A Practical Application of Modern Portfolio Theory to Capital Allocation

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Abstract

This paper presents and evaluates a capital allocation method that represents a significant part of a more complete modeling process required to fully allocate and release capital for application to pricing or profitability analysis. The complete process is also described at a more general level, including discussion of additional requirements for application to reinsurance. While the specific modeling presented is based largely on concepts from modern portfolio theory, the paper first compares alternative perspectives and practical considerations relevant to all methodologies.

The perspective of a specific portfolio analysis is compared to a market equilibrium perspective adopted for risk load or capital allocation methods. The paper compares the differences in their assumptions and the intended interpretation of their indications. It is then described how both perspectives can add value to decisions by properly integrating them into pricing and financial risk management operations.

The concept of economic valuation is introduced in establishing the total capital to be allocated. Implications of the market equilibrium perspective adopted by the methods proposed in the paper are discussed at the total capital level. These include the relationship between capital and expected loss exposure at the total industry level, as well as the use of required rather than actual capital of insurers.

The proposed method for allocation of capital to segments of the portfolio is based on Capital Asset Pricing Model concepts with specific interpretation of the variances and covariance relationships between segments. Several numerical examples of the method are provided and practical application problems are identified and discussed.

In conjunction with the capital allocated at inception, the release of this capital generates the total capital commitment needed to measure return on equity for application to pricing or profitability analysis. Finally, the paper describes extensions of the proposed methods to reflect excess layers, other contract provisions, and asset risk in a consistent manner.

Biography

John Kulik is an Assistant Vice President with SCOR Reinsurance, where he is responsible for pricing and corporate financial analysis. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. Mr. Kulik is also Vice Chair of the CAS Committee on Review of Papers. He received a B.S.B.A. degree from the University of Florida. Prior to his seven years in reinsurance, he had more than ten years of actuarial experience with several multiline insurers.

[The author would like to thank **Donald Mango** for his thorough review, including several helpful comments and suggestions that improved the paper. Any errors in the final paper are entirely the author's responsibility.]

Introduction

Approaches that actuaries typically use to measure and analyze risk have evolved from actuarial risk theory. Considerable research of alternative premium calculation principles has been applied to the analysis of pricing risk. Risk theory is well known for its ruin theoretic approach to the macro-analysis of underwriting portfolio risk. Originally applied to solvency analysis, ruin theory also proved appealing as a pricing concept.

Financial economics concepts more recently expanded the scope of actuarial risk theory. The concept of capital allocation evolved in the context of these principles and the unique nature of insurer capital. Actuaries have traditionally been hostile to this concept, arguing that it is inconsistent with the operational indivisibility of capital. More recently, however, it has been gaining acceptance as an analytical management tool.

Since capital allocation methods are used to estimate risk margins in the form of return on equity (ROE) measurements and targets, they can be viewed as equivalent alternatives to actuarial risk load methods. However, capital allocation views risk from a "top-down" perspective, while actuarial risk theory tends to view risk from a "bottom-up" perspective. At a macro level, financial theory views capital as the equity capital supplied by investors, while actuarial risk theory views it as protection against insolvency. While reconciliation between individual pricing risk loads and total portfolio risk is not an inherent feature of actuarial risk load methods, some form of reconciliation is intrinsic to capital allocation.

In addition to risk load-equivalent applications of capital allocation, its intrinsic reconciliation property can provide management with analytical insight needed to improve capital productivity, risk-return efficiency, and shareholder value. While it is also feasible for an insurer to manage its financial risk in an alternative equivalent manner, communication with investors would require additional challenging effort. Therefore, capital allocation also provides a convenient vehicle for communication with investors which clarifies the relevance and consistency of an insurer's financial risk management policies with the interests of investors.

Alternative Perspectives

The "equivalence" of capital allocation and risk load methodologies enables us to examine the importance of alternative perspectives through some of the risk load literature. The effectiveness of a capital allocation (or risk load) methodology can only be evaluated within its intended perspective.

Feldblum (1990) and Bault (1995) both examined a variety of risk load methods. Bault focused on several specific methods that have been presented in CAS publications and observed that all of "…the methods are nearly equivalent … if care is taken to use a common set of assumptions." In the midst of considerable debate about these methods, this was a very interesting observation, implying that the debate was primarily about their assumptions rather than the formulas and methods themselves. Indeed, Bault uses Kreps' (1990) equations "…to show that variance, standard deviation, ruin theory, and CAPM describe similar (and nearly equivalent) concepts."

Since the approach presented in this paper is based on some of these same "nearly equivalent" methods, I would prefer to avoid the same confusion in previous debates by clarifying its underlying assumptions at the outset. Some assumptions do not conform to reality, but are needed to develop a practical model. Other assumptions that simplify the formulas may not be necessary, but are useful for practical application. When a more realistic alternative assumption is available, it is fair to debate the cost of the additional complexity weighed against the benefit of the additional accuracy realized. This has certainly been one source of debate. For example, Bault noted that covariance considerations are often ignored or oversimplified, significantly changing the results of some methods.

I believe that two completely different fundamental perspectives have emerged as the leading conceptual frameworks underlying a risk load or capital allocation methodology. I will call them 1) the specific portfolio framework, and 2) the market equilibrium framework.

- The **specific portfolio** framework is related to the traditional actuarial risk theory perspective. This is an internal viewpoint intended to measure the risk of a specific portfolio and the contribution of the individual risks or segments to the total portfolio risk. Thus, the analysis and parameterization under this framework is primarily company-specific and individual risk-specific, also reflecting the interaction between individual risks.

- The **market equilibrium** framework is related to the financial economics perspective. This is an external viewpoint intended to measure the risk margin that a competitive market in equilibrium would provide for the individual or aggregate risk exposures assumed. With some oversimplification, the same risk margin would be provided to any insurer for the same exposures. While such a methodology is applied to specific individual risks and companies, the analytical development and parameterization under this framework is entirely at the market, or industry level.

I will compare these perspectives in both "front-end" pricing analysis and "back-end" corporate profitability analysis. *The discussion which follows assumes that individual prices and total premium fully and accurately reflect expected losses, expenses, and the* ***indicated*** *risk load, or profit margin.*

Pricing Application and Interpretation

When applied to pricing an individual risk, a **specific portfolio** methodology *intends* to indicate the risk load needed to maintain the desired risk-return relationship in the total portfolio. *If the actual prices for all of the individual risks reflect exactly these indicated risk loads*, there would be no need to actively manage the risk of the portfolio through diversification or reinsurance. Aggregate premium income would provide exactly the desired expected portfolio ROE. Thus, the total portfolio risk would be "self-managed" through the pricing process.

In contrast, a **market equilibrium** methodology applied to pricing an individual risk *intends* to indicate the "fair premium" risk load that the competitive market will allow. It can also be viewed as the price that an informed, rational insured would be willing to pay. If the actual prices for all of the individual risks reflect exactly these indicated risk loads, then aggregate premium would be extremely unlikely to result in the desired expected portfolio ROE without active management of the portfolio risk through diversification or reinsurance. Thus, the pricing process would not provide any insight into the total portfolio risk resulting from the accumulation of the actual exposures assumed.

Profitability Analysis Application and Interpretation

When allocating capital to analyze ROE profitability by segment of a company portfolio, the company-specific nature of the analysis might appear to render the market equilibrium framework irrelevant. However, if a market equilibrium method is applied *to pricing*, then it may also play a legitimate role in the "back-end" profitability analysis.

While underwriting managers are responsible for their individual pricing decisions, the management of total portfolio risk required to balance total risk-return performance is a corporate financial management responsibility. For most large insurers, it is not practical for the same underwriters responsible for individual pricing decisions to be simultaneously accountable for total portfolio risk management.

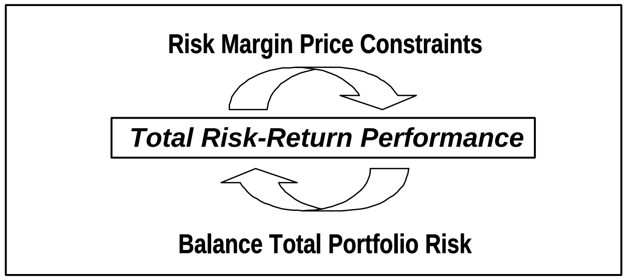
The responsibilities for these functions are generally separated between different individuals in different departments. Therefore, if the performance of underwriting segment managers is evaluated using ROE measurements by segment, then ROE can only be fairly measured using the same market equilibrium criteria applied to their pricing decisions.

Although there is a problem in applying a specific portfolio method to *performance evaluation*, it can be argued that the *actual profitability* of the segments is based on the specific portfolio framework. While there is merit to this argument, the results of such a profitability analysis would be more productively applied to portfolio risk management decisions than to an evaluation of performance or price adequacy by segment. Another application further discussed in the next section would be to provide strategic direction to the underwriting segment managers.

Strategic Integration of Alternative Perspectives and Applications

Regardless of our ability to precisely explain and model the insurance markets, the reality is that the intense competition in these markets places very real and significant constraints on price levels. The financial risk management challenge facing insurers is the need to appropriately balance the total portfolio risk resulting from their accumulation of assumed exposures against the constrained aggregation of the individual risk margins in total premium. This process is described in Figure 1.

Figure 1: Financial Risk Management Process



Therefore, insurers need a set of decision-making tools which simultaneously recognize the constraint of aggregate risk margins in total premium and the actual portfolio risk resulting from their accumulation of assumed exposures. This need is just as valid under favorable competitive market conditions as under severely unfavorable conditions. Pricing and profitability methodologies under the alternative frameworks can be used simultaneously as a part of this toolkit.

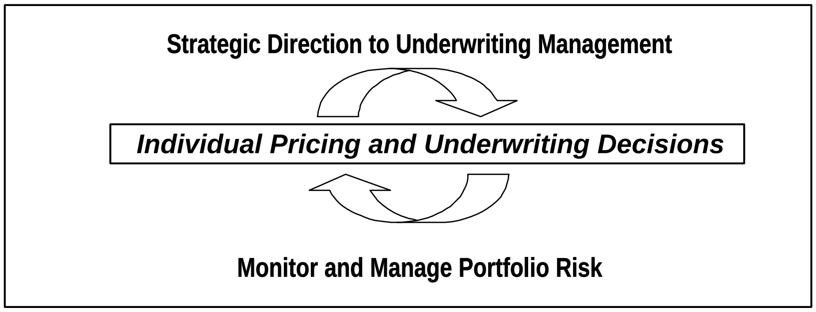
The financial risk management needs described in Figure 1 are not fully addressed by either alternative framework; thus all such methods provide incomplete information. A **specific portfolio** method such as Kreps' (1990) "marginal surplus" methodology intends to calculate the risk margin needed to assume the exposure without consideration of competitive market constraints. When the indication exceeds the corresponding market equilibrium indication, a lower price will be adequate for some competitors with different portfolio characteristics. Therefore, a rational insured would not be willing to pay this price.

While the mechanical development of a **market equilibrium** method can be less explicit in reflecting the constraints illustrated above, Meyers (1991) very explicitly models the risk management process as a constrained optimization problem in his well known "competitive market equilibrium" methodology. Nevertheless, *when a market equilibrium method is actually applied to the pricing of an exposure*, it still provides incomplete information to company management. When the indication is below the corresponding specific portfolio indication, it does not recognize the adverse impact that assuming the exposure at this price would have on its total portfolio risk because it intends to calculate only the risk margin that the competitive market will allow.

However, the incompleteness of each method is not a valid criticism of the method because it results directly from the *intended* interpretation of each method's indication. The incompleteness of each method also reveals a complementary strength of the other method. What is actually important is the manner in which management applies and integrates the alternative methods and applications. It is very important to keep in perspective that these methods only develop *indications* as a decision-making tool. Both of the alternative frameworks can add value to decisions, provided that the decision-maker understands them well enough to properly interpret their indications.

As previously noted, corporate financial responsibility for the active management of total portfolio risk is very difficult to share with the responsibility for individual pricing and underwriting decisions, regardless of the pricing methodology used. In addition to passively monitoring the accumulation of exposures and taking action needed to manage the resulting total portfolio risk, corporate financial management should also use this information to provide strategic direction to the underwriting segment managers responsible for individual pricing and underwriting decisions, as described in Figure 2.

Figure 2: Integration of Corporate and Underwriting Decisions



Corporate risk management action could include structuring efficient reinsurance programs, while strategic direction could include identifying segments for which growth should be curtailed and/or pricing criteria should be tightened, perhaps to the level of specific portfolio method indications. Capital allocation techniques with alternative perspectives applied to both pricing and profitability analysis provide a set of decision-making tools that can be used effectively in the management of an insurer's financial risk.

Considerations for Practical Application

Researchers and actuarial practitioners alike have long recognized certain problems with the application of traditional risk load approaches. While these issues have sometimes been viewed in the context of the alternative conceptual perspectives discussed, they are also recognized as practical problems, regardless of the conceptual framework applied.

