The Determination of Pure Premiums for Minor Classifications on which the Experience Data is Insufficient for Direct Estimate.

BY

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In the several papers in our Proceedings and in the Transactions of The Actuarial Society of America dealing with compensation premium or rate making, the starting point has been a classification pure premium derived by the well-known formula, $\pi = L/P$. It has been generally recognized that it will be impossible to determine the pure premiums in this way for each classification, and that some process of association must be resorted to in order to develop premiums for those classifications where the data is insufficient.*

No one has yet come forward with a study of how, after such association has been determined upon, the data should be combined to develop the premiums for the minor classifications. Yet if this is the only way to determine rates for these classifications, there must be some rule or method of procedure which will stand the test of scientific analysis and criticism. It was with a view to discovering such a method that the present study was undertaken, and it is in the hope that the result thereof may be subjected to such a test that it is now presented here.

Faults of Methods Heretofore Used.

It is well known to the members of this Society that last year when rates for compensation insurance in New York were being made, it was announced that they would be obtained by applying a multiplier of 3.24 (covering law differential and expense loading) to the pure premiums developed from Massachusetts experience and recorded in Schedule Z; but that when the published rates were divided by this factor and the basic pure premiums so found were

* See Rubinow, Proceedings, I, 10–23; Mowbray, *T. A. S. A.*, XV, 92–93; Ryan, *T. A. S. A.*, XV, 369; Whitney, *T. A. S. A.*, XVI, 215.

multiplied into the Schedule Z payrolls it was found that the "expected losses" so found exceeded the actual losses as shown in the schedule by approximately 23 per cent. Considerable publicity was given the matter at the outset, but later, on the statement of the Insurance Department that there was no chicanery involved, the matter was allowed to drop without further press comment. Aside, however, from the statement that Massachusetts experience was not strictly followed, that personal judgment of experienced underwriters aided by data from other sources was also used and that the difficulty presented by scanty data was bridged by grouping analogous hazards, no clear explanation was furnished why this loading of the pure premium occurred, although it was stated it was not unexpected.

It is generally understood that these rates were made by a conference of the best underwriters looking upon the general questions involved from more or less divergent viewpoints and the product was the result of a study of the data in the light of thorough discussion among them. As I understand it, they considered the classifications and associated those in which the hazard was thought so far analogous as to justify it. Premiums for the group were then fixed largely on the basis of the experience of the most prominent classifications though modified by personal judgment for some classifications and the assumption of minimum premiums in certain cases. Such a process carried out by conservative men may be expected to produce results which will deviate more or less from the fundamental data and, fortunately, generally on the safe side as in the case in question.

It would seem, however, that much experience data was, to all intents and purposes, thrown away at least as far as direct use for determination of rates is concerned, because it dealt with a large number of classifications with so small an exposure upon each that it apparently was considered unavailable. Even at best our experience data for many years will be all too scanty and to further restrict it by absolutely discarding a considerable part seems unjustifiably wasteful if a method can be found to utilize it. Addition of data where the hazard is considered to be the same may utilize some of it, yet even with this help we lose much unless we can find some more scientific way of working over our material.

Personal Judgment Must be Used to Supplement Statistical Data.

If the writer's conclusions as to mathematical risk* are sound and the limitations therein set forth must be further restricted for other considerations such as the number, size and character of establishments entering into the experience, it is evident that so long as the manual in use follows present lines there must always be many classifications upon which the experience will be so limited that equitable rates cannot possibly be made from classification experience. Yet, if we are not to discriminate in favor of one industry by transferring too large a part of its accident cost, or against another by placing upon it a burden it should not bear, we must discriminate in rates according to hazard so that, subject to the disturbing influence of accidental variation, each classification shall stand substantially on its feet. We cannot do this without making use of able personal judgment.

The problem before the profession in attempting to formulate a plan of scientific rate making (and for one, I believe that if we are to create and maintain confidence in our profession and its work we must before long place before the public something which will appeal to them as more scientific than methods heretofore in vogue) is to combine personal judgment and experience data in such a way as to use all the light available and produce results which can be predicted in advance with a considerable degree of accuracy, and further, can be tested and checked up as the work progresses.

