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RISKINESS LEVERAGE MODELS

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Abstract

Rodney Kreps has written a paper that is a major contribution to the CAS literature on the central topics of risk load and capital allocation for profitability measurement, which is a core component of an enterprise risk management system. He has given us a rich class of mathematical models that satisfy two very desirable properties for a risk-load or surplus-allocation method: They can allocate risk down to any desired level of definition and they satisfy the additivity property. Tail Value at Risk and Excess Tail Value at Risk reasonably satisfy the properties that management would likely want of such a model, while still satisfying the properties of a riskiness leverage model and the properties of coherent measures of risk.

Donald Mango's ground-breaking work in developing the concepts of insurance capital as a shared asset and Economic Value Added [2] are discussed. A Risk Return on Capital model is suggested as an integration of the approaches presented by Kreps and Mango. This method measures returns on capital after reflecting the mean rental cost of rating agency capital. Reinsurance alternatives are compared using both the Return on Risk Adjusted Capital approach presented by Kreps and this integrated approach.

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1. INTRODUCTION

Rodney Kreps begins his paper by describing the generic problem as a situation where a company holds a single pool of shared capital to support a number of random liabilities and assets. The reserves are ordinarily meant to support their mean value, while the surplus is meant to support their variability around their means. Kreps, and by reference Gary Venter [5], first allay actuarial concerns about allocation of capital (discussed in [3]) by pointing out that return on equity (ROE) methods of computing pricing risk loads are really allocating the return on capital. If a line of business is returning 10% on allocated capital, one should ask whether this is a sufficient return to compensate the providers of that capital. Kreps then enumerates two desirable qualities for allocable risk load (the product of allocated surplus and a target rate of return):

1. It should be allocable down to any desired level.
2. It should be additive, in that risk load or capital allocated to components of the portfolio sum to the total risk load or capital need for the portfolio. This would be true for subsets of the portfolio as well.

Kreps does not insist that a risk load or capital allocation method satisfy all the requirements for a coherent risk measure [1], as he believes the risk measure should emerge from the fundamental economics of the business rather than the desired mathematical properties. Thus, Value at Risk (VAR) and Tail Value at Risk (TVAR) are both examples of riskiness leverage models, while VAR is not a coherent risk measure [1] and TVAR is well known to be a coherent risk measure according to Kreps. The fact that TVAR satisfies the subadditivity requirement of a coherent risk measure (the risk of a combination of exposures should not exceed the sum of the risks of the components) may increase the confidence of many actuaries that TVAR is measuring insurance risk appropriately.

Kreps develops the framework for a rich class of models for determining risk loads and allocating capital that possess the above desirable qualities. He then selects a particular example, TVAR, and demonstrates through a spreadsheet model how management can use such a model (once comfortable with the parameterization) to quantitatively evaluate alternative decisions, such as selecting among alternative reinsurance programs to enhance the risk-reward characteristics of a portfolio.

2. SUMMARY WITH COMMENTS

2.1. *The Framework*

This section summarizes the framework for the riskiness leverage models. Let X_k, $k = 1,\ldots,n$, represent losses associated with n risks or portfolio segments, whose sum represents the total loss to the company:

$$X = \sum X_k \qquad \text{where} \quad k = 1,\ldots,n.$$

If μ represents the mean of X, A is the total premium collected for this portfolio of risks, and R is the total risk load collected, then $A = \mu + R$. Alternatively, A may be interpreted as the total assets and R would represent the capital or surplus supporting this portfolio. (I am using A instead of C which was used in the paper, so as to avoid confusion of assets with capital. I use the terms capital, surplus, and equity interchangeably.)

Correspondingly, let μ_k represent the mean of X_k, let A_k represent the assets (or premium collected), and let R_k represent the surplus allocated to X_k (or risk load collected). Then

$A_k = \mu_k + R_k$ and $\mu = \sum \mu_k$ (where $k = 1,\ldots,n$) because expectations are additive. Riskiness leverage models have the form $R_k = E[(X_k - \mu_k)L(X)]$, where the riskiness leverage $L(X)$ is a function that depends only on the sum X of the individual variables and the expectation is taken with respect to that sum. Similarly, $R = E[(X - \mu)L(X)] = E[r(X)]$. Allocated capital and risk loads are probability-weighted averages of risk loads over outcomes of the total net loss. Riskiness leverage models can reflect the fact that not all loss outcomes are equally risky.

From their definitions, $R = \sum R_k$ and $A = \sum A_k$ where $k = 1,\ldots,n$, no matter what the joint dependence of the variables may be. Analogous to the relation of covariance to variance, the R_k will be referred to as co-measures of risk for the measure R. Since additivity follows automatically for these co-measures, Kreps searched for appropriate forms for the riskiness leverage $L(X)$.

Kreps points out that the capital allocations for risk loads may be efficiently computed through Monte-Carlo simulation. One simply simulates the quantity for which we want the expectation for a large number of years, and then averages the results from these scenarios. Kreps generalizes the covariance concepts to suggest the mathematical form of riskiness leverage models.

2.2. *Properties*

The desirable allocation properties for risk load or surplus allocation listed in the Introduction (allocable down to any desired level and additivity) are clearly satisfied for any choice of $L(X)$.

