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Spring 1997
Including the Reinsurance Call Papers

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NOTICE

The Casualty Actuarial Society *Forum*
Spring 1997 Edition
Including the Reinsurance Call Papers

To CAS Members:

This is the Spring 1997 Edition of the Casualty Actuarial Society *Forum*. It contains 10 Reinsurance Call Papers (see the following note from the CAS Committee on Reinsurance Research).

The Casualty Actuarial Society *Forum* is a non-refereed journal printed by the Casualty Actuarial Society. The viewpoints published herein do not necessarily reflect those of the Casualty Actuarial Society.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper, and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.

The CAS *Forum* is printed periodically based on the number of articles submitted. The committee's goal is to publish two editions during the calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial *Forum*.

Sincerely,

Robert G. Blanco, CAS *Forum* Chairperson

The 1997 CAS Reinsurance Call Papers Presented at the 1997 CAS Seminar on Reinsurance June 1-3, 1997 Marriott's Castle Harbour Resort Tucker's Town, Bermuda

The Spring 1997 Edition of the CAS *Forum* is a cooperative effort of the CAS Continuing Education Committee on the CAS *Forum* and the Research and Development Committee on Reinsurance Research. This edition of the *Forum* focuses on the 1997 Reinsurance Call Paper Program conducted by the Committee on Reinsurance Research.

The CAS Committee on Reinsurance Research was pleased to present for discussion 10 papers prepared in response to its 1997 Reinsurance Call Paper Program. The topics addressed included: excess reinsurance pricing, property catastrophe risk load, workers compensation reinsurance commutations, the relationship of capital and risk to reinsurance programs, pricing extra-contractual obligations and excess of policy limits exposure, variations of contract terms, reserving issues, and contracts with multi-year limits. These papers were discussed by the authors at the 1997 CAS Seminar on Reinsurance, June 1-3, in Tucker's Town, Bermuda.

The Committee on the CAS *Forum* would like to encourage authors to submit papers and articles for further editions.

CAS Committee on Reinsurance Research

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Reinsurance Call Papers

A Simulation Approach in Excess Reinsurance Pricing

by Dmitry E. Papush, ACAS

A Simulation Approach in Excess Reinsurance Pricing

Dmitry Papush

There are numerous papers in the actuarial literature dealing with the different aspects and applications of aggregate loss models. The great demand for research in this area stems from the increasing popularity of insurance and reinsurance arrangements involving aggregate limit and aggregate deductible provisions. The estimates of aggregate loss distributions are also important in the pricing of contracts containing retro adjustments, and profit and contingent commission features.

Some excellent practical methods are available to estimate aggregate loss distributions, including Heckman-Meyers [2] and Panjer [5]. The common assumption used in these methods is that all claims have the same loss size probability distribution. While this assumption is reasonable for many insurance contracts, there are situations where such an assumption becomes impractical.

As an example, one can consider the reinsurance program involving several layers of reinsurance coverage. Each of these layers may have both per occurrence and annual aggregate limits with a possibility to "drop down" if the underlying layers are exhausted, creating a quite difficult "two-dimensional" structure. This type of reinsurance program is quite common for large medical professional organizations. A specific example is considered later in the paper.

Pricing such programs can be challenging for reinsurance actuaries. From a theoretical standpoint, the major difficulty involved is that the reinsurer's loss severity distribution function is changing, depending on the exhaustion of the underlying layer coverage. This makes derived aggregate loss model techniques (Heckman - Meyers, Panjer) difficult to apply. One possible solution is to use stochastic simulation.

The simulation method can also be used successfully in place of Heckman - Meyers' or Panjer's method to build an aggregate loss distribution from estimated frequency and severity distributions. This paper systematically describes the stochastic simulation approach that involves the following steps:

1) Data preparation
2) Selection of frequency and severity distributions; goodness-of-fit tests
3) Estimation of the number of simulations required
4) Simulation of the excess losses
5) Pricing recommendations

This paper outlines some theoretical and practical considerations which may be useful in utilizing this approach. A pricing example will illustrate the application of the method.

1. Pricing Example.

1.1. Description of Coverage.

Our main example deals with the coalition of several hospitals (Alpha Hospital Union, AHU) which purchases a multi-layer reinsurance program to protect itself from catastrophic medical malpractice losses. AHU retains the first $3,000,000 per each and every occurrence, and wants to reinsure the excess. Coverage is claims made; the effective date for the coverage is January 1, 1997.

We will consider the pricing of the first two excess reinsurance layers. The first layer covers $3,000,000 in excess of $3,000,000 for each and every occurrence and is subject to an annual aggregate limit of $9,000,000. The second layer covers $3,000,000 for each and every occurrence in excess of the first layer coverage and is subject to annual aggregate limit of $12,000,000. In other words, the second layer covers $3 Mil xs $6 Mil before the first layer of excess coverage is exhausted, and $3 Mil xs $3 Mil after that.

Exhibit 1 shows the design of the coverage. After the first excess layer is exhausted, the second layer "drops down" to replace it. It makes the pricing of the second layer very difficult, because the severity distribution can change in the course of a year. We will demonstrate how to use simulation to estimate expected loss for the first and the second excess layers.

1.2. Data.

We assume that the following information is provided by the client:

- The complete list of all claims for report years 1983 through 1993 that exceed $1,000,000 at 12/31/95 evaluation date (see Exhibit 2);
- Incurred and paid loss development triangles by report year (see Exhibits 3-1 and 3-2);
- Paid claim count development triangle by report year (see Exhibit 4);
- Historical exposure (Basic class Full Time Equivalents) for years 1985 through 1993 and exposure projection for year 1997 (see Exhibit 5).

The loss and exposure data for report years 1994 and 1995 are also available but not used because of their immaturity.

1.3. Pricing Approach.

Our pricing approach is consistent with one described by Patrik [6]. The following main formula (a modification of Formula 6.2.1 from [6]) will be used:

$$RP = \frac{RLC \times DF}{(1 - CR - BF)\,(1 - IXL)\,(1 - TER)} \qquad (1.3.1)$$

Here RP = reinsurance premium (gross),
RLC = reinsurance loss cost,
DF = discount factor,
CR = reinsurance ceding commission rate,
BF = brokerage fee (if any),
IXL = reinsurer's internal expense loading,
TER = reinsurer's target economic return.

We will concentrate on the estimation of RLC; the other elements of the above formula are determined using other sources. Usually IXL is a function of the size of the account, and TER is a function of the level of risk (or potential volatility of account loss experience). While our methodology does provide a tool to measure potential account volatility, this topic is outside the scope of this paper.

The simulation method is used to estimate RLC. We model the loss severity and loss frequency distribution functions to simulate a statistically representative sample of loss experience in the reinsurance layers; the mean of this sample should give a good proxy for the expected loss in the layer. The details of the method follow.

1.4. Simulation Method - Step By Step.

When simulating loss experience one should be convinced that the severity and frequency loss distributions used in the simulations reflect reality to the greatest extent possible. To assure that, a good amount of meticulous work should be done.

First, historical individual losses should be trended and developed.

Second, loss frequency and loss severity distributions for the projected coverage period should be constructed based on adjusted loss data. Different types of loss severity curves (e.g., lognormal, Pareto, Weibull) fitted to the data should be examined. The Maximum Likelihood or the Least Squares methods may be used for curve fitting.

Next, a rigorous test of the goodness-of-fit needs to be performed. Percentile matching is probably the most important, but other tests (χ^2 - test, Kolmogorov - Smirnov) can also be performed.

Before starting the actual simulation process one needs to estimate the number of simulations required to achieve a certain precision depending on his goal. We recommend a relatively easy formula based on the application of the Central Limit Theorem.

When one is comfortable with the frequency and severity curves selected and the estimated number of simulations, one can run the simulation process.

The following sections explain in detail all the steps mentioned above.

2. Data Preparation.

2.1. Trending Individual Losses.

When trending the historical losses to the prospective experience period claim cost level it is important to select a proper severity trend factor. If underlying experience data is credible, it is better to select a trend factor using the account's own experience. One way of doing so involves the following steps:

- Develop the total incurred losses by year to ultimate;
- Develop the number of claims paid by year to ultimate;
- Calculate (untrended) average loss size by report year (divide the total ultimate loss by the ultimate number of claims);
- Fit an exponential regression to such averages.

This procedure is documented in Exhibit 6. The corresponding annual severity trend factor is 4.4%. Given the size of the account and regression characteristics we have decided to use this trend factor to bring individual losses to 7/1/97 level.

Alternatively, one can look at industrywide trend for Hospital Professional Liability from relevant sources. If necessary, one can adjust it for the difference in medical inflation for the state of the client's primary operations versus countrywide.

2.2. Developing Individual Losses.

Some individual claims in excess of $1,000,000 from the database illustrated in Exhibit 2 are still open at 12/31/95. The ultimate values of these claims might be different from their reserved values which we observed. Generally, it is not easy to adjust individual claim values for possible development using aggregate development data only. The major complication stems from the fact that aggregate loss development is driven by two different forces - the appearance of new claims and the adjustment of values for already outstanding claims. Fortunately, for claims made coverage usually there are no new claims which appear after the first year, and all the development is attributable to the reserve adjustments for outstanding claims only. This makes it possible *for claims made coverage* to use aggregate loss development data to approximate the development of individual claims. A procedure similar to the one described below can be used to develop individual claims for *occurrence coverage*; however, more information would be necessary.

The following technique could be used to develop individual losses which are open at 12/31/95 at its n^{th} evaluation (n=1 for claims reported in 1995, n=2 for claims reported in 1994, etc.):

- For each report year and fixed n (n=1,2,...) create a development triangle for claims <u>open at n^{th} evaluation only</u>. This can be done by subtracting column n of Exhibit 3-2 (paid losses at n^{th} evaluation) from columns n and subsequent of Exhibit 3-1 (reported losses at n^{th} evaluation and subsequent);
- Select appropriate loss development factors;

- Apply selected n-to-ultimate development factor to open claims outstanding at n^{th} evaluation.

For claims that were reported in 1992 ($n = 4$) this procedure is illustrated in Exhibit 7; the corresponding factor to be applied to report year 1992 claims open at 12/31/95 is 1.075. Please note that no loss development adjustment is applied to closed claims.

Alternatively, one can fit a series of curves to claim values at 1st, 2nd, and subsequent evaluations, and investigate the movement of the parameters. This methodology is consistent with one currently used by ISO (Pareto soup).

3. Selection of Frequency and Severity Distributions.

To calculate the expected losses in both reinsurance layers (see Exhibit 1) we need to project the number of claims in excess of $3,000,000, and the claim severity for such claims. Because AHU retains the first $3,000,000 of each and every claim, we should concentrate on the portion of claims in excess of this amount.

3.1. Selection of Number of Claims Distribution.

For the Excess Claim (in excess of $3,000,000) Frequency distribution we use the Negative Binomial. This discrete distribution has been utilized extensively in actuarial work to represent the number of insurance claims. Since its variance is greater than its mean, the Negative Binomial distribution is especially useful in situations where the potential claim count can be subject to significant variability. As Exhibit 5 Column (5) illustrates, this is the case in our example.

To estimate parameters for the Negative Binomial distribution we start with the estimate of expected number of claims in excess of $3,000,000. Exhibit 5 summarizes our approach.

First, we select the total claim frequency based on the historical exposure information and our estimates of ultimate number of paid claims; this selected number is **0.40** claims per one Full Time Equivalent (FTE) of exposure and is shown at the bottom of column (4). Second, we select the probability that the paid claim exceeds $3 Mil; our selection of **1.50%** is shown at the bottom of column (6). Based on these two numbers and the estimation of **840** FTE exposure for year 1997 provided by AHU, we expect **5.00** claims in excess of $3,000,000 for the coming year.

In order to estimate both parameters of the Negative Binomial, we need to estimate the variance of the claim count distribution. One possible approach is to look at the sample of historical claims in excess of $3,000,000 *at a 1997 exposure level* and estimate the second moment of that distribution. This approach is documented in Exhibit 8; the estimated variance-to-mean ratio is 4.46.

The result of 4.46 would be appropriate to use had we estimated it from an *observed* statistical sample. However, since we manipulated the data (trending, loss development, etc.), there was a parameter risk involved. As a result, the actual variability of the number of excess claims from the estimated expectation may have been larger than predicted in Exhibit 8. Meyers [4] addressed this problem. He suggested considering the *mean* of the Number of Claims distribution to be a random variable. The principal effect of this assumption is to increase the potential variability of the number of claims distribution around its expected value. To attain the same effect, while avoiding unnecessary complications, one can judgementally increase the indicated variance-to-mean ratio.

Based on our evaluation of possible errors in the estimation procedure used to price medical malpractice accounts, we have judgementally chosen to increase the variance-to-mean ratio to 6.0.

In translating the results of our estimates of mean and variance-to-mean ratio to standard parameters (p,r) of the Negative Binomial distribution (see, for example, [3], p. 52), we have $p = 0.167$; $r = 1$.

3.2. Selection of Severity Distribution.

To select a loss severity distribution we apply the maximum likelihood method to fit a curve to individual claim data. Some caution is necessary in dealing with this particular data. The problem is that we do not have the complete set of historical information but only claims whose (untrended and undeveloped) values exceed \$1,000,000. This means that for different years we only have information about the incurred claims which exceed some threshold (equal to the trended and developed value of \$1,000,000). For example, for report year 1983, we only have information about claims whose values in 1997 dollars are greater than \$1,000,000 x 1.827 x 1.000 = 1,827,000 (see Exhibit 2). In this case our likelihood function can be written in the form:

$$L = \prod_i f(x_i,\Lambda) / [1 - F(t_i,\Lambda)] \qquad (3.2.1)$$

Here Λ is the set of parameters describing a member of particular family of distribution functions (for example, for the lognormal distribution, Λ consists of the two standard parameters, μ and σ), $f(x_i,\Lambda)$ - pdf of loss severity distribution given the set of parameters Λ, x_i - the value of (trended and developed) claim i, $F(t_i,\Lambda)$ - distribution function, t_i - corresponding threshold value (1,827,000 for 1983 claims, etc.). The maximum likelihood estimators are the set of parameters Λ_0 that maximizes the function (3.2.1).

It is recommended to try different types of loss distribution to fit the data and select the one that has the best fit. Also, one can fit the curve to the portion of the data in excess of different retention points, such as \$2 Mil, \$2.5 Mil, etc.; this approach is consistent with one suggested by Finger [1]. The next section describes our approach in comparing different distributions. Exhibit 9 contains the list of distribution functions fitted to different portions of the data we used in pricing the AHU account.

3.3. Goodness-of-Fit Tests for Severity Distributions.

To select which distribution to use one can use the percentile matching test. The idea is to compare the theoretical excess probabilities for the fitted loss distributions with the empirical excess probabilities. This approach is illustrated in Exhibit 10. Comparing the excess probabilities for five fitted curves with empirical data, we have selected the distribution **Lognormal-2** as having the best fit; this lognormal distribution was fitted to individual claims greater than $2,000,000 (see Exhibit 9). Please note that only excess *conditional* probabilities should be considered; it is not that important how good the fit is for claim values below the retention of $3,000,000.

Finally, one can perform the χ^2 - test to assure a good fit to empirical data for the selected distribution (see Hogg and Klugman [3], p. 103 for the description of the χ^2 - test). For **Lognormal-2** this test is documented in Exhibit 11; we tested the goodness-of-fit on the interval in excess of $2,000,000. The test statistic value of 3.776 is smaller than 20% critical value of 9.803 for χ^2 - distribution with 7 degrees of freedom. This indicates an acceptable fit.

4. Estimation of the Number of Simulations Required.

Before starting the simulation process one should approximate the number of simulations to perform in order to achieve the intended goal. Different people may select different goals depending on their pricing philosophy. While we concentrated on the estimation of the expected reinsurer's loss cost only (the first moment of the aggregate loss distribution for both excess layers), one may want more information. For example, one may wish to price the account based on its expected variability (e.g., to select a profit load as a function of the variance of expected loss cost), or based on established expected deficit standards. Utilizing such approaches, one would need to perform enough simulations to approximate higher moments, or even percentiles of the aggregate loss distribution, with some reasonable degree of accuracy. The number of simulations required to achieve that is much larger than for an estimation of the first moment only. However, we focused on the simulation procedure and not on sophisticated pricing techniques. Thus we selected the number of simulations necessary to estimate the expected RLC with an acceptable degree of precision.

To describe our approach we first need to define some terms and values. "One simulation" is equivalent to the aggregate loss experience for a one year period in both reinsurance layers. Exhibit 12 shows the results of one simulation. First, we generate a random number n for claims in excess of $3,000,000; this number is taken from the Negative Binomial distribution as specified in section 3.1. Secondly, we generate n claim values; all these values are taken from **Lognormal-2** distribution truncated at 3,000,000, as specified in section 3.2. Next, each claim value is apportioned to two reinsurance layers according to the terms described in section 1.1. Finally, the aggregate loss for each of reinsurance layers is calculated by adding the appropriate portions of n individual claim values.

