Modeling Parameter Uncertainty in Cash Flow Projections

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by

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Abstract

In order to be complete dynamic financial analysis (DFA) models should deal with both the amount and timing of future loss and loss adjustment expense payments. Even more than asset cash flows, these future payments are very uncertain. However, even with this uncertainty, one would expect to see payments that are somewhat stable from year to year.

This paper presents an approach that can deal with this seeming contradiction. By separating total uncertainty in future cash flows into its parameter and process components we present a method to model future liability cash flows that maintains the desired total uncertainty characteristics. However, it will also result in specific payment flow "paths" having less variation from year to year than would a completely random sample from the expected total payout would indicate.

There is also a companion of this paper, titled "Estimating Uncertainty in Cash Flow Projections" that considers the problem of estimating the distributions, including separate consideration of process and parameter uncertainty.

Biography

Roger is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries, and Consulting Actuary in the Pasadena, California office of Milliman & Robertson, Inc. with over twenty-one years of casualty actuarial consulting experience. Roger is a frequent speaker on reserve and DFA related topics and has authored several papers dealing with considerations and estimates of uncertainty in reserve projections. Roger is currently the chair of the CAS Research Policy and Management Committee and has served as chair of both the CAS Committee on Theory of Risk and the CAS/AAA Joint Committee on the Casualty Loss Reserve Seminar.

MODELING PARAMETER UNCERTAINTY IN CASH FLOW PROJECTIONS

Introduction

With the increased focus on dynamic financial analysis (DFA) as a tool to assist in quantifying the financial strength of insurers and other risk bearing entities, comes increased demands on tools for use in those models. As with reserves, insurer cash outflows representing those liabilities are subject to considerable uncertainty. Capturing and appropriately modeling this uncertainty will greatly enhance the accuracy and reliability of DFA models.

The purpose of this paper is to outline a simple approach that can be used to capture various sources of uncertainty and incorporate them into stochastic cash flow models. A simple example should help illustrate this point.

Consider two insurers, both with expected reserves of $90 million, assets of $110 million, ignoring interest, and experiencing the following future payment possibilities:

Table 1: Distribution for Stable Insurer, Inc.

	Year		
Probability	1	2	Total
50.0%	$80	$40	$120
50.0%	40	20	60
Expected	$60	$30	$90

Table 2: Distribution for Random Insurer, Inc.

	Year		
Probability	1	2	Total
25.0%	$80	$40	$120
25.0%	80	20	100
25.0%	40	40	80
25.0%	40	20	60
Expected	$60	$30	$90

Each insurer experiences the same distribution of possible payments in each year. However, the first insurer has a 50% chance of becoming insolvent at the end of two years while the second has only a 25% chance.

The primary difference is that Random Insurer is allowed to experience all possible "futures" with either $80 or $40 paid in the first year and either $40 or $20 paid in the second. Stable Insurer is only allowed two possible "futures," the best and the worst. As we will see, these are simple examples of two approaches to modeling liability cash flows.

If historically the second year's payments were always half of those in the first year, then it could be argued that Stable Insurer's pattern is closer to "reality" than that of Random Insurer. The challenge, then, is to develop methods of modeling liability cash flows that capture the full variation that can be expected in future payments, without "unrealistic" swings in payments from year to year. That is the purpose of this paper.

Types of Uncertainty

There are many ways to categorize uncertainty. Here we will divide uncertainty faced by actuaries into three categories:

1. *Process* – uncertainty present simply from the random nature of a particular process, even if the process itself is known with certainty,

2. *Parameter* – uncertainty that parameters selected for a particular model accurately reflect the reality to be modeled, and

3. *Specification and/or Model* – uncertainty that the models selected themselves accurately reflect the reality to be modeled.

Sometimes the third category is divided into two parts, model and specification where specification refers to the selection of distributions and model refers to the selection of the underlying model itself.

For example, if we throw a fair die, even though we know the underlying physical model with (relative) certainty, there is still an equal chance of each of the six sides showing up. This is an example of process uncertainty.

If, however, the die may be "loaded," but that we know we are observing the throw of a die, we have added parameter uncertainty to the situation. Here we know we will observe throws from one through six, but with one result potentially having higher probability than the others do.

