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SOURCE OF EARNINGS ANALYSIS FOR PROPERTY-CASUALTY INSURERS

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Abstract

Source of earnings analysis has long been a staple of life insurance policy pricing and profitability monitoring. It has grown in importance with the advent of universal life insurance and similar contracts with non-guaranteed benefits or charges. Statement of Financial Accounting Standard (SFAS) 97 requires insurers to use source of earnings analysis for Generally Accepted Accounting Practice (GAAP) reporting of universal life-type contracts.

Source of earnings analysis is not a specific ratemaking method, like the loss ratio method or the pure premium method. Rather, source of earnings analysis is a reporting structure that reveals the sources of gain and loss on a block of business, highlighting errors in the pricing parameters, as well as the sensitivity of profit and loss to various pricing factors, and enabling more accurate selection of new parameters and factors.

This paper applies source of earnings analysis to workers compensation and personal automobile insurance. The uncertainty in many casualty insurance pricing factors, such as loss development factors and loss trend factors, makes source of earnings analysis particularly important for casualty products.

The paper shows how to use the source of earnings exhibits to better analyze insurance profitability. The private passenger auto illustration divides the difference between actual and expected results between estimation error, which is within the purview of the pricing actuary, and random errors, which result from stochastic fluctuations in loss occurrences, inflation rates, or interest rates.

The workers compensation illustration focuses on the spread between the earned and credited interest rates, the solicitation costs for not-taken business,[1] *and the amortization of initial expense and loss costs by policy year.*

Analysis of the variances from previous years' predictions is a means of improving next year's predictions. Sources of earnings analysis provides the needed postmortem to judge the accuracy of the pricing assumptions.[2]

1. INTRODUCTION

This paper illustrates source of earnings analysis for property-casualty insurance. Source of earnings analysis is a staple of life insurance policy pricing. It is mandated by National Association of Insurance Commissioners (NAIC) regulations for par-

[1]"Not-taken" business is business that is underwritten and for which an insurance offer is made but not accepted. The importance of not-taken business for determining fixed expense provisions by classification is discussed in Feldblum [1996], which deals with policy pricing. This paper shows the methods to test for variance of actual results from the pricing assumptions.

[2]I am indebted to Jill Petker, Ruy Cardozo, and John Conners, for extensive comments on an earlier draft of this paper.

ticipating policies issued by mutual life insurance companies, and it is required by SFAS 97 for amortization of deferred policy acquisition expenses on universal life policies and policies with non-guaranteed benefits or charges.

We discuss source of earnings analysis for private passenger automobile and workers compensation ratemaking. Personal auto ratemaking is well suited to source of earnings analysis, since the volume of business is large enough for the effects of estimation error and random error to be distinguished. In addition, private passenger automobile has high retention rates and different acquisition costs for new policies vs. renewal policies, making profitability highly sensitive to persistency patterns.

Workers compensation retrospectively rated policies are analogous to universal life insurance contracts in that expected profits stem from margins in the pricing assumptions. The casualty actuary prices the components of the retrospectively rated policy, such as the insurance charge and the excess loss charge, even as the life actuary prices the components of the universal life policy.

Large commercial policies have high not-taken rates, various premium payment plans, and much investment income, all of which require pricing expertise. Comparing total premiums with total costs may not yield the information needed to improve the pricing process. Source of earnings analysis is better suited to identifying the causes of superior and inferior performance.

Structure of This Paper

Section 2 provides a description of source of earnings analysis as applied to life insurance products, with specific reference to (i) the calculation of policyholder dividends by means of the contribution principle for mutual life insurance companies and (ii) the SFAS 97 accounting for universal life-type products. This section is background; it may be skipped by readers who are already familiar with source of earnings analysis or those who wish to focus on only the casualty applications.

Section 3 applies source of earnings analysis to private passenger automobile ratemaking. This section explains the difference between estimation error and process error; the handling of credibility; and the difference between implicit and explicit profit margins.

Section 4 applies source of earnings analysis to workers compensation ratemaking for retrospectively rated contracts. This section discusses static versus dynamic amortization of deferred policy acquisition costs, and the source of earnings exhibits showing charged, expected, and actual results.

Section 5 summarizes the implications of the paper for pricing paradigms and the effects of random variations.

2. CLASSICAL SOURCE OF EARNINGS ANALYSIS

Source of earnings analysis was first used to set policyholder dividends for participating life insurance sold by mutual insurance companies. Source of earnings analysis is also needed to amortize the GAAP deferred policy acquisition expenses for universal life-type contracts (SFAS 97) and for participating policies sold by mutual life insurance companies (SFAS 120).

Policyholder Dividends

The contribution principle, which is required by the NAIC model act on policyholder dividends and by the American Academy of Actuaries *Standards of Practice*, mandates that the amount of divisible surplus used to pay policyholder dividends on a block of business reflect the contribution of that block to company earnings.[3] Although simple and elegant, this principle

[3]See particularly Actuarial Standard of Practice #15, "Dividend Determination and Illustration for Participating Individual Life Insurance Policies and Annuity Contracts," and Actuarial Standard of Practice #24, "Compliance with the NAIC Life Insurance Illustrations Model Regulation."

is difficult to apply rigorously, since it requires the actuary to quantify the long-term contribution to profit from variations in the pricing assumptions.

The major elements affecting life insurance profitability and used in source of earnings analysis are persistency rates (or withdrawal rates), interest earnings, and mortality ratios. Each of these is also applicable to property-casualty business.

Illustration—Persistency Rates

Suppose the expected withdrawal rates were 10% for the second year of a cohort of permanent life insurance policies, but the actual withdrawal rates are 15%. The surrender charges and the takedown of conservative statutory reserves cause an increase in statutory profits in the second year. But the smaller block of persisting business leads to lower profits in succeeding years. These lower profits offset the statutory gain from the second year. If the initial acquisition costs are not fully recovered by the surrender charges, policyholder dividends may have to be reduced. Source of earnings analysis helps quantify the equitable change in the dividend rate.

For casualty products, we use a simpler adjustment for persistency changes. Solicitation costs on not-taken business, as well as high first year acquisition expenses, are amortized over the expected policy lifetimes. If withdrawal rates increase, the amortization period is reduced and profitability declines.[4]

Illustration—Interest Earnings

Suppose that the expected Treasury bill yield for the second year of a cohort of permanent life policies was 6% but the actual yield is 5% per annum. The change in statutory investment earnings during this year may be slight, since (i) the coupons on existing bonds have not changed, (ii) bonds are valued at amor-

[4]Casualty products do not show the temporary increase in statutory profitability from higher terminations stemming from surrender charges and the release of policy reserves, so decreased persistency shows a drop in both immediate and long-term profits.

tized cost in statutory statements, and (iii) invested assets are still small in the second year of a cohort of permanent life insurance policies. The change in long-term profitability depends on the duration and inflation sensitivity of the liabilities. For a guaranteed cost block of traditional whole life business, the expected long-term profitability might drop (since liability durations are generally longer than asset durations), possibly causing a decrease in policyholder dividends.

The effects of changing interest rates are more complex for casualty products, since inflation affects loss payments and interest rates affect asset returns.[5] A full source of earnings exhibit shows the effects of variation in loss cost trends side-by-side with the effects of variation in the investment yield. The difference is the net effect on profitability.

Mortality

Variations in mortality ratios highlight the importance of distinguishing estimation error from process error. Suppose the ratio of actual-to-expected mortality in the second year of a cohort of business is 150%. If the higher than expected mortality reflects random deaths, policyholder dividends paid to the remaining insureds should not be changed. If the higher than expected mortality reflects a poor quality book of business, the policyholder dividends may have to be reduced.

For casualty business, loss frequency and severity are similar to life insurance mortality rates. Higher than expected loss frequency or severity may reflect either random loss occurrences or estimation error. Severe estimation errors call for re-examination of the pricing assumptions.

Amortization of the Deferred Policy Acquisition Cost (DPAC)

In statutory statements, acquisition costs are written off when they are incurred. In GAAP statements for traditional life insur-

[5]See Feldblum ["Investment Strategy," forthcoming].

ance policies, deferred policy acquisition costs (DPAC) are expensed as the premium is earned. For universal life-type policies, there is no set premium, so one cannot amortize the DPAC asset in relation to premiums. SFAS 97 mandates that the DPAC asset be amortized as a proportion of future expected gross profits.[6]

To illustrate the use of source of earnings analysis in FAS 97 accounting, consider an unexpected increase in the withdrawal rate from 10% to 15% in the second year of a cohort of policies. If this cohort consists of universal life-type policies, the DPAC asset would be amortized in relation to future expected gross profits. Suppose that originally the second year profits were expected to be 10% of all future profits. After the withdrawal rate increase, the actual second year profits increase and the future expected profits decrease. The second year profits are now higher than 10% of all profits, and a correspondingly larger amount of deferred policy acquisition costs is amortized in the second year.[7]

Extension to Casualty Products

Source of earnings analysis is applicable to any insurance product whose returns depend on conditions subsequent to policy pricing. This is true of all property-casualty products, since their returns depend on random loss occurrences, interest rates, and inflation rates.

Profitability also depends on the persistency of the business, particularly for direct writing insurers (D'Arcy and Doherty [1989]). Prospective pricing of products whose profitability depends on persistency patterns relies on asset share models; see Feldblum [1996]. Subsequent monitoring of product perfor-

[6]The term "universal life-type" is the GAAP term for policies with benefits or charges that are not fixed. Gross profits are profits before the amortization of deferred policy acquisition costs; net profits are profits after the amortization of deferred policy acquisition costs. The amortization of these costs in relation to expected gross profits, rather than in relation to premiums, makes sense for all policies, not just universal life. The Financial Accounting Standards Board (FASB) did not wish to change accounting practice for existing policies, so the new rules apply only to universal life-type policies.

[7]For a thorough analysis of SFAS 97, along with illustrations of the source of earnings exhibits, see Tan [1989] and Eckman [1990].

mance uses dynamic amortization of the deferred policy acquisition costs by means of multi-year source of earnings exhibits. We examine the dynamic amortization of solicitation costs for not-taken business in retrospectively rated workers compensation policies.

Workers compensation retrospectively rated policies have premiums based on the total exposure, but they provide insurance coverage for only certain layers of loss. The cost of the coverage is based on an insurance charge calculation that considers premium bounds, loss limits, the risk size, and hazard group. Profitability depends on implicit margins in the insurance charge and on the investment income from the underwriting cash flows. Source of earnings analysis allows the actuary to monitor the performance of the business in terms of the pricing assumptions.

As these illustrations show, source of earnings exhibits can deal even with gains and losses that are not generally reflected in profitability monitoring. But the primary benefits of source of earnings analysis are more general. Source of earnings analysis serves as a postmortem of previous reviews, evaluating the accuracy of the assumptions, and uncovering the causes of poor performance.

3. PRIVATE PASSENGER AUTOMOBILE

The structure of the source of earnings analysis depends on the factors affecting the rates for each line of business. Most life insurance products use a four-factor analysis, focusing on withdrawal rates, mortality ratios, interest rates, and expense ratios. For property-casualty products, mortality ratios are replaced by loss assumptions, such as loss development and loss trend, or loss frequency and loss severity.

There are three levels of the source of earnings analysis: individual factor, policy year, and policy cohort:

- The individual factor level shows the application of source of earnings analysis to each earnings factor. For private passen-

ger automobile, we examine loss severity trends in this paper, differentiating between estimation error and process error. For workers compensation, we examine several earnings factors: non-ratable losses, acquisition costs, and interest earnings.

- The source of earnings exhibits for a single policy or a single policy year combine the earnings factors but do not consider policy persistency (retention rates). These exhibits are appropriate for blocks of business with (i) low persistency rates, (ii) little difference between first year and renewal year loss and expense costs, and (iii) low solicitation costs for not-taken business.
- The source of earnings exhibits for a cohort of policies considers both the new writings and all the renewals. These are the standard exhibits required for universal life-type policies and for participating policies issued by mutual life insurance companies.

Maintenance expenses are not discussed in this paper. Maintenance expenses are generally stable, and they are more easily analyzed by direct examination than by source of earnings exhibits.

Individual Factor Level: Estimation Error and Process Error

We illustrate source of earnings analysis with loss cost trend adjustments. For private passenger automobile, whose exposure base (car-years) is not inflation sensitive, trend factors are critical for rate adequacy.

Actual results frequently differ from expected results. Source of earnings analysis relates this difference to the underlying earnings factors (or "sources"). For each factor, there are two potential reasons for the difference: estimation error and process error.

- Estimation error is the difference between the forecast and the true expectation.

- Process error is the difference between the true expected result and the actual realization.

These errors emerge over time, from the date of the rate review to the final settlement of claims. Estimation error can often be controlled by the pricing actuary, whereas process error is an unavoidable element of insurance operations.[8]

The personal auto trend illustration here uses an experience period of accident year 20X4 to set rates for annual policies written in 20X6. Thus the trend period is 2.5 years (7/1/20X4 to 1/1/20X7). Suppose the projected trend rates estimated from countrywide fast track data are +7% severity and +1% frequency.[9]

Errors may result from three sources:

1. predicting future countrywide loss trends based on historical fast track experience,
2. applying countrywide trends to a particular state, and
3. using loss trend estimates to predict the changes in actual losses incurred.

Estimation Error: Suppose that several months after the policy year expires, the source of earnings analysis shows that the actual fast track trend rates were +8% for severity and +2% for frequency. The fast track estimates, which we used as a proxy for the actual loss trends, were too low. This is estimation error.[10]

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[8]Separating estimation error from process error is not always easy; see the comments in footnote 9.

[9]Numerous data sources are available for trend estimates. The illustration in the text assumes that the pricing actuary uses countrywide fast track data to estimate trend factors, since this allows a clear demarcation between estimation error and process error. The same two sources of error exist when one extrapolates future trend factors from the company's historical statewide experience, though it is harder to separate the two sources of error.

[10]The concepts are important, not the mechanics. Conceive of this illustration as an initial derivation of a 7% annual trend by fitting an exponential curve to 1996–1999 experience. Two and a half years later we retrospectively find that the actual fit was an 8% annual trend.

TABLE 1

ESTIMATION ERROR AND PROCESS ERROR

	Estimated Fast Track	Actual Fast Track	Loss Cost Change
Loss Severity	+7%	+8%	+5%
Loss Frequency	+1%	+2%	+4%

State Differences: Differences between countrywide and statewide trends are not easily discerned. When there is no change in state compensation systems or other exogenous factors, no difference would be expected. When there is a change in compensation systems or in other exogenous factors (such as attorney involvement in insurance claims), trend differences can be significant. To simplify the presentation, we do not analyze countrywide-statewide differences.[11]

We examine the average loss severities and frequencies in the experience period and in the new policy period. Our initial numbers are estimates, since (i) the figures for the new policy year are immature, and (ii) even for the experience period the loss severities are not yet final. We won't have actual loss severity and loss figures for the new policy period until all the policies have expired, and these figures will change further as the losses are settled. For the first source of earnings exhibit, we use a mix of actual data and revised estimates. For subsequent source of earnings exhibits, the actual data are more complete.

