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RATEMAKING: A FINANCIAL ECONOMICS APPROACH

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1. INTRODUCTION

In their 1997 *Proceedings* paper, Stephen D'Arcy and Michael Dyer survey the property-casualty ratemaking landscape from a financial economics perspective. While they summarize several approaches, including target total return, CAPM, and discounted cash flow, this discussion is devoted entirely to Section 8 of their paper, which deals with a method that draws upon option pricing theory (OPT).

D'Arcy and Dyer base their exposition on the approach first presented by Doherty and Garven [2] in 1986 and updated by Garven [3] in 1988. Unfortunately, their presentation falls short as a primer on the OPT approach to ratemaking for several reasons. The first problem is that they make some mystifying mistakes with the options, mischaracterizing both the policyholders' claim (wrongly calling it a call option) and the government's tax claim (correctly calling it a call option but wrongly parameterizing it). This might stem from the cumbersome notation they employ (largely borrowed from Doherty-Garven), which they do not use consistently. Second, while the discourse is ostensibly about ratemaking, the authors provide neither a formula for nor an example of the calculation of the fair premium that is the objective of the ratemaking exercise. Finally, most of their discussion treats the aggregate amount of insurance claims as a fixed and known quantity, essentially as a loan. They merely note that allowing claims to vary stochastically "complicates the calculation" and provide a formula but no example. Unfortunately, the formula they give (which is wrongly attributed to Doherty-Garven) does not adequately address the stochastic claims scenario.

Despite these and other shortcomings, the D'Arcy-Dyer paper is still a useful springboard for discussing the OPT approach. The purpose of this discussion is to correct, clarify, and extend their work. We will 1) point out and correct what we see as the paper's shortcomings, 2) rework and extend the examples, and 3) expand the exposition to allow for a more complete treatment of taxes and claims that vary stochastically.

2. THE OPTION PRICING THEORY RATEMAKING FRAMEWORK

Essentially, the Doherty-Garven idea is that, given some simplifying assumptions, the pre-tax value of an insurance company can be represented by a call option on the company's assets. This call option has a variable strike price equal to the aggregate amount of claims. The government's tax claim can also be represented as a call option. The implication for ratemaking is that the appropriate rate level is the one that results in equality between beginning policyholders' surplus and the value of the call option representing the shareholders' interest, net of tax.

Doherty and Garven assume the insurer's assets consist of tradable investments on which it is possible to price an option. They entertain asset-value distributions in both the normal and lognormal family. They also allow for the possibility of tax-advantaged investments within the company's investment portfolio, though they do not allow for tax-loss carryforwards or carrybacks. In addition, their analysis considers the possibility that premium funds might be held for more than one year while claims are being negotiated.

D'Arcy and Dyer simplified some assumptions in order to make their illustration easier to follow:

1. All policies are written for a one-year term at a common date.

2. Claims totaling L are paid exactly one year from policy inception.

3. Premium funds (net of expenses) are received at policy inception.

4. Premium receipts, P_0, and initial surplus, S_0, are invested solely in taxable assets initially valued at Y_0.

3. NOTATION

In a bid to clarify key concepts, this discussion uses the basic D'Arcy-Dyer notation with some refinements. For example, we will use numerical subscripts to refer to time only: 0 meaning inception, 1 meaning one year after inception. Variables l and y refer to the random variables representing aggregate claims and invested assets, respectively, one year from inception. The equation $\text{call}_1(y \mid Y_1, L)$ refers to the expiry value of a one-year European call option on y, given a price at expiry of $y = Y_1$ and an exercise price of $l = L$. The equation $\text{call}_0(y \mid Y_0, L)$ refers to the value of the same option at inception, when the value of invested assets is Y_0. We use a similar notation for European puts. Other notation will be introduced and defined as needed. In Appendix A, we restate (and, where necessary, correct) the D'Arcy-Dyer formulas (8.3) through (8.9) in our notation.

