# An Introduction to Basic Credibility by Howard C. Mahler, FCAS

# AN INTRODUCTION TO BASIC CREDIBILITY

# TALK BY HOWARD C. MAHLER MARCH 1996 CAS RATEMAKING SEMINAR

In my talk, I will try to reinforce and expand on the ideas Gary Dean presented in his talk.

I will start off my talk by using the following set of graphs taken from my "Student's Guide to Buhlmann Credibility and Bayesian Analysis" to illustrate some simple credibility ideas in terms of experience rating or individual risk rating. The goal of experience rating is to use an individual insured's experience to help predict the future. Assuming the individual risk's experience were observed to be worse than average, we would predict his future experience would also be likely to be <u>somewhat</u> worse than average. Therefore, we would be likely to charge this insured somewhat more than average.

As mentioned by Gary Dean, credibility <u>quantifies</u> how much worse or better an insured's <u>future</u> experience is expected to be based on a particular deviation from average observed in the <u>past</u>. These graphs should illustrate some of the ideas Gary Dean mentioned, such as why more weight is given to an individual's experience in certain situations. Also, those of you familiar with linear regression should see much that is familiar. (With the widespread use of personal computers, anyone can do a linear regression.)

2757

The first graph, Exhibit 1, shows simulated claim counts for 100 insureds divided into two equal groups. In this graph, the "Good Risks" are labeled with crosses and the "Bad Risks" with circles. In both the real world and many of the subsequent graphs, the risks come without such labels attached. (If they did come with such labels, we would not need to use credibility.) Assume we have 100 insureds all in the same risk classification, territory, etc.

The 50 Bad Risks each have an expected Claim Frequency of 15 while the 50 Good Risks each have an expected Claim Frequency of 10. For each of the 100 insureds I have plotted a single prior year against a single subsequent year of claim counts. (For example, one of the Good Risks had 4 claims in the prior year and 5 claims in the subsequent year. This is indicated by a cross at the point (4, 5)). There is considerable overlap between the groups. Nevertheless, the Good Risks are more likely to be in the lower left while the Bad Risks are more likely to be in the upper right.

The next graph, Exhibit 2, shows the same 100 insureds without labels. Here we have fit a least squares regression line to the points. One could use this fitted line to predict a future year's experience based on an observation. Since the line slopes upwards, we see that a bad former year would lead one to predict a worse than average subsequent year.

So if one observed 20 claims for an insured, one might predict about 15 claims for that insured next year, compared to the overall average of 12.5. This least square's line is approximately:

#### Y = .40X + 7.6

This can be put in the form of the "Basic Formula" discussed by Gary Dean:

# Estimate = Z (observation) + (1 - Z) (overall average)

#### with the credibility Z = 40%

With only 100 insureds, this result is subject to considerable random fluctuation. The simulation with many more insureds would give a credibility of 1/3. (This can be derived using methods taught on the CAS, Part 4B Exam, which were touched on by Gary Dean.)

The credibility is just the slope of the straight line.. It is the weight given to the observation.

Note the way that the fitted line passes through the point (12.5, 12.5), denoted by a plus. Average experience in the prior year yields an estimate of average experience in the subsequent year.

Note that the line Y = X, with a slope of unity, would correspond to 100% credibility, while the line Y = 12.5 with a slope of zero, would correspond to zero credibility. In general, the slope and the credibility will be between zero and one.

These general features displayed in Exhibit 2, will carry over to subsequent exhibits. The least squares line will slope upwards and pass through the point denoting average experience in the prior and subsequent period. The slope will be (approximately) equal to the credibility.

The next graph, Exhibit 3, is similar to Exhibit 2 but shows three years of prior experience rather than one. Note that the X-axis is now the <u>annual</u> claim frequency observed over three years. We expect three years of data to contain more useful information and thus be given more weight than would one year. In fact, when we fit a straight line we see a larger slope of about 60% (actually 58%) corresponding to a

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credibility of 60%. As Gary Dean noted, one way to increase the credibility of data is to increase the volume of data.

