# *A Comprehensive System for Selecting and Evaluating DFA Model Parameters*

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# A Comprehensive System for Selecting and Evaluating DFA Model Parameters

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**Abstract**

Stochastic scenario generators for assets and liabilities are critical components of a robust DFA model. Vital to any stochastic scenario generation system is the selection of the underlying parameters. The process of parameter estimation is second only to model structure in the quest for generating reasonable results. If the model is simple, we can use standard statistical methods such as maximum likelihood to estimate parameters. However, for very complex models, we need to establish criteria for evaluation and find the parameters that are best with respect to those criteria.

In this paper, we discuss a parameter estimation system called American Re-Insurance Company's Constraint Evaluator System. This system allows modelers to define a multitude of targets and to assign a weight to each target to create a comprehensive objective function. Each target represents a quality that the model should possess with an assigned level of significance (weight). The targets are based on historical analysis or on some rational vision for future relationships. We discuss the analysis involved in setting appropriate targets including the monitoring of relationships between variables in a multi-period environment.

Our goal is to minimize the deviation between the user-defined targets and the model output. This is a non-convex optimization problem, which we use a combination of techniques to solve. Finally, we study the robustness of our parameter estimates as it relates to the number of scenarios and the observed model outputs.

## 1. Introduction

Stochastic scenario generators for assets and liabilities are important components of a robust DFA system. These generators will forecast asset and liability distributions over time as part of the development of income statement and balance sheet projections. These forecasts are developed as a collection of individual scenarios. Each scenario represents one possible future, and by looking at many scenarios, distributions can be calculated at any point in time. Examples of such systems can be found in Berger and Mulvey (1998), Dempster and Thorlacius (1998), Wilkie (1986), and Mulvey and Thorlacius (1998).

In developing this scenario-based approach, modelers try to understand fundamental economic and asset market structures. For example, when inflation is increasing, how will the stock and bond markets react? By understanding fundamental relationships, more realistic scenarios can be generated. These relationships can be modeled with mathematical equations, thus grounding the model in some amount of economic theory. The danger, however, is that the resulting scenarios don't exhibit characteristics seen in the market historically. For instance, we would not want a model that produces scenarios with negative interest rates.

After the underlying economic relationships are determined and modeled, we control the scenario output by the selection of model parameters, called *calibrating the model*, or *calibration*. Model parameters could include mean reversion level for interest rates, volatility for stock returns, and expected inflation growth. For simple models, standard statistical methods such as maximum likelihood estimation are appropriate. For complex models, we need to employ more sophisticated methods to determine the best parameters.

The calibration method described in this paper allows the user to specify characteristics the scenarios should have, referred to as targets. Each target represents a quality that the scenarios should exhibit, such as a range of bond returns over time, and an accompanying level of significance (weight). The targets can be based on historical analysis or some rational vision of future relationships. We then utilize an optimization procedure to determine best parameter settings to meet the targets.

This paper focuses on an economic scenario generator and the calibration process employed by American Re-Insurance Company headquartered in Princeton, NJ. In the next section, we briefly describe the entire DFA system, of which the scenario generation is one important piece. Section 3 focuses on the economic modeling system, the different types of economic models, and characteristics of a good model. In Section 4, we discuss how to set targets for the calibration process. Section 5 presents an example of the calibration process, utilizing software developed by Lattice Financial. Some final thoughts are in Section 6.

## 2. A Dynamic Financial Analysis System

American Re-Insurance Company's Risk Management System (ARMS) is an integrated compilation of models. The system is applied to determine internal capital allocation for the Company. The system is also used to assist both Munich Re[1] and American Re-Insurance Company clients in evaluating and setting up efficient re-insurance structures. The structure of the system is laid out in Figure 1.

