# CLASSIFICATION RATEMAKING—FURTHER DISCUSSION

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*Abstract*

*Classification ratemaking is one of the most important elements in the process of a property-casualty rate calculation. It is here that the pricing actuary moves from a rate change that is appropriate for an entire portfolio of policyholders, to prices that attempt to be fair and equitable for each policyholder in the portfolio.*

*Classification ratemaking is so important that is has its own chapter in the textbook Foundations of Casualty Actuarial Science (Chapter 6, authored by R. Finger). Other sources of P&C study material also present lengthy analysis of this topic [e.g., Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance (2nd edition) by Brown and Gottlieb (2001)].*

*This paper illustrates that these two important references do not arrive at exactly the same results for a classification ratemaking situation where some cells have less than full credibility. The paper then goes on to attempt to isolate the reason for the differences, and in so doing, sheds new light on the process itself.*

ACKNOWLEDGEMENT

The author would like to acknowledge the assistance of David Oakden in making substantive improvements to the first version of this paper.

## 1. INTRODUCTION

For more than a decade now, students of the CAS syllabus have learned classification ratemaking from R. Finger's chapter, "Risk Classification" in the textbook *Foundations of Casualty Actuarial Science*, currently Chapter 6 in the 4th edition.

However, this is not the only source of study material on this topic. The Society of Actuaries also introduces their students to some P&C topics through their Part 5 course, and they use the textbook *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, authored by Brown and Gottlieb.

Interestingly, it will be shown that these two text references do not arrive at exactly the same solution for a classification ratemaking question where some classes in the analysis do not have full credibility.

By analyzing the reason for the differences in the two answers, this paper attempts to elucidate the entire process of classification ratemaking.

## 2. THE PROBLEM BY ILLUSTRATION

In *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, authors Brown and Gottlieb present an algebraic proof that the two classical methods to calculate class differentials (namely, the loss ratio method and the loss cost method) are equivalent. This example, however, only covers the case where all risk classes being analyzed have credibility equal to one [see Brown and Gottlieb, pp. 173–175].

It is also the case that for a portfolio of risks where every class has full credibility, the class relativities produced by Finger are equal to the relativities produced by Brown and Gottlieb.

This will now be illustrated with a simple example.

*Example, Part I*

The pricing actuary has decided upon a statewide-adopted rate level increase of +6%. Given the following data, show the new adopted rates for Classes 1, 2, and 3.

The existing base rate is $100 in Class 1.

All classes have full credibility ($Z = 1$).

You also have the following data by class:

TABLE 1

| Class | Existing Relativity | Exposure Units | Earned Premium | $ Loss | Loss Cost | Loss Ratio |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 500 | $50,000 | $30,000 | $60.00 | 0.6000 |
| 2 | 1.25 | 150 | 18,750 | 12,750 | 85.00 | 0.6800 |
| 3 | 1.50 | 200 | 30,000 | 15,900 | 79.50 | 0.5300 |
| Total | | 850 | 98,750 | 58,650 | 69.00 | 0.5939 |

*Method I*

We will use the loss cost method using Class 1 as the base rate for the calculation. Remember that $Z = 1$ throughout. We will use seven decimal accuracy in all calculations, even if fewer decimal place accuracy is displayed.

TABLE 2

| Class | Existing Relativity | Loss Cost | Indicated Relativity |
|---|---|---|---|
| 1 | 1.00 | 60.00 | 1.000 |
| 2 | 1.25 | 85.00 | 1.416 |
| 3 | 1.50 | 79.50 | 1.325 |

Since $Z = 1$ in all cells, the existing relativity does not have any impact on the answer and could be ignored (as it is in some examples below).

We have set the class relativity for Class 1 equal to 1.000. This means that our overall rate change may not balance to +6%. So, we need to balance back, as follows:

$$\begin{aligned}\text{Old Average Relativity} &= [500(1.00) + 150(1.25) + 200(1.50)]/850\\ &= 1.1617647\\ \text{New Average Relativity} &= [500(1.00) + 150(1.416) + 200(1.325)]/850\\ &= 1.15\\ \text{Balance-Back Factor} &= 1.1617647/1.15 = 1.0102302,\end{aligned}$$

giving us:

TABLE 3

| Class | New Rate | Exposure Units | Premium Income |
|---|---|---|---|
| 1 | $107.08 | 500 | $53,542 |
| 2 | 151.70 | 150 | 22,755 |
| 3 | 141.89 | 200 | 28,377 |
| Total | | 850 | 104,675 |

Now, $\$104,675 = \$98,750 \times (1.06)$, so, everything is as it should be.