In the case of Exhibits 2 and 3, the credibility is equal to N / (N + K) where N = # of years of data and K = 2. As mentioned by Gary Dean, this formula is used quite often, with the "Buhlmann Credibility Constant" K dependent on the statistical properties of the particular situation. Note that for Exhibit 2,  $Z = \frac{1}{1+2} = \frac{1}{3}$ , while in Exhibit 3,

 $Z = \frac{3}{3+2} = 60\%$ . (In the next set of graphs, K will equal .22.)

The next graph, Exhibit 4, shows 100 risks divided this time into Excellent Risks and Ugly Risks. The Excellent Risks are shown by asterisks and the Ugly Risks by wedges. The mean frequencies are 5 and 20 rather than 10 and 15 as in the previous Exhibits. Therefore, the two groups are much more spread apart. Since there is more dispersion between risks, each risk's data will be given more credibility than in the first graph.

This can be seen in the next graph, Exhibit 5, where a straight line has been fit to these points. The line has a much larger slope than the first line, corresponding to higher credibility of about 82%. (Again the results of an experiment with only 100 drivers differs from the theoretical result due to random fluctuation.) So due to the larger variation in hypothetical means (holding everything else equal) in Exhibit 5 versus Exhibit 2, the credibility increased from 33% to 82%. The value of the individual risks information increased relative to the information contained in the grand mean. Conversely, the relative value of the information contained in the grand mean decreased.

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The next graph, Exhibit 6, combines the four different types of insureds. This starts to approach the real world situations where risks' expected claim frequencies are along a continuous spectrum, rather than being of unique types. (One could approach a continuous situation similar to the Gamma-Poisson frequency process.) We can see plenty of overlap between the four types, although since we labeled the insureds, we can discern the grouping of different types.

The next graph, Exhibit 7, shows a line fit to all four types. There the slope of 72% is between the slopes of either 40% and 78% we got when dealing with just two groups. This makes sense since the variation of the hypothetical means is in between those two situations.

The following graphs will all involve 125 Excellent and 125 Ugly Risks, but rather than dealing with just claim frequency will deal with claim severity as well. By looking at dollars of loss rather than numbers of claims, as can be seen on the next graph, Exhibit 8, we introduce more random fluctuation. Therefore, the relative value of the observation is less compared to average; the credibility goes down. As mentioned by Gary Dean, one way to <u>decrease</u> the credibility of data is to <u>increase</u> the variability of the data.

As can be seen on the next graph, Exhibit 9, the slope of the fitted line is 51.5%. The theoretical credibility is 53% compared to 82% for the corresponding claim frequency situation. The greater random fluctuation, which is quantified by the larger "process variance" has decreased the credibility assigned to the observations.

In practical applications, one often limits the size of claims entering into Experience rating. As Gary Dean mentioned, one way to decrease the variability of the data is to cap losses. The final graph, Exhibit 10, shows the results of capping each claim at \$25,000.

(This capping can be either just for the purposes of experience rating or could involve an actual policy limit.) The fitted line between prior limited losses and subsequent limited losses is 71.4%. The theoretical credibility of 70% when using limited losses compares to 53% for total losses. Capping the losses has reduced the random fluctuations, i.e., has reduced the process variance, thereby increasing the credibility assigned to the experience. (Basic limit losses are less volatile than total limits losses.) (For more on how to analyze Experience Rating Plans, see for example, *"An Analysis of Experience Rating"* by Glenn Meyers in PCAS 1985 and my discussion in PCAS 1987.)

So far my talk has illustrated the concept of using credibility for individual risk rating. As Gary Dean mentioned, credibility is also used in classification rating, reserving, trending, and other areas. Whenever an actuary wishes to make an estimate, credibility can be useful to overcome the problem of limited data.

