

Using Utility Theory for Describing Best Estimate Reserves

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ABSTRACT

Actuarial analysis has been described as a blending of science and art. Mathematical modeling involving probability, statistics, regression, and basic arithmetic form the scientific component. Business knowledge, insight, and experience regarding the influence of internal and external conditions and events on insurance data provide the art, also known as judgment.

In the context of best estimate claims reserving for property casualty insurance, the quantitative and qualitative viewpoints are brought forward by the various professionals associated with the reserving process. These professionals include actuaries, management, regulators, auditors, and others. Their associations may be at the time the reserve provisions are being decided, or through subsequent review or testing of the reserves.

Utility theory provides a useful framework in which to identify and describe the various interests and motivations of the various parties with an interest in the recorded reserves. We discuss each party's view as to the preferred or optimal estimates, for which each party's utility is maximized. We conclude by defining a best estimate as the one that maximizes the aggregate utility for the parties directly involved in the process.

DISCLAIMER

The views and examples contained in this paper are intended to generate discussion of the topic of best estimate reserving. They may not necessarily represent the professional viewpoint of the author nor the viewpoint of the author's employer.

ABOUT THE AUTHOR

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INTRODUCTION

Background

The actuarial profession acknowledges the uncertainty inherent in estimates for unpaid property and casualty claims liabilities and that a range of estimates can be considered reasonable. The draft of NAIC Issue Paper No. 55 regarding unpaid loss and loss adjustment expenses indicated that “management shall record its best estimate of its liability.” The draft further stated that, if “it is determined that no point within the range is a better estimate than any other point, the midpoint within the range shall be accrued.”

The NAIC draft did not explicitly define the term “best” nor “better.”

We propose that the definition depends on the interest of the party with an interest in the reserves. Certain estimates may be preferred over others, due to qualitative and quantitative considerations, which will depend on the interest of the party to the reserve provisions. Stated another way, a “best estimate” is that which maximizes the utility to an individual with an interest in the reserves. To illustrate,

- ◇ one actuary may propose that the best estimate is the one where it is equally likely that the actual subsequent payout is more or less than reserve;
- ◇ another actuary may propose that it is the estimate most consistent with analysis guided by signals from diagnostic testing on current data or hindsight testing of method(s);
- ◇ a regulator may consider a best estimate to be the reserve that provides for a high degree of confidence that outstanding obligations will be met, and that reflects proper consideration of the principles of conservatism and reserve adequacy.

From this short list of examples of the views on reserves of just actuaries and regulators, it is apparent that some views may be in conflict with other views. Nevertheless, they all demonstrate a consistency in that it is not necessarily the theoretical correctness of a reserve estimate that matters most, but what the implications (utility) are of the provision. And the utility of the provision is dependent on the particular party.

Description of Paper

We describe a variety of views of selected parties with an interest in the reserves for unpaid losses and loss adjustment expenses of an insurance company. The motivations of each party have direct influence on the criteria each would propose for determining a best estimate. The paper explores these issues, in the context of utility theory, in order to provide a framework for evaluating “best estimates.”

The paper is organized as follows:

- ◆ a brief review of utility theory and some of its basic applications
- ◆ citations of other work that discuss the concepts in insurance settings
- ◆ a description of the “characters” with an interest in reserves and their viewpoints
- ◆ a general discussion of the decision-making process
- ◆ a case study to illustrate the application of this utility-based framework

A REVIEW OF UTILITY THEORY

Utility theory has been used in a number of applications of decision theory. Its objective is to explain the behavior of the rational decision maker, either in face of certainty or uncertainty.

Under Certainty

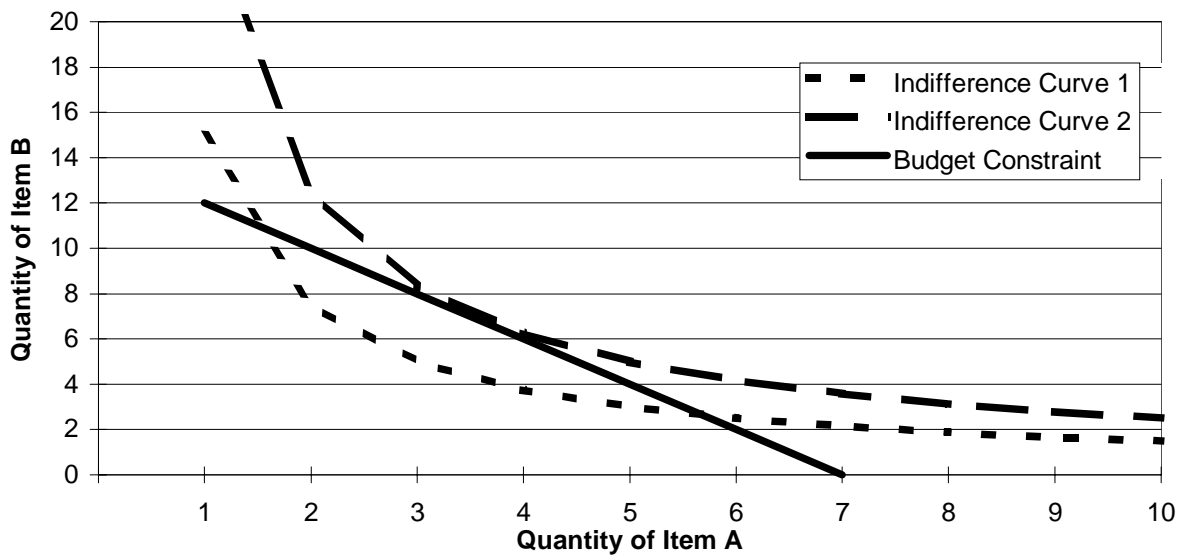
A consumer’s decision to purchase from among a choice of products or services, given a fixed budget, is an example of decision-making under certainty. The consumer must express her or his relative preferences of different combinations of the available products or services. A graphical

mapping of the alternate combinations that are perceived to yield the same utility (satisfaction) forms “indifference curves.” Indifference curves describe the relative quantities of the choices to which the consumer is indifferent.