[1] American Re-Insurance Company is a member of the Munich Re Group

# ARMS Structure

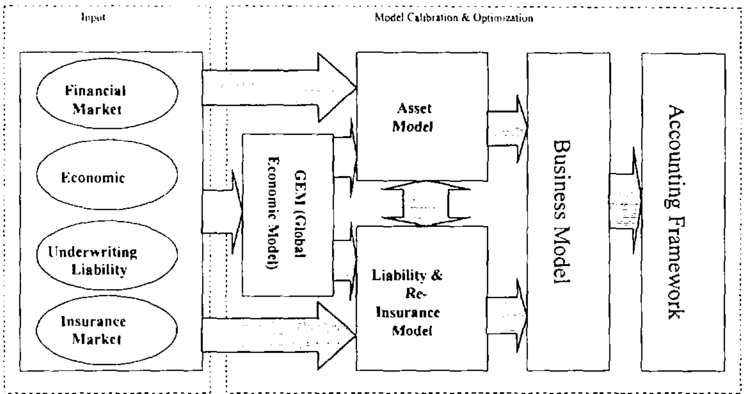


Figure 1. American Re-Insurance Company's Risk Management System (ARMS) is an integrated compilation of models. Historical data from financial and economic markets, underwriting decision processes, and insurance market trends are inputs to the system (left). Output includes balance sheet and income statements, optimal investment mixes and reinsurance structures.

The **Global Economic Model**[2] generates plausible time series outcomes of future economies based on user specifications and parameter settings. The user specifications are inputs reflecting the current economic environment and expectations for long-term median trends. The parameter settings are referred to as calibration parameters and those are set via the **Constraint Evaluator System**.

Each of the economic time series scenarios are fed to the **Asset Model** as well as the **Liability and Re-insurance Model**. These two models project different asset and liability classes along each economic scenario. It is important to recognize that the economic scenario generator lays the foundation for the calibration of the liability and asset models. Although the liability losses are based on fitted frequency and severity distributions (see Hogg & Klugman (1984), Panjer & Willmot (1992)), our analysis of loss data shows dependency on inflation for many lines of business. Therefore, inflation scenarios from the economic model define the trend in the prospective severity distributions over time. Similarly, the prospective premium is trended with inflation. Any discounting for future pricing purposes is based on output from the economic model. We consider thousands of scenarios for many years in the future and thus develop distributions for our underlying asset and liability returns in a multi-period environment.

The **Business Model** considers the underlying strategy of the business managers. It models the decisions we make as the business moves forward through time. For example, how will the

[2] Global Economic Model (GEM) is under development. At the time of writing and for the foreseeable future, the only country modeled is the United States.

business grow if gross margins are reduced by 10% next year? This also includes any change in asset allocation or in re-insurance structure.

The **Accounting Framework** refers not only to accounting but also to tax implications. There are several advantages to separating this functionality. They include the facilitation of operating in a multi-country (and therefor multi-regulatory) environment.

Wrapped around all this functionality is a non-convex optimization engine – the driving force behind the **Constraint Evaluator System**. Since each of these models must be calibrated in one form or another, access to a non-convex optimization system minimizes traditional trial and error attempts to ensure the reasonability of results. Ideally, we want to back-test the models with historical data and ensure optimal performance before we start modeling prospectively.

To better understand the calibration process, we will focus on calibrating the economic scenario generator. A description of the generator in the context of previous modeling efforts is in the next section.

## 3. Scenario Generator

### 3.1 What Makes A Good Scenario Generator?

Unfortunately, there are no agreed upon standards for scenario generation techniques. For some, the model must be a series of mathematical equations that are solved analytically (e.g., Black-Scholes option pricing model). Others have a more empirical approach, preferring to forecast future returns directly on current and past conditions (e.g., vector auto-regressive and kernel regression approaches).

The Global Economic Model (GEM) scenario generator strikes a balance between the two. Relationships among economic variables are modeled with explicit stochastic difference equations and the equation parameters are based on historical data via the calibration process[3]. The set of equations is too complex to have a closed form solution. Thus, Monte Carlo simulation is utilized to generate a multitude of paths (scenarios).

American Re-insurance defined the following criteria for the GEM system:

- Must be logically defensible – relationships among the economic variables must be consistent with economic theory and be statistically defensible given historical data.
- Must produce the proper relationships over time - movements in the economic variables must be reasonable across long time horizons and across different time steps. That is, the statistical properties of the factors must be consistent whether the model is run monthly, quarterly, or annually.

A good model must be able to capture risk both within and across time. This can be accomplished with a multi-period model. As a counter-example, the traditional Markowitz model is a one-period asset allocation model based on statistical observations of means, variances and correlations and as such, the Markowitz model does not address risk over time. One of the key

[3] We could calibrate for pricing purposes, but in our experience this does not generate reasonable results for future economies.

statistics for risk over time is serial correlation (sometimes referred to as auto-correlation) which is any time series correlation with itself lagged one (or more) time periods.