In his explanation of the pricing formula for converting the expected loss cost to the rate or premium, Patrik (1990) correctly states that the "…target economic return, or profit and risk charge, is properly related to the loss potential on each contract, and should be appropriately stated in relation to the loss expectation." *Relative to expected losses*, however, the **pricing** risk loads indicated by traditional actuarial approaches, including specific portfolio methods, typically vary by:

- the **size of the individual risk** priced
- the **size of the portfolio** into which the risk is placed
- the **correlation between the individual risk with the *company-specific* portfolio**
- the **order of entry** between individual risks into the portfolio

The practical implications of these observations may be more easily understood for reinsurers, where the acquisition and/or non-renewal of several large accounts can significantly impact a reinsurer's total portfolio. As a result of these risk load fluctuations, the rate indication would change from the previous year for a renewal account with no change in terms or loss exposure, reflecting changes in the total portfolio. Even for new accounts, rate indications would differ based on the time of submission, reflecting portfolio changes during the lapse of time. Several small accounts which aggregate to the identical total loss exposure of one large account would generate a different aggregate premium than the large account. Any of these situations would complicate contract negotiations if the indications were used in isolation without other decision-making tools.

Size of risk variation and order dependence are generally considered the most problematic of these issues. Practitioners using a **specific portfolio** method address some of these problems by making certain adjustments to the method. For the purpose of determining a risk margin, for instance, loss exposures can be scaled to a common size through the claim count variable. The other problems can be addressed to some extent by defining a fixed total portfolio against which all individual risks are measured. While these adjustments result in indications that depart from the fundamental interpretation of the underlying specific portfolio framework, they are made as one part of the overall decision-making analysis.

Actuarial risk theory has long recognized these problems and identified various mathematical properties that more rigorously address them. Several examples of these properties, including sub-additivity, scale invariance, and linearity are summarized by Wang (1995). Such properties have been used to evaluate various premium calculation principles for their vulnerability to these problems. While this type of analysis appears to be theoretical, the examples of practical problems discussed clearly indicate that the need to address them is not merely an exercise in conceptual eloquence.

Market equilibrium methods are not vulnerable to these particular problems because their underlying conceptual framework assumes that the market inefficiencies associated with these problems do not exist. While it is possible to model certain inefficiencies, in this paper *I begin with the assumption of no-arbitrage insurance markets*. Venter (1991) clearly demonstrated that insurance "…markets without arbitrage must charge additive premiums for…" all risks. In other words, the premiums for multiple risks must sum to the independently calculated premium for the aggregation of these risks. Otherwise, arbitrage opportunities would always be present in the market for these risks. This additivity of premiums is the key to addressing the problems discussed. For the premiums to be additive, their risk margins must also be additive.

This discussion of practical considerations may have suggested that these issues pertain entirely to pricing. For an extension of these considerations to **profitability analysis**, we will briefly revisit the previous example of performance evaluation by underwriting segment. If we substitute "underwriting segment" for "individual risk" in the above list of problems, they will also apply to the use of a specific portfolio method to measure ROE profitability by segment. The practical implication is that the profitability of each segment manager would be impacted by the profitability of every other segment manager. Clearly, this would be an untenable *performance evaluation* process that *could* also be addressed by using a market equilibrium method. Such a method would result in additive capital allocations and profit margins by underwriting segment. This is merely an example to illustrate the relevance of these considerations to other applications of risk load and capital allocation.

Total Required Capital

The capital allocation approach developed in this paper adheres to the market equilibrium framework by applying basic financial economics concepts from modern portfolio theory (MPT). Before allocating capital to segments of business, we first need to establish the total capital to be allocated. The surplus on the company's financial statements, whether statutory or GAAP, is not the capital to be allocated for primarily two reasons:

- capital must be stated on an economic basis to measure true profitability
- the actual economic capital may not equal the required economic capital

Economic Valuation of Capital

In this analysis, the **economic capital** of an insurer is defined as the market value of the insurer's *balance sheet* surplus. Thus:

$$\textbf{Economic Capital = Market Value of Assets - Market Value of Liabilities} \quad \textbf{(1)}$$

It is important that this is not confused with the total market capitalization of the insurer, or the total publicly traded market value of all outstanding shares of the company's stock. The theoretically "correct" amount of total market capitalization is often called **shareholder value,** which exceeds economic capital by an amount known as the franchise value (which can be negative). Thus:

$$\textbf{Economic Capital + Franchise Value = Shareholder Value} \quad \textbf{(2)}$$

The significance of this distinction is that:

- Franchise value represents *future* earnings expectations (further detail is beyond the scope of this discussion). Shareholders can trade their ownership in a company for their share of this value at the publicly traded consensus, but it is not available to company management.

- Economic capital represents the amount that is actually available to management to invest in the business. The productivity of this capital is reflected in investor expectations and, thus, the franchise value. Therefore, management can increase shareholder value by improving the productivity of economic capital.

Actual vs. Required Capital

The actual economic capital must first be segregated between required capital and excess capital. The definition of required capital differs significantly between the specific portfolio and market equilibrium perspectives:

- The **specific portfolio** framework represents the traditional solvency viewpoint and is conceptually easier to understand. Required capital is the "capital at risk" mathematically needed to support the actual portfolio risk. Ruin theory is the traditional actuarial risk theory approach to its estimation, which requires a specified level of financial strength. Well known examples of simplified, pragmatic approaches are the NAIC risk-based capital (RBC) formulas and rating agency models designed for expedient application to nearly all insurers.

- The **market equilibrium** framework represents the investor viewpoint and is more abstract. Required capital is the capital for which the competitive market will reward the cost of capital for the portfolio of assumed exposures. This represents the intersection between the level of security demanded by insureds and the level of profitability required by investors for these assumed exposures. Its estimation is based on actual capital levels observed in the insurance industry, consistent with the use of securities market data for MPT applications to securities.

The inclusion of excess capital may be optional in a specific portfolio method, but excess capital is excluded in a market equilibrium method because its indications are not company specific. Therefore, the approach in this paper will not allocate an individual insurer's actual total economic capital, but rather the required market equilibrium economic capital. Although actual capital would need to be included to measure the insurer's actual total ROE, this is not the intent of the market equilibrium approach.

Assumptions for Estimation of Total Required Capital

I have established that we wish to estimate total market equilibrium required capital, valued on an economic basis. A corollary of market, or economic, valuation is that financial returns should be measured as returns to *economic* capital. Therefore, the capital to be allocated for ROE measurement and corporate financial risk management is the economic capital.

The two major risk categories that are supported by nearly all of an insurer's capital are:

- **underwriting risk**, the risk of lower than expected underwriting returns, and
- **asset risk**, the risk of lower than expected asset returns and market valuations

I now introduce two simplifications that will be used to develop the capital allocation method:

- Only **underwriting risk** will be considered; therefore, we assume that the amount of capital supporting asset risk can be segregated from the capital supporting underwriting risk, and

- Only first dollar, or **ground-up**, loss exposure will be considered; therefore, we assume that the losses and/or capital in our data can be properly adjusted to reflect the impact of contract terms including high deductibles, excess layers, and various loss-sensitive premium provisions, on the risk in the losses supported by the capital.

While these are *conceptual* simplifications, the analysis required to reflect them is anything but simple. Both of these simplifications will be revisited later, but our analysis of required capital will first focus entirely on underwriting risk for ground-up loss exposure. This required "underwriting" capital should be related to the expected losses supported by this capital. This general relationship is consistent with the RBC formulas and the earlier Patrik (1990) statement that target economic *return* should be related to expected losses.

The market equilibrium approach requires an estimate of the *economic* capital required to support the *market value* of expected losses *for the total industry*. This relationship will establish the anchor from which adjustments will later be made when allocating capital. The relevant historical data for this analysis are primarily the loss reserves, unearned premium reserves, and surplus on insurer balance sheets. Despite the availability of this accounting data, estimation of this relationship requires a very significant amount of judgment for several reasons, including:

- limited accuracy in conversion of the data from its accounting basis to an economic basis
- difficulty in reflecting asset risk and ground-up loss exposure adjustments in the aggregate
- difficulty in reflecting individual company differences in asset risk and contract provisions
- difficulty in reflecting individual company differences in property catastrophe loss exposure
- distortions from major loss reserve strengthening, capital infusions, or share repurchases
- distortions of cyclical deviations from market *equilibrium* levels

Estimation of total market equilibrium required economic capital clearly does not lend itself easily to a purely quantitative analysis. Instead, we analyze industry data only as a starting point in making an informed judgment about the economic capital required to support the market value of expected losses.

Formulas and Estimation Procedures

Butsic (1988) derived a formula for the appropriate risk-adjusted discount rate needed to calculate the market value of loss reserves. He demonstrates that this rate adjustment and, thus, the market value of loss reserves is independent of an insurer's actual investment portfolio, but instead depends upon a risk-free market rate which has the same duration as the loss reserves (which includes the expected loss portion of the unearned premium reserve). Since the market value of assets is determinable, this risk-adjusted discounting of the loss reserves provides a means of estimating the economic capital from Formula (1).

The Butsic formula for the risk-adjusted discount rate is:

$$i_a = i - (S/L)(ROE - i) \tag{3}$$

where

i_a = *risk-adjusted* discount rate
i = *risk-free* market rate
S = *economic* capital
L = *market value of* [(UPR x expected loss ratio) + loss reserves]
ROE = return on *economic* capital
UPR = unearned premium reserves

Note that the relationship between economic capital and the market value of loss reserves is a ratio in this formula. Since the market value of the loss reserves is the value of the loss reserves *discounted at the risk-adjusted rate*, it can be seen that the solution of the formula requires an iterative analysis. Butsic provides a complete application of Formula (3), in conjunction with other data estimation procedures, to estimate the total industry risk-adjustment reflecting historical industry profitability.

I would recommend also using informed judgment to *select* an appropriate ROE for this analysis, but it is particularly important to recognize that:

- the ROE target *for this formula* should specifically reflect *risk-free* investment income, and
- the actual industry ROE target should be higher to reflect the actual risk of industry assets

Since the discounting of losses requires separate accident years of reserves and estimated loss payment patterns, there are various degrees of accuracy that can be attempted with this analysis. However, Butsic also shows that, if we reasonably assume that the ROE target is a constant spread above the risk-free market rate, then the risk-adjustment is independent of the level of interest rates. Thus, while this process is tedious, it doesn't require frequent analysis and revision.

Data Analysis

Rather than reviewing industry data only in the aggregate, the data should be analyzed separately for at least the largest individual insurers. This facilitates the process of addressing various distortions, such as those discussed above, and the considerable need for judgment. Since the most readily available data for this kind of analysis will generally be statutory data, the adjustment of this data to market, or economic, valuation can be conveniently divided into two major steps:

- conversion from statutory to GAAP valuation including estimation of deferred acquisition cost
- conversion from GAAP to economic valuation including discounting of expected loss reserves

Exhibit 1 summarizes the results of such an estimation process and application of Formula (3) for several major insurers. In this summary, the relationship between discounted losses and the economic capital required to support those losses is expressed as the leverage ratio **(L/S),** or the reciprocal of the ratio in Formula (3). This leverage ratio will later become the basis for application of the market equilibrium method to individual insurers, since the determination of market equilibrium required capital is based on total insurance market information.

Following a review of individual company data, aggregation into logical major categories is useful after excluding insurers that would appear to distort the averages. Broad classes of candidates for exclusion are reinsurers because of the focus on ground-up loss exposures and mutual insurers because of their unique capitalization issues. Specific individual insurers are vulnerable to a variety of distortions.

Exhibit 2 provides a summary of the same data aggregated for primary insurers of predominantly personal lines coverages and insurers of predominantly commercial lines coverages, with some individual insurers excluded from the totals. Clearly, such a review of industry capitalization can be used as a management benchmarking tool, regardless of its role in a capital allocation method.

Final Adjustments

The economic capital estimates in the exhibits are based on total balance sheet data. Therefore, two major adjustments are still needed which are more practically handled at the total industry level:

- The capital supporting asset risk must be subtracted from the total economic capital to estimate the capital supporting underwriting risk. This will be revisited later.

- All of the expected losses reflected in the balance sheet have been included, but the balance sheet capital also supports expected losses for some *future* business yet to be written.

Feldblum (1996) explains that "written premium risk" in the NAIC Risk-Based Capital method represents this *future* underwriting loss exposure. An approach consistent with this RBC method would be to add one *future* year of expected losses to the balance sheet expected losses, also discounted at the risk-adjusted discount rate. This would significantly increase the leverage ratios shown in Exhibit 2.