The purchase decision must also take into account the amount of available funds and the relative prices of the choices; these determine a budget constraint. The rational consumer will choose the combination of goods and services that can be purchased and that maximize her or his overall utility.

An illustration of indifference curves and a budget constraint is presented in Figure 1 below:

Figure 1
Indifference Curves and Budget Constraint



In this example, the consumer may choose from 2 items, A and B. The consumer is indifferent between having 1 unit of Item A and 15 units of Item B, or having 5 units of Item A and 3 units of Item B. These combinations (and others) that generate the same level of satisfaction are described by Indifference Curve 1. The consumer is indifferent between any combination of units of Items A and B, just so long as the product of the number of units is 15.

Indifference Curve 2 illustrates the same relative preferences, but at a higher level of consumption; the utility derived from the combinations along Curve 2 is greater than the utility associated with Curve 1.

The Budget Constraint represents the different combinations of the 2 items that can be purchased with the amount of funds available. Assuming that the consumer wishes to expend all the available funds, the objective of the decision process is to find the point on the budget constraint that maximizes utility. The indifference curve that is tangential to budget constraint will provide the solution.

Under Uncertainty

For decision making under uncertainty, probabilities enter the equation. For example, the consumer may be faced with a 90% chance of not being involved in an automobile accident. In the other 10% of the situations, a crash occurs, with an uncertain amount of damages. The damages could be repair costs only for a fender, repair costs for more extensive vehicle damage and medical expenses for minor bodily injuries, or significant reparations and medical expenses in the event of severe bodily injuries. In the risk assessment process, bad outcomes generally carry greater weights than those suggested by their probabilities of occurrence. This is because people are generally risk-averse. Although the expected value of the risk exposure may be tolerable, the expected utility-adjusted value of the risk exposure may be greater than the fixed premium offered by an automobile insurance underwriter. In this situation, the consumer maximizes personal utility by purchasing the insurance policy and transferring the risk. The consumer may not save “real dollars” but saves the costs of anxiety.

Relevance for the Reserve Decision Process

The framework of evaluating the relative desirability of alternate decisions can be used to describe the behaviors of various parties with an interest in the loss reserve decision process. Each party's objective is to maximize utility. When multiple parties are involved, the objective is to maximize the aggregate utility of all parties combined.

The utility curves of the reserves-interested parties, however, may not be so easily presented in terms of indifference curves and budget constraints. We utilize a 2-dimensional chart, presenting

the utility values (the dependent variable) as a function of the alternate values for reserve provisions (the independent variable).

OTHER LITERATURE

References to the general concepts discussed herein in connection with insurance applications have been made in at least two previous publications.

Elizabeth Grace, of San Jose State University, wrote an article titled, "Property-Liability Insurer Reserve Errors: A Theoretical and Empirical Analysis," and printed in the March 1990 edition of the Journal of Risk and Insurance. The analytical testing was rooted in a hypothesis that "an insurer has as its objective the maximization of cash flow subject to smoothing constraints and uncertainty." Within this framework, reserve development (considered to be errors) was viewed in the context as potentially having been influenced by certain considerations in the original establishment of the reserve. Although the word "utility" did not appear in the article, the concepts were.

Lee Van Slyke prepared a paper titled "Regulatory Standards for Reserves" and published in the 1978 edition of the Casualty Actuarial Society Discussion Paper Program. He specifically wrote that:

the thrust of this paper is that the utility of a cash flow should be considered because any reasonable decision or evaluation should weigh bad outcomes more severely than favorable outcomes. Expected value calculations are merely a special case of utility calculations in which the evaluator's aversion to risk is negligible. Expected value calculations fall short of the needs of regulators, investors, product developers, and others because it is appropriate for these decision-makers to be averse to risk.

Mr. Van Slyke goes on to explore this extension of expected value calculations, primarily from the view of an insurance regulator. The calculations are used to determine risk-adjusted values of the company, which he proposes may be more useful for regulatory purposes than the expected values.

THE CAST OF CHARACTERS

A variety of characters have interest in the reserve decision making process. They include:

- the reserving actuary,
- regulators (state insurance departments),
- the company's independent auditors,
- shareholders and investors,
- Wall Street analysts and rating agencies, and
- senior management of the company.

In the following sections, we discuss the perspectives of each party on the reserves decision process and present illustrative utility curves.

The reserving actuary

For the purpose of this paper, we assume that the actuary responsible for the reserving analysis is also the actuary issuing the statement of actuarial opinion to be filed with insurance regulators.

We recognize that the views of an actuary recommending reserve amounts may be different from the views of an actuary setting reserves, which may differ from the views of an actuary examining the reserves. In this section, we do not make a distinction.

The actuarial estimation process, we propose, is not sufficiently refined to associate individual probabilities with discrete point-estimates of the reserves. For example, the process cannot assign individual probabilities to discrete point-estimates such as \$2,000,000 and \$2,000,005. A range of estimates that is considered reasonable can be constructed, but the assignment of probabilities to each point within the range is either impossible or meaningless. The application of stochastic methods may yield probabilities assigned to various intervals within the range. Such analytics may suggest that intervals closer to the middle of a range may have a higher likelihood, but a meaningful, distinctive, maximum likelihood point-estimate will not be found.