Let X be the quantity we wish to estimate. For example, X might be the expected losses for a Workers' Compensation class relative to the statewide, i.e., X is the class relativity. In my previous example, X was a risk's future expected experience relative to average.

As shown in Exhibit 11, in the "Basic Formula" we weight together two estimates of the quantity X. In that case we usually write:

$$X = Z Y_1 + (1 - Z) Y_2$$

where Z is called the credibility and 1 - Z is called the complement of credibility. In the experience rating example,  $Y_1$  was the risk's observed experience while  $Y_2$  was the overall average experience.

As listed on Exhibit 12, the estimators Y<sub>i</sub> can have many sources. (This subject is discussed in more detail in Joseph Boor's paper "*The Complement of Credibility*" in the Fall 1995 CAS Forum.)

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For example:

- 1. The recent observation(s) of X.
- 2. The recent observation(s) of the same quantity as X, but for a superset.
- The recent observation(s) of a similar quantity to X; there may be an adjustment necessary.
- Past estimates(s) of X. There may be an adjustment for the intervening period of time.
- 5. The result of a model.
- The result of judgment.

Exhibit 13 shows those rules I think will aid you in using credibility for practical applications.

#### Rule\_1A;

Spend a lot of time and effort deciding on or choosing the  $Y_i$ . Each  $Y_i$  should be a reasonable estimate of X.

So for example, if trying to estimate a medical claim cost trend it may not make much sense to assign the complement of credibility to an estimate based on the general overall rate of inflation. It might make sense to look at some other measure of medical inflation rather than a measure of general inflation.

#### Rule 1B;

Spend a lot of time and effort computing, collecting data on, or estimating each Y<sub>i</sub>.

2757

If you are going to include a value in your weighted average, it makes sense to try to carefully quantify that value.

#### Rule 2A:

The procedure is generally forgiving of small "errors" in the weights. Therefore, do not worry overly much about getting the weights exactly right.

In our experience rating example, you can confirm that for most risks, small changes in the credibility do not result in major changes in the estimate of their future experience.

This is discussed in my paper "An Actuarial Note on Credibility Parameters" in PCAS 1986. Exhibit 14 illustrates the effect of changing K, the Buhlmann credibility parameter, on the credibility. As can be seen, changes in K of less than a factor of 2 would result in relatively small changes in credibility. In turn, these small changes in credibility usually result in small changes in estimates of the quantities of interest.

#### Rule 2B:

The concept of credibility is a <u>relative</u> concept. A relative weight can only be assigned to any single estimator, if you know what all the other estimators are.

For example, assume you have two estimators each of which has been assigned "only" 50% credibility. This merely indicates that the two estimators are equally good or equally bad, not whether they are good or bad in some absolute sense.

#### Rule 2C:

The less random variation in an estimate, the more weight it should be given. In other words, the more useful information and the less noise, the more the weight. We

saw that limited losses were given more weight than unlimited losses, since the limited losses had less random variation.

#### Rule 2D:

The more closely related to the desired quantity, the more weight an estimator should receive.

For example, observations more distant in time usually deserve less weight. A given quantity of data from the same state would probably receive more weight than data from outside the state.

#### <u>Rule 3:</u>

Cap the changes in relativities that result from the use of credibility.

A properly chosen cap may not only add stability, but may even make the methodology more accurate by eliminating extremes.

An example of a practical use of credibility involves revising the definitions of automobile insurance territories in Massachusetts. Each town's relative loss potential is determined based on four years of data and a relatively complicated credibility methodology. For frequency, the complement of credibility is given to a road density model. For severity, the complement of credibility is given to a combination of the county average severity and the state average severity. Then towns with similar estimated loss potential are grouped together. Here we will ignore the details of the procedure which are explained in Robert Conger's paper, *"The Construction of Automobile Rating Territories in Massachusetts"* in PCAS 1987, and discuss one aspect of the results of the reviews conducted over the last decade.

2757

It has been demonstrated that use of this credibility technique produces "better" predictions on average. However, credibility is a linear process, and thus the extreme cases may not be dealt with as well as they might.