Finally, we could be observing a series of digits from 1 through 6 without knowing the underlying process generating the series. We can still use a loaded die model. However, there is the possibility that some other process is generating the digits that cannot be modeled using a loaded die. For example, the digits could be the last digit from a Geiger counter reading with 1 substituted for 7 and 8 and 6 substituted for 9 and 0. Here we have specification or model uncertainty.

Modeling Process Uncertainty

These categories of uncertainty are increasingly difficult to estimate. Reserves for insurers, or other risk bearing entities, are often set using non-statistical actuarial forecasting methods, including broad application of "actuarial judgment."

Even when statistical methods are used, the information regarding the resulting uncertainty is usually limited to conclusions within the framework of the model. For example, two different statistical models may result in two different probability ranges about their estimates with possibly little or no overlap in the ranges.[1] The same statistical model applied to two different sets of data, paid and incurred losses for example, could even give widely different results and ranges.

Statistical projection methods also tend to concentrate on "squaring the triangle" for a single set of data, usually paid losses. As Berquist and Sherman[2] and many other papers dealing with reserve estimation indicate, there is valuable information in many different insurer statistics. Claim count statistics are extremely valuable in a reserve analysis. Frequency and severity methods are often less volatile than development factor (or link ratio) methods for less mature exposure periods. In addition, claim counts, in conjunction with other insurer data, can help identify changes that could affect one or another projection method. For example, changes in average case reserves per open claim could signal a change in relative reserve adequacy thus affecting projections

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[1] See, for example, *Transcripts of the 1992 Casualty Loss Reserve Seminar*, pp. 1123-1150. This Advanced Case Study presented two actuaries with the same set of data and asked them to develop reserve and variability estimates. One estimated reserves to be $239 million with a $12.7 million standard error. The other estimated reserves to be $178 million with a standard deviation of $10.7 million.

[2] Berquist, J.R. and Sherman, R.E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *Proceedings of the Casualty Actuarial Society*, LXIV, 1977, pp. 123-184.

based on incurred loss development. Similarly, changes in the rate at which claims are closed will affect methods based on paid losses. The author is unaware of any statistical method that incorporates all these items of information in estimating ultimate losses.

The collective risk model offers a rather easily understood framework to model insured uncertainty. Briefly the collective risk model is based on the following algorithm:

Algorithm 1 – Collective Risk:

1. Randomly select N, the number of claims that will occur.
2. Randomly select N independent claims, $X_1, X_2, \ldots, X_N$ from the selected claim size distribution.
3. Total the amounts $T = \sum_{i=1}^{N} X_i$.
4. Repeat steps 1 through 3 "many" times.

With a minimum of additional assumptions we can derive some very useful relationships between the distributions of the number (N) and size (X) of individual claims and that of the total. In particular, if sufficient moments exist for the various distributions and if all random variables are independent then we have:

$$
\begin{aligned}
\mathrm{E}(T) &= \mathrm{E}(N)\mathrm{E}(X) \\
\mathrm{Var}(T) &= \mathrm{E}(N)\mathrm{Var}(X) + \mathrm{Var}(N)\mathrm{E}^2(X)
\end{aligned} \tag{1}
$$

Similar formulae also hold for higher moments.[3]

The collective risk model also seems to be a logical choice to model process uncertainty in the distribution of insured losses. There has been considerable attention paid to this basic model in the literature and several algorithms have been developed to calculate the distribution of T given distributions of N and X. Probably of greatest interest to

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[3] See, for example, Mayerson, A.L., Jones, D.A., Bowers, N.L. (Jr.), "The Credibility of the Pure Premium." *Proceedings of the Casualty Actuarial Society*, LV, 1968, p. 179 for these and formulae for third and fourth moments.

practicing casualty actuaries are references by Heckman and Meyers,[4] Panjer and Willmot,[5] Robertson,[6] and the text about to appear by Klugman, Panjer, and Willmot.[7]

The attractiveness of the collective risk model, aside from its description of the insurance process is that it breaks the problem of estimating process variation into more manageable parts, i.e., to estimating the distribution of claim counts and the distribution of the size of claims. As with any model, the collective risk model is an approximation of reality. Many actuaries are concerned with some of its inherent assumptions, not the least of which is the assumption of independence among claims and between the claim size and the claim count distributions. Recent work by Wang[8], sponsored by the Casualty Actuarial Society, addresses this issue. Although derived independently, the methods here follow closely with those presented by Wang.

