

THE PRACTICE OF EXPERIENCE RATING.

BY

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The practice as opposed to the theory of experience rating may mean either of two things. It may mean the actual application of a plan to the routine rating of individual risks, or it may imply the development of a practical plan from fundamental theoretical principles. It is the purpose of this paper to deal almost exclusively with the second phase. The subject will be taken to embrace all the interesting considerations found in connection with the formulation of a plan of experience rating from the theory recently developed by the Actuarial Section of the National Reference Committee.*

To a certain extent the theory and practice of experience rating are separate and distinct. The theory of experience rating is fundamental. The practice of experience rating may take on any one of a number of different aspects, depending entirely upon the amount and kind of statistical information available and the particular underwriting considerations to which the plan must give expression. The development of the theory has simplified the practice of experience rating. In the past, experience rating suffered from the lack of a fundamental basis. Then discussions of the subject were, in reality, arguments concerning the practice of experience rating. The scheme or the framework was not fixed; the essential principles had not been reduced to formulae. The various elements were, therefore, matters depending upon the judgment of underwriters and actuaries for their determination.

The theory has narrowed the application of judgment. It is surprising to find that when a point is reached in the practice of experience rating which formerly gave rise to extended discussion, the problem has been so thoroughly analyzed that there is apparently but one solution, the form of which is dictated by the theory and the application of which depends largely upon statistical

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data and only to a limited extent upon actuarial and underwriting judgment. Therefore, instead of an elaborate discussion, a paper on the practice of experience rating today will be a description of the procedure whereby the theoretical basis can be transformed into some practical and workable method.

THE FORM OF EXPERIENCE RATING.

The form of experience rating is represented by the following formula:

$$x = P + z(p - P),$$

where for practical purposes the various elements may be defined as follows:

P = the average manual rate for the risk.

This will be an average rate in all cases, for the reason that no risk is completely described by a single manual classification. The average manual rate for the risk is obtained by weighting the several individual manual rates for the respective classifications with the payroll exposure assigned to each.

p = the rate indicated by the risk's own experience.

Inasmuch as experience rating involves a comparison of p with P , p will be developed from the experience for the risk in exactly the same way that the manual rate for any individual classification is developed from the experience of the classification.

z = the allowed percentage of the difference between the manual and indicated rates.

The form of " z " depends upon the nature of the assumption made in the theory. Practically, " z " may be said to embody all of the mathematical or actuarial theory.

x = the final adjusted rate for the risk.

Inasmuch as experience rating involves a comparison of risk and class experience, it is obvious that P , p and x may be expressed not only in terms of rate or pure premium, but also in terms of premium, provided all factors are treated alike. As a matter of fact, in the practical method of rate modification, it is necessary to use premiums throughout. A comparison is made, finally, of the adjusted premium and the premium computed at manual or schedule rates to determine the experience modification. This in turn is

applied to the individual manual or schedule rates to obtain the adjusted rates for the several classifications involved.* In this paper, for the sake of simplicity, rates will be used exclusively.

It is evident that with this definition of the form of experience rating, the actual construction of a plan is a comparatively direct procedure. It is not as simple as might be expected, however, for there are several practical difficulties to be overcome, and there is still necessity for the exercise of personal judgment because of the limitations of the experience data at present available for the solution of the problem.

DIVISION OF THE RATING PROCEDURE INTO SEVERAL PARTS IN RATING INDIVIDUAL RISKS.

The first question which presents itself for determination is whether, in practice, the rating procedure as applied to the individual risk should consist in a strict comparison of the entire risk experience with the complete manual rate. That is to say, should there be but a single comparison of risk and class experience or should the comparison be in several parts?

In the development of the theory of experience rating, the factors P , p and x were considered in various ways. At no point, however, were they made to represent the collective hazard of the manual classification or of the risk. A logical analysis of the hazard into several elements was assumed and the theory was developed with reference to one individual element, such as the death hazard.

The total hazard in workmen's compensation insurance arises out of the occurrence of accidents resulting in several different types of injuries. The most common analysis of injuries is one which divides them into four different types—fatal, permanent total, permanent partial, and temporary.

Strictly, the theory would necessitate a separate analysis of each of these elements of the hazard. The necessity for some refinement is evident as the probabilities involved range from one in five to one in ten thousand. It is not conceivable that logical results could be obtained if all these hazards were considered as a unit. On the other hand, it is impracticable to make too much refinement as this would tend to complicate the rating procedure. The problem is, therefore to group those elements which naturally go well

* This procedure is clearly described in the example on page 324.

together because of similarity of probability and thus to simplify the plan as much as possible.

The practical effect of considering all the hazard elements as a unit would be to permit abnormal fluctuations of the adjusted rates for individual risks depending upon the occurrence of those accidents the probability of which is very low. Thus, in the case of a small manufacturing risk a death case is not expected except once in a considerable number of years. If the cost of a death case were permitted to influence the rate in proportionately the same degree as a temporary disability case, the result would be an abnormally high rate as soon as the death case entered the risk experience. This would destroy the equity of the plan. In the final analysis, workmen's compensation insurance is carried by an assured for the purpose of protecting himself against those losses which occur with comparatively low frequency. The occurrence of temporary and permanent partial disability cases is so certain that an assured might, after a period of experience, carry his own risk as regards the cost of such cases. The large element of uncertainty, however, which death or permanent total disability cases present, forces him to protect himself by insurance. It is the function of insurance to distribute the cost of cases of this character so that the burden will not fall too heavily upon one individual. It follows, therefore, that one of the fundamental principles of experience rating is that it should not excessively penalize an assured for the occurrence of an accident which, as regards the individual risk, may be considered fortuitous. The fact that a death or permanent total disability case appears in the experience of a risk is not as good evidence as the presence of a large number of temporary or permanent partial disability cases aggregating the same loss.

It is apparent, therefore, at the outset, that there must be some division of the problem in order that proper relative weights may be assigned to the different types of accidents.

In the Actuarial Section several proposals were considered. In the discussion these were finally narrowed down to two. One of these would have divided the problem into two parts, as follows:

1. (a) Death cases.
- (b) Permanent partial disability cases.
- (c) Major dismemberment cases, such as the loss of a hand, arm, leg, eye, etc.

2. (a) Permanent partial disability cases, excluding major dismemberments.
- (b) Temporary disability cases, total and partial.
- (c) Medical cost.