For example, Exhibit 15 shows the results of applying the same methodology consistently over time to two small towns, each with somewhere around 5,000 exposures per year.

	Estimated Loss Potential Relative to Statewide Average									
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review			
Acushnet	.84	.87	.88	.87	.93	1.00	.97			
Brewster	.74	.84	.70	.61	69	.69	.58			

	Indicated Territory (Prior to Capping)								
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review		
Acushnet	5	6	6	6	7	8	8		
Brewster	3	6	2	1	2	2	2		

The results for the first town, Acushnet, are typical. The relative loss potential varies somewhat from review to review, with a change in indicated territory of plus or minus one from time to time. In this particular case there is an upward drift over time relative to the statewide average.

The results for the second town, Brewster, are not typical. In fact, Brewster was chosen as the most extreme case of fluctuating experience over the period of time from

the 1984 review to the 1989 review. As you can see in Exhibit 16, the estimated relative loss potential swung up and then down. This in turn resulted in large changes in the indicated territories. This occurred in spite of relying on four years of data, so that the data periods used in the reviews overlap. This occurred in spite of the use of credibility, which ameliorated the effect of the large fluctuations in the experience of this town.

Such large swings are unlikely. However, when dealing with 350 towns, something that only has a .3% chance of happening per town, on average occurs for one town.

This problem is dealt with by capping territory movements. The actual cap chosen was to restrict movements in any one revision to at most one territory either up or down. This is an example of the third rule I discussed earlier.

Another example of a practical use of credibility, is the Workers' Compensation Experience Rating Plan. This is an individual risk rating plan conceptually similar to my first set of graphs involving Excellent, Good, Bad, and Ugly risks. Around 1989 or 1990, the National Council on Compensation Insurance made a major revision to their Workers' Compensation Experience Rating Plan. Among the changes was a major revision to the credibilities assigned to an individual insured's loss experience relative to average. This was based on an extensive and detailed study by the NCCI actuaries. (See for example, William R. Gillam's paper "Parametrizing the Workers' Compensation Experience Rating Plan" in PCAS 1992 and my discussion in PCAS 1993.) Without getting into any details, Exhibit 17 shows you the overview.

Primary Losses are the first layer of losses while Excess Losses are those above them.  $Z_p$  is the credibility assigned to primary losses. For the prior plan, it is shown by

dots; for the revised plan by circles. Similarly,  $Z_x$  is the credibility assigned to excess losses. For the prior plan, it is shown by solid squares; for the revised plan, it is shown by open squares. In each case, the credibility assigned to the primary losses is greater than that assigned to the excess losses, since excess layers are more volatile than basic limits losses.

Note that the credibility varies by size of risk. The more expected losses, the more credibility is assigned to the insured's own experience and the less that is assigned to the manual rate. (Note that the maximum credibility for the revised plan is less than 100%. The credibilities for the revised plan are based on a refinement of the Buhlmann Credibility formula discussed by Gary Dean.)

Exhibit 18 shows the changes in credibilities. For smaller risks, the revised plan assigns higher credibilities than the prior plan. For larger risks, the revised plan assigns lower credibilities than the prior plan. Thus, large insureds with good experience get smaller credits under the revised plan, while large insureds with bad experience get smaller debits under the revised plan. The theoretical credibility work by the Actuaries at the National Council that led to this revision, had a major impact on thousands of businesses across the country. So "theoretical credibility" can have immense practical impact.

A final example of a practical use of credibility, is the estimation of relative average claim costs for workers compensation classes. Exhibit 19 shows the calculation of the observed average claim costs for the classes in the Office and Clerical Industry Group for one year. We divide losses by the number of claims. Then for each class we calculate the <u>relative</u> average claim cost by dividing the classes' average claim cost by that for the

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industry group. Note that I have not limited the size of claims, but that I have excluded the large lifetime claims which would produce the most random fluctuation.

