilon + \tau)} + \gamma$$

would represent the manual rate for such a classification.

The manual rates will be adjusted, for some classifications at least, by schedule and experience rating. R_S is offered as a symbol for manual rate adjusted by schedule application or as the schedule rate. Similarly, R_E should be used to indicate an experience rate. According to the present practice schedule rating is applied before experience rating and, therefore, the experience rate would oftentimes be a modification of the schedule rate. However, this does not effect the symbol.

In all the states in which experience rating is applied, the plan in use provides for a division in the premium producing partial premiums, each of which is designed to cover some particular group

of losses. It is generally understood that all but one of these states will continue on such a basis, and the writer believes this to be certain enough to warrant the inclusion of a series of notations applicable to experience rating terms and processes.

Because of the slight differences in the exact grouping of the losses in the various state plans, it is recommended that the groups be referred to by subscripts 1, 2, 3, etc., the first in each case to refer to the group including the losses of the most serious nature with the provision only that the last group shall consist of the catastrophe losses if they are reported separately. Thus, if ${}_xP$ represents, in general, premium subject to experience rating, ${}_xP_1$, ${}_xP_2$, ${}_xP_3$, etc., would represent the partial premiums designed to cover the losses of the various loss groups in the descending order of their severity. *Premium subject to experience rating* is the premium computed by extending the actual payrolls at the manual rates for classifications which are not subject to schedule rating and by extending at the appropriate schedule rates the actual payrolls subject thereto. Similarly, *premium subject to schedule rating* would be indicated by sP .

The actual losses reported are subdivided into the loss groups as indicated and these are brought to a premium level by the application of the same factors as would apply to pure premiums established on the bases of those losses in order to construct rates. The factors which translate losses to a premium level are called *modification factors*. When the losses have been translated to premium level, they are known as *indicated partial premiums* and should be symbolized as I with the proper subscripts.

The differences between the indicated premiums and the corresponding partial premiums subject to experience rating are known as *indicated departures* and may be represented by Y_1 , Y_2 , Y_3 , etc. These indicated departures are given a weight, depending upon the size of the risk being rated. This weight measures the credibility of the risk experience and the factors which indicate the exact weights are called *credibility factors* and are indicated by z_1 , z_2 , z_3 , etc. The product of the credibility factors and the indicated departures produce the *partial premium modifications*. The algebraic sum of these premium modifications represents the actual premium modification allowed for the particular risk. The symbol X represents this premium modification. Utilizing the above notation, an

experience rate for a risk would be indicated by

$$R_E = R \left(1 + \frac{X}{{}_E P} \right).$$

For a risk subject to schedule rating $R_S = R_E \left(1 + \frac{X}{{}_E P} \right)$ where R_E represents the experience rate, with X representing the premium modification and ${}_E P$ representing the premium subject to experience rating. The expression ${}_E P + X$ would represent the adjusted premium as a result of experience rating.

A paper on workmen's compensation notation would not be complete without some mention of loss reserves. These may be set up on the basis of accumulated individual reserves or as a percentage of earned premiums less paid losses and claim expenses. The majority of compensation states require that reserves be set up on the former basis for the earlier policy years and on a percentage basis for the later years. The symbol Q serves to indicate in general a special reserve for unpaid workmen's compensation losses as found in Part II of Schedule "P." A prefixed superscript is used to denote the basis upon which the reserve is set up, v indicating individual claim basis and the numerical value of the percentage being used to denote the latter basis, as well as the percentage used. If the former method is employed and a system of loading is applied to the results obtained on the individual claim basis, v' should then be used. A superscript gives the territory to which the reserve is applicable, as well as the policy years for which it is set up. If a particular classification is being dealt with, this is indicated by a subscript. Thus, ${}^{65}Q_{3632}^{US:11-16}$ represents the reserve for workmen's compensation losses on Classification No. 3632 for the United States for policy years 1911 to 1916 inclusive, the reserve being set up on a 65 per cent. basis. Similarly, ${}^V Q_{3632}^{US:11-16}$ represents the corresponding reserve on an unloaded individual claim basis.

The subject of workmen's compensation notation has been presented in the narrative form with the thought that, if the reader were following mentally the constructive procedure outlined herein, the context would clarify and possibly emphasize the need of the suggested symbols. Undoubtedly a subject as comprehensive as workmen's compensation notation cannot be covered thoroughly in such a brief article, but this paper has been presented with the hope

that it may serve as a basis of discussion and that it might lead ultimately to the adoption of some system of notation by the Society.

APPENDIX.

For purposes of reference a list of the recommended symbols and codes has been prepared in the order in which they appear in this paper. Instead of listing each symbol separately specific instances of the application of each symbol have been used as they were used in the main body of the paper.