*Method II*

We will use the loss cost method but the base class will be the state (loss cost).

TABLE 4

| Class | Loss Cost | Indicated Relativity | Relativity with Class 1 = 1.000 |
|---|---|---|---|
| 1 | 60.00 | 0.8695652 | 1.000 |
| 2 | 85.00 | 1.2318841 | 1.416 |
| 3 | 79.50 | 1.1521739 | 1.325 |
| State | 69.00 | 1.0000000 | |

This gives us the same relativities as does Method I, and there is no reason to go further (i.e., there is no reason to do the balance-back calculation).

*Method III*

This method follows the loss ratio approach with the base class being Class 1.

TABLE 5

| Class | Loss Ratio | Existing Relativity | Indicated Change $[LR_i/LR_1]$ | Indicated Relativity |
|---|---|---|---|---|
| 1 | 0.6000 | 1.00 | 1.000 | 1.000 |
| 2 | 0.6800 | 1.25 | 1.133 | 1.416 |
| 3 | 0.5300 | 1.50 | 0.883 | 1.325 |

Again, the same answer. Thus, it has been shown that under the set conditions, the loss ratio method and the loss cost method do provide the same answer (as proven algebraically by Brown and Gottlieb).

*Method IV*

We will follow the loss ratio approach again, but now the base class will be the state (loss ratio).

TABLE 6

| Class | Loss Ratio | Existing Relativity | Indicated Change $[LR_i/LR_S]$ | Indicated Relativity | Indicated Relativity with Class 1 = 1.000 |
|---|---|---|---|---|---|
| 1 | 0.6000 | 1.00 | 1.0102302 | 1.0102302 | 1.000 |
| 2 | 0.6800 | 1.25 | 1.1449275 | 1.4311594 | 1.416 |
| 3 | 0.5300 | 1.50 | 0.8923700 | 1.3385550 | 1.325 |
| State | 0.5939 | | 1.0000000 | | |

Again, the same answer.

*Method V*

Finally, we follow the template presented in Chapter 6 of the *Foundations* textbook (Finger). Remember that the overall rate change is +6%.

TABLE 7

| Class (1) | Existing Relativity (2) | Adjusted Exposures [(2)× Given Exp] (3) | Adjusted Loss Costs [$ Loss/(3)] (4) | Indicated Adjustment [(4)/(4) Total] (5) | Extension [(5) × Old Rate × 1.06] (6) | Adopted Relativity† (7) |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 500 | 60.00 | 1.0102302 | 107.08 | 1.000 |
| 2 | 1.25 | 187.5 | 68.00 | 1.1449275 | 151.70 | 1.416 |
| 3 | 1.50 | 300 | 53.00 | 0.8923700 | 141.89 | 1.325 |
| | | 987.5 | 59.39 | | | |

†This is not produced by Finger, but is clearly consistent.

Obviously, we had identical answers for the adopted relativities and the new rates from all the approaches attempted. This should be gratifying and should create a level of comfort among users. Further, because all Classes have full credibility, in Methods I and III, we could have chosen Class 2 or Class 3 as our base and the results would have been the same.

*Example, Part II*

We now stir the pot somewhat by stipulating credibility factors for the different classes where only Class 1 has full credibility.

We will use the following data in this illustration:

TABLE 8

| Class | Existing Relativity | Exposure Units | Earned Premium | $ Loss | Loss Cost | Loss Ratio | Credibility $Z$ |
|---|---|---|---|---|---|---|---|
| 1 | 1.00 | 500 | $50,000 | $30,000 | 60.00 | 0.6000 | 1.000 |
| 2 | 1.25 | 150 | 18,750 | 12,750 | 85.00 | 0.6800 | 0.500 |
| 3 | 1.50 | 200 | 30,000 | 15,900 | 79.50 | 0.5300 | 0.600 |
| State | | 850 | 98,750 | 58,650 | 69.00 | 0.5939 | 1.000 |

Again, we will find the new Class 1, 2, and 3 (base) rates with an overall +6% rate increase.