So far we have not used credibility. However, since some classes have very few claims in a single year, I would not want to rely on the results of one year of observations. Exhibit 20 puts together the results of seven years of observations. We observe considerable random fluctuation in the relative claim costs. I take an average over the seven years for each class and then use credibility.

For each class its observed relative claim cost is given credibility equal to the square root of its number of claims divided by 2,500. A class with 2,500 or more claims over 7 years is assigned full credibility. The Complement of credibility is assigned to unity, an average claim cost equal to the overall average for the Industry Group. Applying the Basic Formula on Exhibit 11 to this case the estimated relative average claim cost is:

Z (observed average claim cost) + (1 - Z)(1)

as shown in Column 12 of Exhibit 20.

Exhibit 21 graphs the Credibility in this case. Exhibit 22 compares the credibility from the use of the square root formula to that using Z = N/(N + K) with K = 350 claims. The credibilities are similar.

I have tried to illustrate a few of the many applications of credibility. I've given a number of general rules which you should find useful in your own work with credibility.

The theory behind the use of credibility can be complex. However, the use of credibility itself is set up precisely so that it can be understood by a layman. While ratemakers may differ in their knowledge of credibility theory, all ratemakers should be completely familiar with credibility practice.













50 Excellent Risks (Poisson 5), 50 Good Risks (Poisson 10), 50 Bad Risks (Poisson 15), and 50 Ugly Risks (Poisson 20)



50 Excellent Risks (Poisson 5), 50 Good Risks (Poisson 10), 50 Bad Risks (Poisson 15), and 50 Ugly Risks (Poisson 20)







Exhibit 11

# **"BASIC FORMULA"**

 $X = Z Y_1 + (1 - Z) Y_2$ 

# where X is the quantity to be estimated

- $Y_1$  and  $Y_2$  are estimators of X
- Z is credibility

Exhibit 12

The estimators  $Y_i$  can have many sources. For example:

1. The recent observation(s) of X.

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- 2. The recent observation(s) of the same quantity as X, but for a superset.
- 3. The recent observation(s) of a similar quantity to X; there may be an adjustment necessary.
- 4. Past estimate(s) of X. There may be an adjustment for the intervening period of time.
- 5. The result of a model.
- 6. The result of judgement.

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### <u>Rule 1A:</u>

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# <u>Rule 1B:</u>

Spend a lot of time and effort computing, collecting data on, or estimating each  $Y_{I}$ .

# <u>Rule 2A:</u>

The procedure is generally forgiving of small "errors" in the weights. Therefore, do not worry overly much about getting the weights exactly right.

# Rule 2B:

The concept of credibility is a <u>relative</u> concept. A relative weight can only be assigned to any single estimator, if you know what all the other estimators are.

# <u>Rule 2C:</u>

The less random variation in an estimate, the more weight it should be given. In other words, the more useful information and the .

Exhibit 13

# <u>Rule 2D:</u>

The more closely related to the desired quantity, the more weight an estimator should receive.

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# <u>Rule 3:</u>

Cap the changes in relativities that result from the use of credibility.

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# **Massachusetts Private Passenger Automobile**

	Estimated Loss Potential Relative to Statewide Average										
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review				
Acushnet	.84	.87	.88	.87	.93	1.00	.97				
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	Indicated Territory (Prior to Capping)									
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review			
Acushnet	5	6	6	6	7	8	8			
Brewster	3	6	2	1	2	2	2			

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Exhibit 15







# MASSACHUSETTS WORKERS' COMPENSATION Relative Average Claim Costs Industry Group: Office & Clerical Composite Policy Year 85/86 @2nd Report

