Loss Estimates Using S-Curves: Environmental and Mass Tort Liabilities

by Bruce E. Ollodart, FCAS

Loss Estimates Using S-Curves

Environmental and Mass Tort Liabilities

Abstract

This paper discusses the application of S-Curve modeling for estimating certain environmental and mass tort liabilities. Emphasis is placed on pollution and asbestos liabilities, which are a significant component of the total environmental and mass tort liabilities for many insurance companies and manufacturers. The general concept of S-Curve modeling is discussed, followed by a technical discussion explaining its application to asbestos and pollution liabilities. Included are comments on the advantages and disadvantages of the technique.

Biography

Bruce Ollodart is a consulting actuary with the firm of Arthur Andersen LLP, and works out of their Hartford, Connecticut office. A Fellow of the Casualty Actuarial Society and Member of the American Academy of Actuaries, he has responsibility for Andersen's environmental and mass tort knowledge base in the actuarial and insurance services area.

Acknowledgments

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Introduction

Manufacturers, their insurers and reinsurers, as well as many other commercial enterprises have environmental and mass tort liabilities that must be estimated and managed. Such liabilities arise from many sources including environmental pollution, asbestos, medical implants, carcinogenic toxins, lead, radiation and other toxic exposures. Typically, these liabilities can be characterized by a historical period of exposure to a substance or process that produces latent health problems or property conditions that result in legal liabilities for bodily injury and/or property damage. The latency period can be many years, adding to the difficulty of estimating the exposure. For example, a chemical manufacturer legally dumped toxic wastes from 1940 to 1975 and then became legally liable for the property damage caused by these wastes as a result of 1980 superfund legislation. Similarly, a medical device manufacturer made artificial mandibular joints that were implanted in thousands of patients and later stopped sale of the devices once it was discovered they produced serious side affects for which the manufacturer was held liable.

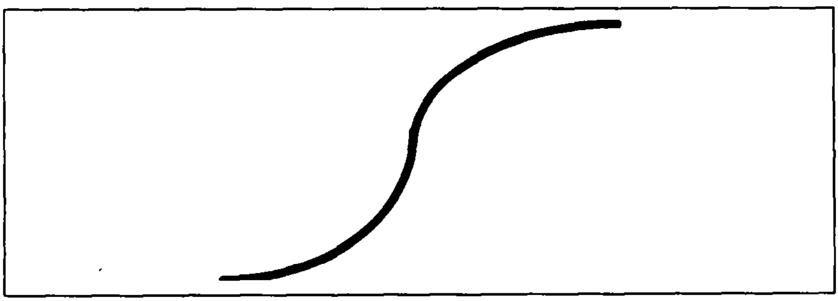
Environmental and mass tort liabilities typically arise suddenly as a result of long term exposure to a given agent or process (for example, asbestos or dumping industrial waste). Problems with data, including lack of historical precedents, poorly defined exposure periods, and improper data capture are common difficulties of estimating the value of these liabilities. Often, only calendar year data is available. Pollution claims, for example, have been attributed to multiple accident or policy periods by court decisions. Estimating the ultimate liability for these claims is often not feasible using traditional actuarial techniques, and highly sophisticated procedures involving a large number of claim by claim reviews are expensive and so time consuming that once performed, cannot be easily updated, but can quickly become outdated due to legislative and judicial changes.

The S-Curve approach, because it assumes a general pattern for loss emergence, can overcome many of these problems, is easy to apply, and can be updated readily as new information becomes available. As demonstrated in this paper, the S-Curve is a projection technique that has many of the characteristics of traditional loss development techniques.

S-Curves have been proposed by other actuaries as a method for evaluating pollution liabilities. However, technical difficulties with the sensitivity of the underlying assumptions halted most serious pursuits in this area. This paper provides techniques for overcoming these problems and increasing the objectivity, flexibility, and usefulness of the S-Curve approach for actuarial analysis.