The other proposal, which was adopted,* provides for two divisions and includes major dismemberments in the second division.

In connection with the first proposal, it may be argued that major dismemberment cases do not occur with great frequency; that they cost as much as many death and permanent total disability cases and that their effect, therefore, would be to produce as much inequity, if they were thrown into the classification of accidents which might be considered normal, as the inclusion of death or permanent total disability cases. The answer to this argument is that no statistical information is available for the purpose of developing the problem along these lines, and that as a matter of fact, the total number of these cases in a state experience of considerable volume would be very slight and the possibility of a single accident of this character falling in the experience of a small risk, and thus creating a serious influence on the adjusted rate, very remote.

Having decided upon a two-way division of the problem, the Actuarial Section next established two separate and distinct formulæ. The first is applied to the hazard represented by fatal and permanent total disability cases, the other to the hazard represented by the remaining cases, namely, permanent partial disability, temporary disability and medical cost. The results of the two formulæ are combined to determine the total adjusted rate for the risk.

As a matter of practice, then, the plan of experience rating is applied independently to each division of the hazard and a combination of the results, produced in this manner, is necessary before the final adjusted rate can be ascertained. Symbolically, this procedure may be described as follows:

The general formula for the determination of an adjusted rate is

$$x = P + z(p - P).$$

The adjusted rate for the death and permanent total disability hazards is determined by the following formula:

$$x_1 = P_1 + z_1(p_1 - P_1),$$

* Subsequently, Pennsylvania adopted the first division for use in connection with its plan.

where,

P_1 = the death and permanent total disability partial manual rate for the risk.

p_1 = the death and permanent total disability partial rate indicated by the risk's experience.

z_1 = the experience rating factor.

The adjusted rate for the remaining hazards is then obtained by a similar formula:

$$x_2 = P_2 + z_2(p_2 - P_2).$$

The final adjusted rate, X , is the sum of the two adjusted partial rates:

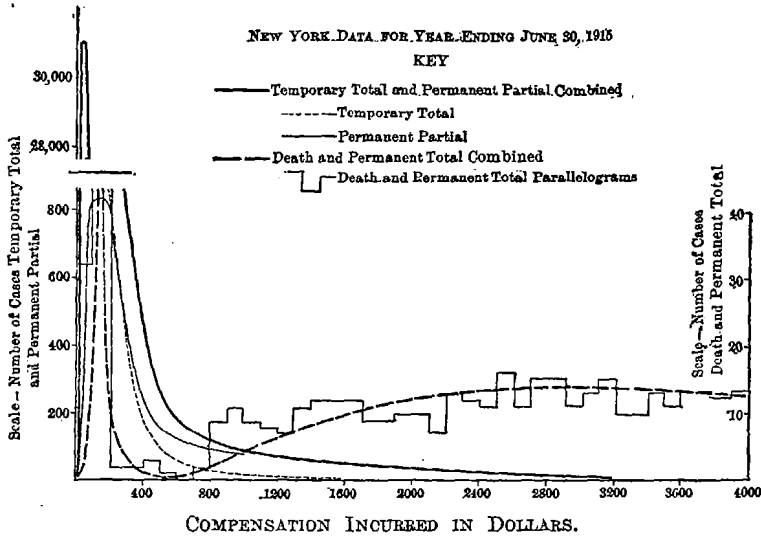
$$X = x_1 + x_2.$$

An estimate which has been made of the probabilities involved in these two divisions indicates that in the first group the range of probabilities is from .0001 to .01 and in the second, from .001 to .5. There is evidently little overlapping.

The validity of the adopted division of the problem is apparent from several statistical studies which have been made of accident experience.

Offhand it would be assumed that if temporary disability cases, permanent partial disability cases, death cases and permanent total disability case were plotted on one graph in accordance with the amount of compensation paid, the maxima of the curves for the individual accident types would appear, starting with the origin, in the order named. It is reasonable to expect, further, that there would be a considerable hiatus between the curves for the temporary and permanent partial disability cases on the one hand and the death and permanent total disability cases on the other, and that it would be possible to demonstrate that the two sets of curves on either extreme might very well each be represented by a single curve or frequency distribution. As a matter of fact, some interesting results were obtained from New York and California experience which in a general way substantiated this reasoning. The results of the investigation for New York* are presented in the following graph which is self-explanatory.

* The New York data, used in this investigation, were secured through the courtesy of Mr. L. W. Hatch, of the State Industrial Commission.



FACTORS FOR THE DIVISION OF MANUAL RATES INTO PARTIAL RATES.

The division of the problem into two parts requires the use of partial manual rates to represent the two elements of the hazard. The second problem, therefore, is the utilization of the available experience for the purpose of obtaining factors to be used for the computation of such partial rates.

It is obvious, first of all, that these factors will vary for the different manual classifications and that it will be necessary to establish more than one set, if proper division of the manual rate is to be made in all cases. Thus, the percentage of the manual rate which is designed to provide for the payment of death losses will be very much different in the textile industry from what it is in the contracting industry, and further there will be many variations within the contracting industry itself.

Some practical method must be devised, therefore, by means of which several sets of factors will be determined for the classifications. Here the problem is limited by the available statistical information. Apparently, one method of procedure would be to have factors for each manual classification. Another would be to produce factors for broad divisions of industry and then to apply them to the rates for all manual classifications within the division. A

third would be to group the manual classifications by rate, to produce factors for each group, and then, by some process of graduation, to translate these group factors into a series of factors to be applicable to the individual rates. If the last method of procedure is followed, the assumption must be made that the relative proportion of losses representing the two elements of the hazard is a function of the size of the rate.

As a matter of practical convenience it has been necessary to employ this third method.

The first approximation to the factor for any classification is obtained by establishing the ratio of the particular loss elements in question to the total losses of an actual experience. The broadest experience available for this purpose is the experience used by the Augmented Standing Committee at the time of the last manual revision as the basis for the establishment of basic pure premiums. If it is assumed that the ratio of death and permanent total disability losses to total losses is a function of the pure premium, factors for the division of the basic pure premium may be obtained by grouping this experience according to the size of the basic pure premiums and establishing ratios for each group.

Permanent total disability losses were not separated in this experience. Neither was it possible to obtain a complete distribution of death losses throughout. It is necessary, therefore, to provide for these deficiencies in the investigation. To provide for the fact that the death losses were not separately stated in all cases, it is necessary to obtain the payroll of all experiences where the death cases were properly segregated and to use this payroll, rather than the total payroll, for the purpose of computing death pure premiums. To provide for the fact that the permanent total disability losses were not separately stated, it is necessary to supply this element by an independent calculation.