The Markowitz model also does not create a direct link between underlying economic variables and the asset model. Thus, the Markowitz model cannot consistently create an asset liability framework as there is no direct link between assets and liabilities. A more preferable approach is to build an underlying economic framework and then evaluate both assets and liabilities based on that framework. As an example, an increasing inflation environment will affect both equity markets and certain insurance liabilities.

Many interest rate models do not build a term structure per se, but rather build short-term rates and short-term forward rates. The forward rates imply a term structure at a given point in the future, and the term structure implied based on forward rates today can be viewed as the market's expectation of the future yield curve. However, this is not necessarily a good predictor or even estimator of future yield curves.

Brennan and Schwartz (1979) propose using stochastic differential equations to price bonds. They start with a model for short-term interest rates and long-term interest rates with some inter-dependencies. Based on these two models, they apply Ito's Lemma to derive the necessary structure of the stochastic equations to create a no-arbitrage condition. This is a pricing application.

While the approach we propose is similar in some regards, we do not solve algebraically to create no-arbitrage stochastic equations. Rather, we monitor the modeled results for reasonability and arbitrage opportunities. Clearly any model that creates persistent and significant arbitrage opportunities must be questioned.

Though the yield curve today is a poor predictor of future rates, it is reasonable to assume that the short-term rate will co-move to some extent with the long-term rate, as the long-term rate holds information about the future expected values of the short-term rate. Brennan-Schwarz captures this through a joint Gauss-Markov process and this reflects both the pure expectations hypothesis and the liquidity premium hypothesis. GEM utilizes a similar methodology - though employing it with forward rates rather than with yields or spot rates.

The Wilkie interest rate model breaks interest rates into two components, specifically a real interest rate, which tends to be fairly stable, and inflation, which can be quite volatile at times. Wilkie notes that equity dividend yields and inflation tend to be highly correlated. He views inflation as driving interest rates rather than the opposite. Note, that Brennan-Schwartz does not consider inflation or other indicators in a larger economic context.

Heath-Jarrow-Morton (Heath et al., 1990) has received much attention during the past few years. The HJM model is a more recent extension of the arbitrage-free pricing model. HJM cleverly extends the single factor (short interest rate only) to a multi-factor environment (two or three) but the complexity increases dramatically. In addition, just because the market expects a given term structure in the future does not by any means suggest that this is at all a reasonable estimator of the future. The market changes its expectations almost instantaneously and continuously. The HJM model is based on forward rates from which spot rates and yields can be derived. There are some advantages to basing a stochastic model (pricing or strategic) on forward rates. Namely, if a reasonable forward rate curve is modeled, it is likely that spot rates and yields look reasonable as well. The reverse is not true (Tilley, 1992).

The GEM system incorporates ideas from all of the above. In addition, we have complemented with our own analysis as shown in the pages that follow.

## 3.2 Types of Models

We distinguish between two types of asset modeling approaches. Pricing models are entirely based on the notion that any risk-free profit (above the risk-free rate – this is known as arbitrage) will be exploited in the market place until it no longer exists. The very nature of this action eliminates the risk-free profit. Pricing models generally work in the risk-neutral world, which is particularly useful for pricing liquid contingent options that can be replicated through other vehicles that are also liquid (and can be shorted). But the risk-neutral approach falls short when trying to determine reasonable returns for asset classes and interest rates in general over multiple time horizons. Specifically, the inherent assumptions that all asset classes return the risk-free rate[4] is not satisfactory for a risk management system where one at least should have the option to specify different risk premia for different asset classes. There are also practical implications in terms of "exploding" lattice models, which require a geometrically increasing number of branches with increasing number of time periods.

Strategic models consider an almost infinite series of possibilities. The more scenarios one creates through the Monte Carlo simulation, the more possibilities one can explore. These scenarios depict plausible paths for the future. Some paths have high equity returns, some have low returns. Some have rising interest rates. Some have falling interest rates. On average, the asset class returns reflect the risk-premiums associated with the economic environments under which they are modeled. There is no reason that this should be the risk-free rate – just like in the real world.

Pricing models give a pricing snap-shot at a point in time of certain contingent claims. Strategic models provide a view over time that can be used to design strategies that manage risk and return. The GEM system utilizes Monte Carlo simulations.