Suppose the new loss severity and frequency figures show a change of +5% for severity and +4% for frequency, as shown in Table 1.[12]

[11]The 1991 compensation system changes in Massachusetts showed the effect of structural changes on expected loss frequency and loss severity; see Marter and Weisberg [1992]. On the importance of these regional differences as private passenger automobile cost drivers, see Conners and Feldblum [1998].

[12]Table 1 refers to the observed change as the "loss cost change" expressed as an annual trend. An observed change in the statewide average loss cost per claim of +12.97% over the $2\frac{1}{2}$ period is shown as a +5.0% actual annual change ($1.050^{2.5} = 1.1297$).

We underestimated loss severity by 1 percentage point (+7% versus +8%), and we underestimated loss frequency by 1 percentage point (+1% versus +2%). For a 2.5 year trend period, this causes the rates to be inadequate by 4.9% $[((1.08 \times 1.02)/(1.07 \times 1.01))^{2.5}]$. This is the estimation error.

The actual loss severity change was +5% per annum, and the actual loss frequency change was +4% per annum. We do not call this the actual trend, since it may be influenced by random losses. Lacking other information, we presume that the true severity trend is +8% per annum, and the true frequency trend is +2% per annum. The low observed severity trend may stem from unusually large claims in the experience period or a lack of large claims in the new policy period. Similar random effects may account for the large change in claim frequency.

If compensation system changes and structural changes are not explicitly considered, they are subsumed under the process risk component of the source of earnings exhibits. For instance, there may be an influx of nuisance claims in the new policy period that are settled for small amounts.[13]

We group the various explanations of the difference between the observed patterns in the state and the "hindsight" trend observed in the fast track data as the process error in the trend estimate. This term is not ideal, since not all of the causes of the observed difference result from process error. We simply mean that the observed difference does not stem from misestimating the expected trend.

As the new policy year develops and actual data replaces estimates, the observed loss trends may change. The changes can be large until the new policy year is fully earned, followed by smaller changes as losses are settled. For a single policy year, the first few years of the source of earnings exhibits are most

[13]The phenomenon has plagued private passenger automobile insurance for the past twenty years, and it must always be considered when the frequency change is large and the severity change is small.

TABLE 2

PRIVATE PASSENGER AUTO LOSS SEVERITY (ONE YEAR)

Valuation Date	Projection		Estimation		Process		Total
	Initial Projection	Implicit Profit	Revised Estimate	Estimation Error	Actual Change (Annualized)	Process Error	Total Variance
12/20X7	+7%	$0	+8%	−$250K	+5%	+$750K	+$500K

valuable. For a cohort of business whose profitability depends (in part) on persistency, the year-by-year source of earnings exhibits are more important.

Extending the Exhibits

To analyze the sensitivity of profits to trend errors, we convert the estimation and process errors into dollar amounts. Assuming $10 million of annual losses and using the figures above, we begin the source of earnings exhibits, as shown in Table 2.

The figures are simplified for ease of presentation. We assume a 2.5 year trend period, so a 1% understatement of the trend causes a loss of $250,000 on a $10 million book of losses.[14] Some estimation error is unavoidable; some estimation error reflects poor work and can be corrected by better pricing techniques. The conscientious actuary examines past estimation errors to check for biases in the rate review.

The $0 profit in the initial projection of +7% severity trends means there is no implicit profit margin in this pricing assump-

[14]For clarity's sake, we use rough numbers. "Book of losses" is not an ideal measure of volume, since the size of the losses depends on the trend factors. The gain or loss is the difference in profits under the two trend assumptions. In this analysis, we use nominal losses for the trend figures, and we separately quantify the gain or loss from investment earnings. When an increase in trend stems from higher inflation that is associated with higher interest rates, the loss from trend may be offset in part by a gain from interest; see the discussion below in the text.

TABLE 3

PRIVATE PASSENGER AUTO LOSS SEVERITY (MULTIPLE YEARS)

Valuation Date	Projection		Estimation		Process		Total
	Initial Projection	Implicit Profit	Revised Estimate	Estimation Error	Actual Change (Annualized)	Process Error	Total Variance
12/20X7	+7%	$0	+8%	−$250K	+5%	+$750K	+$500K
12/20X8	+7%	$0	+8%	−$250K	+6%	+$500K	+$250K

tion; contrast the workers compensation source of earnings exhibits in the second half of this paper.

The analysis of process error is important for two purposes:

1. The management of an insurance company must know whether differences of actual results from expected arise from misestimation of future costs or random loss fluctuations. Random differences may mean the business is unstable, but systematic differences indicate possible ratemaking biases.
2. Analysis of process error may uncover effects of exogenous factors, such as changes in compensation systems and in attorney involvement.

Full source of earnings exhibits use a multi-year format. Suppose that by 12/31/20X8, the actual severity increase is +6%, stemming from adverse development on reported claims. A second line would be added to the source of earnings exhibit as shown in Table 3.

Estimation error is the difference between projected and revised; process error is the difference between revised and actual. The projection is the original pricing assumption. Since the trend assumption has no implicit margin, the original "gain or loss" is $0. The projection columns do not change as additional years are added.

The revised estimate shows the actual trend rate in fast track data. The estimation error is the difference between the actual trend rate and the projected trend rate translated into dollars of gain or loss. In this example, the actual fast track trend is 1 percentage point per annum greater than the projected trend rate. For a trend period of 2.5 years and a $10 million book of losses, the estimation error is a loss of $250,000.

To keep the exposition simple, the actual fast track trend does not change from 20X7 to 20X8.[15] When the first row of the exhibit is completed before final fast track data are available (as is true in this example), the estimation error may change between the first and second rows.

The "actual change (annualized)" shows the actual severity change in the company's ratemaking data for that state. If no exogenous changes affect loss severity trends in this state, the difference between the fast track trend and the actual severity change stems from random loss occurrences in either the experience period or the policy period. The average severity in both the experience period and the policy period may change as the losses mature, so the difference stemming from process error changes as years are added to the exhibit.

Revisions stem from both actual data and revised estimates of the future. Consider the first row in Table 3. The "projection" column shows the estimated trend for 7/1/20X4 through 1/1/20X7 at the time of the rate analysis. The fast track trend is a mix of actual and expected figures: if the rate analysis is done in the middle of 20X5, the fast track trend for 7/1/20X4 through 12/31/20X4 may be actual and the remaining trend is an estimate. A revised analysis at a valuation date of December 31, 20X6, might use actual data for 7/1/20X4 through 6/30/20X6 and a revised estimate for 7/1/20X6 through 12/31/20X7.

The source of earnings exhibits trace the replacement of prior assumptions by actual data and revised assumptions. We need

[15]December 20X7 and December 20X8 are the valuation dates; at each valuation date, the fast track trend refers to the same period (July 1, 20X4 to January 1, 20X7).

not wait for final data to form the exhibits. For instance, if the actual fast track trend is higher than the projected trend for the first half of the trend period, we might expect that it will be higher for the second half of the trend period as well.

Distinguishing Sources of Error

Distinguishing estimation error from process error is not easy. For personal auto, with high frequency low severity losses, the actual fast track trend is a reasonable estimate of true loss trend. Other insurance coverages are more complex. When estimating hurricane loss costs for Homeowners, we may never know the true expected losses, since hurricane frequency and severity are difficult to predict.

The postmortem analysis used in source of earnings analysis works best for lines with high claim frequency and little variability in the size of loss distribution. Examples are life insurance, medical insurance, private passenger automobile, and workers compensation. It is more difficult when loss are large and highly variable, as is true for excess of loss reinsurance, commercial property, and catastrophe coverages.[16]

Credibility

Unlike casualty ratemaking, life insurance pricing does not use credibility adjustments. Source of earnings exhibits are more complex when credibility is used.

For other pricing assumptions, actual values are known after the policy expires and the experience is mature. For credibility, there is no actual value. The source of earnings analysis does not compare the initial credibility assumption with a subsequent (revised) value. Rather, the credibility value is used to adjust the initial assumptions.

[16]Even for the more stable lines, separation of estimation error from process error relies somewhat on actuarial judgment.

We focus here on statewide credibility factors. The credibility factors adjust the past experience to be a better proxy for the true expected losses in the experience period.[17]

Illustration: Suppose the underlying pure premium during the experience period of accident year 20X4 was $500 per car, based on a rate filing effective January 1, 20X3, and intended to be in effect for one year. The current rate review has an effective date of January 1, 20X6, and is intended for policies written in 20X6. Because of administrative problems, no rate changes were filed between January 1, 20X3, and January 1, 20X6.

Suppose the pure premium trend is 10% per annum, the experience (indicated) pure premium during accident year 20X4 is $600, and the credibility for the experience pure premium is 50%. The pure premium used in ratemaking is an equal weighting of the trended experience pure premium and the trended underlying pure premium. We adjust the source of earnings exhibits to reflect the 50% credibility factor.

The trend factor is the same whether it is applied to the experience pure premium or to the underlying pure premium. The credibility factor implies that the true expected loss during accident year 20X4 is a 50:50 average of the information from the accident year 20X4 experience and the rates underlying the accident year 20X4 writings.

Since the $500 rate was intended to be adequate for 20X3, the adequate rates underlying the accident year 20X4 losses are $\$500 \times 1.10^{0.5} = \$524.40$.

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[17]Statewide credibility factors are traditionally applied to the developed and trended experience loss ratios, perhaps giving the impression that credibility adjusts the development factors, the trend factors, or the future expected values. This is not correct. Separate credibility factors may be applied to trend and development factors. The statewide credibility factors adjust the actual data to be a better proxy of the expected experience in the past.

The discussion here is based on the "greatest accuracy" justification for credibility. Venter [1992] argues that the justification for classical credibility is to limit rate fluctuation and that the Bayesian-Bühlmann credibility procedure is designed to optimize rate accuracy. Mahler [1997] argues that even traditional credibility procedures improve rate accuracy; see also Mahler and Dean [2001].

The credibility weighted average experience rates are ($600 + $524.40)/2 = $562.20.[18] On the source of earnings exhibits, this is reflected in the actual loss cost change. The initial trend assumption is 10% per annum. The actual trend rate based on hindsight is whatever the trend index reveals. The actual loss cost change is the change between $562.20 and the observed pure premium during the new policy year (20X4).

In sum, the source of earnings analysis accepts the credibility adjustment and tests the loss cost change; it does not test the credibility value itself.[19]

Implicit and Explicit Profit Margins

Actuaries may use implicit or explicit profit margins.

- For explicit profit margins, best-estimate assumptions (for development, trend, investment income) are used in the ratemaking process and a full profit margin is included in the rates.
- For implicit profit margins, conservative assumptions are used in the ratemaking process and a lower explicit profit margin is included in the rates.

To illustrate the difference, we contrast trend factors with discount factors.

- *Trend Factors:* Suppose that fast track data imply a loss severity trend of +5% per annum. This estimate is uncertain, not only because it is a future projection but also because the fast track data may not be comparable to the ratemaking data (dif-

[18]The $500 rates were intended for policies written in the 12-month period from January 1, 1998, through December 31, 1998. The losses on these policies extend from January 1, 1998, through December 31, 1999, for an average loss date of January 1, 1999. The average loss date in the experience period of accident year 1999 is July 1, 1999, or half a year later than the average loss date expected in the filing. For a more complete exposition, see Feldblum [1998: discussion of "The Complement of Credibility"].

[19]This is not to imply that credibility procedures are impervious to empirical testing. Mahler [1990] gives three methods for testing the accuracy of credibility estimators. However, Mahler tests the accuracy of the credibility estimator; one cannot test the accuracy of a particular credibility factor. There is no such thing as the variance between the actual credibility and the assumed credibility.

ferent companies, different states, accident year versus calendar year, closed claims versus incurred claims, and so forth). We presume the trend rate is between 4% and 6% per annum.

The explicit profit method would use a +5% trend and a full explicit profit margin. The implicit profit method might use a +6% trend and a lower profit margin. Some actuaries prefer the use of explicit profit margins to better monitor the adequacy of the rates; other actuaries prefer the use of implicit profit margins to prevent overly aggressive pricing.[20] Rate filing exigencies sometimes compel companies to use lower explicit profit margins offset by conservative assumptions.

- *Discount Factors:* Suppose that losses are discounted to present value at the expected risk-free interest rate in a discounted cash-flow pricing model. The estimate of future interest rates, based on an analysis of the current yield curve and of any mean-reverting tendencies in the assumed interest rate paths, is 5% per annum. This estimate is uncertain because we are projecting a future rate and because the interest rate model may itself be flawed. We might presume that the future interest rate will probably be between 4% and 6% per annum.

  The explicit method would use a 5% assumed interest rate with a full explicit profit margin. The implicit method might use a 4% assumed interest rate with a lower profit margin.[21]

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[20]See Benjamin [1976], page 238: "The explicit method seems natural and right in contrast to the implicit method which appears to have no good or credible foundation. But in life insurance actuaries have come down very strongly in favor of the implicit margin method." See also Anderson [1959], page 368: With the inclusion of specific contingency margins and profit objectives, it is proposed that other assumptions necessary to calculate gross premiums be introduced on the basis of "best estimates" rather than "conservative estimates."

[21]The use of an implicit profit margin in the interest rate is not the same as a risk adjustment to the discount rate. For example, Myers and Cohn [1987] use a CAPM-based risk-adjusted loss discount rate that reflects the covariance of loss returns with market returns, following procedures used by Fairley [1979] and Hill [1979]. The CAPM-based risk adjustment is intended to reflect the true present value of the loss payments, not "conservatism" or an implicit profit margin. Similarly, Butsic [1988] uses a risk adjustment to the loss reserve discount rate to estimate the true economic value of the loss reserves.

Investment Income

The expected investment income on the assets supporting the book of business is an important component in pricing. Banks and life insurance companies often model the interest rate spread on assets versus liabilities.

- The pricing of universal life and variable life products uses the spread between the earned rate on invested assets and the credited rate in the policy.
- Annuity writers model the spread between interest earned on the policyholder's account balance and the accrual rates stipulated in the contract.
- Depository institutions (commercial banks, savings and loans, credit unions, thrifts) monitor the spread between the yield on loans and the interest paid on deposits.