Seven groups of classifications according to basic pure premiums have been established for the determination of death ratios, as follows:

Group.	Experience.
I. Less than .05:	
1. Total Payroll	\$542,085,485
2. Death Payroll	386,662,207
3. Death Losses	12,497
4. Total Losses	81,707

5. Death Pure Premium, $\left[\frac{(3) \div (2)}{100} \right]$003
6. Total Pure Premium, $\left[\frac{(4) \div (1)}{100} \right]$015
7. Ratio, [(5)÷(6)]200
II. .05 to .20:	
Total Payroll	1,213,317,650
Death Payroll	851,708,050
Death Losses	84,957
Total Losses	1,314,152
Death Pure Premium010
Total Pure Premium108
Ratio092
III. .21 to .50:	
Total Payroll	1,240,742,468
Death Payroll	791,470,265
Death Losses	352,481
Total Losses	4,148,760
Death Pure Premium045
Total Pure Premium334
Ratio135
IV. .51 to 1.01:	
Total Payroll	635,979,389
Death Payroll	483,877,485
Death Losses	629,815
Total Losses	4,146,160
Death Pure Premium131
Total Pure Premium652
Ratio201
V. 1.02 to 2.03:	
Total Payroll	222,844,884
Death Payroll	164,569,917
Death Losses	526,988
Total Losses	3,058,374
Death Pure Premium320
Total Pure Premium	1.372
Ratio233
VI. 2.04 to 2.56:	
Total Payroll	105,877,017
Death Payroll	84,485,998
Death Losses	447,213
Total Losses	1,985,152
Death Pure Premium529
Total Pure Premium	1.875
Ratio282
VII. 2.57 and over:	
Total Payroll	25,758,587
Death Payroll	21,124,065

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Death Losses	201,093
Total Losses	738,786
Death Pure Premium952
Total Pure Premium	2.868
Ratio332

The results of these computations are repeated below, in simplified form:

Group.	Ratio of Death Losses to Total Losses. (Basic Pure Premium Experience).
I200
II092
III135
IV201
V233
VI282
VII332

Having ascertained the ratios for death losses, it is next necessary to introduce an estimate of permanent total disability cost. In doing this it must be borne in mind that the experience utilized for the determination of the death ratios has been reduced to the level of the cost of the original Massachusetts Workmen's Compensation Act.

An examination of the differential calculation for this act discloses the following figures:

Cost of death and burial benefits	96,964 weeks' wages
Cost of permanent total disability benefits	22,618 weeks' wages
Total cost	<u>119,582</u> weeks' wages

Assuming the accuracy of the differential calculations, if the total cost of these benefits is desired and the death cost is given, the total cost may be determined by increasing the death cost 23.3 per cent.

The differential calculations rest upon the assumption that the distribution of accidents according to severity is the same for all industries. There is, therefore, no means whereby the relationship between death and permanent total disability cost can be ascertained by industries. Consequently, it is necessary to assume that the ratio of permanent disability losses to death losses is constant for all classifications. Proceeding upon this basis, death and permanent total disability factors can be obtained by increasing the death factors for each group 23.3 per cent. If this is done the following table of factors will be the result:

Group.	Death and Permanent Total Disability Factors. (Basic Pure Premium Experience.)
I246*
II114
III166
IV248
V288
VI348
VII409

Given this table, the corresponding table of ratios for "all other" indemnity and medical losses may be obtained by taking the complement of the value for each group as follows:

Group.	"All Other" Indemnity and Medical Factors. (Basic Pure Premium Experience.)
I754
II886
III834
IV752
V712
VI652
VII591

These tables contain factors which can be applied to the basic pure premiums to obtain partial pure premiums for the two loss elements in question. They represent the basis for the construction of a curve from which factors for each individual pure premium symbol have been obtained by graphical interpolation. The details of this transformation need not be described here, as the values for the groups indicate, in a general way, what the trend is.

The next step in the problem is to translate these factors into corresponding factors for any state.

Tables of factors for any state may be computed from the basic tables by a simple calculation. Let us assume that the following information is available:

1. Death and permanent total disability losses on the basic pure premium level.
2. Total losses on the basic pure premium level.
3. Death and permanent total disability partial law differential for each state.
4. Complete law differential for each state.

* This particular factor is abnormal and has been modified for practical use. The modification is not introduced here and the actual results of the experience are used throughout.

If the death and permanent total disability losses, on the basic pure premium level, were multiplied by the proper death and permanent total disability partial law differential, the result, theoretically, would represent death and permanent total disability losses under the state act in question; likewise if the total basic losses were multiplied by the complete differential the result would be the theoretical total losses for the given state. The ratio of the transformed death and permanent total disability losses to the total losses would then represent the proper factor to employ in dividing the manual rates of the state in question into partial rates. The death and permanent total disability factor for the state of New York, for example, would be obtained by the following:

$$\frac{[\text{D. \& P. T. D. losses (basic state)}] \times [\text{D. \& P. T. D. partial differential (N. Y.)}]}{[\text{Total losses (basic state)}] \times [\text{Complete differential (N. Y.)}]}$$

or

$$\left[\frac{\text{D. \& P. T. D. losses (basic state)}}{\text{Total losses (basic state)}} \right] \times \left[\frac{\text{D. \& P. T. D. partial differential (N. Y.)}}{\text{Complete differential (N. Y.)}} \right]$$

However, the first part of this expression is nothing more than the ratio for the basic pure premiums which already has been established. It remains, therefore, to establish the ratio of the death and permanent total disability partial differential to the complete differential for New York. The application of this law differential ratio to the death and permanent total disability factor in the basic table will produce the corresponding state factor. For example, the death and permanent total disability partial differential for New York is 2.48. The complete differential for New York is 1.89. The ratio is 1.31. If the death and permanent total disability factor for the basic pure premiums is multiplied by this value, the result will represent the corresponding factor to employ in the division of the New York manual rates. If this is done, the following table of factors will be obtained:

Group.	Death and Permanent Total Disability Factors (New York).
I322
II149
III217
IV325
V377
VI456
VII536

The factors for the computation of the "all other" indemnity and medical partial rate will then be obtained by taking the complement of each value in this table as follows:

Group.	"All Other" Indemnity and Medical Factors (New York).
I678
II851
III783
IV675
V623
VI544
VII464

It may be argued that these factors are obtained by a method which is based largely upon assumptions. However, it can be demonstrated that the results are not far out of line when compared with actual experience. For instance, corresponding ratios based upon New York Schedule "Z" experience for policy year 1914 may be determined as follows:

RATIOS OF DEATH AND PERMANENT TOTAL LOSSES TO TOTAL LOSSES BY PURE PREMIUM GROUPS.