Some Approaches to Parameter Uncertainty

Probably the most intuitive approach to modeling parameter uncertainty would be Bayesian. Generally one would assume the distribution we wished to model, that of aggregate losses, had a particular distribution with one or more of its parameters being uncertain, itself having a separate distribution. There are many distribution pairs of conditional and prior distributions that mix to closed form mixed distributions. In the appendix to his chapter in *Foundations of Casualty Actuarial Science*, Venter[9] for example has assembled of useful distribution pairs.

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[4] Heckman, P.E., Meyers, G.G., "The Calculation of Aggregate Loss Distributions From Claim Severity and Claim Count Distributions," *Proceedings of the Casualty Actuarial Society*, LXX, 1983, pp. 22-61

[5] Panjer, G., Willmot, G. *Insurance Risk Models*, Society of Actuaries, Chicago, 1992

[6] Robertson, J.P., "The Computation of Aggregate Loss Distributions," *Proceedings of the Casualty Actuarial Society*, LXXIX, 1992, pp. 57-133

[7] Klugman, S.A., Panjer, H.H., Willmot, G.E., *Loss Models: From Data to Decisions*, John Wiley & Sons, New York, 1998.

[8] Wang, S.S., "Aggregation of Correlated Risk Portfolios: Models & Algorithms," Casualty Actuarial Society at www.casact.org (part of the Committee on Theory of Risk page of the Research portion of the web site).

[9] Venter, G.G., "Credibility," *Foundations of Casualty Actuarial Science*, Casualty Actuarial Society, 1992, Chapter 7, pp. 375-483.

One example here may be helpful. Suppose X has a lognormal distribution with parameters μ and σ^2. By this we mean that X has the probability density function:

$$(2) \qquad f(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)}{x\sigma\sqrt{2\pi}}$$

It is well known that the random variable X is lognormal if and only if the random variable $\ln X$ is normal. In this parameterization the variable $\ln X$ has a normal distribution with mean μ and variance σ^2. If, now, we assume μ is uncertain but has a normal distribution with mean m and variance τ^2, then the random variable X is still lognormal with parameters m and $\sigma^2+\tau^2$. We note that the inclusion of parameter uncertainty in this way has the effect of increasing both the mean and variance of the distribution. This follows from the following results for a lognormal distribution with parameters μ and σ^2.

$$(3) \qquad \begin{aligned} E(X) &= \exp\left(\mu + \tfrac{1}{2}\sigma^2\right) \\ Var(X) &= \exp\left(2\mu + \sigma^2\right)\left(\exp\left(\sigma^2\right) - 1\right) = E^2(X)\left(\exp\left(\sigma^2\right) - 1\right) \end{aligned}$$

As an aside, the reader should note that Venter's parameterization of the lognormal distribution differs from what we use here. The first parameter in our parameterization is the mean of the normal distribution of $\ln X$ whereas Venter's parameter is the exponential of this amount. Thus in the appendix Venter assumes the prior distribution of the *parameter* is lognormal to conclude the mixed distribution is lognormal. Because of the log transformation between the two paramterizations, and the fact that a variable X is lognormal if and only if the variable $\ln X$ is normal, the two results are actually identical.

Thus one intuitive way to model parameter uncertainty would be to select a pair of distributions (lognormal and normal in this example), use the lognormal to model process uncertainty (as an approximation to the results of a collective risk model). Parameter uncertainty could then be built in by allowing the μ parameter to have a distribution of its own. In this paper we will label method this the Bayesian approach.

Another approach to modeling parameter uncertainty is discussed in Heckman and Meyers.[10] In their approach they separate parameter and process uncertainty by use of additional random variables. The following is a slight modification of the algorithm they present:

Algorithm 2 – Refined Collective Risk:

1. Randomly select N, the number of claims that will occur from a distribution with mean λ and variance $\lambda+c\lambda^2$.

