Errata and Additional Material Related to "Accounting for Risk Margins"

by Stephen W. Philbrick, FCAS

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TO CAS MEMBERS,

In the Spring 1994 edition of the CAS Forum, is the paper "Accounting for Risk Margins". That paper has been read by a number of people who have identified a few areas where formulas or numbers are either in error or potentially misleading. While several people brought this to my attention, and I am grateful to each for identifying these items, I would particularly like to thank Andrew Rippert who brought most of these items to my attention.

The following four pages summarize the appropriate corrections. On the first three pages, I have explained in narrative form most of the suggested changes. In particular this narrative contains the intellectual justification for an alternative formula for NRM_t that may be more intuitive to some.

One the last page, I have provided a quick summary which can be used as a reference, or used to make corrections to the original text.

My apologies to any who were misled by any of the errors, and my thanks again to those that took the time to bring these issues to my attention.

Stephen W. Philbrick

Errata and Additional Material related to Accounting for Risk Margins,
Stephen W. Philbrick, Casualty Actuarial Society Forum, Spring 1994

On page 26 and in footnote 12 on page 27, there is a reference to the coefficient of variation of the assumed aggregate distribution. The CV value used to create the example is shown as .128, which represents the actual value rounded to three decimal places. Anyone attempting to reproduce the calculations may prefer to use the value carried out to more decimal places. The assumed value of the CV is .12848, to five decimal places. (I created the portfolio from a number of individual risks, each of which had "round" values for the mean and CV. However, the individual risk detail was not relevant to the rest of the paper, so I omitted the details of the calculation of the portfolio parameters.)

Similarly, on page 36 (and subsequent calculations) the factor used to calculate the total asset need is shown to three decimal places as 1.233. This factor, carried out to six decimal places, is 1.233475. This factor is not shown explicitly on page 27, but is used to calculate the value of $359.42.

On page 27, the footnote contains two formulas. The first is stated as:

$$\int_A^\infty (z - A)dF = .003$$

This formula is correct under the assumption that the distribution has been normalized such that the expected losses are equal to 1.0. A more general formula is obtained by multiplying the right side of the equation by the mean losses. Alternatively, the mean loss amount could be placed in the denominator of the left side. Thus, we solve for A such

that: $$\int_A^\infty (z - A)dF = .003 \int_0^\infty z dF$$

or: $$\frac{\int_A^\infty (z - A)df}{\int_0^\infty z dF} = .003$$

The second formula contains an alternative representation. Unfortunately, the limits of integration were shown incorrectly. The limits of integration should be zero to A. In addition, to make the formula general rather than normalized, multiply the right side by the mean:

$$\int_0^A z dF + A[1 - F(A)] = .997 \int_0^\infty z dF$$

On page 37, the formula shown as: $$NRM_t = (ROR - i)\sum \frac{BRM_t}{(1 + ROR)^t}$$

Should be shown as: $$NRM_t = (ROR - i)\sum_{j=t}^{\infty}\frac{BRM_j}{(1 + ROR)^{j+1}}$$

This formula is the easiest one to use in practice. However, an alternative formulation is easier to understand conceptually. This formula is written as:

$$NRM_t = (ROR - i)\sum_{j=t}^{\infty}\frac{SRM_j}{(1 + i)^{j+1}}$$

This formula can best be understood by thinking about the process of establishing an insurance company to take on this specific risk. The insurance company will need an amount over and above the mean(discounted) losses to account for the possibility that actual losses exceed the expected. The amount in addition to the mean losses will be contributed by both the investor and the insured, such that the investor can earn a fair rate of return on the investment.

Assuming that the insured will pay the mean losses, we now examine how the amount over and above the mean losses should be apportioned between the two parties. The investor will contribute an initial amount of surplus, SRM_0, into the company at inception. However. our losses are not fully extinguished by the end of the first year. so our investor is committed to leaving surplus in the company in subsequent years.