From an actuarial view, we propose that a best estimate is one that is “most preferred” by the actuary. The Committee on Property and Liability Financial Reporting of the American

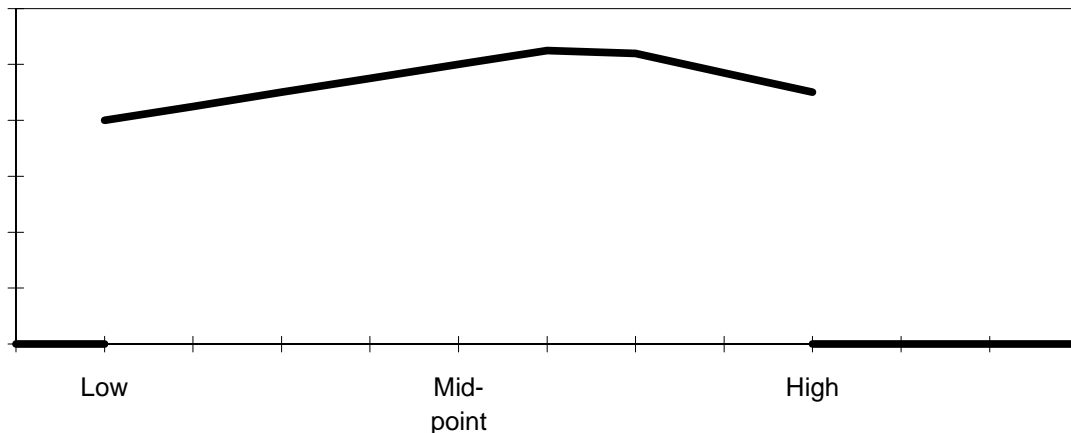
Academy of Actuaries, in its Practice Note regarding Statements of Actuarial Opinion on P&C loss Reserves as of December 31, 1997, suggested that a reasonable estimate is an estimate that would be produced by a set of assumptions that the actuary judges to be reasonable (page 23). An individual actuary may have a preferred set of data, assumptions, and judgments in order to evaluate preliminary point-estimates and derive a final selected point-estimate. Nevertheless, one actuary must acknowledge that another actuary's preferred set may yield a different point-estimate.

Once an actuary has determined a range of reasonable estimates and the most preferred point estimate, we can begin to describe the utility curve associated with alternate reserve provisions. The greatest utility is associated with the most preferred estimate. The preferred estimate may be slightly greater than the mid-point of the range. Such a provision provides greater utility to the actuary than that associated with a provision equal to the mid-point estimate, by reducing the likelihood of adverse development and thereby reducing the likelihood of the actuarial study being called in question in an adversarial proceeding.

The utilities associated with points that surround the preferred estimate do not differ by a significant degree. The difference between the utility associated with the preferred point estimate and the least utility assigned (presumably with the lower bound estimate) may not be considered significant. In other words, the actuary is relatively indifferent to the estimates that are within the actuary's range of reasonability.

For provisions less than the low end or greater than the high end, the actuary's utility is zero, as such provisions would warrant an adverse opinion. Based upon these considerations, an actuary's utility curve may be as shown in Figure 2 below:

Figure 2
Utility Curve of an Actuary



We observe that the maximum utility is for a point-estimate between the mid-point and high-end of the range of reasonable estimates. The curve is discontinuous, as a positive utility is assigned to each end of the range, but no utility is assigned to points immediately outside the range. Further, the utility of the high estimate is slightly greater than the utility associated with the low estimate.

In the example above, we have shown a range that is symmetrical with respect to the mid-point; that is, the distance from the mid-point to the high end is the same as the distance from the mid-point to the low end. We acknowledge that some actuarial ranges may be asymmetrical, for example with a low point that is 5% less than the “mid-point,” and a high point that is 10% greater than the “mid-point.”

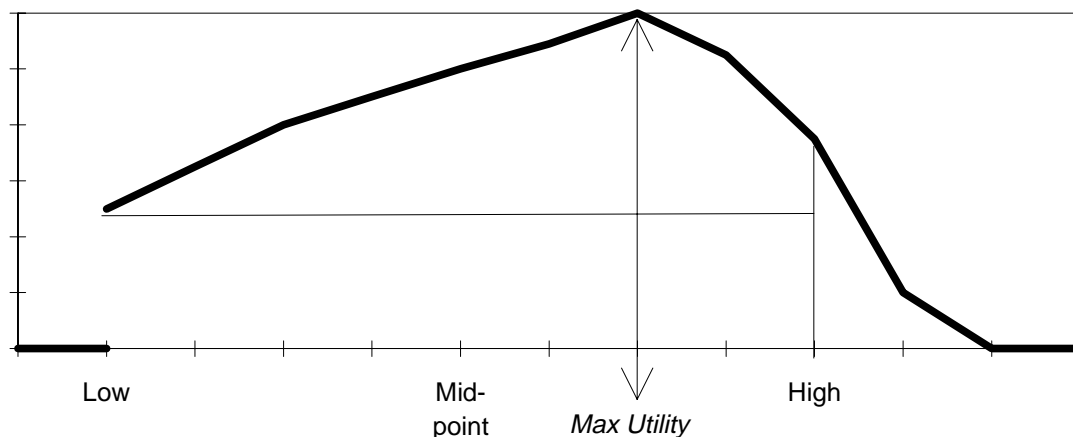
We offer for discussion the notion that an actuary may still assign positive utility to reserve provisions “slightly” above the high end of the actuary’s range. An actuarial opinion on provisions less than the low end may use the words “unreasonable” and “less than the low end.” On the other hand, an actuarial opinion for provisions greater than the high end may use words such as “adequate,” “sufficient,” or “conservative.” In the latter situation, these words, although they may not have the same meaning or connotation as “reasonable,” may still be viewed favorably.

Regulators

With their main objective to ensure the solvency of insurance companies within their jurisdiction, regulators would generally assign lesser utility to recorded provisions less than mid-point of actuarial ranges and greater utility to provisions greater than the midpoint. The draft NAIC paper on unpaid losses and LAE referred to the principles of conservatism and reserve adequacy.

Maximum utility may be assigned to a point, for example, that is the 75/25 weighting of the high or low points of the range provided by the opening actuary. Greater utility may be assigned to the high end of the range than the low end of the range. These relative preferences are illustrated in Figure 3 below:

Figure 3
Utility Curve of a Regulator



The difference in the maximum and minimum utilities within the range is likely to be greater than the corresponding difference for the actuary's utility curve. The regulator may be more concerned than the actuary with provisions less than the midpoint of the range.

We offer for discussion that notion that the regulator may assign some positive utility to provisions that are greater than the high end of the actuary's range. Such provisions may be viewed in light of principles of conservatism and reserve adequacy, rather than only in light of reasonableness.

