RISK AND RETURN: UNDERWRITING, INVESTMENT AND LEVERAGE

PROBABILITY OF SURPLUS DRAWDOWN AND PRICING FOR UNDERWRITING AND INVESTMENT RISK

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Abstract

The basic components of the risk/return model applicable to insurance consist of underwriting return, investment return and leverage. A pricing approach is presented to deal with underwriting and investment risk, guided by basic risk/return principles, which addresses the policyholder and shareholder perspectives in a consistent manner. A methodology to determine leverage is also presented, but as a distinct and separate element, enabling the pricing approach to be applied either with or without allocation of surplus to lines of business. Since the leverage is also developed within a total risk/return framework, the approach provides a means to integrate what are often disjointed rate and solvency regulatory activities.

Risk is controlled by a focus on the likelihood that total return falls short of the target "fair" return by an amount which results in a specified drawdown of surplus. Thus rate adequacy and solvency are dealt with simultaneously. A shift away from probability of ruin and expected policyholder deficit approaches to solvency and ratings is proposed and explained.

An "Operating Rate of Return" is defined and suggested as the appropriate rate of return measure that should be used for measuring the charge for risk transfer from the policyholder to the company, rather than other measures such as profit margin, return on premium, etc.

1. SUMMARY

Rate of return and *risk* in return represent the dimensions of expectation and uncertainty, respectively. The tradeoffs between them are real and faced by individuals and businesses frequently. The decision to invest involves a choice among alternatives having anticipated variation in both return and risk. Being generally averse to risk, individuals and businesses choose the least risky investment for a given level of anticipated return, or require a greater return when investments are riskier. The investor perspective with respect to risk tends to be one of concern with the degree to which returns might depart (or vary) from the expected level.

The policyholder perspective, as represented by regulators and rating agencies, is typically more concerned with the dimension of risk having to do with the occurrence of extreme and adverse events, and whether the level of capital available is adequate given the probability and magnitude of such events occurring. However, the risk transfer that occurs from the policyholder to the company is governed by much the same risk/return principles as those that govern the relationship between the company and the shareholder. When viewed within the risk/return context, the linkage between the policyholder and shareholder perspectives becomes clear, and the means for determining both fair premiums to the policyholder and fair returns to the shareholder is provided.

In employing its equity and setting prices, insurance company management is making an investment choice among alternative lines of business and investment asset classes based on knowledge of expected returns coincident with the risks associated with those choices. These risks reflect both the shareholder and policyholder perspectives. The assessment of the tradeoff between these risks and returns and the level of surplus either required or available, is guided by the company's desire to achieve a reasonably balanced portfolio of businesses with a controlled risk/return profile for the company in aggregate.

This paper will explain the basic components of the risk/return model applicable to insurance, as comprised of underwriting return, investment return and insurance leverage. It will discuss a pricing approach to deal with underwriting and investment risk (i.e., variability) that addresses the concerns of both the policyholders and the shareholders. A risk charge is shown as a function of underwriting and investment risk, and the sensitivity of price changes to them is demonstrated. Operating return (i.e., return on underwriting and investment of policyholder funds) coupled with the specification of "probability of surplus drawdown" (PSD) is a focal point in this approach.

The PSD is a fundamental aspect of the risk/return relationship that is applicable to both the policyholder and the shareholder. Although consistent with the probability of ruin and expected policyholder deficit concepts, it differs in that its focus is more on the degree to which returns depart from expected levels, rather than simply on the extreme adverse outcomes.

The "operating return–probability of drawdown" method presented in this paper is suggested as a replacement for the return on premium concept by an operating return measure which extends shareholder risk/return principles to the policyholder level. As a consequence, the method demonstrates how risk can be reflected in the pricing mechanism *without* varying the allocation of surplus to individual lines of business, through the focus on operating return. The result is a unified and consistent framework for establishing fair returns that reflects the transfer of risk from the policyholder to the company and from the company to the shareholder.

Importantly, issues of leverage and surplus allocation are removed from the pricing process. The need for surplus is viewed primarily as an overall company issue with respect to financial strength and ratings. The result is a mechanism for establishing prices which recognizes the policyholder and shareholder perspectives centered around their respective risk/return tradeoffs, without requiring that surplus be allocated to lines of business.

Varying leverage ratios by line of business is shown to be an optional risk adjustment step that translates rates of return to a common level, such as a specified cost of capital.

With respect to style and focus, this paper will avoid an overly-detailed and mathematically-oriented presentation in favor of simpler demonstrations focused on the most basic of principles. These principles are essentially:

1. Functionally and mathematically, insurance is composed of underwriting, investment and leverage.
2. Interactions among the policyholder, company and shareholder are governed by the fundamental risk/return relationship, in which higher risk requires higher return and vice versa.
3. The transfer of risk either from the insured to the company or from the company to the shareholder are both essentially investment-like decisions, which involve a charge for this transfer to occur. In the policyholder case, this results in a premium payment to the company; in the case of the company, this results in an expected "payment" to the shareholder via dividends or stock price appreciation (i.e., the cost of capital).
4. The amount and timing of policyholder-related liabilities and cash flows that will eventually be paid are uncertain. The price for the transfer of this underwriting risk from the policyholder to the company must be incorporated into the premium charged when insurance is sold.

These fundamental principles apply broadly to all ratemaking models. Unfortunately, unnecessary confusion exists with respect to the many ratemaking models presented in the literature, for two basic reasons. First, because the relevance of these basic risk/return principles may not be recognized in each of the models, the assumptions and parameters used in them are determined in various ways, causing their output to diverge substantially.

Second, because many of the models differ in construction and output, comparisons to one another are made difficult. It is important to note that the many ratemaking models (such as underwriting profit margin, target total rate of return, insurance capital asset pricing model, discounted cash flow, Myers–Cohn, and internal rate of return, etc.) *are all essentially equivalent*. A single well-constructed total return model, supported by the full complement of balance sheet, income and cash flow statements, and further valued both nominally and on a discounted basis, encompasses them all and will produce identical results when the same input assumptions are used (as discussed in the material in the References).

2. BACKGROUND

2.1. Rate of Return

Rate of return (often referred to more simply as return) reflects the amount of income produced on an investment in relation to the investment itself. This ratio is usually expressed as an annual rate, although the investment period may be more or less than one year. Insurance decisions to invest in underwriting operations, in particular, usually involve a multi-year commitment (e.g., losses may take many years to settle) and the rate of return that results must reflect this timeframe as well. This is much like an investment with a holding period of several years, wherein both the level of investment and return might vary over time, requiring that some form of composite annual percentage rate of return (APR) be calculated.

Insurance companies deploy (i.e., invest) their surplus in either of two essential operating activities—underwriting or investing. Each of these activities carries with it an anticipated rate of return. The amount of insurance written on the one hand and the amount of surplus/capital provided from financing activities on the other, result in an operating leverage that magnifies the underwriting and investment returns in relation to surplus. The following expression provides a simple yet accurate representation of the way that underwriting and investment return, in conjunction with their respective leverage, contribute to total return:

$$R = (Ru)(L/S) + (Ri)(L/S + 1). \qquad (1)$$

Total return on surplus (R) is the sum of the respective products of return and leverage from underwriting and investment. The return on underwriting (Ru) measures the profitability from underwriting operations (absent investment income). The return on underwriting can be measured in various ways, depending on whether the view is historical or prospective, or whether it is relative to calendar or ultimate accident year. The return on investment (Ri) is essentially a yield on total invested assets, which include assets generated from both underwriting liability "float" and surplus.

Each of these returns is magnified by the leverage employed by the company. The underwriting leverage (L/S) is the net liability to surplus ratio. Liabilities consist primarily of loss reserves, but other liabilities must be considered, such as premiums receivable (a negative liability), reinsurance balances payable, taxes, etc. Since invested assets (I) are equal to net liabilities (L) plus surplus (S), $L/S + 1$ in the above expression is equivalent to the ratio of invested assets to surplus, or investment leverage. Viewed in this way, *the total return is seen to be dependent simply on underwriting return, investment return, and insurance leverage*. (It is noted that statutory surplus and GAAP equity differ in their definitions. For purposes of risk transfer pricing and in the context of this paper, surplus is better thought of as a required risk-based "benchmark" amount. This is discussed in [3].)

Underwriting income (after-tax) is expressed as a rate of return (Ru) and can be determined in either of two ways. The first is to use a common finance tool, the internal rate of return (IRR), which is based on the *underwriting cash flows* that evolve over time. The second is to relate *underwriting income* to the *balance sheet investment* that is derived from the same insurance liabilities that produce the underwriting income. This is approximately the ratio of after-tax underwriting income to underwriting float (i.e., primarily loss reserves). Both of these alternatives are demonstrated by way of example in the Appendix, and are discussed in detail in the reference material.

Underwriting return, Ru, is not the same as return on premium. While return on premium may be a useful statistic, a ratio to sales does not capture the dynamics as fully as a return on funds invested statistic does, when the magnitude and time periods of the investment differ widely. Returns on premium are not comparable between short- and long-tailed lines of business, since the magnitude and time commitment of supporting policyholder funds are dramatically different. The underwriting rate of return (Ru) fully reflects this dimension and presents a statistic that is comparable across lines of business.

Investment return is dependent on returns (yields on fixed income investments, stock market dividends and capital gains, etc.) available in financial markets, together with the selection of various asset classes in which investments are made. In the case of fixed income investments, investment return is also affected by the maturity selected (which entails added interest rate risk as well).

Options exist within both underwriting and investment to select lines of business and/or investments that entail varying returns and associated risks. The above formula (1) refers to a single underwriting return and a single investment return when, in reality, there are numerous options within each of them.