The source of earnings analysis considers the difference between the spread assumed in the pricing analysis and the spread that is actually achieved.

Illustration: Suppose the benchmark investment yield (the casualty equivalent of the credited interest rate) used in policy pricing is 7%, and the company expects to earn 7.5% per annum on its invested assets (the projected earned rate). The actual investment yield varies with market interest rates and capital gains or losses.

The source of earnings exhibits use three sets of figures:

1. the investment yield originally assumed for the future pricing period (assumed earned interest rate), or IY_0;
2. the credited interest rate (CR), or the investment yield used in the pricing model; and
3. the actual investment yield during the period that reserves are held, or IY_t.

The actual investment yield includes dividends, interest, rents, and capital gains and losses. For investment management purposes, the source of earnings exhibits differentiate market yields from realized plus unrealized capital gains and losses.

The interest spread is most important for the long-tailed lines of business. We estimate the invested funds for each year (IF_t).[22]

The source of earnings analysis quantifies the implicit profit margin in the investment yield assumptions and the subsequent unfolding of the actual profit margin. Each year's implicit expected profit margin is the invested funds times the difference between the expected investment yield and the investment yield used in pricing, or $IF_t \times (IY_0 - CR)$. The total profit is the sum of the annual profits discounted at the cost of capital.[23]

Illustration: Suppose we are analyzing a $10 million book of business, with average invested funds of $3 million during the policy year, $4 million the next year, and declining by $1 million a year until all losses are settled.[24] The company expects an investment yield of 8% per annum, and it prices the business assuming an investment yield of 7% per annum and a 12% cost

[22]Most casualty pricing models estimate the invested funds by projecting premium collection patterns, loss payment patterns, and expense payment patterns. Life actuaries use the term "account balance" instead of invested funds. In life insurance and annuities, the account balance belongs to the policyholder and may be withdrawn on demand, sometimes with a surrender charge deducted or a market value adjustment. In casualty products, the policyholder does not own the funds used to support the reserves. The term invested funds refers to the assets supporting the unearned premium and loss reserves.

[23]This formula assumes that IY_0 is the pricing assumption for all future years; that is, the actuary assumes a constant future investment yield.

[24]This progression of the invested funds reflects a policy year of writings. With a prepaid acquisition expense ratio of 20%, a net premium of $8 million collected up-front on some policies and by installment plans on others, and some losses paid during the policy year, the average invested funds during the policy year are about $3 million. The invested funds peak about 12 months after inception of the policy year, since premiums have been collected but losses remain in reserves. During the next 12 months, the invested assets remain relatively constant, as the remaining premium is collected and some losses are paid. The invested funds decline to zero as losses are settled. To keep the illustration simple, we use an expected policy lifetime of four years; actual lifetimes for long-tailed lines of business are longer.

TABLE 4

SOURCE OF EARNINGS ANALYSIS FOR INTEREST SPREAD AT POLICY INCEPTION

Year	Invested Funds	Expected Invest Yield	Credited Interest Rate	Interest Rate Spread	Interest Rate Margin	PV of Margin
0	3,000,000	8%	7%	1%	30,000	30,000.00
1	4,000,000	8%	7%	1%	40,000	35,714.29
2	3,000,000	8%	7%	1%	30,000	23,915.82
3	2,000,000	8%	7%	1%	20,000	14,235.60
4	1,000,000	8%	7%	1%	10,000	6,355.18
Total						110,220.89

of capital.[25] The implicit profit margin in the investment yield assumption is shown below. The present values are taken to the middle of the initial policy year (year 0) as shown in Table 4.

Illustration: Average investable funds in year 3 are \$2,000,000. With a 1 point spread, the interest margin is \$20,000. Discounting to the middle of year 0 at the 12% cost of capital gives $\$20{,}000/1.120^3 = \$14{,}235.60$.

Between initial pricing and final settlement of claims, several items may change.

1. Interest rates may change, causing immediate (unrealized) capital gains or losses in GAAP statements and market values (though not in statutory accounting) and revised investment yields in future years.

2. The amount of invested funds may differ from the initial assumption.

[25]To keep the arithmetic simple, we ignore federal income taxes in this paper. In practice, they must be considered, particularly since different investments have different tax rates (see Feldblum and Thandi [2003]). For prospective pricing, one often assumes that the present value of future investment income does not depend on the type of investment; see Derrig [1994]. In contrast, the source of earnings analysis focuses on the defaults and market value changes of risky investments.

TABLE 5

SOURCE OF EARNINGS ANALYSIS FOR INTEREST SPREAD AFTER ONE YEAR

Year	Invested Funds	Investment Yield	Credited Interest	Interest Spread	Interest Margin	Capital Gain/Loss	PV of Margin
0	$2,500,000	9.5%	7%	2.5%	$62,500	−$50,000	$12,500.00
1	$3,500,000	10%	7%	3.0%	$105,000	$0	$93,750.00
2	$3,000,000	10%	7%	3.0%	$90,000	$0	$71,747.45
3	$2,000,000	10%	7%	3.0%	$60,000	$0	$42,706.81
4	$1,000,000	10%	7%	3.0%	$30,000	$0	$19,065.54
Total							$239,769.81

3. There may be unexpected capital gains or losses for reasons other than interest rate changes.

The new entries in the source of earnings exhibits are a mix of actual figures and revised estimates.

Illustration: In Table 5, the investment yield rises to 10% per annum between the rate review and the end of the policy year. Year 0 shows a 9.5% average actual yield, and years 1 through 4 show 10% as the revised (estimated) yield. More insureds used installment payment plans; the actual investable assets in year 0 and the estimated investable assets in year 1 are reduced.

The investment yield increase from 8% at the rate review date to 10% by the end of the policy year causes the $50,000 capital loss in year 0. Since most of the investment yield increase occurred before assets were bought, the capital loss is small and the greater future investment income more than offsets it.[26]

Inflation Rates and Interest Rates

The full effect of interest rate changes requires a combined analysis of assets and liabilities. If inflation rates rise along with interest rates, loss severity increases. The revised expected loss

[26]If the investment yield increase occurs after fixed income assets are bought, the capital loss may more than offset the higher reinvestment rate for coupon payments.

ratio exceeds the target loss ratio, but this loss may be offset by the rise in the investment yield (see Butsic [1981]).[27]

Inflation rates and interest rates are correlated, but they do not move in lock step. The source of earnings exhibits provide a year-by-year analysis of the gains and losses from inflation and interest, allowing a better analysis of net profitability.

Illustration: Interest rates and inflation rates rise shortly before the inception of the policy year. Losses are larger than initially projected, but investment income is greater than initially projected as well; the net profit variance shows the combined effects of both. This analysis is particularly important for retrospectively rated workers compensation policies and large dollar deductible policies, since inflation has a leveraged effect on losses above the deductible. Equal increases in interest rates and inflation rates generally reduce the net profits on this business.

Persistency

Of the four life insurance earnings factors—mortality, maintenance expenses, interest, and persistency—persistency is the least well understood but often the most important. Mortality rates change slowly over time; maintenance expenses are equally stable. Interest earnings come from the spread between earned rates and credited rates. Although the earned rates may vary from year to year, many companies try to keep the spreads stable.

Persistency rates can only be estimated. Differences of actual from expected persistency can be large, and they strongly affect profitability; see Tan [1989] and Eckman [1990].

Persistency patterns greatly affect property-casualty profitability as well. For a variety of reasons, casualty actuaries have not always given persistency patterns the attention they deserve.

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[27]Traditional profitability measures of loss ratios and combined ratios can be misleading. Statutory measures of total profitability, as reflected in the investment income allocation procedure in the Insurance Expense Exhibit, are distorted by the use of portfolio investment yields and amortized values of fixed income securities; see Feldblum [1997].

- Acquisition expense differences between new business and renewal business are not as great for property-casualty insurance as for life insurance. First-year commissions for permanent life insurance may exceed the annual premium; for casualty products, commissions are rarely more than 25% of the premium.
- Life and health insurers must renew their permanent policies as long as the policyholder pays the premium. They have more incentive to quantify the effects of persistency on product profitability. A property-casualty insurer may cancel the policy or decline to renew it.
- For companies using the independent agency distribution system, the agent owns the renewals, and commissions are level from year to year. Persistency patterns are not under the control of the company, and they have less effect on expense ratios.
- Rating bureaus, which set the traditional workers compensation ratemaking procedures in the twentieth century, have less interest in persistency patterns than competitive insurers have. Life and health insurers do not use rating bureaus.

Ideally, persistency patterns are incorporated in prospective ratemaking by asset share pricing models. The source of earnings analysis evaluates the profits achieved from a cohort of policies.

Illustration: A personal auto direct writer has had a 90% retention rate in past years. The retention rate drops to 80% for the new policy year. Acquisition expenses are 20% for new business and 5% for renewal business. The expected loss ratio is 80% for new business and 70% for renewal business. The total spread between new and renewal business is 25% of premium.

The drop in the retention rate reduces profitability. The decline in profitability may be estimated as the reduction in renewal business times the spread between new and renewal business, or $(90\% - 80\%) \times 25\% = 2.5\%$ of premium each year.

The traditional premium, loss, and expense exhibits show higher than expected loss and expense ratios. But neither ex-

pense costs nor loss costs have changed.[28] If the pricing actuary did not consider persistency effects, the source of earnings analysis is all the more necessary to tease apart the underlying sources of profit or loss.

We illustrate below one way of amortizing acquisition expenses. Traditional property-casualty ratemaking combines acquisition expenses with on-going maintenance expenses and treats the sum as either an additive factor (fixed expenses) or a multiplicative factor (variable expenses). This obscures the effects of expense items. In the illustration here, acquisition costs and solicitation costs on not-taken business are treated separately and amortized over the expected lifetimes of the insurance policies.

4. RETROSPECTIVELY RATED POLICIES

Policy Economics

Pricing and accounting should reflect the underlying economics of the insurance product. The FASB introduced SFAS 97 to make the accounting for universal life and variable life contracts consistent with their economic structure. This section applies the FASB's distinction between traditional whole life and universal life policies to prospectively priced private passenger automobile versus retrospectively rated workers compensation.[29]

For a traditional whole life policy (SFAS 60), the premium due is an income statement revenue and the increase in the policy reserve plus any death benefit in excess of reserves is an income statement expense. For casualty products, earned premium is the revenue and incurred losses are the expense. Greater earned premium reflects additional profits and greater losses reflect decreased profits. The pricing actuary sets the premium rate (the

[28]The business growth illustration in Feldblum [1996, "Personal Automobile"] analyzes these profitability effects.

[29]Over the past decade, many insurers have shifted much of their retrospectively rated workers compensation business to large dollar deductible policies. The money paid by the employer to cover losses below the deductible is termed an assessment, not a premium, and it is generally paid shortly before or after the benefits are paid. In most states, premium taxes and involuntary market burdens are not levied on assessments. The discussion in the text applies to both retrospectively rated contracts and to large dollar deductible contracts.

revenues) based on estimates of the ultimate losses and expenses (the income statement expenses).

When a universal life policyholder pays premiums, the money belongs to the policyholder, not to the life insurance company. The insurance company is a financial intermediary, investing the policyholder's money. It deducts a management fee for investment services and specified charges for underwriting protection, such as the mortality charge and the maintenance expense charge. Premiums are a deposit, not a revenue.

- The policy charges plus the investment income earned on the account value are revenues.
- Benefit payments in excess of the account value, interest credited to the account value, and expenses paid are expenditures.

A workers compensation retrospectively rated policy is similar in substance. The insurer uses the policy premium to pay losses and to cover the various charges, such as the insurance charge and the other components of the basic premium. If the losses do not materialize, the insurer returns part of the premium to the insured. If actual losses exceed the original expectations, the insurer collects additional premium.[30]

For retrospectively rated policies, additional incurred losses lead to additional retrospective premiums, with the net effect depending on the premium sensitivity (Teng and Perkins [1996], Feldblum [1997], Bender [1994], Mahler [1994]). A change in losses or in premiums does not by itself signal higher or lower profitability. Traditional exhibits of premiums and losses are not always an appropriate means of monitoring the profitability of this business.

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[30]The various charges in a universal life policy, such as the mortality charge, asset management charge, surrender charge, and expense charge, are noted in the policy and in periodic reports to the policyholder, particularly if the asset accumulation rate is tied to external investment indices. For the retrospectively rated workers compensation policy, the pricing actuary knows the individual charges, but the insured may not be aware of them.

Retro Policies vs. Universal Life

The source of earnings analysis for workers compensation retrospectively rated policies has two differences from the analysis for universal life-type policies.

1. The *insurance charge* takes the place of the *mortality charge*, and *non-ratable losses* takes the place of *policyholder benefits in excess of the account value*. The mortality charge in a universal life policy pays for policyholder benefits in excess of the account value; the insurance charge in the retrospectively rated workers compensation policy pays for non-ratable losses.[31]
2. SFAS 97 amortizes deferred acquisition costs in relation to expected gross profits, with a year-by-year unlocking of assumptions as actual experience emerges. We use a simpler amortization procedure here but the amortization schedule is still dynamic, so that persistency is reflected in the source of earnings exhibits.

Evaluation of Results

Pricing for retrospectively rated policies depends on four sources of earnings: (a) investment income, (b) non-ratable losses, (c) expense levels, and (d) retention rates.

Standard reports of premiums and losses do not show the expected profits on retrospectively rated policies stemming from these earnings factors or the variations in profit caused by changes in these factors. The reports do not show if the ratemaking assumptions accurately reflect the expected experience on the book of business.

If profits are unexpectedly low, we do not know if the cause is (i) higher than anticipated non-ratable losses, (ii) lower than expected investment income, (iii) excessive expenses, or (iv) higher than anticipated lapse rates or not-taken rates.

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[31]Non-ratable losses are losses above the loss limit or losses that would cause premium above the maximum.

Amortization of Deferred Acquisition Costs

For two reasons, the amortization of deferred policy acquisition costs is essential for monitoring universal life profitability.

- Deferred acquisition costs are as much as 50%–60% of gross profits for many universal life contracts.[32] In the first one or two policy years these products show large statutory losses from acquisition costs and low investment income, since (i) first year agents' commissions are high (often 100% of the annual premium), and (ii) invested assets from policyholder funds are zero in the initial policy year and low in the first renewal year. There is little profit from the interest spread in these years.
- Retention rates greatly affect long-term profitability. Statutory accounting distorts the effects, since only the surrender charge (a gain) is shown for the current calendar year. Dynamic amortization of deferred policy acquisition costs reveals the effects of retention rates on long-term profitability.