When Personal Judgment is at its Best—Grouping of Classifications.

No matter what the field in which personal judgment is to be exercised, it is axiomatic that if that judgment is to be quantitative in character and not merely qualitative it will be much more accurate if confined to discrimination between the elements of a relatively small and homogeneous group than when like discrimination must be made between individuals in a large heterogeneous group. For example, we will form much more accurate judgment of the relative heights of a group of men than of the relative heights of a group of an equal number of objects composed of trees, buildings, men, animals, etc. And again even within a homogeneous group,

* Proceedings, I, 24–30.

judgment of relative values will be more accurate than judgment of absolute values. We will then probably gain, if the exercise of personal judgment is along these lines. This presumes a grouping of manual classifications according to nature of hazard, and where there is a wide variation in degree with further subdivision into sections according to degree.

Grouping of classifications has been the subject of considerable discussion recently and there seem to be two rather divergent lines of approach. The State Industrial Commissions accustomed to census grouping by products are inclined to follow that precedent, and in this they are encouraged by the phraseology of our manual classifications, which are generally expressed in terms of product. Of course, from an actuarial point of view the only grouping which is of value is a grouping according to hazard, and if the data are to be combined without modification only those classifications should be associated which are alike in both kind and degree of hazard within pretty narrow limits. It is proposed, however, to modify the data so as to justify its use in combination when the hazard is alike in kind, but varies in degree.

Theory Underlying Proposed Method.

It is an old mathematical trick in attempting the solution of a problem to assume that it has been solved and study the relations between the several elements from which very often a method will be developed which will furnish a ready solution of the problem. This method may be advantageously applied to the matter in hand.

If it be assumed that there is available a large volume of data on a group of, say ten, kindred classifications such that for each the exposure is in all respects sufficient to give a thoroughly dependable pure premium, then obviously we can express the hazard of each as a percentage of one taken as the standard of reference. If then, we take the reciprocal of each of these percentages and multiply the losses in each classification by it we will obtain a hypothetical loss figure which, when divided by the payroll, will give a pure premium equal to that for the base classification. Of course, the addition of the payrolls and hypothetical losses will give aggregate figures from which the same pure premium would be derived. Multiplication of this pure premium by the percentages expressing the relation of the several classifications to the base will again, obviously, reproduce the original pure premiums.

Let it now be assumed that this data has been lost and that only the percentages remain, but that we know the percentages are accurate and were derived from a dependable experience. In place of the large volume of data for the group we may now have only a small exposure on each classification, though the aggregate exposure if on one classification would be sufficient to furnish a dependable pure premium. Even though the pure premiums in such limited data bore very different relations to each other from the percentages derived from the large experience, it would seem we would be amply justified in assuming that this difference was entirely accidental and that if we multiplied actual losses by the reciprocals of our percentages and added the results, dividing them by the aggregate payroll, we would have a true base pure premium from which the classification pure premiums could be found by multiplying by the percentages known to be based upon ample experience. Under certain circumstances, which could be suggested, this process might somewhat distort results, but it is believed that these circumstances under which serious distortion would occur are very unusual.*

Statement of Proposed Method.

Of course, in actual practice, we will never have a series of percentages derived from dependably large, statistical observation of payroll exposure, and then be compelled to determine pure premiums from limited data because the more extensive data is lost. But it would seem that if we can find the percentages from any source, for example, from personal judgment, which we regard as dependable that the process may still be used.

This, then, is the proposed method. Let the relative hazards within a group be determined by the kind of personal judgment hereinafter referred to and expressed as percentages of a given classification. Let the losses on the several classifications be multiplied by the reciprocals of these values, and the sum of the products be taken and be divided by the sum of the payroll exposures. Let the pure premiums thus produced be multiplied by the judgment percentages and the results taken as the classification pure premiums, subject to test by comparing the expected losses for the group produced by their use with the actual losses, and subject to such further adjustment as this test shows to be necessary.*

* See appended note.