## 3.3 Global Economic Model

The Global Economic Model (GEM) is based on a series of stochastic difference equations. The difference equations have an underlying structure as graphed below (Figure 2). We adopt this structure as a way to capture the complex relationships that the real world offers.

The structure demonstrates how the model is developed within each time period. Although the time increments in the model are flexible, the default is monthly. Each month the system simulates values for each item in accordance with this structure.

We use stochastic differential equations to build our underlying framework. The examples in Figure 3 below show the most basic form of Brownian motion. The "dZ" is a Wiener process, which is generated from a standard normal distribution. "$l_t$" represents the long interest rate (for example, the one-period 30 year forward rate) and "$l_\mu$" is the long-term equilibrium for $l_t$. "$a_l$" and "$\sigma_l$" are calibration parameters. They control the movement and overall volatility of the stochastic process. $a_l$ is often referred to as the "mean reversion parameter", while $\sigma_l$ is the

[4] Arguably one could replace the risk-neutral probabilities with "real-world" probabilities to generate "real-world" scenarios.

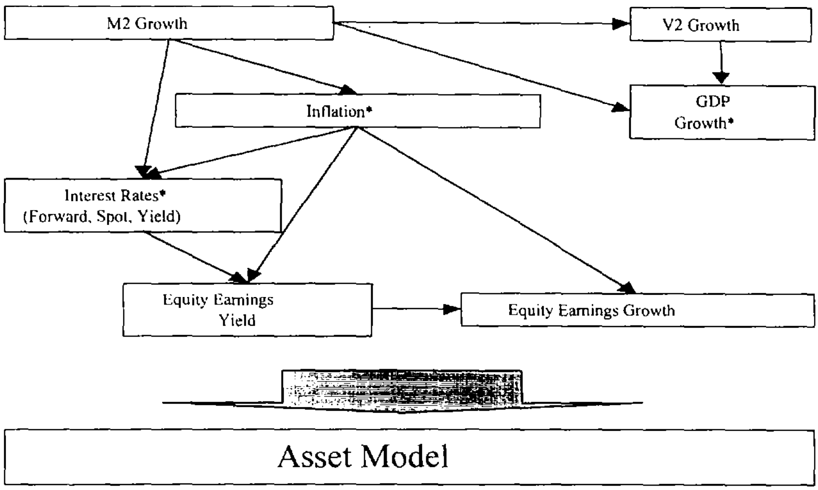


Figure 2. The economic framework underlying GEM. Each variable is modeled via stochastic difference equations. Asterisks indicate variables that could have links to other countries.

volatility parameter. Note, that we are modeling the difference from one time period to the next. This captures the basic notion that economic time series tend to exhibit significant serial correlation over time, while any change in the series tend to be more independently distributed. Similar observations apply to the short rate process shown below the long rate process.

Figure 3:

Long interest rates

$$dl_t = a_l(l_\mu - l_t)dt + l_t\sigma_l dZ_l$$

Short interest rates

$$dr_t = a_r(r_\mu - r_t)dt + r_t\sigma_r dZ_r$$

We normally start the process with the economic environment today (for prospective simulations). Specifically, we get $l_0$ and the rest of the starting yield curve from publicly available data. When calibrating the model (back-testing), however, we start the model in the economic environment that matches the data starting point.

Generally, we will define a stable long-term economic environment that looks very much like the current environment except for a change in the short interest rate to create a more normal looking yield curve. The normal yield curve spread is assumed to be 150 basis point (bp), and short real yields are assumed to be 200 bp. Based on this information, we develop our base line simulation ("base"). We calibrate to fit our targets to the base.

Once the base has been fitted, we change the long-term median assumptions. Clients will often want to explore the risk they are facing if the median environment differs from the one assumed. What happens if interest rates are most likely to increase over the next ten years? What if they are most likely to fall? We can explore all of these options separately or together, and we must ensure that the model holds up to these stress tests and still generates acceptable results (Mulvey and Madsen, 1999).

## 4. Setting Targets

Targets are properties we would like the generated scenarios to possess. The statistic is the actual value calculated from the scenarios. To fix the idea, a target could be the average value (across scenarios) for the annualized standard deviation of stock prices, such as 20%. The statistic would be the calculated average standard deviation of stock prices from the generated scenarios, which we would hope would be close to 20%. Our goal could be to have the statistic as close as possible to the target. Alternatively, we can specify a range of acceptable values and penalize statistics outside the target range.