4. FINDING THE OPTIONS

As D'Arcy and Dyer point out, Y_1, which represents the value of the investment portfolio after one year, is the amount the insurer has available to pay the claims, L. If $Y_1 \geq L$, the insurer will pay the policyholder claims in full. If $Y_1 < L$, the insurer will pay the policyholder claims up to the extent of its available assets, i.e., Y_1. If Y_1 is limited to a minimum value of zero, this policyholders' interest can be summarized as:

$$H_1 = \max[\min(Y_1, L), 0]. \tag{8.3}$$

While (8.3) is correct, D'Arcy and Dyer *incorrectly* describe H_1 as equivalent to the payoff at expiry from a European call option on the invested assets with a strike price of L. Formula (8.3) does not define a call option. The call option of the authors' description belongs to the *shareholders*, not the policyholders. The sale of the insurance policies in exchange for premiums is equivalent to the sale of the company's assets (including the premiums) to the policyholders in exchange for a call option to reacquire the assets at a price of L. If $Y_1 \geq L$, the insurer will exercise the option and reacquire the assets. This results in a gain of $Y_1 - L$. If $Y_1 < L$, the insurer will not exercise the option. The gain is 0. The pre-tax shareholders' interest can be summarized as:

$$\begin{aligned} C_1 &= \max(Y_1 - L, 0) \\ &= \text{call}_1(y \mid Y_1, L). \end{aligned} \tag{1}$$

C_1 matches the payoff value at expiry of a European call option on y, given invested assets at expiry of Y_1 and an option strike price of L.[1]

The policyholders' interest at expiry, H_1, can be characterized as a long position in y and a short position in the call or, alternatively, a long position in y net of the pre-tax shareholders' interest:

$$H_1 = Y_1 - \text{call}_1(y \mid Y_1, L) = Y_1 - C_1. \tag{2}$$

Formula (2) is equivalent to formula (8.3).

Neither D'Arcy-Dyer nor Doherty-Garven mention the formulation of the policyholders' interest in terms of a put option. Since put-call parity implies $Y_1 - \text{call}_1(y \mid Y_1, L) = L - \text{put}_1(y \mid Y_1, L)$, the policyholders' interest at the end of the period can also be characterized as:

$$H_1 = L - \text{put}_1(y \mid Y_1, L), \tag{3}$$

[1]Note that we use C_1 to denote the value of the pre-tax shareholders' interest at time 1, whereas D'Arcy and Dyer confusingly use C_1 to denote its value at time 0.

where $\text{put}_1(y \mid Y_1, L)$ denotes the payoff value at expiry of a European put option on y, given invested assets at expiry of Y_1 and an option strike price of L. Clearly, it is in the policyholders' best interest to minimize the value of the put option, since they are paying for the full recovery of L. Failing that, it seems fair that they should receive a premium discount to reflect the fact they are not receiving full coverage. A fair premium would then be:

$$P_0 = H_0 = Le^{-rt} - \text{put}_0(y \mid Y_0, L). \tag{4}$$

The fair premium is equal to the present value of L less the value of the default put option. Since $\text{put}_0(y \mid Y_0, L)$ is a function of P_0, formula (4) must be solved by numerical methods.

The value of the pre-tax shareholders' interest, C_0, at policy inception is:[2]

$$C_0 = Y_0 - H_0 = Y_0 - Le^{-rt} + \text{put}_0(y \mid Y_0, L). \tag{5}$$

Since $C_0 = \text{call}_0(y \mid Y_0, L)$ and y represents an asset whose behavior is consistent with the conditions required by the Black-Scholes call option pricing formula, the value of C_0 can easily be calculated. D'Arcy and Dyer illustrate its calculation with an example. Given $S_0 = \$100$ million, $P_0 = \$160$ million, $L = \$150$ million, risk-free rate $r = 4\%$, $k = t = 1$ year, and asset volatility $\sigma = 50\%$ (reflecting an *extremely* aggressive investment strategy!), they show the Black-Scholes value of the pre-tax shareholders' interest at inception is \$121.41 million. (This discussion calculates it at \$121.42 million, but we will use their number.) D'Arcy and Dyer note that this is surprisingly high, since "adding the initial equity to the underwriting profit totals \$110 million." They attribute the difference to the "default option" considered by the option methodology. Qualitatively, this is correct, but it is wrong to compare the \$110 to the \$121.41, since the first number is valued at the end of the period (but without interest) while the latter is valued at the