Background

S-Curves can be used to analyze cumulative distributions for paid losses, reported losses, and claim counts. For purposes of this discussion, S-Curves will represent cumulative calendar year amounts for paid losses. The techniques and assumptions used work equally well for other cumulative forms of data. S-Curves have the following general shape:



The x-axis represents time, while the y-axis represents the cumulative amount paid. As a cumulative distribution, the first half of the curve indicates an accelerating rate of payment up to the inflection point of the curve, then the incremental payments begin to taper off and eventually stop. For a given S-Curve equation, the inflection point will be the point at which the first derivative reaches its maximum value and the second derivative changes sign. Depending on the type of exposures modeled, the representative S-Curve can be very steep in the center or almost flat. The particular S-Curve that best fits a company's historical data will depend on several factors including the length of exposure, the beginning period of exposure, the claim settlement practices of the company, the time since claims were first reported, and the legal process that affects policy coverage.

S-Curves can effectively represent the pattern of emergence for environmental and mass tort claims. A typical scenario involves detection of a health problem and/or a property condition, discovery of the agent or process that caused the situation, a period of statutory and legal developments that establish legal liability regarding the agent or process, an exodus from the production of the agent or process, a period in which policyholders and their insurers find themselves reacting to mounting claims activity related to the agent or process, a change in insurer coverage (usually eliminating future exposure to these claims), a period of increasing reserves and loss payments, then a long period of run-off of these claims. In terms of cumulative calendar year paid loss activity, it is easy to picture the resulting S shaped curve such scenarios produce.

S-Curve Functions

Previously, it has been suggested that the arc-tangent curve, because of its S shape and finite tail, be used for modeling purposes. Our research has determined that the arc-tangent is not flexible enough for environmental and mass tort liability modeling purposes. An alternative family of S-Curves based on power and gamma functions works much better and provides much more flexibility in curve selection. In this paper we deal primarily with the power functions, as they are easier to model. An example of a gamma function application is included for reference.

The general form of the power function is:

$$y=s(x-b)^{p}+c$$

The dependent variable y represents the cumulative paid losses, s is a scalar coefficient greater than zero, x is the year of projection (or year corresponding to the historical data), b represents the time at which the curve's inflection point occurs, p is an odd power between zero and one, and c is a constant representing the projected cumulative paid loss at time b.

The power p is typically chosen from among the family of fractional powers 1/3, 1/5, 3/5, 1/7, 3/7, 5/7, 1/9, etc. Testing of the various powers indicates that a few of them can adequately represent most of the S-Curves required for analyzing environmental and mass tort data. It is not necessary to fit all possible values of p. In our models, we fit approximately ten different values of p and select the best fits from among them.

When x is less than b, the odd power returns a negative value. When x equals b, the value of y is equal to c, which occurs at the inflection point. When x is greater than b, which occurs after the inflection point, the difference between x and b is positive. These relationships give the curve its S shape.

The s parameter determines the change in height of the curve for each time increment, and p determines the shape of the curve.

A positive c parameter is a constant that brings the curve above the x-axis and is selected such that y is equal to zero at the beginning period of claims emergence. For example, if c equals zero, then b, the inflection point, would occur where y equals zero (that is, the x-axis would cut the curve at b).

The power curve does not converge for large values of x. Therefore, a maximum number of years of run-off must be selected. Otherwise, the model will produce an infinite ultimate loss. We select our maximum number of run-off years at a point when incremental changes in the S-Curve become small, typically after about 30 years for pollution and 20 years for asbestos, a runoff period that we feel is reasonable based on other factors.

Power curves are symmetrical around the inflection point, a property that is useful when the inflection point is not observable in the data. A gamma function can be derived that is asymmetrical around the inflection point providing added flexibility to the curve fitting process.

Several actuaries have suggested fitting curves to the incremental paid data. The first derivative of the power curve, dy/dx, is given by the following equation and represents the shape of the curve corresponding to the calendar year incremental paid losses:

$$y' = ps(x-b)^{p-1}$$

This is a bell shaped curve that has an undefined value at its inflection point (where x equals b) when p is less than one. This implies that curve fitting using the incremental data cannot be achieved for the power curve for values of p less than one, as no value of b will minimize the squared error for the fit in these cases. Curve fits using other types of functions (gamma, lognormal) may work on incremental data.

