ACCRUED RETROSPECTIVELY RATED PREMIUMS BY INDIVIDUAL POLICYHOLDER ACCOUNT

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Abstract

The literature that the author has seen to date, Berry[1] and Perkins & Teng[2], concerning the accrued retrospectively rated premium focuses on calculating the asset in bulk by policy year, but not by policyholder account. In order to derive the asset by individual policyholder account, the bulk asset would have to be allocated after the fact. Another way to view this process is to first allocate (or calculate) the ultimate loss and allocated loss adjustment expense by individual policyholder plan or account. Such an approach would have strong intuitive appeal, since average retrospective rating parameters may change over time due to market pressures or a changing mix of business. This paper describes a method that was developed to calculate the accrued retrospectively rated premium using this approach.

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I. INTRODUCTION

Roy K. Morell in his discussion of Berry[1] wrote highly of calculating the accrued retrospectively rated premiums or retrospective reserve by individual policyholder account. The following is a quote of the paragraph. "Although possibly of theoretical value only, consideration should also be given to the perfect method of setting a reserve for retrospective rating adjustments. By this I mean a procedure which establishes a retrospective reserve for each individual account. Such a method could explicitly recognize all the individual characteristics of the policies which make up the group of policies for which a reserve is being set. Since any method which develops a reserve for a group is unavoidably imperfect, such an ideal system at least deserves mention. Obviously such a system would have many practical and some theoretical obstacles to it. However, given the computer technology available, these obstacles may be overcome."[1] This paper attempts to take a step towards this "perfect" system. In designing this system and developing a methodology, many of these practical and theoretical obstacles were encountered. The solutions were reasonable and practical, but often not theoretically nor statistically ideal. Since loss development is analyzed for some group of policies in bulk, this method is actually a sophisticated procedure to allocate the ultimate loss and allocated loss adjustment

expense by individual policyholder account and then calculate the resulting accrued retrospectively rated premium by individual policyholder account.

II. REASONS

There are several reasons (other than the one mentioned by Mr. Morell) for calculating this accrual by policyholder account or even by policy. One reason to calculate the asset by policyholder account is due to statutory reporting requirements. A portion of the amount reported in the statutory annual statement for accrued retrospectively rated premium must be categorized as nonadmitted. According to the NAIC Annual Statement Instructions[3], if the accrued retrospectively rated premium related to bulk IBNR is allocated to individual policyholder accounts, then the nonadmitted portion may be calculated using alternative methods. This can result in a significantly smaller charge to statutory surplus. Other reasons include:

- 1) increased accuracy when average retrospective rating parameters are changing,
- 2) improved management information for use in decision making,
- availability of information in order to give customers guidance concerning their future retrospectively rated premiums,
- 4) more accurate measure of exposure to credit risk,
- 5) the ability to more accurately time cash flows based on customers retrospectively rated premium billing dates, and
- 6) intuitive appeal.

Assuming that none of the direct accrued retrospectively rated premium is recoverable from an insured for whom any agents' balances or uncollected premiums are classified as nonadmitted, then a company can choose between two methods for calculating the nonadmitted portion of the direct accrued retrospectively rated premium. The first step in each method is to determine what portion of the accrued premiums are "not offset by retrospective return premiums, other liabilities to the same party(other than loss and loss adjustment expense reserves), or collateral, not otherwise used."[3] The first method applies an across the board 10% nonadmitted percentage to the remaining asset, while the second method applies a percentage based on the Quality Rating of the account. In order for an insurer to switch from one method to another, approval is required "from the insurer's domiciliary state and such change must be disclosed in Note #1 of the annual statement Notes to Financial Statements."[3]

Depending on the Quality Rating of an insurer's accounts, there can be a significant difference in the charge to statutory surplus based on which method an insurer uses. Consequently, if an insurer does not allocate accrued retrospectively rated premiums to individual policyholder account, then the charge to statutory surplus can be larger than necessary.

Example 1 – Charge to Statutory Surplus.

An insurer's accrued retrospectively rated premium is 100 million. None of this premium is recoverable from an insured for whom any agents' balances or uncollected premiums are classified as nonadmitted. None of this premium is offset by retrospective return premiums, other liabilities to the same party(other than loss and loss adjustment expense reserves), or collateral, not otherwise used. The insurer has allocated the 100 million by individual policyholder account, data shown is in millions of dollars.

Method 1- Across the board 10% Nonadmitted:

(1)	(2)	(3)		
Accrued	Nonadmitted	Nonadmitted		
Premium	Percentage	Portion		
		(3)=(1)x(2)		
100	10%	10.00		

Method 2 - Nonadmitted based on Quality Rating:

(1)	(2)	(3)	(4)	
Quality	Accrued	Nonadmitted	Nonadmitted	
Rating	Premium	Percentage	Portion	
			(4)=(2)x(3)	
1	60	1%	0.60	
2	20	2%	0.40	
3	5	5%	0.25	
4	5	10%	0.50	
5	0	20%	0.00	
6	0	100%	0.00	
unrated	10	20%	2.00	
Total	100		3.75	

In this example, the charge to statutory surplus for method 1 is 6.25 million greater than the charge to statutory surplus for method 2. This difference would not be known if the accrued retrospectively rated premium was not allocated by individual policyholder account.

III. RETROSPECTIVE RATING FORMULA

This paper assumes a prior knowledge of retrospectively rated policies, but a brief description is given to refresh memories and define some formulas and assumptions used in this paper. A retrospectively rated policy is one where a subject standard premium is collected during the policy period. Six months after policy expiration, a premium is calculated based on the experience. If this formula premium is less than the premium previously collected, then the redundancy is returned to the insured, and if it is greater than the premium previously collected then the insured is billed for the shortfall. This process is repeated annually until the insured and insurer mutually agree that there will be no more retrospective rating adjustments processed. Usually a policyholder plan includes all retrospectively rated policies for that account for more than one year. A policyholder plan is important because the maximum premium and minimum premium are applied on a plan level rather than a policy level. In this paper, it is assumed that all policies cover a 12 month policy period. If three year policies are written, they should either be broken up into three 12 month pieces or appropriate adjustments should be made to formulas.

In this paper, the retrospective rating formula that will be used for each policy is:

 $URP = [BASIC + ELP + CL] \times TM$ $CL = SL \times LCF$

URP = Unlimited Retrospectively Rated Premium BASIC = Subject Standard Premium x Basic Premium Factor ELP = Excess Loss Premium Factor x Subject Standard Premium x Loss Conversion Factor CL = Converted Loss SL = Subject Loss LCF = Loss Conversion Factor TM = Tax Multiplier The unlimited retrospectively rated premiums for each policy in the plan are summed up. The total unlimited retrospectively rated premium for the plan is then adjusted to ascertain that it is greater than the minimum premium and less than the maximum premium. The resulting premium is the retrospectively rated formula premium.