1. Group.	2. Basic Pure Premiums.	3. Total Losses.	4. Death and Permanent Total Losses.	5. Ratio of Column 4 to Column 3.
I..	Less than .05	\$ 13,261	\$ 1,841	.139
II..	.05- .20	399,927	77,540	.194
III..	.21- .50	1,340,118	311,517	.232
IV..	.51-1.01	1,458,502	476,969	.327
V..	1.02-2.03	1,246,665	424,843	.341
VI..	2.04-2.56	613,917	291,318	.474
VII..	2.57 and over	380,927	189,796	.498

If the ratios in column 5 of the above table are compared with the ratios established for experience rating purposes, close similarity will be noted. As a matter of fact, if the experience rating factors are projected into the total New York losses for the several groups, it will be found that the expected death and permanent total disability losses, on this basis, exceed the actual death and permanent total disability losses of the Schedule "Z" experience by the narrow margin of \$8,970.

WHAT MANUAL RATE SHOULD BE EMPLOYED AS THE BASIS FOR EXPERIENCE RATING?

It is a question whether or not, inasmuch as experience rating involves the use of past experience, the manual rate used for the determination of the experience modification should be the one actually charged during the time the experience was developed. Thus, if experience rates are to be determined in 1918, experience may be available for policy years 1915, 1916 and 1917, and a comparison will be made of the risk experience and a manual rate. The question is whether in this comparison the manual rates which obtained in 1915, 1916 and 1917 should be used, or whether the manual rate at present in existence—the manual rate on the basis of which the policy will be written—should be employed.

So far as this point is concerned, it is only necessary to say that rates in workmen's compensation insurance are becoming more and more accurate as time passes. With each manual revision the volume of data increases, greater knowledge is gained concerning the hazards of the classifications, and the underwriting procedure becomes more firmly established.

It is evident that the very latest manual rate is the most accurate so far established. The manual rate to be used for experience rating purposes, therefore, should be the *present* manual rate.

This simplifies the problem; it provides a single criterion rather than several, for in workmen's compensation insurance the manual rates have changed, on the average, once a year and it follows, therefore, that if the manual rates of the past were used there would be as many different rates for each classification as there were changes during the experience period.

MODIFICATION FACTORS.

Experience rating involves a comparison of risk and classification experience, or what is the same thing, a comparison of the rate indicated by the risk's experience with the manual rate.

The rate used for experience rating purposes is the present manual rate. This rate represents certain cost conditions differing from those represented by the losses of the risk. For instance, it is known that the cost of compensation has gradually increased since the inception of compensation laws. The present manual rate rep-

resents the cost conditions assumed to exist at the present time. The experience losses reflect the conditions which obtained in the past. Again, the statutory provisions of workmen's compensation laws are changing rapidly. The present manual rate measures the cost of the present benefits of the state law. The experience losses may measure the cost of benefit provisions entirely different because of intervening amendments. Further, the industrial conditions which affect the cost of compensation and which are reflected in the present manual rate may be entirely dissimilar to those which existed at the time the risk experience was accumulated.

If the manual rate and the rate indicated by the experience of the risk are to enter into the determination of the experience modification, it is essential that these rates should be comparable, that is, that they should represent the same cost conditions. For this reason modification factors must be introduced to take account of such differences between the cost conditions represented by the experience losses and those reflected in the present manual rate which can be detected, analyzed and evaluated. In addition, because the manual rate contains a loading for management expenses, a similar factor must be applied to the risk pure premium.

The factors which are employed to produce the indicated risk rate from the experience losses are, therefore, of two kinds: first, factors which measure differences in cost conditions between the experience losses and the present manual rates; second, a factor to provide the necessary loading for management expenses, taxes and profit.

Factors of the first class take into consideration the following elements:

- (a) A possible underestimate of the outstanding losses of the risk experience.
- (b) Changes in the interpretation and administration of the compensation law and in the attitude of claimants toward the compensation law.
- (c) Changes in industrial conditions which may affect the production of accidents and their severity.
- (d) Amendments to the compensation law.

The loading for management expenses, taxes and profit is merely a duplication of the corresponding factor employed in the preparation of state rates.

The indicated risk rate will, therefore, be obtained as follows:

$$p = \frac{100 LM}{R}$$

where

p = the indicated risk rate, for any hazard element, per hundred dollars of payroll exposure.

L = the actual risk losses, paid and outstanding, for the hazard element in question.

M = the modification factor to reflect the difference between the cost conditions represented by the experience and the present manual rate, and to provide a loading for management expenses, taxes and profit.

R = the payroll for the risk.

As a matter of fact, there must be a value of " M " for every policy year, and in addition, distinction must be made between the value of " M " for indemnity losses, and for medical losses. The reasons for this arise out of the fact that amendments to compensation laws do not always affect the medical benefits. It is also difficult to estimate medical losses which have been incurred but not paid. Because of this condition the medical cost of the risk is based upon the actual medical losses paid. The paid medical losses for years preceding the current year represent substantially the medical cost for the reason that medical claims mature rapidly and are usually paid in a lump sum and not over an extended period as in the case of indemnity benefits. For the last year, however, it is obvious that if the actual paid medical losses are taken as the basis, a considerable factor to provide for outstanding losses must be applied before the losses are used in the calculation. For these reasons the modification factors for medical losses, although they correspond in general with those used to modify the indemnity losses, differ in some degree.

The following modification factors are given as an example:

TO BE APPLIED TO INDEMNITY LOSSES.

Policy Year.	Underestimate of Outstanding Losses.	Changing Conditions			Management Expenses, Taxes and Profit.	Total.
		Administrative; Attitude of Claimants.	Industrial Conditions.	Amendments to Act.		
1914..	1.00	1.09	1.10	1.04	1.64	2.04
1915..	1.02	1.04	1.10	1.04	1.64	1.99
1916..	1.02	1.02	1.05	1.03	1.64	1.76
1917..	1.08	1.00	1.00	1.00	1.64	1.77

TO BE APPLIED TO MEDICAL LOSSES.