[2]Note that this discussion uses C_0 instead of D'Arcy-Dyer's C_1 to denote the pre-tax shareholders' interest.

beginning. From formula (5) it is easy to see that, if the value of the put is zero, the pre-tax shareholders' interest is equal to $Y_0 - Le^{-rt} = (\$100 + \$160) - \$150e^{-0.04} = \$115.88$. The amount attributable to the default option arising from the investment in risky securities is the difference, $C_0 - (Y_0 - Le^{-rt}) = \text{put}_0(y \mid Y_0, L) = \$121.41 - \$115.88 = \$5.53$. This is a substantial amount, but not nearly as large as D'Arcy and Dyer's wording suggests.

Doherty and Garven observe that in equilibrium the present value of the shareholders' interest must equal the initial surplus, so in the pre-tax case this implies:

$$C_0 = S_0. \tag{6}$$

Combining (8.1), which is $Y_0 = S_0 + P_0$, with (5) and (6), we have

$$(S_0 + P_0) - Le^{-rt} + \text{put}_0(Y_0, L) = S_0$$

$$P_0 = Le^{-rt} - \text{put}_0(Y_0, L), \quad \text{which is formula (4).}$$

Solving (4) for P_0 in the authors' example results in $P_0 = \$136.44$ at equilibrium.

In this example, there is no underwriting risk to the insurer, since L is fixed at \$150. To pay the \$150 at the end of the year, the insurer needs $\$150e^{-0.04} = \$144.12$ at inception to meet that obligation, reflecting an interest credit of \$5.88. The difference $\$144.12 - \$136.44 = \$7.68$, the value of the put, represents a credit to the policyholders to reflect the risk the insurer will default on claim payments.

This illustrates one of the interesting aspects of the option-based approach, namely, that it automatically incorporates the claim default risk into the insurance rate. It highlights the solvency implications of investment strategy and underwriting leverage. Other ratemaking methods implicitly assume the default risk is immaterial. The Doherty-Garven equilibrium is premised on the idea that an insurer's shareholders should not receive the windfall benefit of the default option that arises from a pursuit of a risky investment strategy and/or high underwriting leverage. Instead, the policyholder premium should be reduced. However, that has the paradoxical implication that insurers most at risk of insolvency are *required* to charge premiums that are less than the expected value of their claim obligations, which clearly can only hasten their demise. That hardly seems like the right recipe for rehabilitation of a financially weak or poorly managed insurer! It would seem to be better public policy for regulators to establish investment and underwriting leverage standards that avoid anything beyond a negligible risk of insolvency. Where the risk of insolvency is found to be material, the remedy should be to correct the insurer's financial or strategy weaknesses, rather than to require it to reduce its rates. In that light, it seems entirely appropriate that other ratemaking methods ignore the risk of insolvency, since effective regulation should make it remote.

Note that D'Arcy and Dyer chose an unrealistic investment volatility parameter, σ, for their example. The standard deviation of U.S. stock market returns from 1900–2000 was 20.2% [1]. The authors' choice of $\sigma = 50\%$ implies an investment strategy much riskier than investing 100% of assets in a diversified portfolio of U.S. equities, which itself is a strategy that an insurer and regulators would find far too risky. The example of $\sigma = 50\%$ was undoubtedly chosen in order to illustrate a material default-risk credit.