We will now repeat the original five methods of calculation to see if they again produce identical answers.

*Method I*[*]

Remember that this is the loss cost method with the base class being Class 1.

TABLE 9

| Class (1) | Existing Relativity (2) | Loss Cost (3) | Indicated Relativity (4) | $Z$ (5) | Adopted Relativity $[Z(4)+(1-Z)(2)]$ (6) |
|---|---|---|---|---|---|
| 1 | 1.00 | 60.00 | 1.000 | 1.000 | 1.000 |
| 2 | 1.25 | 85.00 | 1.416 | 0.500 | 1.333 |
| 3 | 1.50 | 79.50 | 1.325 | 0.600 | 1.395 |

Again we have created off-balance, so we balance back:

$$\text{Old Average Relativity} = [500(1.00)+150(1.25)+200(1.50)]/850$$
$$= 1.1617647$$
$$\text{New Average Relativity} = [500(1.000)+150(1.333)+200(1.395)]/850$$
$$= 1.1517647$$
$$\text{Balance-Back Factor} = 1.1617647/1.1517647 = 1.0086823.$$

This produces the following new "base" rates:

TABLE 10

| Class | New Rate | Exposure Units | Premium Income |
|---|---|---|---|
| 1 | $106.92 | 500 | $53,460.16 |
| 2 | 142.56 | 150 | 21,384.06 |
| 3 | 149.15 | 200 | 29,830.77 |
| Total | | 850 | 104,674.99 |

or, $104,675, which is what we want.

[*]Refers to methods with credibility-weighted relativities.

*Method II**

This is the loss cost method, but with the "base" being the state (loss cost).

TABLE 11

| Class (1) | Existing Relativity (2) | Loss Cost (3) | Indicated Relativity [(3)/(3) Total] (4) | Ind. Rel. Class 1 = 1.00 $[(4)/(4)_1]$ (5) | $Z$ (6) | Adopted Relativity $[Z \times (5) + (1 - Z) \times (2)]$ (7) |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 60.00 | 0.8695652 | 1.000 | 1.000 | 1.000 |
| 2 | 1.25 | 85.00 | 1.2318841 | 1.416 | 0.500 | 1.333 |
| 3 | 1.50 | 79.50 | 1.1521739 | 1.325 | 0.600 | 1.395 |
| | | 69.00 | | | | |

This is the same answer as *Method I**.

However, it is possible to get an incorrect answer by changing the order of the arithmetic operations. For example, one might do the following erroneous calculation:

TABLE 12

| Class (1) | Existing Relativity (2) | Loss Cost (3) | Indicated Relativity [(3)/(3) Total] (4) | $Z$ (5) | Adopted Relativity $[Z \times (4) + (1 - Z) \times (2)]$ (6) | Rate Manual Relativity (Class 1 = 1.00) (7) |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 60.00 | 0.8695652 | 1.000 | 0.8695652 | 1.0000000 |
| 2 | 1.25 | 85.00 | 1.2318841 | 0.500 | 1.2409421 | 1.4270834 |
| 3 | 1.50 | 79.50 | 1.1521739 | 0.600 | 1.2913043 | 1.4850000 |
| Total | | 69.00 | | | | |

This answer is different than those found in the previous two calculations, and it is wrong.

It is wrong because in the formula for the adopted relativity $[Z \times (4) + (1 - Z) \times (2)]$, you do not have the relativities on the same basis. Column (2) has the relativities "normalized" such that the relativity for Class 1 equals 1.000, but in Column (4) the data have not been "normalized." Thus, in the formula for the adopted relativity, we are taking the weighted average of "apples" from Column (2) and "oranges" from Column (4). One could extend the analogy to consider one vector as degrees Fahrenheit and the other, degrees Celsius. These should not be commingled in a weighted average. Obviously, this would lead to an incorrect result.

*Method III**

This is the classical loss ratio method with Class 1 being the base.