2.2. Risk in Return

Risk is a measure of the uncertainty of achieving expected returns (which encompasses the possibility of a complete loss of the investment itself). The most common measure of risk is the standard deviation statistic, which provides a means of quantifying the degree of likely variation of actual return relative to the return expected. The larger the standard deviation, the greater the chance that the actual return will deviate from the expected return (either above or below it).

Underwriting and investment returns both involve a degree of uncertainty (i.e., volatility). The expression below reflects how the standard deviation in total return (σ_R) is affected by the standard deviation in underwriting return (σ_{Ru}) and the standard deviation in investment return (σ_{Ri}). This formula makes use of the square of the standard deviation (known as the variance) for simplicity:

$$\sigma_R^2 = \sigma_{Ru}^2(L/S)^2 + \sigma_{Ri}^2(L/S+1)^2 + 2r(L/S)(L/S+1)\sigma_{Ru}\sigma_{Ri}. \tag{2}$$

Leverage has a similar compounding effect on variability as it does on return. In addition, the interaction (i.e., correlation) between underwriting and investment is a critical component of the total risk, as captured by the last term in (2).

The correlation coefficient (r) measures the degree that underwriting and investment performance move in tandem with each other. Underwriting and investment returns that move together in lock step in the *same* direction, both up or both down, will have a perfect positive correlation ($r = +1$). Underwriting and investment returns that move in exact *opposite* directions, one up and the other down, will have a perfect negative correlation ($r = -1$). When underwriting and investment returns are independent of one another, there is no correlation ($r = 0$). Thus, in terms of total variability, when underwriting and investment move together (positive correlation), risk is greater. Conversely, when underwriting and investment move opposite to one another (negative correlation), risk is less. The same principles apply at a finer level among the lines of business within underwriting and among alternative investments.

In insurance circles, when the topic of a company's surplus requirements is discussed, the term covariance is often used. This is simply another term for describing the interaction among underwriting lines of business and investments, and the effect this may have on the overall need for surplus and the risk to the company as described above (i.e., the benefit of diversification).

It is important to note that of the three basic factors affecting risk and return, leverage stands alone in that it can be controlled directly by management; underwriting and investment, on the other hand, involve given levels of risk which are largely uncontrollable. (This risk can be managed to some degree through diversification.) The selected leverage at which a company chooses to operate has a significant influence on both the level and variability of reported total returns, and is subject to practical regulatory and rating agency constraints.

This process is more complex than can be reviewed here, especially if the correlations among many lines of business and alternative investments were to be considered simultaneously.

2.3. Leverage

The leverage employed by a company is subject to many constraints, including ratings, cost of capital, and most importantly in insurance, the probability of ruin. Insurance, unlike most other businesses, involves selling a product whose costs can only be estimated at the time the product is sold, and whose ultimate value has a significant potential to cause financial loss to an insurer well in excess of premiums charged. Recognizing this financial exposure and the additional limits imposed on leverage by rating agencies and financial markets, insurers have traditionally considered the probability of ruin in determination of surplus requirements. This concept results in the establishment of surplus levels in such a way as to keep to an acceptable minimum probability the chance that surplus will be exhausted by unfavorable loss or other developments. More recently, the concept of expected policyholder deficit (EPD) has been used to further quantify the amount of ruin.

Leverage plays a direct role in the risk/return tradeoff as noted previously, since it simultaneously magnifies both return and risk as shown in formulas (1) and (2). To demonstrate this relationship, it is helpful to express formula (1) differently as follows:

$$R = Ri + (Ru + Ri)(L/S). \tag{3}$$

This is the expression for a straight line, with an intercept of Ri (the return on investment) and a slope of $(Ru + Ri)$. If no insurance were written (i.e., $L/S = 0$), the only return would be on investments, with a return equal to the average yield (Ri). Assuming a consistent level of profitability, as writings and leverage increase, total return increases at a rate of $(Ru + Ri)$. This term has special meaning in that it represents the *operating return* from insurance. Operating return reflects the income from underwriting operations plus the investment income related to the assets generated from underwriting operations (i.e., insurance liability float). It excludes income from investment of surplus, captured in the above formula by the intercept Ri. The meaning and measurement of the underwriting, investment and operating returns is discussed in the reference material and recapped briefly with an example in the Appendix.

Repeating the important point—*leverage simultaneously affects both return (shown by formula 3) and variability in return (shown by formula 2)*. Apart from product or geographic diversification, returns cannot be increased by raising leverage without also increasing variability. Similarly variability cannot be reduced without also reducing returns. Since *insurance uncertainty cannot be eliminated*, some combination of policyholder and/or shareholder pricing mechanisms is needed to deal with this risk transfer.

Predominant drivers of overall variability are: (1) variability in the amount of liabilities, (2) variability in the timing of liability payments, and (3) variability in interest rates. The greater the variability in these three basic drivers, the greater the variability in return. While reinsurance and investment hedges can be used to reduce some of this variability, there will always be a degree of variability remaining which cannot be eliminated, and this should be an important input into the pricing and leverage setting processes.

The following chart (Figure 1) presents key relationships among balance sheet, income and cash flows and the risk transfer activities within the insurance firm. Within this structure the total company is delineated into policyholder versus shareholder related components. Note that the left side of the balance sheet consists of invested assets only. Non-invested assets are portrayed as a negative liability, and included within net liabilities on the right side of the balance sheet.

Several alternatives exist for setting leverage. As noted previously, controlling the probability of ruin has been a traditional approach. More recently the expected policyholder deficit (EPD) has been developed. Controlling the variability in total return, of more interest to the shareholder, is another criterion that is often addressed either by modifying the leverage ratio or by changing the target rate of return.

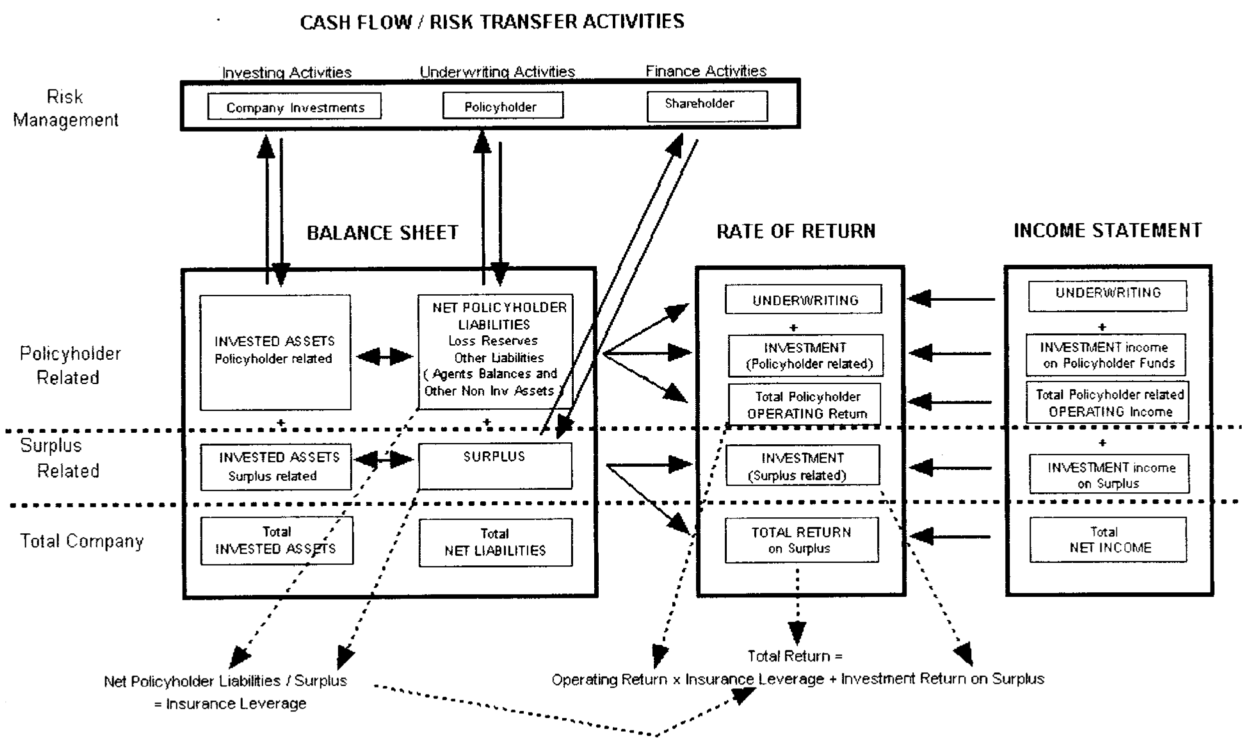
2.4. The Probability of Ruin

The probability of ruin represents the likelihood that the combined effect of variability in liabilities and variability in the timing of liability payments will cause surplus to be exhausted. To keep this probability to an acceptable minimum, surplus can be established at a level which is sufficient to cover the adverse conditions that can occur (e.g., losses larger than expected or payable sooner than anticipated) all but, say, 1% of the time in an individual line of business.

Variability in the amount of loss and variability in the timing of loss payments are most critical in terms of influencing the leverage level and the variability in total return. Variability in loss has an even greater impact, due to a tendency to be

FIGURE 1

KEY BALANCE SHEET, INCOME, CASH FLOW AND RETURN RELATIONSHIPS



skewed, with the possibility of a very large loss (e.g., a natural catastrophe). However, the probability of ruin approach has shortcomings in that it does not typically reflect the impact of taxes and other components of total net income, and may not consider sources of variability other than from losses. A large loss payable shortly after policy issuance is much more serious than is the same loss payable many years later, since, in the latter case, substantial investment income is generated in the meantime. Also, the tax credit generated by losses reduces the out-of-pocket cost to the company. Variability in premium, expense and investment are also potentially significant contributors to overall risk, which should be considered.