PAYROLL—Classification No. 3632, New York State, 1916-1917

policy years *W*₃₆₃₂^{Yr.:16:17}

STATES

Alabama	<i>Ab</i>
Alaska	<i>Al</i>
Arizona	<i>Ar</i>
Arkansas	<i>Ark</i>
California	<i>Ca</i>
Colorado	<i>Cl</i>
Connecticut	<i>Ct</i>
Delaware	<i>Dl</i>
Florida	<i>Fl</i>
Georgia	<i>Gr</i>
Idaho	<i>Id</i>
Illinois	<i>Il</i>
Indiana	<i>In</i>
Iowa	<i>Io</i>
Kansas	<i>Ks</i>
Kentucky	<i>Ky</i>
Louisiana	<i>La</i>
Maine	<i>Me</i>
Maryland	<i>Md</i>
Massachusetts	<i>Ms</i>
Michigan	<i>Mg</i>
Minnesota	<i>Mn</i>
Mississippi	<i>Mp</i>
Missouri	<i>Mo</i>
Montana	<i>Mt</i>
Nebraska	<i>Nb</i>
Nevada	<i>Nv</i>
New Hampshire	<i>Hm</i>
New Jersey	<i>Jr</i>
New Mexico	<i>Mx</i>
New York	<i>Yr</i>

North Carolina	<i>Cn</i>
North Dakota	<i>Dn</i>
Ohio	<i>Oh</i>
Oklahoma	<i>Ok</i>
Oregon	<i>Or</i>
Pennsylvania	<i>Pa</i>
Rhode Island	<i>Rd</i>
South Carolina	<i>Cs</i>
South Dakota	<i>Ds</i>
Tennessee	<i>Tn</i>
Texas	<i>Tx</i>
Utah	<i>Ut</i>
Vermont	<i>Vt</i>
Virginia	<i>Va</i>
Washington	<i>Wa</i>
West Virginia	<i>Vw</i>
Wisconsin	<i>Ws</i>
Wyoming	<i>Wy</i>
TERRITORIAL GROUPS	
Eastern Group	<i>EG</i>
Central Group	<i>CG</i>
Western Group	<i>WG</i>
Southern Group	<i>SG</i>
United States	<i>US</i>
PREMIUM—Classification No. 3632, New York State, 1916–	
1917 policy years	$P_{3632}^{Yr:16:17}$
LOSSES—Death losses, Class No. 3632, New York State, 1916–	
1917 policy years	$DL_{3632}^{Yr:16:17}$
NATURE OF INJURY	
Death	<i>DC</i>
Permanent Total	<i>PT</i>
Major Permanent	<i>MP</i>
Minor Permanent	<i>PM</i>
Indeterminate	<i>IC</i>
Temporary Total	<i>TT</i>
Temporary in Permanent Partial	<i>TP</i>
Medical	<i>MC</i>
Death and Permanent Total	<i>DP</i>
All Other	<i>AO</i>
CONVERSION FACTOR—New Jersey 1916 and 1917 to basic level	
for All Other losses	${}^{AO}C_B^{Jr:16:17}$
PARTIAL PURE PREMIUM	
Death and Permanent Total basic partial pure pre-	
mium for Classification No. 3632	DP_{3632}
New Jersey 1917 Death and Permanent Total partial	
pure premium for Classification No. 3632	$DP_{3632}^{Jr:17}$

TRANSLATION FACTORS

From basic to New Jersey 1917 policy year level..... C_{Jr}^B 17

LOSS RATIO

New York 1919 loss ratio ρ $Yr:19$

LOSSES

Incurred L_i

Paid L_p

Outstanding L_o

PREMIUMS

Written P_w

Ultimate Earned P_u

Earned P_e

Unearned P_o

PROJECTION FACTOR

For adjustment of basic pure premiums to New York
1919 level β $Yr:19/17$

AMERICAN ACCIDENT TABLE DATA

Number of temporary total cases per 100,000 industrial accidents of more than 30 weeks' duration... | $_{30}TT$

Number of temporary total in permanent partial cases per 100,000 industrial accidents of 30 weeks' or less duration | TP

Total number of:

Temporary total cases TT

Permanent partial cases PP

Major permanent partial cases..... MP

Minor permanent partial cases..... PM

Dismemberments PP

Permanent partial non-dismemberment cases PP

Permanent total cases PT

Death cases DC

THEORETICAL COST IN WEEKS' WAGES

Cost of temporary total cases in American Accident Table under New Jersey 1917 Act $OTTJr:17$

PRESENT VALUE OF:

Temporary Annuity

Temporary annuity payable m times a year for n years age x | ${}_n a_x^{(m)}$

Deferred Annuity

Age x deferred n years payable m times a year..... | $a_x^{(m)}$

Life Annuity

Age x payable m times a year..... $a_x^{(m)}$

Annuity to Widow until death or remarriage..... a_x^{18}

AMENDMENT FACTOR

Effect of an amendment on "Death and permanent total" loss group	$DP\alpha$
EXPENSE LOADING in per cent. of gross rate	ϵ
TAX LOADING in per cent. of gross rate	τ
CATASTROPHE LOADING in cents to be added to gross rate....	γ

RATE

Manual Rate	R
Adjusted by schedule rating	R_S
Adjusted by experience rating	R_E
PREMIUM SUBJECT TO EXPERIENCE RATING	E^P
PREMIUM SUBJECT TO SCHEDULE RATING	S^P
INDICATED DEPARTURES	Y
CREDIBILITY FACTOR	z
PREMIUM MODIFICATION	X

RESERVES

Reserve for losses under Classification No. 3632 for the United States, policy years 1911 to 1916 inclusive set up on a 65 per cent. basis.....	${}^{65}Q_{3632}^{US: 11-16}$
Reserve for losses under Classification No. 3632 for the United States, policy years 1911 to 1916 inclusive set up on an unloaded individual claim basis	$VQ_{36}^{US: 11-16}$