The method seems to have the following advantages:

1. It combines personal judgment and statistical data according to a scientifically developed theory.

2. It uses every bit of usable data and does not distort the result by allowing only a part of it to have influence upon the final result.

3. It produces results which are subject to check at intervals throughout the work.

4. It keeps competitive and commercial aspects of the problems in the background during this part of the work, and this makes for calmer and fairer judgment.

5. It calls for the use of judgment under such circumstances as to make it most accurate and dependable.

The only requirement the theory makes as to the determination of the percentage relationship within the group is that it shall be so fixed that its accuracy may be considered as dependable as though such determination rested upon the most acceptable statistical basis. Apparently simple, this requirement is, however, most exacting, but not more so, nor indeed as much so, as the methods heretofore in use if best results are to be obtained. It is the writer's personal view that when the problem is restricted, as here, to discriminate between degree of hazard among the members of relatively small groups where the hazard is of essentially like quality, best results will be obtained if this work is first done by high grade engineers and then subjected to the criticism of competent underwriters. I believe the application of engineering judgment to the problem in this form is of fundamental importance.

Why Engineering Judgment Preferred.

As it has been heretofore considered that rate-making is the function of the underwriter primarily, with such assistance as the actuary may render, there should probably be some explanation furnished of the proposal to transfer the most important part of this work to the engineering department which it has been considered should have little if any part in rate-making. This explanation is to be found in a study of the work performed in the three departments and the way in which this work tends to qualify those performing it for exercising this judgment.

The function of the actuarial department is the recording of payrolls and losses by classifications so as to form experience tables,

and the making of computations of rates and reserves from such tables. Usually this department also compiles any other statistics the office may require. Clearly, this work does not qualify for the exercise of the type of judgment here required since the actuary can only work from the statistics before him, and in the absence of adequate statistics he has no basis of judgment.

The service the underwriter performs for his company is the selection of business so as to secure a satisfactory profit for his stockholders, or margin of dividends for his policyholders if his company be a mutual company. This calls for the exercise of judgment in discriminating between risks in the same class, having regard to the promulgated rate for that class. The underwriter, therefore, is constantly studying statistics of experience with individual risks in order to determine which of a given group will probably give a favorable loss ratio for his company and which should be rejected because the loss ratio will probably be unfavorable. The statistical data with which he works are not primarily group data of payrolls and losses, but individual loss ratios compared with general loss ratios. If, due to the intrusion of a few bad risks the experience of his company with a given classification has proven unfavorable over a term, he obtains the impression that this business is poor business and should be discriminated against, and on the other hand, if certain lines of business prove especially profitable to his company, this seems to him good business and in rate-making he will be disposed to give it a low rate. It may be that his favorable experience has been due entirely to unusual skill of his own or his agency organization in securing desirable business, or to his engineering department's work in improving the character of the particular examples of this type of business which comes to him. His judgment throughout is formed by impressions of financial transactions with all the elements of luck and skill which enter into them. His attention is cencentrated on the abnormal (good or bad) not the normal or average.

The duty of the safety engineer (and by this is meant a distinctly different type of man from the viewpoint of education and experience than a mere safety inspector) is to prevent accidents where possible, and where it is not possible to prevent them to minimize the seriousness of their consequences. To do this efficiently in any industry he must know the types of machinery used in that industry and their inherent hazards, the processes carried

on, the raw material used, chemical combinations involved, etc., and the average as well as the superior and under-average conditions of operating practice. In order that he may not recommend the expenditure of money on improvements which will not give a commensurate return, he must know the relative exposure to each type of hazard.

The theory of classification rate-making, as the writer understands it, is that the hazard determines the loss so that when a satisfactory individual rating schedule has been developed for distinguishing between individual risks, those having the same hazard in kind and degree may take the same classification rate. If this is the principle which the making of classification rates is presumed to follow, then it would seem that because of his knowledge of inherent hazard the engineer is primarily the one best qualified to exercise the kind of judgment called for in this process, to accurately distinguish relative hazard of average risks in similar but not equally hazardous lines of industry.

