Statistical Modeling Techniques for Reserve Ranges: A Simulation Approach

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The goal of this paper is to develop simple, quantitative methods to generate a range of reserves for an aggregate insurance portfolio, and provide a basis for selecting the best estimate of the aggregate reserves, given assumptions by accident period or by line of business. The basic assumption of the paper is that the range of reserves generated by the application of various actuarial techniques (for example projection of paid losses, projection of incurred losses, Bornhuetter-Ferguson techniques) can be used to generate parameters for loss distributions by accident year or by line of business. Accident year or line of business parameters generated based on the reserve projections are then used, along with simulation techniques, to generate a range for the aggregate reserves.

The first section of the paper describes some of the current statistical, and ad-hoc methodologies used by actuaries to generate reserve ranges. The second section describes some distribution functions that could be used in the simulation process. It focuses on four relatively simple and common distribution functions (uniform, triangle, normal, and lognormal) and describes situations where they would be most suitable. The third section describes the simulation methodology and explores techniques for generating the parameters to be used in the actual simulations. In addition, it explores the issue of what would be an appropriate number of simulations. The final section presents results from simulations for three different types of insurance portfolios. Results are plotted graphically, and comparisons are made of the simulated and unsimulated ranges.

Keywords: Simulation, Reserve Ranges, Aggregate Insurance Portfolios

STATISTICAL MODELING TECHNIQUES FOR RESERVE RANGES: A SIMULATION APPROACH

The purpose of the paper is to identify simple, rational methods of producing a range of reserves for an aggregate insurance portfolio given accident year or line of business projections from a typical reserve analysis. The paper starts with a description of some of the current methodologies used by actuaries to produce reserve ranges, describes distribution functions and simulation techniques, and finally presents results from three diverse insurance portfolios. The goal of the paper is to assist the actuary in generating a range of reserves at specific confidence intervals, and to produce a statistical best estimate of reserves for an aggregate insurance portfolio.

Current Methodology

According to the Casualty Actuarial Society's Statement of Principles Regarding Property And Casualty Loss and Loss Adjustment Expense Reserves:

- An actuarially sound loss reserve for a defined group of claims as of given valuation date is a provision, based on estimates derived from reasonable assumptions and appropriate actuarial methods, for the unpaid amount required to settle all claims, whether reported or not, for which liability exists on a particular accounting date.
- The uncertainty inherent in the estimation of required provisions for unpaid losses or loss adjustment implies that a range of reserves is actuarially sound.
- Ordinarily the actuary will examine the indications of more than one method when estimating the loss and loss adjustment expense liability for a specific group of claims.

Accordingly, in a typical reserve analysis the actuary produces a range of reserve projections by accident year or line of business by the application of several standard actuarial methodologies. Although the Statement of Principles states that a range of reserves may be actuarially sound, it does not specify how this range may be determined. Based on current practice, the range of reasonable reserves is largely based on actuarial judgment. Listed below are some of the methodologies the authors have used or come across in their review of other actuaries' work. The list is not exhaustive; however, it does give a flavor of the varied techniques currently used to generate reserve ranges.

- Provide only a point estimate of the reserves, and stipulate verbally that the ultimate projection could vary from the point estimate due to various parameters affecting the reserve process. This method does not provide a numerical range, however it does state that a range exists.

- Provide a point estimate of the reserves, select an empirical percentage variation from the point estimate (for example +/- 5%), and use this variation to generate a numerical range. This methodology does provide a numerical range, however it is highly judgmental. Various methods can be used to apply the selected empirical percentage variation. For example:

    - ♦ It may be applied on a total portfolio basis. The variation can be a function of the company size, and it can be larger for smaller companies relative to larger companies.

    - ♦ It may be applied by accident year. The variation can be function of the maturity of an accident year, and more recent accident periods can have a greater variation than mature accident periods

    - ♦ It may be applied by line of business. The variation can be a function of the characteristics of the business being analyzed, and it can be larger for longer tailed umbrella or excess coverage relative to short tailed property coverages.