IF MIN \leq URP \leq MAX \Rightarrow RP = URP IF URP < MIN \Rightarrow RP = MIN IF MAX < URP \Rightarrow RP = MAX

RP = Retrospectively Rated Premium

MIN = Subject Standard Plan Premium x Minimum Premium Factor

MAX = Subject Standard Plan Premium x Maximum Premium Factor

Subject Standard Premium and Subject Loss are the policy standard premium and policy losses which are subject to the above retrospective rating formula. BASIC, LCF and TM provide for commissions, other acquisition, general underwriting expenses, various loss adjustment expenses (not included in subject loss), premium taxes, assessments, profit, contingencies and any other expenses that should be included in the price of the insurance coverage. The subject loss may or may not include allocated loss adjustment expense and may or may not include an adjustment for the application of a per occurrence limit. An occurrence will be defined in the policy, but, generally, it is a single event which triggers one or more claims. If more than one claim arises from a single occurrence, then the amounts of those claims are aggregated before the application of the per occurrence limit. For a more detailed explanation of retrospectively rated policies, see Tiller[4] or Gillam & Snader[5].

Retrospective rating parameters will be used to refer to the basic premium factor, the excess loss premium factor, loss conversion factor, tax multiplier, minimum premium factor, maximum premium factor and definition of subject loss. These parameters are specified in the retrospectively rated policy endorsement.

IV. METHODOLOGY

The methodology for calculating the accrued retrospectively rated premiums is to calculate the ultimate retrospectively rated premium for each account and then subtract the premium previously collected. The ultimate retrospectively rated premium is derived by calculating the ultimate loss and allocated loss adjustment expense (ALAE) for each active retrospectively rated policy and then applying the retrospective rating formula using all the parameters of the individual policies and plans. The parameters that affect the ultimate converted loss include loss limit per occurrence, expenses not subject to the retrospective rating calculation and application of the loss conversion factor. The ultimate converted loss is added to the basic and excess loss premium (if applicable), then the sum is multiplied by the tax multiplier to obtain the ultimate unlimited retrospectively rated premium by policy. The ultimate unlimited retrospectively rated premium for each plan is calculated and then adjusted to reflect the plan maximum and minimum to produce the ultimate retrospectively rated premium.

A. Basic and Excess Loss Premium

For each policy, the basic is calculated by multiplying the earned subject standard premium by the basic factor. The excess loss premium is calculated by multiplying the earned subject standard premium by the excess loss premium factor and the loss conversion factor for each policy. For many policies there will not be an excess loss premium factor; in this case, a factor of zero can be used.

The following sections have examples with policy, retrospective rating parameter, development, ratio of severities, excess severity, loss ratio, ALAE ratio and loss reserve liability IBNR data that is for illustrative purposes only. The derivation of this data is not included in the paper. All data in the examples for this paper is assumed to be evaluated as of 12/31/98.

Example 2 – Basic and Excess Loss Premium.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	
	Earned		Excess				
	Subject		Loss	Loss		Excess	
Policy	Standard	Basic	Premium	Conversion	Basic	Loss	
Number	Premium	Factor	Factor	Factor	Premium	Premium	
					(6)=(2)x(3)	(7)=(2)x(4)x(5)	
WCP1	1,000,000	0.200	0.040	1.065	200,000	42,600	
GLP2	1,000,000	0.150	0.000	1.100	150,000	0	
ALP3	750,000	0.180	0.000	1.080	135,000	0	

B. Ultimate Converted Loss

The ultimate converted subject losses are calculated by multiplying the ultimate subject losses for each policy by the appropriate loss conversion factor. In order to define subject losses for a policy, there are usually at least three pieces of information that are needed:

- 1) whether or not ALAE is included in the definition of loss for purposes of applying the retrospective rating formula,
- 2) the per occurrence limit, and
- 3) whether or not ALAE is included in the definition of loss for limiting purposes.

If ALAE is not included in the definition of subject loss, then it will be referred to as a Nonretro Expense. The loss and, if applicable, ALAE which exceed a per occurrence limit will be referred to as Excess Loss. The ultimate subject losses are equal to the ultimate gross losses and ALAE less the ultimate nonretro expenses less the ultimate excess losses.

Example 3 – Ultimate Converted Losses.

(1)	(2)	(3)	(4)
	Ultimate	Loss	Ultimate
Policy	Subject	Conversion	Converted
Number	Loss	Factor	Loss
	Example 12	Example 2	(4)=(2)x(3)
WCP1	728.232	1.065	775.567
GLP2	951,000	1.100	1,046,100
ALP3	378,975	1.080	409,293

1. Ultimate Gross Loss And ALAE For Occurrences That Have Been Reported

This section discusses calculating the future development only on occurrences that have already been reported using the gross incurred loss and gross incurred ALAE development methods. In order to separate this development from the development arising from all occurrences, report year triangles are used instead of accident or policy year triangles. If report year is not available, then accident year or policy year development factors should be adjusted to derive development factors for future development only on occurrences that have already been reported. If the assumption is made that the average severity of occurrences that have not been reported is equal to the average severity of occurrences that have not been reported is the accident year cumulative loss or ALAE development factor at age x divided by the cumulative occurrence count development factor at age x. If report year is not an available or not a reliable field in the data, then it can be derived. One possibility is to use the year in which the incurred loss or loss and ALAE is greater than zero.

In this paper, loss and ALAE will be developed to ultimate separately. If they are not, then for policies where ALAE is a nonretro expense, the ultimate loss and ALAE will need to be split into the separate components so that the ALAE can be subtracted out. Likewise, in the case where ALAE is included in the definition of loss, but not for limiting purposes, the ultimate ALAE will need to be subtracted out, the per occurrence limit applied and the ultimate ALAE added back to the ultimate limited loss. For purposes of applying a per occurrence limit, this methodology calculates the ultimate gross loss and ALAE per occurrence. When applying development to individual occurrences, it is important to realize that for many books of business, on average, large losses have a lower ultimate development

factor than small losses of the same age. (The author first became aware of this during a presentation of Angelina & Buchanan [6].) When the same average development factor is applied to large losses as small losses, more case development ends up in excess of the loss limitation than should for the large occurrences as illustrated in Example 4. This excess is not subject to the retrospective rating calculation and thus, not converted to premium which may cause an understatement of the asset. On the other hand, when an average development factor is applied to a small occurrence, it usually will not exceed a per occurrence limit. This means that a redundant amount of IBNR may be allocated below the loss limitation, which is then converted to premium causing an overstatement of the asset. Unfortunately, these two amounts are not always completely offsetting for the total book of retrospectively rated policies and are even less likely to be offsetting for an individual policy or policyholder account. These issues are probably most appropriately addressed by a fully stochastic model, but in lieu of that, they can be adequately addressed using empirical methods as is done in this paper. Developing losses differently according to size will be discussed in this section; while adjusting for small losses which will eventually exceed the limit will be discussed in the Excess Loss section.

Example 4 – Large Loss Development.