Table 1: Leverage Ratio Adjustment Example

Balance Sheet Indications		
(1) Discounted Loss Reserves		**148,003,973**
(2) Selected Leverage Ratio		**3.00**
(3) Required Economic Capital	**(1)/(2)**	**49,334,658**
Expected Loss Adjustments		
(4) Future Discounted Expected Losses (One Year)		**51,801,391**
(5) Total Discounted Expected Losses	**(1)+(4)**	**199,805,364**
Economic Capital Adjustments		
(6) Capital Supporting Asset Risk		**2,960,079**
(7) Required Underwriting Capital	**(3)-(6)**	**46,374,578**
(8) Adjusted Economic Leverage Ratio	**(5)/(7)**	**4.31**

The dollars shown in Table 1 are scaled to the level of the predominantly commercial lines insurers in Exhibit 2. In actual practice, the adjustments for future expected losses and economic capital supporting asset risk could be made using multiplicative factors.

Application to Individual Insurers

In the market equilibrium framework, the same relationships determined at the total industry level are applicable to all individual insurers. The allocation of only the required "underwriting" capital for individual insurers would represent a complete allocation of total required capital for an insurer with investments held *entirely* in risk-free assets. This places all insurers, regardless of their different investment strategies, on a common basis for various applications of the methodology and is consistent with the following established principles:

- For **pricing** applications, Myers and Cohn (1987) outlined a financial insurance principle that only *risk-free* investment income on premiums is credited to policyholders, regardless of an insurer's actual investments. The rationale is that insureds should not bear the risk associated with a specific insurer's investment strategy and, conversely, shareholders assume all of the investment risk and reward. Therefore, alternative risky investment strategies have no impact on prices in competitive insurance markets.

- For profitability measurement and other corporate **financial management** applications, Woll (1987) established the consistent and insightful principle that the profitability of an insurer's *underwriting* operations should include investment income *at risk-free rates*. The rationale is that insurers secure the use of their investment funds by writing insurance. Woll explains that "…the investment department of an insurance company functions like a bank. It takes funds provided by the underwriting operation and pays for them at risk-free rates. It invests the funds in risky assets and expects to earn a profit from the risk it is taking. … The underwriting operation gets credited for the funds it provides to the investment department at risk free rates."

Capital Allocation to Segments of the Portfolio

The major concepts to be further explored are based on the well known Capital Asset Pricing Model (CAPM). While the CAPM was the first major pricing model for financial securities, financial economics has since advanced to more sophisticated versions of the original CAPM, the Arbitrage Pricing Model (APM), and the Option Pricing Model (OPM) for derivative securities. Despite these advances, the limitations in its underlying assumptions and predictive ability, and parameterization difficulties, the CAPM is still the most widely used model in practical applications by financial decision-makers. Clearly, therefore, there is value in its simplicity and ease of application.

Previous Applications of CAPM Principles to Insurance

The earliest applications of the CAPM to insurance were collectively known as *the insurance CAPM*. The general concept behind *the insurance CAPM* was to treat insurance underwriting in a manner identical to financial assets, measuring its systematic risk with the same total market of financial assets. The practical failure of this approach lies in the essential absence of correlation between insurance underwriting returns and financial asset prices, thus providing no significant risk loading for the assumption of insurance underwriting risk. Cummins (1990) and D'Arcy and Dyer (1997) provide a thorough development of the original CAPM, its application to insurance, and explanation of its failure to be applied successfully to insurance.

In recognition of *the insurance CAPM* problems, Ang and Lai (1987) and Turner (1987) introduced an important generalization into expanded versions of the model. The new idea included in both models was the explicit recognition of the *insurance* market from the insurance industry viewpoint. This generalized the concept of systematic risk to include the covariability among insurance losses, providing a risk loading to reflect correlation of losses with the *insurance market underwriting portfolio*, even where these losses are uncorrelated with the market portfolio of traded financial securities.

While the Ang-Lai and Turner models *added* the concept of systematic *insurance* risk to the original concept of systematic *investment* risk, Feldblum (1990) advocated the *replacement* of systematic investment risk with systematic insurance risk, thus focusing entirely on insurance underwriting risk. He further argues that, unlike investors, an insurer does not have the same option of investing its capital in other securities because "…it would subject its stockholders to double income taxation…." Bault (1995) contributed the rationale that market equilibrium pricing would require using "…the *industry* portfolio, rather than an individual company portfolio..." to measure the total market return in the CAPM.

The result of this Feldblum and Bault simplification is an insurance market analogue of the traditional CAPM. Meyers (1991) applied similar principles to the development of his "competitive market equilibrium" risk load methodology. He similarly described the rational risk management behavior of insurers as an insurance market analogue of traditional CAPM principles that describe the rational behavior of investors. It is this insurance analogue of CAPM that I will examine further in this section, but unlike Feldblum and Meyers, I will also apply it as a capital allocation method.

Capital Allocation Assumptions

Our application of the insurance analogue of CAPM assumes that we have divided the total insurance industry portfolio of loss exposure into segments, or lines of business, with different risk characteristics. We will apply the method to allocate the market equilibrium required capital for the total industry to the different segments, or lines. The following additional assumptions are made:

- Consistent with the market equilibrium required capital estimated in the preceding section, only this required "underwriting" capital will be allocated. This is also consistent with Butsic's (1988) derivation of the risk-adjusted discount rate and the Myers-Cohn (1987) and Woll (1987) principles explained in the preceding section.

- Consistent with the required underwriting capital estimated in the preceding section, the capital allocated to each segment will support the first dollar, or ground-up, loss exposure in each segment. Therefore, at the total industry level, this capital will be allocated to direct business, adjusted where necessary to reflect ground-up loss exposure.

- The insurance analogue of CAPM in this section of the paper will allocate *only* the capital required to support the *prospective* ground-up loss exposure *on a common contract inception date with a common policy term* for all segments, or lines.

The capital required to support the loss and unearned premium reserves from *prior* underwriting commitments is not considered in this section, but will be revisited later. At contract inception, expected *ultimate* losses are fully at risk for all segments, or lines. Therefore, the capital allocated to each segment *at inception* should reflect the risk of the expected *ultimate* losses for each segment.

Practical Considerations

Historical financial insurance data is often problematic for several reasons:

- readily available data is generally stated on a statutory calendar year accounting basis
- readily available data of reasonable quality is generally only available at annual intervals
- price adequacy changes distort the movements in premiums and loss ratios over time
- loss reserve adequacy changes distort the movements in incurred losses over time
- current loss reserves and, thus, adequate prices are only estimates with some subjectivity

Therefore, where applicable, it is important to make reasonable estimates of the changes in premium and loss reserve adequacy to appropriately adjust historical financial insurance data. Rather than calendar year data, accident year or underwriting year data is needed to measure the true underlying results in underwriting performance. To the extent possible, loss reserves in this data should be adjusted to a consistent, adequate level for all years and lines.

The central statistic used in the application of the CAPM is the beta. In traditional securities applications, linear least-squares regression analysis is used to calculate the betas of each security. The CAPM formula requires the betas to determine the cost of capital for each security. The beta is simply the slope of the regression line in which the independent variable represents the total market price and the dependent variable represents the individual security price. Financial securities data has two major advantages over financial *insurance* data that make least-squares regression analysis feasible:

- closing securities prices are objectively determined without any subjective estimates
- closing securities prices are available every trading day, providing voluminous data

Since the beta estimate from the least-squares regression analysis directly measures the covariance between the individual security with the total market, it reflects all of the covariances between the individual security with all of the other individual securities included in the definition of the total securities market. The most important practical advantage of least-squares regression analysis is that these *individual* covariance measurements are not required to calculate the beta.

However, least-squares regression analysis is not feasible for application to financial *insurance* data. Even with accurate adjustments, the very limited number of annual data points cannot provide sufficiently reliable estimates. While the maturity of the older years provides more objectivity than recent years for lines with a development tail, this advantage is offset by their loss of relevance (i.e., the classic actuarial dilemma between stability and responsiveness).

Capital Allocation Formulas and Methodology

Since least-squares regression analysis is not feasible for *insurance* data, the application of CAPM requires that we estimate the *individual* covariances needed. In our insurance analogue of CAPM, financial securities prices are replaced by the ultimate first dollar, or ground-up, loss exposure of our selected segments or lines of insurance.

Rather than working with total industry loss *dollars*, it will be more practical to analyze losses related to a defined unit of exposure. The exposure definition should be inflation-sensitive to remove scale distortions from the historical data. *(In the remainder of this section, we will refer to* *the ultimate ground-up* *losses* *per exposure* *simply as "losses.")*

If we restrict our selected segment definitions to standard insurance lines and sublines, then the required number of individual covariance estimates should be manageable, unlike financial securities. Greater detail, of course, is always an option.

We will denote the losses for the $\mathbf{i^{th}}$ segment or line by the random variable $\mathbf{L_i}$ and the losses for the total insurance market will be denoted by $\mathbf{L_m}$. The variance of $\mathbf{L_i}$ will be denoted by $\mathbf{Var(L_i)}$, while the covariance between $\mathbf{L_i}$ and $\mathbf{L_j}$ will be denoted by $\mathbf{Cov(L_i, L_j)}$. Thus, for a total of $\mathbf{n}$ segments:

$$\mathbf{L_m = \sum_{i=1}^{n} w_i L_i} \tag{4}$$

where $\mathbf{w_i}$ ≡ percentage of exposure (weight) in $\mathbf{i^{th}}$ segment

The insurance analogue of CAPM requires that we estimate the covariances between the losses for each of the **n** segments with the losses for the total industry. This, in turn, requires that we estimate the individual covariances between the losses *for each combination* of the **n** segments. The covariance between the losses for the $\mathbf{i^{th}}$ segment, $\mathbf{L_i}$, with the total industry losses, $\mathbf{L_m}$, is estimated by the following formula:

$$\mathbf{Cov(L_i, L_m) = w_i Var(L_i) + \sum_{j \neq i}^{n} w_j Cov(L_i, L_j)} \tag{5}$$

$$\mathbf{= \sum_{j=1}^{n} w_j Cov(L_i, L_j)} \tag{6}$$

since $\mathrm{Var}(L_i) = \mathrm{Cov}(L_i, L_i)$

Since losses are scaled to a common exposure base for all segments, the relative risk between segments is indicated by a comparison of their covariances. The variance of total industry losses, $\mathbf{L_m}$, is estimated by the following formula:

$$\mathbf{Var(L_m) = \sum_{i=1}^{n} w_i^2\, Var(L_i) + 2\sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j Cov(L_i, L_j)} \tag{7}$$

$$\mathbf{= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(L_i, L_j)} \tag{8}$$

Since this total variance is also scaled to the same level as the covariance for each segment, the relative risk between each segment with the risk of the total industry portfolio is also indicated by a comparison between their covariance with the total variance. In fact, when expressed as ratio for the $\mathbf{i^{th}}$ segment, this comparison is the well known *beta* statistic of the CAPM:

$$\beta_i = \frac{\mathbf{Cov(L_i, L_m)}}{\mathbf{Var(L_m)}} \qquad \textbf{(9)}$$

In our insurance analogue of CAPM, the *beta* of each segment can be viewed as the relative *contribution* of each segment to the risk of the total insurance market portfolio of underwriting exposure. As clearly seen in formulas (5) and (7), each segment contributes to the total portfolio risk not only by its individual variance, but also by its covariance with all of the other segments. Since the denominator is identical for all segments, the relative risk contribution between segments is indicated by a comparison of their *betas*.

Gogol (1996) suggested a capital allocation methodology using the above total variance and covariance formulas that appears to be remarkably similar, but there are some differences. The most fundamental difference is that he adopted a specific portfolio framework rather than market equilibrium framework. He also defined the random variable differently to be *dollars of return*. While he did not calculate *betas* for each segment directly, he equivalently applied these formulas using dollar amounts separately for each of the segments instead of their weights. Thus, the separate dollar covariances sum to the total dollar variance of the total portfolio capital. Although his emphasis was on the risk of excess layers and he did not explicitly allocate capital, there is also similarity with some of Meyers' (1991) formulas.

Numerical Computation of Data

In order to organize the data for analytical and computational convenience, we will restate the above formulas in more detail based on the following formula:

$$\mathbf{Cov(L_i, L_j)} = \rho_{ij}\sigma_i\sigma_j \qquad \textbf{(10)}$$

where $\sigma_i \equiv$ standard deviation of $\mathbf{i}^{th}$ segment, and

$\rho_{ij} \equiv$ coefficient of correlation between the $\mathbf{i}^{th}$ and $\mathbf{j}^{th}$ segments, and

Therefore, from formulas (6) and (10), we can restate formula (6) as follows:

$$\mathbf{Cov(L_i, L_m)} = \sum_{j=1}^{n} \mathbf{w_j}\, \rho_{ij}\sigma_i\sigma_j \qquad \textbf{(11)}$$

From formulas (8) and (10), we can restate formula (8) as follows:

$$\mathbf{Var(L_m)} = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathbf{w_i}\,\mathbf{w_j}\,\rho_{ij}\sigma_i\sigma_j \qquad \textbf{(12)}$$

We can facilitate the above covariance calculations by constructing vectors and a correlation matrix representing the **n** segments as follows:

$$\begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \\ \vdots \\ \mathbf{w}_n \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \\ \sigma_n \end{pmatrix} \begin{pmatrix} 1.0 & \rho_{12} & \rho_{13} & \dots & \rho_{1n} \\ \rho_{21} & 1.0 & \rho_{23} & \dots & \rho_{2n} \\ \rho_{31} & \rho_{32} & 1.0 & \dots & \rho_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \dots & 1.0 \end{pmatrix}$$

Segment Variance and Correlation Analysis

There is some question as to the significance of covariance between the losses for different segments of insurance exposure. Price movements of financial securities tend to be highly correlated with the average price movement for the total market of financial securities, but this is generally not the case for different segments of insurance losses.