- Generate a range based on the application of a number of standard actuarial methodologies such as paid loss development, incurred loss development, and Bornhuetter-Ferguson methods. These projections, and ranges, are available on an accident year basis. Hence a summation of the range by accident year can be used to generate the range for the aggregate portfolio. A possible variation of this is to sum up the results by method for all accident years and select a range by line of business based on the accident year/methodology sum.

- Generate a range based on selected variations in parameters used to project the reserves. For example, parameters such as loss development factors, loss ratios, loss trend can be varied to generate a range of projections.

- Generate a range using stochastic models. More recently, reserving techniques that consider the reserve data as stochastic variables have emerged. Sophisticated statistical models are used to model the reserve process and then a range is generated based on the model parameters.

The Simulation Approach

Theory

For the purposes of this paper, we consider the amount of the ultimate loss or allocated loss adjustment expense for a specific line of business and specific accident year to be a random variable. Where it is important to be precise we will use the symbol X(ay) to distinguish which accident period is being discussed (line of business should be clear from the context). Otherwise the generic symbol X will be used.

Then for a given line of business, b, the random variable for the total loss is:

$$T(b) = \Sigma\ X(ay),$$

where the sum is over all accident periods.

For the aggregate portfolio, the total loss is:

$$T = \Sigma\ T(b),$$

where the sum is over all lines of business.

It follows that the random variable for the claim reserve is:

$$R = T - k,$$

where k is a known value. If R represents the total claim reserve, then k is the claims paid to date. If R represents just the IBNR, then k would be the case incurred claims to date.

Information with respect to the random variable R is the object of our quest. However, it is clear that information with respect to the random variable T can be easily translated into information with respect to R. So the analysis in this paper will focus on T, which in turn causes the focus to be on the individual variables X(ay).

In an effort to limit the range of the discussion, we have chosen four families of continuous distributions from which the X(ay) can be chosen. These families, and examples of when the family would be an appropriate choice, are described below.

Distribution Functions

Uniform [a,b]

Definition

X is defined on the closed interval [a,b]. The probability density function (pdf) is 1/(b-a) over this range, and zero elsewhere.

Application

This distribution is applicable when the actuary has generated a range of projections based on various actuarial methodologies and considers each estimate within the range of projections to be equally likely. In other words, no value within the range based on actuarial projections is more likely than any other value within the range.

Triangle [a,b,c]

Definition

X is defined on the closed interval [a,b]. In addition to the range, the definition of X requires the selection of a mode, c. Then the pdf is given by:

$\{(x - a) * 2\}/\{(c-a)*(b-a)\}$ when $a \leq x \leq c$,

$\{(b - x) * 2\}/\{(b-c)*(b-a)\}$ when $c \leq x \leq b$, and 0 elsewhere.

Application

This distribution is applicable when the actuary has generated a range of projections based on various actuarial methodologies and considers each estimate within the range of projections to be possible. However, the actuary assigns the highest probability to a particular value within the range of projections. The value with the highest probability may be described as the best estimate, the selected estimate or the point estimate. By the nature of the probability curve, the probability associated with the other values within the range decrease linearly as they move away from the point estimate. It should be noted that the shape of the distribution curve need not be symmetrical, and the best estimate may be positioned anywhere (high or low) within the range. This shape is particularly useful when the actuary is most comfortable with the projection based on one particular methodology, and assigns that projection the highest probability.

Normal (μ,σ)

Definition

X is defined on the entire real line, with pdf given by:

$$(1/\sigma*\mathrm{sqrt}(2\pi)) * \exp(-(1/2)* ((x-\mu)/\sigma)^\wedge 2)$$

where μ is the mean of X and $\sigma > 0$ is the standard deviation of X.

Application

This distribution is most useful when the probability associated with the range of estimates around the selected or point estimate is symmetrical. The shape of the curve results in several assumptions that may present disadvantages in comparison to the previous curves.

- The parameters of the curve require the actuary to choose a mean value within the range. Since the curve is symmetrical, the selected mean value will most likely correspond to the average value of the projected range. If the mean does not correspond to the midpoint of the range, and the selected standard deviation is small, there may be portions of the range that are assigned negligible probabilities.
- Since the curve is defined over the entire real line, there are finite probabilities associated with negative values of X.