Fitting S-Curves

To fit an S-Curve, numerical methods are used in our model. By minimizing the sum of the squared errors between the fitted curve and the historical data, a numerical algorithm is used to determine the best fitting parameters s, b, and c. As noted above, approximately ten values of p are selected and separate fits are made for each p value. The fit is performed on the cumulative data. Depending on the relationship between the data and the fitted S-Curve, this approach may give more weight to the squared error in the most recent data points as these points will contain the cumulative errors from all prior years. We believe this has a positive influence on the fit as it helps minimize error in the most crucial part of the curve (the most recent points). That is, precedence is given to minimizing the cumulative error over minimizing error for all points on the curve.

The S-Curve, depending on the value of p, can be very sensitive to the selection of the b parameter. To make the selection of b less subjective, we constrain the numerical algorithm as follows:

1. The year in which y first becomes positive is fixed based on the earliest date that the losses are first paid. This gives the curve a realistic starting point. This point can be varied plus or minus a few years to improve the goodness of fit, but should be within a reasonable range of the known starting date.

2. The value of b is constrained to be at least four years after the year in which payments are first made. This constraint keeps the algorithm from selecting b unreasonably close to the starting date, an outcome that may minimize squared errors but is not reasonable for projection purposes. The four year period should be used as a guide, as varying the parameter value may provide improved fit without sacrificing reasonability.

3. The parameters s and c must be positive.

For a given value of p, the other parameters are selected, subject to the above constraints, such that the sum of the squared errors is minimized.

Once a series of S-Curves have been fitted to the historical data, the best fits must be selected. Standard measures of goodness of fit do not work well with S-Curves because of their non-linearity. We developed several relative goodness of fit tests. These tests, along with graphical representations of the fit, help to determine which S-Curves provide the best fit to the data. Two of these tests are as follows:

$$R_1=\Sigma(y_f-y_d)^2/\Sigma y_f^2$$

$$R_2=1-\Sigma(\ln y_f-\ln y_d)^2/\Sigma(\ln y_d-\Sigma\ln y_d/n)^2$$

The variable y_f indicates fitted values, y_d indicates data values, and n is the number of data values in the fit. R_1 compares the squared error of the fitted values to the squared fitted values, with lower values indicating better relative fit. R_2 compares the squared error of the natural logarithms of the fitted values from the data to the squared error of the natural logarithms of the data from the average, with higher values indicating better relative fit. A third alternative, based on the R_1 statistic, is to use an absolute difference in the numerator of R_1 instead of a squared difference and drop the square in the denominator, with lower values indicating better relative fit.

In practice, we have experienced problems where two fits of the same data using the same value of p both minimize the squared error. This may occur when the data does not fit a particular S-Curve well, is extremely volatile, or is too immature. In such cases, there is enough "slack" in the shape of the curve to obtain more than one best fit. This is caused by some interdependence between the b and c parameters where, for certain data sets, several combinations of b and c can result in minimized squared error. Our numerical algorithm stops when it finds the first of these solutions. To address this limitation, we run our numerical algorithm twice. The first run determines an initial set of parameters. The second run uses the output of the first run for seed values. In almost all cases, the second fit is either identical to the first fit or is improved and subsequent fittings do not yield improved results. This approach essentially eliminates the "slack" problem.

In the final selection process, actuarial judgment must be used to determine which fits best represent the data and are reasonable for the purpose(s) intended. We typically select the best two or three fits from our analysis to determine a range of ultimate values. Consideration is also given to the quality of the underlying data and its applicability for extrapolation into the future.