The surplus commitment is represented by the string of surplus values, SRM_j. (Ignore the denominator for the present.) While the investors surplus commitment is in the future for all years other than SRM_0, the loss amounts used in the calculation of the SRM_j have been calculated by discounting future losses back to time zero, so the implied surplus value is the present value of the future surplus commitment. The investor wishes to earn a return of ROR on the investment, so we must pay the investor a total of ROR times the present value of the surplus commitment. However, the surplus placed in the company will earn investment income at rate i, so we can reduce the amount required to be paid by the policyholder by this amount, hence the (ROR-i) factor. Finally, the amount paid by the policyholder (NRM_0) will be paid into the company at time zero. and this amount will earn investment income over its life. Some of the narrow risk margin will be returned to the investor at each year end, so the calculation of the investment income earned over the life of the policy is not straightforward. However, it works out that the adjustment for investment income earned by NRM_0 can be handled by dividing the SRM_j values by $(1+i)^{j+1}$

It is tempting to presume that this factor in the denominator is the factor to discount the surplus requirements back to time zero. in which case the exponent seems wrong. However. each SRM_j value is ultimately derived from L_j, which represents unpaid losses discounted back to time zero. Instead, this factor accounts for the investment income earned on the narrow risk margin over its lifetime.

On page 38, the following four quantities are shown:

$BRM_0 = .233$

$L_0 = \$131.00$

$NRM_0 = \$16.14$

$SRM_0 = \$114.87$

The last two are correct, but the first two quantities should have been shown as:

$L_0 = \$561.09$

$BRM_0 = .233 \times L_0 = \$131.00$

On page 44, in the section labeled "**Balance Sheet (Year X+1)**", there are references to BRM_1, NRM_1, and SRM_1. These should be references to BRM_2, NRM_2, and SRM_2, respectively.

On page 79, the statement is made that $P = E(z) + R$. In the middle of the page it says "On average (or over the long run), the company will pay E(z), leaving profits of R on capital of C. Thus:

$$\frac{R}{C} = \text{return on capital"}$$

This statement is correct if we ignore investment income. The inclusion of investment income does not affect E(z), because we have defined our loss variable to be on a discounted basis. However, part of the return to the investor arises from investment income earned on surplus, as well as investment income earned on the Narrow Risk Margin. Denote these as ii_S and ii_R, respectively. Consequently, it would be more accurate to state that "On average (or over the long run), the company will pay E(z), leaving a profit consisting of R plus ii_S plus ii_R. Thus:

$$\frac{R + ii_S + ii_R}{C} = \text{return on capital"}$$

Summary of Changes and Additions

Location	Shown As	Should be
On page 26 and in footnote 12 on page 27, reference to CV	.128	.12848
Expected Deficit Assumption, page 27	2%	.3%
Factor used to calculate the total asset need, page 36 (and other pages)	1.233	1.233475
First Expected Deficit formula in Footnote 12	$\int_A^\infty (z-A)dF = .003$	$\int_A^\infty (z-A)dF = .003 \int_0^\infty zdF$
Second Expected Deficit formula in Footnote 12	$\int_A^\infty zdF + A[1-F(A)] = .997$	$\int_0^A zdF + A[1-F(A)] = .997 \int_0^\infty zdF$
Formula on page 37	$NRM_t = (ROR - i)\sum \frac{BRM_t}{(1+ROR)^t}$	$NRM_t = (ROR - i)\sum_{j=t}^{\infty} \frac{BRM_j}{(1+ROR)^{j+1}}$
Equivalent formula to the one shown on page 37	Not shown	$NRM_t = (ROR - i)\sum_{j=t}^{\infty} \frac{SRM_j}{(1+i)^{j+1}}$
Quantities on page 38	$BRM_0 = .233$ $L_0 = \$131.00$	$L_0 = \$561.09$ $BRM_0 = .233 \times L_0 = \$131.00$
Variables on page 44	BRM_1 NRM_1 SRM_1	BRM_2 NRM_2 SRM_2
Formula on page 79	$\frac{R}{C}$ = return on capital	$\frac{R + ii_S + ii_R}{C}$ = return on capital