Lognormal (μ,σ)

Definition

X is defined for positive numbers. X is said to have a lognormal distribution when the random variable Y = ln(X) has a normal distribution, as defined above.

Application

The lognormal distribution is appropriate for lines of business such as umbrella or excess reinsurance where there is significant probability of large losses and, accordingly, the range is more skewed towards the high end of the estimates.

The Simulation Process

For any distribution, X, the cumulative distribution function (cdf) is given by

$$F(x) = Pr(X \le x).$$

For each of the families of continuous distributions chosen above, the cdf is a known function. The key fact is that each of the cdf's has a computable inverse, G(z). For any value of z, $0 \le z \le 1$, G(z) gives the value of x for which F(x) = z. The simulation method uses this as follows. For each trial, generate a random number between zero and one for each accident period, r(ay,n), where n corresponds to the trial number. Then, based on the selected distribution and its parameters, calculate the simulated value for the random variable X, X(ay,n) = G(r(ay,n)). For each trial, find the sum of the simulated values,

$$T(n,b) = \Sigma\ X(ay,n), \text{ and}$$

$$T(n) = \Sigma\ T(n,b)$$

T(n,b) corresponds to the sum of the simulated values for all accident years corresponding to the n th trial for a particular line of business. T(n) corresponds to the sum of simulated values for all accident years for all lines of business.

The final step is to use the array of values T(n,b) or T(n) to pick the desired range and the best estimate. For example, the range can be defined as the values encompassed in the 5th and the 95th percentile of the array T(n,b) or T(n). The best estimate may be selected using a number of different criteria. It may correspond to a selected percentile (for example the 50th percentile) of the array T(n,b) or T(n). It may also correspond to the mean or the mode of the array T(n,b) or T(n).

The important point to note is that the array T(n,b) or T(n) provides the actuary with a set of values that can be used to derive overall portfolio variance, ranges at various confidence intervals, and a basis for choosing the best estimate.

Parameter Selection

A required step in the simulation process is the selection of parameters for the selected distribution functions. Outlined below are suggested methods that could be used for selecting parameters.

Uniform [a,b] : Selecting parameters for the uniform distribution is relatively straight forward. For this distribution, parameters [a,b] correspond to the minimum and maximum of the projections based on selected, applicable actuarial methodologies. Although an actuarial methodology may be used for projections, it is possible that the projected value corresponding to the methodology may be excluded from the range for particular reasons. For example, if there is a significant increase in claim closures, and the estimates based on the associated paid losses are abnormally high, they may be excluded in selecting the high end of the range.

Triangle [a,b,c] : Parameters a and b can be selected on the same basis as the uniform distribution. Parameter c can be selected by assigning the highest probability to a projection based on a particular method, to a judgmentally selected best estimate, or to the midpoint of the range.

Normal (μ,σ) : If the distribution is to assign appropriate probability to all portions of the range, the mean, μ, should be selected as the mid-point of the range. The standard deviation, σ, can be selected in two different ways:

- By specifying that the range based on the actuarial projections corresponds to a certain number of standard deviations around the mean (for example, three, where the standard deviation would correspond to one sixth of the difference between the maximum and minimum actuarial projections)
- By specifying that the maximum (or minimum) of the range corresponds to a single, selected percentile of the distribution. Given the mean, the standard deviation can then be computed.

For the normal distribution, it is important to specify parameters such that negative values of loss have negligible probability.

Lognormal (μ,σ) : The mean can be based on a projection resulting from a particular actuarial method, and the standard deviation can be selected using the percentile methodology described above. To generate a more symmetrical distribution, specifying the range as a certain number of standard deviations can be used also.

Practical Considerations

Listed below are some practical issues to be considered in the simulation process. The list is not exhaustive, however it does give a flavor of the types of issues encountered.