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	(1) (2)		(3)	(4) = (2)/(3)	(5) = (4)/T⊤(4)
Phraseology	Class	Losses (Indemnity+Medical)	Number of Claims	Average Claim Cost	Relative Average Claim Cost
Photographer-All Emp-Clerical Sales-& Dr	4361	231 122	33	7 004	0.680
Radio or TV Broadcast-All Emp. Cler-& Dr	7610	702,919	42	16 736	1.625
Engineer or Architect-Consulting	8601	1.356.461	134	10,123	0.983
Salesperson, Collector, Messenger-Outside	8742	8,771,008	703	12.477	1.211
Auto Sales or Service Accy-Salesperson	8748	1,552,606	73	21,269	2.065
Mailing or Addressing Co-& Clerical	8800	245,229	38	6,453	0.626
Auditor Accountant Etc-Traveling	8803	184,289	43	4,286	0.416
Clerical Office Employees NOC	8810	24,323,122	2,404	10,118	0.982
Attorney-All Emp-Clerical Messenger & Dr	8820	741,565	40	18,539	1.800
Physician-& Clerical	8832	1,444,953	136	10,625	1.031
Hospital-Professional Employees	8833	11,760,162	1,199	9,808	0.952
School-Professional Emp & Clerical	8868	5,263,573	634	8,302	0.806
Telephone/Telegraph Co-Office Emp & Cl	8901	146,908	14	10,493	1.019
Theatre-Players, Entertainers, Musicians	9156	131,147	26	5,044	0.490
	Total	56,855,064	5,519	10,302	

(2),(3): Losses and claims are as reported under the Unit Statistical Plan, but excluding any Fatal, Permanent Total, or Medical Only Claims. (Losses are paid plus case reserves and are neither limited nor adjusted.)

# MASSACHUSETTS WORKERS' COMPENSATION Estimated Relative Average Claim Cost Industry Group: Office & Clerical

(1)	(2)	(3)	(4)	(5)	(6)	ന	(8)	(9)	(10)	(11)	(12) = 1+(11)x[(9)-1]
	85/86	86/87	87/88	88/89	89/90	90/91	91/92	Combined	Number		Estimated
Class	Relative	of		Relative							
Code	ACC	Claims	Credibility	ACC							
4361	0.680	0.920	0.640	0.708	1.087	0.428	1.002	0.785	323	0.359	0.923
7610	1.625	1.351	0.839	0.934	1.127	0.969	0.858	1.059	364	0.382	1.023
8601	0.983	1.440	1.169	1.069	1.026	0.919	0.915	1.100	939	0.613	1.061
8742	1.211	1.161	1.031	1.221	1.028	1.017	1.444	1.143	5,829	1.000	1.143
8748	2.065	1.747	2.151	1.967	2.130	1.626	1.215	1.895	452	0.425	1.380
8800	0.626	0.725	1.025	0.830	0.883	1.365	0.721	0.889	325	0.361	0,960
8803	0.416	1.124	0.472	1.893	0.830	1.109	2.268	1.029	188	0.274	1.008
8810	0.982	1.021	1.044	1.040	1.066	1.113	1.005	1.040	17,195	1.000	1.040
8820	1.800	1.307	1.630	1.639	1.238	1.216	1.540	1.450	426	0.413	1.186
8832	1.031	1.233	1.536	1.176	1.051	1.037	1.096	1.150	1,478	0.769	1.115
8833	0.952	0.773	0.814	0.792	0.863	0.884	0.774	0.837	6,819	1.000	0.837
8868	0.806	0.905	0.828	0.675	0.796	0.724	0.711	0.774	5,211	1.000	0.774
8901	1.019	0.556	1.128	1.068	0.788	0.567	0.386	0.817	173	0.263	0.952
9156	0.490	0.668	1.005	1.066	0.701	0.604	1.281	0.803	170	0.261	0.949

(2)-(8): Calculated as per Exhibit 19.

(9): Seven Years of relative average claim costs are combined by taking a weighted average using claim counts as weights.

(10): Total of Seven Years of claim counts.

(11): Credibility = square root of (7-yrs-claim-count by class / 2,500) limited to unity.

(12): Relative Average Claim Costs are credibility weighted with unity.





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