2. Randomly select N independent claims, $X_1, X_2, \ldots, X_N$ from the selected claim size distribution.

3. Randomly select a mixing parameter β from a distribution with mean 1 and variance b.

4. Total the amounts and divide by β, $T = \left(\sum_{i=1}^{N} X_i\right) \Big/ \beta$.

5. Repeat steps 1 through 4 "many" times.

Actually, in Heckman and Meyers the authors assume the claim count distribution is a mix of a Poisson prior distribution with a gamma uncertainty distribution for a negative binomial posterior distribution. Their results, however, generalize to situations where the parameter c is negative, which does not make sense in terms of mixed distributions. The algorithm they present for calculating the aggregate distribution does require either a Poisson, binomial, or negative binomial claim count distribution, but the results we use here do not need that assumption.

The primary result we will use, however, is that given *Algorithm 2*, and assuming all the distributions are independent from each other, then we have the following relationships:

$$\begin{aligned} \mathrm{E}(T) &= \lambda \mathrm{E}(X) \\ \mathrm{Var}(T) &= \lambda(1+b)\mathrm{E}\left(X^2\right) + \lambda^2(b+c+bc)\mathrm{E}^2(X) \end{aligned} \tag{4}$$

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[10] Heckman, P.E., Meyers, G.G., ibid

We note that these formulae reduce to formulae (1) in the case that b=0. Rearranging terms in the variance formula we obtain:

$$\begin{aligned} \text{Var}(T) &= \lambda \text{E}(X^2) + b\lambda \text{E}(X^2) + \lambda^2 c \text{E}^2(X) + \lambda^2 b \text{E}^2(X) + \lambda^2 bc \text{E}^2(X) \\ &= \lambda \text{E}(X^2) + \lambda^2 c \text{E}^2(X) + b\left(\lambda \text{E}(X^2) + \lambda^2 c \text{E}^2(X) + \lambda^2 \text{E}^2(X)\right) \\ &= \text{Var}(T|b=0) + b\left(\text{Var}(T|b=0) + \lambda^2 \text{E}^2(X)\right) \end{aligned} \tag{5}$$

Which can be used to obtain the following useful relationship for the coefficient of variation (ratio of the standard deviation to the mean) of the respective distributions:

$$\begin{aligned} \text{cv}^2(T) &= \frac{\text{Var}(T)}{\text{E}^2(T)} \\ &= \frac{\text{Var}(T|b=0) + b\left(\text{Var}(T|b=0) + \lambda^2 \text{E}^2(X)\right)}{\text{E}^2(T|b=0)} \\ &= \frac{\text{Var}(T|b=0)}{\text{E}^2(T|b=0)} + b\left(\frac{\text{Var}(T|b=0)}{\text{E}^2(T|b=0)} + \frac{\lambda^2 \text{E}^2(X)}{(\lambda \text{E}(X))^2}\right) \\ &= \text{cv}^2(T|b=0) + b\left(\text{cv}^2(T|b=0) + 1\right) \end{aligned} \tag{6}$$

Solving for b we obtain:

$$b = \frac{\text{cv}^2(T) - \text{cv}^2(T|b=0)}{\text{cv}^2(T|b=0) + 1} \tag{7}$$

Recalling that b=0 refers to the situation with only process variation, this formula provides a way to model parameter uncertainty given knowledge of the coefficient of variation for the final distribution and that for the distribution with only process uncertainty.

From this point on we will assume that we know the various means and variances of the distributions with and without parameter uncertainty and concentrate on practical considerations in modeling these sources of uncertainty.

Moving to the example with a lognormal prior distribution mixed with a normal distribution let us consider two different ways of modeling the amounts. We will identify two methods to generate random loss amounts.

Intuitive Method:

1. Randomly pick ν from a normal distribution with mean m and variance τ^2.

2. Randomly pick X from a lognormal distribution with mean ν and variance σ^2.

"Smarter" Method:

1. Randomly pick X from a lognormal distribution with mean m and variance $\sigma^2 + \tau^2$.

As we saw above, both methods give exactly the same result. The *Intuitive Method* is simply the Bayesian statement of the problem and the *Smarter Method* is the posterior distribution.