Risk load or surplus allocated will scale with a currency change if $L(X)$ is independent of a currency change: $R(\lambda X) = \lambda R(X)$ if $L(\lambda X) = L(X)$. This will be true if L is a function of ratios of currencies such as x/μ, x/σ (where σ is the standard deviation of X), or x/S (where S is the total surplus of the company). It is intuitively appealing to select the riskiness leverage to be a function of the ratio of the difference of the outcome from the mean to the surplus.

However, the general formulation of risk load or surplus allocation may not yield a coherent risk load or surplus allocation [1]. The major reason is the subadditivity requirement that the risk load for the portfolio not exceed the sum of the risk loads for the components.

2.3. Examples of Riskiness Leverage Models

Risk-Neutral: If the riskiness leverage $L(X)$ is a constant, then the risk load is zero. This would be appropriate for risk of ruin where potential losses are small relative to capital, or for risk of not meeting plan if you are indifferent to the consequences of not meeting plan.

Variance: If $L(x) = (\beta/S)(x - \mu)$, then it can be shown that required surplus or risk load is a multiple of the standard deviation of the aggregate loss distribution. This model suggests that there is risk associated with favorable outcomes as much as there is with unfavorable outcomes, and that the risk load or surplus need increases quadratically with deviations of the loss from its mean.

Tail Value at Risk: Let $L(x) = [\theta(x - x_q)]/(1 - q)$, where the quantile x_q is the value of x where the cumulative distribution of X is q and $\theta(x)$ is the step function (1 when the argument is positive, 0 otherwise). Kreps shows that the assets needed to support the portfolio would be the average portfolio loss X when it exceeds x_q (the definition of TVAR).

He reminds us that this is a coherent risk measure [1] and states that only the part of the distribution at the high end is relevant for this measure. Kreps calculates the assets needed to support a line of business k as the average loss in line k in those years where the portfolio loss X exceeds x_q. This quantity is referred to by Kreps as a co-measure, and is defined by Venter [5] as co-Tail VaR (co-TVAR). (I further refer to the Venter paper below because it is very helpful in clarifying concepts in the Kreps paper.)

Venter also discusses Excess Tail Value at Risk (XTVAR) defined to be the average value of $X - \mu$ when $X > x_q$. The same properties that Kreps proved for TVAR and co-TVAR can be shown to hold for XTVAR and co-XTVAR.

Venter notes that if capital is set by XTVAR, it would cover average losses in excess of expected losses for those years where the portfolio losses X exceed the qth quantile x_q. It is assumed that expected losses have been fully reflected in pricing and in loss reserves. The capital allocated by co-XTVAR to a line would be the line's average losses above its mean losses in those same adverse years. Venter notes that there should be some probability level q for which XTVAR or a multiple of it makes sense as a capital standard. He points out that co-XTVAR may not allocate capital to a line that didn't contribute significantly to adverse outcomes. That is, the deviations from the mean for a line of business may average to approximately zero when total losses exceeded the qth quantile. Venter believes this makes sense if capital is being held for adverse outcomes only.

Value at Risk (VAR): Kreps defines a riskiness leverage model that produces the quantile x_q as the assets needed to support a portfolio of risks. This measure says that the shape of the loss distribution does not matter except to determine the one relevant value x_q. The VAR measure is known not to be coherent [1].

Semi-Variance: Kreps defines a riskiness leverage model that yields needed surplus or risk load as a multiple of the semi-deviation of the aggregate loss distribution. This is the standard deviation with all favorable deviations from the mean ignored (treated as zero). This measure implies that only outcomes worse (greater) than the mean should contribute to required risk load or surplus. This measure is consistent with the usual accounting view that risk is only relevant for adverse outcomes and further implies that the risk load or surplus required increases quadratically with adverse deviations of the loss from its mean.

Mean Downside Deviation: Kreps defines another riskiness leverage model that produces a multiple of mean downside deviation as the risk load. This is really XTVAR with $x_q = \mu$. Kreps notes that this measure could be used for risks such as not meeting a plan, even though ruin is not in question.

Proportional Excess: Finally, Kreps defines a riskiness leverage model that produces a capital allocation for a line that is pro rata on its average contribution to the excess over the mean.

The wide range of risk loads that can be produced by these riskiness leverage models suggests that this is a very flexible, rich class of models from which one should be able to select a measure that not only reflects one's risk preferences but also satisfies the very desirable additivity property.

2.4. Generic Management of Risk Load

Kreps points out that there are many sources of risk, such as the risk of not making plan, the risk of serious deviation from plan, the risk of not meeting investor analysts' expectations, the risk of a rating agency downgrade, the risk of regulatory notice, the risk of going into receivership, the risk of not getting a bonus, etc. Given these risks, he states that it seems plausible that company management's list of desirable properties of the riskiness leverage ratio should be as follows:

1. It should be a down-side measure (the accountant's point of view).
2. It should be more or less constant for excess that is small compared to capital (risk of not making plan, but also not a disaster).
3. It should become much larger for excess significantly impacting capital.

4. It should go to zero (or at least not increase) for excess significantly exceeding capital (once you are buried, it doesn't matter how deep).

Concerning (4), he notes that a regulator might want more attention paid to the extreme areas and might list desirable properties for the riskiness leverage ratio as follows:

1. It should be zero unless capital is seriously affected.
2. It should not decrease with loss significantly exceeding capital, because of the risk to the state guaranty fund.

Kreps points out that TVAR could be such a risk measure if the quantile is chosen to correspond to an appropriate fraction of surplus. However, he notes that at some level of probability, management will have to bet the whole company.