TABLE 13

| Class (1) | Loss Ratio (2) | Existing Relativity (3) | Indicated Change $[LR_i/LR_1]$ (4) | Indicated Relativity (5) | $Z$ (6) | Adopted Relativity $[Z \times (5) + (1 - Z) \times (3)]$ (7) |
|---|---|---|---|---|---|---|
| 1 | 0.6000 | 1.00 | 1.000 | 1.000 | 1.000 | 1.000 |
| 2 | 0.6800 | 1.25 | 1.133 | 1.416 | 0.500 | 1.333 |
| 3 | 0.5300 | 1.50 | 0.883 | 1.325 | 0.600 | 1.395 |

This agrees nicely with all of our previous work.

*Method IV**

Again, this is the loss ratio method with the base "class" being the state (loss ratio).

TABLE 14

| Class (1) | Loss Ratio (2) | Existing Relativity (3) | Indicated Change $[LR_i/LR_S]$ (4) | Indicated Relativity $[(4)\times(3)]$ (5) | Indicated Relativity Class 1 = 1.00 (6) | $Z$ (7) | Adopted Relativity $[Z\times(6)+(1-Z)\times(3)]$ (8) |
|---|---|---|---|---|---|---|---|
| 1 | 0.6000 | 1.00 | 1.0102302 | 1.0102302 | 1.000 | 1.000 | 1.000 |
| 2 | 0.6800 | 1.25 | 1.1449275 | 1.4311594 | 1.416 | 0.500 | 1.333 |
| 3 | 0.5300 | 1.50 | 0.8923700 | 1.3385550 | 1.325 | 0.600 | 1.395 |
| State | 0.5939 | | | | | | |

Obviously, this is an acceptable answer. But again, the order of calculation and the use of factors that are "normalized" to the same base are of the essence. For example, we could erroneously do the following:

TABLE 15

| Class (1) | Loss Ratio (2) | Existing Relativity (3) | Indicated Change $[LR_i/LR_S]$ (4) | Indicated Relativity $[(4)\times(3)]$ (5) | $Z$ (6) | Adopted Relativity $[Z(5)+(1-Z)\times(3)]$ (7) | Rate Manual Relativity (Class 1 = 1.000) (8) |
|---|---|---|---|---|---|---|---|
| 1 | 0.6000 | 1.00 | 1.0102302 | 1.0102302 | 1.000 | 1.0102302 | 1.000 |
| 2 | 0.5280 | 1.25 | 1.1449275 | 1.4311594 | 0.500 | 1.3405797 | 1.327 |
| 3 | 0.5400 | 1.50 | 0.8923700 | 1.3385550 | 0.600 | 1.4031330 | 1.389 |
| State | 0.5676 | | | | | | |

This is an incorrect answer because in our adopted relativity calculation, Column (3) has been "normalized" so that Class 1 has a relativity equal to 1.00, but Column (5) has not. Thus, we are attempting to do a weighted average of "apples" and "oranges."

*Method V**

This uses the template found in Chapter 6 of the *Foundations* text as authored by Finger (2001).

TABLE 16

| Class (1) | Existing Relativities (2) | Exposure Units (3) | Earned Premiums (4) | Adjusted Exposures [(3) × (2)] (5) | Adjusted Loss Costs [$ Loss/(5)] (6) | Indicated Adjustment [(6)/(6) Total] (7) | $Z$ (8) |
|---|---|---|---|---|---|---|---|
| 1 | 1.00 | 500 | 50,000 | 500 | 60.00 | 1.0102302 | 1.000 |
| 2 | 1.25 | 150 | 18,750 | 187.5 | 68.00 | 1.1449275 | 0.500 |
| 3 | 1.50 | 200 | 30,000 | 300 | 53.00 | 0.8923700 | 0.600 |
| Total | | 850 | 98,750 | 987.5 | 59.39 | | |

Continuing with the template:

TABLE 17

| Class | Credibility Weighted Adjustment $[Z \times (7) + (1 - Z)]$ (9) | Extension [(9) × (4)] (10) | Balanced Adjustment [(9)/(9) Total] (11) | New Rates [(11) × Old × 1.06] (12) | Extension [(12) × (3)] (13) |
|---|---|---|---|---|---|
| 1 | 1.0102302 | 50,511.51 | 1.0109174 | 107.16 | 53,578.62 |
| 2 | 1.0724638 | 20,108.70 | 1.0731934 | 142.20 | 21,329.72 |
| 3 | 0.9354220 | 28,062.66 | 0.9360583 | 148.83 | 29,766.65 |
| Total | 0.9993202† | 98,682.87 | | | 104,674.99 |

†0.9993202 = 98,682.87/98,750.