Furthermore, it should also be noted that control of probability of ruin does *not* result in a uniform variability in total return. Neither does it reflect the magnitude of policyholder deficit if ruin does occur. Note, for example, that a highly skewed loss distribution may result in a greater policyholder shortfall than would a normal distribution, yet have the same probability of ruin.

2.5. Expected Policyholder Deficit (EPD)

EPD is a broader concept than is the probability of ruin, in that it includes both the frequency and severity of extreme adverse consequence. Whereas the probability of ruin specifies the chance that company surplus may be exhausted, the EPD further estimates how much this amount is likely to be on average. Clearly policyholders and regulators are concerned with both the probability and potential magnitude of loss, should surplus be exhausted. While shareholders are concerned with the probability of ruin, EPD is of little relevance since shareholder loss is limited to the amount of their investment.

The EPD concept has gained prominence in recent years and is being incorporated in some rating agency methodologies. However, this approach will have the same shortcomings as the probability of ruin, if it does not reflect the impact of taxes and other components of total net income.

Of more serious concern, however, is a basic principle of statistics and probability distributions that cautions against use of the "tail," or low probability outcomes in frequency distributions. Most statistical methods rely on the "middle" of the distribution, where the vast majority of the values occur. The probability of ruin and EPD approaches rely on the areas of the data distribution having the least credibility and reliability. While of interest to policyholders, shareholders are instead concerned with how returns might vary from that expected (that is, with the middle of the distribution).

This shareholder perspective is one of risk versus return and is more appropriate within a context of risk transfer pricing. The probability of ruin and EPD, while important from a solvency standpoint, are not as well-suited to this end.

2.6. Variability in Return

Shareholder investments reflect a tradeoff between the level of return required and the uncertainty of such return. Shareholders expect returns commensurate with uncertainty—if returns are more variable, then investors will expect a higher absolute return, all else being equal. This in essence reflects the middle of the distribution of returns about the expected value. In this regard the shareholder perspective inherently embodies more statistical credibility.

Fortunately, the probability of ruin, EPD and variability in return viewpoints are connected. The concept of "probability of surplus drawdown" will be discussed in this regard.

2.7. Value at Risk and Probability of Surplus Drawdown (PSD)

The distribution of total return encompasses all financial components of an insurance company and the variability inherent in them. This is the distribution that is of concern to the shareholders, or investors, who provide capital to support the operations of the company. In fact the traditional probability of ruin and EPD, when expanded to include all sources of underwriting and investment income and taxes, are captured in the tail of this distribution. Ruin occurs when the total rate of return is -100% or worse, with EPD being the average magnitude of such events. Thus the first step in bridging the gap between the policyholder and shareholder measures is the conversion of ruin and EPD to a net income basis, and their expression as a rate of return.

The second step is to view the distribution of returns as a continuum from the expected value downward to the ruin threshold of -100% return. *Economic* surplus drawdown occurs along this continuum when total returns fall below the rate of return that could be achieved on alternative (typically risk-free) investments. Alternatively, this is equivalent to the point at which *operating returns* fall below 0% as shown in (3). This rate of return is most properly defined on an economic basis to reflect the point at which investors lose money in economic terms. Thus the PSD represents the likelihood that an investor will experience an economic loss, when time value is considered. This is a specific case within the more general *value at risk* approach, which deals with a reduction in surplus of any specified amount (i.e., below zero) together with the probability of its occurrence, rather than just simply the single threshold of 0% return at which surplus drawdown occurs.

It is important to note that the points of surplus drawdown and ruin, and their respective probabilities, both lie on the same distribution. Actions which alter the return distribution will simultaneously and similarly improve or worsen both the policyholder and shareholder positions. This is shown more clearly by examining the basic risk/return relationship.

2.8. The Basic Risk/Return Tradeoff

The basic risk/return relationship is shown schematically in Figure 2. As variability in return increases along the x-axis, the return required to compensate for this risk also increases. Beginning at the origin (the point of "no risk, no return"), a risk/return line exists such that the probability of a negative return, or surplus drawdown, is the same at all points along the line. This probability is controlled by increasing or decreasing the slope of this line. A higher return (steeper slope) will reduce the probability of surplus drawdown by moving the distribution at each point on the line farther up and away from negative return territory.

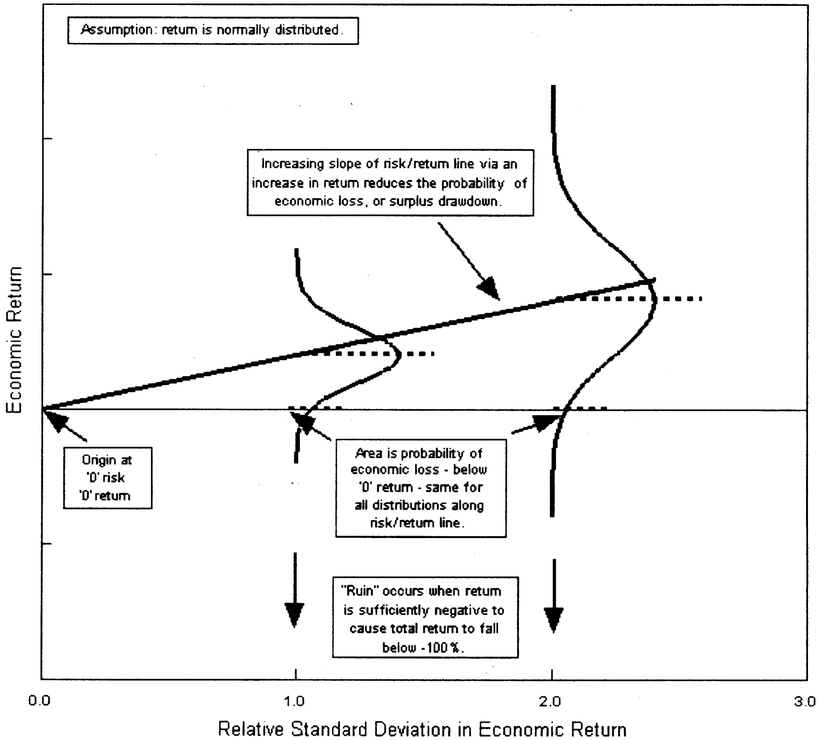
This essential relationship, that increased risk requires an increased return, is at work governing the risk transfer process that takes place between the policyholder and the company and between the company and the shareholder. Referring back to the basic relationship shown in (3), the operating return components, particularly its expected value and variability (i.e., mean and standard deviation) define the essentials of the risk/return relationship between the policyholder and the company. When leverage is applied and the investment of surplus (Ri) is included, the risk/return relationship between the company and the shareholder is established on a total return basis. Consistency in these two risk transfer pricing activities is important in order to simultaneously establish fair policyholder premiums and fair shareholder returns. A focus on operating return, in particular how risk and variability are priced, will be presented first, with total return following.

3. OPERATING RETURN–PRICING FOR RISK AND VARIABILITY

As shown previously, operating return on insurance operations, driven by both its underwriting and investment components and coupled with the magnifying effect of leverage, defines the total risk and return profile of the insurance enterprise. The particular characteristics of a line of business, such as the amount

FIGURE 2

RISK VERSUS RETURN



and variability of its loss payouts, specify its operating return profile with respect to risk and return (i.e., the two dimensions of expected value and variability). This profile has policyholder implications, with respect to risk transfer and pricing, which can be assessed separately from leverage.

3.1. The Policyholder Risk/Return Tradeoff

The traditional shareholder (investor) risk/return perspective is one that reflects the need to provide returns consistent with risk. Greater risk requires greater returns, which must be comparable to other investment opportunities. The essence of the policyholder risk versus return relationship can be viewed similarly as reflected in Figure 3, which portrays variability in operating return and average operating return. Regardless of the underlying underwriting or investment uncertainty, this basic relationship must hold. In fact, for a given PSD, all combinations of loss variability and business tail length are shown here to lie on the same risk/return line. That is to say that all businesses conform to a uniform risk/return relationship, regardless of the variety of characteristics possessed by them.

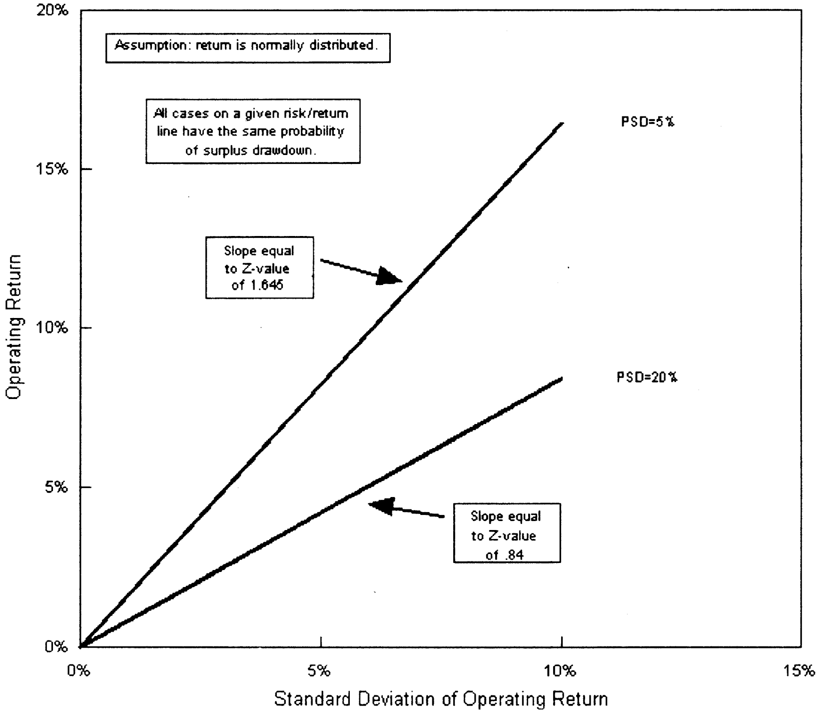
Since losses are assumed here to be normally distributed, each line has a slope equal to the normal distribution "Z-value." This is the number of standard deviations from the mean corresponding to specified probabilities from a normal distribution. For example, a Z-value of 1.645 corresponds to a 5% probability of occurrence (in each tail) from the mean. Thus using the Z-value provides an easy shortcut to determine the necessary operating return required to compensate for risk, with a specified PSD.