Examples Using Insurance Industry Data

Power Function

To show how the S-Curve model utilizing a power function performs using actual data, we have prepared examples based on insurance industry pollution and asbestos claim information. This data is based on information from a select group of companies and does not represent an industry-wide composite. Exhibits 1 and 2 show these results for asbestos and pollution claims, respectively. The input data, the results of the numerical algorithm, best fit statistics, graphical representations of the fit, and resulting estimates of ultimate loss are shown on the exhibits.

Observations regarding these examples include:

1. The curve fits are still showing fairly large payouts at the end of our projection period. The length of the projection period could be lengthened, the curve forced to zero over a period near the end of the selected projection period, or the curve can be truncated as in our example. In certain cases, the present value of loss payments beyond our projection period will not be significant.

2. The asbestos and pollution paid losses through 1995 in the projection are approximately 60 and 40 percent of the projected ultimates, respectively.

3. The fit statistics are based on 1981 to 1995 and 1984 to 1995 for asbestos and pollution, respectively. This period was selected for practical reasons to reflect differences in the emergence of asbestos and pollution and to emphasize goodness of fit over a certain period of years. It may be more appropriate to test goodness of fit over the entire data set or a different portion of the data set depending on the application.

Gamma Function

There are cases where use of a gamma function may improve the fit or at least offer a good alternative to the power function. In practice, we found the power function to be reasonable in most cases. Cases that may be improved using a gamma function usually involve asymmetrical S-Curve shapes where the data is already fairly mature and an inflection point is clearly visible in the data. One form of the gamma function used was

$$y(\sigma,\iota)=\Gamma(\tau,\alpha,\lambda)+c=\int_0^{\tau}\lambda^{\alpha}x^{\alpha-1}e^{-\lambda x}\,dx+c$$

where λ is a scalar, α is the shape parameter, c is a constant, ι is the initial year of payment, σ is the projection year, and τ represents the number of years from the first year of payment to the projection year plus one (e.g., if the initial year of payment is 1980 and you are estimating the 1995 value, then ι is 1980, σ is 1995 and $\tau = \sigma - \iota + 1$ is 16). Both λ and α must be greater than zero. Parameters α and λ have roles in the gamma function that are comparable to the corresponding parameters p and s in the power function. The c parameter is included to improve the fit in certain cases and is optional. The inflection point for this gamma function is given by $(\alpha-1)/\lambda$, as determined by setting the second derivative equal to zero and solving for τ.

On Exhibit 3, we show a gamma function S-Curve fit to the asbestos data used in Exhibit 1. The parameter c produces a disjointed looking change in the fit near the beginning years but improves the overall fit for the latter years. The curve turns faster in the projection years than the power curve used in Exhibit 1 and runs off fairly well during the truncated projection period. The fit statistics are also comparable in quality to the power curve.

Advantages and Disadvantages of the S-Curve Approach

The following lists are based on practical application of the model as well as feedback we have received from other actuaries. The advantage or disadvantage of using this approach is dependent on the type of application involved.

The advantages of the S-Curve approach include:

1. Uses readily available data
2. Is a pure actuarial approach in the sense that it does not have to depend on claim department estimates

3. Comparable to a loss development approach as it performs aggregate loss projections rather than individual claim or policy projections
4. Can be used with paid and reported data for both dollars and counts
5. Is easy to update with more current information as the data matures
6. Provides a basis for testing the sensitivity of key assumptions including judgment concerning future changes in judicial or legislative practices
7. Can be performed fairly quickly
8. Appears to produce reasonable results for many environmental and mass tort liabilities
9. Does not require analysis and testing of a large number of assumptions and variables

The disadvantages of the S-Curve approach include:

1. May be impossible to select best fitting curves with a reasonable range of outcomes
2. Some data sets will be too immature for valid application of the model
3. Comparable to loss development methods applied to new lines of business - the ultimate pattern of runoff for the tail remains uncertain until the data becomes fairly mature