If we rework the authors' example using $\sigma = 20\%$, which is consistent with 100% of assets in U.S. equities and thus still very aggressive for an insurer, a premium of \$160 implies $C_0 = \$115.90$. Since the value of the underwriting profit alone indicates a value of \$115.88, the default put option is worth only \$0.02! For more realistic and prudent investment strategies with $\sigma < 20\%$, the value of the default put option is essentially zero.

If we solve equation (4) to find the equilibrium value of P_0, given $\sigma = 20\%$, we obtain $P_0 = \$144.07$. Since the present value of L is \$144.12, this implies the value of the default option is \$0.05. The risk of default is slightly higher with a premium of

\$144.07 than it is with a premium of \$160, resulting in an increase in the default-risk credit from \$0.02 to \$0.05.

5. EFFECT OF TAXES

Let's now consider the effect of taxes. If taxes apply only to income and no tax credits arise from losses, then the government's tax interest in the insurer's income can also be characterized as a call option. D'Arcy and Dyer correctly describe the payoff value of this tax interest as:

$$T_1 = \max\{\mathrm{tax}\cdot[i\cdot(Y_1 - Y_0) + P_0 - L], 0\}. \tag{8.4}$$

Setting the tax rate = 35% and the proportion of taxable assets $i = 100\%$ in their example with $P_0 = \$160$, they claim this corresponds to the payoff value of 0.35 European call options with a total value of \$16.05. Unfortunately, the parameters they use in the Black-Scholes formula do not make sense. Their parameters and the \$16.05 correspond to $0.35\cdot \mathrm{call}_0(y \mid Y_1 - Y_0 + P_0, L)$, implying the value of invested assets at time zero is $Y_1 - Y_0 + P_0$, when clearly it must be Y_0. The correct value of the tax call is \$20.96, as we show below.

Consistent with (8.4) with $i = 100\%$, the value of the insurer's income at the end of the period is:

$$\begin{aligned} I_1 &= (Y_1 - Y_0) + (P_0 - L) \\ &= Y_1 - (S_0 + P_0) + (P_0 - L) \\ &= Y_1 - (S_0 + L). \end{aligned} \tag{8}$$

If we focus only on positive outcomes, $\max(I_1, 0)$ is the payoff profile at expiry of a call option on invested assets, y, with a strike price of $S_0 + L$. Assuming the investment return is 100% taxable, the present value of the government's positive tax interest is equal to the tax rate times this call option:

$$T_0 = \mathrm{tax}\cdot \mathrm{call}_0(y \mid Y_0, S_0 + L). \tag{9}$$

The correct value of the tax call in the authors' example is

$$\begin{aligned} T_0 &= 0.35 \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L) = 0.35 \cdot \mathrm{call}_0(y \mid 260, 250) \\ &= (0.35)(\$59.89) = \$20.96. \end{aligned}$$

Then the shareholders' interest, net of tax, is

$$C_0 - T_0 = \$121.41 - \$20.96 = \$100.45$$

instead of the \$105.36 given by D'Arcy and Dyer.

To find the fair premium in these circumstances, we solve for the value of P_0 that meets the condition $C_0 - T_0 = S_0$. Since $C_0 = Y_0 - [Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L)]$ and $Y_0 = S_0 + P_0$, then

$$C_0 - T_0 = S_0 \quad \text{implies}$$

$$(S_0 + P_0) - [Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L)] - \mathrm{tax} \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L) = S_0$$

$$\text{and} \qquad P_0 = Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L) + \mathrm{tax} \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L). \tag{10}$$

This implies a fair premium $P_0 = \$159.33$ in the authors' after-tax example.

The Doherty-Garven model deliberately ignored tax-loss carryforward and carryback provisions. However, they are actually easy to deal with within the simple framework presented by D'Arcy and Dyer.