This all seems to check out just fine. The final answer ($104,675) is a +6% rate increase as requested. However, the "New Rates" are different than what we got in the other four methods.

One can also see that the new class relativities created by Finger (but never actually displayed) are as follows and differ from those calculated by Methods I* to IV*.

TABLE 18

| Class | Relativity with Class 1 = 1.00 |
|---|---|
| 1 | 1.000 |
| 2 | 1.327 |
| 3 | 1.389 |

Why is this?

With a little bit of work (and some insight) the differences are easily reconciled.

In Methods I* to IV*, we calculated all relativities using a base relativity of 1.000 for Class 1. What Finger does is to calculate all relativities using a base relativity of 1.000 for the state. We can show that this is true by recalculating Methods I* to IV* using a relativity of 1.000 for the state.

In our existing examples, the following hold:

TABLE 19

| Class | Relativity |
|---|---|
| 1 | 1.000 |
| 2 | 1.250 |
| 3 | 1.500 |
| State | 1.1617647 |

Switch these values to equivalent values with the state relativity equal to 1.000 and you get:

TABLE 20

| Class | Relativity |
|---|---|
| 1 | 0.8607595 |
| 2 | 1.0759494 |
| 3 | 1.2911392 |
| State | 1.0000000 |

Now, calculate your credibility-weighted new relativities using the above as starting points. You will get the following:

TABLE 21

| Class (1) | Existing Relativity (2) | Loss Cost (3) | Indicated Relativity (4) | $Z$ (5) | Adopted Relativity (6) |
|---|---|---|---|---|---|
| 1 | 0.8607595 | $60.00 | 0.8695652 | 1.00 | 0.8695652 |
| 2 | 1.0759494 | 85.00 | 1.2318841 | 0.50 | 1.1539167 |
| 3 | 1.2911392 | 79.50 | 1.1521739 | 0.60 | 1.2077600 |
| Total | | 69.00 | | | |

With this change in format, you will arrive at the premiums and relativities derived by Finger. Just recreate the adopted relativities in Table 21 with Class 1 = 1.000, and you get:

TABLE 22

| Class | Adopted Relativity with Class 1 = 1.000 |
|---|---|
| 1 | 1.000 |
| 2 | 1.327 |
| 3 | 1.389 |

Thus, one cannot conclude that one methodology is correct and the other incorrect. They are merely two versions of the same analysis that happen to result in slightly different answers. However, there are some implications to these findings, including:

- Regulators cannot guarantee that two actuaries will arrive at the same answer given the same data without prescribing the methodology in extreme detail.
- Educators, like me, who have to create questions for term tests and final examinations, cannot guarantee a uniquely correct answer to a question unless the method of solution is defined in extreme detail.

- The pricing actuary who is aware of these differences might then be able to use them to his or her advantage.

For example, assume you have two large classes (A and B) that are fully credible and a few smaller classes with little credibility. If we assume that A increases by 10% and B declines by 10%, then the choice of A or B as the base class will drive the rates of the classes with little credibility. If we choose A, their rates will go up and if we choose B their rates will go down. If we choose the statewide average, their rates will change little (all else being equal).

## 3. CONCLUSION

As stated in the introduction, classification ratemaking is one of the most important steps in arriving at new rate manual rates.

This topic has been presented in a variety of forms, templates, and methodologies over the years. Unfortunately, the different methods presented to students do not necessarily produce the same unique result.

It is the belief and hope of this author that a full understanding of the consequences as presented in this paper will bring the level of knowledge of future students to a new high in this important area.

## REFERENCES

[1] Brown, R. L., and L. R. Gottlieb, *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, 2nd ed., (Winstead, Ct.: ACTEX Publications Inc., 2001).

[2] Finger, R. J., "Risk Classification," Chapter 6 in *Foundations of Casualty Actuarial Science*, 4th ed., (Arlington, Va.: Casualty Actuarial Society, 2001).