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# PROCEEDINGS
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## THE STANARD-BÜHLMANN RESERVING PROCEDURE: A PRACTITIONER'S GUIDE

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*Abstract*

*The Stanard-Bühlmann reserving method, commonly used by reinsurance actuaries, combines the stability of expected loss methods with the adherence to empirical data that is characteristic of the chain ladder method. This paper is a teaching guide for the reserving actuary: it explains the intuition for the Stanard-Bühlmann reserving method, shows an algebraic derivation from the Bornhuetter-Ferguson reserving method, uses a series of illustrations to explain the needed premium adjustments, and compares the Stanard-Bühlmann reserving method with the Bornhuetter-Ferguson and chain ladder methods.*

## 1. INTRODUCTION

An ideal loss reserving method would rely primarily on observed data but not be subject to random loss fluctuations. The chain ladder reserving method relies entirely on historical loss triangles, but it is sensitive to random loss fluctuations in the most recent years. The Bornhuetter-Ferguson expected loss reserving method sometimes provides more stable reserve indications, but it requires an a priori estimate of the expected losses. The Stanard-Bühlmann reserving method has the stability of an expected loss method, yet it draws all the needed information from the observed experience.

The Stanard-Bühlmann procedure has been a major advance in actuarial loss reserving methods. It has proved especially useful for reinsurers lacking the pricing data to use other expected loss methods. Primary companies may benefit equally from this technique, particularly if the reserving actuary does not have a good estimate of the expected loss ratio.[1]

This paper explains the intuition for the Stanard-Bühlmann reserving method. We begin with the assumption underlying most reserving techniques—that historical patterns may be repeated in the future—and we differentiate among the patterns that chain ladder and expected loss methods use. We provide two derivations of the Stanard-Bühlmann method: an algebraic derivation from the Bornhuetter-Ferguson expected loss method and an intuitive derivation based on the speed of "processing" premiums. Finally, we explain the premium adjustments made in the Stanard-Bühlmann method and we give illustrations for several scenarios.

### *1.1. Patterns of Stability*

Most actuarial reserving techniques assume that certain loss reporting patterns or loss settlement patterns remain relatively stable over time. The observed patterns, adjusted (if necessary) for changes in the insurance environment and company claims practices, are a reasonable predictor of future experience.

[1] The Stanard-Bühlmann technique is also called the "adjustment to total known losses" (Stanard [1985]). Patrik [2002] provides a general introduction to this method.

TABLE 1.1

| Development Months | 0–12 | 12–24 | 24–36 | 36–48 | 48–60 |
|---|---|---|---|---|---|
| Incremental Paid Losses | $100,000 | $200,000 | $150,000 | $75,000 | $25,000 |

Illustration: Most formulations of the stability principle are chain ladder relationships, such as

- Reported losses as of 24 months since inception of the accident year are expected to be 50% higher than reported losses as of 12 months for that accident year.
- Cumulative paid losses as of 48 months since inception of the accident year are expected to be 20% higher than cumulative paid losses as of 36 months for that accident year.

The format of the two statements above is that the cumulative losses (of whatever type) as of development period $i+1$ are $X\%$ greater or lower than the same cumulative losses as of development period $i$. This is the chain ladder format; the format differs for expected loss reserving methods.

Suppose $550,000 of accident year losses were paid over a five-year period as shown in Table 1.1.

We formulate the observed pattern:

A. Incremental Development: Losses paid between 12 and 24 months are twice the losses paid between 0 and 12 months. Losses paid between 24 and 36 months are three quarters of the losses paid between 12 and 24 months.

B. Cumulative Development: The cumulative losses paid from 0 to 24 months are three times the cumulative losses paid from 0 to 12 months. The cumulative losses paid from 0 to 36 months are one and a half times the cumulative losses paid from 0 to 24 months.

TABLE 1.2

| Development Months | 0–12 | 12–24 | 24–36 | 36–48 | 48–60 |
|---|---|---|---|---|---|
| Incremental Paid Losses | \$100,000 | \$200,000 | \$150,000 | \$75,000 | \$25,000 |
| Cumulative Paid Losses | \$100,000 | \$300,000 | \$450,000 | \$525,000 | \$550,000 |
| Incremental Ratio | | 2.000 | 0.750 | 0.500 | 0.333 |
| Cumulative Ratio | | 3.000 | 1.500 | 1.167 | 1.048 |
| Loss Development Factor | | 5.500 | 1.833 | 1.222 | 1.048 |
| Incremental Portion of Ultimate | 0.182 | 0.364 | 0.273 | 0.136 | 0.045 |
| Cumulative Portion of Ultimate | 0.182 | 0.546 | 0.819 | 0.955 | 1.000 |
| Future Portion of Ultimate | 0.818 | 0.454 | 0.181 | 0.045 | 0.000 |

C. Percentages of Ultimate: Of the \$550,000 total paid losses, 18.2% are paid in the first 12 months, and 36.4% are paid in the next 12 months.

These patterns differ in their measurement bases. The patterns are shown in Table 1.2.

For the expected losses, the different bases can be converted into one another. Given the incremental ratio pattern, we can derive the cumulative ratio and the percent of ultimate.

The chain ladder method uses cumulative ratios. The paid loss link ratio from 36 to 48 months is the 1.167 in the 36 to 48 months column of the Cumulative Ratio row. The cumulative product of the link ratios from a given development date to ultimate is the ultimate loss development factor. The loss development factor from 36 months to ultimate is $1.167 \times 1.048 = 1.222$.[2]

The Incremental Portion of Ultimate row is used for expected loss methods. The Bornhuetter-Ferguson factor is the sum of the portion of ultimate figures from a given development date forward. For instance, the factor from 36 months to ultimate is $0.136 + 0.045 = 0.181$. The Bornhuetter-Ferguson factor equals $1 - (1/\text{the link ratio})$. In this example, $0.181 = 1 - (1/1.222)$.

[2]The chain ladder and expected loss reserving methods are described in Wiser [2001], Salzmann [1984], Peterson [1981], and Feldblum [2002].

The portion of ultimate factor used in the Stanard-Bühlmann method is the complement of the Bornhuetter-Ferguson factor. The portion of ultimate factor at 36 months is $1 - 0.181 = 0.819$.

### *1.2. Estimated Ultimate Losses*

The Bornhuetter-Ferguson method needs an estimate of the ultimate losses. For primary companies, the pricing actuary estimates ultimate losses to set premium rates. The reserving actuary can use the estimate provided by the pricing actuary.

The estimate of ultimate losses is the premium times the expected loss ratio. This estimate is suitable when the premium charged is the indicated premium. It must be adjusted if the manual premium differs from the indicated premium or if underwriters provide schedule credits and debits to individual insureds. These adjustments demand business acumen, but a knowledgeable actuary can often make a reasonable estimate of the ultimate losses.

The reinsurer's reserving actuary may not have this underwriting information. The reinsurer's reserving book of business may consist of disparate pieces with different expected loss ratios. The reinsurer may not have the information to adjust for the adequacy level of the primary premiums or for schedule credits and debits provided by the primary underwriters. This is also true for primary insurers if the reserving actuary does not have access to the pricing actuary's estimates, to manual deviations from indicated rates, or to the underwriters discretionary price modifications. This is often the case for large commercial lines insurers.

James Stanard and Hans Bühlmann provided a solution to this quandary. If we have sufficient past experience, they argued, we do not need to know the expected loss ratio. We simply adjust all premiums in the historical period to the same level of adequacy, so the expected loss ratios are the same in each year. We first provide the intuition underlying their method, and then we show the premium adjustments.