Policy Year.	Underestimate of Outstanding Losses.	Changing Conditions			Management Expenses, Taxes and Profit.	Total.
		Administrative; Attitude of Claimants.	Industrial Conditions.	Amendments to Act.		
1914..	1.00	1.09	1.10	1.75	1.64	3.44
1915..	1.00	1.04	1.10	1.75	1.64	3.28
1916..	1.10	1.02	1.05	1.50	1.64	2.76
1917..	1.25	1.00	1.00	1.02	1.64	2.09

"z" FORMULAS.

Formerly, a discussion of experience rating was incomplete without lengthy reference to such subjects as maximum debits and credits, the graduation of debits and credits and the neutral zone. These artificial limitations were necessary because, in the absence of a well-conceived and thoroughly balanced fundamental basis of experience rating, it was essential that safeguards should be thrown about the plan in order that illogical and abnormal results might be avoided. Naturally, because of their importance, these subjects were always matters for protracted discussion and argument. Now that we have a theory of experience rating, they drop out of sight, for the "z" factor provides all the working power of the experience rating formula.

The determination of the form of "z" is an actuarial problem and properly belongs to the theory of experience rating. The actual application of the "z" formula, however, and the establishment of a practicable method of determining "z" are matters which fall within the scope of this paper.

Three forms of z were considered by the Actuarial Section:

I. The second approximation;*

* See Mr. A. W. Whitney's paper, formula (15A).

II. The first approximation where

$$z = \frac{A}{A + P - P^2}$$

and

$$A = N\epsilon^2.$$

P = the hazard index for the elements under consideration. (Or, from a practical point of view, the partial manual rate for the hazard elements involved.)

N = the number of employees exposed. (Or, from a practical point of view, the payroll exposure for the risk.)

ϵ = a constant of the general form CP^t .

III. Another form of first approximation where

$$z = \frac{PN}{PN + K}$$

and

PN = the gross premium for the risk for the hazard elements under consideration.

K = a constant determined by judgment.

From a theoretical point of view, it is desirable that an experience rating plan should be so designed that it will properly measure every conceivable variation in hazard which may be found in practice. From this point of view there is no limit to the complications which may be introduced in the formulae. As a matter of practice, however, the first essential is simplicity, for the reason that those who actually apply experience rating are not familiar, as a general rule, with mathematical terms or actuarial formulae.

From a practical point of view, therefore, in deciding upon a law for " z ," it is necessary to choose a law which produces the most accurate results in the greatest number of cases and which at the same time is simple not only from the standpoint of the mathematics involved but also with reference to its interpretability by underwriters, raters, agents and assured.

The so-called second approximation form was found to be impracticable because it was too complicated. It was never seriously considered, for it was found impossible to reduce the formula to such terms that it could be readily applied in practice. It was thought, at one stage of the proceedings, that it might be employed

to treat certain abnormal cases which would not receive logical and equitable treatment by a method of first approximation. But this idea was dropped later as unnecessary.

A form of first approximation was, therefore, resorted to as a practicable method for the calculation of "z." In Mr. Whitney's form the essential problem centered around the determination of a law for "ε." As a matter of theory, "ε" should be determined by a statistical investigation. However, for this purpose it would be necessary to have a large number of individual risk experiences and to know more about the true hazard of individual risks than we do at present. Several laws were, therefore, assumed. Experiments were also made with a view to ascertaining whether it was necessary to have two laws of "ε," one for each element of the hazard. In these investigations underwriting and actuarial judgment was relied upon entirely as a guide. Finally, the following laws which seemed to produce results most nearly in conformity with good actuarial and underwriting judgment were taken as a basis for tests:

For the death and permanent total disability hazard,

$$\epsilon^2 = .0006P^{5/4};$$

For the "all other" indemnity and medical hazard,

$$\epsilon^2 = .0015P^{5/4}.$$

Tables were then constructed by means of which values of "z," based upon these two laws, might be obtained by inspection. These were two-way tables, so arranged that, given the payroll exposure for the risk and the partial manual rate for the hazard in question, the value of "z" might be readily located.

Sample values, selected from the Illinois tables, are given below for illustration.

D. & P. T. D.

$$z = \frac{N\epsilon^2}{N\epsilon^2 + P(1 - P)} \quad \epsilon^2 = .0006P^{5/4}.$$

Payroll.	D. and P. T. D. Rate.										
	.018.	.036	.091	.1	.188	.218	.294	.563	.750	1.164	1.455
50,000.....	.003	.003	.004	.004	.005	.005	.005	.006	.007	.008	.008
100,000.....	.006	.007	.008	.009	.009	.010	.011	.013	.014	.015	.016
500,000.....	.026	.032	.038	.042	.045	.048	.051	.060	.065	.072	.075
1,000,000.....	.051	.062	.073	.081	.086	.091	.098	.114	.122	.134	.140
5,000,000.....	.216	.247	.280	.301	.317	.331	.347	.386	.405	.430	.444

“ALL OTHER”

$$z = \frac{N\epsilon^2}{N\epsilon^2 + P(1 - P)} \quad \epsilon^2 = .0015P^{5/4}$$

Payroll.	"All Other" Rate.										
	.082	.164	.409	.614	.832	1.082	1.456	1.687	2.250	2.836	3.545
50,000.....	.023	.028	.036	.041	.044	.048	.053	.056	.064	.072	.081
100,000.....	.046	.055	.069	.077	.084	.091	.100	.106	.120	.133	.150
500,000.....	.194	.224	.269	.294	.314	.333	.366	.371	.402	.432	.466
1,000,000.....	.325	.367	.425	.456	.479	.501	.526	.541	.575	.604	.636
5,000,000.....	.703	.740	.784	.805	.818	.831	.845	.853	.869	.882	.896

The “z” values were then tested by actual application to 190 risks for the state of Illinois. On the whole, the results were entirely satisfactory, but the Section still had to make a decision concerning the practicability of the formula. Depending as it did upon two elements and requiring a two-way table for its application, there was some question as to whether or not values of “z,” based upon laws of “ε,” should be taken. It was decided that if any other first approximation formulae, producing results substantially in accordance with those of the “ε” formulae, could be developed, they should be adopted, provided it could be demonstrated that they were more simple.