- Distribution by Accident Year

Based on the methodology described, it is possible, and actually desirable, to vary the selected distribution function by accident year. For example, for the most recent accident year, there is greater variance associated with the projections. For such an accident year, it may be reasonable to conclude that all outcomes in a given range are equally likely and therefore a uniform distribution is most appropriate. For older accident periods, where projections are more certain, a normal distribution with a small standard deviation may be appropriate. For intermediate accident years, the triangular distribution can be a choice that temporizes between choices more appropriate for recent and mature accident years.

It is also possible that, coincidentally, the results of the projection methods may be very close for a particular accident year. This does not imply that the range is narrower for that year versus other years. Based on greater uncertainty associated with more recent years, it is appropriate that the range associated with a more recent year should be wider than the range associated with an older year, even if the projection methods yield results that imply a narrower range.

- Statistical dependence between accident years

In our methodology, we have assumed that the X are independent random variables. Under certain circumstances this may not be true, or at a minimum there is additional information available which is being ignored by the use of the independence assumption. For example, it might be found that the ultimate loss for the most recent accident year is correlated with the ultimate losses for the previous N accident years.

$$X(ay) = \Sigma\ A(i) * X(ay-i),$$

where the sum is for i equal 1 to N, and A(i) is a collection of numeric coefficients. In this case the current accident period can be modeled as the linear sum of the prior accident years. In practical terms, A(i) correspond to parameters such as loss trend, rate changes, impact of underwriting actions, etc.

A systemic correlation can exist between all accident years as well. For example, if a company has undergone significant case reserve weakening, it is possible that the incurred loss projection is biased low for every accident year. A possible method to handle such bias is to use projection techniques that compensate for the bias, and not include projections with the suspected bias in the range projections.

- Statistical dependence between lines of business

Statistical dependence can exist between various lines of business as well. There can be systemic bias across all lines of business (for example incurred loss projections in an environment of case reserve weakening), a common external factor may cause dependence (for example, inflation), or the quantity of losses between two lines may be related due to common exposures. Such correlations between lines of business can be reflected using appropriate mathematical relationships, in a manner similar to the methodology used to reflect accident year correlation.

- Lack of data for more recent accident years

If the line of business under consideration is so slow in developing for a recent accident year, that there is no paid or incurred loss data to be projected to ultimate, a modification of the previous solution can be applied. Here the focus needs to change to loss ratios. Correlation between the loss ratios for the various accident periods can be utilized to develop a formula which expresses the ultimate losses for the current accident period as a function of the ultimate losses for the previous N accident periods, and the approach discussed above will apply.

- Theoretical basis for projecting aggregate reserves

For certain types of distributions, it is possible to determine the distribution for the sum of the X's and therefore avoid performing the simulations. For example, the sum of a collection of normally distributed random variables is also normally distributed. However, the convenience of this summation process is not available for the other three families of distributions. Also, it is not theoretically possible to determine the statistical distribution of the aggregate portfolio when the type of distribution varies by accident year without performing a simulation.

- Number of trials required

In performing the simulations, the question of how many trials are enough may arise. One way of establishing this is to determine the ranges which result from simulations with various numbers of trials, and test for consistency and convergence in the results between trials. We tested simulation results using 100, 200, 400 and 800, and 1,000 trials. Based on our convergence criteria, it was found that the distribution derived from the simulations do not appear to be significantly different from each other when 1,000 trials are performed. As a practical matter, the time required to run a simulation of 1,000 trials is often quite short. This argues that even if a smaller number of trials can be viewed as adequate, the extra cost in time and effort to run 1,000 trials is not material, and the larger simulation should provide greater comfort with respect to the conclusions reached based on the simulation.

Application of the Simulation Approach

Company A

We first used the simulation approach for Company A which is a small company with only three lines of business. In addition, the lines of business are reasonably uncomplicated with relatively short payout patterns, and the associated experience has been stable for several years. The three lines of business are Homeowners, SMP & Landlords, and Fire & Inland Marine.

As indicated above, for each of the three lines of business, the first step is to apply some standard methods of estimating the liability for unpaid claims. For Company A the following methods were utilized:

Paid projection method,

Incurred projection method,

Paid Bornhuetter-Ferguson method, and

Incurred Bornhuetter-Ferguson method.