TABLE 1.3

| Development Date | Percent Reported | Development Date | Percent Reported |
|---|---|---|---|
| 12 months | 30% | 72 months | 85% |
| 24 months | 50% | 84 months | 90% |
| 36 months | 65% | 96 months | 94% |
| 48 months | 75% | 108 months | 97% |
| 60 months | 80% | 120 months | 99% |

## *1.3. Derivation*

We first derive the Stanard-Bühlmann method from the Bornhuetter-Ferguson method, which is better known to many readers; we then proceed to the intuition for the method. The Bornhuetter-Ferguson method defines the bulk reserve[3] as

adequate premium × expected loss ratio × percentage unreported.

Illustration: Tables 1.3 and 1.4 show the expected loss reporting percentages from inception of the accident year and the premiums and losses by accident year at year-end 20X9. A slow reporting pattern is common for casualty excess-of-loss reinsurance, products liability, and professional liability.

We explain the derivation of the Adjusted Premiums after explaining the reserving method.

If the premiums are at the same adequacy level, then the multiplicative factor needed to arrive at the expected losses is the same for all accident years. For instance, if the premiums are all 20% inadequate, then the expected losses in each accident year equal

$$\text{premium} \times 1.200 \times \text{expected loss ratio.}^{4}$$

[3]The bulk reserve, or the actuarial reserve, covers the emergence on unreported claims and adverse development on reported claims.

[4]The terms "premium adequacy" and "expected loss ratio" have numerous interpretations. When used in a pricing context, premium adequacy generally has an economic meaning: premiums are adequate if they provide a reasonable return to the insurance enter-

TABLE 1.4

AMOUNTS AS OF 20X9

| Accident Year | Adjusted Premiums | Reported Losses | Accident Year | Adjusted Premiums | Reported Losses |
|---|---|---|---|---|---|
| 20X0 | \$200,000,000 | \$150,000,000 | 20X5 | \$300,000,000 | \$185,000,000 |
| 20X1 | \$220,000,000 | \$155,000,000 | 20X6 | \$320,000,000 | \$205,000,000 |
| 20X2 | \$240,000,000 | \$200,000,000 | 20X7 | \$340,000,000 | \$155,000,000 |
| 20X3 | \$260,000,000 | \$175,000,000 | 20X8 | \$375,000,000 | \$185,000,000 |
| 20X4 | \$280,000,000 | \$215,000,000 | 20X9 | \$400,000,000 | \$75,000,000 |

Let $Z$ = the expected loss ratio times the factor needed to bring premiums to adequate levels.

Let $Y_i$ = the bulk reserves for year $i$.

Let $Y$ = the total bulk reserve; that is, $Y = \sum Y_i$.

The index $i$ ranges from 0 to 9, corresponding to accident years 20X0 through 20X9.

The Bornhuetter-Ferguson expected loss method defines the bulk loss reserves as

adequate premium $\times$ expected loss ratio $\times$ percentage unreported.

For year 20X0, the expected percentage already reported is 99%, so the Bornhuetter-Ferguson estimate of the bulk reserves is $\$200{,}000{,}000 \times Z \times (1 - 99\%) = Y_0$. Similarly, for the 20X9 accident year, the estimate is $\$400{,}000{,}000 \times Z \times (1 - 30\%) = Y_9$. We sum all 10 equations to get $Z \times [\$200{,}000{,}000 \times (1 - 99\%) + \cdots + \$400{,}000{,}000 \times (1 - 30\%)] = \sum Y_i = Y$.

If the expected loss ratio is accurate, the total reported losses plus the total bulk reserves should be close to the total expected

prise. Statutory reserving uses undiscounted losses. By "premium adequacy" and "expected loss ratio" in this paper we mean figures such that ultimate (undiscounted) losses equal adequate premiums times the expected loss ratio.

losses. We write the equation for this statement as

$$[\$150{,}000{,}000 + \cdots + \$75{,}000{,}000] + Y$$
$$= Z \times [\$200{,}000{,}000 + \cdots + \$400{,}000{,}000].$$

We have a pair of simultaneous linear equations. We compute the sums in these equations.

- The sum of the adjusted premiums is \$2,935,000,000.
- The sum of the reported losses is \$1,700,000,000.
- The sum of the adjusted premiums × (1 – percentage reported) is \$817,500,000. Then

$$Z \times \$817{,}500{,}000 = Y.$$

$$\$1{,}700{,}000{,}000 + Y = Z \times \$2{,}935{,}000{,}000.$$

We need to find $Y$, the total bulk reserve. We eliminate $Z$ by substituting $Z = Y/\$817{,}500{,}000$.

$$\$1{,}700{,}000{,}000 + Y = Y \times \$2{,}935{,}000{,}000/\$817{,}500{,}000.$$

$$\$1{,}700{,}000{,}000 \times \$817{,}500{,}000 = Y \times \$2{,}117{,}500{,}000.$$

$$Y = \$1{,}700{,}000{,}000 \times \$817{,}500{,}000/\$2{,}117{,}500{,}000$$
$$= \$656{,}320{,}000.$$

### *1.4. Intuition*

First, we explain the intuition for the chain ladder reserving method versus the Stanard-Bühlmann reserving method. Consider year 20X9. The adjusted premium is \$400,000,000. By 12 months from the inception of the accident year, 30% of the adjusted premium, or \$120,000,000, has been processed into reported losses. The other 70% of the adjusted premium, or \$280,000,000, has not yet been processed into reported losses.

The word *processed* warrants explanation. The adjusted premium does not become reported losses. Rather, think of the verb *process* as connoting emergence or development or settlement. It denotes the relationship between the premium collected and the loss activity.

There is some relationship between the $400,000,000 of premium and the ultimate reported losses. At 12 months of development, 30% of the losses should have been reported. $120,000,000 of premium has the same relationship to the losses that have already been reported as the other $280,000,000 of premium has to the losses that are yet to be reported.

The chain ladder reserving method uses this relationship for each accident year. Let $X$ be the bulk reserve. Then

$$\frac{\$75{,}000{,}000}{\$120{,}000{,}000} = \frac{X}{\$280{,}000{,}000}.$$

We solve for the bulk reserve:

$$X = \$75{,}000{,}000 \times \$280{,}000{,}000/\$120{,}000{,}000, \qquad \text{or}$$

$$X = \$175{,}000{,}000.$$

This is the chain ladder reserving method. The bulk reserve for the chain ladder technique is directly dependent on the losses that have been reported so far. If the reported losses at 12 months were twice as high, $150,000,000 instead of $75,000,000, the bulk reserve would be twice as large. We verify this by writing

$$\$120{,}000{,}000/\$150{,}000{,}000 = \$280{,}000{,}000/X, \qquad \text{or}$$

$$X = \$350{,}000{,}000.$$

If LDF is the loss development factor, the bulk reserve in the chain ladder technique is the reported losses times (LDF – 1). In the equation above, the bulk reserve equals the reported losses times (1 – portion of ultimate)/(portion of ultimate). The loss lag is the reciprocal of the loss development factor. We rewrite the expression above:

$$(1 - \text{portion of ultimate})/(\text{portion of ultimate})$$

$$= (1 - 1/\text{LDF})/(1/\text{LDF}) = \text{LDF} - 1.$$

The Stanard-Bühlmann reserving method argues that losses are volatile, and that we may not want to give too much credence to the \$75,000,000 of losses that have been reported as of 12 months for accident year 20X9. Instead, we combine the processed premium from each year, and we combine the reported losses from each year. The total processed premium is \$2,117,500,000. The total reported losses are \$1,700,000,000. The total premium that remains to be processed is \$817,500,000. We form the equation

$$\frac{\$1{,}700{,}000{,}000}{\$2{,}117{,}500{,}000} = \frac{X}{\$817{,}500{,}000}.$$

We solve for $X$, the total bulk reserve, as

$$\begin{aligned} X &= \$1{,}700{,}000{,}000 \times \$817{,}500{,}000/\$2{,}117{,}5000{,}000 \\ &= \$656{,}300{,}000. \end{aligned}$$

## 2. ADJUSTED PREMIUMS

The premium adjustments differ for dollars of loss versus number of claims. We explain the premium adjustments by means of a series of illustrations. The experience period consists of two accident years, 20X1 and 20X2, with premium of \$100,000,000 in 20X1 and \$120,000,000 in 20X2. For simplicity, all policies are effective on January 1, and all rate changes occur on January 1; we relax these assumptions at the end of Section 2.1. In each illustration, we adjust the earned premiums to the same adequate level. Unless specified otherwise, text and formulas apply to dollars of loss, not the number of claims.