Development Based on All Occurrence Sizes Combined

(1)	(2)	(3) Cumulativa	(4)	(5)	(6)
Reported Occurrence Loss	Report Age in Months	Loss Development Factor	Ultimate Occurrence Loss (4)=(1)x(3)	Per Occurrence Limit	Ultimate Excess Loss (6)=(4)-(5)
1,250,000	12	2.00	2,500,000	250,000	2,250,000

Development Based on Occurrences > \$1,000,000

(1)	(2)	(3) Cumulative	(4)	(5)	(6)
Reported Occurrence Loss	Report Age in Months	Loss Development Factor	Ultimate Occurrence Loss (4)=(1)x(3)	Per Occurrence Limit	Ultimate Excess Loss (6)=(4)-(5)
1,250,000	12	0.97	1,212,500	250,000	962,500

Difference in ultimate excess:

1,287,500

The issue of large versus small loss development will be addressed by applying report year development factors by size of loss. This can be accomplished by looking at report year development triangles for different sizes of gross incurred loss such as:

Range 1	<=50,000
Range 2	>50,000 and <=100,000
Range 3	>100,000 and <= 250,000
Range 4	>250,000 and <=500,000
Range 5	>500,000 and <=1,000,000
Range 6	>1,000,000

These triangles can be developed by line of business, by hazard group, by industry type, by state or along any other data definition lines that are desired and have enough credibility to give reliable results. The system will be more complicated if many different sets of development factors are used. Conceptually, ALAE development could be separated by size of occurrence based on loss only, loss and ALAE combined or ALAE only. The author does not know the benefits nor the practicalities of doing this, since she has not researched whether ALAE develops differently by size. This method will assume that ALAE report year development for all occurrence sizes combined can be used without too much loss of accuracy.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) Cumul	(9) ative
		Reported			Report	Size of	Loss	Ultimate
Policy	Occurrence	Occurrence	Accident	Report	A op in	Loss	Development	Occurrence
Number	Number	Loss	Date	Year	Months	Range	Factor	Loss
munioer	Tuntoer	Loss	Duic	rear	11011115	nunge	1 40101	(9)=(3)x(8)
WCP1	WCC1	5,000	08/20/97	1997	24	1	1.75	8,750
WCP1	WCC2	55,000	10/05/97	1997	24	2	1.60	88,000
WCP1	WCC3	1,250,000	<i>01/27/98</i>	1998	12	6	0.97	1,212,500
WCP1	WCC4	25,000	09/30/97	1997	24	1	1.75	43,750
WCP1	WCC5	180,000	04/18/98	1998	12	3	1.40	252,000
		1,515,000						1,605,000
GLDA		6.000	0.7.1.5.10.7	1007			2.25	12 500
GLP2	GLC6	6,000	0//15/9/	1997	24	1	2.25	13,500
GLP2	GLC/	2,000	02/01/98	1998	12	1	3.00	6,000
GLP2	GLC8	500,000	08/03/97	1998	12	4	1.01	505,000
GLP2	GLC9	7,500	10/17/97	1997	24	1	2.25	16,875
GLP2	GLC10	12,000	03/22/98	1998	12	3	1.50	18,000
		527,500						559,375
ALP3	ALC11	10,000	11/13/97	1997	24	1	1.85	18,500
ALP3	ALC12	750	05/11/98	1998	12	1	2.00	1,500
ALP3	ALC13	750.000	01/26/98	1998	12	5	0.98	735.000
ALP3	ALC14	17.500	08/06/97	1997	24	1	1.85	32.375
ALP3	ALC15	2.500	05/16/98	1998	12	1	2.00	5.000
	112010	780.750	00,10,90	1770		-	2.00	792.375
		,						,
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) Cumul	(9) ative
(1)	(2)	(3) Reported	(4)	(5)	(6) Report	(7) Size of	(8) Cumul ALAE	(9) ative Ultimate
(1) Policy	(2) Occurrence	(3) Reported Occurrence	(4) Accident	(5) Report	(6) Report Age in	(7) Size of Loss	(8) Cumul ALAE Development	(9) ative Ultimate Occurrence
(1) Policy Number	(2) Occurrence Number	(3) Reported Occurrence ALAE	(4) Accident Date	(5) Report Year	(6) Report Age in Months	(7) Size of Loss Range	(8) Cumul ALAE Development Factor	(9) ative Ultimate Occurrence ALAE
(1) Policy Number	(2) Occurrence Number	(3) Reported Occurrence ALAE	(4) Accident Date	(5) Report Year	(6) Report Age in Months	(7) Size of Loss Range	(8) Cumul ALAE Development Factor	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8)
(1) Policy Number WCP1	(2) Occurrence Number WCC1	(3) Reported Occurrence ALAE 250	(4) Accident Date 08/20/97	(5) Report Year 1997	(6) Report Age in Months 24	(7) Size of Loss Range NA	(8) Cumul ALAE Development Factor 1.25	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313
(1) Policy Number WCP1 WCP1	(2) Occurrence Number WCC1 WCC2	(3) Reported Occurrence ALAE 250 2,750	(4) Accident Date 08/20/97 10/05/97	(5) Report Year 1997 1997	(6) Report Age in Months 24 24	(7) Size of Loss Range NA NA	(8) Cumul ALAE Development Factor 1.25 1.25	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438
(1) Policy Number WCP1 WCP1 WCP1	(2) Occurrence Number WCC1 WCC2 WCC3	(3) Reported Occurrence ALAE 250 2,750 35,000	 (4) Accident Date 08/20/97 10/05/97 01/27/98 	(5) Report Year 1997 1997 1998	(6) Report Age in Months 24 24 24 12	(7) Size of Loss Range NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.25 1.60	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000
(1) Policy Number WCP1 WCP1 WCP1 WCP1	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4	(3) <i>Reported</i> <i>Occurrence</i> <i>ALAE</i> 250 2,750 35,000 1,500	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97	(5) Report Year 1997 1998 1997	(6) Report Age in Months 24 24 24 12 24	(7) Size of Loss Range NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.25 1.60 1.25	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5	(3) <i>Reported</i> <i>Occurrence</i> <i>ALAE</i> 250 2,750 35,000 1,500 6,000	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98	(5) Report Year 1997 1998 1997 1998	(6) Report Age in Months 24 24 12 24 12 24 12	(7) Size of Loss Range NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.25 1.60 1.25 1.60	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98	(5) Report Year 1997 1997 1998 1997 1998	(6) Report Age in Months 24 24 12 24 12	(7) Size of Loss Range NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.25 1.60 1.25 1.60	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97	(5) Report Year 1997 1998 1997 1998 1997	(6) Report Age in Months 24 24 12 24 12 24 12 24	(7) Size of Loss Range NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC6 GLC7	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98	 (5) Report Year 1997 1997 1998 1997 1998 1997 1998 1997 1998 	 (6) <i>Report</i> <i>Age in</i> <i>Months</i> 24 24 12 24 12 24 12 24 12 24 12 	(7) Size of Loss Range NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC6 GLC7 GLC8	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97	 (5) Report Year 1997 1997 1998 1997 1998 1997 1998 1997 1998 1998 1998 1998 1998 1998 	 (6) <i>Report</i> <i>Age in</i> <i>Months</i> 24 24 12 24 12 24 12 24 12 12 12 	(7) Size of Loss Range NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97	(5) Report Year 1997 1998 1997 1998 1997 1998 1997 1998 1997	 (6) Report Age in Months 24 24 12 24 12 24 12 24 12 24 12 24 	(7) Size of Loss Range NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75 2.50	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500	 (4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 	 (5) Report Year 1997 1998 	 (6) <i>Report</i> <i>Age in</i> <i>Months</i> 24 24 12 	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75 2.50 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98	(5) Report Year 1997 1997 1998 1997 1998 1997 1998 1997 1998	 (6) <i>Report</i> <i>Age in</i> <i>Months</i> 24 24 12 14 12 14 15 	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 2.50 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97	 (5) Report Year 1997 1998 1997 	 (6) <i>Report</i> <i>Age in</i> <i>Months</i> 24 24 12 24 12 24 12 24 12 24 12 24 12 24 	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75 2.50 3.75 1.45	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11 ALC11 ALC12	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000 450	 (4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97 05/11/98 	 (5) Report Year 1997 1998 	 (6) Report Age in Months 24 24 12 	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75 2.50 3.75 1.45 2.00	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250 900
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11 ALC11 ALC12 ALC13	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000 450 150,000	 (4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97 05/11/98 01/26/98 	 (5) Report Year 1997 1998 	 (6) Report Age in Months 24 24 12 12 12 12 12 12 14 14	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.25 1.60 1.25 1.60 2.50 3.75 2.50 3.75 2.50 3.75 2.50 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250 900 300,000
(1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11 ALC12 ALC13 ALC14	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000 450 150,000 12,500	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97 05/11/98 01/26/98 08/06/97	 (5) Report Year 1997 1997 1998 1997 	 (6) Report Age in Months 24 24 12 24 	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 2.50 3.75 2.50 3.75 2.50 3.75 2.50 3.75	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250 900 300,000 18,125
 (1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP2 GLP3 ALP3 	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11 ALC12 ALC13 ALC14 ALC15	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000 450 150,000 12,500 1,200	 (4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97 05/11/98 01/26/98 08/06/97 05/16/98 	 (5) Report Year 1997 1997 1998 	 (6) Report Age in Months 24 24 12 14 12 14 14	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 3.75 2.50 3.75 1.45 2.00 2.00 1.45 2.00	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250 900 300,000 18,125 2,400
 (1) Policy Number WCP1 WCP1 WCP1 WCP1 WCP1 GLP2 GLP2 GLP2 GLP2 GLP2 GLP3 ALP3 	(2) Occurrence Number WCC1 WCC2 WCC3 WCC4 WCC5 GLC6 GLC7 GLC8 GLC9 GLC10 ALC11 ALC12 ALC13 ALC14 ALC15	(3) Reported Occurrence ALAE 250 2,750 35,000 1,500 6,000 45,500 4,000 1,200 100,000 5,000 3,500 113,700 5,000 450 150,000 12,500 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,200 1,500 1,500 1,500 1,500 1,500 1,500 1,200 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,3,500 1,200 1,3,500 1,3,500 1,200 1,3,500 1,3,500 1,200 1,200 1,200 1,3,500 1,3,500 1,200	(4) Accident Date 08/20/97 10/05/97 01/27/98 09/30/97 04/18/98 07/15/97 02/01/98 08/03/97 10/17/97 03/22/98 11/13/97 05/11/98 01/26/98 08/06/97 05/16/98	 (5) Report Year 1997 1997 1998 	 (6) Report Age in Months 24 24 12 14 12 14 14	(7) Size of Loss Range NA NA NA NA NA NA NA NA NA NA NA NA NA	(8) Cumul ALAE Development Factor 1.25 1.60 1.25 1.60 2.50 3.75 2.50 3.75 2.50 3.75 1.45 2.00 2.00 1.45 2.00	(9) ative Ultimate Occurrence ALAE (9)=(3)x(8) 313 3,438 56,000 1,875 9,600 71,225 10,000 4,500 375,000 12,500 13,125 415,125 7,250 900 300,000 18,125 2,400 328,675