Exhibit I
Sheet 1

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1979
Power Curve 7
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0	362	0	
1979	57,426	17,918	57,426	17,556
1980	117,252	33,987	59,826	16,069
1981	179,775	84,014	62,523	50,026
1982	245,358	193,596	65,584	109,583
1983	314,454	258,994	69,095	65,397
1984	387,632	284,030	73,178	25,037
1985	465,635	324,534	78,003	40,504
1986	549,452	374,068	83,818	49,534
1987	640,459	612,636	91,007	238,568
1988	740,659	752,146	100,199	139,509
1989	853,169	898,011	112,511	145,866
1990	983,338	1,026,623	130,169	128,612
1991	1,141,855	1,259,167	158,517	232,543
1992	1,357,474	1,585,463	215,618	326,296
1993	2,095,513	2,078,939	738,040	493,476
1994	2,591,167	2,470,635	495,654	391,696
1995	2,802,383	2,835,848	211,216	365,213
1996	2,959,027		156,644	
1997	3,088,106		129,080	
1998	3,199,887		111,781	

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1999	3,299,556		99,669	
2000	3,390,156		90,599	
2001	3,473,648		83,492	
2002	3,551,384		77,736	
2003	3,624,338		72,954	
2004	3,693,242		68,904	
2005	3,758,660		65,418	
2006	3,821,037		62,378	
2007	3,880,734		59,697	
2008	3,938,045		57,311	
2009	3,993,215		55,170	
2010	4,046,451		53,236	
2011	4,097,929		51,478	
2012	4,147,800		49,870	
2013	4,196,194		48,394	
2014	4,243,225		47,032	
2015	4,288,996		45,770	
2016	4,333,593		44,598	
2017	4,377,097		43,504	
2018	4,419,578		42,481	
2019	4,461,100		41,522	
2020	4,501,720		40,620	
2021	4,541,489		39,770	
2022	4,580,456		38,967	
2023	4,618,663		38,207	
2024	4,656,149		37,486	
2025	4,692,951		36,802	
2026	4,729,102		36,151	
2027	4,764,633		35,531	

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit I
Sheet 2

First Year of Loss Payments = 1979
Power Curve 7

Fulcrum Year* =	1993

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981	84,014	179,775	9,170,196,298
1982	193,596	245,358	2,679,308,189
1983	258,994	314,454	3,075,820,634
1984	284,030	387,632	10,733,322,993
1985	324,534	465,635	19,909,448,322
1986	374,068	549,452	30,759,505,110
1987	612,636	640,459	774,112,539
1988	752,146	740,659	131,949,576
1989	898,011	853,169	2,010,788,515
1990	1,026,623	983,338	1,873,600,295
1991	1,259,167	1,141,855	13,761,879,071
1992	1,585,463	1,357,474	51,978,969,483
1993	2,078,939	2,095,513	274,716,290
1994	2,470,635	2,591,167	14,528,079,463
1995	2,835,848	2,802,383	1,119,904,087
Total			162,781,600,866
	(4) Sum of value squared	25,619,226,658,823	
		(5) R_1:	0.006

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981	11.339	12.099	0.579	4.2240
1982	12.174	12.410	0.056	1.4895
1983	12.465	12.659	0.038	0.8638
1984	12.557	12.868	0.097	0.7008
1985	12.690	13.051	0.130	0.4954
1986	12.832	13.217	0.148	0.3156
1987	13.326	13.370	0.002	0.0047
1988	13.531	13.515	0.000	0.0187
1989	13.708	13.657	0.003	0.0986
1990	13.842	13.799	0.002	0.2005
1991	14.046	13.948	0.010	0.4251
1992	14.276	14.121	0.024	0.7787
1993	14.547	14.555	0.000	1.3303
1994	14.720	14.768	0.002	1.7583
1995	14.858	14.846	0.000	2.1430
Total			1.090	14.8469
Ave	13.394	13.526		
			(10) R_2:	92.7%

(1) = Sheet 1, Column 2
(2) = Sheet 1, Column 3
(3) = [(1) - (2)]^2
(4) = sum of square (2)
(5) = Sum of (3) / (4)

(6) = Natural Log of (1)
(7) = Natural Log of (2)
(8) = [(7) - (6)]^2
(9) = [(6) - Ave of (6)]^2
(9) = 1 - [Sum of (8) / Sum of (9)]

* The fulcrum year is the point in the power curve when the slope changes from positive to negative -- the inflection point.