Assume the tax code allows for tax-loss carryforwards and carrybacks. If $Y_1 < S_0 + L$, which implies a loss, the insurer earns a tax credit of $\mathrm{tax} \cdot (S_0 + L - Y_1)$. If $Y_1 \geq S_0 + L$, which implies a profit, the insurer earns a tax credit of zero. That tax-credit pattern matches the payoff profile of tax European-put options on y having a strike price of $S_0 + L$. Thus the tax credit equates to a long put-option position owned by the insurer, and a short put-option position on the part of the government. If the insurer becomes insolvent ($Y_1 < L$), then it won't be in a position to use its tax credit. Therefore, the portion of the credit arising from insolvency scenarios must be removed.

The government's net tax-option position in this symmetrical tax scenario is:

$$T_0^* =$$
$$\text{tax} \cdot \{\text{call}_0(y \mid Y_0, S_0 + L) - [\text{put}_0(y \mid Y_0, S_0 + L) - \text{put}_0(y \mid Y_0, L)]\}.$$

In the authors' $P_0 = \$160$ example with tax = 35%, the tax put has a value of

$$\begin{aligned} 0.35 \cdot [\text{put}_0(y \mid Y_0, S_0 + L) &- \text{put}_0(y \mid Y_0, S_0 + L)] \\ &= 0.35 \cdot (\$40.08 - \$5.54) = \$12.09; \end{aligned}$$

and the symmetrical after-tax value of the shareholders' interest, $V_0^*(P_0 \mid L)$, is

$$\begin{aligned} V_0^*(160 \mid 150) = C_0 - T_0^* &= \$121.41 - \$20.96 + \$14.03 - \$1.93 \\ &= \$112.55. \end{aligned}$$

The formula for T_0^* can be simplified. Since put-call parity implies $\text{call}_0(y \mid Y_0, S_0 + L) - \text{put}_0(y \mid Y_0, S_0 + L) = Y_0 - (S_0 + L)e^{-rt}$, the symmetrical tax obligation can be expressed as

$$T_0^* = \text{tax} \cdot [Y_0 - (S_0 + L)e^{-rt} + \text{put}_0(y \mid Y_0, L)]. \tag{11}$$

If the tax treatment of profits and losses is symmetrical, we can determine the fair premium by substituting T_0^* for the tax term in (10):

$$\begin{aligned} P_0 &= Le^{-rt} - \text{put}_0(y \mid Y_0, L) \\ &\qquad + \text{tax} \cdot [Y_0 - (S_0 + L)e^{-rt} + \text{put}_0(y \mid Y_0, L)] \\ &= Le^{-rt} - \text{put}_0(y \mid Y_0, L) + \frac{\text{tax} \cdot (1 - e^{-rt})}{1 - \text{tax}} \cdot S_0. \end{aligned} \tag{12}$$

Formula (12) implies a fair premium of $P_0 = \$138.80$, given the non-tax parameters used in the authors' example combined with symmetrical taxation.

6. STOCHASTIC CLAIMS

D'Arcy and Dyer discuss the application of the Doherty-Garven approach to the real insurance world in which claims vary stochastically only very briefly. They focus mainly on the scenario where the aggregate claim amount, L, is an amount certain; in effect treating the transaction as a loan. In fact, for realistic insurance-ratemaking applications the claim amount is a random variable, which we will denote l.

If we know $f(l)$, we can determine the unconditional expected value of the shareholders' interest. Assuming symmetrical tax treatment of profits and losses, the expected value of the after-tax shareholders' interest, $\mathrm{E}[V_0^*(P_0)]$, is given by:

$$\begin{aligned} \mathrm{E}[V_0^*(P_0)] &= \int_0^\infty (C_0 - T_0^*)\cdot f(l)dl \\ &= \int_0^\infty \{Y_0 - [le^{-rt} - \mathrm{put}_0(y \mid Y_0, l)] \\ &\quad - \mathrm{tax}\cdot[Y_0 - (S_0 + l)e^{-rt} + \mathrm{put}_0(y \mid Y_0, l)]\}\cdot f(l)dl \\ &= (1-\mathrm{tax})\cdot[Y_0 - \mathrm{E}(l)e^{-rt}] + \mathrm{tax}\cdot S_0 e^{-rt} \\ &\quad + (1-\mathrm{tax})\int_0^\infty \mathrm{put}_0(y \mid Y_0, l)\cdot f(l)dl. \end{aligned} \tag{13}$$