Auditors

The external auditors that provide an opinion on the overall financial statements have their own set of criteria for reserves that are based on the scope and objectives of their audit engagement. The criteria include testing whether the reserves have been computed and recorded in conformance with applicable actuarial and accounting standards of practice. Further, the auditors evaluate the entire set of financial statements for presentation of the company's financials in accordance with the applicable accounting standards. The set of financial statements includes not only the balance sheet, but also the income statement and cash flow statement.

We briefly re-visit the considerations of the reserving actuary, to set the stage for highlighting areas of difference with an auditor's view.

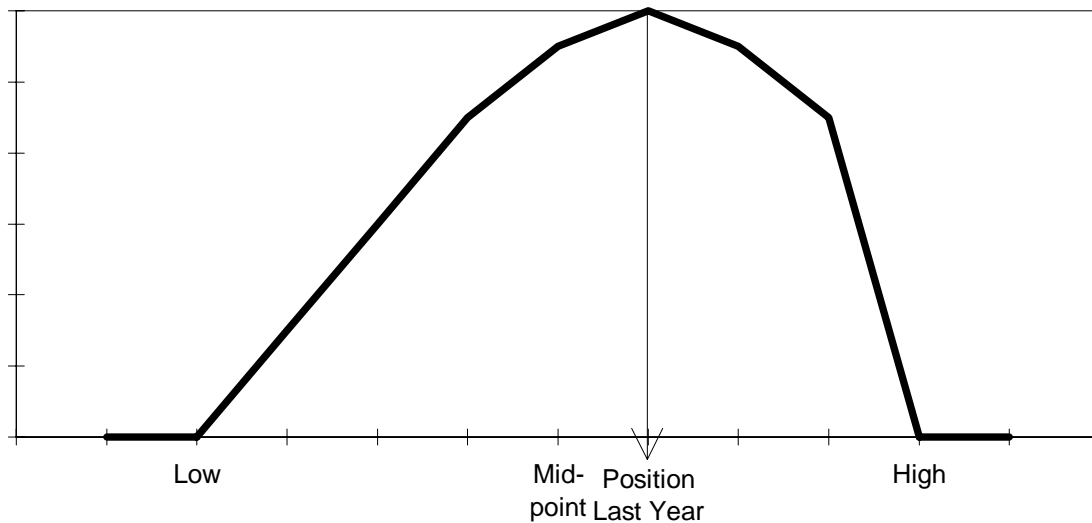
The reserving actuary generally has the balance sheet as her or his frame of reference. She or he generally evaluates the reasonableness of the recorded provisions by comparing the provisions to an actuarial point-estimate, or a range of reasonable estimates. The actuary generally evaluates differences in relation to the company's reported surplus or equity. The frame of reference for the reserving actuary is mainly the balance sheet.

The auditing firm evaluates the figures presented in the balance sheet, but must weigh issues and findings with regard to the balance sheet with issues and findings connected to the figures presented in the income statement as well. Thresholds of materiality are often described in terms of a certain percentage of surplus or net income. Movements in the relative position of the recorded reserves to actuarial point-estimates or ranges are considered by auditors in terms of their effects on the income statement. Depending on the leverage of the company's business and the type of business underwritten, movements in relative position may appear insignificant from a balance sheet view, but can be significant from an income statement view.

The utility of the auditing firm's interest in the recorded reserves depends on the significance of the current recorded provisions and movements in the provisions and relative position from the prior reporting period. The utility may be maximized for provisions that are consistent with the relative position from the prior period's financial statement. Thus, the utility curve is dependent

not only on the current range but last year's position in range. We illustrate such a utility curve in Figure 4 below:

Figure 4
Utility Curve of an External Auditor



Reserve provisions that correspond with positions in range that are “far” from last year's position are assigned lesser utility, as they are viewed as having a distorting influence on the current year's income statement. Additionally, points that approach the low and high ends are assigned decreasing utility values, in a more continuous manner than in the illustration for an actuary as in Figure 2 above.

Movements in ranges are not disallowed by audit firms. To the extent that management's decision last year reflected conditions or concerns about reserve adequacy or actuarial indications, and such concerns came about during the following year, and current conditions warrant different considerations, movements can be acceptable.

To give an example, suppose that at last year-end, management was concerned about the actuarial indications due to staffing and workload issues in the claim department for the months preceding the year-end closing. Therefore, management recorded a provision above the actuarial mid-point estimate to cover potential shortcomings in the estimation process. Assume that one

year later, the runoff of the claims experience revealed the effects of the staffing and workload issues, in the direction and general magnitude as judged by management. Current conditions are different than at last year-end, and therefore, management records a provision closer to the mid-point estimate determined by the actuary. From an auditor's view on this example, the charge to income was appropriately made in last year's income statement; and the current income statement was not affected by the claims runoff, as it was (presumably) offset through reductions in IBNR.

Shareholders and Investors

Shareholders and investors (potential shareholders) do not like surprises. In particular, unexpected charges to income are not looked upon favorably. Consistency in earnings growth is rewarded with a premium, in terms of price to earnings ratios or multiples of book value. Volatility in earnings is generally not rewarded. The reward or penalty emerges, in part, through the different rates at which future earnings streams are discounted in valuing a company.

Paragraph 65 of Statement of Financial Accounting Standard 5 expresses these ideas as follows:

. . . some investors may have a preference for investments in enterprises having a stable pattern of earnings, because that indicates lesser uncertainty or risk than fluctuating earnings. That preference, in turn, is perceived by many as having a favorable effect on the market prices of those enterprises' securities.

While stability in earnings is desirable, there may also be situations where a one-time large fluctuation (generally, an unfavorable one) is also rewarded. For example, a significant charge to earnings arising from a significant increase to claims reserves for exposures that have been perceived to be inadequately funded is generally rewarded in the market. The volatility of future earnings is perceived to have decreased, and therefore the valuation of the future earnings stream increases. (We observe that a 2nd significant increase, following an increase that purported to address the exposures, may be punished, as the credibility of the company's management becomes called into question.)

From strictly a shareholder and investor view, maximum utility is associated with reserve amounts that maximize the valuation of the enterprise. These amounts would contribute to a steady stream of increasing earnings, which are highly valued in the market.