As a second step, it is necessary to select distributions by accident year and choose the corresponding parameters. It should be noted that the methods used to select parameters are for illustrative purposes only, and judgment should be exercised in selection procedure.

For the uniform distribution, it was desired that the mean be approximately the selected ultimate, and that the results of the four projection methods fall within the range. The minimum value for each accident year was therefore taken to be the selected ultimate loss less the maximum difference between the selected ultimate and the estimated ultimates from the four projection methods. The maximum value for each accident year was taken to be the selected ultimate loss plus the maximum difference. This creates a range which is symmetric around the selected ultimate amount. It should be noted that some rounding of values was done.

For the triangle distribution, the minimum and maximum values were taken to be the same as the values used in the uniform distribution. The mode was taken to be the selected ultimate loss.

For normal distribution, the mean was taken to be selected ultimate loss. The standard deviation was determined by taking the maximum difference between the selected ultimate and the estimated ultimates from the four projection methods, and dividing by three. This choice was designed to cause all four estimated ultimates to be within three standard deviations of the selected mean. This approach appears reasonable given the short tailed nature of the lines of business being reviewed.

For the lognormal distribution, the mean and standard deviation were taken to be the same as for the normal distribution.

The last assumptions necessary for the simulation are the choices as to which distribution appears to best fit the information available by accident year. There are several factors that need to be considered in selecting the appropriate distribution functions. First, one would think that the standard deviation of the ultimate loss should decrease as the accident period ages. It was observed that this is generally true for the choices made for the simulation of Company A, but not uniformly so. Second, the choice of distribution for the mixed simulation should reflect the fact that in spite of having applied the same four estimation methods to each accident year, we clearly know less about the more recent accident years than the older accident years. Therefore we chose the uniform distribution for the more recent accident years, the triangle distribution for the middle accident years, and the normal distribution for the older accident years.

The simulation was run with 1,000 trials for each combination of accident period and distribution. The results of the simulations are summarized in Appendix A.

It should be noted that, not surprisingly, the ranges associated with the total company simulation are smaller that the result of adding the ranges for the separate lines of business. That is, the sum of the 5% limits for the three single line simulations is less than the 5% limit of the total simulation, and similarly for the 95% limits. Naturally, the 50% limits do tend to add, as the mean of the sum of independent random variables is the sum of the means of those variables.

It should also be noted that the range for the mix of distributions is very similar to the range for the uniform distribution. This is true because the mix of distributions used the uniform distribution for the most recent accident years, which are the source of most of the variation in this instance.

Company B

Company B was chosen as the next test case because it had several sizable blocks of workers' compensation business. These lines were long tailed and subject to greater variance than the lines of business within Company A. The three sub-lines were treaty casualty, specific excess and aggregate excess.

For each of these sub-lines the following reserve methods were utilized to estimate the projected ultimate loss by accident period:

Incurred Loss Development (based on Company data) projection method;

Loss Ratio Development method;

Incurred Loss Development (based on RAA data) method;

Expected Loss Ratio method; and

Incurred Bornhuetter-Ferguson method.

As with Company A's lines of business, the selected ultimate loss was taken as the mean of the normal distribution for each accident year. The standard deviation was calculated as one third of the maximum difference between the selected ultimate loss and the projected ultimate loss based on the actuarial methods listed above. The range for the uniform distribution was taken to be the mean from the normal distribution, plus and minus 2.5 times the standard deviation, subject to the lower limit always being at least as great as the case incurred amount as of the valuation date. The range for the triangular distribution is the same as for the uniform distribution, and the expected value was set equal to the selected ultimate loss. The mean and standard deviation of the lognormal were taken to equal to those of the normal distribution.