Of course, when the percentages have been fixed for the several groups the remainder of the process becomes largely mechanical and under proper supervision can be performed by ordinarily competent clerks, although if when the test proposed is applied to any group it indicates that further adjustment is necessary, men of high ability will be required on the problem.

The method herein presented has been brought to the attention of the Manual Committee of the Massachusetts Inspection and Rating Bureau and although not formally adopted by them, it has been considered of sufficient value that the Engineering Committee (Safety and Inspection Committee) of that Bureau has been requested to undertake the first problem, viz., of ascertaining the relative percentages within the several groups. It is not unlikely that the method will at least be tested in the construction of the new Massachusetts Manual and that at some subsequent meeting of the Society it will be possible to present the results of that test of practical application.

NOTE

Since this paper was written, but before its presentation to the Society, the method described was presented for consideration of the Pure Premiums Committee of the Joint Conference on Workmen's Compensation Rates. During a discussion of it there, a

slight change in mode of operation was suggested by Dr. Rubinow, which makes it reproduce, in the projected loss, the actual losses with a fidelity limited only by the number of places to which the computation of pure premiums is carried.

Dr. Rubinow proposes that instead of multiplying the losses by the reciprocals of the judgment percentages, adding the results and dividing by the sum of the pay rolls to produce the pure premium for the classification rated as unity, we multiply the pay rolls themselves by the judgment percentage and use the sum of the products for a divisor against the actual losses for the group.

The theory underlying this procedure corresponding to the explanation given above of the method first proposed is as follows:

If any classification has a hazard value equal to X per cent. of the classification chosen as the standard of reference then, the same factors of chance variation being present, the recorded losses would have been produced by a pay-roll exposure on the standard classification only X per cent. as large.

The reason why Dr. Rubinow's modification of the formula gives more accurate results than the formula as first presented is that the latter operates on the numerator of the fraction L/P, which is subject to large chance fluctuations from its true value. Hence the chance fluctuations to be distributed over the group are magnified or reduced according as they occur in classifications less or more hazardous than the average of the group. The denominator which is operated on under the modified formula is the constant term for the purpose for which it is used. Hence there is no change in the amount to be distributed.

The following hypothetical group worked out by both methods will illustrate the process and the difference between the two methods:

ORIGINAL FORMULA.

Classification.	Judgment Rating.	Reciprocal.	Observed Losses.	Mod. to Stand. Basis.	Pay Roll.	Pure Prem.		Projected Losses
						Original Experience.	Adj. by Formula.	
A...	.50	2.00	$ 2,500	$ 5,000	$ 500,000	.50	.064	$ 320
B...	.75	1.33	1,000	1,333	1,000,000	.10	.096	960
C...	1.00	1.00	100,000	100,000	80,000,000	.125	.127+	101,600
D...	1.25	.80	5,000	4,000	5,000,000	.10	.158	7,900
			$108,250	$110,333	$86,500,000	.125	.127+	$110,780

Excess Projected over Actual Losses approx. 2 per cent.

MODIFIED FORMULA.

Classification.	Judgment Rating.	Observed Pay Roll.	Corresp. P. R. Standard Basis.	Observed Losses.	Pure Prem.		Projected Losses.
					Original Experience.	Adj. by Formula.	
A.....	.50	$ 500,000	$ 250,000	$ 2,500	.50	.062	$ 310
B.....	.75	1,000,000	750,000	1,000	.10	.093	930
C.....	1.00	80,000,000	80,000,000	100,000	.125	.124	99,200
D.....	1.25	5,000,000	6,250,000	5,000	.10	.155	7,750
		$86,500,000	$87,250,000	$108,250	.125	.124	$108,190

Deficiency Projected under Actual Losses approx. $\frac{6}{100}$ per cent.