I also believe that rating agencies would not look favorably on the fourth item in management's hypothetical list of desirable properties of a riskiness leverage ratio. While I believe that management would have to take into account the regulatory and rating agency views, I believe they might well not prefer the Variance or Semi-Variance models, which increase quadratically to infinity. It would seem that TVAR and XTVAR reasonably satisfy the properties that management would likely want of such a model, while still satisfying the properties of a riskiness leverage model (additivity, allocable down to any desired level) and the requirements for a coherent measure of risk (including the subadditivity property for portfolio risk).

He states that management might typically formulate its risk appetite as satisfaction of two VAR requirements (limit chance of losing all capital to 0.1%, while limit chance of losing 20% of capital to 10%). In this case, one would take the larger of the two required capital amounts.

For his simulation examples, the author selects the criteria that "we want our surplus to be a prudent multiple of the average bad result in the worst 2% of cases." He notes that Gary Venter has suggested that the prudent multiple could be such that the renewal book could still be serviced after a bad year. Thus, Kreps selects TVAR with a prudent multiple of 150%.

2.5. Simulation Application

As he includes investments as a separate line in his model, TVAR is calculated for net income rather than portfolio losses. He has two insurance lines, one low risk and the other high risk. He shows that surplus can be released by writing less of the risky line, but this may not be possible if one is writing indivisible policies or if one is constrained by regulations. He demonstrates that an excess-of-loss reinsurance treaty can reduce required capital significantly and improve the portfolio's return on allocated surplus. Note that expected profit has decreased due to the cost of reinsurance, but capital needed to support the portfolio has decreased by a larger percentage.

In his simulation example, Kreps notes that the percentage allocation of surplus to line based on the co-TVAR measures is consistent for a wide range of quantiles x_q. That is, when the tail probability varies between 0.1% and 10%, the capital allocation percentage for a given line doesn't change very much. Kreps also tested his simulation model on VAR and power measures, such as mean downside deviation and semi-variance. He discovered that as the power increases, the measure is more sensitive to extreme values and the allocation to line of business moves toward the TVAR allocations.

3. INSURANCE CAPITAL AS A SHARED ASSET

In a paper submitted to the *ASTIN Bulletin* [2], Donald Mango treats insurance capital as a shared asset, with the insurance contracts having simultaneous rights to access potentially all of that shared capital. Shared assets can be scarce and essential public entities (e.g., reservoirs, fisheries, national forests) or desirable private entities (e.g., hotels, golf courses, beach houses). The access to and use of the assets is controlled and regulated by their owners; this control and regulation is essential to preserving the asset for future use. The aggregation risk is a common characteristic of shared asset usage, since shared assets typically have more members who could potentially use the asset than the asset can safely bear.

He differentiates between consumptive and non-consumptive use of an asset. A consumptive use involves the transfer of a portion or share of the asset from the communal asset to an individual, such as in the reservoir water usage and fishery examples. Non-consumptive use involves temporary, limited transfer of control that is intended to be non-depletive in that it is left intact for subsequent users. Examples of non-consumptive use include boating on a reservoir, playing on a golf course or renting a car or hotel room.

While shared assets are typically used in only one of the two manners, some shared assets can be used in either a consumptive or non-consumptive manner, depending on the situation. Mango gives the example of renting a hotel room. While the intended use is benign occupancy (non-consumptive), there is the risk that a guest may fall asleep with a lit cigarette and burn down a wing of the hotel (clearly consumptive).

Mango notes that rating agencies use different approaches in establishing ratings, but the key variable is the capital-adequacy ratio (CAR) that is the ratio of actual capital to required capital. Typically, the rating agency formulas generate required capital from three sources: premiums, reserves, and assets. Current year underwriting activity will generate required premium capital. As that premium ages, reserves will be established that will generate the required reserve capital. As the reserves are run off, the amount of required reserve capital will diminish and eventually reach zero when all claims are settled. As there are usually minimum CAR levels associated with each rating level, Mango points out that a given amount of actual capital corresponds to a maximum amount of rating agency required capital. For given reserve levels, this implies a limit to premium capital and thus to how much business can be written. Mango summarizes by stating that an insurer's actual capital creates underwriting capacity, while underwriting activity (either past or present) uses up underwriting capacity.

Mango notes that the generation of required capital, whether by premiums or reserves, temporarily reduces the amount of capacity available for other underwriting. Being temporary, it is similar to capacity occupancy, a non-consumptive use of the shared asset. Capacity consumption occurs when reserves must be increased beyond planned levels. Mango points out that this involves a transfer of funds from the capital account to the reserve account, and eventually out of the firm. Mango recaps by stating that the two distinct impacts of underwriting an insurance portfolio are as follows:

1. Certain occupation of underwriting capacity for a period of time.
2. Possible consumption of capital.

He notes that this "bi-polar" capital usage is structurally similar to a bank issuing a letter of credit (LOC). The dual impacts of a bank issuing a LOC are as follows:

1. Certain occupation of capacity to issue LOCs, for the term of the LOC.
2. Possible loan to the LOC holder.

Mango notes that banks receive income for the issuance of LOCs in two ways:

1. An access fee (i.e., option fee) for the right to draw upon the credit line.
2. Loan payback with interest.

Mango notes that every insurance contract receives a parental guarantee: should it be unable to pay for its own claims, the contract can draw upon the available funds of the company. He states that the cost of this guarantee has two pieces:

1. A Capacity-Occupation Cost, similar to the LOC access fee.
2. A Capital-Call Cost, similar to the payback costs of accessing an LOC, but adjusted for the facts that the call is not for a loan but for a permanet transfer and that the call destroys future underwriting capacity.