The choice of the low end of the range for the uniform and triangle distributions was made to avoid the possibility of a negative reserve values. While this makes a certain amount of sense, several other observations become apparent when looking at the results of the simulation of the total company. The mean of the uniform distribution is no longer the middle of the range. The truncation of the low side of the range causes the mean of the trials for the uniform distribution to be higher than intended. This is most noticeable when looking at the graph of the cumulative distributions for the various distributions for the aggregate excess workers' compensation simulations (see Appendix B). A way to avoid this is to force the upper end of the range to be symmetric with the lower range. This, however, will reduce the standard deviation of the simulation of the uniform distribution which implies more knowledge concerning the distribution of possible losses than is intended when making the choice of the uniform distribution. The aggregate excess sub-line has no case incurred losses on the valuation date for the most recent four accident periods. This makes it a candidate for the application of one of the linear combinations discussed above.

Appendix B contains the details of the results of applying the simulation method to the lines of business from Company B. The results for specific excess are interesting because, although smaller, they line up reasonably well with the high and low range selected based on the arithmetic sum of the actuarial projection methods. This contrasts with the more typical result that the simulation based range is noticeably narrower than the range based on arithmetic sum of actuarial methodologies.

Company C

Company C was selected because it has a large block of umbrella business. The following reserve methods were used to estimate the projected ultimate loss by accident period:

Paid projection method;

Incurred projection method;

Expected loss ratio method;

Paid Bornheutter-Ferguson method;

Incurred Bornhuetter-Ferguson method;

Limited loss with increased limits factor method; and

Frequency/severity method.

The method of choosing parameters for the simulation is the same as was used for Company B.

Appendix C contains the details of the results for the Umbrella business.

Based on the results, it was observed that the low end of the simulation ranges vary from just under the selected low end to about 20,000 above the selected low end. On the other hand, the high end of the simulation ranges are from 90,000 to 105,000 lower than the selected high end. Hence the simulation had the impact of narrowing the range in an asymmetrical fashion. Due to the statistical nature of the simulation method, rather than the judgmental nature of picking the range based on actuarial projections, it appears as if this effect may be due to some relative bias in selecting the high end of the range. For each accident year, the low estimate was generally selected from the same projection method. The source of the high estimate varied, and the selection methodology indicated that the high end tended to include more outliers than the low estimates did.

Conclusion

Techniques using a combination of actuarial judgment and statistical simulation to generate a range of reserves have been presented. The input parameters required for the modeling can be derived with relative ease. Application of the methodology to actual data yield some predictable and some surprising results. It is hoped further development of the methodology presented will lead to more robust actuarial techniques for generating reserve ranges.

Possible Areas For Further Research

Part of the intent of this paper is to stimulate additional thought in the area under discussion. To that end, the following ideas are offered for consideration:

- It is possible to more rigorously model each accident year using frequency/severity models. Each accident year would then be specified as a unique combination of model parameters and the simulation would then proceed as outlined above.

- The correlation between accident years and lines of business needs to be examined more closely. The approach outlined has the capability of reflecting any type of linear or non-linear statistical dependence that may exist. However, the exact nature of this dependence needs to be explored further and needs to be reflected in the simulation model.

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Appendix A

Company A has three primary blocks of business, each of which is comparatively short tailed: homeowners, SMP and landlords, and fire and inland marine. For each line of business, several standard methods of estimating the reserve were applied to the company's loss data. The following table summarizes the indicated IBNR, where the IBNR equals the difference between the indicated ultimate loss and losses paid to date. The ranges were estimated based on an arithmetic sum by accident year.

	Low	Selected	High
Homeowners	2,172	2,366	2,575
SMP & landlords	1,910	2,154	2,431
Fire & inland marine	589	709	777
Total	4,671	5,229	5,783

A simulation was performed against each line of business, and then for the company in total. The simulations were each based on 1,000 trials. The following tables summarize these simulations.