Some targets we specify are:

- Arithmetic means
- Compound means
- Standard deviations
- Skewness and kurtosis (though we generally place less weight on these)
- Tails of non-normal distributions
- Minimum and maximum observations
- Correlations
- Serial correlations
- Yield curve statistics

David Becker of Lincoln National studied US interest rates (Becker, 1995). He used the period 1955 --1994 and made a number of interesting observations. Based on his observations, he developed a number of "stylized facts" that an interest rate model should possess:

- Rates are non-negative
- Rates do not go to zero nor do they go low and stay low
- Rates do not go to infinity nor do they go high and stay high
- Rates neither increase nor decrease rapidly with significant frequency
- Rates have the appearance of a random walk
- Rates have the appearance of mean reversion, i.e. when rates fall they rebound to "normal" levels, and similarly when rates rise
- Rates tend to cluster in trading ranges, or narrow bands, before breaking out to a higher or lower range
- Periodic movements in rates are not independent, but are correlated to a limited number of prior period movements
- Levels of serial correlation tend to decrease with maturity
- Short term and long term rates are highly correlated, but not perfectly correlated
- Generally, rates tend to rise and fall together. Thus, shifts in term structure are largely "parallel"

- Higher absolute interest rate levels are associated with higher absolute interest rate volatility
- Rate volatility declines with maturity
- Yield curve inversions:
  - Frequency: Less than 16% absolute
  - Infrequent and of limited duration
  - Occur during severe economic stress, geopolitical and/or policy volatility
- Yield spreads decrease with maturity, i.e. 1 year – 3 month spread > 3 year – 1 year spread and so on
- Correlation between increase in CPI and Treasuries declines as maturity increases
- *In general, as rates rise spreads narrow such that the yield curve flattens; and as rates fall, spreads widen such that the yield curve steepens*

We designed our model targets to capture these stylized facts as well as other calibration targets.

Cash tends to have a high serial correlation as does inflation, whereas stocks tend to have slightly negative serial correlation. Even these general observations, however, change over time as is illustrated by the example below.

Example of Target:

The correlation between long-term yields and inflation has ranged from –35% to 70% (Figure 4).

**10 Year Correlation Between LT Yields and Inflation**

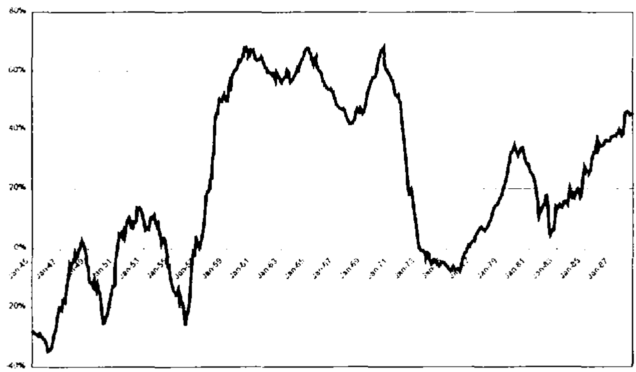

Figure 4. Historical correlation between long-term government bond yields and inflation.

How do we set a reasonable target based on this information? Our target becomes a distribution with an expected value of 30%-40%. We still create some paths with correlation of –40%, but they occur less frequently than paths with 30% correlation between the two variables (Figure 5).

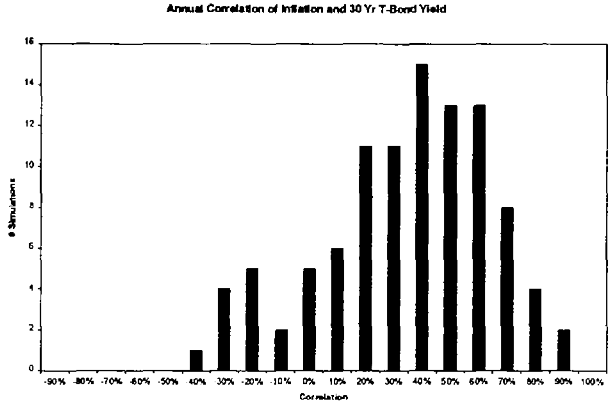


Figure 5. Simulated correlation between long-term government bond yields and inflation (distribution looks "choppy" as only 100 simulations were run).