The capitalization and amortization of acquisition and issue costs is also important for retrospectively rated policies. First year agents' compensation, initial underwriting, workplace inspection, loss engineering, and policy issue costs are the major expenses for retrospectively rated policies.[33]

For large account retrospectively rated business, not-taken rates can be high. There are a limited number of large workers compensation accounts, with $2 million or more of annual premium. The risk manager of each insured might put the account out to bid every five years or so. Developing the bids is costly, but each bid may have only a 10% to 20% chance of being accepted, leading to an 80% to 90% not taken rate.

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[32]"Gross profits" are the present value of lifetime profits from the block of business before consideration of prepaid acquisition costs; see SFAS 97.

[33]This is especially true for direct writers, with large first year commissions and low renewal commissions.

The costs of not-taken policies must be included with acquisition costs. Some companies spread these costs over related books of business, thereby raising the apparent profitability of the book being priced and lowering the profitability of the related books. For instance, some companies spread the costs of not-taken business over the entire workers compensation line of business.[34]

The high acquisition expense costs—including the cost of not-taken policies—must be amortized over the policy lifetimes. It is tempting to overestimate persistency rates and underestimate not-taken rates. Source of earnings analysis with dynamic amortization of policy acquisition costs counteracts this temptation.

Static vs. Dynamic Amortization

Static amortization schedules, like static depreciation schedules, do not change with the passage of time. The rate of amortization or depreciation may vary over time, as with double declining balance depreciation schedules, but the amortization schedule is not re-estimated as more is learned about the business.

Static amortization schedules distort profitability analyses if actual persistency rates or investment yields differ from those assumed in pricing. Dynamic amortization allows for revision of the schedule as actual experience becomes known and as future expectations change.[35]

DPAC amortization schedules use an implicit interest rate, so that the present value of the expenses amortized equals the deferred expenses incurred. To simplify the illustrations here, we use pro rata amortization with a 0% amortization interest rate.

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[34]This leads to incorrect pricing and marketing decisions. There may be strategic reasons for this practice, such as a desire to break into the large account market. More often this practice stems from data limitations that hamper the allocation of expense costs.

[35]For the universal life-type policies covered by SFAS 97, the deferred policy acquisition costs are amortized in proportion to future expected gross profits. The amortization schedule is revised whenever actual experience or future expectations differ from assumptions for any of three items: persistency rates, investment yield, and expected or actual gross profits.

Illustration—Static Amortization: If all policies are expected to persist eight years, one eighth of a policy's deferred policy acquisition costs is amortized each year (assuming a zero interest rate for amortization). If after two years of experience, the average policy lifetime is expected to differ from eight years, the amortization schedule is not changed.

Illustration—Dynamic Amortization: Suppose the excess of first year over renewal acquisition costs on a $100 million block of business is 20% of premium, and the solicitation cost for not-taken business is 10% of the not-taken premium. The pricing actuary assumes an 8 year average policy lifetime and a 20% not-taken rate.

Computation: The excess acquisition costs in the first year are $20 million. The not-taken rate is 20%, so the premium solicited but not taken is $100 million $\times$ (20%/(1 − 20%)) = $25 million. The solicitation costs for not-taken business are $25 million $\times$ 20% $\times$ 50% = $2.5 million. The total acquisition expenses are $22.5 million. Since policies last an average of 8 years, the annual cost is $22.5 million/8 = $2,812,500.

The assumptions used for the amortization schedule are uncertain, though they become known with the passage of time. The not-taken rates and the solicitation costs for not-taken business are known once the new policies are written. The average policy lifetime is re-estimated two or three years after the expiration of the initial policy year (by projecting from early retention rates).

If these figures are revised after the policies are written to an average lifetime of 5 years and a not-taken rate of 60%, the annual acquisition cost is revised as well as shown in Table 6.[36]

[36]Table 6 is simplified. If the anticipated not-taken rate is 20% and the actual rate is 60%, the insurer has written about $125 million $\times$ (1 − 60%) = $50 million of premium. The dollar amortization figure in the exhibit is overstated, but the ratio of the amortization amount to the premium is correct.

TABLE 6

DPAC DYNAMIC AMORTIZATION SCHEDULE: SOLICITATION COSTS FOR NOT-TAKEN BUSINESS

	Assumptions	
	Initial	Revised
A. Premium	$100,000,000	$100,000,000
B. Excess acquisition costs	$20,000,000	$20,000,000
C. Not-taken rate	20%	60%
D. Not-taken premium [$= A \times C/(1-C)$]	$25,000,000	$150,000,000
E. Not-taken acquisition costs [$= \frac{1}{2} \times D \times 20\%$]	$2,500,000	$15,000,000
F. Total acquisition costs [$= B + E$]	$22,500,000	$35,000,000
G. Average policy lifetime	8 years	5 years
H. Annual amortization [$= F/G$]	$2,810,000	$7,000,000

Invested Capital

The SFAS 97 source of earnings exhibits for universal life policies do not consider invested capital. Before the advent of risk-based capital requirements, this approach was reasonable, at least for GAAP statements.

- Little capital is embedded in the policy reserves, which do not much exceed the account balance.
- Deferred policy acquisition costs are amortized on GAAP statements, so the initial underwriting loss is small.
- Little surplus was needed to satisfy regulatory requirements. Even with the advent of risk-based capital requirements, the surplus requirements for life insurance products are lower than for casualty products.

The capital contributed by investors is much smaller than the policyholder premium.

In contrast, the capital invested for workers compensation is large. Much investors' capital is embedded in undiscounted loss reserves and gross unearned premium reserves. Additional capi-

tal is needed to meet the NAIC's risk-based capital requirements or rating agency capital formulas. The investment spread in the source of earnings analysis applies to the investment income on both policyholder-supplied funds and investors' funds.[37]

Charged, Expected, and Actual

For the private passenger automobile source of earnings analysis, we showed three values for the loss severity trend factors:

1. initial (*ex ante*) trend factors,
2. revised (*ex post*) trend factors, and
3. the actual loss cost change.

The change from estimated trend to actual trend is estimation error; the change from actual trend to actual loss cost change is process error. The same three-level analysis applies to loss development factors, loss frequency trends, and other ratemaking items.

Judging the adequacy of the insurance charge is more difficult. The insurance charge is based on size of loss distributions developed from a large volume of industry experience. The actual policy-year experience tells us the actual non-ratable losses, not the proper insurance charge. The credibility of the excess loss experience for a given block of business is hard to measure.

Personal auto policies are sold for a single premium. The underwriter does not assemble a policy for a given insured with separate charges for development, trend, and expenses. In contrast, a retrospectively rated policy is assembled by the underwriter or sales agent, given values for the insurance charge, the excess loss charge, and other plan parameters. For each earnings

[37]For source of earnings analysis applied to financial pricing models, see E. Schirmacher and S. Feldblum [forthcoming].

TABLE 7A

RETROSPECTIVE RATING COSTS—CHARGED, EXPECTED, ACTUAL

Date	Insurance Charge	Expected Non-Ratable losses	Expected Gain	Actual Non-Ratable losses	Variance (Actual from Expected)	Actual Gain
1/20X1	$500,000	$450,000	+$50,000	—	—	—

element there are three values:[38]

1. the amount charged in the pricing analysis,
2. the expected cost at policy inception, and
3. the actual (realized) cost.

Illustration: A policy is issued on January 1, 20X1, with an insurance charge (including the excess loss charge) of $500,000, and with expected non-ratable losses of $450,000.[39] The initial report at policy inception is shown in Table 7A.

On December 31, 20X1, at the expiration of the policy, the estimated non-ratable losses (including bulk reserves) are $470,000. The variance of actual from expected is −$20,000,

[38]The charges for the various pricing components do not sum to the policy premium, since much of the policy premium serves as a deposit to pay ratable losses. Compare universal life policies, much of whose premium is an investment designed for tax-deferred accumulation, not for insurance protection.

[39]Some actuaries use an insurance charge equal to the expected non-ratable losses along with a separate profit provision. Other actuaries use a more conservative insurance charge. The insurance charge minus the expected non-ratable losses is an implicit profit margin. Life insurance pricing often uses implicit mortality and interest margins, or conservative mortality tables and a spread between the earned interest rate and the credited interest rate. Similarly, the exhibits here use conservative assumptions and implicit profit margins. A company that uses explicit profit margins with no spreads in the pricing components would show zeroes as the initial profit from each source. The gains and losses are shown here as dollar amounts. In pricing the policies, many of these items—such as the insurance charge—are shown as percentages of standard earned premium.

TABLE 7B

RETROSPECTIVE RATING COSTS—CHARGED, EXPECTED, ACTUAL

Date	Insurance Charge	Expected Non-Ratable losses	Expected Gain	Actual Non-Ratable losses	Variance	Actual Gain
12/20X1	$500,000	$450,000	+$50,000	$470,000	−$20,000	+$30,000

TABLE 7C

RETROSPECTIVE RATING COSTS—CHARGED, EXPECTED, ACTUAL

Date	Insurance Charge	Expected Non-Ratable losses	Expected Gain	Actual Non-Ratable losses	Variance	Actual Gain
1/20X1	$500,000	$450,000	+$50,000	—	—	—
12/20X1	$500,000	$450,000	+$50,000	$470,000	−$20,000	+$30,000
12/20X2	$500,000	$500,000	+$50,000	$515,000	−$65,000	−$15,000

and the actual gain is +$30,000.[40] Table 7B shows the entries for December 20X1.

Actual non-ratable losses increase to $515,000 by December 31, 20X2, and Table 7C shows the updated figures.

We comment on each source of earnings in this table.

Insurance Charge

The insurance charge illustrates the difficulty in assigning gains and losses to sources. Ideally, we should separate the difference between (i) actual and expected excess losses and (ii) the earnings from interest. But the insurance charge is stated in nominal dollar terms, not in present value terms, whereas the actual excess losses are paid many years after the premium is collected. A zero dollar initial variance is an implicit profit margin.

[40]The term *variance* is used in the accounting sense, meaning the difference between expected and actual.

Disentangling the insurance charge from the time value of money is a general problem in retrospective rating. Retrospectively rated policies can be priced in several ways:

1. In theory, the insurance charge should reflect the present value of excess losses, though since the loss limit and the maximum and minimum premiums are stated in nominal dollars, present values are rarely used.

2. The insurance charge is based on the ultimate values of losses, but it is reduced for the expected investment income on the excess losses. Some actuaries presume that this is done implicitly, since the insurance charge is a percentage of standard premium, whose profit provision considers the expected investment income. The resultant insurance charge may be less than the expected (nominal) excess losses. But this assumes that the loss payment pattern for excess losses is similar to that for ratable losses. In fact, excess losses have slower payment patterns, leading to an implicit profit margin in the insurance charge.[41]

3. The insurance charge is based on ultimate losses, and a separate investment income factor calculated from all insurance cash flows reduces the basic premium.

For simplicity, this illustration uses a single policy. Actual source of earnings analyses use blocks of policies, such as all large account business written by a particular sales office in policy year 20XX. Since non-ratable losses have great random fluctuation, a report showing variances is more meaningful on a block of business basis. The subsequent examples are for policy year blocks of business.

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[41]The explanation in the text is simplistic: the consideration of investment income in the underwriting profit provision has no mathematical relation to the lag between collection of the insurance charge and the payment of excess losses.

TABLE 8

SOURCE OF EARNING ANALYSIS FOR RETROSPECTIVELY RATED POLICIES ($000)

Valuation Date	Non-Ratable Losses	Interest Earned	Persistency	Maintenance Expenses	Explicit Profit	Total Profit
1/1/20X1	$2,000	$2,500	−$1,500	$750	$1,250	$5,000
12/31/20X1	$1,400	$3,400	−$2,500	$750	$1,100	$4,150
12/31/20X2	$2,100	$3,600	−$2,900	$750	$1,100	$4,650

Expenses

Expenses are divided into two components:

1. underwriting and acquisition expenses, including solicitation costs for not-taken business, and
2. policy maintenance expenses, including unallocated loss adjustment expenses.

The effect of acquisition and underwriting expenses on profitability depends on the difference between expected and actual (i) not-taken rates and (ii) renewal rates. We speak of these as earnings from persistency. Maintenance expenses are rarely a material source of gain or loss, and they are not discussed further here.

Combining the Earnings Factors

The first row in Table 8 shows the profit from each factor in the pricing assumptions. Subsequent rows show the variance resulting from actual data and revised estimates.

Pricing Assumptions

At January 1, 20X1, the inception of the policy year, the figures show the implicit and explicit profit margins. Most of the expected profit ($3.75 million out of $5 million) is embedded in the pricing assumptions.

- The insurance charges exceed the expected non-ratable losses by $2 million.
- The company expects an average lag of about one year between premium collection and loss payment, with a small spread between the interest earned and the interest credited to policyholders in the pricing assumptions.[42] The actual investment income is expected to exceed the investment income assumed in pricing by $2,500,000.
- The company expects actual maintenance expenses (including unallocated loss adjustment expenses) to be $750,000 below the amount assumed in pricing.[43]
- The company loses money from solicitation costs on not-taken business. Some of this money is recouped from acquisition expense charges in the basic premium. The amount that is not recouped is a negative implicit profit margin of $1,500,000.[44]
- The company builds an explicit profit component of $1,250,000 into the rates.

Underwriting

The first row shows the pricing assumptions at the inception of the policy year. Rarely are all pricing assumptions realized. The second row shows the revised values at the end of the policy year. The variances from expected profits stem from two causes:

- If the sales price differs from the actuarial indications, the charges embedded in the policy components may differ from those anticipated by the actuary. For instance, the indicated

[42]Incurred loss retros may have long lags between premium collection and loss payment; paid loss retros and large dollar deductible policies have short lags. The one-year lag is an average.

[43]We include unallocated loss adjustment expenses (ULAE) with underwriting expenses because both reflect operating efficiency.

[44]It is hard to persuade policyholders that they should reimburse the costs of soliciting other business, and the company does not expect to recover all the costs from expense charges in the premium.

insurance charge may be $25,000, but the company may use only a $15,000 charge.

- Fluctuations in losses or interest rate changes affect the costs. Even if the company uses the $25,000 insurance charge, a large loss may eliminate the expected profit.

In the illustration, interest rates have risen and the marketplace has softened, but the underwriters have adhered closely to the pricing recommendations.

- The rising interest rates lead to greater excess losses, since inflation has a leveraged effect on higher layers of loss, reducing the implicit profit from non-ratable losses by $600,000.
- A few insureds are given premium credits, reducing the explicit profit margin by $150,000.
- Because of the soft market, not-taken rates increase, leading to an additional $1 million loss from unfulfilled solicitation costs.
- Interest rates rise before the company invests the premiums, leading to $900,000 additional implicit profit from the interest spread.[45]

Actual Experience

Subsequent revisions arise from random loss occurrences and from interest rate changes. For instance, the 12/31/20X2 row shows an increase in the expected profits from non-ratable losses. By December 31, 20X2, all policies have run their course, and there have been fewer large losses than expected. This may result from stringent underwriting or random loss fluctuations.[46]

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[45]The pricing actuary must take care to reflect the higher interest rate, and the potentially higher inflation rates, in the insurance charge. If this is not done, the implicit profit margin from non-ratable losses may be overstated.