Example 5 - Ultimate Loss and ALAE for Occurrences That Have Been Reported.

2. Ultimate Loss And ALAE For Occurrences That Have Been Incurred But Not Yet Reported

Next, the ultimate loss and ultimate ALAE for occurrences that have been incurred but not yet reported must be calculated. This can be accomplished by applying loss factors and ALAE factors to an exposure base. A convenient measure of exposure is earned subject standard premium, which is used elsewhere in the calculations. Another option would be to use the rating basis, if available. The formula is as follows:

IBNYR_x = loss or ALAE for occurrences that have not yet been reported at age x NYRC_x = number of occurrences that have not yet been reported at age x UL = ultimate loss or ALAE UC = ultimate occurrence count EXP = earned portion of the measure of exposure RATE = ultimate loss or ALAE per exposure CDF_x =cumulative occurrence count development factor at age x

Assume that the average severity of occurrences that have not yet been reported at age x is equal to some factor Y_x times the average severity of all occurrences. In formula terms, this is $IBNYR_x/NYRC_x = Y_x(UL/UC)$.

Then,

 $IBNYR_x / EXP = (IBNYR_x / NYRC_x)(RATE/UL) NYRC_x$ $= Y_x(UL/UC)(RATE)(NYRC_x / UL)$ $= Y_x(RATE)(NYRC_x / UC)$ $= Y_x(RATE)(1-1/CDF_x)$

In practice, an expected ultimate loss or ALAE per exposure would be used for RATE.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Earned			Cumulative	Expected		Expected	Expected
	Subject	Policy	Policy	Occurrence	Ultimate		Loss	Ultimate
Policy	Standard	Effective	Age in	Development	Loss	Ratio of	IBNYR	IBNYR
Number	Premium Example2	Date	Months	Factor	to Premium	Severities	<i>Factor</i> (8)=(7) <i>x</i> (6) <i>x</i> [1-1/(5)]	<i>Loss</i> (9)=(8) <i>x</i> (2)
WCP1	1,000,000	07/01/97	18	1.10	0.65	1.50	0.09	88,636
GLP2	1,000,000	<i>07/01/97</i>	18	1.20	0.40	2.00	0.13	133,333
ALP3	750,000	07/01/97	18	1.05	0.55	1.75	0.05	34,375
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Earned			Cumulative	Expected		Expected	Expected
	Subject	Policy	Policy	Occurrence	Ultimate		ALAE	Ultimate
Policy	Standard	Effective	Age in	Development	ALAE	Ratio of	IBNYR	IBNYR
Number	Premium Example2	Date	Months	Factor	to Premium	Severities	<i>Factor</i> (8)=(7)x(6)x[1-1/(5)]	ALAE (9)=(8)x(2)
WCP1	1,000,000	07/01/97	18	1.10	0.10	1.75	0.02	15,909
GLP2	1,000,000	07/01/97	18	1.20	0.30	2.25	0.11	112,500
ALP3	750,000	07/01/97	18	1.05	0.20	1.75	0.02	12,500

Example 6 – Ultimate Loss and ALAE for Occurrences That Have Not Yet Been Reported.

Note: The data in columns (6) and (7) cannot be obtained from elsewhere in this paper.