The special case of the first approximation where

$$z = \frac{PN}{PN + K}$$

was, therefore, tested with the idea of ascertaining how closely, if at all, the results would approximate those obtained by the “ε” formulae. It was discovered that, with the proper choice of constants, the fit was reasonably close, although it was impossible to produce absolutely corresponding results in all cases. The Section decided, however, that the simplicity of the formula overbalanced the fact that it did not produce the very best result in every individual case.

The constants which were finally chosen for the state of Illinois are as follows:

- For the death and permanent total disability hazard 18,000
- For the “all other” indemnity and medical hazard 8,000

For the purpose of comparison and in order that the general fit of this and the other form of first approximation may be observed,

simplified tables of "z" for the two hazard elements are given below.

D. & P. T. D.

$$z_1 = \frac{P_1N}{P_1N + 18000}$$

Payroll.	D. and P. T. D. Rate.										
	.018	.036	.091	.136	.168	.218	.294	.563	.750	1.164	1.455
50,000.	.000	.001	.003	.004	.005	.006	.008	.015	.020	.031	.039
100,000.	.001	.002	.005	.007	.009	.012	.016	.030	.040	.061	.075
500,000.	.005	.010	.025	.036	.043	.057	.076	.135	.172	.244	.288
1,000,000.	.010	.020	.048	.070	.085	.108	.140	.238	.294	.393	.447
5,000,000.	.048	.091	.202	.274	.318	.377	.450	.610	.676	.764	8.02

"ALL OTHER"

$$z_2 = \frac{P_2N}{P_2N + 8000}$$

Payroll.	"All Other" Rate.										
	.082	.164	.409	.614	.832	1.082	1.456	1.687	2.25	2.836	3.545
50,000.	.005	.010	.025	.037	.049	.063	.083	.095	.123	.151	.181
100,000.	.010	.020	.049	.071	.094	.119	.154	.174	.220	.262	.307
500,000.	.049	.093	.204	.277	.342	.403	.476	.513	.584	.639	.689
1,000,000.	.093	.170	.338	.434	.510	.575	.645	.678	.733	.780	.816
5,000,000.	.339	.506	.719	.793	.839	.871	.901	.913	.934	.947	.957

COMPUTATION OF "K" VALUES FOR THE COMPENSATION STATES.

Having determined the form of "z," the problem next to be solved is that of computing constants for the "z" formulas for the different states.

The basic principle assumed in this work is that, in general, the "z" value for a particular risk should be the same in all states, that is, that the percentage of the deviation allowed for a given risk should be independent of the state in which the risk is experience rated. Inasmuch as the premium for the hazard element varies from state to state, it is necessary to make a corresponding modification of the constants in order that there may be consistency in the values of "z."

The "z" formulas are:

Death and permanent total disability

$$z_1 = \frac{P_1N}{P_1N + K_1}$$

“All other” indemnity and medical cost

$$z_2 = \frac{P_2N}{P_2N + K_2}.$$

Given Illinois values of 18,000 and 8,000 for K_1 and K_2 , respectively, the problem is to determine corresponding values for the remaining compensation states.

In solving this problem the following notation will be adopted:

(pp) = basic pure premium,

P = Illinois manual rate (Illinois partial manual rates will then be designated by P_1, P_2),

M = Illinois multiplier,

L = Illinois law differential,

D = Illinois death and permanent total disability differential,

A = Illinois “all other” indemnity and medical law differential,

P' = state manual rate (state partial manual rates will then be designated by P_1', P_2'),

M' = state multiplier,

L' = state law differential,

D' = state death and permanent total disability law differential,

A' = state “all other” indemnity and medical law differential.

Consider the “ z ” formula for the death and permanent total disability hazard. Let z_1 refer to Illinois and z_1' to any other compensation state. Then in accordance with the fundamental assumption,

$$z_1 = z_1' \text{ or,}$$

$$\frac{P_1N}{P_1N + 18,000} = \frac{P_1'N}{P_1'N + K_1},$$

from which

$$\frac{N}{N + \frac{18,000}{P_1}} = \frac{N}{N + \frac{K_1}{P_1}}.$$

It follows, therefore, that

$$\frac{K_1}{P_1'} = \frac{18,000}{P_1} \text{ or } K_1 = \frac{18,000P_1'}{P_1}.$$

where

$$P'_1 = \frac{M'(pp)D'}{L'} \quad \text{and} \quad P_1 = \frac{M(pp)D}{L};$$

therefore,

$$K_1 = 18,000 \frac{M'}{M} \times \frac{L}{L'} \times \frac{D'}{D}.$$

Evaluating for Illinois,

$$M = 3.32,$$

$$L = 1.37,$$

$$D = 1.38.$$

Substituting these values in the general formula,

$$K_1 = 5382 \frac{M'}{L'} \times D'.$$

By a similar process,

$$K_2 = 2428 \frac{M'}{L'} \times A'.$$

It will be seen that all that is necessary for the determination of K_1 and K_2 for any state is the state multiplier, the state law differential (complete) and the state partial law differentials for death and permanent total disability and "all other" indemnity and medical losses.

If this formula is applied to New York, for instance, the constants, in round numbers, will be:

$$K_1 = 28,800,$$

$$K_2 = 8,600,$$

and the "z" formulas will be,

for death and permanent total disability,

$$z_1 = \frac{P_1 N}{P_1 N + 28,800}.$$

for "all other" indemnity and medical cost,

$$z_2 = \frac{P_2 N}{P_2 N + 8,600}.$$

The use of these formulas does not require tables, although the results might be worked out in advance and presented in one-way tables. As a matter of practice, all that is necessary is the determination, for each hazard element, of the partial premium at manual rates. Once this has been determined the value of "z" can be ascertained by substituting this partial premium in the appropriate "z" formula. To take a simple example, if a partial "all other" indemnity and medical premium of 8,600 were obtained in the case of an individual risk in New York, the value of "z" would be 50 per cent. That is to say, in this case, 50 per cent. of the difference between the indicated and manual rates would be allowed as the experience modification for "All Other" indemnity and medical cost. Thus, if there were no losses at all, the plan, on this part of the rate, would produce a 50 per cent. credit.

QUALIFICATIONS.

In the plans that have been used in the past there have been three important qualifications: a qualification as to the experience period and certain limitations concerning the payroll and premium required before a risk could be experience rated.