*Illustration 1: Rate Change*

Earned premium is \$100,000,000 in 20X1 and \$120,000,000 in 20X2. On January 1, 20X2, there was a +10% rate change. The exposure base is not inflation-sensitive. There is no loss trend: neither a loss severity trend nor a loss frequency trend.

A. If the 20X1 premiums are exactly adequate, the 20X2 premiums are 10% redundant. To bring the premiums to the same adequacy level, we divide the 20X2 premiums by 1.100.

B. If the 20X2 premiums are exactly adequate, the 20X1 premiums are deficient by a factor of $1/1.100$. To bring the premiums to the same adequacy level, we multiply the 20X1 premiums by 1.100. These two scenarios give the same result in the Stanard-Bühlmann technique. Multiplying the numerator of a ratio by a constant has the same effect as dividing the denominator of the ratio by the same constant.

C. There are a variety of other possibilities. The 20X1 premiums might be deficient by 5% or they might be redundant by 5%. They all lead to the same Stanard-Bühlmann result.

Given the various possibilities, which should we choose? The actuarial convention is to leave the most recent year unadjusted and to adjust prior years to the level of the most recent year.[5] We multiply the 20X1 premium by unity plus the January 1, 20X2, rate change amount.

These various scenarios give the same result in the Stanard-Bühlmann technique. If all premiums are at the same adequacy level, we can multiply all premiums by a constant $Z$ to convert premiums into expected losses. Suppose the expected loss ratio is 70%, the 20X1 premiums are exactly adequate, and the 20X2 premiums are 10% redundant.

1. If we multiply the 20X1 premium by 1.100, the premiums in both years are 10% redundant. The value of $Z$

[5]This is a general actuarial convention. The readers of the reserving actuary's report may not understand the Stanard-Bühlmann technique. In most situations, other company personnel believe that the current year is "correct." It is easier to explain an adjustment of prior years to the adequacy level of the current year than to explain an adjustment of the current year to the adequacy level of past years.

is 70%/1.100. In combination, we have multiplied the 20X1 premium by $1.100 \times 70\%/1.100 = 70\%$. We have multiplied the 20X2 premium by 70%/1.100.

2. If we divide the 20X2 premium by 1.100, the premiums in both years are exactly adequate. The value of $Z$ is 70%. In combination, we have multiplied 20X1 premium by 70%. We have multiplied the 20X2 premium by 70%/1.100.

*Illustration 2: Loss Trends*

The loss severity trend is +10% per annum. There have been no rate changes, and the exposure base is not inflation-sensitive.

A. If the 20X1 premium is adequate, the 20X2 premium is deficient by 10%, since losses increased by 10% per exposure unit in 20X2 and there was no rate change. We multiply the 20X2 premiums by 1.100 to bring them to an adequate level.

B. If the 20X2 premium is adequate, the 20X1 premium was redundant, since the 20X2 losses were 10% higher per exposure unit and there was no rate change. We divide the 20X1 premiums by 1.100 to bring them to an adequate level.

C. The absolute premium adequacy level does not affect the result. By convention, we adjust prior years' premiums to the adequacy level of the most recent year.

In general, we determine the loss cost trend factors to bring prior years' losses to the level of the most recent year, and we divide the prior years' premiums by the trend factors.

*Illustration 3: Rate Changes and Loss Trends*

The loss severity trend is +10% per annum. A rate change of +10% was effective on January 1, 20X2. The exposure base is not inflation-sensitive.

Both premium rates and losses increased by 10% between the two years. The premiums are at the same adequacy level. Using the general rules, we multiply the 20X1 premium by 1.100 for the rate change, and we divide the 20X1 premium by 1.100 for the loss trend. The net adjustment is no change.

*Illustration 4: Exposure Trends*

The loss severity trend is +10% per annum. The exposure base is inflation-sensitive, and the exposure trend is 10% per annum. No rate changes were taken.

The exposure trend of +10% offsets the loss cost trend of +10%. We conceive of an exposure trend as the reciprocal of a loss cost trend. The net trend is 0% per annum.

### *2.1. General Rules*

Premiums: The illustrations above assume January 1 effective dates for rate changes and policies. That is not necessary. Rather, we determine calendar year on-level factors to bring the earned premium in each calendar year to the current rate level.[6]

The loss severity trend is 0%. Policies are written evenly through the year. We took a rate change of +10% on July 1, 20X1. The exposure base is not inflation-sensitive.

The calendar year on-level factors are 1.075 for 20X1 and 1.025 for 20X2. We multiply the 20X1 premium by 1.075 and the 20X2 premium by 1.025.

Losses: We trend all losses to a common date with the net trend factors. The net trend equals the loss frequency trend times

[6]The Stanard-Bühlmann technique is commonly used by reinsurance actuaries. Most excess-of-loss reinsurance treaties are effective on January 1, and reinsurance rate changes are effective on January 1 as well. The underlying policies written by the ceding company may be written evenly during the year, and the ceding company's rate changes may have occurred during the year. The on-level factors are taken into account to determine the reinsurance rate changes; they need not be recomputed for the reserve estimate.

the loss severity trend divided by the exposure trend. However, we adjust the premiums, not the losses, so we divide the premiums by the net trend factors.

### *2.2. Claim Counts*

The Stanard-Bühlmann technique can be used with claim counts in place of dollars of loss. Suppose claims are reported quickly but claim severities are highly variable and may remain uncertain for many years. The reserving actuary may project ultimate claims by a development procedure and the average claim severity by a trend procedure.

Illustration: Workers compensation permanent disability claims are reported quickly, though it may take years before the severity of the injury is clear. The claims are paid over the remaining lifetime of the injured worker. Both the indemnity (loss of income) benefits and the medical benefits extend over decades, and they are difficult to estimate.

The reserving actuary may project ultimate claim counts by a development year procedure and ultimate claim severities by an accident year trend. Suppose we are estimating accident year 20X9 workers compensation reserves for permanent disability claims. Within a year or two after the expiration of the 20X9 accident year, we have a preliminary estimate of the ultimate claim count. Since we have only a year or two of payments on these claims—each of which may extend for 20 or 30 years—we cannot estimate claim severities from the 20X9 data.

Instead, we estimate ultimate claim severities for the more mature accident years, such as 20X0 through 20X7. We use the workers compensation loss cost trend factors derived from shorter-term injuries to extend the claim severity trend through 20X9. This procedure is suited for excess-of-loss reinsurance reserving, since most of the claims are permanent injuries.

*Illustration 5: Claim Frequencies*

When we deal with reported losses, the ratio of reported losses to unreported losses is set equal to the ratio of processed premium to unprocessed premium. The unreported losses are the bulk reserve. When we deal with reported claims, the ratio of reported claims to unreported claims is set equal to the ratio of processed premium to unprocessed premium.