In practice loss distributions are typically skewed, and the standard deviation alone does not adequately describe risk. In such cases it is important that the area under the tail within each respective total return distribution be used to measure risk (i.e., the PSD), and in turn be used in the pricing process. The Z-value shortcut based on the standard deviation is not appropriate. While Figure 3 would not appear as a straight line in such cases, the approach remains valid with the downside risk to surplus controlled consistently.

If the operating return above is converted to total return by multiplying by a leverage factor and adding Ri to account for investment yield on surplus, Figure 4 emerges. In this scenario that assumes no interest rate variability, the probability of surplus drawdown is now the probability of a total return below Ri. This is the shareholder view that can be used to provide a comparison to alternative investments, and guidance as to whether rates are

FIGURE 3

POLICYHOLDER OPERATING RISK/RETURN TRADEOFF
(WITH VARYING PSD)



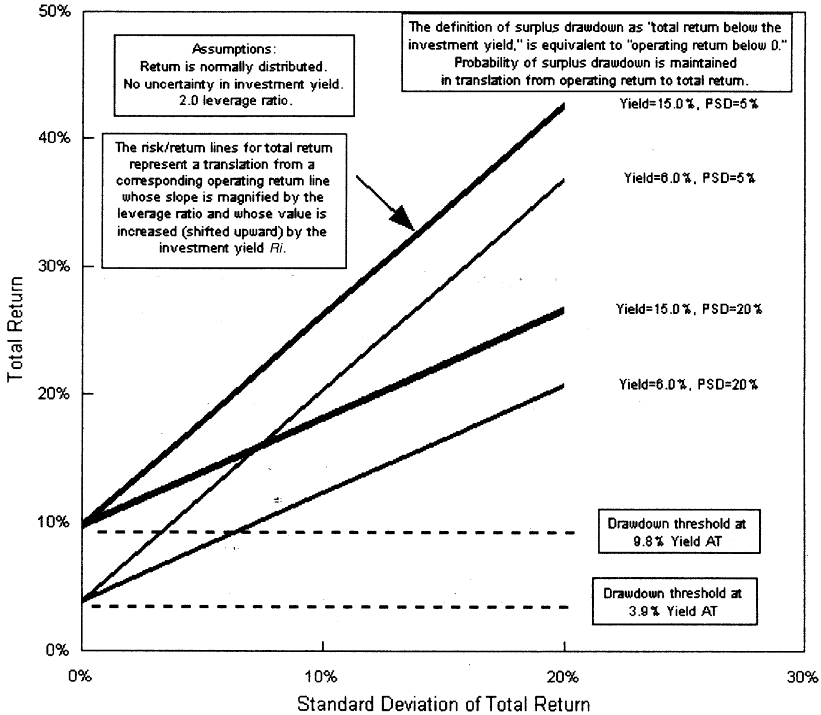
adequate from a shareholder perspective. This will be discussed in more detail later.

In practical terms these steps equate to the use of a constant Sharpe ratio to control risk. The Sharpe ratio, which is calculated by dividing the difference between the total return and the risk-free return by the standard deviation in return, is in effect a Z-value.

It is important to note that the introduction of leverage does not change the probability of drawdown. (This is not true if risky

FIGURE 4

SHAREHOLDER RISK/RETURN TRADEOFF
(WITH VARYING PSD & INVESTMENT YIELD)



investments are assumed.) *Since leverage similarly magnifies both return and risk, increasing leverage simply causes total return to move from lower left to upper right while remaining on the same line. Leverage thus becomes a factor that provides a translation from internal measures of risk-based operating return to total shareholder return, while maintaining a specified probability of surplus drawdown.*

The significance of this characteristic bears amplification, and explains why this risk pricing approach is largely independent of the level of actual company surplus and does not require surplus allocation to lines of business *as long as returns are sufficiently positive*. The premium necessary to generate a total return large enough to keep the downside risk to surplus sufficiently low is the same regardless of the leverage factor utilized, due to the balanced and simultaneous effect leverage has on both risk and return. The *stated* total return (as well as the variability in total return) will of course be higher as leverage increases, but the PSD will remain the same. *Reducing leverage does not improve the joint risk/return profile, and increasing leverage does not worsen it.*

Practically speaking, however, it is much easier to present a rate filing based on a lower rate of return than a higher one, *even if the premium is identical in both cases*. In a total return ratemaking environment, the leverage utilized must be such so as to produce a rate of return within an acceptable range while satisfying other rating criteria. This is one of the considerations in the determination of total company surplus requirements to reflect the concerns of rating agencies and regulators. Furthermore, since premiums often are not sufficient to ensure fair profits, the risk of surplus loss is increased and a greater level of supporting surplus (i.e., lower leverage) is necessary to provide an adequate ruin safety margin.

The primary goals of state regulators, fair premiums and solvency, are simultaneously addressed by this risk transfer pricing process. Fair returns are determined which simultaneously guard against the probability of loss of surplus and ruin. As noted previously, almost any reasonable risk-based level of operating return provides an adequate safety margin, and a very small probability of ruin. *Fair risk-adjusted returns provide the direct connection to solvency and the means by which solvency is ensured.*

3.2. Policyholder Pricing for Underwriting Risk

Operating return is a financial measure which reflects the basic nature of insurance—the fact that it incorporates the activities of underwriting and investment and that it is subject to substantial variability in result. Operating return quantifies the return realized by the insurance company for the transfer of risk from the insured. While some may view insurance simplistically as the spreading of risk from a single policyholder to several policyholders in order to reduce the cost to a more stable per policyholder basis, it is more than this. No matter how large the cohort of policyholders might be, a degree of uncertainty as to the total cost will remain, due to the highly variable and uncertain nature of insurance. A proper price must be determined for this transfer of risk from the insured to the insurer.

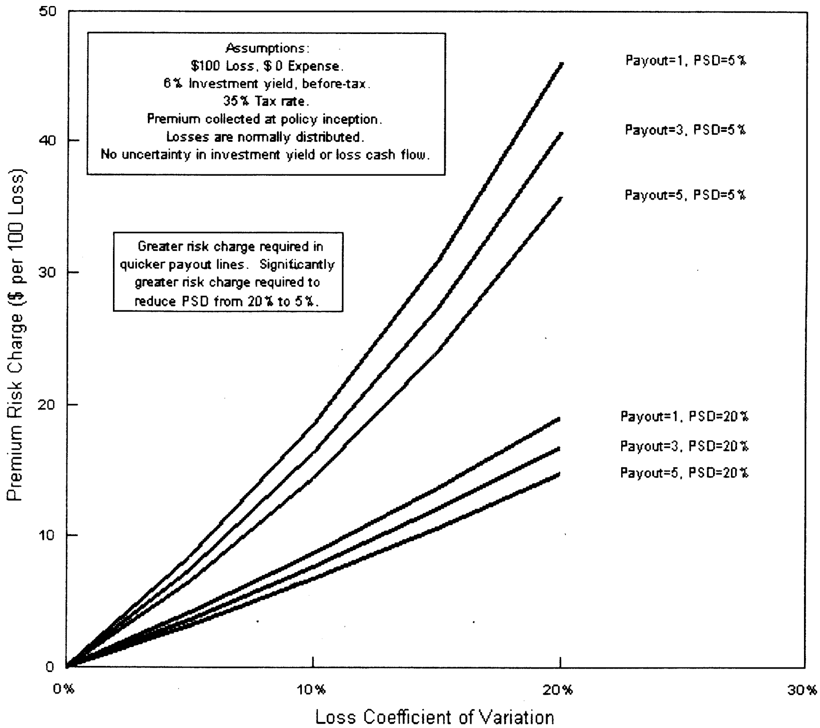
The primary financial drivers which determine the *expected* operating return are the amount and timing of cash flows related to premium, loss, expense and taxes, as well as interest rates. The *variability* in operating return is primarily driven by the variability in loss amount, timing of loss payments and interest rates. These factors must be reflected in the pricing process. The nature of the distribution of operating returns provides such a means, and one by which a degree of objectivity and consistency among lines of business can be maintained by utilizing the basic risk/return relationship.

The probability of surplus drawdown, or negative operating return, can be set at a desired level. Simultaneously the probability of ruin is altered in the same direction. Figure 5 presents the price increase required as the loss variability increases, for lines of business having average loss payouts of one, three, and five years, and for PSD levels of 5% and 20%. Note that the lines for a given drawdown scenario intersect at the origin, since no incremental risk implies no incremental return (in principle). The mathematics for this risk charge are provided in the Appendix.

In this pricing approach, the risk charge is a direct function of loss variability, subject to the specified probability of negative return (i.e., that the charge will prove to be inadequate to cover the risk). How this probability is set should consider both the policyholder and shareholder perspectives.