[46]Because the claim severity distribution is highly skewed, most years show fewer large losses than expected, offset by a few years with more large losses than average.

The December 31, 20X2, figures are a combination of actual figures and estimates:

- The investment yield in 20X1 and 20X2 is known.
- The effect of acquisition costs on policy profitability still depends somewhat on future persistency rates.
- The ultimate amount of large losses may remain uncertain for years.

The source of earnings exhibits are updated until most of the losses have been settled or until subsequent changes in estimated earnings are not material.

Non-ratable losses: When pricing retrospectively rated contracts, some actuaries rely on aggregate industry figures, such as NCCI Table M data. Individual company data may not be considered sufficiently credible for revising Table M figures, and the needed adjustments for inflation and for changes in the size of loss distribution are complex.

Ideally, Table M charges should be reviewed periodically to ensure their adequacy. The source of earnings analysis provides a hindsight view of the adequacy of the insurance charges that can be especially valuable for the pricing actuary. The challenge for the pricing actuary is to discern from the emerging experience how much of the variance stems from estimation error and how much stems from process error.

Interest: The earnings from interest depend on the investment yield received versus that used to price the policy and the collection dates for premium and losses. Large accounts often want customized cash flow plans to retain more of the investment income. For these accounts, the expected earnings from interest may be determined on a plan-by-plan basis.

The interest earnings factor troubles some practicing actuaries, who say:

> This analysis presupposes an investment yield assumption in the rate analysis. But that is not how we develop rates. We price to a target combined ratio, or a target underwriting profit provision. This target is set by company management, not by the pricing actuary doing the rate review. The target combined ratio may have been set by an internal rate of return model or a discounted cash flow model. Even in these models, there may be no simple interest assumption. Our pricing procedure does not fit into the source of earnings mold.

This criticism is dismaying. It has been more than twenty years since actuaries began using financial pricing models for casualty insurance products. The parameters of these models—such as the assumed investment yield, the target return on capital, the surplus requirements, and the implied equity flows—greatly affect the final premiums. Yet some actuaries who are expert in other pricing issues cannot figure out what their pricing model says. They can tell you the effect of a one-point increase in the trend factor, but they can't tell you the effect of a one point increase in the investment yield.

The source of earnings analysis compares the investment income actually received with the investment income assumed in pricing. The analysis of this difference, along with related interest rate changes and capital gains, helps the practicing actuary understand the implications of the financial pricing model.

Persistency: For large account retrospectively rated business, the solicitation costs for not-taken business and the persistency of insured business greatly affect overall profitability.[47] The source

[47]The full effects of interest rate changes and persistency changes take several years to play out. Some pricing actuaries disclaim responsibility for interest rate changes, not-taken rates, and persistency rates, since traditional ratemaking procedures do not deal with these items. The common disclaimer is that "the investment yield is the responsibility of the Investment Department; we simply use the projections that they provide us." Similarly one hears that "the persistency rate, or the not-taken rate, is the responsibility of the sales force; we simply use the projections that they provide us." This retort is disingenuous.

of earnings analysis ensures that pricing actuaries incorporate these effects in the ratemaking formulas.

Combined Effects

Implementing source of earnings analysis requires some means of dealing with combined (non-linear) effects.

Illustration: Suppose the developed and trended losses are $100 million. The source of earnings analysis shows that the loss development factor should have been 10% higher and the loss trend factor should have been 10% higher. A simple source of earnings exhibit might show a (negative) gain of −$10 million from development and a similar −$10 million from trend. But the total variance is −$21 million, not −$20 million.

The allocation of the extra $1 million to earnings sources is problematic. When there are multiple non-linear factors, the problem is more complex. We may use three types of solutions:

1. *Assign the linear component of the variance to the individual factors, and assign the non-linear components to a "combined" bucket.*
2. *Compute the variances by the order of application of the ratemaking factors*. This solution is arbitrary, since there is no inherent order to the calculations. For example, either loss trending or loss development may precede the other.
3. *Spread the non-linear components over the individual factors on a formula basis*. This method is the most sophisticated, but it is the most complex.

The mathematics of source of earnings analysis is not as simple as one might infer from the example in this paper, partic-

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The source of earnings analysis does not bring investment policy or marketing philosophy under the purview of the actuary. Nevertheless, just as the reserving actuary does not rely solely on the claims department's loss estimates, the pricing actuary cannot rely solely on others' estimates for the pricing assumptions.

ularly when multi-year persistency effects are considered. The appendix shows more realistic source of earnings exhibits for retrospectively rated workers compensation business. When the total variance is small, the non-linear components (or the "second order" components) are small enough that they do not affect the analysis. When the total variance is large, one of the above procedures may be used for the non-linear components.

5. CONCLUSIONS

Two topics run through this paper: the unbundling of the insurance contract, and the differentiation between estimation error and random fluctuation. We summarize the two topics below and their implications for practicing actuaries.

Pricing Paradigms

A premium-loss pricing paradigm now dominates casualty actuarial ratemaking. The actuary determines policy premiums to cover expected losses and expenses.

With the life insurance policy revolution of the 1980s, life actuaries moved to a credit-charge paradigm. The new interest-sensitive policies were unbundled into their components. The actuary determines charges and credits for the policy components, which may be rearranged into full policies to meet customer needs.

The flexibility of the credit-charge paradigm makes it ideal for large account workers compensation pricing. The employer purchases a customized policy with specialized components: deductibles, premium payment plans, retrospective rating, loss engineering services, claims handling services, self-insured retentions, excess coverage, and so forth.

The actuary prices the components, which are assembled by the underwriter into the policy. For instance, the actuary deter-

mines the appropriate insurance charge for a set of plan parameters, or the appropriate interest credit for a given plan type and premium payment pattern. Source of earnings analysis enables the actuary to monitor the adequacy of the charges and credits.[48]

The shift from a premium-loss pricing paradigm to a credit-charge pricing paradigm brought "universal" contracts to the life insurance industry. We may conceive of universal policies as retrospectively rated contracts where the premium adjustment depends on the investment yield achieved, not on the loss experience.[49]

By unbundling the policy into its components, the casualty insurer can offer varied product designs, such as universal policies for lines with long term claim payments. The actuary sets the investment spread; the actual premium for the coverage varies with the investment income actually earned. Such policies may be particularly attractive to large accounts seeking aggressive investment returns and reluctant to pay the premium before the losses come due.

Random Variations

Actuaries often attribute differences between expected and actual results to random loss fluctuations, to unforeseeable changes in inflation, or to unanticipated market pressures on underwriters and agents. The work pressures on actuaries are so great, and the potential causes of adverse results are so diverse, that many pricing actuaries never examine the variances in past re-

[48]An analogy with computer manufacturing is instructive. IBM once built machines in pre-set models. Dell builds machines to consumer desires, with the price based on the components that are included. Insurers used to offer pre-determined policies to all insureds. Now insurers offer flexible policy design to large commercial accounts. Actuarial pricing must be equally flexible, so that the customized policies are priced by sound economic principles.

[49]There are differences, of course. Universal life policies allow more management discretion in setting the credited interest rate; workers compensation retrospectively rated policies have contractually determined premium adjustments. Universal life contracts depends on the insurer's investment yield or on an external interest index; retrospectively rated policies depend on the individual insured's loss experience.

sults. Some actuaries believe that their time is too valuable to be spent re-examining their past analyses.

In truth, efficient examination of past results is a requisite for accurate prospective pricing. The source of earnings exhibits enable the actuary to quantify the contribution of each earnings factor to changes in profitability and to differentiate between estimation errors and process errors within the earnings factors. This "policy postmortem" may reveal biases in earnings factors or unstable pricing procedures.

Actuarial Productivity and Alice's Rabbit

Practicing actuaries are busy, busier than Alice's White Rabbit. These busy actuaries are forever computing things, crunching numbers, forming exhibits. There is never time to review previous work, since current tasks are pressing.

All too often, actuaries are computing numbers that do not get used, because they do not accurately reflect the values that they purport to measure. The busy actuaries do not realize this, because they do not have time to evaluate the accuracy of their work.

This is the actuary's destiny: the incessant computation of complex exhibits that bewilder the audience and sometimes entrap even the actuary, so that when errors creep in and lead the results astray, no one can distinguish right from wrong.

Source of earnings analysis is crucial to good actuarial work. Source of earnings analysis asks whether the assumptions are borne out by actual results. Some assumptions, like trend factors, development factors, credibility factors, seem trivial. One wonders: "How can one get these factors wrong?" But as actuarial procedures get more sophisticated, the work on trend factors, development factors, and credibility factors may lead to erroneous results, unbeknownst to the actuaries. Source of earnings analysis enables the practicing actuary to examine the accuracy of the efforts.

Other assumptions are more elusive. The pricing actuary's rate indications rely on investment income assumptions, persistency patterns, acquisition cost assumptions, and loss discount rates. Sometimes the assumptions are explicitly worked into the underwriting profit margin or the underwriting expense ratio; sometimes the assumptions are implicit in the actuary's target loss ratio or target combined ratio. Year after year these implicit assumptions are repeated in the rate reviews. Rarely—if ever—does the actuary examine the validity of the assumptions.[50]

The practicing actuary may object that it is difficult to implement the source of earnings analysis for a particular factor, such as the interest earnings factor or the persistency factor. What the actuary is saying is that it is hard to determine whether the factors being used are correct. Let us rephrase this: if it is hard to determine whether the factors are correct, then it is quite possible that the factors are not correct. If the factors are not correct, then not only has the actuary wasted time computing these factors, but the actuary has wasted more time performing the analyses that rely on these factors. Source of earnings analysis is not an impediment to productivity; it is crucial to making the actuarial time become more productive.

Data Availability

A common complaint about source of earnings analysis is that the data are not available. Regarding retrospectively rated policies, the pricing actuary might say:

> "We don't have the data needed for the analysis of expenses. We don't keep track of our not-taken rates,

[50]Two examples illustrate the questionable paths along which actuarial science has progressed. (i) Casualty actuaries have produced a plethora of financial pricing models, some of which are at odds with financial theory. With no way of checking their validity, rate makers use these models over and over again. (ii) Auto pricing actuaries churn out rate indications in state after state, repeating the cycle year after year. Yet the incessant work may miss the true cost drivers of auto insurance losses; see. Conners and Feldblum [1998]. Source of earnings analysis forces the actuary to rethink the pricing assumptions.

> we don't quantify the solicitation costs for the not-taken business, we don't separately evaluate the first year acquisition costs, and we don't keep records of policy persistency."

One wonders: "If you don't know your expenses, how do you price the business?"

The pricing actuary adds:

> "We don't have the data needed for the analysis of the interest factor. We track loss cash flows, but not premium cash flows. We have incurred loss retros and paid loss retros, and we have all sorts of premium payment patterns; we don't know when the average premium comes in. We don't know when the expenses are paid; all we have are aggregate calendar year figures. We estimate our new money rates, but we don't know how much we actually earn on a given book of business. We don't have the data to quantify the interest we actually earn."

One wonders: "If you don't know your interest earnings, how do you price the business?"

The answer to these questions is straightforward: "We price the business as well as we can, using estimates and guesses when we don't have data."

If an assumption is not material, then it can be ignored in the source of earnings exhibits. An example is maintenance expenses, which are ignored in this paper.

If an assumption is critical to the pricing analysis, such as the acquisition expense assumption or the interest earnings assumption, then it cannot be ignored in the source of earnings analysis. But it cannot be ignored in the original pricing analysis either. The source of earnings analysis tells the actuary the work that must be done. One wonders: "Why do some pricing actuaries credibility weight loss development link ratios that are

computed to three decimal places while they are oblivious of the acquisition expenses or the interest earnings on their book of business?"

Actuarial Rates and Market Prices

Some readers have commented on an earlier draft of this paper that the actuarial indications are not the only problem. An additional problem is that the sales force or the underwriters cut the prices below the indications, either to meet peer company competition or to retain valued customers.

The source of earnings analysis explicitly incorporates such price adjustments. A market decision to change the price is an adjustment to the explicit profit provision.

Illustration: If the underwriting profit margin, after incorporation of investment income, is 8% of premium, and the underwriter grants a 10% premium reduction, the revised explicit profit provision is a negative 2.2% $[= 1 - (1 - 8\%)/(1 - 10\%)]$.[51]

One critique of this analysis is that price-cutting is not done arbitrarily. The 10% rate reduction may have been offered to retain market share or to keep a valued customer who may turn more profitable in subsequent years. The source of earnings analysis does not tell us if the 10% rate reduction is justified.

This is correct. A single policy year perspective is not sufficient. Both pricing and profitability measurement must be done using "lifetime" methods. This does not mean that we must wait several years to measure profitability. On the contrary, source of earnings analysis enables us to examine long-term profitability reasonably quickly, since we can examine whether original pricing assumptions are validated by experience.

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[51]We should adjust for expenses that vary directly with premium. If the variable expense ratio is 15%, expenses are reduced by 1.5% of the original premium, and the new underwriting profit margin is $-2.2\% + 1.5\% \times 10/9 = -0.5\%$.

Ratemaking is prospective; we price next year's business, not last year's business. The pricing actuary succeeds by peering into the future, not by looking back.

Yet our ratemaking procedures are not infallible. Sometimes our methods are defective and our predictions are erroneous. Ever afraid of looking back, we try to outrun the errors.

We cannot outrun our errors. If we never look back, we never know the causes of our errors. We never learn if a variance of actual from expected results from random loss fluctuations or from improper ratemaking assumptions.

Our actuarial expertise is built on our past efforts. By examining our past efforts, we strengthen our current work.

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APPENDIX

SOURCE OF EARNINGS ILLUSTRATION

Prepared by

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INTRODUCTION

This appendix focuses on three aspects of the source of earnings exhibits.

- It presents the source of earnings exhibits in sufficient detail that the practicing actuary can implement the procedure.
- It outlines the deferral and amortization of acquisition costs over the life of the business, in contrast with the standard GAAP amortization for property-casualty contracts over one year.
- It shows the effects of renewal rates in the book of business.

ASSUMPTIONS

The model assumptions are summarized in Table 9, Table 10 and Table 11.