Mango states that there is an opportunity cost to capacity occupation, and thinks of it as a minimum risk-adjusted hurdle rate. He computes it as the product of an opportunity cost rate and the amount of required rating agency capital generated over the active life of the contract. Mango also develops a formula for computing capital-call costs, which are his true risk loads, and defines the expected capital-usage cost to be the sum of the capacity-occupation cost and the expected capital-call cost. He defines his key decision metric Economic Value Added (EVA) to be the NPV Return minus the expected capital usage cost:

$$\text{EVA} = \text{NPV Return} - \text{Capacity-Occupation Cost}$$
$$- \text{Capital-Call Cost.}$$

Mango's shared-asset view eliminates the need for allocating capital in evaluating whether the expected profit for a contract is sufficient to compensate for the risks assumed. He also shows how this approach can be used to evaluate portfolio mixes. His approach permits stakeholders great flexibility in expressing risk reward preferences. As Mango, Kreps, and David Ruhm jointly contributed to the development of the RMK (Rhum, Mango, and Kreps) algorithm, which is a conditional risk allocation method [4], it is no surprise that the Capital-Call Costs satisfy the key properties of a riskiness-leverage model (additivity, allocable down to any desired level).

4. INTEGRATION OF APPROACHES

This reviewer sees a limitation in the return on risk-adjusted capital (RORAC) approach as applied by Kreps that can easily be corrected by borrowing a concept from EVA. RORAC based upon riskiness leverage models does not reflect rating agency capital requirements, particularly the requirement to hold capital to support reserves until all claims are settled. This is especially important for long-tailed casualty lines. In the RORAC calculation as applied by Kreps, Expected Total Underwriting Return is computed by adding the mean NPV of interest on reserves from the simulation, interest on allocated capital, and expected underwriting return (profit and overhead). RORAC is computed as the ratio of Expected Total Underwriting Return to allocated risk capital and represents the expected return for both benign and potentially consumptive usage of capital.

As an alternative, I have developed a modified RORAC approach, which I call a risk-return on capital (RROC) model. A mean rating agency capital is computed by averaging rating agency required capital from the simulation (capital needed to support premium writings is added to the NPV of the capital needed to support reserves on each iteration of the simulation). The mean rental cost for rating agency capital is calculated by multiplying the mean rating agency capital by the selected rental cost percentage, which serves the same function as Mango's opportunity cost rate. Expected underwriting return is computed by adding the mean NPV of interest on reserves and interest on mean rating agency capital to expected underwriting return (profit and overhead). The expected underwriting return after rental cost of capital is computed by subtracting the mean rental cost of rating agency capital.

In my comparisons of RORAC and RROC, risk capital is a selected multiple of XTVAR. Capital is allocated to line of business based upon Co-XTVAR. RROC is computed as the ratio of expected underwriting return after rental cost of capital to allocated risk capital. It is assumed that expense items like overhead and taxes, as well as returns from any capital excess of the rating agency required capital or from riskier investments that would require additional rating agency capital, would be handled within corporate planning.

RROC represents the expected return for exposing capital to risk of loss, as the cost of benign rental of capital has already been reflected. It is analogous to the Capital-Call Cost in the EVA approach, here expressed as a return on capital rather than applied as a cost. In the discussion of Tail Value at Risk, it was observed that Venter has noted that co-XTVAR may not allocate capital to a line of business that didn't contribute significantly to adverse outcomes. In such a situation, the RORAC calculation based upon riskiness leverage models may show the line to be highly profitable, whereas RROC may show that the line is unprofitable because it did not cover the mean rental cost of rating agency capital.

In the EVA approach, risk preferences are reflected in the function selected and parameterized in computing the Capital-Call Cost. In the RROC approach, risk preferences are specified in the selection of the riskiness leverage model used to measure risk. This riskiness leverage model in practice would be parameterized to equal the total capital of the company, which would be maintained to at least cover rating agency capital required to maintain the desired rating. Both approaches utilize the RMK algorithm for allocating risk (measured as a Capital Call Cost in EVA and as risk capital in RROC) to line of business.

5. SIMULATION EXAMPLE

The RORAC and RROC approaches were tested and the results are summarized in the attached exhibits. Exhibit 1.1 summarizes the examples tested, including underlying assumptions, while Exhibit 1.2 summarizes the technical differences between the two approaches. In the base case, Example 1, the lines 1 and 2 are 50% correlated while being uncorrelated to line 3, and no reinsurance is purchased. Equal amounts of premium are written in the three lines, and pricing is assumed to be accurate with the plan loss ratio equaling the true Expected Loss Ratio (ELR) of 80% for each line. Aggregate losses are assumed to be modeled accurately by lognormal distributions with coefficients of variation of 80%, 20% and 40% for lines of business (LOB) 1–3, respectively. In Example 2, a stop-loss reinsurance treaty is purchased for line 1 covering a 30% excess of 90% loss ratio layer for a 10% rate. In Example 3, a 50% quota share is purchased for line 1 with commissions just covering variable costs.