FIGURE 5

PRICING FOR UNDERWRITING RISK-LOSS VARIABILITY
(WITH VARYING LOSS PAYOUT & PSD)



As noted earlier, a lower operating return (and premium) will bring with it an increased probability of negative return and probability of ruin. In most instances, any reasonable price level and risk charge will have a very small probability of ruin and EPD. Clearly, long run financial strength and solvency cannot be maintained without adequate rates. In other words, adequate rates are *the true* means by which solvency is made secure, at least with respect to current business writings (i.e., excluding other balance sheet risks).

3.3. Policyholder Pricing for Investment Risk

Risks exist in both underwriting and investment. Figure 5 presents the impact of variability in underwriting (incurred loss) only. Investment risks range from a low involving government "risk-free" investments (which experience only relatively modest fluctuations in yield) to higher-risk investments which have a far greater potential to vary, as well as an exposure to loss. A further component of a risk-averse investment strategy would be to match investment maturities with the timing of expected underwriting cash flows. While higher fixed-income investment yields can be achieved by investing at longer maturities, this creates risk should cash flows not emerge as expected.

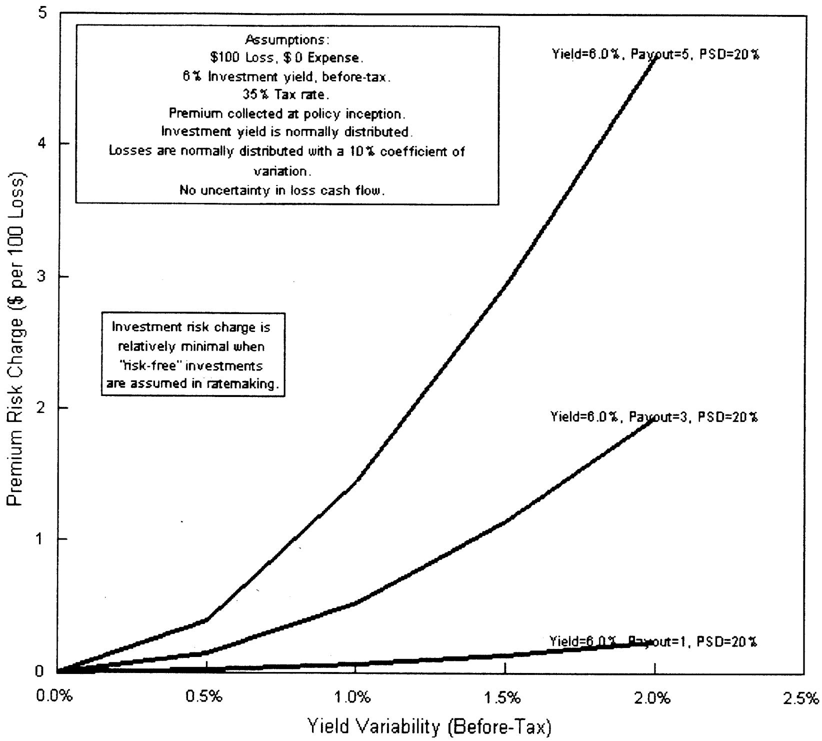
A controversial issue is whether or not insurance prices should be based on a risk-free investment strategy. Should policyholders be credited with risk-free rates or something more in line with the higher-risk investments that insurers are making. If it is the latter, then the increased yield carries with it an increase in risk. The mechanism presented here provides a framework in which the return and risk characteristics of investment can be priced along with those from underwriting.

Figure 6 presents the price increase required as the investment yield variability increases, for lines of business having average loss payouts of one, three, and five years for a PSD of 20%. The variability in yield is very small, as might be expected with risk-free investments. A maturity matching policy is assumed, and the loss variability is assumed to be 10%. Once again note that the lines for a given drawdown scenario intersect at the origin, since no incremental risk implies no incremental return (in principle). The mathematics for this risk charge are also provided in the Appendix.

When risk-free investments are assumed, the risk charge for investment risk is very minor in comparison to that required to cover underwriting risk, since such investments are subject to

FIGURE 6

PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–"MINIMAL" INVESTMENT RISK)

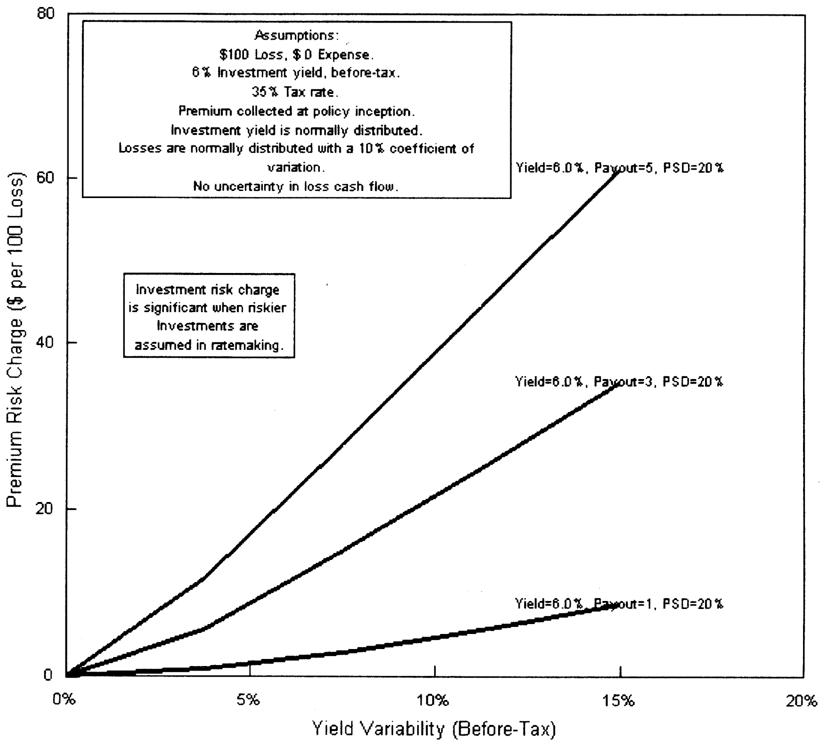


interest rate risk only. However, this picture changes dramatically if higher credit risk investments are assumed.

The charge for higher investment risk becomes substantial, as shown in Figure 7. This presents the additional premium required to reflect increases in investment risk for lines of business having average loss payouts of one, three, and five years with a PSD of 20%, when investment variability is substantial.

FIGURE 7

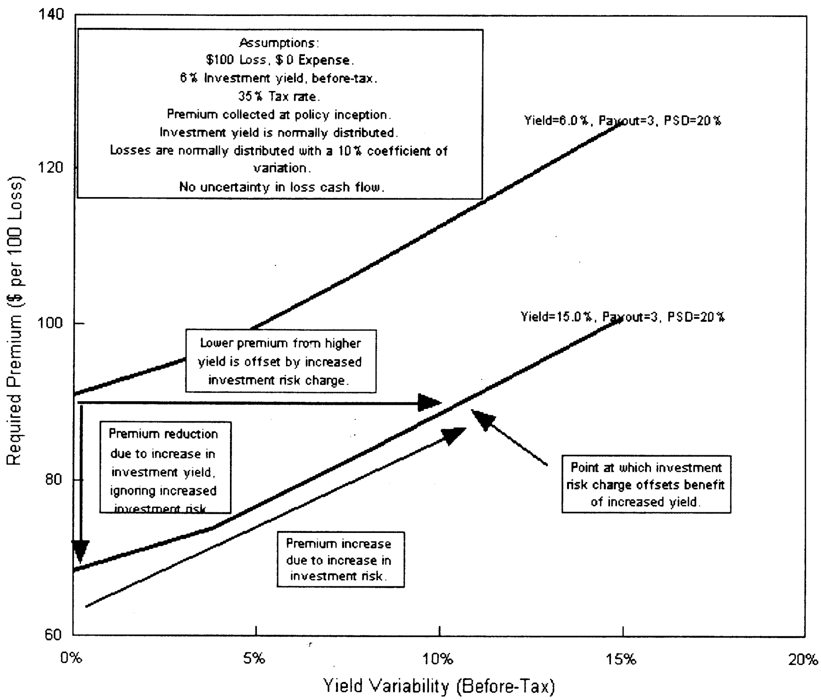
PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–RISKIER INVESTMENTS)



However, the key issue is to judge to what degree the increased benefit from higher yields (via a reduction in price) is offset by the increase in price due to the higher risk. Figure 8 presents an example of such an assessment. (Mismatching, which would increase risk and required premiums, has not been factored into this analysis.) A line of business with a three-year average loss payout in which yields increase from a risk-free 6% to 15% (before-tax) will lose the entire benefit from this increase

FIGURE 8

PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–RISK CHARGE OFFSETTING HIGHER YIELD)



if the attendant variability increases to a standard deviation of approximately 10%.

Unfortunately a further complication arises in that, in the translation from operating to total return, the variability of *Ri* adds greater variability to total return as seen by the shareholder above that reflected and priced into the operating return (based on (3)). In other words, the variability in investment income on surplus itself adds variability beyond that coming from operating return, and additional total return is required to compensate the shareholder for this additional risk.

An alternative approach is to view operating returns in insurance on a risk-free investment basis, with higher-risk investment strategies being introduced incrementally after this for total return purposes. Such a step moves the risks and rewards of higher risk investments to the shareholder, and issues of risk, return and leverage are addressed separately for this component. This also provides a useful delineation between the underwriting and investment functions, permitting the investment function to be managed incrementally on a value-added risk/return basis.

The basic risk-charge mechanism functions well without introducing higher-risk investments into the equation. Furthermore, as practical policy, it is difficult to see why two identical insurance policies should be priced differently simply because the insurance companies offering them have a different investment mix, assuming that policyholders should be insulated from investment risk. A mechanism for dealing separately with investment risk will be explored further from the total return shareholder perspective.