Exhibit I
Sheet 3

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Power Curve 7

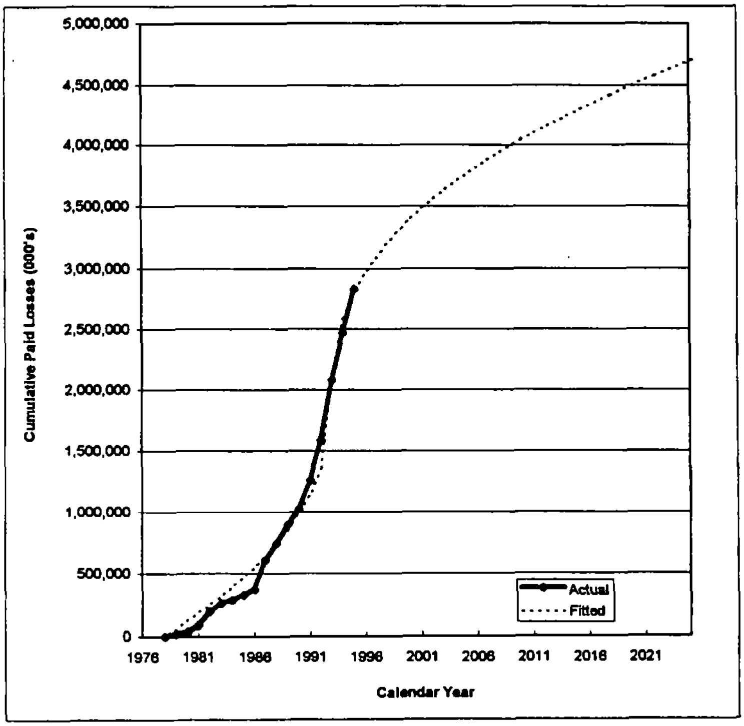


Exhibit II
Sheet 1

Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1984
Power Curve 9
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0		0	
1979	0		0	
1980	0		0	
1981	0		0	
1982	0		0	
1983	0	135,953	0	
1984	160,048	172,946	160,048	36,994
1985	326,616	222,134	166,567	49,188
1986	500,739	407,273	174,123	185,139
1987	683,772	579,370	183,034	172,097
1988	877,553	914,273	193,780	334,903
1989	1,084,678	1,150,537	207,126	236,263
1990	1,309,057	1,410,354	224,379	259,817
1991	1,557,103	1,613,107	248,045	202,753
1992	1,840,889	1,951,047	283,786	337,940
1993	2,189,868	2,334,475	348,979	383,428
1994	2,791,813	2,779,049	601,944	444,574
1995	3,557,863	3,373,188	766,050	594,139
1996	3,914,670		356,807	
1997	4,202,122		287,453	
1998	4,452,431		250,309	

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1999	4,678,395		225,964	
2000	4,886,715		208,319	
2001	5,081,439		194,724	
2002	5,265,244		183,805	
2003	5,440,014		174,770	
2004	5,607,135		167,121	
2005	5,767,665		160,530	
2006	5,922,432		154,768	
2007	6,072,103		149,670	
2008	6,217,219		145,116	
2009	6,358,232		141,013	
2010	6,495,522		137,290	
2011	6,629,411		133,889	
2012	6,760,177		130,766	
2013	6,888,061		127,884	
2014	7,013,275		125,213	
2015	7,136,002		122,728	
2016	7,256,409		120,407	
2017	7,374,642		118,233	
2018	7,490,832		116,190	
2019	7,605,098		114,266	
2020	7,717,546		112,449	
2021	7,828,275		110,729	
2022	7,937,373		109,098	
2023	8,044,921		107,548	
2024	8,150,993		106,072	
2025	8,255,658		104,665	
2026	8,358,979		103,321	
2027	8,461,015		102,036	