A Dilemma?

Consider a very simple extension of our Bayesian type of algorithm with a lognormal mixed with a normal but for multiple years.

Algorithm 3. Multiple Year Bayesian

1. Assume X_i has a lognormal distribution with parameters μ_i and σ_i^2, with σ_i^2 known but

2. $\mu_i = m_i\beta$ where β has a normal distribution with mean b and variance τ^2, with both b and τ^2 known.

Here the parameter β provides "global" parameter uncertainty. The above discussion leads us to conclude that each X_i has a lognormal distribution with parameters bm_i and $\sigma_i^2 + m_i^2\tau_i^2$. Thus we are tempted to use either the *Intuitive Method* or the *Smarter Method* in modeling. In this case we would have the methods described as:

Intuitive Method:

1. Randomly pick β from a normal distribution with mean b and variance τ^2.

2. Randomly pick X_i from a lognormal distribution with parameters $\mu_i = m_i\beta$ and σ_i^2.

"Smarter" Method:

1. Randomly pick X from a lognormal distribution with parameters bm_i and $\sigma_i^2 + m_i^2\tau_i^2$.

Our reasoning above could lead to the conclusion that the two methods give the same answer. In fact the distributions for each year are identical. However, consider the example where m_i=0.25i, all the σ_i^2=0, and $b=\tau^2$=1. The following graphs make it clear that, at least in this case, the two methods give considerably different answers:

Figure 1: Intuitive Method, First Example

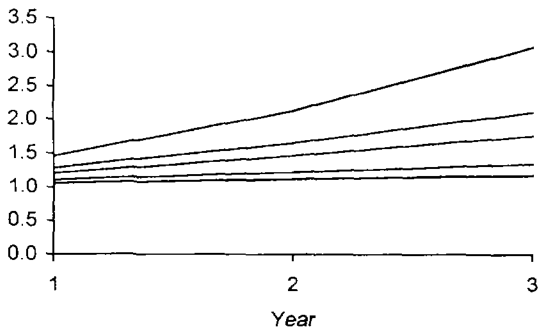
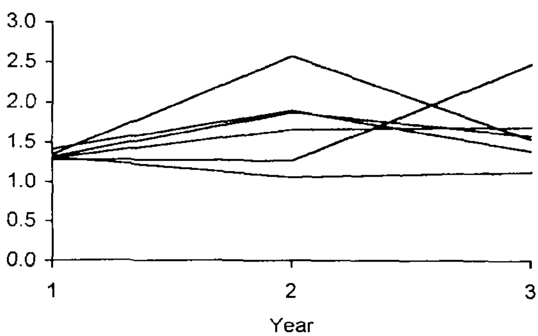


Figure 2: "Smarter" Method, First Example



Even though each year has a lognormal distribution by itself, the structure does not imply that each year is independent of the others. That is the major difference between the *Intuitive* and "*Smarter*" methods. It is also the difference between Stable Insurer and Random Insurer in the *Introduction*.

The above statement of the multiple year algorithm may lead to some ambiguity regarding the role of the uncertainty parameter. The following restatement may help clarify the ambiguity and provide us with a more explicit means to move *Algorithm 2* to a multiple year setting.

Algorithm 4, Refined Multiple Year Bayesian Algorithm

1. Select p with $0 < p < 1$.

2. Set $\mu_i = m_i + \tau_i\Phi^{-1}(p)$, where $\Phi^{-1}(p)$ represents the inverse normal distribution, that is the value such that $P\left(Z < \Phi^{-1}(p) \middle| Z \sim N(0,1)\right) = p$.

3. Randomly select X_i from a lognormal distribution with parameters μ_i and σ_i^2, σ_i^2 are known.

4. Repeat steps 2 and 3 for each year to be modeled.

5. Repeat steps 1 through 4 "many" times

We recognize a slight inconsistency in the parametizations of these two versions. Strictly speaking we should have $\mu_i = m_i\left(b + \tau_i\Phi^{-1}(p)\right)$ to be consistent with the first, but this parameterization leads directly to the conclusions for each year individually exactly parallel to those of the single year case.