- All policies have January 1 effective dates.
- At the end of each year, some policies lapse and some policies renew.
- Acquisition costs are amortized over the lifetime of the policies. The profitability of the business depends on the acquisition costs. The income reported in each accounting period depends on the amortization schedule for these costs.
- Loss costs are higher on new business than on renewal business, but they do not vary by renewal year.

TABLE 9

NEW BUSINESS ASSUMPTIONS

Time	Premium	Expenses		Loss Adj. Expenses	Losses
		Acq. + UW	Not-Taken		
0					
1	1500	−200	−50		
2				−12.50	−100
3				−18.75	−150
4				−31.25	−250
5				−43.75	−350
6				−18.75	−150

TABLE 10

RENEWAL BUSINESS ASSUMPTIONS

Time	Premium	Expenses		Loss Adj. Expenses	Losses
		Acq. + UW	Not-Taken		
+1	1500	−50	0		
+2				−10.00	−80
+3				−15.00	−120
+4				−25.00	−200
+5				−35.00	−280
+6				−15.00	−120

Investment income is 8% of the assets required at the beginning of the year. The required assets are the discounted value of the reserves at year-end, using the investment yield of 8% as the discount rate.

Some of the policies lapse each year. The lapse rate assumptions are summarized in Table 11. The lapse rate times the in-force number of policies is the number of policies that leave the cohort at the end of the year.

Illustration: The cohort contains 100 policies in year one, with lapse rates of 1/10, 1/9, 1/8, 1/7, and 1/1 in years 1 through

TABLE 11

LAPSE RATE ASSUMPTIONS

Time	1	2	3	4	5+
Lapse rate	1/10	1/9	1/8	1/7	1

5. Ten policies lapse at the end of year 1, and 90 continue into year 2. Ten more policies lapse at the end of year 2, and 80 continue into year 3. At the end of year 5, the remaining 60 policies all lapse. The abbreviated amortization schedule simplifies the exhibits in this appendix. In practice, a 15 or 20 year amortization schedule would be used.

DEFERRED ACQUISITION COSTS (DAC)

The deferrable first-year acquisition costs are 16.6% of the first-year premium. We amortize these costs over the five year expected lifetime of the cohort of policies.

The Expected DAC Schedule

The illustration in the text of this paper amortizes the acquisition costs over a fixed number of years with a 0% valuation rate. The actual GAAP amortization schedule for deferred policy acquisition costs differentiates between FAS 60 policies and FAS 97 policies.

- FAS 60: Deferrable expenses for long-duration contracts are amortized at a constant percentage of premium income.
- FAS 97: Deferrable expenses for universal life type contracts are amortized against gross profits (see below). The amount amortized each year is a constant percentage of book profits.

Two concepts underlie the DAC schedule.

1. The amortization percentage equals the ratio of the present value of deferrable expenses to the present value of the amortizing stream.

2. The amortization schedule takes into account the time value of money by considering the present value of the expenses written off each year.

For these exhibits, the DAC valuation rate, or the discount rate for computing the present value of book profits and deferrable expenses, is the 8% investment yield. Book profits are premium plus investment income less expenses, paid losses, and the increase in loss reserves.

Mechanics of The DAC Schedule For One Policy Year

We determine the DAC schedule for one policy year. The DAC schedule for a book of business is the sum of the DAC schedules over all policy years.

First, we determine which expenses are deferrable and separate them from other expenses. We then project book profits for each policy year.

Next, we compute the present value of book profits (PVBP) and the present value of deferrable expenses (PVDE). The ratio PVDE/PVBP = k is the percentage of book profits that we use to amortize the DAC in each year. A k value larger than one implies that book profits are not sufficient to pay for the deferred acquisition costs.

The DAC amortization proceeds in three steps.

1. Deferrable expenses in the current year are added to the DAC balance at the end of the previous year.

2. The new DAC balance is accumulated for interest for one year.

3. The accumulated DAC balance is reduced by the product of k and the book profits for the year.

Algebraically,

$$\mathrm{DAC}_t = \{\mathrm{DAC}_{t-1} + \mathrm{DE}_t\} \times (1 + r) - k \times \mathrm{BP}_t \quad \text{for } t = 1, 2, 3, \ldots,$$

where

DAC_t is the deferred acquisition cost asset balance at the end of year t,

DE_t is the deferrable expenses in year t,

BP_t is the book profit for year t.

The DAC balance at year zero is defined to be zero.

THE INCOME STATEMENT

The income statement has two components: book profits and the charge due to the amortization of the DAC.

1. Book profits equal premium plus investment income less expenses, paid losses, and the increase in nominal reserves.
2. The charge due to amortization of the DAC is the difference in the DAC balance at two adjacent valuation dates. Table 12 shows the income statement, along with symbols that we use further below.

SOURCES OF EARNINGS

We track five sources of earnings: premium, investment income, expenses, incurred losses, and persistency.[52] A reduction in the persistency rate, or a higher than expected lapse rate, reduces the profitability of the business by forcing the initial acquisition costs to be spread over a smaller number of policies

[52]The text of the paper does not include premiums as a source of earnings, since most of the illustrations in the text do not include variances in the lapse rate.

TABLE 12

INCOME STATEMENT

	Income Statement Item	Symbols
Profit =	Premium +	Premium(t) +
	Investment Income –	Investment Income(t) –
	Acquisition & UW expenses –	Expenses Acq & UW(t) –
	Not-taken costs –	Expenses NT(t) –
	Loss Adjustment expenses –	Expenses Loss Adj(t) –
	Losses –	Paid Losses(t) –
	Change in Loss Reserve –	[Nominal Reserve(t) – Nominal Reserve($t-1$)] –
	Amortization of DAC	[DAC($t-1$) – DAC(t)]

TABLE 13

ANALYSIS OF SOURCES

		Variation in …
Actual profit(t) =	Expected profit(t) +	
	[actual premium – expected premium] +	Premium
	[actual investment income – expected investment income] +	Investment income
	[actual expenses – expected expenses] +	Expenses
	[actual losses – expected losses] +	Losses
	[actual change in reserves – expected change in reserves] +	Change in Reserves
	[actual DAC amortization – expected DAC amortization]	DAC amortization

or policy years. This is true even if other pricing assumptions remain valid.

The source of earnings exhibits measure the deviation between actual results and initial expectations, as shown in Table 13.

We divide the variation between expected and actual results into two components.

1. The variation between expected and "accumulated past experience"
2. The variation between "accumulated past experience" and actual results.

We focus on two aspects of the source of earnings analysis:

1. The revision of the pricing assumptions for future years based on information gathered up to now.
2. The division of the variances into those stemming from past year events and those arising from current year events.

TRACKING ACTUAL EXPERIENCE

We record experience as it emerges and adjust the DAC schedule based on the new information.

At inception of the cohort, we project expected results for all future years based on the pricing assumptions. This benchmark projection does not change as actual experience comes in.

Analysis of sources of earnings is a continuous process with the following steps:

1. Projection of results for the current year, taking into account all past events.
2. Analysis of deviations between the initial benchmark projection and the projection from step (1). We call these deviations "variation due to past accumulated experience."
3. Capturing actual experience over the current year.
4. Recalculating the DAC schedule.
5. Analysis of deviations between the projection from step (1) and the actual results. These deviations are called "variation due to current year experience."

As a final step, we project the results for the upcoming valuation date, incorporating all available information from accumulated past experience and any new estimates for future years. This projection is the best estimate of actual experience over the next valuation period. Since this projection incorporates more information, it may differ substantially from the pricing benchmark. These deviations are "deviations due to past accumulated experience."

As the new year's experience emerges, there may be additional "deviations stemming from current experience." These are deviations between the projected experience at the beginning of the year and the actual experience that emerges.

We separate these two sources of deviation to better understand their causes. The sum of the two sets of deviations gives the total deviation between the pricing benchmark and the actual results.

The deviations show the dollar differences between pricing assumptions and actual experience. Analysis of the deviations enables the pricing actuary to refine the ratemaking procedure and the pricing assumptions.

The recalculation of the DAC schedule is the most complex part of the analysis. The DAC is amortized in proportion to book profits in each year. As actual experience emerges, the book profits change, and the percentage of book profits used to amortize the DAC changes as well.

At each valuation date, we recalculate the remaining DAC schedule. The calculation for the DAC ratio is the same as in the first year except that we have a non-zero previous DAC balance. The DAC ratio is equal to the ratio of:

1. the previous DAC balance plus present value of remaining deferrable expenses, to
2. the present value of remaining book profits.

TABLE 14

INITIAL DAC SCHEDULE AND BOOK PROFITS

Time	Book Profit	DAC Balance
1	88.9	120.2
2	81.5	75.0
3	66.7	36.2
4	44.4	9.2
5	14.8	0.0
6	0.0	0.0

TABLE 15

REVISED BOOK PROFITS

Time	Book Profit	DAC Balance
1	88.9	120.2
2	81.5	75.0
3	56.7	
4	44.4	
5	14.8	
6	0.0	

Illustration: The book of business has the stream of book profits and the DAC schedule shown in Table 14. The first two periods reflect actual results. The remaining periods are projected results based on all information available at the end of period two.

The expected book profit in period three is 66.7. The expected DAC balance at the end of period three is 36.2. There are no additional deferrable expenses in period 3 through 6.

Adverse loss experience in period 3 alters the book profit from 66.7 to 56.7, as shown in Table 15.

The new DAC ratio equals 75.0/102.31 = 73.31%

TABLE 16

REVISED DAC SCHEDULE

Time	Book Profit	DAC Balance
1	88.9	120.2
2	81.5	75.0
3	56.7	39.4
4	44.4	10.0
5	14.8	0.0
6	0.0	0.0

- The numerator is the sum of the previous DAC balance of 75.0 and the present value of remaining deferrable expenses, which are zero.
- The denominator is the present value of remaining book profits (i.e., 56.7, 44.4, 14.8). The present values are computed at an 8% discount rate.

The DAC balance at the end of period three equals $75.0 \times 1.08 - 73.31\% \times 56.7 = 39.43$. The remaining amortization schedule is shown in Table 16. The depressed book profits in period 3 increases the DAC ratio from the original 67.25% to 73.31% and changes all remaining values.

CHANGES IN INVESTMENT INCOME AND INCURRED LOSSES

We track the evolution of a hypothetical example with exhibits and commentary. The initial assumptions are the same as those described above. Table 17 shows the pricing actuary's projection for the block of business.

Table 9, Table 10 and Table 11 show the pricing assumptions for a cohort of business. Premium, expense, and expected loss ratio assumptions are provided by the pricing actuary. Table 17 shows the new business plus four renewal years. To simplify the

TABLE 17

PRICING EXPECTATIONS FOR BLOCK OF BUSINESS

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−100.0	697.5	630.0	−36.1	593.9
3	1,200.0	169.8	−40.0	0.0	−27.8	−222.0	470.3	609.8	−34.8	575.1
4	1,050.0	181.3	−35.0	0.0	−52.8	−422.0	155.3	566.3	−26.3	540.1
5	900.0	164.5	−30.0	0.0	−85.3	−682.0	−227.3	494.5	−9.6	484.9
6	0.0	106.7	0.0	0.0	−86.8	−694.0	−780.8	106.7	−6.7	100.0
7	0.0	61.3	0.0	0.0	−68.0	−544.0	−612.0	61.3	−3.9	57.4
8	0.0	27.0	0.0	0.0	−51.5	−412.0	−463.5	27.0	−1.8	25.2
9	0.0	6.0	0.0	0.0	−31.5	−252.0	−283.5	6.0	−0.4	5.6
10	0.0	0.0	0.0	0.0	−9.0	−72.0	−81.0	0.0	0.0	0.0

TABLE 18

PREMIUM CALCULATION

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Losses
0						
1	1500					1500.0
2		1500				1350.0
3			1500			1200.0
4				1500		1050.0
5					1500	900.0
6						0.0
7						0.0
8						0.0
9						0.0
10						0.0
Persistency factors (based on lapse assumptions)						
	1	0.9	0.8	0.7	0.6	

illustration, we assume that all remaining policies lapse at the end of the fifth year. Years 6 through 10 show the run-off of the remaining reserves. In practice, the table would show policy renewals and run-off of reserves until the figures were not material. For a book of workers compensation business, this would be about 30 years.

Before proceeding with the analysis of deviations, we document the procedures used to create the projection for the block of business. The projection includes the initial year of production plus four years of renewals. Of the original number of policies, only a fraction renew into the first year. Similarly, of those policies in-force during the second calendar year, only a fraction renew into the third year. Table 18 below shows the premium per policy that is collected for the first year of production and each renewal year. The last column of Table 18 shows the total premium collected for each calendar year. The bottom row shows the fraction of the *original* polices that are

TABLE 19

TOTAL LOSS CALCULATION

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Losses
0						
1	0					0.0
2	−100	0				−100.0
3	−150	−80	0			−222.0
4	−250	−120	−80	0		−422.0
5	−350	−200	−120	−80	0	−682.0
6	−150	−280	−200	−120	−80	−694.0
7		−120	−280	−200	−120	−544.0
8			−120	−280	−200	−412.0
9				−120	−280	−252.0
10					−120	−72.0
Persistency factors (based on lapse assumptions)						
	1	0.9	0.8	0.7	0.6	

in-force through the various renewal years. The total premium is equal to the sum of each row times the appropriate persistency factor. For example, the total premium at time 3 equals $1 \times 0 + 0.9 \times 0 + 0.8 \times 1500 + 0.7 \times 0 + 0.6 \times 0 = 1{,}200$.

The same procedure is applied to expenses, loss adjustment expenses, and losses. Table 19 shows the total losses, in each calendar year, for this block of business. The losses shown is the first 5 columns are on a per policy basis. For the first year, the entries come from Table 9. For the renewal years they come from Table 10. The total loss of −682 at time 5 is equal to

$$(-350) \times 1 + (-200) \times 0.9 + (-120) \times 0.8 + (-80) \times 0.7 + 0 \times 0.6 = -682.$$

The calculations necessary to obtain the total nominal reserves are more complex. The total nominal reserve is equal to the to-

TABLE 20

TOTAL LOSS RESERVE

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Reserve
0						
1	1000					1000.0
2	900	720				1620.0
3	750	648	640			2038.0
4	500	540	576	560		2176.0
5	150	360	480	504	480	1974.0
6	0	108	320	420	432	1280.0
7	0	0	96	280	360	736.0
8	0	0	0	84	240	324.0
9	0	0	0	0	72	72.0
10	0	0	0	0	0	0.0
Persistency factors (based on lapse assumptions)						
	1	0.9	0.8	0.7	0.6	

tal loss reserve plus the total loss adjustment expense reserve. To calculate the total loss reserve we first compute the nominal reserves for the first year of production and each renewal year. Table 20 shows the nominal reserves for each year and the grand total.