We repeat N independent simulations resulting in samples of size N for the annual aggregate loss in both reinsurance layers; then we use the sample mean $\bar{X}$ as an estimate of the expected reinsurer's loss costs. If N is large enough, we can use the Central Limit Theorem to estimate the difference between $\bar{X}$ and the true expectation μ of the aggregate loss cost. Namely, according to the Central Limit Theorem, even though the aggregate loss distribution is skewed and not normal, for large N the distribution

$$(\bar{X} - \mu) / (\sigma / \sqrt{N}),$$

being derived from the sum of N independent aggregate loss distributions, converges to the standard normal distribution (σ is the standard deviation of the aggregate loss distribution). Therefore, at 95% confidence level,

$$|\bar{X} - \mu| \leq 1.96 * \sigma / \sqrt{N} \qquad (4.1.1)$$

Now, if we select T to be an acceptable tolerance for the difference $|\bar{X} - \mu|$, we can estimate the number N of simulations required to assure that this difference is less than T at the 95% confidence level:

$$N \geq (1.96 * \sigma / T)^2 \qquad (4.1.2)$$

For the practical use of the formula (4.1.2) σ and T need to be approximated.

When pricing a reinsurance contract, an actuary often knows a proposed price or existing terms for it. This knowledge can help to select T (5% of existing price, for example). Even if the actuary does not know an amount of premium anticipated for an account, he or she can easily approximate such an amount by running a relatively small number of simulations (say, 1000). The mean of the resulting sample could be used to reasonably select T. The same approach could be recommended to approximate the value of σ.

For our AHU example after 1000 simulation we have: for the 1-st Excess Layer $\bar{X}$ = \$4,532,000, σ = \$3,510,000; for the 2-nd Excess Layer $\bar{X}$ = \$1,788,000, σ = \$3,403,000. Selecting T = \$50,000 and approximating σ = \$3,500,000 we have by formula (4.1.2):

$$N \geq (1.96 * 3{,}500{,}000 / 50{,}000)^2 = 18{,}824.$$

Therefore, at a 95% confidence level, performing 20,000 simulations for an annual aggregate loss should assure that the sample mean differs from the true expected annual aggregate loss by less than \$50,000 (for either reinsurance layer).

Alternatively, one can monitor the convergence of the simulation process and stop it when the change in the sample mean (and, possibly, higher moments) in between simulations becomes reasonably small.

The third approach [1] is to use an upper bound for σ. For example, it can be proven that the standard deviation σ of any distribution whose values are concentrated on the finite segment [0;A] is less than A/2. For the 2-nd Excess layer, using T = $50,000 and A = $12,000,000, formula (4.1.2) implies that

$$N \geq (1.96 * 12{,}000{,}000 / 2 / 50{,}000)^2 = 55{,}320.$$

The indicated number of simulations for this method is usually significantly higher than it is really necessary to obtain a required tolerance level.

5. Simulation Results for the Excess Loss Distribution.

The simulation process has been described in Section 4; the results for one simulation are shown in Exhibit 12. Different software packages could be utilized for simulation. We use a package called *@RISK*; this one is designed to be used with standard spreadsheets, like *Lotus 1-2-3* or *Excel*. Exhibit 13 shows the settings for the simulation procedure; the number of simulations to run (20,000) has been specified in Section 4.

The simulation results are shown separately for the 1-st and the 2-nd reinsurance layers in Exhibit 14. Please note that the aggregate loss distributions for both reinsurance layers, although shown in detail (the four first moments and percentiles), should be used with great caution. The number of simulations we went through has been selected to achieve our goal, which is to obtain a reasonably accurate estimator for the expected aggregate loss. There is no warranty that the percentile statistics shown are accurate estimates of the true percentiles of the aggregate loss distribution; to achieve that, it might be necessary to run more simulations.

Using formula (4.1.1) we can refurbish our estimate of $|\bar{X} - \mu|$. Namely, using estimated results for the 1-st Excess layer, we can conclude that

$$|4{,}481{,}577 - \mu_1| \leq 1.96 * 3{,}498{,}020 / \sqrt{20{,}000} = 48{,}480,$$

where μ_1 is the expected annual aggregate loss for the 1-st Excess layer. For the 2-nd Excess layer the same approach leads to estimate

$$|1{,}779{,}283 - \mu_2| \leq 1.96 * 3{,}433{,}117 / \sqrt{20{,}000} = 47{,}580,$$

where μ_2 is the expected annual aggregate loss for the 2-nd Excess layer.

[1] The idea of this method has been suggested to the author by Marc Shamula.

To insure the quality of the results produced by the simulation method one could compare them to the results obtained by using another known technique if it is possible. To do such a comparison we estimated the annual aggregate loss for the 1-st Excess layer using the Panjer method. Using the Number of Claims and Severity distributions specified in Section 4, and the unit length of $25,000 for discretization, we obtained the estimate of $4,482,940. The difference of this result from the one produced by simulation method is about 0.03%.

6. Pricing Recommendations.

The final step in the process is to convert the estimated loss cost to a recommended price for reinsurance coverage by using formula (1.3.1). We will not attempt to give a recipe on how to select corresponding factors. However, we will briefly discuss their relationship with the simulation pricing approach.

CR and BF are external variables suggested by a broker or client and often are not under the control of the reinsurer; we will not discuss them.

IXL reflects the reinsurer's expenses; it might be a separate load or it might be combined with the TER under the concept of "risk based capital". If a reinsurance company uses a separate load for IXL in its pricing formula, it is usually expressed as a function of the size of account (reinsurance premium net of commission and brokerage fees).

TER for the contract should, at least theoretically, reflect the level of risk that the reinsurer is taking by writing a particular contract. Usually the risk of the contract is measured by the potential variability of its loss experience. If a reinsurance company utilizes some unified approach to reflect risk in the pricing formula (e.g., use risk load proportional to the variance of the expected loss cost), the simulation method is an ideal provider of information. Exhibit 14 shows various characteristics of the expected aggregate loss distributions (higher moments, mode, and percentiles) one can use to measure the risk. However, as discussed earlier, one must make sure to run enough simulations to obtain reliable estimates for these characteristics.

DF is a function of the expected payout pattern for the account's losses and interest rates. While some information can be extracted from the historical loss emergence pattern for the account (see Exhibit 3.2), the estimated payout pattern may not be a good predictor for the high attaching reinsurance layers. For example, one can anticipate a significant delay in payments for the 2-nd Excess layer, because the payments in this layer would intensify considerably after the coverage of the 1-st Excess layer is exhausted. According to Exhibit 14, the probability that the coverage of the 1-st Excess layer will be depleted is about 25%. An alternative way to deal with this problem is to simulate the payment date of each excess loss in addition to its value. Then calculate the present value of such payments in 1997 dollars while applying the corresponding discount factor to the simulated claim value. Using this approach one can omit the DF multiplier in formula (1.3.1) because the produced RLC is already discounted.

Exhibit 15 displays the recommended reinsurance premiums derived by application of formula (1.3.1) for both reinsurance layers of coverage. The loading factors used in this exhibit are for illustrative purposes only, and are not actual factors used for pricing.

7. Final Remarks and Conclusions.

This paper illustrates the application of a simulation method in excess reinsurance pricing. Our considerations were intentionally limited by the data described in Section 1.2; having more detailed information one can achieve much more accurate results. For example, getting the individual development information for large claims, one can use it to estimate the development factor more accurately. There are countless variations of the types of data which reinsurance actuaries can find available for a pricing analysis. We have not even tried to reflect these variations. Rather, we attempted to show the application scheme of the simulation method in reinsurance pricing emphasizing its critical points.

We have considered the simulation approach in computing aggregate loss distributions. As we demonstrated, the scope of the applicability of the simulation method is more broad than for other aggregate loss distribution techniques. It combines easy programming with highly accurate results. Although it currently requires a substantial amount of computer resources, this will become less of an issue with further advancements of computer technology. With the development of efficient simulation software and increasing speed of modern computers, simulation methods promise to become one of the leading tools in actuarial practice.

8. Acknowledgments.

I would like to express my deep appreciation to Marc Shamula for his outstanding effort in reviewing the drafts of this paper. He suggested numerous improvements and provided the author with valuable insights.

I am also very thankful to Gary Patrik for his continuous support and encouragement.

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Exhibit 1

Pricing Example:
Alpha Hospital Union

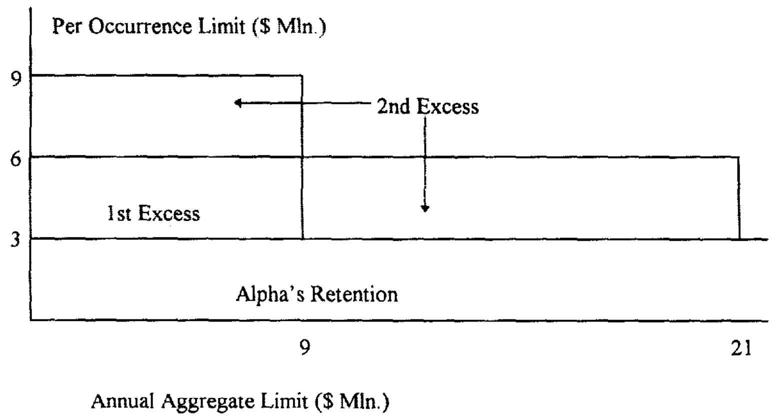


Exhibit 2

ALPHA HOSPITAL UNION

Incurred Cases Over $1,000,000 @ 12/31/95 - Extract

Trended to 07/01/97 Trend = 4.4%

Case #	Total Incurred Loss	Trend Factor	Trended Loss	Develop-ment Factor	Trended & Developed Loss	1-st Excess Layer Loss	2-nd Excess Layer Loss
Report Year 1983							
C83-0988	7,454,310	1.827	13,621,170	1.000	13,621,170	3,000,000	3,000,000
C83-0518	5,854,006	1.827	10,696,954	1.000	10,696,954	3,000,000	3,000,000
C83-0832	4,800,106	1.827	8,771,177	1.000	8,771,177	3,000,000	2,771,177
C83-0021	3,228,345	1.827	5,899,115	1.000	5,899,115		2,899,115
C83-0656	3,157,378	1.827	5,769,438	1.000	5,769,438		329,708
C83-0305	2,093,321	1.827	3,825,099	1.000	3,825,099		0
C83-0441	2,131,311	1.827	3,894,519	1.000	3,894,519		0
C83-0209	2,106,704	1.827	3,849,554	1.000	3,849,554		0
C83-0767	1,911,213	1.827	3,492,337	1.000	3,492,337		0
C83-0008	1,641,695	1.827	2,999,849	1.000	2,999,849		
C83-0390	1,500,234	1.827	2,741,360	1.000	2,741,360		
C83-0962	1,300,452	1.827	2,376,300	1.000	2,376,300		
C83-0481	1,198,792	1.827	2,190,538	1.000	2,190,538		
C83-0190	1,187,056	1.827	2,169,094	1.000	2,169,094		
C83-0271	1,137,370	1.827	2,078,303	1.000	2,078,303		
C83-0450	1,141,698	1.827	2,086,210	1.000	2,086,210		
C83-0393	1,103,989	1.827	2,017,306	1.000	2,017,306		
C83-0468	1,095,040	1.827	2,000,954	1.000	2,000,954		
Total Report Year 1983						**9,000,000**	**12,000,000**
------	-------	------	-------	------	--------		
------	-------	------	-------	------	--------		
Report Year 1992							
C92-0921	3,720,867	1.240	4,614,734	1.000	4,614,734	1,614,734	
C92-0691	3,032,036	1.240	3,760,424	1.075	4,042,456	1,042,456	
C92-0423	2,877,629	1.240	3,568,924	1.075	3,836,594	836,594	
C92-0802	2,376,103	1.240	2,946,916	1.075	3,167,934	167,934	
C92-0331	2,309,169	1.240	2,863,902	1.000	2,863,902		
C92-0669	2,240,742	1.240	2,779,038	1.075	2,987,465		
C92-0473	2,281,805	1.240	2,829,964	1.000	2,829,964		
C92-0698	2,217,662	1.240	2,750,413	1.075	2,956,694		
C92-0721	2,134,174	1.240	2,646,869	1.075	2,845,384		
C92-0205	2,074,380	1.240	2,572,710	1.075	2,765,663		
C92-0075	1,673,136	1.240	2,075,074	1.075	2,230,705		
Total Report Year 1992						**3,661,718**	**0**

Exhibit 3-1

ALPHA HOSPITAL UNION

Reported Losses ($ 000)

Report Year	Evaluation Year 1	2	3	4	5	6	7	8	9	10	11	Ult.
1983				88,420	91,350	93,593	93,723	93,277	93,914	93,888	93,848	93,848
1984			66,200	69,814	70,282	72,664	72,591	72,397	71,077	71,213	71,575	71,575
1985		73,094	77,151	80,754	82,720	83,984	84,278	84,452	85,566	85,405	85,470	85,470
1986	43,357	48,147	51,946	54,388	55,248	56,209	57,079	57,239	56,747	56,859		56,859
1987	60,455	66,167	70,353	74,966	75,122	76,016	76,213	76,032	76,202			76,202
1988	62,839	65,756	79,543	73,818	76,470	76,333	75,521	75,700				75,700
1989	80,524	85,021	90,377	93,878	98,685	100,593	100,794					100,794
1990	60,507	66,776	71,690	73,010	76,054	77,195						77,349
1991	62,216	66,810	69,397	73,249	75,525							76,660
1992	57,860	63,610	69,004	71,596								75,288
1993	59,360	65,386	70,250									76,458

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983				1.033	1.025	1.001	0.995	1.007	1.000	1.000	1.000
1984			1.055	1.007	1.034	0.999	0.997	0.982	1.002	1.005	1.000
1985		1.056	1.047	1.024	1.015	1.004	1.002	1.013	0.998	1.001	
1986	1.110	1.079	1.047	1.016	1.017	1.015	1.003	0.991	1.002		
1987	1.094	1.063	1.066	1.002	1.012	1.003	0.998	1.002			
1988	1.046	1.210	0.928	1.036	0.998	0.989	1.002				
1989	1.056	1.063	1.039	1.051	1.019	1.002					
1990	1.104	1.074	1.018	1.042	1.015						
1991	1.074	1.039	1.056	1.031							
1992	1.099	1.085	1.038								
1993	1.102	1.074									
Last 3	1.092	1.066	1.037	1.041	1.011	0.998	1.001	1.002	1.000	1.002	
Last 5	1.087	1.067	1.016	1.032	1.012	1.003	1.000	0.999	N/A	N/A	
Best 3 of 5	1.092	1.070	1.032	1.036	1.015	1.003	1.001	1.000	N/A	N/A	
Selected	1.092	1.070	1.035	1.036	1.013	1.002	1.000	1.000	1.000	1.000	1.000
Cumulative	1.272	1.165	1.088	1.052	1.015	1.002	1.000	1.000	1.000	1.000	1.000
Percentage Reported	78.6%	7.2%	6.0%	3.2%	3.4%	1.3%	0.2%	0.0%	0.0%	0.0%	0.0%

Exhibit 3-2

ALPHA HOSPITAL UNION

Paid Losses

Report Year	Evaluation Year 1	2	3	4	5	6	7	8	9	10	11	12
1983				30,563	37,828	46,321	52,139	60,068	68,701	75,438	81,548	86,053
1984			10,393	17,962	24,807	30,667	34,428	46,015	50,036	56,452	60,326	65,303
1985		2,234	17,436	25,322	42,617	47,263	54,168	57,949	66,211	70,764	75,231	
1986	211	802	11,621	12,137	18,960	27,538	33,747	37,267	41,887	45,711		
1987	166	830	13,212	19,768	27,687	36,160	41,669	50,569				
1988	390	2,269	19,637	24,774	34,030	45,975	51,405					
1989	726	2,448	24,402	30,784	39,690	62,815						
1990	507	1,818	24,083	27,515	33,241							
1991	381	1,408	19,560	25,554								
1992	466	1,212	21,501									
1993	430	1,510										