## 5. Calibration Methodology

Calibration targets can be monthly, annual or any other time period. A penalty is assigned for each deviation from a target. The goal is to calibrate the model to minimize the assigned penalties. AmRe's Constraint Evaluator System is used in this process. The Constraint Evaluator System utilizes a non-convex optimizer developed by Lattice Financial. See Berger et al. (1998) for an algorithm overview and Berger (1999) for technical information.

Model parameters are set to initial values using linear multiple regression. We take historical data, set up the difference equation, perform the regression, and utilize the results as the starting point for the analysis.

Calibration Example #1:

$$\Delta f_t = A \cdot \Delta I_t + B \cdot \Delta Y_t + C \cdot \Delta u_t + D \cdot \sqrt{t} \cdot \Delta Z$$ [5]

Here $f$ represents the 3-month one-period forward rate, $I$ represents inflation, $Y$ represents the 30 year one-period forward rate, $u$ represents the inflation adjusted mean reversion process, and $dZ$ is Wiener term and $t$ is time unit. A, B, C, and D are the calibration parameters for this difference equation. A controls the effect inflation has on the 3 month forward rate and B controls the relationship with the long end of the forward rate curve. C controls the rate of reversion, while D reflects the volatility added to the stochastic process.

Regressing this on monthly historical data from 1974 through 1998 (Figure 6), we get {A, B, C} = {0.015, 1.3, -0.015}. All parameters have significant t-statistics with 90%

[5] The difference equation offered here is actually a two-part log-linear process.

confidence, and the $R^2$ is 58%. D is added after reviewing the residual standard error, which is 0.004. The ratio of the residual standard error to the mean is 0.06. Since this is a log-linear process, D is 1.06.

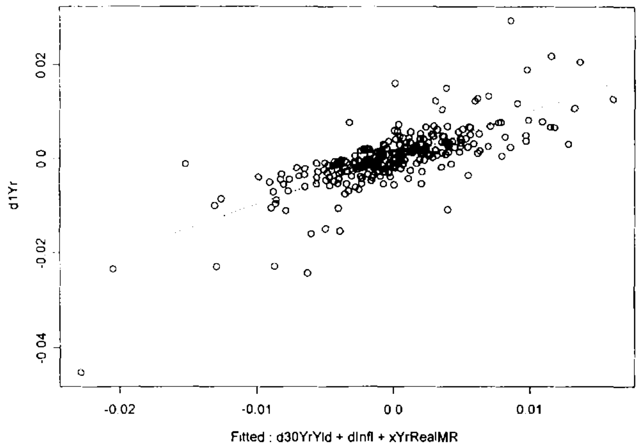


Figure 6. Linear regression results (using historical data) of regressing the change in 3 month yield versus the change in 30 year yield, change in monthly inflation rate, and real inflation-adjusted mean reversion

Now we need to incorporate the regressed results with our simulation goals and simulated data. We have a number of criteria that we monitor with respect to the generated time series. Is serial correlation high enough for shorter term yields? Are we generating a reasonable number of recessions? Are recessions characterized by both inverted yield curves and drops in real GDP? The list goes on to include basic statistics of the modeled indicators.

We code our targets and perform the following optimization described below. Notice that each time series depends on the calibration vector. Specifically, changing the values of {A, B, C, D} will give us different time series, as the difference equations change. We use our regression as a starting point and we want the calibration vector that comes closest to our targets. We run the following optimization:

$$Minimize \sum_{i=1}^{Scenarios} \sum_{j=1}^{Targets} w_j \cdot \left(Statistic_{i,j} - Target_j\right)^2$$

In this case, the result is {0.75, 0.5, -0.04, 1.05}, and we utilize these new values to generate the economic scenarios. The main vector changes were:

- shift weight from the 30 year rate to inflation to increase the correlation between inflation and the 3 month treasury bill

- increase the level of inflation-adjusted mean reversion to avoid "run-away" scenarios (tails were overstated using regression scenarios)
- decrease volatility slightly

If we had wanted to maintain a closer correspondence with the historical regression parameters, we could have penalized deviations from our initial calculated values. In this example, we were more concerned with matching our other calibration targets.