Implications in Modeling Liabilities

Liabilities for most lines of insurance are characterized by fairly (a *very* relative term) stable payments from year to year. Obvious exceptions are lines subject to catastrophe losses and small liability books with large loss exposure. Even large claims may have extended settlement provisions, affecting the timing and variation of future payments.

If we consider only process variation we see that the law of large numbers soon comes into play. From (1) in the case of the collective risk model we have:

$$
(8) \qquad \begin{aligned}
\mathrm{cv}^2(T) &= \frac{\mathrm{Var}(T)}{\mathrm{E}^2(T)} \\
&= \frac{\mathrm{E}(N)\,\mathrm{Var}(X) + \mathrm{Var}(N)\mathrm{E}^2(X)}{\mathrm{E}^2(X)\mathrm{E}^2(N)} \\
&= \frac{\mathrm{Var}(X)}{\mathrm{E}^2(X)\mathrm{E}(N)} + \frac{\mathrm{Var}(N)}{\mathrm{E}^2(N)} \\
&= \frac{\mathrm{cv}^2(X)}{\mathrm{E}(N)} + \mathrm{cv}^2(N)
\end{aligned}
$$

If we make the usual assumption now that N has a Poisson distribution with variance equal to the mean then this becomes:

$$
(9) \qquad \mathrm{cv}^2(T) = \frac{\mathrm{cv}^2(X) + 1}{\mathrm{E}(N)}
$$

Thus, no matter how volatile the claim size distribution is, the total amounts paid could have arbitrarily small relative variation simply by having E(N) sufficiently large. We note the law of large numbers is a special case here where the variance of the number of claims is zero. The same result will follow for any claim count distribution whose standard deviation grows more slowly than the mean, more precisely, whenever

$$
(10) \qquad \mathrm{Var}(N) = o(\mathrm{E}(N)) \text{ as } \mathrm{E}(N) \to \infty
$$

The power of the law of large numbers should not be underestimated. Even if the claim count distribution were fairly "noisy" with a standard deviation of 5 times the mean, it would only require a Poisson distribution with 100 claims to result in the standard deviation of the total to 51% of the total. With 5,000 claims, not unusual for a fairly large insurer, the standard deviation reduces to 7% of the total. If one would use a rule of thumb that results beyond two standard deviations "rare" in this case it would be rare for actual payments to deviate by more than 15% of the mean.

We recognize that "fairly noisy" is a soft term. Many would argue, and quite persuasively, distributions that are interesting to actuaries may not have finite standard deviations, or maybe not even have finite means. However, with policy limits usually in effect, distributions losses faced by insurers usually have finite means and variances.

The conclusion we reach is the same reached by Meyers and Schenker.[11] For insurers, and larger self-insured entities, the law of large numbers gives process variation much less influence on the overall variation of results than other sources of uncertainty. Thus parameter uncertainty and model or specification uncertainty are more significant issues to insurers than simple process uncertainty.

Realistic modeling of liabilities in a dynamic financial model then must balance two realities. First payments for an insurer are often fairly consistent from year to year. Second the liabilities for insurers or self-insureds often have a high degree of uncertainty, often well beyond that which can be attributed to process variation alone.

One way to look at the problem is to consider payments as falling along various future "paths" with relatively little variation in payments from year-to-year on any given path but with potentially widely varying paths or futures. If this is actually the case, modeling future cash payments should be relatively straightforward. We could assume that variation in payments from year to year would be caused by process variation whereas other sources of uncertainty reflect various possible future paths.