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.	
1983				1.238	1.225	1.126	1.152	1.144	1.098	1.081	1.151	
1984			1.728	1.381	1.236	1.123	1.337	1.087	1.128	1.069	1.186	
1985		7.805	1.452	1.683	1.109	1.146	1.070	1.143	1.069	1.063	1.136	
1986	3.801	14.490	1.044	1.562	1.452	1.225	1.104	1.124	1.091			
1987	5.000	15.918	1.496	1.401	1.306	1.152	1.214	0.000				
1988	5.818	8.654	1.262	1.374	1.351	1.118	0.000					
1989	3.372	9.969	1.262	1.289	1.583	0.000						
1990	3.586	13.247	1.143	1.208	0.000							
1991	3.696	13.892	1.306	0.000								
1992	2.601	17.740	0.000									
1993	3.512	0.000										
Last 3	3.269	14.960	1.237	1.290	1.413	1.165	1.129	1.118	1.096	1.071		
Last 5	3.353	12.701	1.294	1.367	1.360	1.153	1.175	1.124	N/A	N/A		
Best 3 of 5	3.490	12.369	1.277	1.355	1.370	1.140	1.157	1.133	N/A	N/A		
Selected	3.500	12.500	1.280	1.350	1.370	1.150	1.160	1.120	1.100	1.070	1.170	
Cumulative	213.098	60.885	4.871	3.805	2.819	2.057	1.789	1.542	1.377	1.252	1.170	
Percentage Paid During Prior Period	0.5%	1.2%	18.9%	5.7%	9.2%	13.1%	7.3%	8.9%	7.8%	7.3%	5.6%	After 11 14.5%

Exhibit 4

ALPHA HOSPITAL UNION

Paid Claim Count

Report Year	1	2	3	4	5	6	7	8	9	10	11	Ult.
1983					354	401	433	466	488	503	511	542
1984				212	265	311	341	379	403	417	443	470
1985			116	176	230	375	313	347	385	390	397	421
1986		51	85	116	145	182	222	260	274	285		307
1987	32	67	105	164	217	251	281	306	324			359
1988	28	59	110	162	208	249	269	296				348
1989	46	101	163	227	269	312	343					443
1990	26	79	125	166	195	234						333
1991	20	63	104	142	175							298
1992	25	56	111	156								330
1993	29	60	102									304

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983					1.133	1.080	1.076	1.047	1.031	1.016	1.000
1984				1.250	1.174	1.096	1.111	1.063	1.035	1.062	1.060
1985			1.517	1.307	1.630	0.835	1.109	1.110	1.013	1.018	
1986		*1.667*	*1.365*	*1.250*	*1.255*	*1.220*	1.171	1.054	1.040		
1987	2.094	1.567	1.562	1.323	1.157	1.120	1.089	1.059			
1988	2.107	1.864	1.473	1.284	1.197	1.080	1.100				
1989	2.196	1.614	1.393	1.185	1.160	1.099					
1990	3.038	1.582	1.328	1.175	1.200						
1991	3.150	1.651	1.365	1.232							
1992	2.240	1.982	1.405								
1993	2.069	1.700									
Last 3	2.486	1.778	1.366	1.197	1.186	1.100	1.120	1.074	1.026	N/A	
Last 5	2.539	1.706	1.393	1.240	1.194	1.071	1.116	1.067	N/A	N/A	
Best 3 of 5	2.491	1.655	1.388	1.234	1.186	1.100	1.107	1.059	N/A	N/A	
Selected	2.500	1.710	1.410	1.240	1.200	1.100	1.100	1.060	1.030	1.015	1.060
Cumulative	12.748	5.099	2.982	2.115	1.706	1.421	1.292	1.175	1.108	1.076	1.060
Percentage Paid During Prior Period	7.8%	11.8%	13.9%	13.7%	11.3%	11.7%	7.0%	7.7%	5.1%	2.7%	7.1%

Exhibit 5

ALPHA HOSPITAL UNION

Statistical Data

Claim Trend = 4.4%

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Report Year	FTE Exposure	Ultimate # of Claims Paid	Claim Frequency	Ult. Number of Trended and Developed Claims > $3M	Probability {Claim > $3M}	Trended and Devel. Loss in 2-nd Excess Layer @12/95
1983		542		9	1.66%	12,000,000
1984		470		7	1.49%	0
1985	762.14	421	0.552	13	3.09%	4,082,847
1986	798.19	307	0.384	7	2.28%	0
1987	773.70	359	0.464	5	1.39%	0
1988	834.66	348	0.417	1	0.29%	0
1989	861.21	443	0.515	6	1.35%	3,914,229
1990	836.91	333	0.397	3	0.90%	0
1991	859.55	298	0.347	0	0.00%	0
1992	834.09	330	0.396	4	1.21%	0
1993	813.45	304	0.374	0	0.00%	0
	All Year Average		0.427	1983-93 Avg.	1.24%	
				1983-89 Avg.	1.65%	
Selected			**0.40**		**1.50%**	
1997-est.	840.00	**333**		**5.00**		

Notes. (2) is Full Time Equivalents for AHU
(3) is from Exhibit 4
(4) = (3) / (2)
(5) and (7) are from Exhibit 2
(6) = (5) / (3)

Exhibit 6

ALPHA HOSPITAL UNION

Trend Estimation

(1) Report Year	(2) Ultimate # of Claims	(3) Ultimate Loss	(4) Ultimate Average Claim Size	(5) Log	(6) Predicted Average Claim Size
1983	542	93,848	173.26	5.1548	169.94
1984	470	71,575	152.42	5.0267	177.36
1985	421	85,470	203.10	5.3137	185.10
1986	307	56,859	185.43	5.2227	193.18
1987	359	76,202	212.23	5.3577	201.62
1988	348	75,700	217.72	5.3832	210.42
1989	443	100,794	227.42	5.4268	219.61
1990	333	77,349	232.56	5.4492	229.20
1991	298	76,660	256.83	5.5484	239.20
1992	330	75,288	228.19	5.4302	249.65
1993	304	76,458	251.36	5.5269	260.55

Regression Output:

(7) Constant	-79.6070
Std Err of Y Est	0.0769
R Squared	0.7904
No. of Observations	11
Degrees of Freedom	9
(8) X Coefficient(s)	0.0427
Std Err of Coef.	0.0073
(9) Annual Trend Indicate	**4.4%**

Notes. (2) is from Exhibit 4
(3) is from Exhibit 3-1
(4) = (3) / (2)
(5) = ln{ (4) }
(6) = exp{ (7) + (1) * (8) }
(9) = exp{ (8) } - 1

Exhibit 7

ALPHA HOSPITAL UNION

The Development of Losses That Were Open At Fourth Evaluation ($ 000)

Report Year	Evaluation Year 4	5	6	7	8	9	10	11
1983	57,857	60,787	63,030	63,160	62,714	63,351	63,325	63,285
1984	51,852	52,320	54,702	54,629	54,435	53,115	53,251	53,613
1985	55,432	57,398	58,662	58,956	59,130	60,244	60,083	60,148
1986	42,251	43,111	44,072	44,942	45,102	44,610	44,722	
1987	55,198	55,354	56,248	56,445	56,264	56,434		
1988	49,044	51,696	51,559	50,747	50,926			
1989	63,094	67,901	69,809	70,010				
1990	45,495	48,539	49,680					

	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983	1.051	1.037	1.002	0.993	1.010	1.000	0.999	1.000
1984	1.009	1.046	0.999	0.996	0.976	1.003	1.007	1.000
1985	1.035	1.022	1.005	1.003	1.019	0.997	1.001	
1986	1.020	1.022	1.020	1.004	0.989	1.003		
1987	1.003	1.016	1.004	0.997	1.003			
1988	1.054	0.997	0.984	1.004				
1989	1.076	1.028	1.003					
1990	1.067	1.024						
Last 3	1.066	1.016	0.997	1.001	1.004	1.001	1.004	
Last 5	1.044	1.017	1.003	1.001	0.999	N/A	N/A	
Best 3 of 5	1.047	1.021	1.004	1.001	0.996	N/A	N/A	
Selected	1.050	1.020	1.003	1.001	1.000	1.000	1.000	1.000
Cumulative	**1.075**	1.024	1.004	1.001	1.000	1.000	1.000	1.000

Exhibit 8

ALPHA HOSPITAL UNION

Number of Claims Distribution Analysis

(1) Report Year	(2) FTE Exposure	(3) Ult. Number of Trended and Developed Claims > $3M	(4) Number of Claims > $3M @ 1997 Exposure
1985	762.14	13	14.328
1986	798.19	7	7.367
1987	773.70	5	5.428
1988	834.66	1	1.006
1989	861.21	6	5.852
1990	836.91	3	3.011
1991	859.55	0	0.000
1992	834.09	4	4.028
1993	813.45	0	0.000
1997-est.	840.00	6.00	

(5)	All Year Average	4.558
(6)	All Year Variance	**20.327**
(7)	Variance-to-Mean Ratio	**4.460**

Notes. (2) is Full Time Equivalents for AHU
(3) is form Exhibit 5
(4) = (3) * 840 / (2), where 840 is estimated FTE exposure for 1997
(5) and (6) are based on column (4)

Exhibit 9

ALPHA HOSPITAL UNION

Severity Curve Fitting Results

Name of Distribution	Lognormal	Pareto
Type of Distribution	Lognormal	Pareto
Data Fitted To	All Claims	All Claims
Parameter Estimators	Mu = 13.580 Sigma = 0.861	B = 4,978,593 Q = 6.313

Name of Distribution	Lognormal - 2	Pareto - 2
Type of Distribution	Lognormal	Pareto
Data Fitted To	Claims in Excess of $ 2 Mil	Claims in Excess of $ 2 Mil
Parameter Estimators	Mu = 14.979 Sigma = 0.371	B = 4,625,321 Q = 6.524

Name of Distribution	Lognormal - 2.5
Type of Distribution	Lognormal
Data Fitted To	Claims in Excess of $ 2.5 Mil
Parameter Estimators	Mu = 15.059 Sigma = 0.356

Exhibit 10

ALPHA HOSPITAL UNION

Severity Curve Fitting Analysis

x	**Empirical**	Pareto	Lognormal	Pareto-2	**Lognorm-2**	Lognorm-2.5
			Prob{X>x}			
2,000,000		11.86%	14.04%	9.59%	**89.76%**	93.92%
2,500,000		7.66%	9.05%	5.97%	**74.74%**	82.13%
3,000,000		5.09%	6.06%	3.83%	**56.93%**	65.83%
3,500,000		3.47%	4.19%	2.53%	**40.48%**	48.98%
4,000,000		2.42%	2.98%	1.72%	**27.39%**	34.42%
4,500,000		1.72%	2.17%	1.19%	**17.91%**	23.21%
5,000,000		1.24%	1.61%	0.84%	**11.45%**	15.19%
6,000,000		0.68%	0.93%	0.44%	**4.51%**	6.17%
7,000,000		0.39%	0.56%	0.24%	**1.74%**	2.42%
			Prob{X>x|X>2,000,000}			
2,000,000	**100.00%**	100.00%	100.00%	100.00%	**100.00%**	100.00%
2,500,000	**79.25%**	64.61%	64.46%	62.21%	**83.27%**	87.45%
3,000,000	**63.21%**	42.94%	43.19%	39.97%	**63.43%**	70.10%
3,500,000	**45.29%**	29.25%	29.88%	26.41%	**45.09%**	52.15%
4,000,000	**27.36%**	20.37%	21.23%	17.89%	**30.51%**	36.65%
4,500,000	**16.04%**	14.47%	15.44%	12.38%	**19.95%**	24.71%
5,000,000	**11.32%**	10.46%	11.44%	8.74%	**12.76%**	16.17%
6,000,000	**4.72%**	5.72%	6.60%	4.59%	**5.02%**	6.57%
7,000,000	**1.96%**	3.30%	4.02%	2.55%	**1.94%**	2.57%
			Prob{X>x|X>2,500,000}			
2,500,000	**100.00%**	100.00%	100.00%	100.00%	**100.00%**	100.00%
3,000,000	**79.77%**	66.46%	67.00%	64.25%	**76.17%**	80.15%
3,500,000	**57.15%**	45.28%	46.35%	42.45%	**54.16%**	59.64%
4,000,000	**34.53%**	31.54%	32.94%	28.75%	**36.64%**	41.91%
4,500,000	**20.24%**	22.40%	23.95%	19.91%	**23.96%**	28.25%
5,000,000	**14.28%**	16.19%	17.75%	14.06%	**15.32%**	18.49%
6,000,000	**5.95%**	8.86%	10.25%	7.38%	**6.03%**	7.51%
7,000,000	**2.48%**	5.11%	6.24%	4.10%	**2.33%**	2.94%

Exhibit 11

ALPHA HOSPITAL UNION

Goodness-of-Fit Test for Lognormal-2 Distribution

Range		Number of Claims		χ^2
From	To	Empirical	Lognorm-2	
2,000,000	2,500,000	22.00	17.74	1.024
2,500,000	3,000,000	17.00	21.03	0.774
3,000,000	3,500,000	19.00	19.43	0.010
3,500,000	4,000,000	19.00	15.46	0.812
4,000,000	4,500,000	12.00	11.19	0.058
4,500,000	5,000,000	5.00	7.63	0.904
5,000,000	6,000,000	7.00	8.20	0.176
6,000,000	Infinity	5.00	5.32	0.020
		106	106	**3.776**

Degrees of Freedom	7
χ^2 (7) 10% Critical Value	12.017
χ^2 (7) 20% Critical Value	9.803

Stochastic Simulation Worksheet

Number of Claims Distribution: Negative Binomial

Parameters:

p	0.167
r	1.000

Number of Claims 14

Severity Distribution: Lognormal

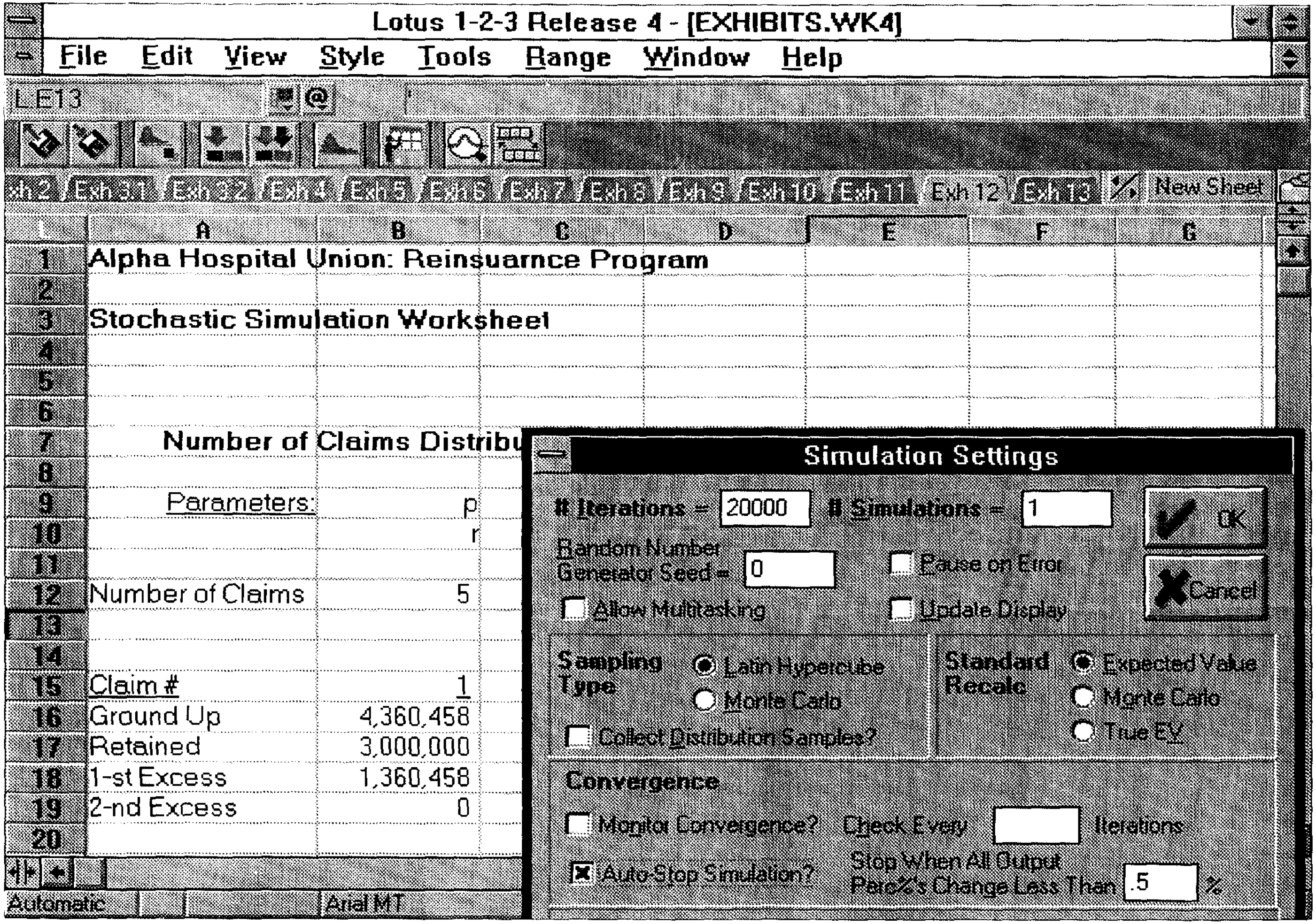
Parameters:

Mu	15.059	Mu-1	3,694,545
Sigma	0.356	Sigma-1	1,358,052

Claim #	1	2	3	4	5	6	7	8	9	10
Ground Up	3,220,292	7,365,376	3,324,321	4,977,541	3,079,357	6,009,490	3,117,650	4,010,786	4,590,674	4,480,066
Retained	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000
1-st Excess	220,292	3,000,000	324,321	1,977,541	79,357	3,000,000	117,650	280,839	0	0
2-nd Excess	0	1,365,376	0	0	0	9,490	0	729,947	1,590,674	1,480,066

Claim #	11	12	13	14	15	16	17	18	19	20
Ground Up	3,674,992	3,346,734	5,064,726	3,929,901	0	0	0	0	0	0
Retained	3,000,000	3,000,000	3,000,000	3,000,000	0	0	0	0	0	0
1-st Excess	0	0	0	0	0	0	0	0	0	0
2-nd Excess	674,992	346,734	2,064,726	929,901	0	0	0	0	0	0

1-st Excess	9,000,000
2-nd Excess	9,191,906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File Edit View Style Tools Range Window Help
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Exh 3.1 Exh 3.2 Exh 4 Exh 5 Exh 6 Exh 7 Exh 8 Exh 9 Exh 10 Exh 11 Exh 12 Exh 13 New Sheet
A B C D E F G
Alpha Hospital Union: Reinsuarnce Program
Stochastic Simulation Worksheet
Number of Claims Distribu
Parameters: p
r
Number of Claims 5
Claim # 1
Ground Up 4,360,458
Retained 3,000,000
1-st Excess 1,360,458
2-nd Excess 0
Automatic
Arial MT
Simulation Settings
Iterations = 20000
Simulations = 1
OK
Random Number Generator Seed = 0
Pause on Error
Cancel
Allow Multitasking
Update Display
Sampling Type
Latin Hypercube
Monte Carlo
Collect Distribution Samples?
Standard Recalc
Expected Value
Monte Carlo
True EV
Convergence
Monitor Convergence?
Check Every
Iterations
Auto-Stop Simulation?
Stop When All Output Perc%'s Change Less Than .5 %

Exhibit 14

ALPHA HOSPITAL UNION

Simulation Statistics

Iterations = 20,000

Name	1-st Excess	2-nd Excess
Cell	L:B28	L:B30
Minimum =	0	0
Maximum =	9,000,000	12,000,000
Mean =	4,481,577	1,779,283
Std Deviation =	3,498,020	3,433,117
Variance =	1.224E+13	1.179E+13
Skewness =	0.092	2.017
Kurtosis =	1.444	5.803
Mode =	9,000,000	0
5% Perc =	0	0
10% Perc =	0	0
15% Perc =	0	0
20% Perc =	417,546	0
25% Perc =	1,029,013	0
30% Perc =	1,591,121	0
35% Perc =	2,168,108	0
40% Perc =	2,805,473	0
45% Perc =	3,334,980	0
50% Perc =	4,088,441	0
55% Perc =	4,837,891	0
60% Perc =	5,682,205	0
65% Perc =	6,615,973	269,680
70% Perc =	7,713,470	813,716
75% Perc =	9,000,000	1,671,010
80% Perc =	9,000,000	2,967,957
85% Perc =	9,000,000	4,741,905
90% Perc =	9,000,000	7,617,267
95% Perc =	9,000,000	12,000,000
Target #1 (Value)=	0	0
Target #1 (Perc%)=	16.67%	62.06%
Target #2 (Value)=	9,000,000	12,000,000
Target #2 (Perc%)=	74.91%	94.70%

Exhibit 15

ALPHA HOSPITAL UNION

Pricing Recommendations

		1-st Excess Layer	2-nd Exces Layer
(1)	ESTIMATED LOSS COST FOR THE LAYER	4,481,577	1,779,283
(2)	COMMISSION	0.00%	0.00%
(3)	BROKERAGE	5.00%	5.00%
(4)	IXL AS % OF RISK PREM	3.50%	5.00%
(5)	TER AS % OF PURE PREM	15.00%	25.00%
(6)	LOSS DISCOUNT FACTOR	0.750	0.550
(7)	RECOMMENDED REINSURANCE PREMIUM	**4,313,425**	**1,445,770**

An Application of Game Theory: Property Catastrophe Risk Load

by Donald F. Mango, FCAS

An Application of Game Theory: Property Catastrophe Risk Load[1]

Donald Mango, F.C.A.S.
Crum & Forster Insurance

Abstract

Two well-known methods for calculating risk load -- Marginal Surplus and Marginal Variance -- are applied to output from catastrophe modeling software. Risk loads for these "marginal methods" are calculated for sample new and renewal accounts. Differences between new and renewal pricing are examined. For new situations, both current methods allocate the full marginal impact of addition of a new account to that new account. For renewal situations, a new concept is introduced -- "renewal additivity". Neither marginal method is renewal additive. A new method is introduced, inspired by game theory, which splits the mutual covariance between any two accounts evenly between those accounts. The new method is extended and generalized to a proportional sharing of mutual covariance between any two accounts. Both new approaches are tested in new and renewal situations.

(1) Introduction

The calculation of risk load continues to be a topic of interest in the actuarial community -- see Bault [1] for a recent survey of well-known alternatives. One area where the CAS literature is somewhat scarce, and the need is great, is calculation of risk loads for property catastrophe insurance.

The new catastrophe modeling products produce modeled "occurrence size-of-loss distributions" for a series of simulated events. Using the occurrence size-of-loss distribution, one can easily calculate expected losses, loss variance and standard deviation. Two of the more well-known risk load methods from the CAS literature -- what I call "Marginal Surplus" (MS) from Kreps [3] and "Marginal Variance" (MV) from Meyers [6] -- use the marginal change in portfolio standard deviation (respectively variance) due to addition of a new account as a means to calculate the risk load for that new account. However, as we shall see, problems arise when we use these marginal methods in calculating the risk loads for the renewal of the accounts in a portfolio.

We apply the MV and MS methods to a simplified occurrence size-of-loss distribution, calculate risk loads both in assembling or building up a portfolio of risks, and in subsequently renewing that portfolio. Then we discuss the differences between build-up and renewal results.

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[1] *I would like to thank Eric Lemieux and Sean Ringsted for their support, editorial suggestions and review of early drafts. I would also like to thank Paul Kneuer for his thoughtful and insightful review which improved the paper.*

We then introduce a new concept to the theory of property catastrophe risk loads -- renewal additivity. However, the concept is not new to the field of game theory, where we will draw inspiration for a new approach.

We begin with a brief outline of the mechanics of catastrophe occurrence size-of-loss distributions, and the calculation of risk loads using the two marginal methods.

(2) The Catastrophe Occurrence Size-of-loss Distribution

For demonstration purposes throughout the paper, we will use a simplified version of an occurrence size-of-loss distribution. It captures the essence of typical catastrophe modeling software output, while keeping the examples understandable[2].

A series of modeled events denoted by identifier i are considered independent Poisson processes each with occurrence rate λ_i. To simplify the mathematics, following Meyers [6], we will employ the binomial approximation with probability of occurrence p_i [where $\lambda_i = -\ln(1 - p_i)$]. This is a satisfactory approximation for small λ_i [3].

For an individual account or portfolio of accounts, the model produces an expected loss for each event L_i. We will refer to a table containing the event identifiers i, the event probabilities p_i and modeled expected losses L_i as an "occurrence size-of-loss distribution."

From Meyers [6], the formulas for expected loss and variance are [Σ_i = sum over all events]:

$$E[L] = \Sigma_i \{ L_i * p_i \} \qquad [2.1]$$

$$Var[L] = \Sigma_i \{ L_i^2 * p_i * (1 - p_i) \} . \qquad [2.2]$$

The formula for covariance of an existing portfolio L (with losses L_i) and a new account n (with losses n_i) is :

$$Cov[L, n] = \Sigma_i \{ L_i * n_i * p_i * (1 - p_i) \} \qquad [2.3]$$

The total variance of the combined portfolio [L + n] is then

2 In particular, we will only be considering single event or occurrence size-of-loss distributions. Many models also produce multi-event or aggregate loss distributions. Occurrence size-of-loss distributions only reflect the *largest* event which occurs in a given year. Aggregate loss distributions reflect the sum of losses for all events in a given year. Clearly, the aggregate table provides a more complete picture, but for purposes of our exposition here, the occurrence table works well and the formulas are substantially less complex.

3 An event with a probability of 0.001 (typical of the more severe modeled events) would have λ = 0.0010005.

$$Var[L] + Var[n] + 2 * Cov[L, n] \quad [2.4]$$

(3) The Marginal Surplus (MS) Method

This is a translation to property catastrophe of the method described in Rodney Kreps' "Reinsurer Risk Loads from Marginal Surplus Requirements" [3].

Consider:

L_0 = losses from a portfolio before a new account is added
L_1 = losses from a portfolio after a new account is added
S_0 = Standard deviation of L_0
S_1 = Standard deviation of L_1

Borrowing from Mr. Kreps, assume needed surplus V is given by

z * *Standard Deviation of loss*[4] *- expected Return* [3.1]

where ***z*** is, to cite Mr. Kreps (p. 197), "a distribution percentage point corresponding to the acceptable probability that the actual result will require even more surplus than allocated." Then

$$V_0 = z * S_0 - R_0$$
$$V_1 = z * S_1 - R_1 \quad [3.2]$$

The difference in returns $R_1 - R_0 = r$, the risk load charged to the new account. The marginal surplus requirement is then

$$V_1 - V_0 = z * [S_1 - S_0] + r \quad [3.3]$$

We determine the risk load based on required return ***y*** on that marginal surplus, which is based on management goals, market forces and risk appetite. The MS risk load would be:

$$r = y * z / (1 + y) * [S_1 - S_0] \quad [3.4]$$

(4) The Marginal Variance (MV) Method

This is based on Glenn Meyers' 1995 CAS Discussion Paper program article "Managing the Catastrophe Risk" [6].

---

4 Mr. Kreps sets needed surplus equal to z * standard deviation of return - expected return. If we assume premiums and expenses are invariant, then Var[Return] = Var[P - E - L] = Var[L].

For an existing portfolio L and a new account n, the MV risk load would be:

$$r = \lambda * \text{Marginal Variance of adding n to L}$$
$$= \lambda * \{ Var[n] + 2 * Cov[L, n] \} \qquad [4.1]$$

where λ is a multiplier similar to y * z / (1 + y) from the MS method, although dimensioned to apply to variance rather than standard deviation[5].

(5) Building Up a Portfolio of 2 Accounts

Now we are prepared to apply the methods to the sample portfolio. Table A shows the occurrence size-of-loss distribution and risk load calculations for building up (assembling) a portfolio of 2 accounts, (X) and (Y). We assume (X) is written first, and is the only risk in the portfolio until (Y) is written.

(5.1) MS Method
Here is a summary of pertinent values from Table A for the Marginal Surplus method:

Table 5.1

Building Up (X) & (Y): Marginal Surplus	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Standard Deviation	4,429	356	4,785	4,785
(2) Risk Load Multiplier	0.33	0.33	-	0.33
(3) Risk Load = (1) * (2)	$1,461.71	$117.43	$1,579.14	$1,579.14

- Item (1) is the change in portfolio standard deviation from adding each account, or *marginal* standard deviation.

- Item (2) is the Risk Load multiplier of 0.33. Using Mr. Kreps' formula, a return on marginal surplus y of 20% and a standard normal multiplier z of 2.0 (2 standard deviations, corresponding to a cumulative non-exceedance probability of 97.725%) would produce a risk load multiplier of

$$y * z / (1 + y) = \quad 0.20 * 2 / 1.20 \quad = \quad 0.33 \text{ (rounded)} \qquad [5.1]$$

- Item (3) is the Risk Load, the product of Items (1) and (2).

---

[5] Mr. Meyers develops a variance based risk load multiplier by converting a standard deviation based multiplier using the following formula:

$$\lambda = (\text{Rate of Return} * \text{Std Dev Mult}^2) / (2 * \text{Avg Capital of Competitors})$$

Since (X) is the first account, the marginal standard deviation from adding (X) equals the standard deviation of (X) (Std Dev [X]) of 4,429. This gives a risk load of $1,461.71.

The marginal standard deviation from writing (Y) equals Std Dev [X + Y] - Std Dev [X], or $356, implying a risk load of $117.43.

The sum of these two risk loads (X) + (Y) is $1,461.71 + $117.43 = $1,579.14. This equals the risk load which this method would calculate for the combined account (X + Y).

(5.2) MV Method
Here is a summary of pertinent values from Table A for the Marginal Variance method:

Table 5.2

Building Up (X) & (Y): ***Marginal Variance***	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Variance	19,619,900	3,279,059	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,353.02	$226.13	$1,579.14	$1,579.14

• Item (1) is the change in portfolio variance from adding each account, or *marginal* variance.

• Item (2) is the Variance Risk Load multiplier λ of 0.000069. To simplify comparisons between the two methods (recognizing the difficulty of selecting a MV-based multiplier[6]), I converted the MS multiplier to a MV basis by dividing by Std Dev [X + Y]:

$$\lambda \quad = 0.33 / 1{,}579.14 \quad = \quad 0.000069 \qquad [5.2]$$

This means the total risk load calculated for the portfolio by the two methods will be the same, although the individual risk loads for (X) and (Y) will differ between the methods.

• Item (3) is the Risk Load, the product of Items (1) and (2).

Since (X) is the first account, the marginal variance from adding (X) equals the variance of (X) (Var [X]) of 19,619,900. This gives a risk load of $1,353.02.

The marginal variance from writing (Y) equals Var [X + Y] - Var [X], or $3,279,059, implying a risk load of $226.13.

[6] Mr. Meyers [6] (p.124) admits that in practice "it might be difficult for an insurer to obtain the (lambdas) of each of its competitors." He goes on to suggest an approximate method to arrive at a usable lambda based on required capital being "Z standard deviations of the total loss distribution."

The sum of these two risk loads (X) + (Y) is $1,353.02 + $226.13 = $1,579.14. This equals the risk load which this method would calculate for the combined account (X + Y).

(6) Renewing the Portfolio of 2 Accounts

Table B shows the natural extension of the Build-up scenario -- renewal of these 2 accounts, in what could be termed a "static" or "steady state" portfolio (one with no new entrants).

As for applying these methods in the renewal scenario, renewing policy (X) is assumed equivalent to adding (X) to a portfolio of (Y); renewing (Y) is assumed equivalent to adding (Y) to a portfolio of (X).

(6.1) MS Method
Here is a summary of pertinent values from Table B for the Marginal Surplus method:

Table 6.1

Renewing (X) & (Y): Marginal Surplus	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Standard Deviation	4,171	356	4,526	4,785
(2) Risk Load Multiplier	0.33	0.33	-	0.33
(3) Risk Load = (1) * (2)	$1,376.27	$117.43	$1,493.70	$1,579.14
(4) Build-up Risk Load	$1,461.71	$117.43	$1,579.14	$1,579.14
(5) Difference	($85.45)	$0	($85.45)	$0

The marginal standard deviation for adding (Y) to (X) is 356, same as it was during Build-up -- see Section (5.1). The risk load of $117.43 is also the same.

However, adding (X) to (Y) gives a marginal standard deviation of Std Dev [X + Y] - Std Dev [Y] , or 4,171. This gives a risk load for (X) of $1,376.27, which is (85.45) **less** than $1,461.71, the risk load for (X) calculated in Section (5.1).

The sum of these two risk loads is $1,376.27 + $117.43 = $1,493.70. This is also (85.45) **less** than the total risk load from Section (5.1).

(6.2) MV Method
Here is a summary of pertinent values from Table B for the Marginal Variance method:

Table 6.2

Renewing (X) & (Y): Marginal Variance	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Variance	22,521,000	3,279,059	25,800,059	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,553.08	$226.13	$1,779.21	$1,579.14
(4) Build-up Risk Load	$1,353.02	$226.13	$1,579.14	$1,579.14
(5) Difference	$200.06	$0	$200.06	$0

The marginal variance for adding (Y) to (X) is 3,279,059, same as it was during Build-up -- see Section (5.2). The risk load of $226.13 is also the same.

However, adding (X) to (Y) gives a marginal variance of Var [X + Y] - Var [Y], or 22,521,000. The risk load is now $1,553.08, which is $200.06 **more** than the $1,353.02 calculated in Section (5.2).

The sum of these two risk loads is $1,553.08 + $226.13 = $1,779.21. This is also $200.06 **more** than the total risk load from Section (5.2).