Premium is \$100,000,000 in 20X1 and \$120,000,000 in 20X2. Policies are effective on January 1, there have been no rate changes, and the exposure base is not inflation-sensitive. The loss severity trend is +10% per annum. We use the Stanard-Bühlmann technique to estimate ultimate claim counts.

We comment on the meaning of premium adequacy with respect to claim counts. If the expected claim frequency is 100 claims for each \$1,000,000 of premium in 20X1, then 20X2 has the same level of premium adequacy if the expected claim frequency is still 100 claims for each \$1,000,000 of premium.

In Illustration 5, there were no rate changes in 20X1 or 20X2. If there were no changes in expected claim frequency, the premiums in 20X1 and 20X2 are at the same level of adequacy with respect to claim frequency.

If the average loss severity rose by 10% from 20X1 to 20X2, the premiums in the two years are not at the same level of adequacy with respect to dollars of losses. For the Stanard-Bühlmann method, we use a premium adjustment if we are dealing with reported losses. We make no premium adjustment in this case if we are dealing with reported claims.

*Illustration 6: Frequency and Severity Trends*

The loss cost trend is +10% per annum, consisting of 7.8% claim severity trend and a 2.0% claim frequency trend. There have been no rate changes, and the exposure base is not inflation-sensitive.

To estimate ultimate losses, we use the total loss cost trend of +10% per annum. To estimate ultimate claim counts, we use the claim frequency trend of 2.0% per annum.

Pricing actuaries have learned to be wary of claim frequency trends. In most lines of business, claim frequency does not follow simple exponential growth patterns. Econometric modeling of claim frequency has generally been disappointing. One might wonder how useful the claim frequency trends would be for the Stanard-Bühlmann reserving technique.

The pricing actuary and the reserving actuary use the trend factors for different purposes. The pricing actuary is projecting future claim frequency; most trend estimates have been poor predictors. The reserving actuary is quantifying the change between two past years. The claim frequency is a historical figure; it is not better or worse than the historical loss cost trend.

1. If we are given both claim frequency trends and claim severity trends, we use the product of these trends when we deal with dollars of loss. When we deal with claim counts, we use only the claim frequency trends.

2. If we have a single loss cost trend, we use the claim frequency portion of the trend. If we do not know the claim frequency portion, we might estimate the claim severity portion from other indices and "back out" the claim severity portion to derive the claim frequency portion.

3. The loss frequency trends in the historical data may reflect shifts in the mix of business, not real changes in claim frequency. Such trends may not be used in pricing, though they may be appropriate for aggregate reserving analyses.

4. For some lines of business, the exposure trends largely offset the loss severity trends, and the net trend is not material. When we are dealing with claim counts, we

ignore loss severity trends but we still include exposure trends to calculate the premium adjustments.

Illustration: Payroll in 20X1 is \$100,000,000. The workers compensation premium rate is 2% of payroll, giving a premium of \$2,000,000. Employment stays the same for 20X2. Wage inflation is 10% per annum, so payroll is \$110,000,000 and premium is \$2,200,000. If nothing has changed in the physical plant, we expect the same number of claims. We increase the 20X1 premiums by +10% to bring them to the adequacy level of the 20X2 premiums.

## 3. CLAIM COUNTS VS. LOSS DOLLARS

We illustrate the Stanard-Bühlmann method's premium adjustments by calculating first an IBNR claim count and then an IBNR loss reserve. In Table 3.1, all policies have effective dates of January 1, and all rate changes occur on January 1. We consider two separate problems: one using the column Reported Claims (Scenario A) and the other using the column Reported Losses (Scenario B).

For clarity, the loss dollars are \$1,000 times the claim count in each year; only the premium adjustments differ between the two scenarios. The processed premium differs for each year because only the losses have a trend, not the claim counts. These are "either-or" columns for two different scenarios; they are not the claim counts and losses in a single scenario.

Annual loss trends and rate changes are shown in Table 3.2 (Scenario A). There is no exposure trend.

There are two premium adjustments: one for rate changes and another for trend. We bring all premiums to the same rate level, and we divide by the appropriate trend factors.

Because all policies are effective on January 1, the rate change on January 1, 20X1 affects all years equally. In Table 3.3 we set

TABLE 3.1

| Cal./Acc. Year | Pure Premium | Reporting Percentage | Reported Claims at 12/31/20X5 | Reported Losses at 12/31/20X5 |
|---|---|---|---|---|
| 20X1 | $40,000 | 38% | 9 | $9,000 |
| 20X2 | $44,000 | 28% | 8 | $8,000 |
| 20X3 | $40,000 | 18% | 8 | $8,000 |
| 20X4 | $45,000 | 9% | 5 | $5,000 |
| 20X5 | $50,000 | 2% | 1 | $1,000 |

TABLE 3.2

| Loss Severity Trends | | Rate Changes | |
|---|---|---|---|
| 20X0 to 20X1 | 15.0% | 1/1/20X1 | 30.0% |
| 20X1 to 20X2 | 12.5% | 1/1/20X2 | 10.0% |
| 20X2 to 20X3 | 10.0% | 1/1/20X3 | −10.0% |
| 20X3 to 20X4 | 10.0% | 1/1/20X4 | 0.0% |
| 20X4 to 20X5 | 10.0% | 1/1/20X5 | 5.0% |

TABLE 3.3

| Date | Rate Change | Rate Level Index | On-Level Factor |
|---|---|---|---|
| 1/1/20X1 | 30.0% | 1.0000 | 1.0395 |
| 1/1/20X2 | 10.0% | 1.1000 | 0.9450 |
| 1/1/20X3 | −10.0% | 0.9900 | 1.0500 |
| 1/1/20X4 | 0.0% | 0.9900 | 1.0500 |
| 1/1/20X5 | 5.0% | 1.0395 | 1.0000 |

the 20X1 rate level to a rate level of 1.000, and we use the other rate changes to bring premiums to the current rate level.

Scenario A—Number of Claims: The rate level index is the cumulative downward product of the rate changes. (When the policy effective dates are distributed through the year and the rate changes occur on different dates, the rate level index is the average rate level during the year.) The on-level factor is the current rate level index divided by the rate level index for the

TABLE 3.4

| Cal./Acc. Year | Pure Premium | On-Level Factor | Adjusted Premium | Reporting Percentage | Processed Premium at 12/31/20X5 | Unprocessed Premium at 12/31/20X5 |
|---|---|---|---|---|---|---|
| 20X1 | $40,000 | 1.0395 | $41,580 | 38.0% | $15,800.40 | $25,779.60 |
| 20X2 | $44,000 | 0.9450 | $41,580 | 28.0% | $11,642.40 | $29,937.60 |
| 20X3 | $40,000 | 1.0500 | $42,000 | 18.0% | $7,560.00 | $34,440.00 |
| 20X4 | $45,000 | 1.0500 | $47,250 | 9.0% | $4,252.50 | $42,997.50 |
| 20X5 | $50,000 | 1.0000 | $50,000 | 2.0% | $1,000.00 | $49,000.00 |
| Total | | | | | $40,255.30 | $182,154.70 |

accident year under consideration. We multiply the premiums by the on-level factors to put all premiums on the same adequacy level. The loss trend is a severity trend; we assume that the claim frequency trend is zero. We use the severity trend adjustment when dealing with loss dollars, not claim counts.

In the claim amount scenario in Table 3.4, 31 claims are reported by December 31, 20X5. We determine the total processed premium and the total unprocessed premium.