4. LEVERAGE AND TOTAL RETURN

4.1. The Shareholder Risk/Return Perspective

Leverage magnifies returns and variability from insurance operations which, with the inclusion of investment income on the surplus itself, results in the total return as shown in (3). Once the operating return profile has been established, leverage merely provides a translation to the shareholder perspective, as shown previously in Figure 4. The probability that the total return will not achieve an economic return—a total return below the yield on surplus Ri which could be achieved without taking insurance risk—is maintained as specified during the determination of the risk charge. In other words, insurance risk is charged to the insured.

Surplus, and thus leverage, is set by balancing the policyholder-related concerns of the regulators and insurance rating agencies (i.e., lower leverage is better) with shareholder-related concerns of the investment rating agencies and analysts (i.e., reasonably higher leverage is generally better). While shareholders should receive a higher return if risk is higher, changing leverage does *not* alter the probability of a negative economic downside risk. Although a leverage increase will raise returns to the shareholder, it also increases risk at the same time, with the result that the PSD remains unchanged.

If returns are low relative to risk and not consistent with other investment alternatives available to the shareholder, then insurance companies will have difficulty raising capital. Essentially, the insurance company is not generating a sufficient return on operations to pay for the transfer of risk from the company to the shareholder under such circumstances. This scenario exists when the risk/return relationship governing the company/shareholder relationship is not supported by a similar risk/return relationship between the company and its policyholders. The only recourse is to increase the underlying policyholder risk charge to bring that risk/return relationship in line with that needed to support a total company risk/return profile comparable to other external investment choices. More specifically, the risk charge and return must be increased and the PSD reduced, so that the risk and returns are made consistent with other investments available to the shareholder.

One important benefit to the aggregate company, and thus to the shareholder, is the reduction in risk and variability that comes from underwriting (line of business) and investment diversification (i.e., covariance). Companies benefit in many ways from offsetting factors which reduce aggregate variability, and thus risk. These offsets occur: 1) in underwriting between lines of business, 2) in underwriting between variables such as expense and loss within a line of business, 3) in investment between asset classes, 4) between underwriting and investment, and 5) in reported calendar year financial results in longer-tailed lines of business (due to an averaging effect on the more volatile policy/accident period results as they flow in). While very difficult to assess, these covariance benefits are of greater benefit to the larger, more diversified insurers. Just how this effective reduction in risk is reflected in the risk transfer pricing mechanisms is a topic that must be addressed at some point.

One of the interesting aspects of this is that surplus allocation to lines of business is not necessary for purposes of pricing, as long as a uniform PSD is maintained among the various insurance products. The probability of ruin and EPD will be similarly controlled, and if prices are adequate, that probability will be sufficiently small and negligible. While this may be a bit of a simplification (since many loss distributions are skewed), the basic principles are valid.

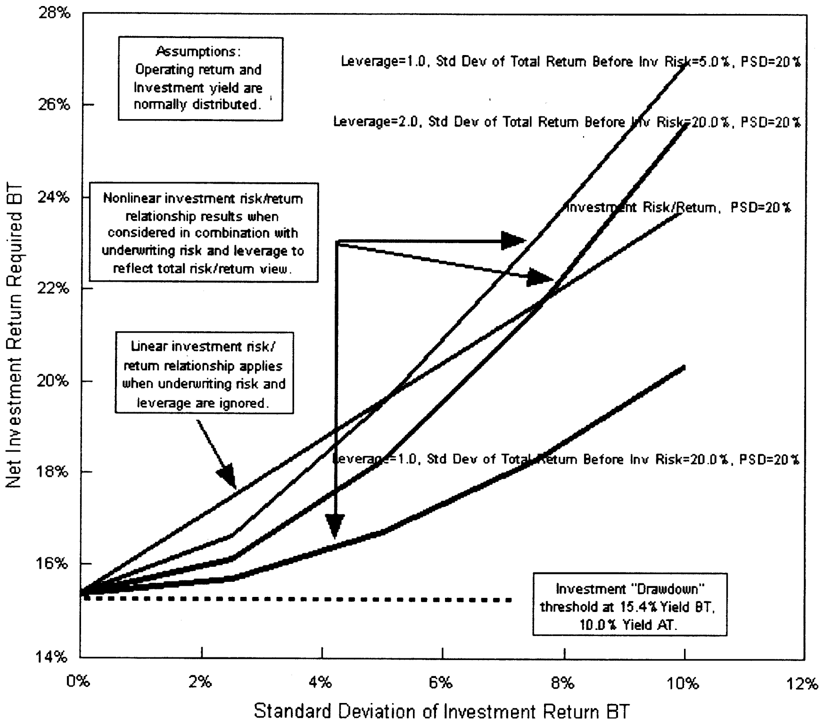
It should be noted that if risky investment strategies are included in the pricing mechanism, it is likely that the degree of risk will vary among the lines of business. For instance, longer-tailed lines might extend maturities to a greater degree than shorter-tailed lines, thus adding more risk.

4.2. Investment Pricing for Investment Risk

The use of operating return, its expected value and distribution, together with the concept of the probability of surplus drawdown provides a basis for setting fair premiums to the policyholder, while at the same time permitting a fair return to the shareholder consistent with the amount of variability in total return. The issue of investment risk remains as an additional component of overall total return variability. A mechanism for including higher investment and a related policyholder premium risk charge for the added investment risk entailed was presented earlier. An alternative approach is to base policyholder premium on an assumed risk-free investment strategy and separately reflect investment in the shareholder total return, with the risks and rewards of investment kept within the shareholder domain.

FIGURE 9

INVESTMENT RISK/RETURN
(WITH VARYING LEVERAGE & TOTAL RETURN
VARIABILITY–TO MAINTAIN PSD)



This perspective recognizes that insurance company investment activities are themselves subject to the same risk/return principles that apply to policyholders and shareholders, facing the same decisions that require greater compensation in return when risk is higher. Investment activities are viewed as an incremental, value-added complement to underwriting activities, which together form insurance operations. Figure 9 presents the basic tradeoff in investment risk and return. (The mathematics for the required increase in investment return are shown in the Appendix.) Here it is assumed that the policyholder premium has been based on a risk-free investment strategy.

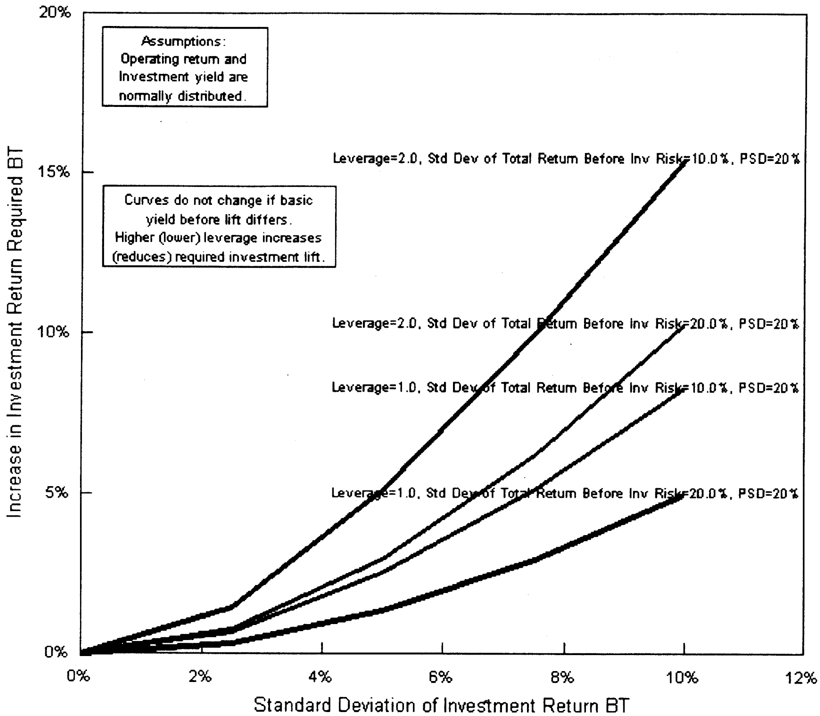
The straight line on this chart reflects the increase in return required to compensate for increase in investment risk in order to maintain a PSD of 20%, when investment is viewed apart from underwriting risk and leverage. Unfortunately, the interplay between underwriting and investment risk and the effect of leverage on total return variability must be considered. This results in the other nonlinear examples shown on the chart. Note that there is a benefit to the firm when investment risk is on the lower side, compared to the independent (i.e., linear) investment risk/return perspective. However, when investment risk continues to increase while the underlying underwriting risk is small or leverage is low, a greater investment return is required. This points out the important connection between underwriting and investment risk and financial leverage.

Figure 10 presents the increase in investment return or "lift" required to maintain a given PSD, as investment return variability increases with the connection between underwriting and investment risk and leverage considered in all cases. This figure provides a frame of reference indicating the degree by which investment returns must improve as investment risk increases. Importantly, the curves shown do not depend on the underlying level of investment yield.

If the lift in investment returns is below those indicated, then the probability of surplus drawdown is increased. If investment returns cannot be improved, then perhaps the risks are too great. Furthermore, higher leverage requires a higher lift due to the magnifying effects of leverage on variability. Thus an alternative to increasing investment return when investment risk increases is to reduce leverage. In other words, increases in investment risk may embody elements of both higher investment return and more conservative (i.e., lower) leverage.