Exhibit II
Sheet 2

Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1984
Power Curve 9

Fulcrum Year* =	1995

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981			
1982			
1983			
1984	172,946	160,048	166,353,410
1985	222,134	326,616	10,916,392,646
1986	407,273	500,739	8,735,786,395
1987	579,370	683,772	10,899,804,671
1988	914,273	877,553	1,348,405,992
1989	1,150,537	1,084,678	4,337,339,580
1990	1,410,354	1,309,057	10,261,027,087
1991	1,613,107	1,557,103	3,136,433,492
1992	1,951,047	1,840,889	12,134,793,722
1993	2,334,475	2,189,868	20,911,063,433
1994	2,779,049	2,791,813	162,916,517
1995	3,373,188	3,557,863	34,104,779,838
Total			117,115,096,782
	(4) Sum of value squared	35,572,402,910,470	
		(5) R_1:	0.003

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981				
1982				
1983				
1984	12.061	11.983	0.006	3.0569
1985	12.311	12.697	0.149	2.2443
1986	12.917	13.124	0.043	0.7955
1987	13.270	13.435	0.027	0.2910
1988	13.726	13.685	0.002	0.0069
1989	13.956	13.897	0.003	0.0215
1990	14.159	14.085	0.006	0.1227
1991	14.294	14.258	0.001	0.2348
1992	14.484	14.426	0.003	0.4553
1993	14.663	14.599	0.004	0.7298
1994	14.838	14.842	0.000	1.0578
1995	15.031	15.085	0.003	1.4939
Total			0.247	10.5100
Ave	13.809	13.843		
			(10) R_2:	97.6%

(1) = Sheet 1, Column 2
(2) = Sheet 1, Column 3
(3) = [(1) - (2)]^2
(4) = sum of square (2)
(5) = Sum of (3) / (4)

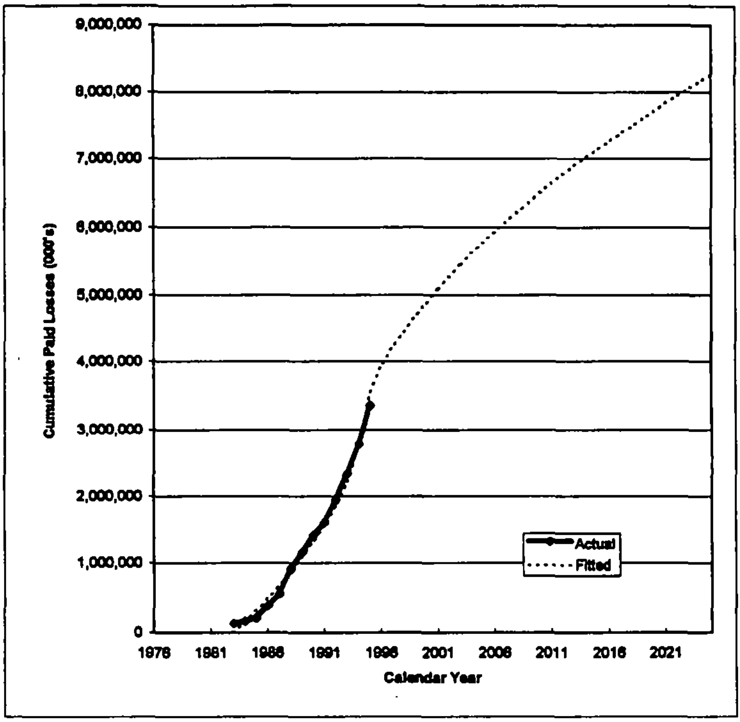
(6) = Natural Log of (1)
(7) = Natural Log of (2)
(8) = [(7) - (6)]^2
(9) = [(6) - Ave of (6)]^2
(9) = 1 - [Sum of (8) / Sum of (9)]

* The fulcrum year is the point in the power curve when the slope changes from positive to negative -- the inflection point.