Wall Street Analysts and Rating Agencies

The perspectives of Wall Street analysts and rating agencies are similar in many respects. Their principal goal is to assess the financial strength of the company in an absolute sense and in a relative sense, in relation to the company's peer group.

A major element of the firms' evaluation of companies is an assessment regarding the embedded value in the claims reserves. Some firms may rely on the report prepared by the opining actuary as their supporting information regarding adequacy of reserves in their evaluation. Other firms may perform actuarial or more simplified calculations in order to derive independent estimates of the liabilities, on an undiscounted and discounted basis. The firms generally allow for recognition of the time value of money, seeking to evaluate the economic value of the liabilities as a component of their evaluation of the market value of the subject company.

Maximum utility would tend to be associated with the "right answer," as the analysts and agencies seek to anticipate potential increases or decreases in reserves in their evaluation of the desirability of the company's securities and the company's financial strength as a going concern.

Management

Public Company

Senior management is at the center of the decision making process regarding claims reserves.

The management of a public company have a myriad of interests to balance: those of shareholders, regulators, and themselves, to name a few. The primary stakeholders to be served are the shareholders and value is generally maximized by steady flow of increasing earnings.

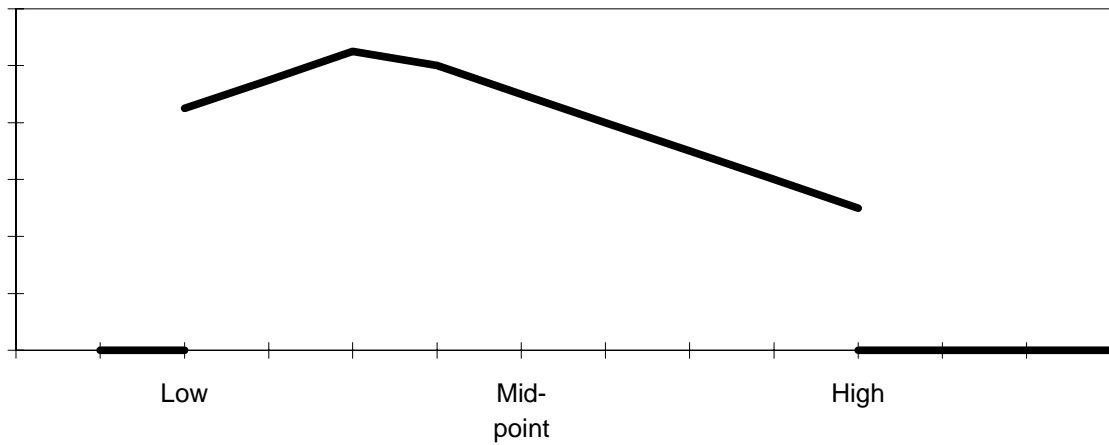
Also, management is aware of the expectations of analysts on earnings, and therefore is mindful of publishing a result that can be reasonably related to the analysts' earnings estimates. From an internal view, department, division, and corporate management teams are mindful of the

relationship of the emerging published earnings to those presented in their respective business plans and performance measures.

- Claims management may prefer a smaller IBNR loading, as they assert the strength of their case reserving practices.
- Underwriting management may prefer a smaller overall package of reserves, to show the profitability of the good risks for which adequate prices have been charged.
- Financial management may prefer a strong package of reserves, to demonstrate the strength of the balance sheet and provide a cushion against future shocks.
- Executive management may prefer a package of reserves that generates income results that are slightly better than the business plan that had been previously presented to the Board of Directors.

The variety of considerations from the different members of senior management may form the basis of a reserving policy whose tendency would be to generate a steady stream of increasing earnings over time, subject to a constraint that the actuary's or a rating agency's opinion is not adverse. Also, the recorded provisions should not trigger any undesirable inquiries from external parties (e.g., the state insurance department or the Securities and Exchange Commission). The descriptive utility curve reflects the combination of internal and external considerations. An example of such a utility curve is illustrated in Figure 5 below:

Figure 5
Utility Curve of Management - Public Company



This utility curve would tend to describe behavior influenced by an attraction to risk. A risk averse member of management, or one whose perennial primary stakeholders are not so concerned with current earnings, would tend to gravitate toward a provision greater than the mid-point. Gravitating toward a point less than the mid-point suggests an attraction to risk.

Other management policies can be described where the descriptive utility curves would be different from that shown above; for example,

- A policy that explicitly sets a long-term target for reserves at a level 5% greater than actuarial point-estimates, with annual step-wise movements toward the target not having more than a fixed amount effect on current year results. (The descriptive curve may look similar to the one shown for an actuary in Figure 2 above.)
- A policy to book mid-point actuarial estimates; this policy may expose the annual income results to the volatility associated with the actuarial estimation process; alternatively, the reserving actuary may perceive pressure to generate the mid-point such that other objectives are achieved.

- A policy that utilizes business plan figures for the current accident year incurred losses, and that directly allows for recognizing a portion of the difference between current and prior estimates of ultimate losses for prior accident years.
- A policy that reflects a blending of underwriting knowledge and actuarial projections. For example, the reserves associated with recent accident years may reflect greater reliance on information from the underwriting process; for earlier accident years, greater reliance may be placed on experience-based projections.

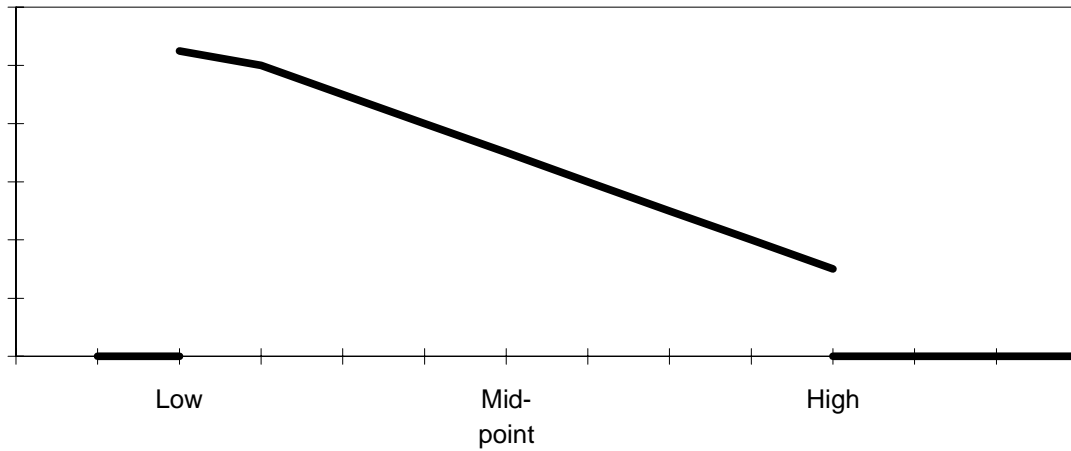
Finally, the considerations of management will reflect conditions and circumstances at that point in time. Assuming a consistency in philosophy and approach, the point at which maximum utility is achieved may shift over time, depending on internal and external conditions. It is unlikely that management will set a reserving policy “in stone,” to be rigidly adhered to over the longer term.