The claims expected to emerge in the future are $31 \times \$182,154.70/\$40,255.30 = 140$.

The reserve indication has great uncertainty. From 31 claims that have been reported so far, we are estimating future emergence of 140 claims. The volatility of the reported claim counts can be seen by a comparison of accident years 20X1 and 20X3. As of December 31, 20X5, the processed adjusted premium for 20X1 is $15,800 and 9 claims have been reported, while the processed adjusted premium for 20X3 is $7,560 and 8 claims have been reported.[7]

Scenario B—Dollars of Loss: In Table 3.5 we adjust for the loss severity trend by forming an index of relative loss costs,

[7] The reserve indication is for five accident years only. For the oldest year in the experience period, only 38% of claims have been reported so far. If the company had business in preceding years, we would still expect much claim emergence for these older years.

TABLE 3.5

| Period | Loss Trend | Index Value | Trend Factor |
|---|---|---|---|
| 20X0 to 20X1 | 15.0% | 1.0000 | 1.497 |
| 20X1 to 20X2 | 12.5% | 1.1250 | 1.331 |
| 20X2 to 20X3 | 10.0% | 1.2375 | 1.210 |
| 20X3 to 20X4 | 10.0% | 1.3613 | 1.100 |
| 20X4 to 20X5 | 10.0% | 1.4974 | 1.000 |

TABLE 3.6

| Cal./Acc. Year | Pure Premium | On-Level Factor | Trend Factor | Adjusted Premium | Reporting Percentage | Processed Premium at 12/31/20X5 | Unprocessed Premium at 12/31/20X5 |
|---|---|---|---|---|---|---|---|
| 20X1 | $40,000 | 1.0395 | 1.497 | $27,776 | 38% | $10,554.71 | $17,220.84 |
| 20X2 | $44,000 | 0.9450 | 1.331 | $31,240 | 28% | $8,747.11 | $22,492.56 |
| 20X3 | $40,000 | 1.0500 | 1.210 | $34,711 | 18% | $6,247.93 | $28,462.81 |
| 20X4 | $45,000 | 1.0500 | 1.100 | $42,955 | 9% | $3,865.91 | $39,088.64 |
| 20X5 | $50,000 | 1.0000 | 1.000 | $50,000 | 2% | $1,000.00 | $49,000.00 |
| Total | | | | | | $30,427.66 | $156,276.85 |

using 20X1 as the base year. The loss trend from 20X0 to 20X1 affects all years equally. The index value for 20X1 is unity, the index value for 20X2 is 1.125, and so forth. The trend factor is the index value for the most recent year divided by the index value for the year under consideration. If we adjust losses to the current level, we multiply by these trend factors. Since we are adjusting premiums here, we divide by the trend factors.

In the claim count scenario, $31,000 of losses are reported by December 31, 20X5. The premium adjustment factors are the on-level factors calculated for rate changes divided by the trend factors. We determine the total processed premium and total unprocessed premium in Table 3.6.

The bulk loss reserve is $31,000 × $156,276.85/$30,427.66 = $159,216.40.

### *3.1. Calendar Year Emergence*

So far we have examined the future emergence of losses, both new claims + adverse development on known claims = bulk reserve, and the future payment of losses, or the total (case + bulk) reserve. The reserving actuary may be asked to show the expected emergence and payment of losses by development period (i.e., by calendar period) subsequent to the valuation date. The emergence and payment patterns have several uses.

1. Reserving: The loss emergence and loss payment in the next calendar period provide a check on the accuracy of the reserve indication. It is difficult to judge the loss reserve indication itself, since the losses may not emerge or settle for many years. By comparing the actual emergence or settlement in the next calendar period with the estimates implied by the reserve indication, one gets a better feel for the accuracy or bias in the indication.

2. Investments: The expected emergence and settlement of claims is necessary for asset/liability management. The insurer's investment department seeks expected liability cash flows in the coming months to optimize its investment strategy. Many insurers structure their investment portfolio in accordance with their insurance liabilities, selecting security types, fixed-income durations, and investment quality to best manage their overall risk. The reserving actuary provides the settlement patterns for the loss reserve portfolio.

- The bulk reserve as of December 31, 20XX, equals the losses expected to emerge in calendar years 20XX+1 and subsequent for accident years 20XX and prior.

- The expected emergence in 20XX+1 equals the losses expected to emerge in calendar years 20XX+1 for only accident years 20XX and prior.

TABLE 3.7

| Cal./Acc. Year | Adjusted Earned Premium | Difference in Percent Reported | Premium Processed in 20X6 |
|---|---|---|---|
| 20X1 | $41,580 | 10.0% | $4,158 |
| 20X2 | $41,580 | 10.0% | $4,158 |
| 20X3 | $42,000 | 10.0% | $4,200 |
| 20X4 | $47,250 | 9.0% | $4,253 |
| 20X5 | $50,000 | 7.0% | $3,500 |
| 20X1–20X5 | | | $20,269 |

We illustrate the method using Table 3.7. We calculate the number of claims expected to emerge for accident years 20X1 through 20X5 during calendar year 20X6.

We estimate the adjusted premium that will be processed in 20X6. For any accident year, the adjusted premium that will be processed in 20X6 is the adjusted premium for that accident year times the difference in the report lag between that accident year and the previous year. For example, the 20X2 adjusted premium processed in 20X6 is $\$41{,}580 \times (38.0\% - 28.0\%) = \$4{,}158$.

We do not know the difference between the reporting percentage as of 60 months and 72 months. For the other figures in Table 3.7, we estimate this difference as 10%.

The adjusted premium for 20X1 through 20X5 processed in 20X6 is \$20,269. The estimated claim emergence in 20X6 is $31 \times \$20{,}269/\$40{,}255 = 16$ claims.

## 4. RESERVE ASSUMPTIONS: CHAIN LADDER VS. EXPECTED LOSS

Chain ladder reserving methods work better in some situations and expected loss methods work better in others. We examine the perspective of each type of method, so that we may judge when a Stanard-Bühlmann reserving method is most appropriate. Brosius, following Hugh White's discussion of the Bornhuetter-

Ferguson paper, explains the differing philosophy of the chain ladder versus the expected loss reserving methods.[8]

- The chain ladder method assumes that unusually high or low cumulative paid losses to date are indicative of similar high or low paid losses in future development periods.
- The Bornhuetter-Ferguson method assumes that unusually high or loss cumulative paid losses to date reflect random loss fluctuations. They are not indicative of unusually high or low paid losses in the remaining development periods.

As Brosius points out, the truth is generally in between these two alternatives. Yet the extreme cases interest us, because certain attributes of the insurance scenario argue for one or the other of these cases.

- When losses are very immature, or when loss severity is large but loss frequency is low, or when the variability of losses is unusually great, the Bornhuetter-Ferguson expected loss method may be favored.
- When losses are mature, or when loss severity is low but loss frequency is high, or when the variability of losses is small, the chain ladder method may be favored.

Excess of loss reinsurance has the former attributes, so many reinsurance actuaries are inclined to use expected loss reserving procedures. Since the reinsurance actuary may not have a good sense of the expected loss ratio, the Stanard-Bühlmann method is often used.

### *4.1. Accident Year Weights*

James Stanard [1985] provides another perspective on the Stanard-Bühlmann method (which he refers to as the "Adjustment to Total Known Losses Method" or the "Cape Cod

[8]See Bornhuetter-Ferguson [1972] and Brosius [1993]. Brosius presents a statistical procedure for selecting the base that allows for multiple bases, such as 60% of one base plus 40% of another base, and he determines the optimal percent of each.