3. Reconciling With Gross Liability For Retrospectively Rated Loss

At this point, the ultimate gross loss and ALAE for all occurrences should be aggregated to produce the ultimate gross loss and ALAE for retrospectively rated policies. The reported incurred loss and ALAE for this book can be subtracted to result in the bulk Incurred But Not Reported (IBNR) reserve underlying the accrued retrospectively rated premium. The actuary must make sure that this IBNR reconciles with the bulk IBNR that goes into the gross liability reserve. If the two IBNR amounts do not match within a selected tolerance, then one or the other must be adjusted. (The selected tolerance would be specific to a company and primarily influenced by actuarial judgement and management philosophy.) As an example, if an actuary is more confident of the IBNR from the loss reserve analysis since more methods were used to calculate it, then the IBNR for the accrued premium calculation would be adjusted. On the other hand, if an actuary is more confident of the IBNR from the accrued premium calculation because development by size of occurrence was used, then the IBNR for the liability would be adjusted. In order to have more confidence in the accrued premium IBNR, a paid development method could theoretically be added, but this may be problematic because the paid amount on a large occurrence can often be zero until it is settled. This reconciliation is best done by accident year or policy year for loss and ALAE separately, but can be done in bulk, if necessary.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Ultimate	Ultimate	Ultimate	Bulk Loss	Bulk	Loss	
			Loss for	Loss for	Loss for	IBNR	IBNR		Difference
	Accident	Reported	Reported	Unreported	All	Underlying	Under	rlying	Percentage
Coverage	Years	Loss	Claims	Claims	Claims	Asset	Liability	Difference	of IBNR
		Example 5	Example 5	Example 6	(6)=(4)+(5)	(7)=(6)-(3)		(9)=(8)-(7)	(10)=(9)/(8)
WC	1997	85,000	140,500	44,318	184,818	99,818	99,900	82	0.08%
WC	1998	1,430,000	1,464,500	44,318	1,508,818	78,818	78,800	-18	-0.02%
GL	1997	513,500	535,375	66,667	602,042	88,542	88,500	-42	-0.05%
GL	1998	14,000	24,000	66,667	90,667	76,667	76,600	-67	-0.09%
Auto	1997	27,500	50,875	17,188	68,063	40,563	40,600	37	0.09%
Auto	1998	753,250	741,500	17,188	758,688	<i>5,43</i> 8	5,400	-38	-0.69%
Total		2,823,250	2,956,750	256,345	3,213,095	389,845	389,800	-45	-0.01%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			Ultimate	Ultimate	Ultimate	Bulk ALAE	Bulk A	ALAE	
			ALAE for	ALAE for	ALAE for	IBNR	IBNR		Difference
	Accident	Reported	Reported	Unreported	All	Underlying	Under	rlying	Percentage
Coverage	Years	ALAE	Claims	Claims	Claims	Asset	Liability	Difference	of IBNR
		Example 5	Example 5	Example 6	(6)=(4)+(5)	(7)=(6)-(3)	(9)=(8)-(7)	(10)=(9)/(8)
WC	1997	4,500	5,625	7,955	13,580	9,080	9,100	20	0.22%
WC	1998	41,000	65,600	7,955	73,555	32,555	32,600	45	0.14%
GL	1997	109,000	397,500	56,250	453,750	344,750	344,600	-150	-0.04%
GL	1998	4,700	17,625	56,250	73,875	69,175	69,200	25	0.04%
Auto	1997	17,500	25,375	6,250	31,625	14,125	14,100	-25	-0.18%
Auto	1998	151,650	303,300	6,250	309,550	157,900	157,900	0	0.00%
Total		328,350	815,025	140,909	955,934	627,584	627,500	-84	-0.01%

Example 7 – Reconciling with Gross Liability IBNR.

If the tolerance for reconciliation is -1.0%, then this example would be reconciled and no adjustments would need to be made.

4. Ultimate Nonretro Expense

For policies which do not include ALAE in the definition of loss, the ultimate ALAE is a nonretro expense. Since loss and ALAE are developed separately and ALAE for occurrences that have not been reported is calculated separately, this is a straightforward calculation. Again, if loss and ALAE are developed on a combined basis, then the ultimate must somehow be split into its separate components for policies which do not include ALAE in the definition of loss.

Example 8 – Ultimate Nonretro Expense.

(1)	(2)	(3)
	ALAE	Ultimate
Policy	a Nonretro	Nonretro
Number	Expense?	Expense
		Example 7
WCP1	Y	87,134
GLP2	Ν	0
ALP3	Ν	0

5. Ultimate Excess Loss

First, the adjustment for small losses that will eventually exceed the limit will be discussed. A simplified way of dealing with this is to take the number of occurrences below the given per occurrence limit at a certain age and apply a factor to derive how many of those are expected to go above the per occurrence limit. Once this expected number of occurrences is known, it can be multiplied by the expected severity greater than the per occurrence limit to calculate the ultimate excess loss. Note that the average severity of large occurrences that are initially reported as large occurrences can be different than the average severity of large occurrences that are initially reported as small occurrences. Consequently, it may be beneficial to do a severity analysis to determine this. The expected percentages of small occurrences which become large are usually different for loss only and loss and ALAE combined.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				Number of	Percentage	Number	Average	Ultimate
	Policy	Per	ALAE	Ultimate	Expected	Expected	Severity	Excess Loss
Policy	Age in	Occurrence	Included	Occurrences	to Exceed	to Exceed	Excess	for Small
							of	
Number	<i>Months</i> Example 6	Limit	in Limit?	Below Limit	Limit	<i>Limit</i> (7)=(5) <i>x</i> (6)	Limit	Occurrences (9)=(7)x(8)
WCP1	18	250,000	Ν	4	0.04%	0.0016	375,000	600
GLP2	18	250,000	Ν	4	0.10%	0.0040	250,000	1,000
ALP3	18	250,000	Y	4	0.01%	0.0004	500,000	200

Example 9 – Ultimate Excess Loss for Small Occurrences That Will Eventually Exceed the Limit.

Note: This calculation can also be done on an individual occurrence level with the same result.

Next, excess loss for occurrences that have already exceeded the per occurrence limit on a reported basis will be discussed. In this case, the ultimate occurrence amount is compared to the per occurrence limit. If it is greater, the ultimate occurrence amount less the limit is the ultimate excess loss. If it is less than the limit, the ultimate excess loss is zero. The ultimate occurrence amount may or may not include ALAE depending on the definition of loss for limiting purposes. This is done only for occurrences that exceed the limit on a reported basis rather than an ultimate basis to prevent overlap with the excess calculated in the previous paragraph.

(1)	(2)	(3)	(4) Reported	(5)	(6)	(7) Ultimate	(8) Ultimate
		ALAE	Occurrence	Per	Greater	Occurrence	Excess
Policy	Claim	Included	Amount for	Occurrence	Than	Amount for	Loss by
Number	Number	in Limit?	Limiting	Limit	Limit?	Limiting	Occurrence
		Example 9	Example 5	Example 9		Example 5	(A)
WCP1	WCC1	N	5,000	250,000	Ν	8,750	0
WCP1	WCC2	N	55,000	250,000	N	88,000	0
WCP1	WCC3	N	1,250,000	250,000	Y	1,212,500	962,500
WCP1	WCC4	N	25,000	250,000	N	43,750	0
WCP1	WCC5	Ν	180,000	250,000	N	252,000	0
			1,515,000			1,605,000	962,500
GLP2	GLC6	Ν	6,000	250,000	N	13,500	0
GLP2	GLC7	N	2,000	250,000	Ν	6,000	0
GLP2	GLC8	Ν	500,000	250,000	Y	505,000	255,000
GLP2	GLC9	Ν	7,500	250,000	N	16,875	0
GLP2	GLC10	Ν	12,000	250,000	N	18,000	0
			527,500			559,375	255,000
ALP3	ALC11	Y	15,000	250,000	Ν	25,750	0
ALP3	ALC12	Y	1,200	250,000	N	2,400	0
ALP3	ALC13	Y	900,000	250,000	Y	1,035,000	785,000
ALP3	ALC14	Y	30,000	250,000	N	50,500	0
ALP3	ALC15	Y	3,700	250,000	Ν	7,400	0
			949,900			1,121,050	785,000

Example 10 – Ultimate Excess Loss for Occurrences That Exceed the Limit.

(A) (8)=(7)-(5) if (6) is "Y" or (8)=0 if (6) is "N".