(7) Exploring the Differences Between New and Renewal

Why are the total Renewal risk loads different from the total Build-up risk loads?

(7.1) MS Method

In Section (5.1) Build-up, the marginal standard deviation for (X), ΔStd Dev [X], was :

$$\begin{aligned}\Delta\text{Std Dev }[X] &= \text{Std Dev }[X] \\ &= \text{SQRT}\left[\,\Sigma_i\ \{X_i^2 * p_i * (1 - p_i)\}\,\right], \qquad [7.1] \\ &\qquad (X_i = \textit{modeled losses for X for event i})\end{aligned}$$

while in Section (6.1) Renewal, the marginal standard deviation was

$$\begin{aligned}\Delta\text{Std Dev }[X] &= \text{Std Dev }[X + Y] - \text{Std Dev }[Y] \\ &= \text{SQRT}\left[\,\Sigma_i\ \{(X_i+Y_i)^2 * p_i * (1 - p_i)\}\,\right] - \\ &\quad\ \text{SQRT}\left[\,\Sigma_i\ \{Y_i^2 * p_i * (1 - p_i)\}\,\right] \qquad [7.2]\end{aligned}$$

For positive Y_i, this value is less than Std Dev [X][7]. Therefore, we *would* expect the Renewal risk load to be less than the Build-up.

---

7 For example, assume Var [X] = 9, Var [Y] = 4, Cov [X, Y] = 1.5; then
ΔStd Dev [X] = Sqrt(Var [X]) = Sqrt (9) = 3 *for X alone*
ΔStd Dev [X] = Sqrt(9 + 4 + 2*1.5) - Sqrt(4) = 4 - 2 = 2 < 3. *for X added to Y*

Unfortunately, when the MS method is applied in the renewal of all the accounts in a portfolio, the sum of the individual risk loads will be less than the total portfolio standard deviation times the multiplier. This is because the sum of the marginal standard deviations (found by taking the difference in portfolio standard deviation with and without each account in the portfolio) is less than the total portfolio standard deviation[8]. This is because **the square root operator is "sub-additive"**: the square root of a sum is less than the sum of the square roots[9].

(7.2) <u>MV Method</u>
In Section (5.2) Build-up, the marginal variance ΔVar [X] was

$$\begin{aligned} \Delta\text{Var}[X] &= \text{Var}[X] \\ &= \Sigma_i \{ X_i^2 * p_i * (1 - p_i) \}, \end{aligned} \qquad [7.3]$$

while in Section (6.2) Renewal the marginal variance was

$$\begin{aligned} \Delta\text{Var}[X] &= \text{Var}[X + Y] - \text{Var}[Y] \\ &= \{ \text{Var}[X] + 2 * \text{Cov}[X, Y] + \cancel{\text{Var}[Y]} \} - \cancel{\text{Var}[Y]} \\ &= \text{Var}[X] + 2 * \text{Cov}[X, Y] \\ &> \text{Var}[X]. \end{aligned} \qquad [7.4]$$

Since 2 * Cov [X, Y] is greater than zero, we *would* expect the Renewal risk load to be greater than the Build-up.

However, when the MV method is applied in the renewal of all the accounts in a portfolio, the sum of the individual risk loads will be more than the total portfolio variance times the multiplier. This is because the sum of the marginal variances (found by taking the difference in portfolio variance with and without each account in the portfolio) is greater than the total portfolio variance. **The covariance between any two risks in the portfolio is double counted**: when each account renews, it is allocated the full amount of its shared covariance with all the other accounts.

(8) <u>A New Concept: Renewal Additivity</u>

The renewal scenarios point out that these two methods are not what I call "**<u>renewal additive</u>**," defined as follows:

> For a given portfolio of accounts, a risk load method is **<u>renewal additive</u>** if the sum of the renewal risk loads calculated for each component account equals the risk load calculated when the combined accounts are treated as a single account.

---

8 The same issue is raised in Mr. Gogol's discussion [2].

9 For example, Sqrt[9 + 16] < Sqrt[9] + Sqrt[16].

Neither the MS nor the MV method is renewal additive: MS because the square root operator is sub-additive; MV because the covariance is double counted. In order for them to be renewal additive, one must assume an **entry order** for the accounts.

It's a puzzling predicament. We apply the risk load formula for the renewal of account (X). The formula makes sense for the renewal of account (X). It also makes sense for the renewal of account (Y). However, the portfolio total does not make sense. We could say that in the renewal context, these methods were "**individually rational**" yet the total was not "**collectively rational**".

I chose these terms deliberately as a segue to the next section. They come from the field of game theory. These concepts and others (including additivity) have been studied extensively by game theorists, and their results will provide us with inspiration for a new approach.

(9) A New Approach from Game Theory

I focused on ideas in two papers by Jean Lemaire: "An Application of Game Theory: Cost Allocation" [4], and "Cooperative Game Theory and Its Insurance Applications" [5]. In both papers, Mr. Lemaire considers the insurance applications of results from "cooperative games with transferable utilities"[10].

The material can be daunting. To facilitate the discussion, I will combine and paraphrase the formal game theory definitions from both of Mr. Lemaire's papers, then follow with translations to our problem[11].

Basics

> "A n-person cooperative game with transferable utilities is a pair [N, v(S)] where N = {1, 2, ..., n} is the set of the players, and v(S), the characteristic function of the game, is a super-additive[12] set function that associates a real number v(S) with each coalition S of players" ([4], p. 68).

[10] Citing Mr. Lemaire [5] (p.20) : "Cooperative game theory analyzes those situations where participants' objectives are partially cooperative and partially conflicting. It is in the participants' interest to cooperate, in order to achieve the greatest possible total benefits. When it comes to sharing the benefits of cooperation, however, individuals have conflicting goals.... Participants are negotiating about sharing a given commodity (such as money or political power) which is fully transferable between players and evaluated in the same way by everyone.... For this reason, the class of games defined here is called 'Cooperative games with transferable utilities.'"

In our case, the conflicting goals arise because all but the largest risks must have catastrophe coverage, and must go for this coverage to an insurance company. Insurance companies write many such risks, which means they have loss covariance created by the pooling of risks exposed to the same potential catastrophic events. The desire for coverage conflicts with the desire to be allocated the least covariance.

[11] Those wishing a more detailed explanation are strongly encouraged to read Mr. Lemaire's papers.

[12] Super-additivity is defined as follows: for S, T any two disjoint coalitions, and a characteristic function v, super-additivity implies v(S) + v(T) <= v(S union T).

Translation:
• Player = account.
• Coalition S = portfolio.
• Characteristic function v(S) = portfolio variance (super-additive because of the covariance component).

Imputation, Individual rationality, additivity

> "An **imputation** is a vector $y = (y_1, ..., y_n)$ such that $y_i >= v(i)$ for every i, and $\Sigma_{i=1 \text{ to } n} y_i = v(N)$" ([5] p. 68).

Translation:
• Imputation = allocation of the coalition total value v(N) back to the individual members.

• The first condition ($y_i >= v(i)$ for every i) is known as "**individual rationality**" -- each member's allocation y_i is no smaller than its value would be were it on its own (= v(i)).

• The second condition ($\Sigma_{i=1 \text{ to } n} y_i = v(N)$) is known as "**additivity**" -- the sum of the individual allocations must add up to the coalition total value.

In our problem, the imputation is each account's marginal variance (under the MV method) from adding it to the remainder of the portfolio. This imputation is individually rational, since the allocations are larger than the individual account variances because of the covariance component. However, as we have seen, it is **not** additive -- the sum of the individual allocations (marginal variances) is greater than the total variance.

Collective rationality and the Core

> "An imputation is **collectively rational** if there is no sub-coalition S' under which the players are better off than they were under S.
>
> "The **core** of the game is the set of all collectively rational imputations." ([5], p. 25)

Translation:
• Collectively rational = the coalition is stable -- there is no incentive for players to split off and form factions.

• The core sets the boundaries for possible, stable allocations.

Shapley value

> "The **Shapley value** is the center of gravity of the core's extremal points." ([4], p. 72)

Translation:
The Shapley value is the only allocation which satisfies the following three axioms ([4], p. 69):

1. Symmetry (Order-independence) - for all permutations P(S) of accounts in a portfolio S, c(S) = c(P(S)). Knowing the combination of accounts is sufficient to have an additive allocation.

2. Inessential Players (Uncorrelated accounts) - if an account generates no covariance with the existing portfolio, it is simply allocated its own variance, and nothing more.

3. Additivity - allocations from distinct games should be additive. This particular condition has no parallel in our situation.

Only one allocation method satisfies these three axioms -- the "**Shapley value**". It equals the average allocation taken over all possible **entrance permutations** -- the different orders in which a new member could have been added to the coalition[13] (i.e. a new account could have been added to a portfolio).

For example, if we had a portfolio of accounts (A), (B), and (C), and we want to add a new account (D), we could consider the marginal variance for adding (D) in all the following entrance permutations:

Table 9.1
Entry Permutations for Account D

(1)	(2)	(3)	(4)
Permutation #	**Entry Order**	**After...**	**Marginal Variance**
1	First	-	Var [D]
2	Second	After (A)	Var [D] + 2*Cov [D, A]
3	"	After (B)	Var [D] + 2*Cov [D, B]
4	"	After (C)	Var [D] + 2*Cov [D, C]
5	Third	After (AB)	Var [D] + 2*Cov [D, A] + 2*Cov [D, B]
6	"	After (AC)	Var [D] + 2*Cov [D, A] + 2*Cov [D, C]
7	"	After (BC)	Var [D] + 2*Cov [D, B] + 2*Cov [D, C]

13 Mr. Lemaire [5] provides this more complete definition of the Shapley value (p. 29): "The Shapley value can be *interpreted* as the mathematical expectation of the admission value, when all orders of formation of the grand coalition are equiprobable. In computing the value, one can assume, for convenience, that all players enter the grand coalition one by one, each of them receiving the entire benefits he brings to the coalition formed just before him. All orders of formation of N are considered and intervene with the same weight 1/n! in the computation. The combinatorial coefficient results from the fact that there are (s-1)!(n-s)! ways for a player to be the last to enter coalition S: the (s-1) other players of S and the (n-s) players of N\S (*those players in N which are not in S - DM*) can be permuted without affecting i's position."

8	Fourth	After (ABC)	Var [D] + 2*Cov [D, A] + 2*Cov [D, B] + 2*Cov [D, C]

The Shapley value is the straight average of Column (4) Marginal Variance over the eight permutations:

Shapley Value = { Sum [Column (4)] } / 8 [9.1]

= { 8*Var [D] +
8*Cov [D, A] +
8*Cov [D, B] +
8*Cov [D, C] } / 8

= Var [D] + Cov [D, A] + Cov [D, B] + Cov [D, C]

Or, to generalize, given

L = losses for existing portfolio
n = losses for new account

Shapley Value = Var [n] + Cov [L, n]. [9.2]

Before seeing this result, we might have been concerned about the practicality of this approach -- how much computational time might be required to calculate all the possible entrance permutations for a portfolio of thousands of accounts? This simple reduction formula eliminates those concerns. The Shapley value is as simple to calculate as the marginal variance.

Comparing the Shapley value to the marginal variance formula from Section 4:

Marginal Variance = Var [n] + **2** * Cov [L, n], [9.3]

we note the Shapley value only takes 1 times the covariance of the new account and the existing portfolio.

We can also calculate the Shapley value under the marginal standard deviation method. However, due to the complex nature of the mathematics -- differences of square roots of sums of products -- no simplifying reduction formula was immediately apparent[14].

Therefore, we will focus going forward on the MV method and the variance-based Shapley value. Life will be much easier (mathematically) working with the variances,

[14] Please contact the author if you can successfully reduce formulas involving the average of the difference of square roots of sums of products.

and we lose very little by choosing variance. Citing Mr. Bault ([1], p. 82), from a risk load perspective, "both [variance and standard deviation] are simply special cases of a unifying covariance framework." In fact, Mr. Bault goes on to suggest "in most cases, the 'correct' answer is a marginal risk approach that incorporates covariance"[15].

(10) Sharing the Covariance

The risk load question, framed in a game-theoretical light, has now become:

> *How do accounts share their mutual covariance for purposes of calculating risk load?*

The Shapley method answers, "Accounts split their mutual covariance equally." At first glance this appears reasonable, but consider the following example.

Assume two accounts, (L) and (M). (M) has 100 times the losses of (L) for each event. Their total shared covariance is

$$\begin{aligned} 2 * \mathrm{Cov}(L, M) &= 2 * \Sigma_i \{ L_i * M_i * p_i * (1 - p_i) \} \\ &= 2 * \Sigma_i \{ L_i * 100L_i * p_i * (1 - p_i) \} \end{aligned} \qquad [10.1]$$

The Shapley value would equally divide this total covariance between (L) and (M), even though their relative contributions to the total are clearly not equal. There is no question that (L) should be assessed *some* share of the covariance. The issue is whether there is a more equitable share than simply half.

We can develop a generalized covariance sharing (GCS) method which uses a weight $W_i^L(L, X)$ to determine (L)'s share of the mutual covariance between itself and account (X) for event i:

$$\mathrm{CovShare}_i^L (L, X) = W_i^L(L, X) * 2 * L_i * X_i * p_i * (1 - p_i) \qquad [10.2]$$

Then (X)'s share of that mutual covariance would simply be

$$\mathrm{CovShare}_i^X (L, X) = [1 - W_i^L(L, X)] * 2 * L_i * X_i * p_i * (1 - p_i) \qquad [10.3]$$

The total covariance share allocation for (L) over all events would be

$$\mathrm{CovShare}_{Tot}^L = \Sigma_Z \Sigma_i \{ \mathrm{CovShare}_i^L(L, Z) \}$$
$$\{ \Sigma_Z = \text{sum over every other account in the portfolio} \} \qquad [10.4]$$

[15] Mr. Kreps [3] also incorporates covariance in his "Reluctance" R (p. 198), which has the formula $R = [yz/(1+y)]/(2SC + \sigma)/(S' + S)$, where C is the correlation of the contract with the existing book. The Risk Load is then equal to $R\sigma$.

The Shapley method is a generalized covariance sharing method with $W_i^L(L, X) = 50\%$ for all (L), (X), and i.

Returning to the example with (L) and (M), we can develop an example of a weighting scheme which assigns the shared covariance by event to each in proportion to their loss for that event. $W_i^L(L, M)$, account (L)'s share of the mutual covariance between itself and account (M) for event i, equals

$$\begin{aligned} W_i^L(L, M) &= [\, L_i / [\, L_i + M_i \,]\,] \\ &= [\, L_i / [\, L_i + 100L_i \,]\,] \\ &= (1 / 101) \\ &= \text{roughly } 1\% \text{ of their mutual covariance for event i} \end{aligned} \qquad [10.5]$$

We will call this the "Covariance Share" (CS) method.

(11) Applying the Shapley and CS Methods to the Example

Now we will see how the Shapley and CS methods perform in our 2 Account example for both Build-up and Renewal.

(11.1) Portfolio Build-up

Table C shows the Build-up of accounts (X) and (Y) from Section 5, but for the Shapley and CS methods. Here is a summary of the pertinent values from Table C for the Shapley value:

Table 11.1

Building Up (X) & (Y): Shapley Value	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Variance	19,619,900	1,828,509	21,448,409	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,353.02	$126.10	$1,479.11	$1,579.14

and for the Covariance Share:

Table 11.2

Building Up (X) & (Y): Covariance Share	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Variance	19,619,900	950,658	20,570,558	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,353.02	$65.56	$1,418.57	$1,579.14

Both Shapley and CS produce the same risk load for (X) as the MV method on build-up - $1,353.02. This is because there is no covariance to share - (X) is the entire portfolio at this point. However, let's compare the results of the three variance-based methods *for account* (Y):

Table 11.3

Comparison of Build-up Risk Loads for Account (Y)	
Marginal Variance (MV) - Section 5.2	$226.13
Shapley Value	$126.10
Difference from MV	***$100.03***
Covariance Share (CS)	$65.56
Difference from MV	***$160.57***

Compared to MV, which charges account (Y) for the full increase in variance (Var [Y] + 2* Cov [X, Y]), the Shapley method only charges (Y) for Var [Y] + Cov [X, Y]. The same can be said for the CS method, although the share of the mutual covariance depends on each account's relative contribution by event, weighted and summed over all events. Let's see what happens to that ***difference from MV*** upon renewal.