For many segments, their historical premium adequacy is generally more highly correlated than their ultimate losses because of the nature of insurance market price competition. Thus, the question arises as to whether we should reflect premium adequacy in the methodology. At least within the market equilibrium framework, neither premium adequacy nor the correlation in premium adequacy between different segments is relevant to capital allocation for several reasons:

- The objective of **pricing** is to *prospectively* determine the appropriate level of prices. Historical price adequacy is not relevant to future capital requirements and the determination of future target price levels.

- Historical **profitability measurement** directly reflects the results of actual historical premium adequacy, but the historical required capital is not affected by this premium adequacy.

- Premium adequacy is arguably *not* a random variable.

- At the option of management, it may be relevant to corporate planning and dynamic financial analysis (DFA), but the future required capital is not affected by future premium adequacy.

Therefore, if premiums were used as the exposure base for the loss random variable defined above, they would need to be adjusted not only for inflation, but also to a *common level of price adequacy* for the purpose of calculating both variances and covariances. This latter adjustment would be needed to focus our analysis on only the risk in the expected losses.

In the previous section, we explained the need to discount losses to estimate their market value. This discounting of the losses complicates the meaning and measurement of their variances and covariances. Therefore, we need to more precisely define the random variable for which we are estimating the variances and covariances before we can assess their significance and how we can approach the estimation process.

As discussed in the previous section, our analysis assumes a risk-free investment portfolio. While the market value of losses is estimated using a *risk-adjusted* discount rate, this rate is fully consistent with the risk-free investment portfolio. The discussion in the previous section implied that asset risk was completely discarded from our model, but there is actually one element of asset risk that remains.

The conventional use of the term "risk-free" for assets refers to the risk of default, or loss of principal. Even for such "risk-free" assets, however, there remains a risk of market fluctuation in the level of risk-free rates. Increases in the level of risk-free interest rates will still reduce the market value of the risk-free assets and, thus, the economic capital. The balance sheet may be fully **immunized** by cash flow or duration matching of assets with the liabilities, but immunization doesn't reflect the dynamics of the *liabilities*. Ferguson (1983) explains immunization concepts in more depth.

The underwriting view of this risk is that, to the extent that such rate increases are inflation-driven, nominal losses may increase to higher than expected amounts when actual inflation exceeds expected inflation, while the cash flows from the bonds intended to fund these losses remain constant. To address this risk, I propose the following assumptions and corollaries of the aforementioned principles established by Woll (1987) and Myers-Cohn (1987):

- Just as *risk-free* investment income should be credited to *underwriting* operations, the market fluctuation risk of risk-free rates *for a fully immunized balance sheet* should be considered a part of underwriting risk. The immunization can be defined on either a cash flow or duration-matched basis.

- Just as *risk-free* investment income is assumed regardless of an insurer's actual investments, this market fluctuation risk will assume a *fully* immunized balance sheet, regardless of the insurer's actual degree of immunization.

To reflect this risk as a part of underwriting risk, we estimate the correlations based on the relationships between the ultimate *nominal* loss dollars for different segments. This reflects the inflationary risk that the nominal fixed income dollars provided by the assets in a fully immunized balance sheet fall short of the ultimate loss dollars resulting from higher than expected inflation. Thus, the losses are *not* discounted for the purpose of calculating the variances, correlations, and betas.

While there is generally little causality in relationships between the losses for different segments, our expanded definition of underwriting risk certainly introduces a source of correlation. This source of correlation is the inflation that operates on the ultimate *nominal* loss dollars in all segments. The losses for many different segments tend to be similarly impacted by inflation, particularly within certain major categories such as property and liability.

The most important variable determining the significance of these correlations is the *duration* of the expected losses, or their average length of time from occurrence to payment. The reason is simply that the full impact of expected inflation operating on the ultimate nominal loss dollars is proportional to their duration. Therefore, we generally expect higher correlations between long-tail casualty lines and lower correlations for combinations involving short-tail property lines.

The expanded definition of underwriting risk may appear to be at odds with our definition of "losses." We had defined losses relative to an inflation-sensitive exposure base, thus removing the impact of inflation on nominal losses, which is now the major source of correlation between segments. The intention of an inflation-sensitive exposure base is to remove scale distortion from data *for different years*. Such an adjustment is needed to estimate variances (and standard deviations) from data over a long historical period, but the losses would not be discounted using our model.

The estimation of correlation between segments requires a much different analysis. If we used historical data to estimate these correlations, then because of the relationships being measured, we would *not* use the same inflation-adjusted losses. However, we would not attempt to estimate these correlations using historical insurance data for the same reasons that least-squares regression analysis is not feasible for application to insurance data.

Under our expanded definition of underwriting risk, correlations between segments would best be estimated using a macroeconomic-based model that reflects the inflation variables that are relevant to insurance losses. Dynamic financial analysis (DFA) models generally include such a macroeconomic component that simulates both interest rates and inflation rates. Any other source of correlation in the losses would need to be included in a DFA model as well. Thus, while the correlations between segments may not be directly calculated in standard applications of the DFA model, they can be calculated using sufficiently detailed simulation output from such a model.

The structure of this modeling process is also ideally suited to the application of informed judgment. While least-squares regression analysis would certainly be convenient, all of the detail in the underlying relationships is omitted or obscured, forcing analysts to evaluate the reasonability of the final betas without such detail. Our model structure requires a more tedious process of analyzing the relationships between each pairing of segments defined for the model. However, in conjunction with the easily understood intuitive meaning of a coefficient of correlation ranging between -1.0 to 1.0, the advantage is that actuarial, underwriting, and claims professionals can intuitively judge their reasonability.

Application of Leverage Ratios

The previous section explained that, in the market equilibrium framework, the same relationships determined at the total industry level are applied to individual insurers. Clearly, the use of industry data requires the determination of factor relationships or percentages for application to individual insurers. The critical relationship in this methodology is the leverage ratio estimated using the type of data shown in Exhibits 1 and 2.

Several authors, such as Bault (1995) and Philbrick (1994), have discussed an alternative in the method of reflecting the risk differences between segments. For segments with greater (lower) risk, the alternatives that have been cited to reflect this greater (lower) risk are to either:

- increase (reduce) the required return on equity (ROE) target on a fixed amount of capital, or
- increase (reduce) the amount of required capital while fixing the required ROE target

Bault and Philbrick both chose to hold the required ROE target constant while varying the required amount of capital to reflect the risk in the expected losses. For several reasons, I agree that this is the more practical approach, not the least of which is the difficulty in defining "fixed amount of capital" for different segments. Bault (1995) treated this mechanically by varying the leverage ratios applied to different segments.

The proposed methodology determines the required capital by applying the appropriate leverage ratio to the *discounted* expected losses. Since I have agreed to hold the ROE constant, it might initially appear that the only option for reflecting differences in risk is to vary the leverage ratio. However, the binary choice between the ROE and leverage ratio is an oversimplification. Formula (3) reveals that, even at the total industry level, the leverage ratio used in this methodology is dynamic, varying with the risk-adjusted discount rate.

Therefore, *even with a constant ROE,* a further choice must be made to mechanically differentiate between the risk of the segments:

- *The leverage ratio can be held constant while the risk-adjusted discount rate varies.* With this approach, the discounted losses would vary with the risk-adjusted discount rate. The required capital would vary appropriately by applying the constant total industry leverage ratio to greater discounted losses for higher risk segments and lower discounted losses for lower risk segments.

- *The leverage ratio can vary while the risk-adjusted discount rate is held constant*. With this approach, the discounted losses would remain constant. The leverage ratio applied to the constant discounted losses would vary appropriately to generate more capital for higher risk segments and less capital for lower risk segments.

The proposed methodology measures the differences in risk between segments by the estimated *beta* for each segment. It is clearly more straightforward to apply the *beta* as an adjustment to the leverage ratio than to the risk-adjusted discount rate. The latter alternative would require iteration since the appropriate differences in the risk-adjusted discount rates would not be proportional to the *betas*.

Therefore, the proposed methodology uses the same risk-adjusted discount rate determined at the total industry level to discount the losses for all segments and applies the beta for each segment as an adjustment to the total industry leverage ratio. The leverage ratio for the $\mathbf{i^{th}}$ segment is calculated as:

$$\textbf{Leverage Ratio}_i = (L/S)/\beta_i \tag{13}$$

where

L and S are defined in formula (3), and
β_i is defined and calculated in formula (9)

Formula (9) clearly demonstrates that the beta for the total industry **(β_m) = 1.0,** appropriately resulting in the total industry leverage ratio and confirming the additivity of the methodology. The application of the different leverage ratios by segment will result in different total leverage ratios for different insurers. This is appropriate because of their differences in loss exposures, although there may appear to be some "circularity" with the total required capital analysis in the previous section. Indeed, this difference in loss exposure between different insurers represents one of the difficulties in estimating the total required capital for the total insurance market.

Numerical Examples

In his numerical example, Feldblum (1990) used statutory line of business definitions. Despite the required adjustments to the data for the proposed methodology, this is a natural choice of definitions since consistent data sources are readily available. At least two reports of the American Academy of Actuaries Property-Casualty Risk-Based Capital Task Force (1993) used similar line of business definitions and data sources with considerable adjustments to the data.

One issue that did not receive adequate attention in Feldblum's example and elsewhere is the impact of property catastrophe exposure on the covariances and betas for property lines. In years with high incidence of natural disasters, insured catastrophe losses will be high, increasing the losses for property lines and all lines in total to higher than normal levels. This suggests that property lines may tend to be more highly correlated with total industry losses than other lines, but this high correlation is driven entirely by the catastrophe exposure.

While it may not be necessary for a total industry analysis, application of the proposed methodology to individual insurers makes it critical to separate catastrophe exposure from non-catastrophe exposure in the segment definitions used. This provides the flexibility needed to reflect their considerable variation in geographical concentration of property exposure and its resulting catastrophe potential. It also provides for different capital allocation treatment of catastrophe exposure as an option.

We now have the assumptions, formulas, and definitions needed to provide numerical examples of the methodology. For greater clarity, the examples shown will exclude property catastrophe exposure from the analysis. Therefore, the resulting betas are not applicable to the total industry leverage ratio, unless it has been adjusted to remove property catastrophe exposure. Property catastrophe exposure issues will be revisited after the numerical examples.

For illustration, we have defined a total of 17 segments, or lines of business for which different betas and leverage ratios are to be calculated. Exhibit 3 shows the covariance and beta calculations under the assumption that all of the segments are completely independent. It can be seen at the bottom of column 3 that this independence results in a total industry standard deviation that is much lower than the individual standard deviations for any of the lines.

At first glance, some of the betas in Exhibit 3 might appear unusual or unexpected. For example:

- The beta for workers compensation (WC) is slightly higher than the total portfolio beta, but its standard deviation is lower than every other commercial line.

- Highly protected (HPR) and technical property risks have one of the lowest betas, but one of the highest standard deviations among all of the lines.

This could happen if the correlation coefficients between WC and other lines are generally very high while the correlation coefficients between HPR and other lines are generally very low. In this example, however, all of the lines are independent. Nevertheless, WC is in fact more highly correlated with total losses than HPR. The reason is that WC comprises more than 10% of total losses, while HPR is less than 0.5% of losses. Therefore, the volatility of WC has a significant influence on the volatility of total losses simply by virtue of its weight.

We can see the impact of the line weights by comparison of these results with a scenario in which all of the lines are equally weighted. Exhibit 4 shows the same covariance and beta calculations under the assumption that all of the segments are completely independent, or uncorrelated, but also equally weighted. These results may appear somewhat more intuitive since the resulting betas are directionally consistent with the standard deviations for each line. Closer inspection reveals that the covariances and betas for each line are directly proportional to their *variances*.

In contrast, Exhibit 5 shows the covariance and beta calculations under the assumption that all of the segments are completely dependent, or perfectly correlated, and also equally weighted. In this case, the covariances and betas for each line are directly proportional to their *standard deviations*. Although he didn't calculate betas, the results in Exhibits 4 and 5 are consistent with the Bault (1995) formulas for the cases of total independence and dependence. In the case of total dependence, the covariances and betas for each line remain directly proportional to their standard deviations regardless of their weights, but the scale may shift.