Payout Patterns were generated based on an exponential settlement lag distribution with mean lag to settlement of one year, five years and ten years for LOB 1–3, respectively. Thus, the payout patterns for LOB 1–3 can be characterized as fast, average, and slow, respectively. Interest is credited on supporting surplus using risk-free rates for bonds of duration equal to the average payment lag in each line of business. In this example, interest rates of 3%, 4% and 5% for LOB 1–3, respectively, were assumed. These are the same rates that are used to calculate NPV reserves, interest on supporting surplus, and the NPV Reserves-Capital component of Required Rating Agency Capital. For simplicity, interest rates and payment patterns are assumed to be deterministic.

For both RORAC and RROC models, capital needed to support the portfolio risk is calculated as 150% of XTVAR. That is, the company wants 50% more capital than needed to support 1-in-50-year or worse deviations from plan. Capital needed to support the portfolio risk is allocated to the lines of business based upon Co-XTVAR.

Exhibit 2 summarizes the test results. Recall that in the base case no reinsurance is purchased. In Example 2, a stop-loss reinsurance treaty is purchased for line 1 that modestly improves both RORAC and RROC measures. (RORAC increases from 17.50% to 17.88%, while RROC increases from 9.95% to 10.05%.) However, in Example 3, a 50% quota share for line 1 improves the portfolio RORAC measure by 47% (from 17.50% to 25.74%), RROC improves by 54% (from 9.95% to 15.36%), and risk capital needed to support the portfolio decreases by over 40% (from $5.71 million to $3.39 million).

Line 1 and the reinsurance line 4 were combined in calculating returns by line of business. It is interesting that the expected returns for lines 1 and 2 did not change very much with the purchase of reinsurance, while the highly profitable returns for line 3 declined because it is now contributing to more of the 1-in-50 year adverse deviations. The portfolio returns with reinsurance improved because a smaller share of capital is now allocated to the marginally profitable line 1 and greater shares of capital are allocated to the highly profitable lines 2 and 3 (this can be seen by reviewing the change in the distributions of allocated capital displayed for the reinsurance examples at the bottom of Exhibit 2). It is also interesting that returns for line 2 improve a little because of its correlation with line 1 and because it has not been allocated any of the cost of reinsurance.

For the portfolio, Exhibit 2 also displays the Cost of Capital Released for the two reinsurance examples, which is the ratio of the cost of the reinsurance (decrease in expected profitability due to reinsurer's profit margin) to the decrease in capital needed to support the portfolio. The Cost of Capital Released was modestly lower than the company's net returns for the stop-loss example (12.6% versus 17.9% for RORAC, and 8.6% versus 10.1% for RROC), but dramatically lower for the quota-share example (5.6% versus 25.7% for RORAC, and 2.1% versus 15.4% for RROC). Thus, the company's cost to release over 40% of its capital for other purposes was a small fraction of its net returns for both metrics in the quota-share example.

However, the net capital allocated to the portfolio based on the 150% of XTVAR standard is less than the mean rating agency required capital computed for the RROC metric. It was determined that a 200% of XTVAR capital standard is consistent with the rating agency required capital, providing sufficient capital, beyond the amounts required to support premium written and loss reserves, to also cover rating agency capital required to cover investments.

The model output is displayed as Exhibit 3 for the quota-share example with a 200% of XTVAR capital standard. Net RORAC declines from 25.74% to 20.22%, while net RROC declines from 15.36% to 11.52%. However, note that RROC has been computed after applying a 10% Rental Cost Percentage to the Mean Rating Agency Capital from the simulation. Net capital required to support the 200% of XTVAR standard is now more than 40% lower than a larger gross requirement, while the Cost of Capital Released has declined for both metrics.

6. CONCLUSIONS

Rodney Kreps has written an important paper on the central topics of risk load and capital allocation. He has given us a class of mathematical models that satisfy two highly desirable properties for a risk load procedure, additivity and allocable down to any desired level. Tail Value at Risk and Excess Tail Value at Risk reasonably satisfy the properties that management would likely want of such a model, while still being coherent measures of risk.

Donald Mango's very innovative work in developing the concepts of insurance capital as a shared asset and Economic Value Added contribute significantly to understanding the way capital supports an insurance enterprise. A Risk Return on Capital model is suggested as a way to integrate desirable properties of the approaches presented by Kreps and Mango. This method measures returns on capital after reflecting the mean rental cost of rating agency capital. Thus, returns for exposing capital to risk are measured after reflecting the cost of carrying capital to support both premium written and loss reserves, which is especially important for long-tailed casualty lines.

While actuarial literature frequently refers to risk preferences of the capital provider, little mention is made of the risk-measurement preferences of the actuary. Good arguments can be made for both approaches to measuring exposure to risk of loss from insured events: The choice is either to allocate costs or to allocate capital. The Return on Risk Adjusted Capital approach based upon riskiness leverage models can be modified to reflect the opportunity cost of holding capital to support written premium and loss reserves, while still providing a metric that is understandable to financially oriented non-actuaries.

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[4] Ruhm, David, Donald Mango, and Rodney Kreps, "A General Additive Method for Portfolio Risk Analysis," accepted for publication in *ASTIN Bulletin.*

[5] Venter, Gary G., "Capital Allocation Survey with Commentary," *North American Actuarial Journal*, 8, 2, 2004, p. 96.

SUPPLEMENTARY MATERIAL

1. Seminar notes from the 2005 Seminar on Reinsurance on "Risk Load, Profitability Measures, and Enterprise Risk Management" may be downloaded from the CAS Web Site.