Method"). The Stanard-Bühlmann method estimates the expected losses from the historical data. There is a simpler method of doing this, but a comparison with the Stanard-Bühlmann method shows one of the latter's advantages.

Given the historical data, the chain ladder method estimates the total losses for each accident year. If there is no trend or exposure change from year to year, we can estimate the expected losses as the simple average of the estimated incurred losses for each accident year and then use a Bornhuetter-Ferguson loss reserving method. The Stanard-Bühlmann reserving method does the same, except that it uses a weighted average, where the weights are the reporting percentage for each accident year. This gives more weight to older accident years, for which the total incurred loss is more certain.[9]

We use two illustrations. Scenario A has actual losses equal to expected losses in each year; the chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann reserving methods give the same reserve indication. Scenario B switches the actual losses between the oldest accident year and the most recent accident year. The total actual losses remain the same, but more than expected occur in the most recent accident year and fewer than expected occur in the oldest accident year.

- The chain ladder method treats each accident year independently. The most recent accident year has the largest loss development factor, so shifting more of the actual losses in that year increases the reserve indication.
- If we use a straight average of the indicated reserves by accident year as the expected losses for the Bornhuetter-Ferguson method, the expected loss for each accident year is higher than in Scenario A. But because the Bornhuetter-Ferguson gives

[9]If the exposures differ by accident year or if there is a loss trend, the accident years must be put on equal cost and exposure bases before taking an average. Exposures are known for each accident year. Placing accident years on the same cost level is discussed in Section 2. For simplicity we do not repeat these adjustments here.

TABLE 4.1

DETERMINATION OF BULK RESERVE—CHAIN LADDER METHOD

| Chain Ladder | Adjusted Pure Premium | Reporting Percentage | Reported Losses | LDF – 1 | Bulk Reserve |
|---|---|---|---|---|---|
| 20X1 | $100,000 | 80% | $80,000 | 0.25 | $20,000 |
| 20X2 | $100,000 | 50% | $50,000 | 1.00 | $50,000 |
| 20X3 | $100,000 | 20% | $20,000 | 4.00 | $80,000 |
| Total | $300,000 | | $150,000 | | $150,000 |

less leverage to the actual losses in determining the reserve, the indicated reserve is lower than the chain ladder indication.

- The Stanard-Bühlmann reserving method weights the chain ladder reserve indications by the reporting percentage in each accident year. If reported losses do not change, the estimated ultimate losses do not change.[10]

Illustration—Scenario A: In Table 4.1 suppose the adjusted pure premiums are $100,000 in each of three accident years. The reporting percentages are 20%, 50%, and 80% as of 12, 24, and 36 months from inception of the accident year. The reported claims in the past three accident years are $80,000, $50,000, and $20,000 at the end of the most recent accident year.

There are no loss trend or exposure changes from year to year. The chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann methods give the same reserve indication of $150,000.

For the chain ladder method, the loss development factor equals the reciprocal of the reporting percentage. The incurred loss is the reported loss times the loss development factor, and the bulk reserve is the reported loss times (LDF – 1).

[10]One Review team member of the CAS Committee on Review of Papers noted that "in SB the exact relationship by year of reported losses to adjusted premiums does not matter. That may be considered an advantage or disadvantage of the method."

TABLE 4.2

DETERMINATION OF BULK RESERVE—BORNHUETTER-FERGUSON

| Bornhuetter-Ferguson | Adjusted Pure Premium | Reporting Percentage | Reported Losses | Expected Loss | Bulk Reserve |
|---|---|---|---|---|---|
| 20X1 | \$100,000 | 80% | \$80,000 | \$100,000 | \$20,000 |
| 20X2 | \$100,000 | 50% | \$50,000 | \$100,000 | \$50,000 |
| 20X3 | \$100,000 | 20% | \$20,000 | \$100,000 | \$80,000 |
| Total | \$300,000 | | \$150,000 | | \$150,000 |

TABLE 4.3

DETERMINATION OF BULK RESERVE—STANARD-BÜHLMANN

| Stanard-Bühlmann | Adjusted Pure Premium | Reporting Percentage | Reported Losses | Processed Premium | Unprocessed Premium |
|---|---|---|---|---|---|
| 20X1 | \$100,000 | 80% | \$80,000 | \$80,000 | \$20,000 |
| 20X2 | \$100,000 | 50% | \$50,000 | \$50,000 | \$50,000 |
| 20X3 | \$100,000 | 20% | \$20,000 | \$20,000 | \$80,000 |
| Total | \$300,000 | | \$150,000 | \$150,000 | \$150,000 |

The chain ladder method estimates the total incurred loss for all three years as \$150,000 (reported loss) + \$150,000 (bulk reserves) = \$300,000.

The Bornhuetter-Ferguson bulk reserve is the expected loss times the complement of the reporting percentage (see Table 4.2).

The Stanard-Bühlmann indicated reserve is \$150,000 × \$150,000/\$150,000 = \$150,000 (see Table 4.3).

Illustration—Scenario B: In Table 4.4 we switch the reported losses in 20X1 and 20X3. The chain ladder method bases the reserve for each accident year directly on the reported losses in that year. Since the most recent year has the highest loss devel-

TABLE 4.4

DETERMINATION OF BULK RESERVE—CHAIN LADDER METHOD

| Chain Ladder | Adjusted Pure Premium | Reporting Percentage | Reported Losses | LDF – 1 | Bulk Reserve |
|---|---|---|---|---|---|
| 20X1 | $100,000 | 80% | $20,000 | 0.25 | $5,000 |
| 20X2 | $100,000 | 50% | $50,000 | 1.00 | $50,000 |
| 20X3 | $100,000 | 20% | $80,000 | 4.00 | $320,000 |
| Total | $300,000 | | $150,000 | | $375,000 |

TABLE 4.5

DETERMINATION OF BULK RESERVE—BORNHUETTER-FERGUSON

| Bornhuetter-Ferguson | Adjusted Pure Premium | Reporting Percentage | Reported Losses | Expected Loss | Bulk Reserve |
|---|---|---|---|---|---|
| 20X1 | $100,000 | 80% | $20,000 | $175,000 | $35,000 |
| 20X2 | $100,000 | 50% | $50,000 | $175,000 | $87,500 |
| 20X3 | $100,000 | 20% | $80,000 | $175,000 | $140,000 |
| Total | $300,000 | | $150,000 | | $262,500 |

opment factor, it has the most leverage, and the reserve indication is higher.

The Bornhuetter-Ferguson method smooths the effects of random loss fluctuations among the years. If we use a straight average of the chain ladder estimates of ultimate losses as the estimate of the expected losses, the Bornhuetter-Ferguson reserve indication increases, though not as much as the chain ladder indication does (see Table 4.5).

The Stanard-Bühlmann reserving method weights the estimated losses in each accident year by the reporting percentage in that year to get its estimate of expected losses. The chain ladder estimated losses above are $25,000 for 20X1, $100,000 for 20X2, and $400,000 for 20X3. Weighting these estimates by the

TABLE 4.6

DETERMINATION OF BULK RESERVE—STANARD-BÜHLMANN

| Stanard-Bühlmann | Adjusted Pure Premium | Reporting Percentage | Reported Losses | Processed Premium | Unprocessed Premium |
|---|---|---|---|---|---|
| 20X1 | $100,000 | 80% | $20,000 | $80,000 | $20,000 |
| 20X2 | $100,000 | 50% | $50,000 | $50,000 | $50,000 |
| 20X3 | $100,000 | 20% | $80,000 | $20,000 | $80,000 |
| Total | $300,000 | | $150,000 | $150,000 | $150,000 |

percentage reported (80%, 50%, and 20%) gives estimated expected losses of $100,000. Changes to reported losses by year are divided by the percentage reported to estimate the change in ultimate losses. The ultimate losses are weighted by the percentage reported, so, if the changes to reported losses in different years offset one another, there is no change to the estimated expected losses (see Table 4.6).