These examples raise serious questions about the weights that should be used and illustrate their significance. The market equilibrium framework and CAPM theory would suggest that actual industry weights should be used to reflect the true total market loss "portfolio." However, in securities applications, equal weighting of securities is also commonly used as an analytical alternative to market capitalization weighting for both the construction of indexes and calculation of betas. For example, while the S&P 500 index is market capitalization weighted, the Dow Jones Industrial Average is equally weighted. We will discuss these issues further with more realistic examples.

Exhibit 6 shows an excerpt of a correlation matrix that has been constructed for the proposed methodology under the expanded definition of underwriting risk discussed earlier. It can be seen that the correlation coefficients are generally higher for pairs of lines which both have long durations. Exceptions can be seen for the auto physical damage coverages that have higher correlation with their liability counterparts. This same correlation matrix will be used for all of the examples that follow.

Exhibit 7 shows the covariance and beta calculations using this correlation matrix with industry weights by line. By comparison with the results of total independence in Exhibit 3, we can see the impact of the correlation between lines. While these correlation coefficients appear to be relatively low, they increase the total industry standard deviation from 2.45% to 2.9% and it can be seen that some of the differences in the betas by line are not insignificant.

Exhibit 8 shows the covariance and beta calculations with the same correlation matrix using *equal* weights by line. The resulting betas differ significantly from the industry-weighted betas. When these betas are weighted using the actual industry weights by line, they no longer average to 1.0. Since several of the largest lines (personal auto liability, personal auto physical damage, and workers compensation) have betas significantly below 1.0, the average beta is also below 1.0 (in this example, .56).

Recall that the betas are used in Formula (13) as an adjustment to the total industry leverage ratio **(L/S)** to calculate the leverage ratios by line. Since **(L/S)** should reflect the actual industry weighting of lines, the betas should average to 1.0 so that the resulting leverage ratios by line average to the total industry leverage ratio. Therefore, if the betas in the equally weighted scenario shown in Exhibit 8 were used, they would need to be rescaled as shown in column 6 of the exhibit.

Segment Definitions and Weighting Issues

The "pure" insurance analogue of CAPM would use the actual industry weights and results in Exhibit 7, but there are some problems with its assumptions. Strict application of CAPM theory assumes that the insurance market is efficient and that insurers are "...free to use an insurance analogue of Markowitz diversification to produce a portfolio of insurance contracts," as described by Gogol (1996). With full awareness of these assumptions and their limitations, a practitioner can certainly gain some insight from this strict version of the model.

Many would prefer to reflect more realistic assumptions about the insurance markets in the model. Empirical observations that contradict the assumptions underlying the strict version of the model are:

- Most insurers do not significantly participate in all, or even most, market segments of coverages. Even the largest multiline insurers, for example, strongly emphasize either personal or commercial lines.

- Insurers of HPR and technical property risks would not price these exposures with lower risk loads than personal auto risks, as suggested by their betas in Exhibit 7. This is one example of a situation in which a small, highly volatile specialty coverage is a separately defined segment.

These observations suggest that there are complexities and inefficiencies in the real insurance markets that are not described by this strict version of the model. It can be argued that the total insurance industry is not merely one large market, but rather a collection of multiple separate and distinct insurance markets with various degrees of distinction in coverages, insurers, and distribution channels. It is difficult to identify the cause and effect logic, but high transaction costs may be a barrier to convergence of these separate markets. To the extent that this describes reality, insurers do not fully diversify across the total market and the strict efficient market model in Exhibit 7 oversimplifies the insurance market.

Despite these complexities and inefficiencies, it may be possible to model a more realistic market equilibrium using a similar model structure. Some would argue that the results of equally weighted lines in Exhibit 8 more realistically describe the capital allocation and pricing behavior of insurers. One interpretation of the equal weighting is a marginal comparison of the risk between equal amounts of additional exposure in the different lines. While correlations are reflected in this particular equal weight model, the emphasis is on a comparison between risks rather than their diversification.

The equal weight model might seem to be a reasonable description of insurer behavior for coverages that fall into the distinctly different markets described above, since such coverages are generally not diversified with each other. This also suggests a hybrid approach for modeling the total insurance market in which the weights between the lines would be based upon the structure of the multiple insurance markets. A simplified hybrid model might equally weight segments that are considered separate and distinct markets, while using actual industry weights for different lines within each distinct segment.

An example of this simplified hybrid version of our model is shown in Exhibit 9. In this example, the 17 lines of business are sorted into 8 distinct markets, which are equally weighted. Six of these segments are individual lines, while the remaining lines fall into either a "standard" commercial or personal lines (excluding non-standard auto) segment. Again, the rescaling of betas is required in this model. Since the distinct markets are not *completely* separate, more complex weighting techniques may be appropriate.

An important problem is that the betas and, thus, leverage ratios are dependent on the *definitions* of the segments, which are subjective to some extent:

- When *actual industry* weights are used, changes in segment definitions with consistent changes in the standard deviations and correlations maintain total market balance. However, the betas applicable to many individual risks can change with these definition changes. Much of the change is attributable to the weight of the applicable segments.

- When *equal* weights are used, changes in segment definitions will usually change the total number of defined segments. The addition or combination of segment definitions will change the betas for all segments, despite the rescaling required to balance the average beta.

Traditional securities applications of CAPM do not have such definition problems in practice because the publicly traded entities are objectively defined and the impact of their weights is reduced by both the very large number of securities and the generally high correlation coefficients underlying the betas.

This sensitivity of the betas to segment definitions requires the exercise of discipline in constructing the definitions. Criteria should be established against which proposed definitions are evaluated on an objective and consistent basis. Entirely different coverages or types of exposure to loss would clearly meet any reasonable criteria, but different classes of risks for the same coverages are problematic. Not only can different classes number in the thousands, but most classes are typically well diversified within a larger segment of coverage. Therefore, the evaluation criteria need to place significant restrictions on the acceptable size and uniqueness of classes to be candidates for separate definition.

Parameter Uncertainty

There is some question as to whether this methodology reflects parameter uncertainty. Feldblum (1990) suggests that *process* risk in insurance is analogous to *diversifiable* risk in the CAPM, while *parameter* risk in insurance is analogous to *systematic* risk in CAPM. Since systematic risk in CAPM is measured by beta, this might imply that the betas calculated in our insurance analogue of CAPM reflect parameter uncertainty. On the other hand, Bault (1995) suggests that parameter risk needs to be separately measured and included.

Feldblum's analogy has some merit, but it is not perfect. In securities analysis, for example, diversifiable price fluctuations in the individual securities are reflected in the beta calculations. In our insurance analogue of CAPM under the market equilibrium framework, process risk in the data is only reflected at the industry level. High correlations between securities drive their covariances and betas, but the correlations between insurance segments are relatively low, thus their individual variances, or process risk at the total industry level drives their betas. A rough illustration of this distinction is the comparison between the total independence and total dependence examples in Exhibits 4 and 5.

I consider this issue to be unresolved, but I am skeptical that this methodology reflects insurance parameter uncertainty, or the risk in the estimate of *expected* losses and their distribution for each line. There would appear to be some relationship between the parameter and process risks for each line. Highly volatile lines would also tend to have more parameter uncertainty, but parameter risk differences would tend to be less pronounced than process risk differences. Therefore, if parameter uncertainty needs to be separately included, it would have a dampening effect on the betas. An example of this type of effect using the industry weighted model in Exhibit 7 is illustrated by the rescaled square root of the betas in Exhibit 10.

Theoretical Support

Cummins (1990) provides the following relevant theoretical background: "Markowitz diversification forms the foundation for the CAPM. Since Markowitz diversification considers only the means and variance of asset returns, it is often called mean-variance diversification…mean-variance diversification is usually justified through the assumption that asset return distributions are multivariate normal. Although the lognormal provides a better empirical model of security returns than the normal, the normality assumption is often adequate as an approximation."

This theory is also applicable to our insurance analogue of CAPM. Like asset returns, aggregate loss distributions for most standard insurance coverages are approximately lognormal *for ground-up exposure*. Thus, our assumption of ground-up loss exposure not only makes the losses comparable for each segment, but also provides similar justification for the lognormal assumption and mean-variance diversification for insurance losses.

The validity of these assumptions is strongest at the total industry level for standard lines of business. Thus, the proposed methodology is ideally suited to the market equilibrium framework using total industry data. However, even at the total industry level, the aggregate loss distributions for some coverages cannot be approximated by a lognormal distribution. The most significant coverage for which this is a problem is **property catastrophe** exposure.

The availability of standard modeling software makes the mechanics of the methodology even easier to apply to property catastrophe coverage because historical data is not required. Means and variances are generally provided as standard output and correlations between any two property lines and regions can be calculated from the model output. However, mean-variance diversification is clearly problematic for the extremely skewed property catastrophe loss distributions.

This problem can also be framed in terms of a comparison between volatility and strictly "downside" risk. This distinction is largely irrelevant for a normal distribution since downside risk can be defined as a function of the same standard deviation that measures its volatility. While the distinction is not irrelevant for lognormal distributions, it is generally not very material. However, it is both relevant and material to property catastrophe risk, which may be appropriately viewed from the downside perspective.

Although he defined risk from a downside, ruin-theoretic perspective, Kreps (1990) derived simplified formulas for his specific portfolio methodology, which assume normal distributions with perfect correlation between risks. These same formulas are sometimes inappropriately applied to calculate risk margins for property catastrophe exposure. Although the formulas would become more complex, the oversimplifications must be relaxed to realistically model property catastrophe risk.

Detailed discussion of "downside" methods and application of catastrophe model output to individual exposures is beyond the scope of this paper. If a separate catastrophe method is used, it should be integrated with the insurance analogue of CAPM for non-catastrophe lines in the following general way:

- Property catastrophe exposure should be reflected at the total industry level.
- Total capital would first be allocated between total catastrophe and total "non-cat" exposures.
- The separate methods would then be applied to further allocate the capital supporting total catastrophe exposure and the capital supporting total non-cat exposure.

Alternatives to the actual industry weights by segment depart from modern portfolio theory in the same manner as practical applications of securities analysis using equal weights. The primary motivation for these alternatives is to attempt to more realistically model the insurance markets with some inefficiency, but they can also provide an alternative perspective as additional decision-making information.

Market inefficiency related to the segment weights might appear to represent arbitrage opportunities, but this would depend upon the significance of the transaction cost barriers for these segments. Prohibitive transaction costs required to enter a new segment would prevent such arbitrage.

Insurers that manage their capital to market equilibrium levels can apply the above model with alternative segment weights to evaluate arbitrage potential related to such inefficiency. The different weights can be used to estimate the differences between inefficient and efficient capital requirements, while transaction costs associated with entering new segments would also need to be estimated.

Venter (1998) suggested that "insurers may want to try to build arbitrage possibilities into prices… Exploiting arbitrage opportunities is usually regarded as improving market functioning, as it tends to compete away those opportunities." Therefore, as industry dynamics lead to changes in market structure over time, the same inefficiencies and arbitrage opportunities would not always exist.

Model parameters and even segment *definitions* may require occasional revision to reflect changes in market structure. Industry dynamics also lead to different opinions about current market structure. While *temporary* arbitrage opportunities may occasionally arise in insurance (and other) markets, a no arbitrage-constrained model that reflects some actual market inefficiency is a realistic and useful tool for decision-making.

Release of Capital and Reconciliation of Total Required Capital

The methodology in the previous section allocated *only* the capital required to support the *prospective* loss exposure on a common contract inception date for all segments, or lines. The total required capital at a particular valuation date supports all expected losses on that date. These expected losses include:

- the ultimate loss exposure from *prospective* underwriting commitments
- the ultimate loss portion of unearned premium reserves from *prior* underwriting commitments
- the outstanding incurred loss reserves from *prior* underwriting commitments

All of these categories were included in our previous estimate of total required capital, which was summarized by the *adjusted economic leverage ratio* in Table 1. The need for total required capital to support the expected losses from *prior* underwriting commitments leads to the additional dimension of time as an essential part of capital allocation.

The commitment of capital at contract inception in the previous section is not the end of the allocation process. Capital is also allocated to the outstanding loss exposures from *prior* underwriting commitments in different time periods. This time dimension of capital allocation can be described as follows:

- Required capital is committed at contract inception to support *ultimate* expected losses.
- This capital is released over the length of time that these losses are paid.
- The unreleased capital at a valuation date supports the outstanding *unpaid* losses on that date.
- The required amount of released capital is used to support *new* prospective loss exposures.
- Any remaining released capital is excluded from market equilibrium total required capital.

This complete capital allocation process generates a set of capital flows over time that can be reconciled with total required capital. Bingham (1993) and Philbrick (1994) also describe this process and some related concepts. In actual practice, the remaining released capital in the last step would either be returned to shareholders or retained as *excess* capital. While retained excess capital would be separately identified by this analysis, it would not be reflected in the market equilibrium indications of this methodology.