Consider, for example, *Algorithm 4* with a multiple year runoff of reserves, as given by the following table, assuming no parameter uncertainty:

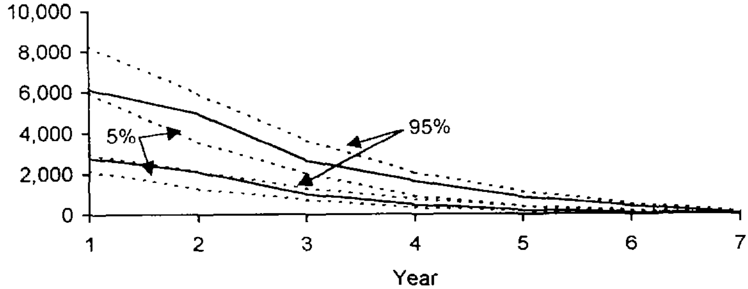
Table 3: Refined Example Data

Year	E(X)	E(N)	E(T)	cv(T)
1	5,000	1,000	5,000,000	0.100
2	11,000	300	3,300,000	0.155
3	13,000	150	1,950,000	0.183
4	20,000	50	1,000,000	0.255
5	25,000	20	500,000	0.316
6	30,000	7	210,000	0.423
7	40,000	1	40,000	1.031

If, now for simplicity, we assume that the payments in each year have lognormal distributions, but with "global" parameter uncertainty as described in *Algorithm 4* with $\tau_i = 0.5$ we can then view alternative future reserve runoffs in the following chart:

[11] Meyers, G.G., Schenker, N., "Parameter Uncertainty in the Collective Risk Model," *Proceedings of the Casualty Actuarial Society*, LXX, 1983, pp.111-143.

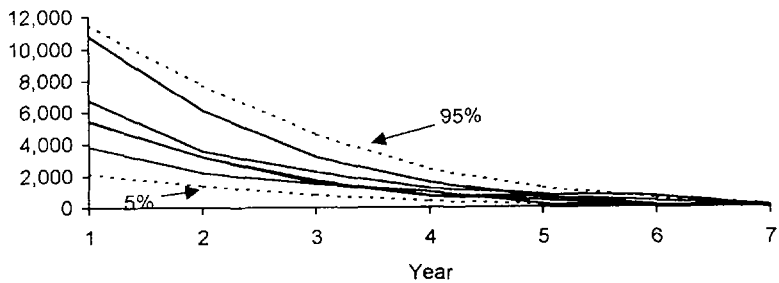
Figure 3: Two "Paths" with Probability Levels



Here the two sets of lines present two of the many possible "futures," corresponding to two different probability levels for the parameter uncertainty. The solid lines indicate the simulated reserve runoff, while the dotted lines represent the 5% and 95% probability bounds accounting only for process uncertainty as defined in the above table. Thus, for these two selected parameter uncertainty levels, we would expect 90% of the possible futures to lie between the dotted lines.

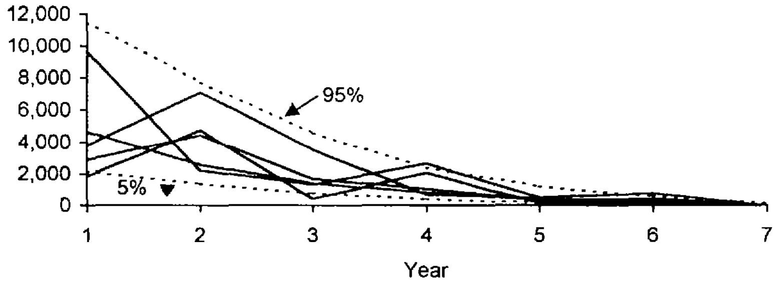
The following graph shows the global 90% range with several simulated runoffs (using our "Intuitive" approach).

Figure 4: Refined Example, Intuitive Method



To show the difference with the "*Smarter*" method the following is a graph showing the fully random lognormal approach:

Figure 5: Refined Example, "Smarter" Method



Again, the *Intuitive* approach gives smoother paths, yet still does provide the total uncertainty expected.

We can also generalize *Algorithm 2* to model multiple year uncertainty.

Algorithm 5 – Multiple Year Refined Collective Risk

1. Assume that payment amount process uncertainty can be modeled by known distributions in each year.

2. Assume that other sources of uncertainty in each year can be reflected by dividing by a "distortion" variable β_i, having mean 1 and known variance b_i.