2. Abbreviations and Notation

CAR, Capital Adequacy Ratio

Co-TVAR, Co-Tail Value at Risk

Co-XTVAR, Co-Excess Tail Value at Risk

ELR, Expected Loss Ratio

EVA, Economic Value Added

LOB, Line of Business

LOC, Letter of Credit

RMK algorithm, a conditional risk allocation method

ROE, Return on Equity

RORAC, Return on Risk-Adjusted Capital

RROC, Risk Return on Capital After Rental Cost of Capital

TVAR, Tail Value at Risk

VAR, Value at Risk

XTVAR, Excess Tail Value at Risk

EXHIBIT 1.1

Summary of Model Assumptions

1. Payout Patterns were generated based upon an exponential settlement lag distribution with mean lags to settlement of one year, five years, and ten years for LOB 1-3, respectively. Thus, the payout patterns for LOB 1-3 can be characterized as Fast, Average, and Slow, respectively. Payments are assumed to be made in the middle of each year.
2. Interest is credited on supporting surplus using risk free rates for bonds of duration equal to the average payment lag in each line of business. In this example, interest rates of 3%, 4% and 5% for LOB 1-3, respectively, were assumed. These are the same rates that are used to calculate Net Present Value (NPV) reserves, interest on supporting surplus, and the NPV Reserves Capital component of Required Rating Agency Capital.
3. For simplicity, interest rates and payment patterns are assumed to be deterministic.
4. Profitability measures are computed before taxes, overhead, and returns on capital excess of the rating agency required capital.

Example	Key Assumptions	Purpose of Example
1	Write equal amounts of premium in three lines of business. Pricing is accurate, as the Plan Loss Ratios equal the true ELR's. The ELR's are equal to 80% for all three lines. No reinsurance is purchased. Aggregate losses are assumed to be modeled accurately by lognormal distributions with coefficients of variation of 80%, 20% and 40% for LOB 1-3, respectively. The correlation between LOB 1 and LOB 2 losses is 50%.	Base example with no reinsurance.
2	Same assumptions as in Example 1, except a 30% xs 90% Loss Ratio Stop Loss reinsurance program is purchased for LOB 1 at a 10% rate.	Test impact of stop loss reinsurance program for LOB 1.
3	Same assumptions as in Example 1, except a 50% Quota Share is purchased for LOB 1 with commission just covering variable costs.	Test impact of quota share reinsurance program for LOB 1.

EXHIBIT 1.2

MODEL SUMMARIES

1. For both models, capital needed to support the portfolio risk is calculated as 150% of Excess Tail Value at Risk (XTVAR). That is, the Company wants 50% more capital than needed to support 1 in 50 year or worse deviations from plan. Capital needed to support the portfolio risk is allocated to line of business based upon Co-Excess Tail Values at Risk (Co-XTVAR).
2. Returns on Risk Adjusted Capital Model (RORAC):
   Expected Total Underwriting Return is computed by adding the mean NPV of interest on reserves from the simulation, interest on allocated capital, and expected underwriting return (profit and overhead). RORAC is computed as the ratio of Expected Total Underwriting Return to allocated risk capital, and represents the expected return for both benign and potentially consumptive usage of capital.
3. Risk Returns on Capital Model (RROC):
   a. Risk Returns on Capital (RROC) may be thought of as a composite of the EVA and RORAC approaches to measuring profitability. The Mean Rental Cost of Rating Agency Capital (an EVA Concept) is subtracted as a cost before applying RORAC concepts to compute the return on allocated capital for exposing capital to potential loss.
   b. Required Rating Agency Capital is computed based upon rating agency premium and reserves capital charge factors assumed appropriate for the Company's desired rating. Somewhat smaller factors were selected for the reinsurance line (LOB 4) under the assumption that the Company would not receive full credit for ceded premium and reserves because a charge for potential uncollectibility would be applied.

      Capital needed to support reserves for a calendar year is the product of the reserves factors and the previous year-end reserves.

      Capital needed to support reserves must be calculated for all future calendar years until reserves run off.

      Required capital to support reserves is the NPV of these capital amounts.
   c. The Mean Rental Cost of Rating Agency Capital is calculated by multiplying the Mean Rating Agency Capital from the simulation by the selected Rental Cost Percentage, an opportunity cost of capacity.
   d. Expected Underwriting Return is computed by adding the mean NPV of interest on reserves and interest on mean rating agency capital to expected underwriting return (profit and overhead). The Expected Underwriting Return After Rental Cost of Capital is computed by subtracting the Mean Rental Cost of Rating Agency Capital. As for RORAC, risk capital is 150% of XTVAR. Capital is allocated to line of business based upon Co-XTVAR. RROC is computed as the ratio of the Expected Underwriting Return After Rental Cost of Capital to allocated risk capital. RROC represents the expected return for exposing capital to risk of loss, as the cost of benign rental of capital has already been reflected.