The reasons for these qualifications were quite obvious. A certain experience period was required to insure a sufficiently long observation of the risk experience to make certain that it was normal and not the result of chance. The payroll requirement was used to insure the application of the plan only to such risks as were of sufficient size to produce a representative and dependable experience. The premium limitation was used to bring in the hazard of the risk and to insure that a sufficient expectation was represented by the experience data to warrant the expense and trouble of determining an experience modification. It was also used as a measure of the dependability of the experience, although not to the same extent as the payroll.

In the new plan, there will be a qualification as to the experience period. In general, it may be said that the minimum experience period will be two years and the maximum four years. Two years was chosen because of the opinion that a period of two years is necessary to render certain the securing of a representative experience. Four years was taken in the belief that past experience more than four years old is of little or no value in the determination of future rates because of the constantly changing conditions in workmen's compensation insurance.

The payroll limitation has been dropped. In the past, a payroll limitation was necessary, just as other artificial limitations were necessary, to guard against abnormal results in exceptional cases. The new theory of experience rating will not produce inconsistent results even in the case of the smallest risks. It is no longer necessary, therefore, to set up a limit which will exclude certain risks because of doubt as to the dependability of the risk experience.

A premium qualification still remains and serves a valuable purpose. While the plan will produce a logical result even in the case of a very small risk, as a matter of practice, this will be an insignificant departure from the manual rate and it comes down to a question of whether the expense of computing an experience modification is warranted by the result. It is necessary, therefore, to depart entirely from the question of the accuracy and to decide upon a premium which will be considered sufficient to warrant the expense and labor of computing an experience modification. The question is purely an administrative one rather than an actuarial or statistical one. In the new plan, a premium limitation of \$500 for the minimum experience period or roughly \$300 per year, is required before a risk can be experience rated.

EXPERIENCE TO BE USED.

Experience rating involves the use of experience for the risk, or in other words, the use of a loss history. Inasmuch as the plan is to be applied only for the modification of workmen's compensation rates, it is obvious that no employers' liability experience should be used. This requirement, coupled with the minimum experience period limitation, makes it impossible to employ experience rating in any workmen's compensation state until the date of the second anniversary of the workmen's compensation act.

In addition, only such experience as can be obtained from insurance carriers is admitted. Thus, experience rating cannot be done upon the basis of experience submitted by a self-insurer. The reason for this qualification is that the compilation of statistics in workmen's compensation insurance is a very technical matter. To be dependable, statistical information must be produced under the proper conditions, with adequate and intelligent supervision. It is probable that the experience of a self-insurer will not represent the same careful methods of compilation as the experience of an insurance carrier, whose business it is to maintain records of this

character. Furthermore, such experience as the self-insurer has available, has been kept for purposes which in no case correspond to the requirements of the experience rating plan. Therefore, its use is not permitted. Thus, in the case of a new risk entering the insurance field, experience rating cannot be applied until the second anniversary of the issuance of a policy of insurance.

TREATMENT OF THE CATASTROPHE HAZARD.

In the theoretical development of experience rating, the theory of probabilities used as the basis for the formulae is the theory of independent events. A catastrophe is not an independent event. The theory, therefore, does not contemplate the inclusion of losses of this character.

From a practical point of view this would require either the entire disregard of catastrophies or the elimination of a certain excess in every case, retaining, for experience rating purposes, only that part of a catastrophe loss which might be considered normal. In either event, it is necessary to give special consideration to those classifications which present a serious catastrophe hazard. In these cases, the normal losses, when compared with the manual rate, will always indicate a low loss ratio because of the large element in the rate providing for the occurrence of catastrophies.

For instance, in the case of powder manufacturing, the normal losses are very low as compared with the manual rate. The normal loss ratio is fictitious, for a large part of the manual rate in this case is devoted to the accumulation of a fund from which heavy losses will be paid when they occur. Therefore, if the plan were applied to this rate without modification, too great an experience credit would be permitted, too much of the premium would be returned and the catastrophe reserve would be impaired.

The Actuarial Section has recommended that a special list be made of all classifications which are considered to present an exceptional catastrophe hazard. In these cases, a part of the premium will be set aside for the catastrophe reserve and will not be subject to experience modification. This is equivalent to making a three-way division of the manual rate. First, the manual rate is divided into the normal rate and the rate to provide for the catastrophe hazard. The catastrophe rate is set aside and is subject to no modification. The normal rate is then divided into the two hazard elements and rated in accordance with the plan. In this way, a ficti-

tious loss ratio is avoided and the adjusted rate reflects the normal hazard of the risk as indicated by the ordinary losses.

In addition, it has been provided that each catastrophe (that is, an individual accident in which five or more people are involved and the cost of which is not less than \$12,500) shall be singled out for special consideration. The loss in any case of this character in excess of \$12,500 will be eliminated entirely. The cost to the limit of \$12,500 will be considered as a normal loss and will be used in the determination of the adjusted rate.

COMBINATION OF EXPERIENCE AND SCHEDULE RATING MODIFICATIONS.

In manufacturing risks both schedule and experience rating are used as methods of rate modification. It is necessary, therefore, to have some method of combining the results of the application of the two plans.

In the past, the usual method of combination was to permit the algebraic addition of the schedule and experience rating modifications with an arbitrary limit of 40 per cent. Expressed in terms of a formula, this is equivalent to the following:

$$R = M[1 - (S + E)],$$

where

R = the final adjusted rate,

M = the manual rate,

S = the schedule modification, expressed decimally,

E = the experience rating modification, expressed decimally.

Thus, in the case of a manual rate of \$1, a schedule credit of 15 per cent. and an experience debit of 5 per cent., the final adjusted rate would be \$.90.

This method is obviously incorrect, if for no other reason than that it leads to a duplication of credits or debits, as the case may be. Proper weight is not given to either the schedule or experience rating modification, nor is there any logical method of combining the results.