For occurrences that have not yet been reported, the ultimate excess can be calculated using an excess loss factor(ELF) for the given per occurrence limit for a policy. ELF's can be obtained from various sources such as NCCI data, ISO increased limit factors or a loss distribution model. There are numerous publications that deal with this subject. The ELF used in the case where loss only is limited is usually different from the ELF used in the case where loss and ALAE combined is limited. When an ELPF is used in the retrospective rating formula , the ELF can be calculated using ELPF x RATE if EXP is earned subject standard premium. The expected ultimate loss or expected ultimate loss and ALAE is multiplied by the appropriate ELF to calculate the expected ultimate excess loss for occurrences that have not yet been reported.

Example 11 – Ultimate Excess Loss for O	ccurrences That Have .	Not Yet Been Re	ported.
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(1)	(2)	(3)	(4)	(5)	(6)
		Expected			Expected
	ALAE	Ultimate	Per	Excess	IBNYR
Policy	Included	IBNYR	Occurrence	Loss	Excess
Number	in Limit?	Amount	Limit	Factor	Loss
	Example 9	Example 6	Example 9		(6)=(3)*(5)
WCP1	Ν	88,636	250,000	0.026	2,305
GLP2	N	133,333	250,000	0.100	13,333
ALP3	Y	46,875	250,000	0.080	3,750

6. Ultimate Subject Loss

The ultimate subject loss for each policy is the total ultimate loss and ALAE less the total ultimate nonretro expense less the total ultimate excess loss.

Example 12 – Ultimate Subject Loss.

(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ultimate		Ultimate	Ultimate	Expected		
Gross	Ultimate	Excess Loss	Excess	IBNYR	Ultimate	Ultimate
Loss and	Nonretro	for Small	Loss by	Excess	Excess	Subject
ALAE	Expense	Occurrences	Occurrence	Loss	Loss	Loss
Example 7	Example 8	Example 9	Example 10	Example 11	(7)=(4)+(5)+(6)	(8)=(2)-(3)-(7)
1,780,770	87,134	600	962,500	2,305	965,405	728,232
1,220,333	0	1,000	255,000	13,333	269,333	951,000
1,167,925	0	200	785,000	3,750	788,950	378,975
	(2) Ultimate Gross Loss and ALAE Example 7 1,780,770 1,220,333 1,167,925	(2) (3) Ultimate Gross Gross Ultimate Loss and Nonretro ALAE Expense Example 7 Example 8 1,780,770 87,134 1,220,333 0 1,167,925 0	(2) (3) (4) Ultimate Ultimate Gross Ultimate Excess Loss Loss and Nonretro for Small ALAE Expense Occurrences Example 7 Example 8 Example 9 1,780,770 87,134 600 1,220,333 0 1,000 1,167,925 0 200	(2) (3) (4) (5) Ultimate Ultimate Ultimate Gross Ultimate Excess Loss Gross and Nonretro for Small Loss by ALAE Expense Occurrences Occurrence Example 7 Example 8 Example 9 Example 10 1,780,770 87,134 600 962,500 1,220,333 0 1,000 255,000 1,167,925 0 200 785,000	(2) (3) (4) (5) (6) Ultimate Ultimate Ultimate Expected Gross Ultimate Excess Loss Excess IBNYR Loss and Nonretro for Small Loss by Excess ALAE Expense Occurrences Occurrence Loss Example 7 Example 8 Example 9 Example 10 Example 11 1,780,770 87,134 600 962,500 2,305 1,220,333 0 1,000 255,000 13,333 1,167,925 0 200 785,000 3,750	(2) (3) (4) (5) (6) (7) Ultimate Ultimate Ultimate Expected (7) Ultimate Ultimate Ultimate Expected (7) Gross Ultimate Excess Loss Excess IBNYR Ultimate Loss and Nonretro for Small Loss by Excess Excess ALAE Expense Occurrences Occurrence Loss Loss Example 7 Example 8 Example 9 Example 10 (7)=(4)+(5)+(6) 1,780,770 87,134 600 962,500 2,305 965,405 1,220,333 0 1,000 255,000 13,333 269,333 1,167,925 0 200 785,000 3,750 788,950

C. Ultimate Unlimited Retrospectively Rated Premium

Once the basic premium, the excess loss premium and the ultimate converted loss are calculated, the ultimate unlimited retrospectively rated premium can be derived using the tax multiplier and the retrospective rating formula for each policy.

Example 13 – Ultimate Unlimited Retrospectively Rated Premium.

(1)	(2)	(3)	(4)	(5)	(6)
					Ultimate
		Excess	Ultimate		Unlimited
Policy	Basic	Loss	Converted	Tax	Retrospective
Number	Premium	Premium	Loss	Multiplier	Premium
	Example 2	Example 2	Example 3		(6) = [(2) + (3) + (4)]x(5)
WCP1	200,000	42,600	775,567	1.035	1,053,803
GLP2	150,000	0	1,046,100	1.050	1,255,905
ALP3	135,000	0	409,293	1.040	566,065

D. Comparison To Minimum and Maximum Premiums

The ultimate unlimited retrospectively rated premium for the plan is calculated by summing across all policies in each policyholder plan. This ultimate unlimited retrospectively rated premium by plan

should then be compared to the minimum and maximum premiums for the plan and adjusted as needed. If the plan has a policy or policies that are less than 12 months, then the ultimate unlimited retrospectively rated premium needs to be annualized. This is so that randomly large losses in the first months of a policy period will not cause a plan to unnecessarily exceed its maximum or conversely so that randomly low losses in the first few months of a policy period will not cause a plan to unnecessarily fall short of the minimum. The ultimate unlimited retrospectively rated premium can be annualized by adding the expected unearned portion of the incomplete policy. Two methods of approaching this will be discussed. The first method is to just add in the unearned portion of the subject standard premium. The second method is to calculate the expected unearned portion of the ultimate unlimited retrospectively rated premium. This can be done by multiplying the unearned portion of EXP (the exposure basis) by the expected loss RATE and expected ALAE RATE to get the expected loss and ALAE. This expected loss and ALAE is reduced for expected nonretro expense and expected excess loss. This will produce the expected subject losses associated with the unearned portion of the subject standard premium. Using the basic factor, excess loss premium factor, loss conversion factor and tax multiplier, the expected unearned portion of the ultimate unlimited retrospectively rated premium is then calculated. This amount is added to the previously calculated earned portion of the ultimate unlimited retrospectively rated premium to get annualized ultimate unlimited retrospectively rated premium. This procedure is used when the expected ultimate retrospectively rated premium does not match the 12 month subject standard premium. The annualized ultimate unlimited retrospectively rated premium is added to the ultimate unlimited retrospectively rated premium for policies in the same plan which are 12 months and older to produce the plan annualized ultimate unlimited retrospectively rated premium.

Example 14 – Annualized Ultimate Retrospective Premium.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Earned			Unearned	Ultimate	Annualized
		Subject	Policy	Policy	Subject	Unlimited	Ultimate
Plan	Policy	Standard	Effective	Age in	Standard	Retrospective	Retrospective
Number	Number	Premium	Date	Months	Premium	Premium	Premium
		Example 2	Example 6	Example 6	0 <= (6) = (3)/(5)x[12-(5)]	Example 13	(8)=(6)+(7)
Plan1	WCP1	1,000,000	07/01/97	18	0	1,053,803	1,053,803
Plan1	GLP2	1,000,000	07/01/97	18	0	1,255,905	1,255,905
Plan1	ALP3	750,000	07/01/97	18	0	566,065	566,065
Plan1	WCP4	525,000	07/01/98	6	525,000	553,246	1,078,246
Plan1	GLP5	525,000	07/01/98	6	525,000	659,350	1,184,350
Plan1	ALP6	393,750	07/01/98	6	393,750	297,184	690,934
Total		4,193,750			1,443,750	4,385,553	5,829,303

Method 1 - Unearned Subject Standard Premium.