For example, the nominal reserve of 750 for the first year (see the second column of Table 20) at time 3 is equal to the persistency factor for the first year times the sum of losses (from Table 19) at time 4, 5, 6, and so forth. That is,

$$750 = 1 \times (250 + 350 + 150).$$

The nominal reserve for the second renewal year at time 5 is equal to

$$480 = 0.8 \times (200 + 280 + 120).$$

The total loss reserve for the book of business is the sum of the rows. For instance, at time 7 the total loss reserve equals $96 + 280 + 360 = 736$.

TABLE 21

TOTAL LOSS ADJUSTMENT EXPENSES

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Adj Exp
0						
1	0.0					0.0
2	−12.5	0.0				−12.5
3	−18.8	−10.0	0.0			−27.8
4	−31.3	−15.0	−10.0	0.0		−52.8
5	−43.8	−25.0	−15.0	−10.0	0.0	−85.3
6	−18.8	−35.0	−25.0	−15.0	−10.0	−86.8
7		−15.0	−35.0	−25.0	−15.0	−68.0
8			−15.0	−35.0	−25.0	−51.5
9				−15.0	−35.0	−31.5
10					−15.0	−9.0
Persistency factors (based on lapse assumptions)						
	1	0.9	0.8	0.7	0.6	

The second component of the nominal reserve is the loss adjustment expenses reserve. Table 21 and Table 22 show the derivation of the loss adjustment expense reserve.

The total loss reserve (Table 20) plus the total loss adjustment expense reserve (Table 22) equals the total nominal reserve. Table 23 shows the nominal reserve and the change in the reserve. This change in nominal reserve is also shown in Table 17.

We now begin the analysis. First year expenses are assumed to be 200/1,500 = 13.3% of premium. Solicitation costs for not-taken business are 25% [= 50/200] of first year expenses.

Loss adjustment expenses are assumed to be 12.5% of paid losses. Since the payout schedule for loss adjustment expenses differs between allocated and unallocated expenses, a more refined schedule would be used in practice.

For simplicity, we assume that the loss cost trend is 0% per annum and that no premium changes are expected over the five-year span of the table. In practice, the appropriate trend rates

TABLE 22

TOTAL LOSS ADJUSTMENT EXPENSE RESERVE

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Adj. Reserve
0						
1	125					125.0
2	113	90				202.5
3	94	81	80			254.8
4	63	68	72	70		272.0
5	19	45	60	63	60	246.8
6	0	14	40	53	54	160.0
7	0	0	12	35	45	92.0
8	0	0	0	11	30	40.5
9	0	0	0	0	9	9.0
10	0	0	0	0	0	0.0
Persistency factors (based on lapse assumptions)						
	1	0.9	0.8	0.7	0.6	

for premiums, losses, and expenses should be included. For the heuristic purposes of this illustration, the simplified model shows the workings of the exhibits without excessive refinements.

The nominal reserves are the sum of future loss and loss adjustment expense payments on this block of policies. The expected loss and LAE ratio is 1,125/1,500 = 75%. Since the previous year reserve is zero, the change in the reserve equals the reserve at the end of the year.

Investment income equals the investment yield of 8% times the required assets at the start of the year. Required assets are defined as the discounted value, at the investment rate of return, of the year-end nominal reserves. For year 1, investment income equals $83.3 = 8\% \times 1{,}125/1.08$.[53]

[53]We use the present value of the year end reserve to illustrate the standard life actuarial use of these exhibits. The traditional property-casualty perspective would use the nominal value of the year end reserve.

TABLE 23

TOTAL NOMINAL RESERVE AND THE CHANGE IN RESERVE

Time	Total Nominal Reserve	Change in Reserve
0		
1	1125.0	1125.0
2	1822.5	697.5
3	2292.8	470.3
4	2448.0	155.3
5	2220.8	−227.3
6	1440.0	−780.8
7	828.0	−612.0
8	364.5	−463.5
9	81.0	−283.5
10	0.0	−81.0

Book profit is equal to premium plus investment income less expenses, paid losses, and the change in nominal reserve. For year 1,

$$208.3 = 1{,}500 + 83.3 - 200 - 50 - 1{,}125.$$

At inception of the cohort of policies, the DAC ratio is the present value of deferrable expenses divided by the present value of all future book profits.

- The present value of the deferrable expenses is 250.
- At inception, the projected book profits are 208.3, 75.0, 62.5, 41.7, and 12.5. Their present value at an 8% investment yield is 345.9.
- The DAC ratio = 72.27% = 250/345.9.

The DAC balance accumulated to the end of year one equals $250 \times 1.08 = 270$. The DAC expenses amortized in year 1 are $72.27\% \times 208.3 = 150.5$. The DAC balance at the end of year

one equals

$$250 \times (1 + 8\%) - 72.27\% \times 208.3 = 119.5.$$

Since the previous DAC balance is zero, the change in DAC equals 119.5. The net profit for year one equals the sum of the book profit and the change in the DAC balance

$$327.8 = 208.3 + 119.5.$$

Years 2 through 5 show the combination of new and renewal business and the use of changes in the nominal reserves. We document the entries for year 3. The first year premium is 1,500, and the lapse rates for year one and two are 1/10 and 1/9. We expect $(1 - 1/10) \times (1 - 1/9) = 80\%$ of the policyholders to renew into year three. The expected premium in year 3 is

$$1{,}200 = 1{,}500 \times 80\%.$$

Underwriting and acquisition costs are $40 = 50 \times 80\%$. The loss adjustment expenses of 27.8 stem from policies written in years 1 and 2 (see Table 21 time 3 row):

$$27.8 = 18.8 \times 1 + 10 \times (1 - 1/10).$$

The first term on the right hand side reflects the loss adjustment expenses from the first year of writings and the second term reflects the loss adjustment expenses from the first renewal year.

Similarly, the losses of 222 in row 3 are the sum of paid losses from two underwriting years (see Table 19 time 3 row):

$$222 = 150 \times 1 + 80 \times (1 - 1/10).$$

The nominal reserve at the end of year three is the sum of all future loss and loss adjustment expense payments from the first three years of writings. Table 20, Table 22, and Table 23 show

the calculation of the total nominal reserve

$$\begin{aligned} 2{,}292.8 = &\{250 + 31.25 + 350 + 43.75 + 150 + 18.75\} \\ &\times 1 + \{120 + 15 + 200 + 25 + 280 + 35 + 120 + 15\} \\ &\times (1 - 1/10) + \{80 + 10 + 120 + 15 + 200 + 25 + 280 \\ &\qquad + 35 + 120 + 15\} \\ &\times (1 - 1/10) \times (1 - 1/9). \end{aligned}$$

The nominal reserve at the end of year two of 1,822.5 is calculated in the same fashion. The change in the nominal reserve at the end of year three is equal to

$$470.3 = 2{,}292.8 - 1{,}822.5.$$

The investment income of 169.8 equals $8\% \times 2{,}292.8/1.08$. The book profit is

$$609.7 = 1{,}200 + 169.8 - 40 - 27.8 - 222 - 470.3.$$

The DAC balance at the end of year two equals

$$\begin{aligned} 83.3 = &\{119.4 \times 1.08 - 72.27\%75.0\} \\ &+ \{45 \times 1.08 - 7.211\%^{54} \times 555.0\}. \end{aligned}$$

The DAC balance at the end of year three equals

$$\begin{aligned} 48.6 = &\{74.8 \times 1.08 - 72.27\% \times 62.5\} \\ &+ \{8.6 \times 1.08 - 7.211\% \times 54.0\} \\ &+ \{40 \times 1.08 - 7.211\% \times 493.3\}. \end{aligned}$$

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[54]For each renewal year we calculate the appropriate DAC ratio. Since all of the renewal years are identical, up to a proportionality factor, the DAC ratios are equal for all the years. The calculation of the renewal DAC ratio is performed as follows:

- The present value of deferrable renewal expenses for the first renewal year is 45.
- The projected book profits for the first renewal year are 555.0, 54.0, 45.0, 30.0, and 9.0. Their present value at a discount rate of 8% is 624.1.
- The DAC ratio, for renewal years, equals 7.211% = 45/624.1.

The entries of 45 and 555.0 in the second summand of the equation above represent the deferrable expenses and the book profit for the first renewal year at time two.

Here the entries of 8.6 and 54.0 in the second summand represent the DAC balance at time 2 and the book profit at time 3 for the first renewal year. In the third summand, the entries of 40 and 493.3 are the deferrable expenses and book profit, respectively, for the second renewal year.

The change in the DAC balance is $-34.7 = 48.6 - 83.3$. The profit for year three equals the book profit of 609.8 plus the change in DAC balance of -34.7; hence, the profit is 575.1.

At the inception of the block of business we project the results for the upcoming year (year one). In this illustration, we assume there is no new information between the pricing of the block and the actual issuing of policies.[55] Table 24 presents the projection based on past accumulated experience.

Over the course of the year we tabulate actual experience. This illustration assumes that actual first year experience exactly matches the initial projections, and all variations are zero (see Table 25, Table 26 and Table 27).

As the final step of the first evaluation, we project the results for the upcoming year, taking into consideration all available information. Table 28 shows the projections for year two.

In year two, actual results do not exactly match expectations (see Table 29). The pricing assumptions project year two loss payments of 100 units; actual year two loss payments are 120 units. This change necessitates a recalculation of the DAC schedule as well.

The variation due to past accumulated experience is zero (see Table 30) because at the start of year two there are no past variances.

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[55]The illustration assumes all policies are written at the start of the year. In practice, policy year writings are spread over the year. As the first policies are issued, we might learn more about the expected experience and thereby alter the projection.

TABLE 24

PAST ACCUMULATED EXPERIENCE AT START OF YEAR ONE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8

TABLE 25

ACTUAL EXPERIENCE AT END OF YEAR ONE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8

TABLE 26

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE AT END OF YEAR ONE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 27

VARIATION DUE TO CURRENT YEAR EXPERIENCE AT END OF YEAR ONE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 28

PAST ACCUMULATED EXPERIENCE AT YEAR TWO

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−100.0	697.5	630.0	−36.1	593.9

TABLE 29

ACTUAL RESULTS AT END OF YEAR TWO

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−120.0	697.5	610.0	−26.6	583.4

Variation due to current year experience (Table 31) shows an increase in paid losses of 20 units and a variation in the DAC balance of 9.4 units. The variation in paid losses is the 20 unit difference between projected and actual loss payments. The change in the DAC balance arises because the book profits change.

The additional 20 units of paid losses depress the book profits in year 2. This increases the DAC ratio, which is the ratio of the present value of total deferred expenses to the present value of total book profits. Since the projected book profits in subsequent years have not changed, the relative book profits in year 2 decline as a percentage of total book profits. Similarly, the amount of DAC amortized in year 2 declines as a percentage of the total DAC as well as in dollar terms.

The general principle is that a change in book profits in a single year is partially offset by a change in DAC amortization. This principle is not applicable to changes in book profits that affect multiple years, as is true for changes stemming from investment yields or retention rates.

Year three shows no deviations from experience expected at the beginning of the year, though there are deviations stemming from past experience. Table 32 shows the projection at the beginning of the year, taking into account all previous deviations.

The actual experience for year three in Table 33 is identical to the projected experience at the beginning of the year. The variances based on past experience and current experience are shown in Table 34 and Table 35.

The DAC schedule changes from the initial projections because of the additional paid losses in year two. 9.4 units less of DAC are amortized in year two and 4.9 units more of DAC are amortized in year three.

For year four, we assume that actual experience equals the projected experience (see Tables 36 and 37). During year five, three events occur.

TABLE 30

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE AT END OF YEAR TWO

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 31

VARIATION DUE TO CURRENT YEAR EXPERIENCE AT THE END OF YEAR TWO

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	−20.0	0.0	−20.0	9.4	−10.6

TABLE 32

PROJECTED EXPERIENCE AT START OF YEAR THREE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−100.0	697.5	630.0	−36.1	593.9
3	1,200.0	169.8	−40.0	0.0	−27.8	−222.0	470.3	609.8	−39.7	570.1

TABLE 33

ACTUAL EXPERIENCE AT END OF YEAR THREE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−120.0	697.5	610.0	−26.6	583.4
3	1,200.0	169.8	−40.0	0.0	−27.8	−222.0	470.3	609.8	−39.7	570.1

TABLE 34

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE AT END OF YEAR THREE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−4.9	−4.9

TABLE 35

VARIATION DUE TO CURRENT YEAR EXPERIENCE AT END OF YEAR THREE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	−20.0	0.0	−20.0	9.4	−10.6
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 36

PROJECTION BASED ON PAST ACCUMULATED EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−100.0	697.5	630.0	−36.1	593.9
3	1,200.0	169.8	−40.0	0.0	−27.8	−222.0	470.3	609.8	−39.7	570.1
4	1,050.0	181.3	−35.0	0.0	−52.8	−422.0	155.3	566.3	−29.7	536.6
5	900.0	164.5	−30.0	0.0	−85.3	−682.0	−227.3	494.5	−10.7	483.8
6	0.0	130.9	0.0	0.0	−86.8	−744.0	−830.8	130.9	−7.0	123.9
7	0.0	75.3	0.0	0.0	−68.0	−544.0	−612.0	75.3	−4.2	71.0
8	0.0	33.1	0.0	0.0	−51.5	−412.0	−463.5	33.1	−2.0	31.1
9	0.0	7.4	0.0	0.0	−31.5	−252.0	−283.5	7.4	−0.5	6.9
10	0.0	0.0	0.0	0.0	−9.0	−72.0	−81.0	0.0	0.0	0.0

TABLE 37

ACTUAL RESULTS OF OUR BLOCK OF BUSINESS

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−120.0	697.5	610.0	−26.6	583.4
3	1,200.0	169.8	−40.0	0.0	−27.8	−222.0	470.3	609.8	−39.7	570.1
4	1,050.0	181.3	−35.0	0.0	−52.8	−422.0	155.3	566.3	−29.7	536.6
5	900.0	206.4	−30.0	0.0	−85.3	−657.0	−177.3	511.4	−9.8	501.7
6	0.0	130.9	0.0	0.0	−86.8	−744.0	−830.8	130.9	−7.0	123.9
7	0.0	75.3	0.0	0.0	−68.0	−544.0	−612.0	75.3	−4.2	71.0
8	0.0	33.1	0.0	0.0	−51.5	−412.0	−463.5	33.1	−2.0	31.1
9	0.0	7.4	0.0	0.0	−31.5	−252.0	−283.5	7.4	−0.5	6.9
10	0.0	0.0	0.0	0.0	−9.0	−72.0	−81.0	0.0	0.0	0.0

1. Paid losses are less than expected.
2. The investment department reports that invested assets will generate more investment income than had been anticipated for year five and for subsequent years (see Table 38).
3. A revised loss reserve analysis raises the estimate of unpaid losses.