(11.3) Renewal

Table D shows the renewal of (X) and (Y) for the Shapley and CS methods. Here is a summary of pertinent values from Table D for the Shapley method:

Table 11.4

Renewing (X) & (Y): Shapley Value	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)
(1) Change in Variance	21,070,450	1,828,509	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,453.05	$126.10	$1,579.14	$1,579.14
(4) Build-up Risk Load	$1,353.02	$126.10	$1,479.11	$1,579.14
(5) Difference	$100.03	$0	$100.03	$0

and for the Covariance Share method:

Table 11.5

Renewing (X) & (Y): Covariance Share	Account (X)	Account (Y)	Account (X) + Account (Y)	Account (X + Y)

(1) Change in Variance	21,948,301	950,658	22,898,959	22,898,959
(2) Risk Load Multiplier	0.000069	0.000069	-	0.000069
(3) Risk Load = (1) * (2)	$1,513.59	$65.56	$1,579.14	$1,579.14
(4) Build-up Risk Load	$1,353.02	$65.56	$1,418.57	$1,579.14
(5) Difference	$160.57	$0	$160.57	$0

With both the Shapley and CS methods, the sum of the risk loads for Account (X) and Account (Y) equals the risk load for Account (X + Y), namely $1,579.14. This means we have two **renewal additive** methods, which also means they are legitimate imputations.

To see what happened to ***difference from MV***, compare the risk loads calculated at renewal for (X) with those at build-up:

Table 11.6

Build-up vs Renewal Risk Loads for Account (X)	**Shapley**	**Cov Share**
Renewal	$1,453.05	$1,513.59
Build-up	$1,353.02	$1,353.02
Additional Renewal Risk Load over Build-up	***$100.03***	***$160.57***
Difference from MV	***$100.03***	***$160.57***

The difference from MV during build-up is simply the portion of (X)'s risk load attributable to its share of covariance with (Y). It was missed during build-up because it was unknown -- account (Y) had not been written.

(12) Conclusion

These new approaches address the concerns with renewal additivity, and point out the issue of covariance sharing between accounts. Perhaps the ideal solution might involve using a marginal method for the pricing of new accounts, and a renewal additive method for renewals. Any number of variations are possible, as long as one avoids double-counting the covariance.

It is hoped that this paper has also set the stage for further discussion of order dependency. This is a complex issue which was only touched on here, but which moves more to the forefront as advances in computer technology and modeling make ever finer levels of analysis possible.

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Table (A) Build a Portfolio of 2 Risks

			Loss for Risk		
Event i	P(i)	1-P(i)	(X)	(Y)	(X + Y)
1	2.0%	98.0%	25,000	200	25,200
2	1.0%	99.0%	15,000	500	15,500
3	3.0%	97.0%	10,000	3,000	13,000
4	3.0%	97.0%	8,000	1,000	9,000
5	1.0%	99.0%	5,000	2,000	7,000
6	2.0%	98.0%	2,500	1,500	4,000

E[L]	1,290	179	1,469
Var[L]	19,619,900	377,959	22,898,959
Std Dev[L]	4,429	615	4,785

Covar	(X)	(Y)
(X)	19,619,900	1,450,550
(Y)	1,450,550	377,959

		(X)	(Y)	(X)+(Y)
Change in Std Deviation		4,171	356	4,526
Risk Load (Std Dev)		1,376.27	117.43	1,493.70
0.33	*Risk Load (A)*	1,461.71	117.43	1,579.14
	Difference	(85.45)		(85.45)

Change in Variance		22,521,000	3,279,059	25,800,059
Risk Load (Variance)		1,553.08	226.13	1,779.21
0.000069	*Risk Load (A)*	1,353.02	226.13	1,579.14
	Difference	200.06		200.06

Table (B) Renew the Portfolio of 2 Risks

			Loss for Risk		
Event i	P(i)	1-P(i)	(X)	(Y)	(X + Y)
1	2.0%	98.0%	25,000	200	25,200
2	1.0%	99.0%	15,000	500	15,500
3	3.0%	97.0%	10,000	3,000	13,000
4	3.0%	97.0%	8,000	1,000	9,000
5	1.0%	99.0%	5,000	2,000	7,000
6	2.0%	98.0%	2,500	1,500	4,000

E[L]	1,290	179	1,469
Var[L]	19,619,900	377,959	22,898,959
Std Dev[L]	4,429	615	4,785

Covar	(X)	(Y)
(X)	19,619,900	1,450,550
(Y)	1,450,550	377,959

		(X)	(Y)	(X)+(Y)
Change in Std Deviation		4,171	356	4,526
Risk Load (Std Dev)		1,376.27	117.43	1,493.70
0.33	*Risk Load (A)*	1,461.71	117.43	1,579.14
	Difference	(85.45)		(85.45)

Change in Variance		22,521,000	3,279,059	25,800,059
Risk Load (Variance)		1,553.08	226.13	1,779.21
0.000069	*Risk Load (A)*	1,353.02	226.13	1,579.14
	Difference	200.06		200.06

Table (C) Build a Portfolio of 2 Risks - Alternatives

			Covariance Share $(Y)	
1	2.0%	98.0%	9,920,635	79,365
2	1.0%	99.0%	14,516,129	483,871
3	3.0%	97.0%	46,153,846	13,846,154
4	3.0%	97.0%	14,222,222	1,777,778
5	1.0%	99.0%	14,285,714	5,714,286
6	2.0%	98.0%	4,687,500	2,812,500

		Total
2,328,401	572,699	2,901,100

Chg in Variance		(X)	(Y)
If added 1st		19,619,900	377,959
If added 2nd	after 1		3,279,059
	after 2	22,521,000	
Average *(Shapley Value)*		21,070,450	1,828,509

Shapley Value		21,070,450	1,828,509	22,898,959
Risk Load (Shapley)		**1,453.05**	126.10	1,579.14
0.000069	*Risk Load (C)*	1,353.02	126.10	
	Difference	**100.03**	*= Deferred Risk Load from (C)*	

Covariance Share		21,948,301	950,658	22,898,959
Risk Load (Cov Share)		**1,513.59**	65.56	1,579.14
0.000069	***Risk Load (C)***	1,353.02	65.56	
	Difference	**160.57**	*= Deferred Risk Load from (C)*	

Table (D) Renew the Portfolio of 2 Risks - Alternatives

			Covariance Share $	
Event i	P(i)	1-P(i)	(X)	(Y)
1	2.0%	98.0%	9,920,635	79,365
2	1.0%	99.0%	14,516,129	483,871
3	3.0%	97.0%	46,153,846	13,846,154
4	3.0%	97.0%	14,222,222	1,777,778
5	1.0%	99.0%	14,285,714	5,714,286
6	2.0%	98.0%	4,687,500	2,812,500

		Total
2,328,401	572,699	2,901,100

Chg in Variance		(X)	(Y)
If added 1st		19,619,900	377,959
If added 2nd	after 1		3,279,059
	after 2	22,521,000	
Average *(Shapley Value)*		21,070,450	1,828,509

Shapley Value		21,070,450	1,828,509	22,898,959
Risk Load (Shapley)		**1,453.05**	126.10	1,579.14
0.000069	*Risk Load (C)*	1,353.02	126.10	
	Difference	**100.03**	*= Deferred Risk Load from (C)*	

Covariance Share		21,948,301	950,658	22,898,959
Risk Load (Cov Share)		**1,513.59**	65.56	1,579.14
0.000069	***Risk Load (C)***	1,353.02	65.56	
	Difference	**160.57**	*= Deferred Risk Load from (C)*	

Levels of Determinism in Workers Compensation Reinsurance Commutations

by Gary Blumsohn, FCAS

Levels of Determinism in Workers' Compensation Reinsurance Commutations

Gary Blumsohn

Abstract

When commuting workers' compensation reinsurance claims, the standard method is to project the future value of the claims using stated assumptions for future medical usage, medical inflation, COLAs, and investment income. The actuary selects a best guess for each variable, and assumes this deterministic number will be realized in the future. To account for the date of death being stochastic, a mortality table is used to model the future lifetime.

By assuming deterministic values for future medical usage, medical inflation, COLAs, and investment income, the calculation ignores the possibilities of higher or lower values. It is shown that these do not generally balance out, and that the standard method produces biased results. In low reinsurance layers, the commutation amount is overstated, and in high layers it is understated. By removing deterministic assumptions from the calculation, bias is removed from the results. The paper gives a detailed, realistic, example to illustrate this.

The implications of the paper reach beyond the narrow realm of workers' compensation reinsurance commutations. The most obvious implications are for workers' compensation reserving, but the essential message applies to pricing and reserving of any excess insurance and reinsurance: deterministic assumptions often lead to biased results.

Biography

Gary Blumsohn, FCAS, is an Associate Actuary with the Liberty Mutual Group, where he is the actuary responsible for reinsurance. Prior to that he worked at Prudential Reinsurance Company. He has a Bachelor of Economic Science degree from the University of the Witwatersrand (Johannesburg, South Africa) and a Ph.D. in economics from New York University. His dissertation was on the impact of changing knowledge on the size of damages in tort and securities law.

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Introduction

Excess reinsurance for workers' compensation generally pays out over many decades. While workers' compensation claims are usually reported to the insurer soon after the accident, and the insurer may soon report them to the reinsurer, the loss payments are slow, being made over the lifetime of the injured worker or even the lifetime of uninjured dependents. Consequently, even for reinsurance with a relatively modest retention, it can take many years to breach the retention, and many more years to exhaust a layer. For example, Gary Venter (1995) has estimated that it takes, on average, over 30 years to pay half the ultimate claim amount.

At some point after an excess reinsurance treaty ends, but before the losses have been fully paid, it is common to commute either the reinsurance treaty or the individual reinsured claims. The commutation is a transaction whereby the reinsurer pays the ceding company a flat amount, in exchange for canceling future liabilities. This saves costs for both parties, since the expense of submitting claims to the reinsurer and the cost of paying these claims are eliminated. It allows the parties to shut their reinsurance files and spend their time on more profitable activities.

The actuarial techniques for evaluating workers' compensation commutations differ from the techniques generally used in commutations of other lines of business. With workers' compensation (and in some other cases, like unlimited medical benefits for no-fault auto) the population of claims is generally known at the time of the commutation — there is very little lag in claims being reported to the primary company. Also, the amount of the payments is not dependent on some future court verdict. The payments are based on a fixed annual indemnity amount, subject, in some states, to an annual cost of living adjustment, and on the actual medical payments to be incurred by the claimant. In the case of permanent-total disability cases, these payments often continue for the rest of the claimant's life. Since the losses are so closely tied to the claimant's life span, it is natural to use the mortality techniques more generally associated with life actuaries than with their property/casualty brethren.

While the actuarial techniques in these calculations are by now well accepted, this paper will argue that the results are systematically biased and can be improved upon. The life-table techniques generally assume that mortality is stochastic, but that various other variables (amount of medical care, inflation rates, investment yields) are deterministic. These deterministic variables can be stripped away, much as earlier actuaries stripped away the assumption of deterministic mortality. By doing this, we improve the accuracy of our calculations and eliminate some biases.

Though this paper will express the issues in terms of commutations, the issues are similar when doing excess workers' compensation case reserving using life-table methods. In other words, even though there are layers that we do not expect to get hit, we should carry reserves for those layers. Over a pool of claimants, some will die before hitting the upper layers, and others will not. The goal should be to get the reserves right on average.

Life-Table Techniques

Method 1: Totally deterministic calculation

The simplest method for performing the calculation is to assume the claimant will live to his life expectancy and then calculate the present value of the future stream of payments for this time. This method, though simple and appealing, is wrong. As actuaries are well aware, and as will be discussed in detail later, assuming a deterministic life-span leads to systematically incorrect results.

Method 2: Stochastic date of death

The actuarial literature contains several papers that discuss the calculation of reserves for long-term workers' compensation cases, and the calculation of a commutation value only differs in minor respects from the calculation of a reserve.[1] It is generally accepted among actuaries, and, to a lesser extent, the wider insurance community, that the right way to reserve these claims is through the life-table techniques routinely used by life actuaries. The big advance of the life-table method over a method that assumes the insured will live to his exact life expectancy is that it takes into account the probabilities of the claimant dying either earlier or later than the life expectancy. This is particularly important when dealing with excess reinsurance, because if the claimant lives beyond his life expectancy, a higher layer may be breached.

The move from a deterministic number of payments to a stochastic number of payments, through the use of a life table, is a crucial advance in the accuracy of the calculation. A life-table approach allows for the possibility that a claimant may live to age 95, and hence pierce reinsurance layers that would not have been pierced if he had died at his life expectancy. Thus, in calculating the value of a commutation for a high reinsurance layer, there may be a positive amount in a layer, even though the layer will not be hit unless the claimant lives well beyond his life expectancy. In other words, if the claimant lives to his life expectancy of, say, 75, a retention of $5 million may not be breached. But if he lives another 10 years, to 85, the total payments in the additional 10 years of life may be enough to breach the $5 million retention.

Put another way, there will be a positive commutation amount in layers that we do not expect to get hit. The commutation is (effectively) a purchase of reinsurance by the reinsurer, covering the possibility of the claimant breaching the retention. There need not be a guarantee that the retention will be breached in order for the expected losses in the layer to be positive.

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1 The classic paper is Ronald Ferguson's *Actuarial Note on Workmen's Compensation Loss Reserves* (1971), which applied life-table methods to excess indemnity reserves. He did not address the issue of the medical portion of the reserve. Richard Snader (1987) applied similar methods to long-term medical claims. A recent valuable addition to the literature is by Lee Steeneck (1996), who uses an analysis very close to the "Method 2" that will be discussed later in this paper.

Assumptions

In doing the commutation calculation, the actuary needs to make a number of assumptions:[2]

- An appropriate *mortality table* must be selected.

- For workers' compensation, the indemnity amount is generally known, but it may be subject to *cost-of-living adjustments*, which depend usually on movements in the average weekly wage in the state.

- The amount of medical expenses must be estimated for each year in the future. This is usually done in two steps: first, estimate the future *annual medical expense* in today's dollars, and, second, estimate what future *medical price inflation* will be, to convert today's dollars into tomorrow's dollars.

- The *rate at which to discount* future dollar payments to present value.

Once assumptions have been chosen, the calculations can be performed, and the parties can agree on an amount for settlement.[3]

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2 In practice, some reinsurance contracts have commutation clauses in which the parties have negotiated some of the parameters at the time the contract is drawn up. For example, the clause may specify what mortality table to use and what rate to use in discounting the future payments.

3 This paper will not address the crucial impact of income tax. In looking at the commutation, one must account for taxes without the commutation, compared to taxes with the commutation.

i) If the claim is not commuted, the reinsurer carries a reserve on its books. For tax purposes, this reserve is discounted by the IRS discount factors, and the unwinding of the reserve is counted into the incurred losses of the company each year. On the other hand, the investment income earned on the reserve is taxable.

Levels of Determinism

The problem, though, as this paper will show, is that the life-table method ignores fluctuations in other key variables. Just as it is wrong to assume a claimant's life-span is fixed, so it is wrong to assume that medical usage and inflation are fixed. Assuming a deterministic life-span leads to inaccurate calculations. Likewise, assuming deterministic medical care and inflation will lead to inaccurate calculations. A deterministic life span implies that high layers of reinsurance will not be hit, when they do, in fact, have a chance of getting hit if the claimant lives long enough. Likewise, deterministic medical care and deterministic inflation understate the costs to the highest reinsurance layers.

Just as Ferguson's paper stripped away one level of determinism from these calculations, so we must strip away further levels of determinism, if we want to get greater accuracy.

A Comprehensive Example

The following section gives a realistic example of how one would strip determinism from the model. The calculations are significantly more

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ii) If the claim is commuted, the reinsurer takes down the reserves it holds for the claim and puts up a paid loss. If the reserve is greater than the paid loss (as it frequently is, because statutory accounting demands undiscounted, or perhaps tabularly-discounted, reserves) the reinsurer's profit rises by the difference between the reserve and the paid loss. This profit is taxable.

The ceding company has the reverse entries on its books.

When commuting, the tax benefits or tax hits are as important as any other cash flows. They are, however, beyond the scope of this paper. For a detailed discussion of the tax effects, see Connor and Olsen (1991).

complex than the standard life-table method. However, using computers, the problems are not insurmountable, and the results are significantly less biased.

The Data

Suppose we are commuting the following claim:

- Joe Soap has been permanently and totally disabled since 1992. On 1/1/97, the effective date of the commutation, he will turn 35 years old.

- Through 12/31/96, the primary company has paid out $300,000 in medical expenses and $70,000 in indemnity payments.[4] This is an unusually large claim, but by no means unheard of. A smaller claim would not affect any of the conclusions.

- In 1996, Mr. Soap received indemnity payments at the rate of $20,000 per year, but these are subject to a cost-of-living adjustment that is effective on January 1 of each year, based on the increase in the state-average-weekly-wage over the previous year.

- The best estimate of his future medical expenses is $70,000 per year, in 1996 dollars. These will increase with medical inflation.