Method 2 - Expected Unearned Ultimate Unlimited Retrospectively Rated Premium.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Earned			Unearned	Ultimate	Ultimate	Unearned
		Subject	Policy	Policy	Subject	Loss	ALAE	Ultimate
Plan	Policy	Standard	Effective	Age in	Standard	to Premium	to Premium	Loss &
Number	Number	Premium	Date	Months	Premium	RATE	RATE	ALAE
		Example 2	Example 6	Example 6	0 <= (6) = (3)/(5)x[12-(5)]	Example 6	Example 6	(9)=(6)x[(7)+(8)]
Plan1	WCP1	1,000,000	07/01/97	18	0	0.650	0.100	0
Plan1	GLP2	1,000,000	07/01/97	18	0	0.400	0.300	0
Plan1	ALP3	750,000	07/01/97	18	0	0.550	0.200	0
Plan1	WCP4	525,000	07/01/98	6	525,000	0.650	0.100	393,750
Plan1	GLP5	525,000	07/01/98	6	525,000	0.400	0.300	367,500
Plan1	ALP6	393,750	07/01/98	6	393,750	0.550	0.200	295,313
Total		4,193,750			1,443,750			1,056,563
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
				Excess		Unearned	Ultimate	Annualized
Excess	Loss	Unearned		Loss		Unlimited	Unlimited	Ultimate
Loss	Conversion	Converted	Basic	Premium	Tax	Retrospective	Retrospective	Retrospective
Factor	Factor	Loss	Factor	Factor	Multiplier	Premium	Premium	Premium
Example 11	Example 2	(A)	Example 2	Example 2	Example 13	<i>(B)</i>	Example 13	(18)=(16)+(17)
0.026	1.065	0	0.200	0.040	1.035	0	1,053,803	1,053,803
0.100	1.100	0	0.150	0.000	1.050	0	1,255,905	1,255,905
0.080	1.080	0	0.180	0.000	1.040	0	566,065	566,065
0.026	1.065	352,528	0.200	0.040	1.035	496,690	553,246	1,049,936
0.100	1.100	363,825	0.150	0.000	1.050	464,704	659,350	1,124,054
0.080	1.080	293,423	0.180	0.000	1.040	378,869	297,184	676,053
						1,340,263	4,385,553	5,725,816

(A) For WC, (12) = [(9)-(8)x(6)-(10)x(9)]x(11). For GL and Auto, (12)=(9)x[1-(10)]x(11). (B) (16) = [(13)x(6)+(14)x(6)x(11)+(12)]x(15)

Note: Example references are for policies WCP1, GLP2 and ALP3 only. Data for WCP4, GLP5 and ALP6 is based on an assumed 5% increase in premium and losses from the prior year. The data for these policies does not appear elsewhere in this paper.

For plans which do not have policies that are less than 12 months, the minimum and maximum premiums can each be calculated as earned subject standard premium for the plan multiplied by the minimum and maximum factors respectively. For plans which do have policies that are less than 12 months, the earned subject standard premium for these policies is adjusted to reflect 12 months, added to the earned subject standard premium for the other policies in the plan, and then multiplied by the minimum and maximum factors respectively.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Earned	Unearned	Annualized				
		Subject	Subject	Subject			Annualized	Annualized
Plan	Policy	Standard	Standard	Standard	Minimum	Maximum	Minimum	Maximum
Number	Number	Premium	Premium	Premium	Factor	Factor	Premium	Premium
		Example 14	Example 14	(5)=(3)+(4)			(8)=(5)x(6)	(9)=(5)x(7)
Plan1	WCP1	1,000,000	0	1,000,000				
Plan1	GLP2	1,000,000	0	1,000,000				
Plan1	ALP3	750,000	0	750,000				
Plan1	WCP4	525,000	525,000	1,050,000				
Plan1	GLP5	525,000	525,000	1,050,000				
Plan1	ALP6	393,750	393,750	787,500				
Total		4,193,750	1,443,750	5,637,500	0.900	1.200	5,073,750	6,765,000

Example 15 – Annualized Minimum and Maximum Premiums.

The plan annualized ultimate unlimited retrospectively rated premium is compared to the plan annualized minimum and maximum premiums. If it is less than the plan annualized minimum, then the annualized minimum is subtracted from the annualized ultimate unlimited retrospectively rated premium and this amount will be called Deficient of Plan. The amount Deficient of Plan is always less than zero. If it is greater than the plan annualized maximum, then the annualized maximum is subtracted from the annualized ultimate unlimited retrospectively rated premium and this amount will be called Exceeding Plan. The amount Exceeding Plan is always greater than zero. If the plan annualized ultimate unlimited retrospectively rated premium and this amount will be called exceeding Plan. The amount Exceeding Plan is neither less than the annualized minimum nor greater than the annualized maximum, then both the amount Deficient of Plan and the amount Exceeding Plan are zero.

Example 16 – Deficient of Plan and Exceeding Plan.

(1)	(2)	(3)	(4)	(5)	(6)
	Annualized				
	Ultimate	Annualized	Annualized		
Plan	Retrospective	Minimum	Maximum	Deficient	Exceeding
Number	Premium	Premium	Premium	of Plan	Plan
	Example 14	Example 15	Example 15	(5)=(2)-(3)<=0	0<=(6)=(2)-(4)
Plan1	5,725,816	5,073,750	6,765,000	0	0

The above procedure might not produce an adequate reserve for plans that may exceed the maximum. A topic for further research could be to fully analyze this and, if necessary, calculate an additional reserve. This reserve could possibly be developed using the policy insurance charge. The additional reserve may not be significant and would probably affect only policies that are not very old.

E. Ultimate Retrospectively Rated Premium

The ultimate retrospectively rated premium is calculated as the ultimate unlimited retrospectively rated premium for a plan less the amount Deficient of Plan less the amount Exceeding Plan.

<u>Example 17 – Ultimate Retrospectively Rated Premium.</u>

(1)	(2)	(3)	(4)	(5)
	Ultimate			
	Unlimited			Ultimate
Plan	Retrospective	Deficient	Exceeding	Retrospective
Number	Premium	of Plan	Plan	Premium
	Example 14	Example 16	Example 16	(5)=(2)-(3)-(4)
Plan1	4,385,553	0	0	4,385,553

F. Accrued Retrospectively Rated Premium

Once the ultimate retrospectively rated premium by plan is calculated, it is summed up by account. The premium previously collected for an account is then subtracted to produce the accrued retrospectively rated premiums by individual policyholder account. Premium previously collected is the sum of subject standard premium and all retrospective rating adjustments that have been collected.

(1)	(2)	(3)	(4)	(5)	(6)
			Earned		Accrued
		Ultimate	Subject	Retrospective	Retrospectively
Account	Plan	Retrospective	Standard	Adjustments	Rated
Number	Number	Premium	Premium	Processed	Premiums
		Example 17	Example 14		(6)=(3)-(4)-(5)
Account1	Plan1	4,385,553	4,193,750	0	191,803

Example 18 – Accrued Retrospectively Rated Premium.