Exhibit II
Sheet 3

Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Power Curve 9



Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1979
Gamma
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0	362	0	
1979	208,151	17,918	208,151	17,556
1980	208,152	33,987	1	16,069
1981	208,193	84,014	42	50,026
1982	208,703	193,596	510	109,583
1983	211,663	258,994	2,960	65,397
1984	222,431	284,030	10,768	25,037
1985	250,870	324,534	28,439	40,504
1986	310,410	374,068	59,539	49,534
1987	414,958	612,636	104,548	238,568
1988	574,924	752,146	159,968	139,509
1989	794,069	898,011	219,145	145,868
1990	1,068,239	1,026,623	274,170	128,612
1991	1,386,219	1,259,167	317,980	232,543
1992	1,732,068	1,585,463	345,849	326,296
1993	2,088,059	2,078,939	355,992	493,476
1994	2,437,385	2,470,635	349,325	391,696
1995	2,766,116	2,835,848	328,732	365,213
1996	3,064,244		298,128	
1997	3,325,877		261,632	
1998	3,548,826		222,949	
1999	3,733,847		185,021	
2000	3,883,756		149,909	
2001	4,002,598		118,843	
2002	4,094,956		92,358	
2003	4,165,435		70,479	
2004	4,218,322		52,887	
2005	4,257,398		39,076	
2006	4,285,859		28,461	
2007	4,306,314		20,456	
2008	4,320,834		14,520	
2009	4,331,022		10,188	
2010	4,338,093		7,072	
2011	4,342,952		4,859	
2012	4,346,259		3,307	
2013	4,348,489		2,230	
2014	4,349,981		1,492	
2015	4,350,971		990	
2016	4,351,623		652	
2017	4,352,049		426	
2018	4,352,325		277	
2019	4,352,504		179	
2020	4,352,618		114	
2021	4,352,691		73	
2022	4,352,738		46	
2023	4,352,767		29	
2024	4,352,785		18	
2025	4,352,796		11	
2026	4,352,804		7	
2027	4,352,808		4	

Exhibit III
Sheet 2

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1979
Gamma

Fulcrum Year* =	1993

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981	84,014	208,193	15,420,630,711
1982	193,596	208,703	228,217,129
1983	258,984	211,663	2,240,169,393
1984	284,030	222,431	3,794,433,145
1985	324,534	250,870	5,426,301,286
1986	374,068	310,410	4,052,397,325
1987	612,636	414,956	39,077,464,111
1988	752,146	574,924	31,407,562,985
1989	898,011	794,069	10,804,079,680
1990	1,026,623	1,068,239	1,731,853,967
1991	1,259,167	1,386,219	16,142,223,992
1992	1,585,463	1,732,068	21,493,136,653
1993	2,078,939	2,088,059	83,191,157
1994	2,470,635	2,437,385	1,105,540,857
1995	2,835,848	2,766,116	4,862,416,337
Total			157,869,618,729
	(4) Sum of value squared	25,488,773,081,811	
		(5) R_1:	0.006

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981	11.339	12.246	0.824	4.2240
1982	12.174	12.249	0.006	1.4895
1983	12.465	12.263	0.041	0.8838
1984	12.557	12.312	0.060	0.7008
1985	12.690	12.433	0.066	0.4954
1986	12.832	12.646	0.035	0.3156
1987	13.326	12.936	0.152	0.0047
1988	13.531	13.262	0.072	0.0187
1989	13.708	13.585	0.015	0.0986
1990	13.842	13.882	0.002	0.2005
1991	14.046	14.142	0.009	0.4251
1992	14.276	14.365	0.008	0.7787
1993	14.547	14.552	0.000	1.3303
1994	14.720	14.706	0.000	1.7583
1995	14.858	14.833	0.001	2.1430
Total			1.289	14.8469
Ave	13.394	13.361		
			(10) R_2:	91.3%

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* The fulcrum year is the point in the curve when the slope changes from positive to negative -- the inflection point.

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit III
Sheet 3

Gamma