EXHIBIT 2

Model Comparisons for All Lines Combined

	Returns on Risk Adjusted Capital (RORAC)				Risk Returns on Capital After Rental Cost of Capital (RROC)			
Example	Gross *	Net	Difference	Cost of Capital Released	Gross *	Net	Difference	Cost of Capital Released
1	17.50%	17.50%			9.95%	9.95%		
2	17.50%	17.88%	0.4%	12.6%	9.94%	10.05%	0.1%	8.6%
3	17.55%	25.74%	8.2%	5.6%	9.97%	15.36%	5.4%	2.1%

Model Comparisons LOB 1 and LOB 4 (Reinsurance) Combined

	Returns on Risk Adjusted Capital (RORAC)			Risk Returns on Capital After Rental Cost of Capital (RROC)		
Example	Gross *	Net	Difference	Gross *	Net	Difference
1	5.84%	5.84%		1.80%	1.80%	
2	5.85%	5.21%	–0.6%	1.81%	1.17%	–0.6%
3	5.86%	6.44%	0.6%	1.82%	2.00%	0.2%

Model Comparisons LOB 2

	Returns on Risk Adjusted Capital (RORAC)			Risk Returns on Capital After Rental Cost of Capital (RROC)		
Example	Gross *	Net	Difference	Gross *	Net	Difference
1	61.41%	61.41%		37.52%	37.52%	
2	62.09%	62.54%	0.4%	37.96%	38.25%	0.3%
3	63.45%	63.83%	0.4%	38.85%	39.10%	0.3%

*Note that Gross simulated returns differ somewhat between Examples for lines allocated a small share of the total capital due to differences in simulation results.

EXHIBIT 2

Continued

Model Comparisons LOB 3

Example	Returns on Risk Adjusted Capital (RORAC) Gross *	Net	Difference	Risk Returns on Capital After Rental Cost of Capital (RROC) Gross *	Net	Difference
1	131.06%	131.06%		93.60%	93.60%	
2	122.28%	112.67%	−9.6%	87.09%	79.95%	−7.1%
3	114.39%	50.28%	−64.1%	81.22%	33.62%	−47.6%

Stop Loss Example 2: Comparison of Capital Requirements						Quota Share Example 3: Comparison of Capital Requirements					
Line	Gross *	Gross Weight	Net	Net Combining Lines 1 and 4	Net Weight	Line	Gross	Gross Weight	Net	Net Combining Lines 1 and 4	Net Weight
1	4,919,918	85.90%	4,892,514	4,468,187	84.24%	1	4,886,073	85.59%	4,069,551	2,034,775	59.99%
2	453,766	7.92%	450,304	450,304	8.49%	2	443,376	7.77%	440,592	440,592	12.99%
3	353,859	6.18%	385,446	385,446	7.27%	3	379,403	6.65%	916,596	916,596	27.02%
4			(424,327)			4			(2,034,776)		
	5,727,543	100.00%	5,303,937	5,303,937	100.00%		5,708,852	100.00%	3,391,963	3,391,963	100.00%
Average RORAC:	17.50%		17.88%			Average RORAC:	17.55%		25.74%		
Average RROC:	9.94%		10.05%			Average RROC:	9.97%		15.36%		

*Note that Gross simulated returns differ somewhat between Examples for lines allocated a small share of the total capital due to differences in simulation results.

EXHIBIT 3.1

Quota-Share Reinsurance Example Comparing Returns on Risk-Adjusted Capital with Returns on Capital After Rental Cost of Capital

1) Loss Generator	Fast Pay LOB 1	Average Pay LOB 2	Slow Pay LOB 3	Reinsurance LOB 4	Net Total	Gross Total
1A) True Expected Loss: Copy and Paste-Special from LOB 4 of 3H).	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
1B) Coefficient of Variation of Assumed Lognormal Loss Distribution	80.0%	20.0%	40.0%			
1C) Standard Deviation	800,000	200,000	400,000			
1D) Profit and Overhead Margin (includes Brokerage on Reinsurance)	9.0%	8.0%	7.0%	9.0%	7.8%	8.0%
1E) Variable Expense Ratio	11.0%	12.0%	13.0%	11.0%	12.2%	12.0%
1F) Plan Premium	1,250,000	1,250,000	1,250,000	(625,000)	3,125,000	3,750,000
1G) Expected Loss Ratio = (1A)/(1F)	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
1H) Expected Underwriting Return (Profit & Overhead)	112,500	100,000	87,500	(56,250)	243,750	300,000
1I) Plan Loss Ratio	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
1J) Plan Expected Loss	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
1K) Pricing Error = ((1J) – (1A))/(1A)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
2) Capital Usage Calculation	**LOB 1**	**LOB 2**	**LOB 3**	**LOB 4**	**Net Total**	**Gross Total**
2A) Required Capital Charge on Premium	40.0%	40.0%	40.0%	35.0%	41.0%	40.0%
2B) Required Capital Charge on Reserves	25.0%	25.0%	25.0%	20.0%	25.2%	25.0%
2C) Rental Fee	10.0%					
2D) Required Premium Capital = (1F) × (2A)	500,000	500,000	500,000	(218,750)	1,281,250	1,500,000
2E) Simulated Required NPV Reserves Capital = (2B) × (NPV Future Reserves)	229,011	1,022,318	1,637,097	(91,604)	2,796,821	2,888,425
2F) Simulated Total Required Rating Agency Capital = (2D) + (2E)	729,011	1,522,318	2,137,097	(310,354)	4,078,071	4,388,425