V. THE SYSTEM

This method requires a database that contains all the retrospective rating parameters for every active retrospectively rated policy written by the company. Also the database needs to contain at a minimum the earned subject standard premium, the effective date, retrospective rating adjustments processed, incurred loss and ALAE for each occurrence arising from these policies, development factors, expected loss ratios(RATE) and ELF. Sample table layouts are displayed in the Appendix. Much of this data should be available either for retrospectively rated premium billing purposes, for loss reserving purposes or for pricing purposes. The data that is probably specific to this system is the report year loss development factors, report year ALAE development factors and percentage of small occurrences that will exceed a limit. Usually, loss and ALAE development factors are not calculated by report year nor by size of occurrence. In some cases, this data may be available for developing losses used in the process of fitting a loss distribution. The task is to gather all this data into one place and develop the system. Once it is in place, it can be used for other purposes such as retrospectively rated premium billing and large deductible reserving and billing (with slight modifications).

In building the computer system, there are various practical aspects to the data and plans that, at times, make the system more complicated than the theory suggests. Some examples of these practical aspects that need to be considered are:

- 1) interpolation of development factors,
- 2) definitions of nonretro expense that are only part of ALAE or include more than ALAE,
- 3) differing definitions of occurrence,
- 4) asbestos and environmental claims that are not developed using a traditional development method, and
- 5) the retrospectively rated book of business for purposes of calculating the liability may contain inactive retrospectively rated plans as well as active retrospectively rated plans. (If inactive plans are added to the system, the accrued retrospectively rated premium for these plans should be set to zero.)

Some of these aspects can be handled in the programming, while some require modifications or additions to the actuarial formulas.

VI. CONCLUSION

This paper has presented a method for allocating the retrospectively rated ultimate loss and ALAE to individual policy and then calculating the resulting accrued retrospectively rated premium by individual policyholder account. This method, like many methods, has assumptions and approximations that are made as a compromise between accuracy and practicality. This method is meant as a starting point, to be modified as necessary to fit the needs of the user. There are more assumptions and approximations that could be made to further simplify this method and the resulting system to fit specific company needs. There are also many refinements that could be made to further enhance the accuracy and responsiveness of this method, which would probably increase the complexity of the system.

The most problematic part of this method has been calculating IBNR by policy rather than in bulk. Several complications in calculating the ultimate losses and ALAE by policy were discussed. These included:

1) reconciling the bulk IBNR back to the liability carried based on the aggregate loss reserve analysis for these policies,

- allocating bulk IBNR between future case development and losses and ALAE for occurrences which have not yet been reported,
- allocating the future case development to individual occurrences so that the per occurrence limits can be applied,
- handling different case development by size of occurrence (this is important in the allocation by occurrence), and
- 5) allocating loss and ALAE for occurrences not yet reported to individual policies in order to apply the correct retrospective rating parameters.

In order to handle all of the complications, an actuary must be familiar with various actuarial, statistical, financial and underwriting practices when working with the retrospective reserve. Through this process it has become clear to the author that the accrued retrospectively rated premium or retrospective reserve is as much an actuarial reserve as IBNR and should be treated accordingly.

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APPENDIX

SAMPLE TABLE LAYOUTS

	1	01	1
Name	Туре	Size	Brief Description
Plan	Text	5	Policyholder plan identification number
Policy	Text	5	Policy identification number
Coverage	Text	5	Coverage identification code
Basic	Number (Double)	5	Basic premium factor
ELPF	Number (Double)	5	Excess loss premium factor
Nonretro Defn	Text	1	"Y" if ALAE a nonretro expense, "N" otherwise
Excess Defn	Text	1	"Y" if ALAE included in loss for limiting, "N" otherwise
Limit	Money		Per occurrence limit
LCF	Number (Double)	5	Loss conversion factor
ТМ	Number (Double)	5	Tax multiplier
MIN	Number (Double)	5	Minimum premium factor
MAX	Number (Double)	5	Maximum premium factor

Table: **Retro Parameters** - Retrospective rating parameters for active policies

Table: Policy Data - Policy data for active retrospectively rated policies

Name	Туре	Size	Brief Description
Account	Text	5	Account identification number
Policy	Text	5	Policy identification number
Effective	Date/Time	8	Effective date
Premium	Money		Earned subject standard premium
Adjustments	Money		Retrospectively rated premium adjustments processed

Table: Occurrence Data - Occurrence data for active retrospectively rated policies

Name	Туре	Size	Brief Description
Occurrence	Text	5	Occurrence identification number
Policy	Text	5	Policy identification number
Accident	Date/Time	8	Accident date
Report	Number (Integer)	4	Report year
Loss	Money		Gross incurred loss
ALAE	Money		Gross incurred ALAE

Table: Loss Development – Report year loss development by occurrence size

Name	Туре	Size	Brief Description
Report Age	Number (Double)	5	Report age in months
Range	Number (Integer)	1	Number in [1,6] which indicates size of loss range
Coverage	Text	5	Coverage identification code
LDF	Number (Double)	5	Report year loss development factors by occurrence size

Table: Size of Loss Range - Definition of size of loss ranges

Name	Туре	Size	Brief Description
Range	Number (Integer)	1	Number in [1,6] which indicates size of loss range
Lower	Money		Lower limit of loss range
Upper	Money		Upper limit of loss range

Name	Туре	Size	Brief Description
Report Age	Number (Double)	5	Report age in months
Coverage	Text	5	Coverage identification code
ADF	Number (Double)	5	Report year ALAE development factors

Table: ALAE Development - Report year ALAE development

Table: Occurrence Development – Occurrence count development

Name	Туре	Size	Brief Description
Policy Age	Number (Double)	5	Policy age in months
Coverage	Text	5	Coverage identification code
CDF	Number (Double)	5	Reported occurrence count development factors

Table: Excess Loss Factors – Excess loss and excess loss and ALAE factors

Name	Туре	Size	Brief Description
Limit	Money		Per occurrence limits
Coverage	Text	5	Coverage identification code
ELF	Number (Double)	5	Excess loss only factor
ELAF	Number (Double)	5	Excess loss and ALAE combined factor

Table: Excess Percents and Severities – Percent of and excess severities for small occurrences expected to exceed limit

Name	Туре	Size	Brief Description
Coverage	Text	5	Coverage identification code
Policy Age	Number (Double)	5	Policy age in months
Percentage	Number (Double)	5	Percentage of small occurrences expected to exceed limit
Severity	Money		Excess severity for small occurrences expected to exceed limit

Table: RATE - Expected loss and ALAE ratios to earned subject standard premium

Name	Туре	Size	Brief Description
Coverage	Text	5	Coverage identification code
Policy Age	Number (Double)	5	Policy age in months
ELR	Number (Double)	5	Expected loss ratio to earned subject standard premium
EAR	Number (Double)	5	Expected ALAE ratio to earned subject standard premium

Table: Severity Ratios - Ratio of severity for occurrences that have not yet been reported to severity of all occurrences

Name	Туре	Size	Brief Description
Coverage	Text	5	Coverage identification code
Policy Age	Number (Double)	5	Policy age in months
Y	Number (Double)	5	Ratio of severity for occurrences that have not yet
			been reported to severity of all occurrences