- Joe's mortality follows that for the overall male population, as shown in the 1990 US census. (Exhibit 1) Based on this mortality, his life expectancy is 39.6 years.[5]

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4 For simplicity, we have ignored ALAE in this example. ALAE is usually covered by the reinsurance, and should be included if this is the case. However, ALAE is usually a small portion of workers' compensation claims, and including it would not change any of the principles discussed in this paper.

5 One may wonder whether it is reasonable to use mortality for the general population, when Joe is presumably rather badly injured. Depending on the claimant's condition, one may wish to use impaired mortality tables. It should be noted, however, that contrary to the usual

- Our best guess of future inflation is 4.2% per year.[6] We assume, for convenience, that changes in the state-average-weekly-wage follow the overall price inflation in the economy. (We generally expect wages to rise faster than prices over the long run. As productivity increases, real wages generally rise.)

- Our best guess of future medical inflation is 5.36% per year.[7] Exhibit 2 shows historical changes in the CPI and medical CPI.

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intuition on the matter, workers' compensation lifetime-pension cases do not, overall, appear to have higher mortality rates than those of the general population. Gillam (1993) shows that at some ages, the mortality of workers' compensation claimants is even below that of the general population. Gillam's technique weights each claimant equally. However, over a large book of business, that may not be the optimal approach, since some claims are bigger than others. In particular, many of the really big claims are for people who are extremely badly injured and require, say, 24-hour attendant care. One might speculate that a dollar-weighted average of mortality could be found to be significantly worse than the general population.

By using the 1990 census table, we are ignoring future mortality improvements, that may result from better medical care in the future. As medical care improves, mortality rates have historically dropped. By ignoring mortality improvements, we are implicitly assuming Joe Soap has impaired mortality.

6 The 4.2% used in the text is the average of actual Consumer Price Index changes from 1935 to 1995, using data supplied by the US Bureau of Labor Statistics. Using this average was a matter of convenience, rather than a matter of believing that it is a good predictor of future inflation. The data, though not a predictor of future inflation, give one a reasonable idea of how inflation could move over the long term.

Steeneck (1996, p. 252), when faced with projecting indemnity inflation into the indefinite future, selects 4.0% as his annual rate.

7 As with CPI changes, this average is based on changes in the Medical component of the CPI from 1935 to 1995. Also, as with the CPI, I am using this number for illustrative purposes,

- The appropriate risk-adjusted discount rate is assumed to be the same as the expected annual inflation rate, namely 4.2% per year. Again, this assumption is for convenience in this illustrative example. In general, discounting should be based on some investment yield, less a risk adjustment to take care of the riskiness in the flows being discounted. (Butsic, 1988) Real interest rates will usually be positive, and I am assuming the appropriate risk adjustment exactly offsets the real interest rate. (This is not the same as assuming that inflation is zero and discounting is done at a zero rate. Assuming zero inflation will ensure that higher reinsurance layers are not touched, when, in fact, there is a great likelihood that they will be hit.)

- The primary insurer has purchased reinsurance in a number of layers:

Layer 1	$130,000 excess of $370,000
Layer 2	$500,000 excess of $500,000
Layer 3	$1 million excess of $1 million
Layer 4	$3 million excess of $2 million
Layer 5	$5 million excess of $5 million
Layer 6	$5 million excess of $10 million
Layer 7	$5 million excess of $15 million
Layer 8	$10 million excess of $20 million
Layer 9	$10 million excess of $30 million
Layer 10	$10 million excess of $40 million
Layer 11	$10 million excess of $50 million

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rather than as a prediction of future medical inflation. Steeneck (1996, p. 252), projects annual medical inflation of 5.5%.

Layer 12	$10 million excess of $60 million
Layer 13	$10 million excess of $70 million
Layer 14	$10 million excess of $80 million
Layer 15	$10 million excess of $90 million
Layer 16	Unlimited excess of $100 million

The first layer is somewhat artificial: since $370,000 has already been paid by the end of 1996, the layer will pay from the first dollar in 1997. This allows us to look at the value of all future payments. Also, the top layer is somewhat unusual. Reinsurers do not usually sell unlimited layers. However, it will be instructive to see the value of reinsurance on the unlimited top layer.

Method 1: Totally Deterministic Calculation

Though actuaries would not use a totally deterministic method (i.e., one that assumes Joe lives exactly to his life expectancy and then dies) it is instructive to see what result this produces. Exhibit 3 shows this calculation, and the table below summarizes the results.

Layer (in $,000s)	Nominal Payments (in $,000s)	Present Value of Payments (in $,000s)
130 xs 370	130	126
500 xs 500	500	430
1,000 xs 1,000	1,000	679
3,000 xs 2,000	3,000	1,358
5,000 xs 5,000	5,000	1,388
5,000 xs 10,000	1,911	399
Higher Layers	0	0
Total, All Layers	**11,541**	**4,380**

Total payments are $11.5 million, exhausting the five layers and part of the sixth. The lack of payments in higher layers implies these layers will not be breached, and no commutation payment is needed. This method ignores the chance of death either earlier or later than one's life expectancy. We correct this by using a life-table approach, following Ferguson.

Method 2: Stochastic date of death

In Method 2, a mortality table is used to model Joe's life span, as shown in Exhibit 4. The table below compares the commutation amounts from Methods 1 and 2.

Layer (in $,000s)	**Expected Nominal Payments (in $,000s)**		**Expected Present-Value Payments (in $,000s)**	
	Method 1	Method 2	Method 1	Method 2
130 xs 370	130.0	129.7	126.0	125.7
500 xs 500	500.0	494.9	430.2	425.9
1,000 xs 1,000	1,000.0	970.6	679.4	659.8
3,000 xs 2,000	3,000.0	2,729.7	1,357.8	1,241.3
5,000 xs 5,000	5,000.0	3,734.8	1,387.7	1,048.5
5,000 xs 10,000	1,910.9	2,647.3	398.7	510.2
5,000 xs 15,000	0.0	1,704.2	0.0	254.6
10,000 xs 20,000	0.0	1,523.1	0.0	177.9
10,000 xs 30,000	0.0	374.7	0.0	33.6
10,000 xs 40,000	0.0	61.0	0.0	4.5
10,000 xs 50,000	0.0	6.5	0.0	0.4
10,000 xs 60,000	0.0	0.4	0.0	0.0
Higher layers	0.0	0.0	0.0	0.0
Total, all Layers	**11,540.9**	**14,376.9**	**4,379.7**	**4,482.5**

Several points are worth noting:

- Using Method 2, twelve layers have non-zero commutation amounts, compared to only six layers using Method 1. This is because Method 2 recognizes that people can live beyond their life expectancies. If the person lives to the outer reaches of the mortality table, say to 110, many more layers will be breached. The highest layer reached is $10 million excess of $60 million, implying that the largest possible claim, for a person living to the maximum number of years in the life table is somewhere between $60 million and $70 million. [Exhibit 4 shows that the maximum possible loss is $78.4 million, but the tiny probability of this happening means that the expected losses in the layers above $70 million are below $1,000, and thus do not show up on the table above.]

- For all layers combined (which translates to the value of all future amounts payable to the claimant) the nominal total from Method 1 ($11.5 million) is considerably lower than the nominal total from Method 2 ($14.4 million). However, the present value from Method 1 ($4.4 million) is only slightly lower than the present value from Method 2 ($4.5 million). How can we explain this?

  i) Nominal Total from Method 2 considerably greater than Method 1
  The easiest way of explaining the relation between the nominal totals is by analogy to a more familiar idea involving annuities. As most actuaries are aware, the present value of a life annuity is less than the present value of an annuity certain for the person's life expectancy. (Bowers, 1986, pp. 149 - 150 (example 5.13) and p. 158 (exercise 5.45).) In other words, the cost of paying someone $1 per year for life is less than the cost of paying $1 per year for a guaranteed period equal to the person's life expectancy. The intuition is that if you pay for the person's actual lifetime, there's a chance of living beyond the life expectancy, and those payments will be discounted at a higher rate than the earlier payments. By contrast, the annuity certain ignores the possibility of these higher discounts.

How does this relate to the nominal payments from Method 1 being much lower than Method 2? In our situation, we have inflation affecting the payments in two ways: the indemnity amounts are increased by the annual cost-of-living increase, and the medical amounts are increased by the annual medical inflation. If the claimant lives to, say, 95 years old, there will be many years of inflation increasing the annual payments, beyond the inflation contemplated in Method 1, which halts at the life expectancy. Thus, without inflation, the nominal amounts from Methods 1 and 2 would be identical; with inflation, the nominal amount from Method 1 will be lower than that for Method 2.

ii) Present value of Method 2 almost the same as Method 1
Without inflation, the payments would be the same each year. Then, as noted above, the present value of Method 1 (an annuity certain for the life expectancy) would exceed the present value for Method 2 (a life annuity). When there is inflation, things are more complicated. The issue is whether the effect of the additional inflation beyond the life expectancy outweighs the effect of the additional discounting. Depending on the rates, the present value of Method 2 could be either higher or lower than the present value of Method 1.

- On the layers that are pierced by Method 1, the commutation value from Method 2 is lower than the value from Method 1. For example, on the $500,000 excess $500,000 layer, the value under Method 1 is $430,200, while under Method 2 it's $425,900. This is because Method 1 assumes the amounts are paid for certain, and discounts only for the time-value of money. By contrast, Method 2 recognizes that the claimant may die early, and that the amounts may not be paid. Of course, in the layers not pierced in Method 1, the commutation value for Method 2 is always higher.

- We can make no general statement about whether a commutation calculated using Method 1 will produce a total amount, for all layers combined, that is greater than or less than the total for Method 2. This will depend on a number of factors. For example, if the primary company buys reinsurance on only very low layers, Method 1 will tend to be higher. If it buys reinsurance only on high layers, Method 2 will tend to be higher.

Determinism and Risk

Once a claim has been commuted, the cedent takes the risk of future losses. If the claimant lives to a ripe old age, the primary company will suffer a loss — it would have been better off not to have commuted. That's not a problem: insurance is about taking risks. The commutation calculation measured the mortality risk, and included it in the commutation price. Though the primary company may not be happy to have to pay higher than expected losses, the mortality risk has been priced into the commutation amount. But, there are other risks faced by the ceding company that have not been priced into the commutation amount. Medical inflation is one such example.

The assumed rate of medical inflation is often a contentious issue in commutation negotiations. The parties may argue over whether we should use the average for the past decade (currently about 7%), a longer term average (about 6% if we average back to World War 2), or an econometrician's projection for medical inflation for the next decade. In many cases we are projecting inflation for 70 years or more, so we cannot expect our numbers to be perfect. But, often, the parties find a number on which they can agree — let us assume it is 5.36%, and let us assume this number is, indeed, the future long-term average medical inflation rate. The parties use Method 2, with 5.36% medical inflation, and agree on the amount. The ceding company, it would appear, has been compensated for future inflation.

The ceding company has not, in fact, been compensated for future inflation. It has been compensated for a fixed 5.36% future inflation. It faces the risk that 2 or 3 years hence there will be very high medical inflation, say 20% or 25% per year, for 3 or 4 years, after which medical inflation will drop back to its long-term average. This period of abnormally high medical inflation will quickly erode the retention, which is in nominal dollars, and breach the excess layers much more quickly than the commutation calculation assumes.

There is, similarly, a chance that medical inflation for the next few years will be lower than the long term average, and high medical inflation may not occur for another 60 years. Over the course of the 70 years, one would expect this all to even out. So, the skeptic may ask, why should we care? If, on average, it evens out, and if a company does a large number of commutations over a large number of years, the overall result will be about right.

The problem is that it will not be "about right." Things do not average out in the long run. Just as Method 1 gave biased results, so Method 2, by assuming certain inputs are deterministic, gives biased results. Method 1 may be labeled "completely deterministic." Method 2 strips away the deterministic life expectancy from Method 1. But there are further layers of determinism that need to be stripped away if we want to get more accurate answers.

The Effects of Variable Inflation

To see why things do not average out, let's examine the effects of variable inflation more closely. Consider an average inflation rate of 5% per year in each of 3 scenarios, and assume the pre-inflation amount payable per year is $100:

	Medical Amount Payable Each Year		
Year	**Scenario 1:** **5% inflation each year**	**Scenario 2:** **20% inflation in year 1; 0% in all other years**	**Scenario 3:** **20% inflation in year 4; 0% in all other years**
0	100.00	100.00	100.00
1	105.00	120.00	100.00
2	110.25	120.00	100.00
3	115.76	120.00	100.00
4	121.55	120.00	120.00
Total	**552.56**	**580.00**	**520.00**

Inflation early on (scenario 2) raises the nominal dollar amounts in all future years, causing the total nominal amount to be higher. If there is reinsurance on these payments, the reinsurance retention would be breached earlier, and perhaps a layer will be breached that would not otherwise have been breached. The average inflation over the 3 scenarios is the same, but Scenario 2 results in more dollars of medical expenses, and Scenario 3 results in fewer dollars of medical expenses.

For a given average inflation rate, the path of inflation over the life of the claim will affect the future payments: high inflation early on will result in higher amounts; low inflation early on will result in lower amounts. While the total amount over all layers of reinsurance may roughly average out to be the same when present-valued, the amounts within the various layers will differ significantly.

If there is high inflation early on, the reinsurance retention will be breached earlier than expected. There is thus a greater chance that the claimant will still be alive to receive the payment. This greater possibility of payment directly affects the commutation calculation.

The standard commutation calculation fails to include certain risks, and thus neglects to price them. Method 2 assumes mortality is stochastic, but that medical inflation is deterministic. It also assumes wage inflation (and hence cost-of-living adjustments, in states that have them), investment income, and the annual medical usage of the claimant are deterministic. This will generally bias the commutation amount upwards for lower layers and downwards for higher layers. This is analogous to Method 1 overstating the lower layers and understating the higher layers, relative to Method 2. ("Higher" and "lower" is relative to the size of an individual claim.) Making each of these factors stochastic will remove some of the bias in the calculation.

Stripping Away Determinism

Method 3: Stochastic economic factors and medical costs

Method 3 incorporates several additional random variables into the calculation:

- Inflation is not constant over time. It will fluctuate from year to year, with the rates not independent from year to year. [A note on terminology: By "inflation," with no modifier, I mean inflation relating to the overall economy, most popularly measured by the CPI. When referring specifically to price rises for medical care, I will refer to "medical inflation."]

- Medical inflation, while roughly tracking the ups and downs of general inflation, will not be the same as inflation.

- Investment yields fluctuate from year to year, but, like inflation, years are not independent.

- The annual medical payment to the claimant will not be a constant real amount each year. As the claimant's health changes, this amount will change. The claimant may take a turn for the worse, and require $200,000 of hospitalization one year; or he may have a stable period where his medical expense is a lot lower than projected.

Each of these variables needs to be modeled. The specific way they have been modeled here is not the only way it could be done. The details of the example are less important than the general point being made, namely, that additional fluctuations need to be taken into account.

1) Inflation

Inflation was modeled using an autoregressive process of the following form:

$$\text{Inflation rate}_{\text{Year } t} = \text{Long-term average inflation rate}$$
$$+ \alpha[\text{Inflation rate}_{\text{Year } (t-1)} - \text{Long-term average inflation rate}]$$
$$+ \text{error}_{\text{Year } t}$$

Daykin, et al. (1994, pp. 218 - 225), discusses this model, and a number of other inflation models that may better fit the data. In the interests of simplicity, I chose to use this model. Using this model, we can start with a known inflation rate for 1995, and simulate a series of future paths of inflation.

Using least-squares fitting of inflation data from the Bureau of Labor Statistics from 1935 - 1995, I obtained the following parameters:

Long-term average inflation = 4.2% per year.
$\alpha = 0.51$

The error term was modeled using a lognormal distribution. Since the error should be positive or negative, but a lognormal is only defined for positive variables, I shifted the lognormal. The best fit was obtained by using a shifted lognormal with parameters $\mu = -2.76$ and $\sigma = 0.51$. To ensure a zero mean for the error term, the lognormal was shifted by the mean of this distribution, or about .072. Exhibit 5 shows the derivation of these parameters.

This inflation variable was used to model the Cost of Living Adjustment to the indemnity payments. COLAs are usually tied to changes in the state average weekly wage, and I assumed that wage inflation is the same as overall price inflation — a convenient simplification, not necessarily correct. Since most COLAs are capped, I assumed the COLA could not be more than 5% in any year. I also assumed that if inflation is negative, the indemnity amount would not go down. Since COLAs are lagged a year, I assumed the COLA in 1998 is based on 1997 inflation, etc.