Calibration Example #2:

The optimization (minimize penalties by changing the calibration parameter set – see equation above) can be reviewed from other perspectives as well. We take a closer look at inflation in the calibration. The starting vectors (except for monetary growth, which is at the top of the structure – Figure 2) are all based on linear regressions using historical data. In this particular case, we can see from the chart below (Figure 7) that the volatility of inflation associated with our starting calibration parameters is understated compared with the historical data.

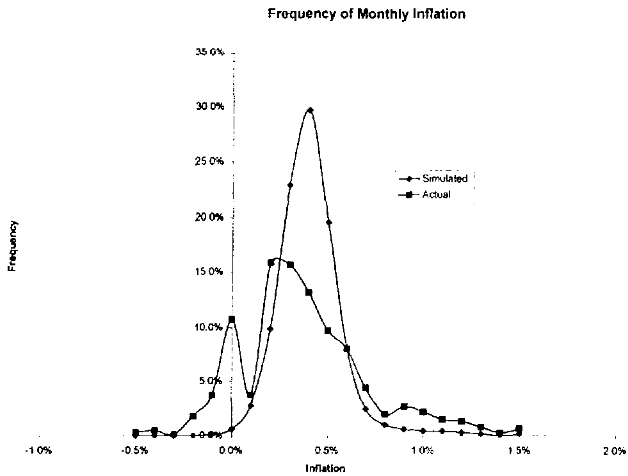


Figure 7. Distribution of monthly inflation levels in generated scenarios based on regression vector has a higher mean and tighter range compared with historical observations from 1974 through 1998.

The differences between simulated and historical results are due to a number of factors. There are sources of variation that are not represented in the regressed data. In addition, estimating the error term from the regression in terms of difference equations is often tricky. Further, statistics such as serial correlation is not monitored through regression, and the relationship may not be perfectly linear. In the graph above, we note that the tails based on historical inflation are much wider.

To address this discrepancy, we specify the volatility of inflation as a calibration target. The historical volatility is 0.33% (3.2% annually) and the volatility from the simulated scenarios above is much less. We specify the historical volatility of 0.33% as a target for the optimization. After optimizing, the resulting inflation levels are shown in Figure 8. The distribution is now much closer to the historical observations. Note that we were able to accomplish this by specifying only one parameter of the distribution (volatility). If this still did not produce the desired results, or if we wish to match more closely, we could specify quantiles on the distribution as targets.

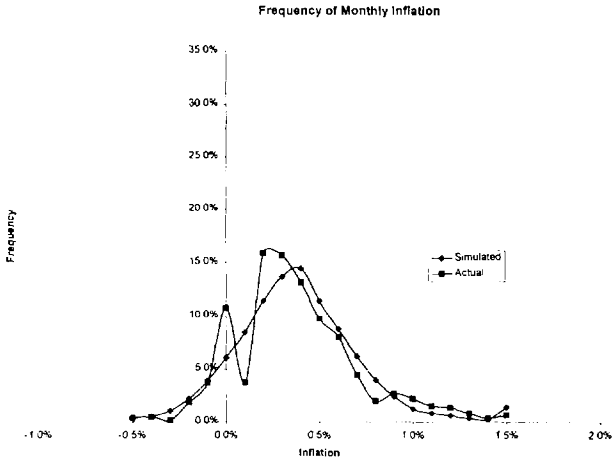


Figure 8. Distribution of monthly inflation levels in generated scenarios based on optimized vector now matches historical observations.

The optimizer helps us fit our model to the available data. Thus, we are able to maintain our economic framework, which is consistently applied to our loss simulation and our asset simulations. We are simultaneously capturing data we would otherwise only be able to capture with more limited methodologies.

In practice, we work with up to 245 calibration parameters for the US model though approximately 50 parameters capture the main process. Optimizing on all these at once has not been practical. Rather, we work our way down the structure shown in Figure 2. We initially calibrate the parameters associated with monetary growth and velocity. Then we calibrate inflation and so on.

## 6. Conclusion

In this paper, we have discussed the scenario generation component of a dynamic financial analysis system. The goal is to produce coherent and comprehensive scenarios for use in modeling an insurance company's financial position over time. American Re-Insurance's GEM system is an example of a generator grounded in economic theory, but one which produces scenarios consistent with historical observations. The calibration process is the mechanism for achieving this: Model parameters are chosen so that the generated scenarios have statistics consistent with user-specified targets. Lattice Financial's optimization software automates the process of determining the best model parameters to meet the desired targets.

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