FIGURE 10

REQUIRED INVESTMENT LIFT
(WITH VARYING LEVERAGE & TOTAL RETURN
VARIABILITY–TO MAINTAIN PSD)

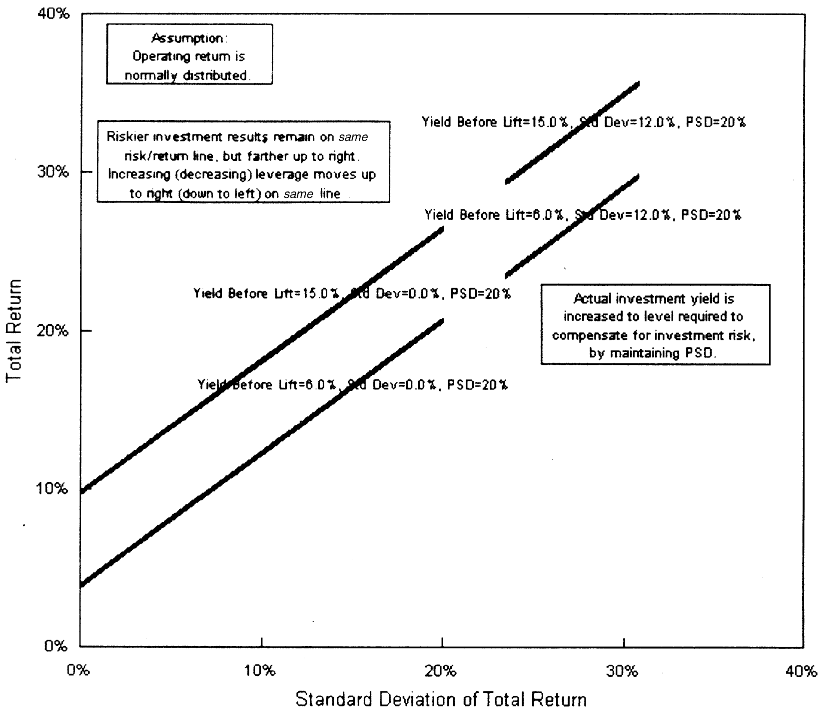


This perspective presents the investment function as subject to the same risk/return principles and PSD that have been applied elsewhere for risk transfer pricing purposes, and provides a means for managing the investment function as an incremental, value-added complement to underwriting.

Figure 11 reflects the risk versus return perspective (similar to Figure 4 shown previously) when the required investment lift is exactly achieved. Note that when investment risk increases, the

FIGURE 11

TOTAL RISK/RETURN
(WITH VARYING YIELD & VARIABILITY AFTER PSD BASED INVESTMENT LIFT)



variability in total return increases as well, but the appropriately increased investment return holds on the same risk/return line (albeit farther up and to the right). Thus the PSD is held. This figure demonstrates how increases in investment risk, followed by an increase in variability, should lead to increases in total return.

Whether actual investment returns are built in at the policyholder or at the shareholder level, the important point is that the attendant increase in risk must be reflected. In the case of the policyholder, this means an increase in premium, possibly by enough to outweigh the benefit of the higher investment return. In the case of the shareholder, this means an increase in the overall total return which recognizes the increase in total return variability.

4.3. Surplus Requirements and Allocation to Lines of Business

A long-running debate continues with regard to the need to allocate surplus to lines of business for the purposes of ratemaking. Those not in favor of surplus allocation and the total return approach to ratemaking usually suggest use of return on premium (i.e., sales) as a preference. This statistic, however, is not a measure of return on *investment*, and it lacks a frame of reference as to what is fair and on what basis it should be set. Also problematic is the fact that it can and should differ markedly among lines of business, due to the length of the tail and the float-generating investment income that results. By way of alternative, operating return as presented in this paper has important attributes including:

1. Both of the operating return components of underwriting and investment rates of return, Ru and Ri, respectively, represent a return on supporting policyholder funds "invested." Thus operating return is truly a return on investment concept.

2. Operating return is an integral component of total return, since mathematically total return is calculated simply as the product of operating return times leverage, plus the investment return on surplus.

3. Operating rate of return fully reflects the differing magnitude and cash flow timing characteristics of individual lines of business. Operating return represents an annualized rate of return, regardless of investment horizon, comparable across all lines of business.

It is suggested here that, at a minimum, operating return be used in place of return on premium.

Arguments which favor the use of total return include the fact that it is a widely recognized benchmark (e.g., 15% ROE) which is readily comparable to other industries in terms of the risk versus return relationship. (It is also clear that every additional policy written entails an increase in risk to the insurance company, and requires some marginal increase in surplus.) The approach presented here extends the same risk (variability in return) versus return principles that govern shareholder actions to a lower operating return level within the insurance company.

In essence the "operating return–probability of drawdown" method presented in this paper is a replacement of return on premium by operating return, and an extension of shareholder risk/return principles to the policyholder level. As a consequence, the method demonstrates how risk can be reflected in the pricing mechanism *without* varying the allocation of surplus to individual lines of business, through the focus on operating return. Yet this remains as a mathematical component of the total return, made complete simply by the application of leverage and the addition of investment return on surplus. The PSD, driven by the connected variability in operating and total return, provides a unifying and consistent framework for establishing fair returns to reflect the transfer of risk from the policyholder to the company and from the company to the shareholder. Furthermore, if the PSD is made the same when pricing the individual lines of business, then leverage can be set uniformly in each line equal to the overall company average for purposes of calculating total return.

Were pricing models able to estimate all prospective financial parameters sufficiently well, then adequate pricing would lay the foundation for financial strength and lessen the need for surplus. However, many factors such as inflation, changing tort law, competitive pricing and catastrophic exposures, introduce uncertainty with respect to balance sheet value and require a substantial surplus cushion. Furthermore, these risks and resulting surplus needs are likely to differ among the lines of business.

The total surplus of an insurance company needs to be sufficient to provide an adequate financial cushion for the many balance sheet risks. The approach presented in this paper supports solvency with respect to current business writings, since the probability of ruin that results is extremely small (given any reasonable PSD and operating return).

4.4. Application Steps to Put Concepts into Practice

The following overview presents the essential steps and capabilities that are necessary to put these concepts into practice.

1. Develop a model framework that provides key balance sheet, income, and cash flow components. If ratemaking is the primary focus, then modeling a single policy period may be sufficient, in which case a single payment approach as presented in [2] and [3] may suffice. If calendar year financials are needed, then a multi-period cash flow model is needed, such as in DFA applications. Ideally this develops calendar period financials as the sum of current and prior policy/accident period contributions.

2. Develop a simulation capability built on top of this model, which can be applied to individual lines of business and then aggregated to a company total. The capability to incorporate key correlations among lines of business and variables may be important.

3. Specify the expected values of all variables and distributions of key variables as necessary. Generally interest rates and the amount and timing of losses, coupled with distributions that reflect the variability in them, are important. Although difficult to determine, key correlations

among lines of business and variables should be specified. Omitting this (i.e., assuming independence) will tend to overstate the benefit of covariance (i.e., diversification) and understate company surplus needs, since correlations are typically positive.

4. Set the (fixed) risk parameter to be used in each line of business. This is the desired probability that the total rate of return in an individual line of business will fall below the risk-free yield. A value in the range of 10% will probably be reasonable for starters. The number of lines of business, and the resultant diversification benefit, will affect this choice. The ruin probability for the total company that results should be verified as sufficiently small.

5. Beginning with underwriting risk/return, initially set a fixed leverage ratio (2 or 3 to 1 for liability to surplus) in all lines, and solve for the premium necessary to satisfy the specified risk parameter. The distributional outcomes from the simulation are used in this step. (A "risk-free" investment yield is suggested at this point.) This will indicate a required underwriting profit margin (i.e., combined ratio). At this point a risk/return line can be viewed for the modelled lines of business.

6. Adjust leverage by line of business to achieve a target total return. Premium is unchanged by this step, since the process is one of simply sliding up or down the risk/return line depending on whether the initial return is below or above the target return. If initially below the target return, leverage is increased (and decreased if above). The risk probability remains the same. The leverage ratios that result provide a risk adjustment mechanism, indicating relative line of business surplus requirements that permit all lines of business to be viewed relative to the same risk-adjusted total return target.

7. If higher risk investments are to be introduced, steps 5 and 6 are repeated for investment risk/return. After estimated investment risk and variability are increased, solve for the investment return necessary to satisfy the specified risk parameter. This will indicate a required investment margin. This should fall on the risk/return line but farther up and to the right (i.e., greater risk, greater return).

8. Adjust leverage by line of business to achieve the target total return. Required investment yields (as well as original premiums) are unchanged by this step, again since the process is one of simply sliding up or down the risk/return line, depending on whether the return is below or above the target return. The risk probability remains the same. The leverage ratios that result provide a risk adjustment mechanism, indicating the relative line of business surplus requirements for underwriting and investment risk combined. The difference from this surplus amount and that in step 6 is the amount required to compensate for investment risk. The leverage ratios that result provide a risk adjustment mechanism that permits all lines of business to be viewed on a comparable total return basis, in which both underwriting and investment risk have been reflected.

The risk-based required premium and investment yield determined in steps 5 and 7 may or may not be achievable. This then becomes part of the company's portfolio investment decision as to whether certain lines of business and/or investments should be written or undertaken.

In summary, this process provides a risk transfer pricing mechanism applicable to underwriting and investment activities, by indicating the premiums and investment returns required given their respective risks. Necessary leverage and relative surplus amounts are also indicated in order to risk-adjust to a common risk/return target.