These three events have partially offsetting effects on total profitability. The source of earnings exhibits enable us to tease apart the effects of each event.

Analysis of Table 38 and Table 39 leads to the following conclusions:

1. Changes from projected experience occur only in years 2 and 5 (see Table 39).
2. The column for "Investment Income" has non-zero entries for year 5 in Table 39 (variation stemming from current experience) and for years 6 through 9 in Table 38 (variation stemming from past accumulated experience). The changed investment yield in year 5 causes increased investment income in that year and the four subsequent years.
3. The column "Δ (Nominal Reserve)" in the current year variation table (Table 39) shows the revised reserve estimate of +50 in year 5.
4. The DAC balance changes for year 5 from current experience Table 39 and in years 6 through 9 from past accumulated experience Table 38. Events that change book profits or deferrable expenses change the DAC schedule for the current year and all subsequent years.[56]

[56]To fully separate the effects of the three events, one could attribute the non-zero entries in the DAC balance column to the various sources (premium, investment income, expenses, paid losses, and change in reserves).

TABLE 38

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−4.9	−4.9
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−3.4	−3.4
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−1.1	−1.1
6	0.0	24.2	0.0	0.0	0.0	−50.0	−50.0	24.2	−0.3	23.9
7	0.0	13.9	0.0	0.0	0.0	0.0	0.0	13.9	−0.3	13.6
8	0.0	6.1	0.0	0.0	0.0	0.0	0.0	6.1	−0.2	5.9
9	0.0	1.4	0.0	0.0	0.0	0.0	0.0	1.4	−0.1	1.3
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 39

VARIATION DUE TO CURRENT YEAR EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	−20.0	0.0	−20.0	9.4	−10.6
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	41.9	0.0	0.0	0.0	25.0	50.0	16.9	0.9	17.8
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CHANGES IN RETENTION RATES

The previous example has no variance of actual retention rates from projected retention rates. The following example (Table 40) shows the effects of changes in retention rates.[57]

We start with the same block of business as in the previous example. At inception of the cohort, we assume that 10% of the policies will lapse at the first renewal date. Actual experience at the end of year two shows the following:

Variances occur in several of the columns, as shown in Table 41.

The corresponding variation in premium and first year expenses suggests a change in the lapse rate.

- The year 1 premium is 1,500 and the projected lapse rate at the end of year 1 is 10%, giving the year 2 projected premium of 1,350.
- The actual year 2 premium is 1,335, implying a lapse rate of 11%.

The variance of -15 implies an excess lapse rate of 1% [$= 15/1{,}500$].

The expenses show the same effect. The underwriting and acquisition cost expense ratio in renewal years is $3\frac{1}{3}\%$ of premium.

- For a premium of 1,500, the expenses equal $3\frac{1}{3}\% \times 1{,}500 = 50.0$.
- For a premium of 1,335, the expenses equal $3\frac{1}{3}\% \times 1{,}335 = 44.5$.

The half unit variance (Table 41) in the expense column reflects the 1% excess lapse rate.

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[57] Life actuaries and casualty actuaries use a variety of terms: retention rates or persistency rates for the percentage of policies that renew and lapse rates or termination rates for the percentage of policies that do not renew.

TABLE 40

ACTUAL RESULTS AT END OF SECOND YEAR

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,335.0	134.3	−44.5	0.0	−12.5	−100.0	688.5	623.8	−36.2	587.7

TABLE 41

VARIATION DUE TO CURRENT YEAR EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	−15.0	−0.7	0.5	0.0	0.0	0.0	−9.0	−6.2	−0.1	−6.3

The calculations shown above for premiums and expenses are more complex when applied to investment income and nominal reserves. A lapse rate deviation in one year affects the number of policies in-force for all future years, thereby changing the reserves and the dollars of investment income (see Tables 42 and 43).

Table 45, "variation due to current year experience," shows variations in year four as well.

Both premiums and expenses have similar deviations, suggesting a change in retentions. Table 42 indicates that expected premium collections were 1,038.3 but actual collections were only 1,008.7, for a variance of 29.7 (see Table 45). The benchmark pricing lapse rate for year three is 12.5%. If everyone had renewed, the collected premium would have been 1,038.3/0.875 = 1,186.7, so the excess lapse rate is 29.7/1,186.7 = 2.5%.

The underwriting and acquisition cost expense deviation is 1.0. The projection based on past accumulated experience indicates that expenses should have been 34.6 (see Table 42) for a renewal rate of 87.5%. If everyone had renewed, the expenses would have been 39.5 (= 34.6/0.875). The indicated excess lapse rate is 1.0/39.5 = 2.5%.

The total profit deviation, or the sum of all entries in the last columns of Table 44 and Table 45, is −53.9.

NOTATION AND FORMULAE

The formulae underlying the exhibits in this appendix are listed below. Policy years are denoted as superscripts and calendar years as subscripts. For example,

$$\mathrm{DAC}_{CY}^{PY}$$

represents the DAC balance at the end of calendar year CY for policy year PY.

TABLE 42

PROJECTION BASED ON PAST ACCUMULATED EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	−45.0	0.0	−12.5	−100.0	697.5	630.0	−36.1	593.9
3	1,186.7	168.6	−39.6	0.0	−27.7	−221.2	463.2	603.8	−34.8	568.9
4	1,038.3	179.8	−34.6	0.0	−52.5	−420.1	150.4	560.5	−26.3	534.2
5	864.6	160.5	−28.8	0.0	−84.6	−676.7	−242.6	477.5	−9.7	467.8
6	0.0	103.5	0.0	0.0	−85.5	−684.2	−769.8	103.5	−6.5	97.0
7	0.0	59.1	0.0	0.0	−66.5	−532.0	−598.5	59.1	−3.8	55.4
8	0.0	25.9	0.0	0.0	−49.8	−398.5	−448.3	25.9	−1.7	24.2
9	0.0	5.8	0.0	0.0	−30.3	−242.1	−272.3	5.8	−0.4	5.4
10	0.0	0.0	0.0	0.0	−8.6	−69.2	−77.8	0.0	0.0	0.0

TABLE 43

ACTUAL EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	1,500.0	83.3	−200.0	−50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,335.0	134.3	−44.5	0.0	−12.5	−100.0	688.5	623.8	−36.2	587.7
3	1,186.7	168.6	−39.6	0.0	−27.7	−221.2	463.2	603.8	−34.8	568.9
4	1,008.7	178.5	−33.6	0.0	−52.5	−420.1	132.6	548.3	−26.5	521.9
5	864.6	160.5	−28.8	0.0	−84.6	−676.7	−242.6	477.5	−9.7	467.8
6	0.0	103.5	0.0	0.0	−85.5	−684.2	−769.8	103.5	−6.5	97.0
7	0.0	59.1	0.0	0.0	−66.5	−532.0	−598.5	59.1	−3.8	55.4
8	0.0	25.9	0.0	0.0	−49.8	−398.5	−448.3	25.9	−1.7	24.2
9	0.0	5.8	0.0	0.0	−30.3	−242.1	−272.3	5.8	−0.4	5.4
10	0.0	0.0	0.0	0.0	−8.6	−69.2	−77.8	0.0	0.0	0.0

TABLE 44

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	−13.3	−1.2	0.4	0.0	0.1	0.8	−7.1	−6.1	0.0	−6.1
4	−11.7	−1.6	0.4	0.0	0.2	1.9	−4.8	−5.8	0.0	−5.8
5	−35.4	−4.0	1.2	0.0	0.7	5.3	−15.3	−17.0	−0.1	−17.1
6	0.0	−3.2	0.0	0.0	1.2	9.8	11.0	−3.2	0.2	−3.0
7	0.0	−2.2	0.0	0.0	1.5	12.0	13.5	−2.2	0.1	−2.0
8	0.0	−1.1	0.0	0.0	1.7	13.5	15.2	−1.1	0.1	−1.0
9	0.0	−0.2	0.0	0.0	1.2	9.9	11.2	−0.2	0.0	−0.2
10	0.0	0.0	0.0	0.0	0.4	2.8	3.2	0.0	0.0	0.0

TABLE 45

VARIATION DUE TO CURRENT YEAR EXPERIENCE

Time	Premium	Investment Income	Expenses		Loss Adj. Exp	Losses	Δ (Nominal Reserve)	Book Profit	Δ (DAC Balance)	Profit
			Acq. + UW	Not-taken						
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	−15.0	−0.7	0.5	0.0	0.0	0.0	−9.0	−6.2	−0.1	−6.3
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	−29.7	−1.3	1.0	0.0	0.0	0.0	−17.8	−12.2	−0.2	−12.4
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The total calendar year value is the sum over all policy years. We denote this total by omitting the policy year superscript. For example, the DAC balance at the end of calendar year CY for all policy years is given by

$$\text{DAC}_{CY} = \sum_{PY=1}^{\infty} \text{DAC}_{CY}^{PY}.$$

Premium

For simplicity, premium is assumed to be paid at the beginning of the year, and there are no subsequent audits or retrospective adjustments. Premium_{CY}^{PY} represents the premium collected. If PY does not equal CY, the premium is zero. When $PY = CY$, the premium is an input parameter from the pricing actuary.

The total premium for the book of business depends on the number policies in-force in each year. $\text{Policies In Force}_{CY}$ denotes the number of policies in-force *at the beginning* of calendar year CY. The total premium at the start of a calendar year equals

$$\text{Premium}_{CY} = \text{Premium}_{CY}^{CY} \cdot \text{Policies In Force}_{CY}$$

Expenses

Expenses are paid at the start of the year. Expenses are classified as:

1. Underwriting and acquisition expenses: Expenses Acq & UW_{CY}^{PY}
2. Solicitation costs for not-taken business: Expenses NT_{CY}^{PY}
3. Loss adjustment expenses: Expenses Loss Adj_{CY}^{PY}

Expenses Acq & UW_{CY}^{PY} and Expenses NT_{CY}^{PY} are zero if PY does not equal CY. For simplicity, we have assumed that no losses or loss adjustment expenses are paid until the policy term expires. Expenses Loss Adj_{CY}^{PY} is zero when $PY = CY$ and non-

zero when $CY > PY$ up to a certain point in time. Once all losses are paid, it is again zero. These quantities are per policy and are an input from the pricing actuary.

Expense_{CY}^{PY} is the sum of the three expense categories. The total expenses in calendar year CY is

$$\text{Expense}_{CY} = \sum_{PY=1}^{\infty} \text{Expense}_{CY}^{PY} \cdot \text{Policies In Force}_{PY}.$$

Losses

$\text{Paid Losses}_{CY}^{PY}$ denotes the amount of losses paid per policy for a given policy year PY and calendar year CY. We sum over all policy years and multiply by the number of policies in-force to get the total losses paid in the calendar year

$$\text{Paid Losses}_{CY} = \sum_{PY=1}^{\infty} \text{Paid Losses}_{CY}^{PY} \cdot \text{Policies In Force}_{PY}.$$

Δ *(Nominal Reserve)*

The nominal reserve is the sum of future losses and loss adjustment expenses. For a given policy year PY and calendar year CY the reserve equals

$$\text{Nominal Reserve}_{CY}^{PY} = \sum_{i=CY+1}^{\infty} (\text{Paid Losses}_{i}^{PY} + \text{Expense Re}_{i}^{PY}) \cdot \text{Policies In Force}_{PY}.$$

The total reserve for calendar year CY is the sum over all policy years:

$$\text{Nominal Reserve}_{CY} = \sum_{PY=1}^{\infty} \text{Nominal Reserve}_{CY}^{PY}.$$

The change in the nominal reserve equals

$$\Delta(\text{Nominal Reserve}_{CY}) = \text{Nominal Reserve}_{CY} - \text{Nominal Reserve}_{CY-1}.$$

Investment Income

The investment income is the product of the investable assets at the start of the calendar year and the investment yield. The investable assets at the start of the year is the nominal reserve at the end of the year discounted to the beginning of the year.[58] This assumption is consistent with traditional source of earnings exhibits for permanent life insurance products. For property-casualty products, a more refined calculation based on the loss payment pattern would be used in practice. The investment income is

$$\text{Investment Income}_{CY}^{PY} = \text{Nominal Reserve}_{CY}^{PY} \cdot \text{Discount Factor}_{CY} \cdot \text{Investment Return}_{CY}.$$

The total investment income for calendar year CY is the sum over all policy years

$$\text{Investment Income}_{CY} = \sum_{PY=1}^{\infty} \text{Investment Income}_{CY}^{PY}.$$

Book Profit

The book profit for a given calendar year CY equals

$$\begin{aligned}\text{Book Profit}_{CY} = {} & \text{Premium}_{CY} + \text{Investment Income}_{CY} \\ & + \text{Expense}_{CY} + \text{Paid Losses}_{CY} \\ & - \Delta(\text{Nominal Reserve}_{CY}).\end{aligned}$$

Δ *(DAC Balance)*

The deferred acquisition cost (DAC) balance is calculated for each policy year. DAC_{CY}^{PY} denotes the balance at the end of calen-

[58]For simplicity, we do not estimate the amount of non-investable assets supporting the nominal reserves. For workers compensation, these include premiums receivable, expected audits, accrued retrospective premiums, and deferred tax assets. For most companies, the amounts are material.

dar year CY for policy year PY. The DAC balance for the entire book of business is the sum over all policy years

$$\text{DAC}_{CY} = \sum_{PY=1}^{\infty} \text{DAC}_{CY}^{PY}.$$

The recursive formula for DAC_{PY+i}^{PY} is given by

$$\begin{aligned}\text{DAC}_{PY+i}^{PY} = (&\text{DAC}_{PY+i-1}^{PY} + \text{Deferrable Expense}_{PY+i}^{PY}) \\ &\cdot (1 + \text{Interest Rate}_{PY+i}) - k \cdot \text{Book Profit}_{PY+i}^{PY}\end{aligned}$$

for i greater than or equal to zero. We define $\text{DAC}_{PY-1}^{PY} = 0$. The change in DAC is

$$\Delta(\text{DAC}_{CY}) = \text{DAC}_{CY} - \text{DAC}_{CY-1}.$$

Profit

The net profit for the book of business takes into account the amortization of the deferred acquisition cost asset. It is given by

$$\text{Profit}_{CY}^{PY} = \text{Book Profit}_{CY}^{PY} + \Delta(\text{DAC}_{CY}^{PY})$$

and

$$\text{Profit}_{CY} = \sum_{PY=1}^{\infty} \text{Profit}_{CY}^{PY}.$$