Homeowners

Distribution	5%	50%	95%
Normal	2,144	2,361	2,592
Uniform	1,969	2,354	2,767
Triangle	2,087	2,357	2,651
Lognormal	2,144	2,359	2,594
Mix	1,983	2,354	2,763

SMP & landlords

Distribution	5%	50%	95%
Normal	2,014	2,157	2,305
Uniform	1,914	2,163	2,405
Triangle	1,988	2,159	2,333
Lognormal	2,014	2,156	2,307
Mix	1,911	2,164	2,405

Fire & inland marine

Distribution	5%	50%	95%
Normal	631	710	782
Uniform	565	711	854
Triangle	605	710	808
Lognormal	633	709	782
Mix	584	711	833

Total company

Distribution	5%	50%	95%
Normal	4,953	5,218	5,519
Uniform	4,746	5,209	5,729
Triangle	4,890	5,221	5,587
Lognormal	4,955	5,218	5,520
Mix	4,765	5,216	5,716

The total company simulation was created by adding the simulations of the three separate lines of business. That is, each simulation for the total company equalled the sum of corresponding simulation for the three lines of business.

Note that the ranges associated with the total company simulation are smaller than the result of adding the ranges for the separate lines of business. That is, the sum of the 5% limits for the three single line simulations is less than the 5% limit of the total simulation, and similarly for the 95% limits. Naturally, the 50% limits do tend to add, as the mean of the sum of independent random variables is the sum of the means of those variables.

The following table provides more complete information with respect to the total company simulation. This table displays the indicated ultimate losses, rather than the indicated reserve. The total paid loss for Company A is $75,703. This amount can be used to confirm the consistency of the two tables:

Projected ultimate loss, 5% level, normal distribution	80,656	
Paid loss		75,705
Reserve, 5% level, normal distribution		4,951

The difference between this amount and the table above (4,953) is in the nature of a rounding difference.

Appendix B

The simulation method was applied to three blocks of workers compensation business from Company B: treaty casualty, specific excess and aggregate excess. For each line of business, several standard methods of estimating the reserve were applied to the company's loss data. The following table summarizes the indicated IBNR, where the IBNR equals the difference between the indicated ultimate loss and losses paid to date. The ranges were estimated based on an arithmetic sum by accident year.

	Low	Selected	High
Treaty Casualty	126,846	155,455	196,894
Specific Excess	386,566	415,119	444,748
Aggregate Excess	33,365	38,258	59,092
Total	546,777	608,832	700,734

A simulation was then performed against each line of business, and then for the company in total. The simulations were each based on 1,000 trials. The following tables summarize these simulations.

Treaty Casualty

Distribution	5%	50%	95%
Normal	139,394	154,777	168,402
Uniform	135,424	156,558	175,824
Triangle	140,810	155,774	169,354
Lognormal	140,661	154,086	168,918
Mix	137,557	156,507	173,568

Specific Excess

Distribution	5%	50%	95%
Normal	383,804	415,442	449,030
Uniform	369,004	416,264	464,888
Triangle	383,613	415,360	449,666
Lognormal	384,668	414,773	449,143
Mix	375,828	415,500	459,027

Aggregate

Distribution	5%	50%	95%
Normal	32,387	38,432	44,616
Uniform	37,309	44,300	51,768
Triangle	37,255	42,389	47,773
Lognormal	32,873	38,039	45,368
Mix	33,659	39,594	45,344

Total company

Distribution	5%	50%	95%
Normal	571,734	607,288	646,060
Uniform	553,907	607,849	661,059
Triangle	570,457	607,561	646,123
Lognormal	571,999	607,452	647,043
Mix	560,753	607,772	655,256

Appendix C

Only one line of business from Company C was analyzed using the simulation method: umbrella. For this line of business, several standard methods of estimating the reserve were applied to the company's loss data. The following table summarizes the indicated IBNR, where the IBNR equals the difference between the indicated ultimate loss and losses paid to date. The ranges were estimated based on an arithmetic sum by accident year.

	Low	Selected	High
Umbrella	382,000	450,807	603,888

A simulation was then performed against the umbrella of business. The simulations were each based on 1,000 trials. The following tables summarize these simulations.

Umbrella

Distribution	5%	50%	95%
Normal	401,344	451,300	498,627
Uniform	381,506	451,177	520,676
Triangle	400,965	450,783	499,158
Lognormal	404,907	449,405	500,782
Mix	387,940	451,780	514,198