Since unreleased capital supports outstanding *unpaid* losses, the specific pattern of these capital flows varies by the segments defined for capital allocation. The capital allocated to short-tail lines will be released quickly because the ultimate losses are paid quickly, while the capital allocated to long-tail lines will be released more slowly. Therefore, the pattern of capital flows will also vary by specific insurer because of their different mixes of segments.

Total Capital Commitment

The allocation of capital at contract inception in the previous section is not sufficiently complete for application to pricing or profitability analysis. The pricing of a contract or profitability measurement of a segment for an underwriting year requires an estimate of the *total capital commitment* to the contract or segment underwriting year. This total capital commitment requires an estimate of the complete pattern of capital releases subsequent to inception.

To illustrate this concept, consider a very long-tail line with *ultimate* expected losses that have the same risk as a short-tail line and, therefore, the same leverage ratio. Using the method in the previous section, an insurer that assumes an equal amount of ultimate expected losses in both lines will allocate equal amounts of capital to both lines *at inception*. However, the capital allocated to the long-tail line cannot be released nearly as quickly as the capital allocated to the short tail line.

At the time that the last loss is paid for the short-tail line, most of the ultimate expected losses for the long-tail line may remain unpaid. While all of the capital allocated at inception will have been released for the short-tail line, most of the capital for the long-tail line may still be held for several years to support the unpaid losses. Therefore, although equal amounts of capital were allocated to both lines *at inception*, the long-tail line requires a greater total capital commitment than the short tail line.

In addition to the method of the previous section to allocate capital at inception, a method for the release of capital is needed to estimate the complete pattern of capital releases subsequent to inception. The total capital commitment needed for application to pricing or profitability analysis can be determined from the combined application of these methods.

The most straightforward method for the release of capital is the assumption used by Myers and Cohn (1987) which releases the capital allocated at inception in direct proportion to the payment of losses. This is equivalent to applying the same leverage ratio used to determine the capital allocated at inception to the *unpaid* losses at any valuation date to determine the remaining capital held to support those unpaid losses.

Exhibit 11 shows an example of a prospective stream of expected loss payments and the required capital flows for these expected loss payments. At inception (t=0), expected *ultimate* losses are discounted at the risk-adjusted rate of $\mathbf{i_a}$ and the leverage ratio is applied to determine the capital allocated at inception. The same calculations are made at each subsequent annual point in time to determine the capital required to support the expected *unpaid* losses at those points in time.

The expected losses subsequent to inception in Exhibit 11 are not discounted to inception (t=0) at the risk-adjusted rate of $\mathbf{i_a}$ because these calculations determine expected *future* capital amounts required to support the unpaid losses at the same future points in time. For prospective application as a pricing target, *the return on equity (ROE) must be measured against all of these expected future capital amounts*. Retrospective application to profitability analysis would measure the ROE by combining the allocated capital flows from actual historical loss payments with the expected *future* capital amounts supporting *future* unpaid losses. The time interval used for the calculations in Exhibit 11 must equal the time period for which the ROE target or estimate is to be established.

The sum of the required future capital amounts at annual intervals can be considered a measure of the *total capital commitment* for an annual ROE target or measurement. However, this sum would generally not be used in practice because the individual annual capital amounts are not valued at the same point in time. One of two alternative methods of structuring the calculations would generally be used in practice:

- The dollars of separate *future* returns corresponding to the separate *future* required capital amounts would be individually measured and discounted back to inception.

- The separate *future* required capital amounts would be discounted back to inception, summed, and the dollars of total present value return would be measured against this total.

The specifics of the application may determine which computational method is more convenient. This discounting process can be viewed as adjusting for the expected risk-free investment income earned by the return in the premium. Therefore, the *risk-free* rate would be used in this latter discounting process rather than the risk-adjusted rate.

Ferguson (1983) defined **duration** as the "weighted average term to maturity where the years are weighted by the present value of the related cash flow." D'Arcy (1984) explained that "the related cash flow" in this formula can be loss payments. Let $\text{duration}_{\mathbf{i}}$ be the duration of the expected loss payments for the $\mathbf{i}^{\mathbf{th}}$ segment. The leverage ratio for the $\mathbf{i}^{\mathbf{th}}$ segment was defined in Formula (13).

If we define the total capital commitment as the *discounted* sum of the required future capital amounts discussed above, the release of capital in proportion to loss payments generates a total capital commitment that is proportional to ($\text{duration}_{\mathbf{i}}$/$\text{leverage ratio}_{\mathbf{i}}$). The leverage ratio represents the impact of the capital allocated at inception, while the duration represents the impact of the subsequent release of this capital. Since the ROE is measured against the *total capital commitment*, the application of a complete ROE methodology to individual segments requires both:

- a methodology for capital allocation at inception
- a method for the subsequent release of this capital

Enhancements to the Method for Release of Capital

The capital committed at contract inception supports the risk of expected *ultimate* losses, but as this capital is released, the remaining unreleased capital supports the risk in the expected *unpaid* losses at a later valuation date. It is not clear that the leverage ratio used to determine the capital supporting the unpaid losses subsequent to contract inception should be the same as the leverage ratio at inception.

Possible enhancements to the simple release of capital in proportion to the payment of losses are discussed in Butsic (1988) and Philbrick (1994). The risk of *future* expected losses, including the expected loss portion of the unearned premium reserve, would generally be considered greater than the risk of the loss reserves on past occurrences. Moreover, the risk of IBNR loss reserves would generally be considered greater than the risk of reported loss reserves.

Reflecting such risk differences would result in different total (i.e., all segments) leverage ratios for these different loss categories of expected losses, which would require segregation of the industry data and total leverage ratio into these separate categories. The leverage ratio applied to future expected losses would be the lowest, while the leverage ratio applied to reported expected losses would be the highest.

Regardless of the level of detail and methods used to measure these risk differences, the same method that was summarized in Table 1 can be used to estimate the total economic leverage ratio. Different leverage ratios applicable to various categories of future and past occurrence losses would still average to the same total adjusted economic leverage ratio in Table 1. An example of how this approach would compare to the releasing capital in proportion to the payment of losses is summarized in Table 2.

Table 2: Alternative Capital Release Methods

		Release of Capital Method	
Balance Sheet Expected Losses		*Proportional to Loss Payments*	*Future Risk> Prior Occ Risk*
(1) Discounted Loss Reserves		**148,003,973**	**148,003,973**
(2) Economic Leverage Ratio		**4.31**	**4.96**
(3) Required Economic Capital	**(1)/(2)**	**34,351,539**	**29,839,511**
Future Expected Losses			
(4) Future Discounted Expected Losses (One Year)		**51,801,391**	**51,801,391**
(5) Economic Leverage Ratio		**4.31**	**3.13**
(6) Required Economic Capital	**(4)/(5)**	**12,023,039**	**16,535,068**
Total Expected Losses			
(7) Total Discounted Expected Losses	**(1)+(4)**	**199,805,364**	**199,805,364**
(8) Total Required Underwriting Capital	**(3)+(6)**	**46,374,578**	**46,374,578**
(9) Total Economic Leverage Ratio		**4.31**	**4.31**

Using the example in Table 2, the lower leverage ratio applicable to future expected losses (3.13) would be used as the leverage ratio **(L/S)** in Formula (13) to determine the leverage ratios by segment. These resulting segment leverage ratios would be used only to allocate capital to each segment *at inception*. The leverage ratios applicable to unpaid losses *subsequent to inception* would increase to reflect the reduced risk of the loss reserves subsequent to their occurrence period.

An alternative to the release of capital in proportion to the payment of losses would alter the relationships between the pattern of capital flows by segment and their total capital commitments would no longer be directly proportional to ($\text{duration}_i/\text{leverage}_i$). Nevertheless, their durations would continue to be a major influence on the total capital commitment conceptually. In order to balance to the total economic leverage ratio, an iterative process may be needed to simultaneously reflect the leverage ratio differences by the categories summarized in Table 2 and the differences by segment throughout their durations.

Any of the capital release methods suggested, in conjunction with the complete capital allocation methodology, will generate a very substantial total capital commitment for a long-tail line which is also risky (i.e., has a low leverage ratio). Although most of the ultimate expected losses are at risk for several years, the question arises as to whether the indicated capital commitment is too burdensome since a significant portion of the loss payments are deferred for many years. There may be some inefficiency in this area of the methodology, but this issue is unresolved.

The need to account for the release of capital, or the time dimension of capital allocation, is not obvious using a traditional actuarial risk load approach to pricing, which would generally focus *only* on the risk of expected *ultimate* losses. This is one of the major advantages offered by the capital allocation perspective.

Extension to Excess Layers, Other Contract Provisions, and Asset Risk

The capital allocation methodology in the previous sections established the capital allocated to ground-up loss exposure. Coverage for excess of loss layers and other contractual modifications of this exposure would require further adjustments. The insurance analogue of CAPM cannot easily be extended to such modifications for both theoretical and practical reasons:

- The aggregate loss distributions resulting from these modifications depart significantly from the lognormal distribution needed to theoretically support mean-variance diversification.

- Despite infinite contractual possibilities, specific excess layers or other contract provisions would need to be defined *in advance*. Supporting data would need to correspond to these definitions.

- The data and parameter estimates required to reflect these contractual provisions would be much more difficult to obtain and would require a more tedious analysis.

A more practical alternative was suggested by Venter's (1991) extension of the no-arbitrage requirement of price additivity to excess layers. The capital allocation methodology presented established the total capital allocated to each segment *for all layers*. To further allocate this capital to separate excess layers, an additive methodology would be consistent with the market equilibrium concept underlying the insurance analogue of CAPM methodology.

Venter demonstrated the effectiveness of using mathematical transformations of the loss distribution as a general additive methodology for excess of loss layers. Based on Venter's analysis, Wang (1995) later proposed the proportional hazards (PH) transform as a specific transform with desirable properties. Wang (1998) provides more detailed insight into various practical implementation issues pertaining to the PH transform. Venter (1998) contributes further insight and also suggests some alternative variations. In addition to excess layers, transforms of the loss distribution can be used to adjust allocated capital for various other contract provisions.

Since excess layers and other contract provisions can also have a significant impact on the loss payment patterns, they will also alter the release pattern of the capital allocated at inception and, thus, the total capital commitment. These effects introduce considerable complexity into the estimation of market equilibrium parameters, since the availability of industry data for contract provisions is very limited. Therefore, judgment will play a major role in this part of the analysis.

Asset Risk

The insurance analogue of CAPM can be logically and easily extended to reflect asset risk. This would simply require an expansion of the correlation matrix to include various segments of assets in addition to the insurance underwriting segments. These asset segments would reflect as many classes of equity and fixed income investments as needed to adequately reflect the covariance relationships in this expanded structure of the model.

Lamm-Tenant (1996) applied this general approach to evaluate the economic rationale of the NAIC Model Investment Law. Although calendar year statutory underwriting data is used, Lamm-Tenant provides a complete numerical illustration of a correlation matrix which includes both the statutory insurance lines and several asset segments. Through the covariance estimates between the insurance underwriting segments and asset classes, such an expanded version of the methodology would also be conceptually similar to the Ang-Lai (1987) and Turner (1987) models.

The betas calculated for the asset segments would be used to determine the capital required to support asset risk. Recall, however, the market equilibrium concept that alternative risky investment strategies have no impact on prices in competitive insurance markets. The need to reflect asset risk for the methodology in this paper is to segregate the *total industry* capital supporting asset risk from the capital supporting underwriting risk. This segregation of capital was assumed in the "total required capital" section of the paper. With the aggregate "underwriting" capital separately identified, the proposed methodologies can then be applied as outlined to underwriting segments only.

Summary and Conclusion

The market equilibrium capital allocation process described in this paper is comprised of several distinct parts. The major parts of this process can be summarized as follows:

- the estimation of total required underwriting capital for ground-up loss exposure
- the allocation of this total required capital to portfolio segments at inception
- the release of capital allocated at inception to estimate the total capital commitment
- adjustments to reflect excess layers and other contract provisions

This entire process cannot be adequately modeled using a single methodology. An integrated set of consistent methods is required to model these distinctly different parts of the process. There are several practical advantages offered by the methodologies proposed in this paper:

- The market equilibrium framework requires only industry data to develop the model and the same results can be used by any insurer or reinsurer.

- The market equilibrium framework avoids several of the practical problems encountered by specific portfolio methods discussed under "considerations for practical application."

- The leverage ratios provide some transparency to management through which they can intuitively evaluate the reasonability of the allocation methodology to some extent.

- Correlation between segments is conveniently reflected in the calculated betas for each segment. Correlation is more difficult to reflect using traditional actuarial methods.