While we are most interested in the symmetrical taxation scenario embodied in (13), before we explore that case further, we will discuss the treatment of stochastic claims in the D'Arcy-Dyer world in which tax-loss carryforwards and carrybacks are not allowed. In that case, the expected value of the after-tax shareholders' interest, $\mathrm{E}[V_0(P_0)]$, is given by:

$$\begin{aligned} \mathrm{E}[V_0(P_0)] &= \int_0^\infty (C_0 - T_0)\cdot f(l)dl \\ &= \int_0^\infty \mathrm{call}_0(y \mid Y_0, l)\cdot f(l)dl \\ &\quad - \int_0^\infty \mathrm{tax}\cdot \mathrm{call}_0(y \mid Y_0, S_0 + l)\cdot f(l)dl. \end{aligned} \tag{14}$$

Let's compare formula (14) to the authors' (8.9), which they describe as applicable when losses are assumed to vary

$$V_e = C[Y_1(P^*);\mathrm{E}(L)] - t \cdot C\{i \cdot [Y_1(P^*) - Y_0(P^*)] + P^*;\mathrm{E}(L)\}, \tag{8.9}$$

where P^* is chosen so that $V_e = S_0$.[3] They attribute (8.9) to Doherty-Garven. As discussed earlier, the second term of (8.9) representing the tax call is wrong. Correcting for that and restating the formula in our notation with $i = 100\%$, the formula becomes:

$$\mathrm{E}[V_0(P_0)] = \mathrm{call}_0[y \mid Y_0, \mathrm{E}(l)] - \mathrm{tax} \cdot \mathrm{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)]. \tag{8.9*}$$

Formula (8.9*) is equivalent to (14) only in the special case where

$$\int_0^\infty \mathrm{call}_0(y \mid Y_0, l) \cdot f(l)dl = \mathrm{call}_0[y \mid Y_0, \mathrm{E}(l)] \qquad \text{and}$$

$$\int_0^\infty \mathrm{call}_0(y \mid Y_0, S_0 + l) \cdot f(l)dl = \mathrm{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)].$$

Clearly, formula (8.9), in either its original or corrected (8.9*) form, cannot represent the value of the shareholders' interest in the stochastic claims case. Both of the call terms depend only on the first moment of the claim distribution, which is a constant, rather than on the whole distribution.

For the sake of illustration, let's assume $f(l)$ is log-normally distributed with $\sigma_l = 11\%$ (a choice inspired by Van Kampen [4]) and $\mathrm{E}(l) = e^{\mu + 0.5\sigma^2} = \$150$. Let all other premium and investment parameters match the authors' original assumptions. Using these parameters, formula (14) with the constraint $\mathrm{E}[V_0(P_0)] = S_0$ indicates a fair premium of \$158.89. Alternatively, if we maintain $\mathrm{E}(l) = \$150$ but let $\sigma_l = 15\%$, the indicated fair premium from (14) with the same constraint is \$158.50. In comparison, the fair

[3]Note that here V_e denotes a time 0 value, while in (8.5) it denotes a time 1 value.

premium indicated by the similarly constrained formula (8.9*) is always \$159.33, irrespective of the value of σ_l. Note also that the indicated premiums arising from the stochastic claims scenarios are lower than the premium indicated by the constant claim scenario because there is a slightly higher risk of insolvency and default when claims can vary.