3. Randomly select $0 < p < 1$

4. Select each β_i from the distortion distributions at probability level p.

5. Randomly select payments in year i, X_i from the assumed distributions.

6. Model amounts by the ratio of X_i and the selected β_i.

We note for each i this model is similar to *Algorithm 2*. The principal difference is the "linkage" between years provided by selecting the distortion variable at the same probability level for each year.

For each year, then, if we can estimate total variation, the variances required in the second step can actually be easily determined using formula (7) above. Of course, estimating total variation is not a trivial matter. There currently may be no agreed-upon method to derive such estimates, however this continues to be an active area of actuarial research.

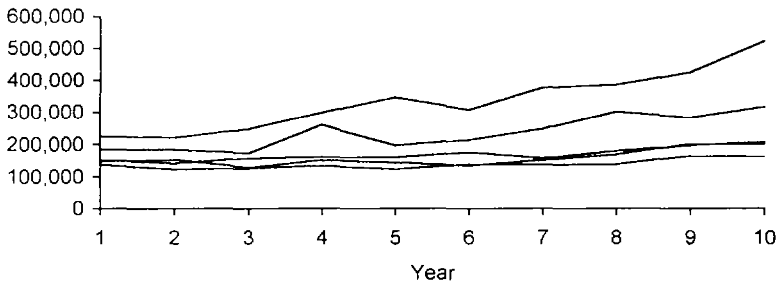
Assuming that we can get the total variance estimates, the following is an example of estimating the b_i values and the resulting graphs. These estimates are based on a fairly comprehensive attempt to estimate process uncertainty as well as other sources of uncertainty in the estimates. All estimates are in current dollars (with the effect of inflation removed) and are for total forecast payments in future years, including those arising from future exposures.

Table 4: Comprehensive Example

	Expected	Standard Deviation		Implied
Year	Paid	Process	Total	b Value
1	\$213,000	\$5,900	\$60,700	0.0804
2	218,000	14,200	96,900	0.1925
3	237,000	22,800	125,000	0.2665
4	255,000	30,700	144,700	0.3031
5	274,000	36,100	167,800	0.3516
6	294,000	38,200	189,300	0.3911
7	316,000	42,900	209,100	0.4118
8	337,000	29,500	228,700	0.4494

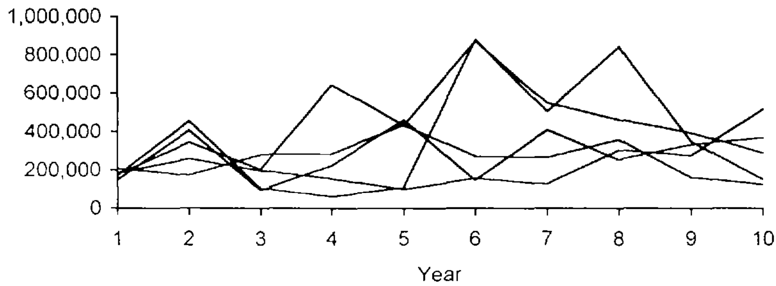
The following graph shows simulations based on *Algorithm 5* using the simplifying assumptions that the uncertainty parameters all have gamma distributions and that process uncertainty can be adequately modeled by a lognormal distribution.

Figure 6: Comprehensive Example, Intuitive Method



This shows relatively moderate variation from year to year but a fairly wide spread of possible outcomes. Both would be expected given the standard deviations shown above. As we compared in other situations, the following graph follows the "*Smarter*" method and results in substantially more variation from year to year than *Algorithm 5*.

Figure 7: Comprehensive Example, "Smarter" Method



As in prior examples of the "*Smarter*" method, there are substantial swings in payments from year to year. If we would expect some predictability of payments then using these simulations in a dynamic financial analysis model may be misleading. In short, the "*Smarter*" model is not really so smart in these situations.

Conclusion

Simply knowing the total distribution of payments in any particular future year does not necessarily give the actuary sufficient information to accurately and adequately model future payments, whether the application be in a full dynamic financial analysis model or in other applications where modeling of reserve payout is important. This paper presents one of many possible alternatives that can be used to separate process variations that will happen even if all information about the model is completely known, from other, potentially more global, influences. Still remaining, however, is significant research into the proper models to be used and in estimating the parameters of those models.