Private Company

The motivations of management of a privately held company may be different, in some respects, from those of management of a publicly traded company. In this section, we assume that the majority owners are involved in the active management of the company. Decisions may be influenced to a greater extent by shareholder dividends through maximization of current net income. A strong balance sheet and steady stream of earnings may be desirable, but may not be necessarily the main priority.

A reserving policy may be to record the lowest level of reserves acceptable to the opening actuary, and which do not result in any adverse regulatory scores. Such a utility curve is illustrated in Figure 6 below:

Figure 6
Utility Curve of Management - Private Company



A different view that management of a private company could take is to place a primary focus on the strength of the balance sheet, and assign a greater utility to reserve provisions toward the upper portion of the range. Such a utility curve would be a mirror image of the above illustration, with respect to the mid-point.

New faces

When there is a change in company management, the utility of the various reserve amounts to the “new faces” may be different from the utility assigned by long-term management teams. The differences may be influenced by the general circumstances at the company surrounding the change in management. For example, if the new management believes there to have been reserving deficiencies in the past, they might assign higher utility to a midpoint or higher estimate, thereby establishing a strong base of reserves against which future development is measured. The consequence is that current year earnings are reduced, but this can be accounted for by their new and good approach for establishing a strong balance sheet and the problems created by the prior management. If the reserve decision proves correct, the new management appear to be heroes. If the reserves prove too much, the provisions are likely to be released into earnings, and with memories not necessarily very long, again, the new management appear as heroes.

THE DECISION PROCESS

The final decision for the reserves to be recorded reflects the net “integration” of the interests of the parties who are present (either literally or figuratively) at the table, based on the relative influence (power) of each party in the process. We do not expect that the company invites regulators, analysts, rating agencies, or their auditors to their management meetings where decisions about reserves are made. Nevertheless, the people in the room are aware of these other parties and implicitly incorporate those views in their own utility preferences.

To the extent that certain parties can impose absolute constraints, beyond which other parties’ utilities decrease so extensively as to accept the constraints, the recorded reserves may fall at the boundary defined by the constraint. For instance, management may assign maximum utility to an estimate that is less than the low end of opining actuary’s range. When confronted with the prospect of an adverse actuarial opinion, however, management re-assesses their overall utility of a provision less than the low end of the actuary’s range and records an amount equivalent to the low end of the range.

CASE STUDY

For the purpose of the case study that follows, we assume that the utility curve of management reflects the influences of external (non-actuarial) parties. We present illustrative viewpoints and utility curves for management and the reserving actuary.

Year-end 1996

The actuary’s analysis of reserves as of year-end 1996 (serving as the prior year’s study) is summarized in Exhibit 1. The figures are shown separately for accident years prior to 1996 and for accident year 1996.

The actuarial analysis produced a range of ultimate loss estimates from \$290 to \$300 for all accident years, with the mid-point estimate being \$295. Cumulative payments were \$210, giving a range of reserve estimates from \$80 to \$90. The mid-point reserve estimate was \$85.

The reserves recorded by the company were \$84, which were \$1 (1%) less than the actuarial mid-point. The relative position within the range was the 40% between the low and high points.

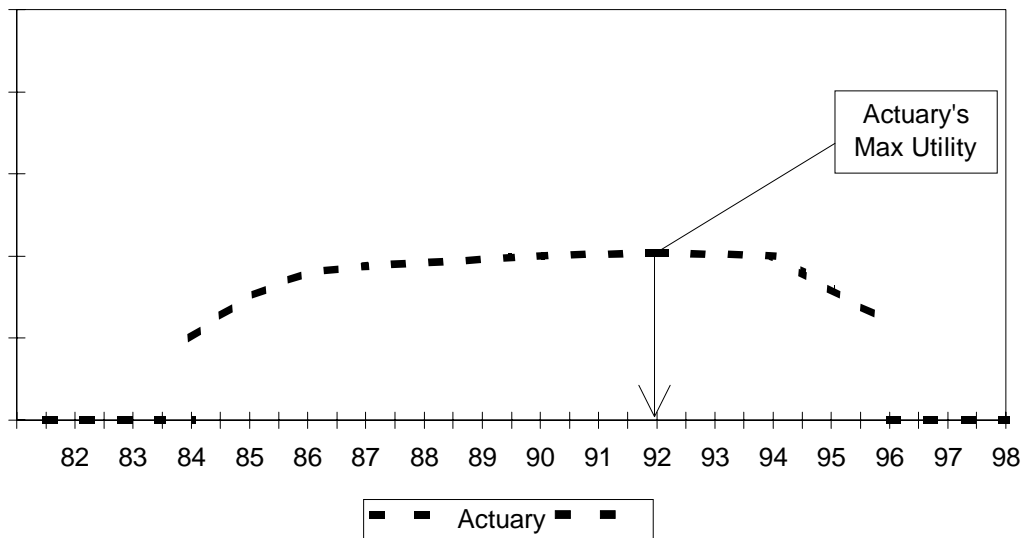
The illustrative income statement is shown in Exhibit 2. The 1996 results reflected a decrease in the provisions related to prior accident years, as payments associated with prior accident years were \$19 during 1996 while the reduction in reserves was \$20. The 1996 accident year was funded to a 78% loss ratio, the same as the actuarial mid-point estimate. The net income published by the company (using simple assumptions for expenses, investments and no taxes), was \$2, which represented a 8% return on average surplus.