Several difficulties with the methodologies were also identified and discussed. The underlying theoretical requirements for CAPM do not support application of the allocation methodology to property catastrophe coverage. Thus, a separate method should be used and integrated with the non-catastrophe segments. Segment definition and weighting problems reveal the difficulties of realistically modeling equilibrium of the insurance markets. Nevertheless, many of these same problems apply to many different methods.

Although the concepts underlying the methods in this paper are applicable to any insurer or reinsurer, they are not necessarily the most efficient capital allocation methods for any particular operation. For example, the risks facing a reinsurer specializing in a relative small number of large, complex contracts may be driven primarily by those complex contract terms. Therefore, despite the more complex analysis required to reflect correlation and other practical difficulties, a contract-specific analysis of risk might be preferable in practice. The methods proposed in this paper are more ideally suited to primary insurers and to reinsurers that must evaluate large numbers of contracts with standard coverages.

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Exhibit 1

Summary of Economic Leverage by Company

pre-tax ia = 1.54% *after-tax ia* = 1.00%

Insurer A		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	12,431,849	11,888,717	12,142,648	12,564,379	12,618,277	13,022,959		
	Economic Capital	5,004,379	4,612,676	4,972,009	5,185,981	5,513,944	5,306,946		
	Discounted Loss/Economic Capital	2.48	2.58	2.44	2.42	2.29	2.45	2.44	3.5%

Insurer B		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	19,494,666	18,525,938	17,692,300	16,758,251	16,175,488	15,193,632		
	Economic Capital	9,735,318	8,405,175	5,979,158	5,686,579	5,395,647	4,504,041		
	Discounted Loss/Economic Capital	2.00	2.20	2.96	2.95	3.00	3.37	2.75	17.5%

Insurer C		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	8,253,393	7,614,302	7,100,284	6,082,551	5,570,187	5,135,924		
	Economic Capital	2,873,916	2,439,678	2,313,674	2,247,541	2,099,888	1,809,516		
	Discounted Loss/Economic Capital	2.87	3.12	3.07	2.71	2.65	2.84	2.88	6.0%

Insurer D		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	7,981,715	6,871,157	6,769,370	6,691,763	6,536,486	6,487,141		
	Economic Capital	2,471,948	2,398,506	2,349,237	2,062,467	1,919,817	1,803,967		
	Discounted Loss/Economic Capital	3.23	2.86	2.88	3.24	3.40	3.60	3.20	8.2%

Insurer E		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	26,497,641	19,622,422	19,011,746	17,810,392	15,576,455	15,084,981		
	Economic Capital	7,267,535	4,436,230	4,616,913	4,082,142	4,779,262	4,126,866		
	Discounted Loss/Economic Capital	3.65	4.42	4.12	4.36	3.26	3.66	3.91	10.8%

Insurer F		1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
	Total Discounted Loss Reserves	N/A	7,603,250	7,105,757	6,869,030	7,109,159	7,142,162		
	Economic Capital	N/A	1,939,234	2,300,629	2,190,911	2,284,340	2,264,305		
	Discounted Loss/Economic Capital	N/A	3.92	3.09	3.14	3.11	3.15	3.28	9.8%

Exhibit 2

Summary of Industry Data Compilation

Predominantly Personal Lines Insurers

Excluding 3 Major Insurers

	1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
Total Discounted Loss Reserves	51,286,472	51,412,629	49,123,751	46,988,413	42,905,967	40,199,582		
Economic Capital	24,302,937	20,901,179	20,466,088	16,928,003	16,976,313	15,082,094		
Weighted Average Economic Leverage	2.11	2.46	2.40	2.78	2.53	2.67	2.49	8.5%
Straight Average Economic Leverage	1.96	2.18	2.20	2.39	2.20	2.30	2.21	6.0%

Predominantly Commercial Lines Insurers

Excluding 2 Major Insurers

	1995	1994	1993	1992	1991	1990	1990 - 1995 Str Avg	1990 - 1995 Coeff of Variation
Total Discounted Loss Reserves	148,003,973	144,733,967	141,898,566	137,986,536	130,718,306	126,255,267		
Economic Capital	56,119,941	48,575,610	46,636,905	43,164,155	43,935,856	39,799,937		
Weighted Average Economic Leverage	2.64	2.98	3.04	3.20	2.98	3.17	3.00	6.1%
Straight Average Economic Leverage	2.72	3.08	3.05	3.18	3.00	3.18	3.04	5.2%

Exhibit 3

Covariance Summary

Total Independence

Line	(1)	(2)	(3)	(4)	(5)	(6)
		(1)/[sum(1)]		(2) x (3)	Formula (11) in text	(5)/[sum(5)]
	Industry Expected Losses	Expected Loss Weights	Standard Deviation	Weighted Standard Deviation	Covariance with Total	***Beta***
Premises/Operations Liability	19,296,363	**11.3163%**	**12.5%**	1.4145%	0.176818%	**2.94**
Products Liability	4,254,365	**2.4950%**	**17.5%**	0.4366%	0.076408%	**1.27**
Commercial Auto Liability	9,278,649	**5.4415%**	**10.0%**	0.5441%	0.054415%	**0.91**
Personal Auto/Standard Liability	35,860,108	**21.0301%**	**5.0%**	1.0515%	0.052575%	**0.88**
Personal Auto/Non-Standard Liab	8,965,027	**5.2575%**	**4.5%**	0.2366%	0.010647%	**0.18**
Workers Compensation	18,542,347	**10.8741%**	**7.5%**	0.8156%	0.061167%	**1.02**
Non-Medical Professional Liab	2,213,311	**1.2980%**	**15.0%**	0.1947%	0.029205%	**0.49**
Medical Professional Liab	3,611,199	**2.1178%**	**15.0%**	0.3177%	0.047650%	**0.79**
Accident and Health	6,823,476	**4.0016%**	**5.0%**	0.2001%	0.010004%	**0.17**
HPR&Technical Property Non-Cat	581,674	**0.3411%**	**20.0%**	0.0682%	0.013645%	**0.23**
Commercial Property Non-Cat	9,814,556	**5.7557%**	**10.0%**	0.5756%	0.057557%	**0.96**
Personal Property Non-Cat	16,443,320	**9.6432%**	**8.0%**	0.7715%	0.061716%	**1.03**
Commercial Phys Dam Non-Cat	2,988,271	**1.7525%**	**4.5%**	0.0789%	0.003549%	**0.06**
Personal Physical Damage Non-Cat	25,843,332	**15.1558%**	**4.0%**	0.6062%	0.024249%	**0.40**
Crop/Agricultural	724,676	**0.4250%**	**20.0%**	0.0850%	0.016999%	**0.28**
Aviation/Ocean Marine	2,177,874	**1.2772%**	**25.0%**	0.3193%	0.079826%	**1.33**
Fidelity, Surety, and Credit	3,099,172	**1.8175%**	**17.5%**	0.3181%	0.055661%	**0.93**
Total All Lines	170,517,720	100%	**2.45%**		**0.060%**	**1.00**

Covariance Summary

Total Independence

Line	(1)	(2)	(3)	(4)	(5)	(6)
		(1)/[sum(1)]		(2) x (3)	Formula (11) in text	(5)/[sum(5)]
	Equal Expected Losses	Expected Loss Weights	Standard Deviation	Weighted Standard Deviation	Covariance with Total	***Beta***
Premises/Operations Liability	1,000,000	**5.8824%**	**12.5%**	0.7353%	0.091912%	**0.87**
Products Liability	1,000,000	**5.8824%**	**17.5%**	1.0294%	0.180147%	**1.70**
Commercial Auto Liability	1,000,000	**5.8824%**	**10.0%**	0.5882%	0.058824%	**0.55**
Personal Auto/Standard Liability	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.014706%	**0.14**
Personal Auto/Non-Standard Liab	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.011912%	**0.11**
Workers Compensation	1,000,000	**5.8824%**	**7.5%**	0.4412%	0.033088%	**0.31**
Non-Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	0.132353%	**1.25**
Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	0.132353%	**1.25**
Accident and Health	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.014706%	**0.14**
HPR&Technical Property Non-Cat	1,000,000	**5.8824%**	**20.0%**	1.1765%	0.235294%	**2.21**
Commercial Property Non-Cat	1,000,000	**5.8824%**	**10.0%**	0.5882%	0.058824%	**0.55**
Personal Property Non-Cat	1,000,000	**5.8824%**	**8.0%**	0.4706%	0.037647%	**0.35**
Commercial Phys Dam Non-Cat	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.011912%	**0.11**
Personal Physical Damage Non-Cat	1,000,000	**5.8824%**	**4.0%**	0.2353%	0.009412%	**0.09**
Crop/Agricultural	1,000,000	**5.8824%**	**20.0%**	1.1765%	0.235294%	**2.21**
Aviation/Ocean Marine	1,000,000	**5.8824%**	**25.0%**	1.4706%	0.367647%	**3.46**
Fidelity, Surety, and Credit	1,000,000	**5.8824%**	**17.5%**	1.0294%	0.180147%	**1.70**
Total All Lines	17,000,000	100%	**3.26%**		**0.106%**	**1.00**

Exhibit 5

Covariance Summary

Total Dependence

Line	(1) **Equal** Expected Losses	(2) (1)/[sum(1)] Expected Loss Weights	(3) Standard Deviation	(4) (2) x (3) Weighted Standard Deviation	(5) Formula (11) in text Covariance with Total	(6) (5)/[sum(5)] ***Beta***
Premises/Operations Liability	1,000,000	**5.8824%**	**12.5%**	0.7353%	1.477941%	**1.06**
Products Liability	1,000,000	**5.8824%**	**17.5%**	1.0294%	2.069118%	**1.48**
Commercial Auto Liability	1,000,000	**5.8824%**	**10.0%**	0.5882%	1.182353%	**0.85**
Personal Auto/Standard Liability	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.591176%	**0.42**
Personal Auto/Non-Standard Liab	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.532059%	**0.38**
Workers Compensation	1,000,000	**5.8824%**	**7.5%**	0.4412%	0.886765%	**0.63**
Non-Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	1.773529%	**1.27**
Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	1.773529%	**1.27**
Accident and Health	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.591176%	**0.42**
HPR&Technical Property Non-Cat	1,000,000	**5.8824%**	**20.0%**	1.1765%	2.364706%	**1.69**
Commercial Property Non-Cat	1,000,000	**5.8824%**	**10.0%**	0.5882%	1.182353%	**0.85**
Personal Property Non-Cat	1,000,000	**5.8824%**	**8.0%**	0.4706%	0.945882%	**0.68**
Commercial Phys Dam Non-Cat	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.532059%	**0.38**
Personal Physical Damage Non-Cat	1,000,000	**5.8824%**	**4.0%**	0.2353%	0.472941%	**0.34**
Crop/Agricultural	1,000,000	**5.8824%**	**20.0%**	1.1765%	2.364706%	**1.69**
Aviation/Ocean Marine	1,000,000	**5.8824%**	**25.0%**	1.4706%	2.955882%	**2.11**
Fidelity, Surety, and Credit	1,000,000	**5.8824%**	**17.5%**	1.0294%	2.069118%	**1.48**
Total All Lines	17,000,000	100%	**11.82%**		**1.398%**	**1.00**

Exhibit 6

Correlation Matrix Excerpt

Line	Wtd Std Dev	Prem/Ops	Products	Comm Auto Liability	Pers Auto Standard Liability	Pers Auto Non-Std Liability	Workers Comp	Non-Medical Prof Liab	Medical Prof Liab	Accident & Health	HPR Technical Non-Cat	Comm Property Non-Cat	Personal Property Non-Cat
		1.415%	0.437%	0.544%	1.052%	0.237%	0.816%	0.195%	0.318%	0.200%	0.068%	0.576%	0.771%
Premises/Operations Liability		1.000	0.100	0.050	0.040	0.030	0.120	0.050	0.050	0.050	0.000	0.000	0.000
Products Liability		0.100	1.000	0.050	0.040	0.030	0.100	0.050	0.050	0.050	0.000	0.000	0.000
Commercial Auto Liability		0.050	0.050	1.000	0.200	0.100	0.120	0.050	0.050	0.050	0.000	0.000	0.000
Personal Auto/Standard Liability		0.040	0.040	0.200	1.000	0.150	0.100	0.040	0.040	0.040	0.000	0.000	0.000
Personal Auto/Non-Standard Liab		0.030	0.030	0.100	0.150	1.000	0.050	0.020	0.020	0.020	0.000	0.000	0.000
Workers Compensation		0.120	0.100	0.120	0.100	0.050	1.000	0.050	0.050	0.050	0.000	0.000	0.000
Non-Medical Professional Liab		0.050	0.050	0.050	0.040	0.020	0.050	1.000	0.100	0.050	0.000	0.000	0.000
Medical Professional Liab		0.050	0.050	0.050	0.040	0.020	0.050	0.100	1.000	0.050	0.000	0.000	0.000
Accident and Health		0.050	0.050	0.050	0.040	0.020	0.050	0.050	0.050	1.000	0.000	0.000	0.000
HPR&Technical Property Non-Cat		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.030	0.030
Commercial Property Non-Cat		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	1.000	0.030
Personal Property Non-Cat		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.030	1.000
Commercial Phys Dam Non-Cat		0.000	0.000	0.300	0.100	0.050	0.000	0.000	0.000	0.000	0.010	0.010	0.010
Personal Physical Damage Non-Cat		0.000	0.000	0.100	0.300	0.200	0.000	0.000	0.000	0.000	0.010	0.010	0.010
Crop/Agricultural		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Aviation/Ocean Marine		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.010	0.010
Fidelity, Surety, and Credit		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.010	0.010