5. CONCLUSION

It should be clear that the returns from underwriting and investment (in terms of expectations and uncertainty) together with the operating leverage employed by an insurance company, establish the essential elements of the risk/return tradeoff. This paper has presented an approach based on the application of very basic risk/return tradeoff principles to the risk transfer process that occurs between the insured and the company and between the company and the shareholder. Risk-based pricing algorithms have been presented to deal consistently with underwriting risk among lines of business and with investment risk. This process is apart from leverage, and does not require a varying surplus allocation to lines of business.

Operating return as presented in this paper is suggested as the fundamental measure that should be used to judge the risk transfer activities and pricing with respect to the policyholder. It is noted that risk is a fundamental element of insurance and it cannot be eliminated. Variability in results is expected, and simply throwing more surplus into the mix does not alter the basic risk/return relationship. Therefore, whether it is underwriting or investment based, some charge for risk transfer is needed whenever it occurs.

The PSD has been introduced as a guide by which the risk/return tradeoff can be managed similarly for both the policyholder and for the shareholder. This is suggested as the appropriate basis by which risk and return should be managed and prices set. Furthermore it is suggested that, while consistent with the probability of drawdown, the probability of ruin and EPD perspectives are different and more appropriate as a means to satisfy company solvency criteria than as a basis for risk transfer pricing. Instead, the risk transfer pricing approach presented here provides a single unified method which simultaneously satisfies regulatory concerns with respect to both setting fair risk-adjusted premiums and maintaining solvency.

Ultimately, insurance companies are faced with investment decisions with respect to the creation of optimum portfolio combinations of underwriting lines of business and investments to increase total return for a given level of risk. This involves application of the basic principles associated with the tradeoff between risk and return, and in which aggregate company diversification and covariance benefits play a role. While this paper has attempted to present concepts in as simple a manner as possible, solutions must extend into those cases which reflect the many insurance variables, how they relate to one another, and how they evolve over time.

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APPENDIX

PROBABILITY OF SURPLUS DRAWDOWN RISK-BASED PREMIUM DETERMINATION

The underwriting risk-based premium based on the PSD in which loss is the only parameter with uncertainty is:

$$P = \{(1-T)-(1-D)\}L/\{(1-T)(1-ZC)\}, \qquad \text{(I)}$$

where:

P = Premium required

Z = Standard normal deviate corresponding to desired probability of drawdown

L = Estimated loss

T = Tax rate

R = Investment interest rate, after-tax

N = Average loss payment date

σ_L = Standard deviation of loss

C = Loss coefficient of variation (σ_L/L)

D = Discount factor$= 1/(1+R)^N$,

assuming:

- Expenses are 0
- Premium is collected at policy inception
- Losses are normally distributed
- Approximation using average loss payment date
- Variability in loss amount only (i.e., certain cash flows and interest rates).

The underwriting risk-based premium based on the PSD in which interest rates and losses are both uncertain and independent is found by solving the following quadratic equation:

$$P^2A + PB + C = 0, \qquad P = (-B - \sqrt{B^2 - 4AC})/(2A), \quad \text{(II)}$$

where:

$$A = \{1 - (ZC)^2\}/L^2$$
$$B = -2(1 - R/M)/L$$
$$C = \{\sigma_R^2 - (M^2 - 2RM + R^2\}/M^2$$
$$M = R(1 - T)/(1 - D).$$

The investment return lift in yield required to maintain the specified PSD from the shareholder perspective is $Ra - Rf$:

$$Ra = Z\sqrt{\sigma_{Ru}^2(L/S)^2 + \sigma_{Ra}^2(L/S + 1)^2} - Z\sigma_{Ru}(L/S), \qquad \text{(III)}$$

where:

Ra = Actual yield

Rf = Risk-free yield

σ_{Ru} = Standard deviation of underwriting return

σ_{Ra} = Standard deviation of actual yield

L/S = Liability to surplus leverage ratio,

assuming policyholder premium does not include a risk charge for investment.

Formula (I) is derived by noting that $Ru = (P/L - 1)M$, and solving for P such that $(Ru + Ri) = Z\sigma_{Ru}$.

Formula (II) is derived by solving for P such that $(Ru + Ri) = Z\sqrt{\sigma_{Ru}^2 + \sigma_{Ri}^2}$.

Formula (III) is derived by solving for Ra by determining the value that results in a shift from the risk-free total return line, given by $Rf + (Ru + Rf)(L/S)$ having a standard deviation of $\sigma_{Ru}(L/S)$, to the riskier total return line, given by $Ra + (Ru + Ra)(L/S)$ having a standard deviation of:

$$\sqrt{\sigma_{Ru}^2(L/S)^2 + \sigma_{Ra}^2(L/S+1)^2},$$

in order to satisfy the same PSD (i.e., Z-value).

The formulae presented here to demonstrate the concepts do not reflect all variables. While loss is generally the key driver in terms of expected return and variability, a more complete extension of this approach should reflect the impact of all parameters and multi-period cash flows (and the relationships among them) on return.

Demonstration Example

The following financial assumptions form the basis for the example presented in Exhibit A-1:

- 101.0% Combined ratio
- $9,900 Premium, collected without delay
- $10,000 Loss, single payment after 3 years
- $0 Expense
- 35% Income tax rate, no delay in payment
- 6.0% Investment interest rate before-tax, 3.9% after-tax
- No loss discount tax
- 3.0 Liability/surplus ratio.

Simplified balance sheet, income and cash flow statements are shown for this example. The rules governing the flow of surplus are as follows: (1) the level of surplus is maintained at a 1/3 ratio with loss reserves, (2) investment income on surplus is paid to the shareholder as earned, and (3) operating earnings are distributed in proportion to the level of insurance exposures in each year (measured by loss reserve levels) relative to the total exposure. Since loss reserves are equal at $10,000 in each of the three years, operating earnings are distributed to the shareholder equally in each year.

Three "levels" of return exist within an insurance company. The first is the underwriting rate of return, which reflects what the company earns on pure underwriting cash flows before reflecting investment income on the float. This is a "cost of funds" to the company. The second, operating return, reflects what the company earns on underwriting when investment income on the float is included. This is the "risk charge" to the policyholder for the transfer of risk to the company. Finally, the total return is the net result of underwriting and investment income from operations together with investment income on surplus.

These rates of return can be determined either by a cash flow-based internal rate of return (IRR) calculation, or by relating income earned to the amount invested. With regard to the shareholder total return perspective, the IRR based on cash flows from and to the shareholder indicates a 14.9% return over the three-year period. The income versus investment approach (i.e., ROE) relates the income over the full three-year aggregate financial life of the business to the shareholder's investment over this same period. This is shown in both nominal (i.e., undiscounted) and in present value (discounted) dollars to produce a 14.9% rate of return on investment. Furthermore, the return realized by the shareholder via dividends is also an identical 14.9% in each year. (This attribute follows from the rules used to control the flow of surplus.)

The operating return, inclusive of underwriting and investment income, is most easily shown to generate a cash flow-based internal rate of return of 3.7%. Equivalently, the operating income of $1,100 is a 3.7% return on the "investment equivalent" of $30,000, the total balance sheet float upon which these earnings were generated.

EXHIBIT A-1

THREE-YEAR DEMONSTRATION EXAMPLE

	PERIOD					Present
	1	2	3	4	Total	Value
BALANCE SHEET (Beginning of Period)						
Invested Assets	13,268	13,289	13,311	0	39,868	37,072
Loss Reserves	10,000	10,000	10,000	0	30,000	27,804
Retained Earnings	−65	−44	−23	0	−132	
Surplus Contributed	3,333	3,333	3,333	0	10,000	9,268
Liabilities/Surplus	3.0	3.0	3.0	0	3.0	3.0
INCOME AFTER-TAX (During Period)						
Underwriting	−65	0	0	0	−65	−65
Inv Inc Retained Earnings	−3	−2	−1	0	−5	
Inv Inc Loss Reserves	390	390	390	0	1,170	1,084
Total Operating	322	388	389	0	1,100	1,019
Inv Inc on Surplus	130	130	130	0	390	361
Total Net Income	452	518	519	0	1,490	1,381
Return on Beginning Contributed Surplus	13.6%	15.5%	15.6%	0	14.9%	14.9%
CASH FLOW (During Period)						
Operating Cash Flows						
Premium Receipt	9,900	0	0	0	9,900	9,900
Loss Payment	0	0	0	−10,000	−10,000	−8,916
Underwriting Tax Paid	35	0	0	0	35	35
Retained Earnings "Funding"	65	−23	−23	−23	−5	0
Total Underwriting	10,000	−23	−23	−10,023	−70	

EXHIBIT A-1

(Continued)

	PERIOD 1	2	3	4	Total	Present Value
Underwriting Return		Underwriting IRR = –0.2%				
Investment Receipts (After-Tax)	0	390	390	390	1,170	
Total Operating	10,000	367	367	–9,633	1,100	1,019
Operating Return		Operating IRR = 3.7%				
Surplus Cash Flows (Beginning of Period)						
Contributed Surplus	3,333	0	0	–3,333	0	
Dividend						
Surplus Inv Inc	0	–130	–130	–130	–390	
Operating Earnings	0	–367	–367	–367	–1,100	
Net Shareholder	3,333	–467	–467	–3,830	–1,490	
Shareholder Rate of Return		Shareholder IRR = 14.9%				
Shareholder "Dividend" Yield		14.9%	14.9%	14.9%	14.9%	

RATE OF RETURN

	IRR	Nominal (Income/Investment)	Net Present Value Basis (Income/Investment)
Underwriting Return	–0.2%	–0.2% = –70/30,000	–0.2% = –65/27,804
Operating Return	3.7%	3.7% = 1,100/30,000	3.7% = 1,019/27,804
Shareholder Total Return	14.9%	14.9% = 1,490/10,000	14.9% = 1,381/9,268