EXHIBIT 3.1

Continued

3) Annual Simulation	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
3A) Simulated Losses	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
3B) Deviations From Plan = (1J) – (3A)	—	—	—	—	—	—
3C) Deviation from Plan at 2nd Percentile: Copy and Paste-Special from (3K), re-run simulation to calculate XTVAR.	(2,310,809)	(472,747)	(1,048,430)	1,155,288	(1,679,627)	(2,671,871)
3D) Deviation from Plan when Exceed 1 in 50 Year Result	—	—	—	—	—	—
3E) Flag to Count Number of Simulations in Excess of 1 in 50 Year Result	—	—	—	—	—	—
3F) Contribution to Gross 1 in 50 Year Result	—	—	—			—
3G) Contribution to Net 1 in 50 Year Result	—	—	—	—	—	

Number of Simulations: 100,000

Loss Simulation Statistics	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
3H) Expected Loss	1,000,009	1,000,003	999,998	(500,005)	2,500,006	3,000,010
3I) Standard Deviation	800,103	200,028	399,974	400,052	657,760	992,642
3J) Coefficient of Variation	80.0%	20.0%	40.0%	-80.0%	26.3%	33.1%
3K) Percentiles of Deviations from Plan (Negatives are Values at Risk)						
0.1 Percentile (1 in 1000)	(5,870,875)	(808,528)	(2,056,781)	2,928,441	(3,538,483)	(6,282,611)
1st Percentile (1 in 100)	(3,010,960)	(554,496)	(1,275,329)	1,505,136	(2,063,235)	(3,412,871)
2nd Percentile (1 in 50)	(2,310,938)	(472,759)	(1,048,428)	1,155,244	(1,692,567)	(2,681,451)
5th Percentile (1 in 20)	(1,483,359)	(358,194)	(749,783)	741,605	(1,210,214)	(1,819,725)
10th Percentile (1 in 10)	(923,290)	(263,894)	(521,248)	461,604	(837,389)	(1,204,122)
50th Percentile (1 in 2)	219,129	19,415	71,515	(109,568)	104,896	195,304
90th Percentile	682,957	239,216	433,300	(341,484)	717,829	996,299

EXHIBIT 3.2

	Risk Capital Standard (Multiple K of XTVAR): 200%					
4) Returns on Risk Adjusted Capital (RORAC)	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
4A) Plan Premium	1,250,000	1,250,000	1,250,000	(625,000)	3,125,000	3,750,000
4B) Expected Underwriting Return (Profit & Overhead)	112,500	100,000	87,500	(56,250)	243,750	300,000
4C) Average Deviation from Plan When Exceed 1 in 50 Year Result (XTVAR)	(3,423,226)	(588,435)	(1,381,853)	1,711,613	(2,260,925)	(3,805,255)
4D) Gross Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	6,514,764	591,168	505,871			7,611,804
4E) Interest Rate Assumed	3.0%	4.0%	5.0%	3.0%		
4F) Interest Earned on Gross Allocated Capital = (4D) × (4E)	195,443	23,647	25,294			244,383
4G) Mean Net Present Value of Interest Earned on Reserves	27,485	163,603	327,516	(13,742)	504,861	518,603
4H) Gross Expected Total Underwriting Return = (4B) + (4F) + (4G)	335,427	287,250	440,310			1,062,987
4I) Gross Return on Risk Adjusted Capital = GRORAC = (4H)/(4D)	5.15%	48.59%	87.04%			13.96%
4J) Net Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	5,426,069	587,457	1,222,128	(2,713,034)	4,522,619	
4K) Interest Earned on Net Allocated Capital = (4E) × (4J)	162,782	23,498	61,106	(81,391)	165,996	
4L) Net Expected Total Underwriting Return = (4B) + (4G) + (4K)	302,767	287,101	476,123	(151,383)	914,607	
4M) Net Return on Risk Adjusted Capital = NRORAC = (4L/(4J)	5.58%	48.87%	38.96%	5.58%	20.22%	
4N) Change in Return Due to Reinsurance = (4L – Net Total) – (4H – Gross Total)	(148,380)					
4O) Change in Allocated Capital = (4J – Net Total) – (4D – Gross Total)	(3,089,184)	4P) Cost of Additional XTVAR Capital = (4N)/(4O)				4.8%

EXHIBIT 3.2

Continued

5) Risk Returns on Capital (RROC) After Rental Cost of Capital	Risk Capital Standard (Multiple K of XTVAR): 200% LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
5A) Mean Rating Agency Capital = Mean of (2F)	729,013	1,522,321	2,137,093	(310,355)	4,078,072	4,388,427
5B) Mean Rental Cost of Rating Agency Capital = (5A) × (2C)	72,901	152,232	213,709	(31,036)	407,807	438,843
5C) Mean Interest Earned on Rating Agency Capital = (5A) × (4E)	21,870	60,893	106,855	(9,311)	180,307	189,618
5D) Expected Underwriting Return After Rental Cost of Capital = (4B) + (4G) + (5C) – (5B)	88,954	172,264	308,161	(48,267)	521,111	569,379
5E) Gross Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	6,514,764	591,168	505,871			7,611,804
5F) Gross Risk Return on Capital = GRROC = (5D)/(5E)	1.37%	29.14%	60.92%			7.48%
5G) Net Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	5,426,069	587,457	1,222,128	(2,713,034)	4,522,619	
5H) Net Risk Return on Capital = NRROC = (5D)/(5G)	1.64%	29.32%	25.22%	1.78%	11.52%	
5I) Change in Return Due to Reinsurance = (5D for LOB 4)	(48,267)					
5J) Change in Allocated Capital = (5G – Net Total) – (5E – Gross Total)	(3,089,184)	5K) Cost of Additional XTVAR Capital=(5I)/(5J)				1.6%