Exhibit 7

Covariance Summary

Line	(1) Industry Expected Losses	(2) (1)/[sum(1)] Expected Loss Weights	(3) Standard Deviation	(4) (2) x (3) Weighted Standard Deviation	(5) Formula (11) in text Covariance with Total	(6) (5)/[sum(5)] ***Beta***
Premises/Operations Liability	19,296,363	**11.3163%**	**12.5%**	1.4145%	0.208507%	**2.50**
Products Liability	4,254,365	**2.4950%**	**17.5%**	0.4366%	0.135033%	**1.62**
Commercial Auto Liability	9,278,649	**5.4415%**	**10.0%**	0.5441%	0.108844%	**1.31**
Personal Auto/Standard Liability	35,860,108	**21.0301%**	**5.0%**	1.0515%	0.078484%	**0.94**
Personal Auto/Non-Standard Liab	8,965,027	**5.2575%**	**4.5%**	0.2366%	0.030802%	**0.37**
Workers Compensation	18,542,347	**10.8741%**	**7.5%**	0.8156%	0.093515%	**1.12**
Non-Medical Professional Liab	2,213,311	**1.2980%**	**15.0%**	0.1947%	0.066571%	**0.80**
Medical Professional Liab	3,611,199	**2.1178%**	**15.0%**	0.3177%	0.083172%	**1.00**
Accident and Health	6,823,476	**4.0016%**	**5.0%**	0.2001%	0.021652%	**0.26**
HPR&Technical Property Non-Cat	581,674	**0.3411%**	**20.0%**	0.0682%	0.024372%	**0.29**
Commercial Property Non-Cat	9,814,556	**5.7557%**	**10.0%**	0.5756%	0.061399%	**0.74**
Personal Property Non-Cat	16,443,320	**9.6432%**	**8.0%**	0.7715%	0.064319%	**0.77**
Commercial Phys Dam Non-Cat	2,988,271	**1.7525%**	**4.5%**	0.0789%	0.019524%	**0.23**
Personal Physical Damage Non-Cat	25,843,332	**15.1558%**	**4.0%**	0.6062%	0.041818%	**0.50**
Crop/Agricultural	724,676	**0.4250%**	**20.0%**	0.0850%	0.016999%	**0.20**
Aviation/Ocean Marine	2,177,874	**1.2772%**	**25.0%**	0.3193%	0.083364%	**1.00**
Fidelity, Surety, and Credit	3,099,172	**1.8175%**	**17.5%**	0.3181%	0.058138%	**0.70**
Total All Lines	170,517,720	100%	**2.89%**		**0.083%**	**1.00**

Exhibit 8

Covariance Summary

Line	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(1)/[sum(1)]		(2) x (3)	Formula (11) in text	(5)/[sum(5)]	(6)/[avg(6)]
	Equal Expected Losses	Expected Loss Weights	Standard Deviation	Weighted Standard Deviation	Covariance with Total	***Beta***	On-Level ***Beta***
Premises/Operations Liability	1,000,000	**5.8824%**	**12.5%**	0.7353%	0.130404%	**1.03**	**1.85**
Products Liability	1,000,000	**5.8824%**	**17.5%**	1.0294%	0.227346%	**1.80**	**3.23**
Commercial Auto Liability	1,000,000	**5.8824%**	**10.0%**	0.5882%	0.102059%	**0.81**	**1.45**
Personal Auto/Standard Liability	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.037279%	**0.29**	**0.53**
Personal Auto/Non-Standard Liab	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.024485%	**0.19**	**0.35**
Workers Compensation	1,000,000	**5.8824%**	**7.5%**	0.4412%	0.063640%	**0.50**	**0.90**
Non-Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	0.171309%	**1.35**	**2.43**
Medical Professional Liab	1,000,000	**5.8824%**	**15.0%**	0.8824%	0.171309%	**1.35**	**2.43**
Accident and Health	1,000,000	**5.8824%**	**5.0%**	0.2941%	0.026956%	**0.21**	**0.38**
HPR&Technical Property Non-Cat	1,000,000	**5.8824%**	**20.0%**	1.1765%	0.247647%	**1.96**	**3.52**
Commercial Property Non-Cat	1,000,000	**5.8824%**	**10.0%**	0.5882%	0.066765%	**0.53**	**0.95**
Personal Property Non-Cat	1,000,000	**5.8824%**	**8.0%**	0.4706%	0.044282%	**0.35**	**0.63**
Commercial Phys Dam Non-Cat	1,000,000	**5.8824%**	**4.5%**	0.2647%	0.023837%	**0.19**	**0.34**
Personal Physical Damage Non-Cat	1,000,000	**5.8824%**	**4.0%**	0.2353%	0.019365%	**0.15**	**0.28**
Crop/Agricultural	1,000,000	**5.8824%**	**20.0%**	1.1765%	0.235294%	**1.86**	**3.34**
Aviation/Ocean Marine	1,000,000	**5.8824%**	**25.0%**	1.4706%	0.373235%	**2.95**	**5.30**
Fidelity, Surety, and Credit	1,000,000	**5.8824%**	**17.5%**	1.0294%	0.184059%	**1.46**	**2.62**
Total All Lines	17,000,000	100%	**3.56%**		**0.126%**	**0.56**	**1.00**

Covariance Summary

Line	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(1)/[sum(1)]		(2) x (3)	Formula (11) in text	(5)/[sum(5)]	(6)/[avg(6)]
	Industry Adjusted Losses	Adjusted Loss Weights	Standard Deviation	Weighted Standard Deviation	Covariance with Total	***Beta***	On-Level ***Beta***
Premises/Operations Liability	19,296,363	**3.5472%**	**12.5%**	0.4434%	0.092924%	**0.35**	**1.51**
Products Liability	4,254,365	**0.7821%**	**17.5%**	0.1369%	0.080919%	**0.31**	**1.31**
Commercial Auto Liability	9,278,649	**1.7057%**	**10.0%**	0.1706%	0.058657%	**0.22**	**0.95**
Personal Auto/Standard Liability	31,203,212	**5.7360%**	**5.0%**	0.2868%	0.034057%	**0.13**	**0.55**
Personal Auto/Non-Standard Liab	67,998,399	**12.5000%**	**4.5%**	0.5625%	0.034856%	**0.13**	**0.56**
Workers Compensation	18,542,347	**3.4086%**	**7.5%**	0.2556%	0.046392%	**0.18**	**0.75**
Non-Medical Professional Liab	67,998,399	**12.5000%**	**15.0%**	1.8750%	0.325019%	**1.23**	**5.27**
Medical Professional Liab	67,998,399	**12.5000%**	**15.0%**	1.8750%	0.325019%	**1.23**	**5.27**
Accident and Health	67,998,399	**12.5000%**	**5.0%**	0.6250%	0.044277%	**0.17**	**0.72**
HPR&Technical Property Non-Cat	67,998,399	**12.5000%**	**20.0%**	2.5000%	0.509175%	**1.93**	**8.25**
Commercial Property Non-Cat	9,814,556	**1.8042%**	**10.0%**	0.1804%	0.029588%	**0.11**	**0.48**
Personal Property Non-Cat	14,307,944	**2.6302%**	**8.0%**	0.2104%	0.025998%	**0.10**	**0.42**
Commercial Phys Dam Non-Cat	2,988,271	**0.5493%**	**4.5%**	0.0247%	0.008016%	**0.03**	**0.13**
Personal Physical Damage Non-Cat	22,487,243	**4.1338%**	**4.0%**	0.1654%	0.016493%	**0.06**	**0.27**
Crop/Agricultural	724,676	**0.1332%**	**20.0%**	0.0266%	0.005329%	**0.02**	**0.09**
Aviation/Ocean Marine	67,998,399	**12.5000%**	**25.0%**	3.1250%	0.788477%	**2.99**	**12.77**
Fidelity, Surety, and Credit	3,099,172	**0.5697%**	**17.5%**	0.0997%	0.022506%	**0.09**	**0.36**
Total All Lines	543,987,189	100%	**5.14%**		**0.264%**	**0.23**	**1.00**

Exhibit 10

Covariance Summary

Line	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		(1)/[sum(1)]		(2) x (3)	Formula (11) in text	(5)/[sum(5)]	(6)^.5	(7)/[avg(7)]
	Industry Expected Losses	Expected Loss Weights	Standard Deviation	Weighted Standard Deviation	Covariance with Total	***Beta***	Square Root ***Beta***	On-Level ***Beta***
Premises/Operations Liability	19,296,363	**11.3163%**	**12.5%**	1.4145%	0.208507%	**2.50**	**1.58**	**1.65**
Products Liability	4,254,365	**2.4950%**	**17.5%**	0.4366%	0.135033%	**1.62**	**1.27**	**1.33**
Commercial Auto Liability	9,278,649	**5.4415%**	**10.0%**	0.5441%	0.108844%	**1.31**	**1.14**	**1.19**
Personal Auto/Standard Liability	35,860,108	**21.0301%**	**5.0%**	1.0515%	0.078484%	**0.94**	**0.97**	**1.01**
Personal Auto/Non-Standard Liab	8,965,027	**5.2575%**	**4.5%**	0.2366%	0.030802%	**0.37**	**0.61**	**0.63**
Workers Compensation	18,542,347	**10.8741%**	**7.5%**	0.8156%	0.093515%	**1.12**	**1.06**	**1.10**
Non-Medical Professional Liab	2,213,311	**1.2980%**	**15.0%**	0.1947%	0.066571%	**0.80**	**0.89**	**0.93**
Medical Professional Liab	3,611,199	**2.1178%**	**15.0%**	0.3177%	0.083172%	**1.00**	**1.00**	**1.04**
Accident and Health	6,823,476	**4.0016%**	**5.0%**	0.2001%	0.021652%	**0.26**	**0.51**	**0.53**
HPR&Technical Property Non-Cat	581,674	**0.3411%**	**20.0%**	0.0682%	0.024372%	**0.29**	**0.54**	**0.56**
Commercial Property Non-Cat	9,814,556	**5.7557%**	**10.0%**	0.5756%	0.061399%	**0.74**	**0.86**	**0.90**
Personal Property Non-Cat	16,443,320	**9.6432%**	**8.0%**	0.7715%	0.064319%	**0.77**	**0.88**	**0.92**
Commercial Phys Dam Non-Cat	2,988,271	**1.7525%**	**4.5%**	0.0789%	0.019524%	**0.23**	**0.48**	**0.50**
Personal Physical Damage Non-Cat	25,843,332	**15.1558%**	**4.0%**	0.6062%	0.041818%	**0.50**	**0.71**	**0.74**
Crop/Agricultural	724,676	**0.4250%**	**20.0%**	0.0850%	0.016999%	**0.20**	**0.45**	**0.47**
Aviation/Ocean Marine	2,177,874	**1.2772%**	**25.0%**	0.3193%	0.083364%	**1.00**	**1.00**	**1.04**
Fidelity, Surety, and Credit	3,099,172	**1.8175%**	**17.5%**	0.3181%	0.058138%	**0.70**	**0.84**	**0.87**
Total All Lines	170,517,720	100%	**2.89%**		**0.083%**	**1.00**	**0.96**	**1.00**

Exhibit 11

Expected Loss and Required Capital Flows

Leverage Ratio = 5.270

i_a = 1.0%

Time (= **t**) in Years	Expected Paid Losses from **t** to **t**+1	Expected Unpaid Losses at **t**	Unpaid Losses at **t** Discounted to **t** at i_a	Required Capital at **t**
0	205,381	1,000,000	973,223	184,672
1	251,657	794,619	776,550	147,353
2	206,972	542,962	531,403	100,835
3	135,350	335,989	328,712	62,374
4	81,697	200,639	195,975	37,187
5	44,012	118,943	115,830	21,979
6	25,501	74,931	72,757	13,806
7	13,750	49,430	47,856	9,081
8	7,497	35,680	34,516	6,550
9	5,390	28,183	27,327	5,185
10	4,792	22,792	22,183	4,209
11	4,000	18,000	17,589	3,337
12	4,000	14,000	13,745	2,608
13	4,000	10,000	9,862	1,871
14	3,000	6,000	5,941	1,127
15	3,000	3,000	2,985	566
Total	1,000,000			602,742