Year-end 1997 - Actuarial Analysis

A summary of the actuarial analysis for year-end 1997 is shown in Exhibit 3. The actuary's mid-point estimate for accident years 1996 and prior was \$296, or \$1 greater than the estimate of \$295 at December 31, 1996. The mid-point estimate for loss ratio for the 1997 accident year was 75%, based on mid-point ultimate losses of \$48 and earned premiums of \$60 (Exhibit 4). For all accident years, the mid-point estimate of reserves was \$90 at December 31, 1997. The range of reasonable reserve estimates was \$84 to \$96.

The actuary's utility of alternate reserve estimates is illustrated in Figure 7 below:

Figure 7
Actuary's Utility of Alternate Reserve Provisions



The actuary assigns utility values that do not differ significantly from the low end of the range (\$84) to the high end (\$96). Reserves associated with points outside the range are assigned 0 utility. The maximum utility is assigned to a reserve of \$92, or 67% of the way from the low end to the high end.

The implications of the actuary's preferred estimate on the income statement for 1997 are shown in the column labeled "Actuary" in Exhibit 4. By changing the relative position of the recorded loss provisions, the current year's income is significantly affected. The adjustment to reserves for prior accident years has an effect of \$(3) (equivalent of 5 points of loss ratio) on the current year's results; this is caused by the increase in actuarial mid-point estimate (\$1) and moving from a position of \$(1) to \$1 relative to the mid-point estimates at each valuation.

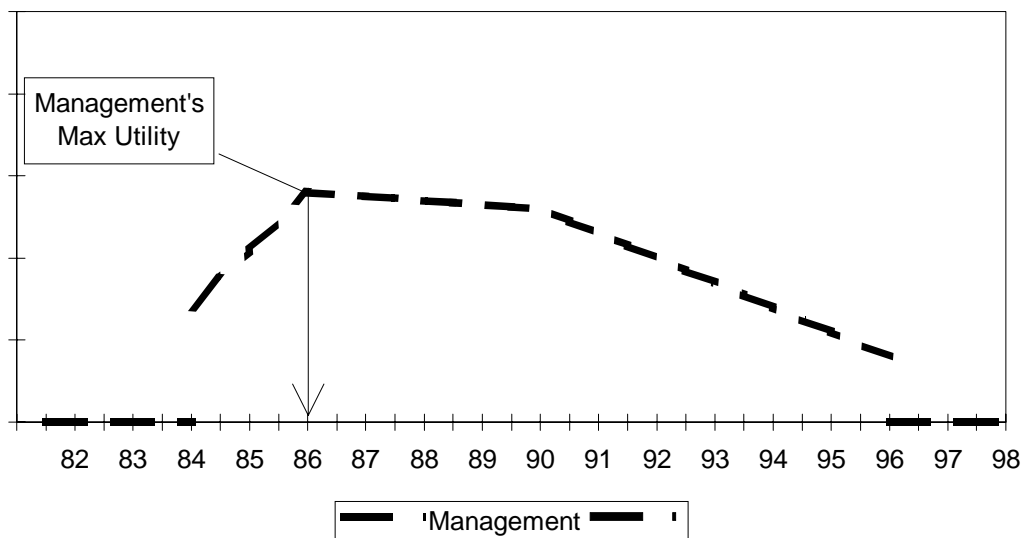
The 1997 accident year is funded to a 77% loss ratio, or 2 points above the mid-point estimate of 75%. The current accident year incurred losses of \$46 are \$1 greater than the mid-point estimate of \$45.

Overall, the net income for the year is a loss of \$1, for a return on average surplus of -5%.

Year-end 1997 - Management Considerations

Management is under pressure to produce results that show increases in earnings as well as a respectable balance sheet. Management considers that the effects of underwriting and claims initiatives have had a greater effect on loss ratios than was taken into account in the actuary's point estimates. In other words, management's range of estimates that are considered reasonable does not coincide with the actuary's range. Imposed upon the actuary's range, however, the utility of alternate provisions to management is illustrated in Figure 8 below:

Figure 8
Management's Utility of Alternate Reserve Provisions



Management's optimal estimate, were they making a unilateral decision, would be \$86. As shown in Exhibit 3 in the section labeled "Management," the reserves for accident years 1996 and prior were \$55, or \$2 less than the mid-point of the actuary's range. The reserve associated with accident year 1997 of \$31 is also \$2 less than the mid-point of the actuary's range for that year. Overall, reserves of \$86 would be \$4 less than the actuarial mid-point, or 17% of the way from the low end to the high end of the range.

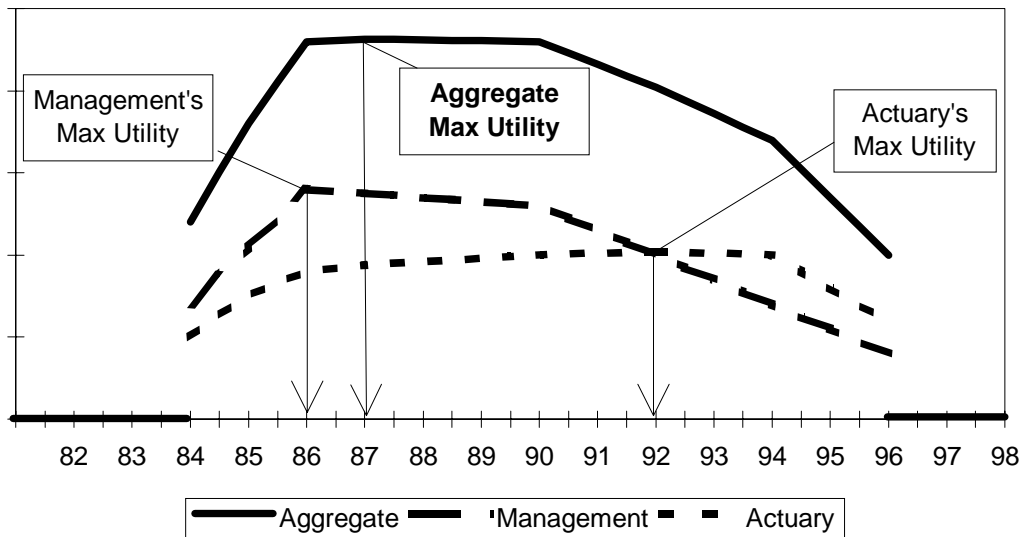
The income statement implied by management's optimal reserve provisions is shown in the column labeled "Management" in Exhibit 4. There is no effect on current year earnings from prior accident years. The current accident year's loss ratio would be 72%, showing improvement

over the 78% loss ratio published for accident year 1996 in the 1996 income statement. Income for 1997 would be \$4, with a return on average surplus of 15%.