From D'Arcy and Dyer's discussion about (8.9) it is clear that they believe that formula is faithful to Doherty-Garven, not only in reflecting stochastic variation in claims, but also in reflecting an underwriting-risk charge. The fact is their formula reflects neither. We suspect their confusion arises from the fact that Doherty and Garven presented a similar but not identical formula (their formula (7)):

$$V_e = C[\tilde{Y}_1(P^*);\tilde{L}] - \tau \cdot C\{\theta \cdot [\tilde{Y}_1(P^*) - \tilde{Y}_0(P^*)] + P^*;\tilde{L}\}. \tag{DG.7}$$

Note the difference in the strike prices of E(L) in (8.9) and $\tilde{L}$ in (DG.7). The former is a constant, while the latter is defined by Doherty-Garven to be a random variable. For that reason alone, the two formulas are clearly different. Since the call options used in DG.7 have variable strike prices *and* embedded underwriting-risk charges, they are not the usual kind of European options that have fixed exercise prices. In contrast, the D'Arcy-Dyer formula (8.9) uses standard European calls and reflects no underwriting-risk charge.

Returning to our formula (13) for the expected value of the after-tax shareholders' interest in the symmetrical taxation case, note that if we solve $\mathrm{E}[V_0^*(P_0)] = S_0$ for P_0, we obtain the following stochastic claims analogue to formula (12):

$$P_0 = \mathrm{E}(l)e^{-rt} - \int_0^\infty \mathrm{put}_0(y \mid Y_0, l) \cdot f(l)dl + \frac{\mathrm{tax} \cdot (1 - e^{-rt})}{1 - \mathrm{tax}} \cdot S_0, \tag{15}$$

which, for the example we have been following, implies a premium $P_0 = \$138.22$ (vs. \$138.80 for the fixed $L = \$150$ case) that reflects both symmetrical tax effects and a credit to policyholders to compensate them for the risk of default by the insurer. However, $P_0 = \$138.22$ does not reflect a risk charge to reflect the stochastic nature of l.

To reflect such a risk charge formula, we need to solve for the value of P_0 that satisfies $\mathrm{E}[V_0^*(P_0)] = S_0 + \lambda$, where λ is the after-tax charge for pure underwriting risk. That implies a fair premium in the stochastic claims case with symmetrical tax treatment of

$$P_0 = \mathrm{E}(l)e^{-rt} - \int_0^{\infty} \mathrm{put}_0(y \mid Y_0, l) \cdot f(l)dl + \lambda + \mathrm{tax} \cdot \frac{(1 - e^{-rt}) \cdot S_0 + \lambda}{1 - \mathrm{tax}}. \qquad (16)$$

We see that the only option in formula (16) is the put option representing the credit for insurer insolvency. If that put option has a value of zero, as it should under any effective regulatory regime, formula (16) reduces to the standard actuarial ratemaking formula.

Note that the approach this discussion has taken with respect to the underwriting-risk charge is slightly different from that of Doherty-Garven. They incorporate the risk charge for underwriting risk into the non-standard call-option formula they derived for calculating the value of the options in (DG.7). We prefer to treat the risk charge for underwriting risk explicitly.

Using the D'Arcy-Dyer parameters (except substituting the more realistic investment volatility value $\sigma = 10\%$ for their 50%), and setting $\sigma_l = 11\%$ and $\lambda = 0.0325P_0$, formula (16) indicates $P_0 = \$153.92$ as the appropriate premium reflecting symmetrical taxation, policyholder credit for insurer default risk, and a risk charge for stochastic claims. In this example with a more realistic investment policy assumption, the value of the default option is zero. Table 1 shows the composition of premium.