## 5. SUMMARY

The Stanard-Bühlmann reserving technique is a simple, intuitive procedure that works well even in situations that don't lend themselves to easy estimates, such as reserving for high layers of loss. It is most useful for recent accident years in lines of business where random loss fluctuations preclude the use of chain ladder reserving methods but uncertainty about the excess loss ratio precludes the use of a Bornhuetter-Ferguson expected loss reserving method.

Illustration: Losses may not be reported on casualty excess-of-loss reinsurance treaties until several years after the accident date, so a chain ladder reserving method is not appropriate for the most recent two or three accident years. In a chain ladder method, we apply the loss development factors to the reported losses for that accident year. If the loss development factors are very high (say, more than 10.000) and the expected reported losses at a early maturity are very low, a change of $1,000,000 in the reported losses causes a change greater than $10,000,000 in the estimate of incurred losses.

But the Bornhuetter-Ferguson expected loss reserving method may also be inappropriate. If the reinsurer is not sufficiently familiar with the book of business and the underwriting practices of the ceding company, the reserving actuary may not know the expected loss ratio to determine expected losses.

Reinsurance actuaries often use a Stanard-Bühlmann reserving method in this situation. If the reinsurer has enough years of historical data, the Stanard-Bühlmann method derives the expected loss ratio from the actual experience. This practitioner's guide should encourage the use of the Stanard-Bühlmann reserving method by primary company actuaries for volatile lines of business, in addition to its current use by reinsurance actuaries.

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## APPENDIX A

## PATTERNS AND PROJECTIONS

This appendix discusses patterns of stability in the historical data. It is geared to the student, though even the experienced reserving actuary may find the review useful.

### *A.1. Weighted vs. Unweighted Averages*

The prospective future pattern is based on the observed patterns. One may use either unweighted or weighted averages of historical observations. There are two reasons for using weighted averages: shifting risk parameters and credibility considerations.

Shifting Risk Parameters: If a more recent year is a better predictor of future experience than an older year, more recent accident years should receive more weight than older accident years. This approach is most important when trends appear in the columns of age-to-age factors.

The consideration of shifting risk parameters is particularly applicable to loss reserving, since the covariance matrix can be estimated from the experience; see Mahler [1990, 1998]. In many scenarios, a broad range of credibility values is close to optimal. A weighting system may be selected one year and used for subsequent reviews as well.

Credibility: The experience years should be weighted in proportion to the real volume of business. The loss amounts in each year differ for two reasons: (i) the real dollar amount of losses may differ, and (ii) inflation causes the nominal amount of losses to differ.

Ideally, one should use weighted averages of the observed link ratios, where the weights are the deflated dollars of loss. If deflated triangles are used in the reserve analysis, weighted averages based on dollars of loss at the earlier of the two development periods should be used. If nominal loss triangles are used, the

TABLE A.1

| Accident Year | At 12 months | At 24 months | Link Ratio |
|---|---|---|---|
| 20X1 | \$10,000,000 | \$15,000,000 | 1.500 |
| 20X2 | \$20,000,000 | \$25,000,000 | 1.250 |
| 20X3 | \$30,000,000 | \$60,000,000 | 2.000 |
| Total | \$60,000,000 | \$100,000,000 | (see text) |

following rule is a reasonable guide. When the dollar amount of losses is consistent with monetary inflation, one should use unweighted averages. When the dollar amount of losses is considerably different from monetary inflation, one should use weighted averages, with an adjustment to offset monetary inflation.

When the weights are the same as the measurement base (e.g., the weights are the losses at the start of the period for the chain ladder link ratios) the weighted average may be computed by summing the losses for several years as the numerators and denominators of the link ratios.

Illustration: Cumulative paid losses in deflated dollars are shown in Table A.1.

The unweighted average of the link ratios is $(1.500 + 1.250 + 2.000)/3 = 1.583$. The weighted average is most easily determined as the sum of losses at 24 months divided by the sum of losses at 12 months: $\$100,000,000/\$60,000,000 = 1.667$.

Suppose that the covariance matrix to determine optimal weights based on shifting risk parameters gives weights of 20%–30%–50%. If there were no changes in the volume of business by year, the weighted link ratio would be $20\% \times 1.500 + 30\% \times 1.250 + 50\% \times 2.000 = 1.675$. In the scenario given above, where the deflated losses are increasing, the weighted link ratio is $(20\% \times 15 + 30\% \times 25 + 50\% \times 60)/(20\% \times 10 + 30\% \times 20 + 50\% \times 30) = 1.761$.

Nominal Dollars: If the dollars in the table above are not adjusted for inflation, and the loss cost trend is 10% per annum, the weighted average (not adjusted for shifting risk parameters) is $(10 \times 1.500 + 20/1.100 \times 1.250 + 30/1.100^2 \times 2.000)/(10 + 20/1.100 + 30/1.100^2) = 1.648$.

### *A.2. Inflation*

Changing inflation rates may bias the projected pattern. The effects of inflation are most significant for the long-tailed commercial casualty lines of business.

Illustration: We are using a paid loss development method for workers compensation (WC) medical benefits. The workers compensation medical benefits severity trend has been 8% per annum during the experience period. Medical inflation has recently risen, and we expect the future workers compensation medical benefits severity trend to be 12% per annum.

In the company's book of business, WC medical benefits are paid about three years after the accident date (on average). However, many medical cases close early. The time until payment for the medical loss reserves is five years, on average. We assume that medical benefits are affected by inflation through the payment date; see Butsic [1981].

The change in the medical severity trend raises the reserve indication by $(1.120/1.080)^5 = 1.199$, or about 20%. If no adjustment to the reserving procedure is made to account for the change in the inflation rate, the reserve indication may be severely understated.

To correct for changes in the inflation rate, one may deflate the historical triangle for past inflation, perform the actuarial analysis on "real dollar" figures, and project forward with future expected inflation or stochastic inflation rate paths.[11]

[11] Hodes, Feldblum, and Blumsohn [1999] use an interest rate generator and a stochastic inflation model with a probability distribution of loss realizations in future calendar years. Feldblum [2002] summarizes the procedure, with an application to Schedule P loss reserve monitoring.

### *A.3. Trend, Outliers, and Credibility*

Trend: When the insurance environment is changing, one might trend the historical figures. Examples are changing attorney involvement in private passenger automobile claims and changing claims management practices in workers compensation insurance.[12]

Outliers: To eliminate outliers, one might use averages that discard the high value and the low value. When discarding outliers, one must be careful not to introduce additional bias.

- If the distribution of link ratios is skewed, eliminating outliers gives a biased average.
- The chain ladder reserving method may have an inherent bias; see Stanard [1985]. The elimination of outliers may offset some of this bias.

Credibility: For small insurers, one might weight company averages with industry averages, or state averages with countrywide averages; see Graves and Castillo [1990].

### *A.4. Stability Patterns: Derivation vs. Application*

The derivation of the stability pattern is similar for all reserving methods. Once we determine any one pattern, we have determined the other patterns as well. One sometimes hears that chain ladder methods and expected loss methods both start with the observed link ratios. We could equally well say that the methods start with the observed percentages of ultimate.

It is in the application of the patterns that the reserving methods differ.

- Chain ladder methods apply the factors to the cumulative paid or reported losses for each experience year. We do not use the estimated ultimate losses.