Year-end 1997 - The Decision

The decision process brings together the actuary and management, with their respective utilities of the alternate choices for reserve provisions. For the purpose of this case study, we allowed management’s utility values to carry greater weight in the aggregation process. The aggregation (sum) of the utility values for management and the actuary is illustrated in Figure 9 below:

Figure 9
Aggregate Utility Curve



The provision at which aggregate utility is maximized is \$87. This reflects the relatively greater weight assigned to the management utility curve as well as the relative indifference of the actuary to the alternate points within the range of reasonable estimates. As shown in the “Aggregate” section of Exhibit 3, reserves of \$87 were \$3 less than the actuarial mid-point, at a point 25% of the way from the low end to the high end of the actuary’s range. As shown in the “Aggregate” column of Exhibit 4, the calendar year results reflect no impact from prior accident years, with the current accident year loss ratio recorded at 73%. Income for the year was \$3, for a return on average surplus of 12%.

As presented in this example, neither party's individual utility was maximized, but the aggregate utility was maximized. Therefore, in this example, the reserve value of \$87 represented the company's best-estimate.

CLOSING REMARKS

A "best" estimate of reserves is somewhat akin to a "fair" rate of return in ratemaking. What is "fair" depends on the interests and motivations of the party associated with the process.

For a definition of a best estimate reserve, we propose that it is the value for which the aggregate utility of the interested parties is maximized.

Utility curves vary by interested party and will tend to vary over time for an individual party.

The aggregation function of the individual utility curves could be interpreted, in a practical way, as the people process of negotiation and compromise. With regard to the claims reserving process for property and casualty exposures, it represents the blending of views from the various professionals involved in the process. In this way, the best estimate reflects the appropriate blending of science and art.

Actuarial Estimates**As of Year-end 1996**

<u>Item</u>	<u>Bound</u>	<u>Prior AY's</u>	<u>AY 1996</u>	<u>Total</u>
Cumulative Payments		200	10	210
Ultimates	Low	253	37	290
	Mid-Point	256	39	295
	High	259	41	300
	Range	+/- 1.2%	+/- 5.1%	+/- 1.7%
	Ult L/R		78%	
Reserves	Low	53	27	80
	Mid-Point	56	29	85
	High	59	31	90
	Range	+/- 5.4%	+/- 6.9%	+/- 5.9%
Recorded Provisions		55	29	84
Estimated Position		-1	0	-1
	vs. Mid-Point	-2%	0%	-1%
	In Range	33%	50%	40%

1996 Income Statement

	<u>Actual</u>
Earned Premium	50
Incurring Losses	
<u>Prior Accident Years</u>	
Beginning Reserves	75
Payments for Prior Years	19
Ending Reserves Prior Years	55
Calendar Year Impact	(1)
Loss Ratio Impact	-2%
<u>Current Accident Year</u>	
Payments	10
Ending Reserves	29
Incurred Losses	39
Curr AY Loss Ratio	78%
<u>Total</u>	
Ending Reserves	84
Incurred Losses	38
Cal Year Loss Ratio	76%
Expenses	14 28%
Investment Income	4
Income	2
Surplus	
Beginning	25
Ending	27
Return on Average Surplus	8%

Actuarial Estimates**As of Year-end 1997**

<u>Item</u>		<u>AY 1996&p</u>	<u>AY 1997</u>	<u>Total</u>
Payments		239	12	251
Ultimates	Low	293	42	335
	Mid-Point	296	45	341
	High	299	48	347
	Range	+/- 1.0%	+/- 6.7%	+/- 1.8%
	Ult L/R		75%	
Reserves	Low	54	30	84
	Mid-Point	57	33	90
	High	60	36	96
	Range	+/- 5.3%	+/- 9.1%	+/- 6.7%

Actuary

Recorded Provisions		58	34	92
Estimated Position		1	1	2
	vs. Mid-Point	2%	3%	2%
	In Range	67%	67%	67%

Management

Recorded Provisions		55	31	86
Estimated Position		-2	-2	-4
	vs. Mid-Point	-4%	-6%	-4%
	In Range	17%	17%	17%

Aggregate

Recorded Provisions		55	32	87
Estimated Position		-2	-1	-3
	vs. Mid-Point	-4%	-3%	-3%
	In Range	17%	33%	25%

1997 Income Statement

	<u>Actuary</u>	<u>Management</u>	<u>Aggregate</u>
Earned Premium	60	60	60
Incurred Losses			
<u>Prior Accident Years</u>			
Beginning Reserves	84	84	84
Payments for Prior Years	29	29	29
Ending Reserves Prior Years	58	55	55
Calendar Year Impact	3	-	-
Loss Ratio Impact	5%	0%	0%
<u>Current Accident Year</u>			
Payments	12	12	12
Ending Reserves	34	31	32
Incurred Losses	46	43	44
Curr AY Loss Ratio	77%	72%	73%
<u>Total</u>			
Ending Reserves	92	86	87
Incurred Losses	49	43	44
Cal Year Loss Ratio	82%	72%	73%
Expenses	17	17	17
	28%	28%	28%
Investment Income	4	4	4
Income	(1)	4	3
Surplus			
Beginning	27	27	27
Ending	26	31	30
Return on Average Surplus	-5%	15%	12%