TABLE 1

COMPOSITION OF FAIR PREMIUM
STOCHASTIC CLAIMS, DEFAULT CREDIT, SYMMETRICAL TAX

		% of P_0
Losses	\$150.00	97.45%
– PV of Interest on Losses	(\$5.88)	–3.82%
= PV of Losses	\$144.12	93.63%
+ PV of Default Option	\$0.00	0.00%
= PV Pure Premium	\$144.12	93.63%
+ PV Taxes T_0	\$4.80	3.12%
= PV Tax-Adj Pure Premium	\$148.92	96.75%
+ PV U/W Risk Charge	\$5.00	3.25%
= Premium (P_0)	\$153.92	100.00%

7. CONCLUSION

D'Arcy and Dyer concluded that the OPT approach is more complex than the CAPM or Discounted Cash Flow approaches, but that it avoids some of the problems associated with CAPM (such as estimating betas). This discussion aims to make it clear that if taxation is symmetrical, which seems more realistic than assuming it is not, and default risk is zero, then the OPT premium formula (16) is the same as the D'Arcy-Dyer Discounted Cash Flow (DCF) premium formula (6.1) with the underwriting-risk charge broken out explicitly. The only real difference from conventional DCF ratemaking in the OPT framework is in the Doherty-Garven approach to the underwriting-risk charge, which they base on the correlation between insurance claims and the stock market, making it similar to the CAPM approach described in Section 4 of the D'Arcy-Dyer paper. Far from avoiding the problems associated with estimating betas, etc., the Doherty-Garven approach to quantifying underwriting risk has *exactly* the same problems as CAPM.

The Doherty-Garven approach is an interesting application of option theory, but it is also much less exotic than it first appears. If insurance regulations aimed at avoiding insolvencies are formulated and executed effectively, then the insolvency put embedded in the fair premium will be zero. There is no need to resort to the option approach. Options can be used, at least conceptually, to describe the effect of a tax law that does not treat profits and losses symmetrically. However, this discussion has shown, if taxation is symmetrical, those options disappear too, and the ratemaking formula reduces to the conventional one, where the remaining debate is about how to calculate the underwriting-risk charge.

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APPENDIX A

D'ARCY-DYER FORMULAS RESTATED IN NOTATION OF THIS DISCUSSION

(8.3) $H_1 = \max\{\min[L, Y_1], 0\}$	$H_1 = \max\{\min[L, Y_1], 0\}$
(8.4) $T_1 = \max\{t[i(Y_1 - Y_0) + P_0 - L], 0\}$	$T_1 = \max\{\text{tax}[i(Y_1 - Y_0) + P_0 - L], 0\}$
(8.5) $V_e = Y_1 - H_1 - T_1$	$C_1 - T_1 = Y_1 - H_1 - T_1$
(8.6) $H_0 = V(Y_1) - C[Y_0; \mathrm{E}(L)]$	$H_0 = Y_0 - C_0$
	$= Y_0 - \text{call}_0(y \mid Y_0, L)$
(8.7) $T_0 = tC[i(Y_1 - Y_0) + P_0; \mathrm{E}(L)]$	$T_0 = Y_0 - \text{tax} \cdot [\text{call}_0(y \mid Y_0, S_0 + L)]^{\dagger}$
(8.8) $V_e = V(Y_1) - H_0 - T_0$	$C_0 - T_0 = Y_0 - H_0 - T_0$
$= C[Y_0; \mathrm{E}(L)]$	$= \text{call}_0(y \mid Y_0, L)$
$- tC[i(Y_1 - Y_0) + P_0; \mathrm{E}(L)]$	$- \text{tax} \cdot \text{call}_0(y \mid Y_0, S_0 + L)^{\dagger}$
$= C_1 - tC_2$	$= C_0 - T_0$
(8.9) $V_e = C[Y_1(P^*); \mathrm{E}(L)]$	$\mathrm{E}[V_0(P_0)] = \text{call}_0[y \mid Y_0, \mathrm{E}(l)]$
$- tC[i(Y_1(P^*) - Y_0(P^*)) + P^*; \mathrm{E}(L)]$	$- \text{tax} \cdot \text{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)]$
$= C_1^* - tC_2^* = S_0$	$= C_0 - T_0^* = S_0{}^{\dagger}$

†for $i = 1$.