During the recent discussion of experience rating, the theory was advanced that schedule rating might be looked upon as a method of refining the classification of a risk and that experience rating should not be applied until after the schedule had first been used to adjust the manual rate. In other words, a logical sequence of rat-

ing was established. In this sequence the manual comes first, the schedule next and the experience rating plan last.*

A risk is presented for rating. The manual is consulted. A classification is found which describes the risk. The rate for this classification, and the terms of the classification itself for that matter, are approximate. With a single manual and a limited number of classifications, this is bound to be the case. To make a better approximation to the actual classification and hazard of the risk, the schedule is applied to the manual rate. If there are peculiar physical conditions in the risk which cause it to be different in some respect from the average or typical risk described by the classification, the schedule measures this difference and reflects it in the schedule modification, so that when the schedule has had an opportunity to measure the risk for the rate which should be charged, a much more accurate estimate is obtained than the manual rate. The experience rating plan is then applied, not to the manual rate, but to the manual rate adjusted by schedule rating. The result is the final adjusted rate. In this method of procedure, the steps are logical and it can be demonstrated that there is little or no duplication of debits or credits.

This method was chosen for the new experience rating plan. It may be demonstrated by the following formula:

$$R = [M(1 - S)](1 - E),$$

it being understood, of course, that $[M(1 - S)]$ will be obtained before the experience rating plan is applied. Thus, in the case of a manual rate of \$1.00, a schedule credit of 10 per cent. will make the rate adjusted by schedule \$.90. Ninety cents will then be used as the basis for the determination of the experience modification. If the experience modification, on this basis, proves to be a 5 per cent. debit, the final adjusted rate will be \$.945.

EXAMPLE OF THE APPLICATION OF THE PLAN TO THE PROBLEM OF RATING AN INDIVIDUAL RISK.

In order that the reader may appreciate the practical significance of the matters described in this paper, an example is given of the actual application of the new plan of experience rating to the

* It should not be assumed that this is the only theory on this subject. A second theory, which can be substantiated by logical analysis, would place experience rating second and schedule rating as the final step in the rating scheme.

problem of modifying the rates for an individual risk. For this purpose an actual Illinois risk has been chosen which involves the following manual classifications:

- 2501—Clothing Manufacturing,
- 8810—Clerical Office Employees (not otherwise classified),
- 8742—Salesmen (Outside), Collectors and Messengers.

Experience is available, for the risk, for the period from January 14, 1914, to October 14, 1917. The expiration date of the policy is January 14, 1918, but because experience rates are promulgated prior to the date of expiration, the experience has been brought down to a date ninety days prior to that date. For this period, the payroll exposure by classifications is as follows:

Classification Number.	Payroll Exposure.
2501	\$1,438,607
8810	174,868
8742	73,843

In this case, let us assume that the schedule rating plan will not be applied. This assumption is contrary to fact, because this risk is one which is subject to schedule rating and it would, therefore, receive such treatment. The assumption is made for the purpose of simplifying the example.

The present manual is consulted and Illinois manual rates are determined for the three classifications involved. The premium at manual rates for the risk is then computed as follows:

Classification Number.	Payroll Exposure.	Present Manual Rate.	Premium.
2501	\$1,438,607	\$.33	\$4,747
8810	174,868	.11	192
8742	73,843	.19	140
Totals	\$1,687,318		\$5,079

If the total premium at manual rates is divided by the total payroll exposure, the average rate for the risk will be found to be \$.301. If this rate is used as an item of entry in the table of factors, the division of the premium at manual rates into two parts will be ascertained to be as follows:

Hazard Element.	Factor.
Death and Permanent Total Disability Cases149
“All Other” Indemnity Cases and Medical Cost851

Accordingly, the two separate parts of the total premium at manual rates will be:

Hazard Element.	Premium.
Death and Permanent Total Disability Cases	\$757
"All Other" Indemnity Cases and Medical Cost	4,322

Turning now to the losses of the risk, we find, first of all, that there are no death or permanent total disability losses. The exhibit of such losses, as were incurred, by policy years and the method of translating these losses into terms of premium so that a comparison may be made of the risk experience and the manual premium will be as follows:

Policy Year.	"All Other" Indemnity Losses, Excluding Medical Losses.			"All Other" Medical Losses.			Total Indicated "All Other" Premium.
	Actual Losses.	Modification Factors.	Indicated Premium.	Actual Losses.	Modification Factors.	Indicated Premium.	
1914....	\$ 53	2.25	\$119	\$ 61	2.25	\$137	\$256
1915....	46	2.10	97	104	2.05	213	310
1916....	79	1.89	148	99	2.01	199	347
1917....	0	1.94	0	33	2.24	74	74
Totals.	\$178		\$364	\$297		\$623	\$987

The next step is the substitution of the available data in the two experience rating formulae.

Rating of Death and Permanent Total Disability Elements.

The formula for this division is,

$$x_1 = P_1 + z_1(p_1 - P_1),$$

where

x_1 = the adjusted D. & P. T. D. premium,

P_1 = the manual D. & P. T. D. premium,

p_1 = the indicated D. & P. T. D. premium (from risk's experience),

z_1 = the experience rating factor for D. & P. T. D. elements.

In this case,

$$z_1 = \frac{[\text{Manual D. \& P. T. D. premium}]}{[\text{Manual D. \& P. T. D. premium}] + K_1}$$

K_1 is found from the Illinois tables to be 18,000.

Assembling these various items and substituting them in the formula at the proper places produces the following:

$$x_1 = \$757 + \frac{(757)}{(757 + 18,000)} (0 - \$757),$$

from which

$$x_1 = \$727.$$

Rating of "All Other" Elements.

The formula for this division is,

$$x_2 = P_2 + z_2(p_2 - P_2),$$

where the symbols correspond to those in the D. & P. T. D. formula.

In this case,

$$x_2 = \frac{[\text{Manual "All Other" premium}]}{[\text{Manual "All Other" premium}] + K_2}$$

Upon consulting the Illinois tables, K_2 is found to be 8,000.

The determination of the adjusted "All Other" premium is, therefore, as follows:

$$x_2 = \$4,322 + \frac{(4,322)}{(4,322 + 8,000)} (\$987 - \$4,322),$$

from which

$$x_2 = \$3,151.$$

Determination of Final Adjusted Rates.

The total adjusted premium, "X," is then obtained by the following formula:

$$X = x_1 + x_2.$$

Substituting for x_1 and x_2 , we find

$$X = \$3,878.$$

The premium at manual rates for the risk is \$5,079. The experience rating plan has produced a total reduction in this premium of \$1,201, which expressed in a percentage is equivalent to a credit of 23.6 per cent. This is the final experience modification.

To ascertain the final adjusted rates for the three classifications involved, this percentage is applied to the manual rate for each classification. In this way the following results are obtained:

Classification Number.	Manual Rate.	Experience Rate.
2501	\$.33	\$.252
881011	.084
874219	.145