[12] Feldblum [2005] discusses trends in workers compensation loss development factors in the 1980s and 1990s.

- Expected loss methods apply the factors to the estimated ultimate losses. We do not use the cumulative paid or reported losses for each experience year.

In the example in Section 1 (Table 1.2), the paid losses in the first 12 months equal 18.2% of the estimated ultimate paid losses. Suppose we are using this historical pattern to estimate the needed reserves for a more recent accident year. What if the paid losses in the first 12 months of this accident year equal 25% of the estimated ultimate losses, not 18.2% of the ultimate losses?

- A chain ladder method says: "Use the cumulative paid losses in the first 12 months; ignore the estimated ultimate losses."
- An expected loss method says: "Use the estimated ultimate losses; ignore the cumulative paid losses in the first 12 months."

### *A.5. Determining the Pattern*

If we determine the incremental ratios or the cumulative ratios, we know the percentages of ultimate. Conversely, if we determine the percentages of ultimate, we know the incremental ratios and the cumulative ratios. We ask: "Which is the easiest pattern to determine?" not "Which pattern do we want to use?"

If we try to determine the percentages of ultimate, we can't use all the data at our disposal. In particular, we can't use the most current data. If we try to determine the incremental ratios or the cumulative ratios, we use all the historical data, including the most recent data.

If we try to determine the percentages of ultimate directly, we can use only mature accident years that have developed to ultimate. The patterns may have changed in the intervening years, as the social, economic, and insurance environments changed.

If we use incremental ratios or cumulative ratios, we can use all accident years, including even the most recent calendar year information in each accident year. This was the advance in casualty loss reserving theory that gave rise to the chain ladder method.[13]

We still must choose between the incremental ratios and the cumulative ratios. At early development periods, neither method is clearly superior. At later development periods, the incremental losses are relatively small. Small figures in the numerators of the link ratios do not distort the estimation procedure. But small figures in the denominator cause ratios that may be unrealistically large, reducing the accuracy of the results and adding significant bias.

Illustration: Table A.2 shows reported loss development in thousands of dollars from ten years to twelve years. Table A.2, Part 1 has five accident years and five columns, showing

- cumulative reported losses at ten years of development,
- incremental reported losses in year eleven,
- cumulative reported losses at eleven years of development,
- incremental reported losses in year twelve, and
- cumulative reported losses at twelve years of development.

The age-to-age link ratio from year eleven to year twelve is stable when using cumulative reported losses but is not stable when using incremental reported losses (Table A.2, Part 2).

This is the rationale for the method of determining the pattern. All three reserving procedures discussed in the text—chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann—begin by estimating link ratios (or cumulative age-to-age factors). Loss development factors are determined as the cumulative products of the link ratios.

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[13]Health actuaries often use "claim completion percentages," which are chain ladder paid loss development factors that rely on mature years only. Since medical claims are settled quickly, the reliance on mature experience periods is not onerous; see Bluhm [2000], chapter 30. For a typology of reserving procedures, see Salzmann [1984].

## TABLE A.2

### PART 1

| Accident Year (1) | Reported Losses at Ten Years (2) | Incremental Losses in Year Eleven (3) | Reported Losses at Eleven Yrs. (4) | Incremental Losses in Year Twelve (5) | Reported Losses at Twelve Yrs. (6) |
|---|---|---|---|---|---|
| 20X0 | 100,000 | 100 | 100,100 | 1,100 | 101,200 |
| 20X1 | 110,000 | 1,100 | 111,100 | 0 | 111,100 |
| 20X2 | 120,000 | 0 | 120,000 | 1 | 120,001 |
| 20X3 | 130,000 | −100 | 129,900 | 1,100 | 131,000 |
| 20X4 | 140,000 | 1 | 140,001 | 100 | 140,101 |

## TABLE A.2

### PART 2

| Accident Year (1) | Age-to-Age Factor using Cumulative Reported Losses (7) = (6)/(4) | Age-to-Age Factor using Incremental Reported Losses (8) = (5)/(3) |
|---|---|---|
| 20X0 | 1.011 | 11.000 |
| 20X1 | 1.000 | 0.000 |
| 20X2 | 1.000 | ∞ |
| 20X3 | 1.008 | −11.000 |
| 20X4 | 1.001 | 100.000 |

- The reporting percentage is the percent of ultimate losses that are expected to have been reported by the development date.

- The paid loss percentage is the percent of ultimate losses that are expected to have been paid by the development date.

- The percentage of ultimate equals the reciprocal of the loss development factor.

- The Bornhuetter-Ferguson factor is the complement of the percentage of ultimate.

TABLE A.3

| Development Months | 12–24 | 24–36 | 36–48 | 48–60 | 60–Ult. |
|---|---|---|---|---|---|
| Link Ratio | 1.500 | 1.250 | 1.100 | 1.050 | 1.020 |

TABLE A.4

| Development Months | 12 | 24 | 36 | 48 | 60 |
|---|---|---|---|---|---|
| Link Ratio | 1.500 | 1.250 | 1.100 | 1.050 | 1.020 |
| Loss Development Factor | 2.209 | 1.473 | 1.178 | 1.071 | 1.020 |
| Loss Lag | 0.453 | 0.679 | 0.849 | 0.934 | 0.980 |
| B-F Factor | 0.547 | 0.321 | 0.151 | 0.066 | 0.020 |

Illustration: Reported loss link ratios for a block of business are shown in Table A.3. We compute the loss development factors, percentages of ultimate, and Bornhuetter-Ferguson factors.

The loss development factors are the cumulative products of the link ratios. The loss development factor from 12 months to ultimate equals

$$1.500 \times 1.250 \times 1.100 \times 1.050 \times 1.020 = 2.209.$$

The percent of ultimate at 12 months equals $1/2.209 = 0.453$. The Bornhuetter-Ferguson factor at 12 months equals $1 - 0.453 = 0.547$ (see Table A.4).

### *A.6. Actuarial Present Values*

The Stanard-Bühlmann reserving method adds premiums and losses from different accident years. Adding dollars from two different years is problematic. In an inflationary economy, a dollar from year $X$ is worth more than a dollar from year $X + 1$.

In theory, we ought to use present values. We can add present values of dollars that have been discounted or accumulated to the same date. If we know the present value of 20X1 premiums as of a given date and the present value of 20X2 premiums as of the same date, we can add them to determine the present value of the combined premiums as of that date.

Calculating present values is not always easy, particularly for reported loss reserving methods. Case reserves are ultimate values, not present values. The reported losses in an accident year may be paid over a dozen years. In some lines of business, such as workers compensation and private passenger automobile no-fault, even individual claim benefits are paid periodically over months or years. Similarly, the premiums may be collected over the policy term, and audit premiums may be collected several months later.

If the Stanard-Bühlmann reserving method were dependent on calculating present values, it would not be practical. But we don't always need the present values. We are comparing premiums to losses. We require only that the change in premiums from year to year should equal the change in expected losses from year to year. Two conditions suffice for this:

i. The expense ratio stays constant from year to year, and

ii. The premiums are at the same level of adequacy from year to year.

The adjustments to premium ensure the adequacy level remains constant from year to year. The constancy of the expense ratio is rarely an issue. Expense ratios don't change much from year to year, and we may assume that they stay constant. A significant change in expense ratios would necessitate additional premium adjustments, but such changes are not common.

We said above that "we don't always need present values." We might rephrase this to say that

> since we are comparing premiums to losses, we can get away with adding nominal amounts from different years. We are not adding apples and oranges; we are

adding golden delicious apples with McIntosh apples. It's not perfect, but it's a practical solution. The cost of getting present values is greater than the improved accuracy we may obtain.