2) Medical Inflation

Medical inflation may be higher or lower than inflation, but there is a link between the two: if there were a 20% inflation rate for a sustained period, one would not expect medical inflation to remain at 2%. I thus selected a model of medical inflation that is tied to the overall inflation rate, but with a degree of error allowed. The model was:

$$
\begin{aligned}
&\text{Medical Inflation}_t \\
&= \text{Inflation}_{\text{Year } t} \\
&\quad + \beta[\text{Medical inflation}_{\text{Year } (t-1)} - \text{Inflation}_{\text{Year}(t-1)}] \\
&\quad + [\text{long-term average medical inflation - long-term average inflation}] \\
&\quad + \text{error term}_{\text{Year } t}
\end{aligned}
$$

The error term is assumed to be normally distributed, with a mean of zero.[8]

I used the longest available data series to get these parameters. The Bureau of Labor Statistics has medical CPI numbers back to 1935. For the period 1935 to 1995, average medical inflation was 1.16 percentage points higher than average inflation. This is what I used for the third term of the above expression. I am assuming these long-term trends will continue, although, there is of course no guarantee of this.

The fitted value for β was 0.38, and the error term was normally distributed with a mean of 0, and a standard deviation of 0.027. Exhibit 6 shows the development of this model.

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8 The inflation model had a lognormal error term, but the medical inflation model has a normal error term. The reason was that I had a strong feeling that the error for inflation was skewed, whereas it is less obvious that the difference between overall inflation and medical inflation (which is largely what drives the medical inflation model) is skewed.

3) Investment Yields

I used a very simple model of investment yields. The firm is assumed to invest in one-year bonds that are held to maturity. Consequently, one would never have investment losses. In general, the bond yield would equal the expected inflation rate plus some small premium. However, one should discount using a risk-adjusted rate, and I simply assumed that the risk adjustment equals the premium over the inflation rate, i.e., the rate used for discounting is the same as the inflation rate. Even if inflation is negative, one would not expect interest rates to drop below some threshold (e.g., 2%), so I assumed the risk-adjusted discount rate could not go below zero, i.e., I set the rate for discounting at the greater of zero or the inflation rate.[9]

4) Medical Services Used By Claimant

Medical usage will fluctuate from year to year. In some years, the claimant will use relatively little, while in other years he may require surgery, with large medical bills. The services from year to year may be correlated. For example, if he has surgery this year, the costs of post-operative treatment may keep the costs higher than average in the next year. One can model this process using a similar autoregressive model to the way we modeled inflation:

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9 This is a rather unrealistic model of investment income, but it will be adequate for our purposes. Insurers usually buy longer term investments, especially if they are investing reserves backing lifetime workers' compensation claims. They may also invest in stocks, or other assets, that do not have fixed yields. These complications are beyond the scope of the paper.

It is also beyond the scope of the paper to address the question of whether discounting should be based on the firm's (either the reinsurer or reinsured's) actual investments, or whether it should be based on market discount rates.

Medical $\text{amount}_{\text{Year } t}$

= Long-term average medical amount

+ γ[Medical $\text{amount}_{\text{Year } (t-1)}$ - long-term average medical amount]

+ $\text{error}_{\text{Year } t}$

The long-term average medical amount for this case is, by assumption, \$70,000. Empirically, there does not appear to be a very strong link between last year's medical amount and this year's, so I used $\gamma = .05$. The error term was modeled by a lognormal with $\mu = 10.80089$ and $\sigma = 0.75$. The mean of this lognormal is 65,000, so I shifted the distribution by 65,000 to ensure the error term has a mean of zero.

Running the Model

Each of these parameters was then put into a simulation model. By simulating inflation, medical inflation, and the annual medical amount, one can get a set of input parameters for each simulation. These parameters are then run through the same model as is used for Method 2. The difference is that each time it is run through with different parameters, so that instead of getting a single present value of the future payments, we get a distribution. (Exhibit 7 shows a single simulation from this distribution.)

The means of these distributions, for each layer, are shown below, compared with the results for Methods 1 and 2:

Layer (in $,000s)	Expected Nominal Payments (in $,000s)			Expected Present-Value Payments (in $,000s)		
	Method 1	Method 2	Method 3	Method 1	Method 2	Method 3
130 xs 370	130	130	130	126	126	125
500 xs 500	500	495	495	430	426	426
1,000 xs 1,000	1,000	971	969	679	660	664
3,000 xs 2,000	3,000	2,730	2,715	1,358	1,241	1,247
5,000 xs 5,000	5,000	3,735	3,701	1,388	1,048	1,053
5,000 xs 10,000	1,911	2,647	2,694	399	510	526
5,000 xs 15,000	0	1,704	1909	0	255	288
10,000 xs 20,000	0	1,523	2,317	0	178	271
10,000 xs 30,000	0	375	1,214	0	34	108
10,000 xs 40,000	0	61	673	0	4	49
10,000 xs 50,000	0	7	394	0	0	24
10,000 xs 60,000	0	0	241	0	0	13
10,000 xs 70,000	0	0	154	0	0	7
10,000 xs 80,000	0	0	102	0	0	4
10,000 xs 90,000	0	0	69	0	0	3
Unlimited xs $100MM	0	0	193	0	0	6
Total, all Layers	**11,541**	**14,377**	**17,970**	**4,380**	**4,483**	**4,815**

It is worth noting a few things regarding these results:

- Unlike Methods 1 and 2, Method 3 hits all the reinsurance layers. A less deterministic approach ensures that higher layers will be hit. Thus, layers that might otherwise have been thought to have no possibility of a loss, are shown to have some commutation value.

- The total nominal value of Method 3 is higher than the nominal value of Method 2 (and Method 2 is higher than Method 1, as discussed earlier).

  This is largely explained by the treatment of inflation. The medical and indemnity amounts paid in some future period depend on the products of (1 + inflation) for all prior periods. For example, the amount paid in period 3 depends on what inflation was in periods 1 and 2. The inflation rates are not independent from period to period: they are positively correlated. Thus, the expected value of the product is greater than the product of the expected values, making the overall nominal payments for Method 3 higher than the payments in Method 2.[10]

- The overall present value factor for Method 2 is 31% (= 4,483 ÷ 14,377), but the present value factor for Method 3 is only 27% (= 4,466 ÷ 16,420). In other words, Method 3 has, on average, a steeper discount applied to it.

  The relationship between the present values of Methods 2 and 3 is complex, largely because the assumptions are not consistent between the two methods. Yes, we tried to make them consistent, but the differences in the assumptions become clear once we examine them more carefully.

  Consider the indemnity cost-of-living adjustments. We said that, based on the historical record, inflation averages 4.2% per annum, and this was the number we used for the COLA in Method 2. In Method 3, inflation varies stochastically, with a mean of 4.2%. But our rules for the COLA said that it couldn't be more than 5%, or less than 0%. In Method 3, the

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10 $E(XY) = E(X)E(Y) + cov(X,Y)$. Thus, if X and Y are positively correlated, the expected value of the product exceeds the product of the expected values.

average inflation rate is 4.2%, but the average COLA is not 4.2% because it is sometimes capped. In fact, it averages about 2.98%.

Likewise, we said the discount rate was equal to the inflation rate, but that the discount rate could never go negative. On average, then, the discount rate is higher than 4.2% — about 4.39%. This higher effective discount rate is the main reason for the total present value factor of Method 3 being less than the total present value for Method 2.

The assumptions between Methods 2 and 3 are not the same: Method 2 assumes higher COLAs than Method 3, and lower discount. Running Method 2 at the same average COLA as Method 3 (2.98%), and the same average discount (4.39%), changes the Method 2 present value to $4.124 million, which is 8% lower than the $4.483 million we originally calculated. (See Exhibit 8.)

In general, the relationship between the present values of Methods 2 and 3 will depend on the particular assumptions, and how they interact with the various caps and correlations.

- In the lowest layers, the nominal value of Method 1 is higher than Method 2, and Method 2 is higher than Method 3.[11] This is because

[11] On the earlier table, the nominal values for Methods 2 and 3 look the same at the low retentions. In fact, however, the numbers in the table are rounded. If the complete numbers had been shown, the nominal values in the low layers would be systematically less (though admittedly by a small amount) for Method 3 than for Method 2:

	Nominal Value (in $Thousands)	
Layer	**Method 2**	**Method 3**
1	129.74	129.70
2	494.89	494.55
3	970.56	969.34
4	2,729.68	2,715.21

Method 1 implies these layers will be hit for certain, whereas Methods 2 and 3 recognize that the claimant could die before the layer is penetrated. In addition, Method 3 recognizes that there could be years of unusually low claim amounts, so that it may take longer than expected to breach the retention. This reduces the commutation amount in two ways:

i) The longer it is until the retention is breached, the greater the chance of the claimant dying before breaching the retention.
ii) The longer it is until the retention is breached, the steeper the effect of present valuing.

In higher layers, which have a lower probability of being penetrated, this situation reverses itself: Method 3 gives higher results than Method 2. The upper layers are most vulnerable to a period of sustained high inflation or high claim levels. Methods 1 and 2 assume inflation and claim levels are fixed, so they do not contemplate periods of sustained high inflation or claim levels.

- For the lower layers, where the chances are good that the claimant will live long enough to breach them, Method 2 gives similar results to Method 3. But as the layers get higher, the Method 2 number gets lower and lower as a percentage of Method 3.

	Method 2 Result as Percentage of Method 3 Result	
Layer	**Nominal**	**Present Value**
1	100%	100%
2	100%	99%
3	100%	97%
4	100%	95%
5	99%	90%
6	94%	83%

7	82%	72%
8	56%	48%
9	22%	19%
10	5%	4%
11	1%	1%
Higher Layers	0%	0%

- Note how the present value factor for the losses declines sharply in the higher layers. For example, for the $5 million excess $5 million layer, the present value is $1.053 million, compared to the nominal value of $3.701 million. This translates to a present value factor of 28%. By contrast, in the $10 million excess $90 million layer, the present value factor is only 4%.

ARE THERE FURTHER LAYERS OF DETERMINISM?

This paper has demonstrated that the commutation calculation is significantly affected by making a variety of variables non-deterministic. Have we now stripped away all determinism? Put another way: does this paper describe "the perfect" commutation calculation, or are there further layers of determinism that can, at least in principle, be stripped away?

There are, indeed, further layers of determinism that can be stripped away from a calculation of this nature, although it will become increasingly more difficult to do so. This paper has shown how we can strip away determinism in the levels of inflation, medical utilization, etc. But to measure the paths for these variables, we have relied on statistical measures on past data. Clearly, these historical data may no longer be valid predictors of the future. For example, the paper assumes that the best predictor of medical inflation is the last 60 years of medical CPI information. One can plausibly argue that what drove medical inflation in the 1930s and 1940s was completely different from what drove it in the 1970s and 1980s, and different from what will drive it in future. And it is quite possible that the drivers of inflation will change periodically over the course of the claimant's lifetime.

This same issue applies to other variables. For example, advances in medical care could affect the medical utilization for the claimant's condition — and perhaps render the assumed mortality table redundant.

The next layer of determinism is the models themselves. We have assumed the model stays fixed over the claimant's lifetime, but we can easily imagine a situation where the parameters of the model shift, or the model itself changes.

The problem is that this next layer of determinism is not easily subject to measurement, and hence is not amenable to quantification by the usual actuarial methods. But not being able to quantify does not allow us to say that these items do not exist, and to simply ignore them.

The Economics Of Uncertainty

Economists distinguish between "risk" and "uncertainty."[12] Risk includes those things that can be measured statistically, and uncertainty includes those things that cannot be measured, but which might occur. For example, if I bet on a fair coin coming up heads, I am facing a risk. But if I bet on the chance of intelligent life being found on an as-yet-undiscovered planet, that is uncertainty — I have no way of measuring the associated probabilities.

Most insurance problems consist of a mixture of risk and uncertainty. Insurers are good at dealing with risk. By measuring the probabilities of loss and pooling the risk, we can largely eliminate the risk and get stable losses in the aggregate. It is far more difficult to deal with uncertainty.

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12 The classic reference on risk and uncertainty is Knight (1921). For a more recent discussion of the economics of uncertainty, see O'Driscoll and Rizzo (1985).

In this paper, we have been measuring risk: we have only dealt with those things that can be measured. (Insofar as they cannot be modeled well, there are elements of uncertainty.) The next layer of determinism consists of uncertainty. We have no way of estimating the chances of the inflation model changing, or what the new model might be.

Without making any attempt to measure the effect of uncertainty, we can make some qualitative statements about its effects on commutations. Just as removing earlier layers of determinism increased the commutation amount in the higher layers, so removing yet another layer of determinism will increase the commutation amount in higher layers, and higher layers that would not otherwise have been pierced, will have some commutation value. Why? Under the inflation model postulated in the example in this paper, it is conceivable, but extremely unlikely, that there will be years where inflation will run above, say, 100% a year. (Actuaries who have dealt with foreign insurance and reinsurance may themselves have been burnt by hyperinflation in places like Israel and Argentina.) We can certainly envision unlikely circumstances where the US economy falls apart and there is hyperinflation. This possibility was not included in the data used for fitting the models, and is thus not contemplated in the resulting commutation amount.

All the other variables in the commutation are subject to similar uncertainty: mortality rates might plummet as cures are found for cancer and heart disease; or mortality rates might soar, as a new virus kills half the population. The annual medical usage might drop, if a cure is found for the claimant's ailment, which was previously thought to be permanent. Or the cost of medical care might soar as a new drug is discovered that greatly improves the claimant's quality of life, at twice the cost. What if the government takes over the entire health-care system, and insurers are no longer responsible for medical care costs?

We can dream up many different situations that will change what insurers owe to claimants. We can put probabilities on none of these, and we also know that there are many possibilities that we may not even think of, until they actually happen.

In commutations, it is common to ignore this uncertainty, and to commute some of the very high layers without payment. This is unwarranted. Commuting reinsurance is really a matter of pricing future possibilities, and reinsurers do not give away free layers, even if they have only a remote chance of being hit. For example, suppose I want to buy workers' compensation reinsurance for a layer of $1 million excess of $800 million. (To avoid catastrophe issues, let us assume the reinsurance is per claim, not per occurrence.) There has never been a workers' compensation claim that large, or even remotely close to it. Yet, would a reinsurer be willing to give the layer away free (assuming they have no costs to service the contract)? Of course they won't. Reinsurers recognize the remote possibility of having to pay on this contract, and they need to charge for that risk. The risk is remote, but remote is not the same as non-existent. The chance of the layer being hit is not measurable, but not-measurable is not the same as zero.

The pricing issues also apply to commutations. There is no reason why a cedent should be willing to commute a layer for nothing, even when the actuarial calculations (at some level of determinism) say there is no chance of hitting the layer. Though there is far less uncertainty at the time of a commutation than there was when the contract was written, there is still enough uncertainty that payment for the cedent re-assuming this risk is warranted.

Other Lines of Business; Pricing and Reserving, Too

The issues discussed in this paper apply more broadly than just to workers' compensation commutations. A commutation for, say, a General Liability treaty would usually develop the expected losses to ultimate, and commute based on the discounted value of those losses. But this ignores certain risks that are transferred back to the ceding company in the commutation. For example, a GL treaty being commuted in 1978 would have relieved the reinsurer for liability for environmental claims that were generated by the Superfund law, which passed a couple of years later. It was unknown, at the time of the commutation, that the cedent was giving up coverage for this risk, but it was not unknown that the cedent was taking the risk of some such change in the future. Just as a company selling GL reinsurance will not give away remote layers free of charge, so the commutation should not be free for these layers either.

Other lines of business have the same levels of determinism as do workers' compensation. The difference is that for workers' compensation we can do the calculations on a claim-by-claim basis, which helps to lay bare many of the underlying assumptions.

And it is not just commutations that are affected by determinism. It applies to regular pricing and reserving work as well. The clearest example would be the reserving of workers' compensation reinsurance, where the methods used in this paper can be directly applied. But for pricing and reserving of any excess insurance or reinsurance, it is important to keep in mind the problems of determinism. If we simply assume the future will turn out to be what was expected, or that the future will follow the patterns of the past, we are bound to be led astray. The scary part of writing insurance is the uncertainty of what the future will bring. The uncertainty cannot be quantified, but all too often we stick our heads in the sand and assume that if something cannot be quantified, it doesn't exist.

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Exhibit 1

1990 US Life Table (Males)

Age	l(x)	Life Expectancy
0	100,000.0	71.8
1	98,969.0	71.6
2	98,894.0	70.6
3	98,840.0	69.7
4	98,799.0	68.7
5	98,765.0	67.7
6	98,735.0	66.8
7	98,707.0	65.8
8	98,680.0	64.8
9	98,657.0	63.8
10	98,638.0	62.8
11	98,623.0	61.8
12	98,608.0	60.8
13	98,586.0	59.9
14	98,547.0	58.9
15	98,485.0	57.9
16	98,397.0	57.0
17	98,285.0	56.0
18	98,154.0	55.1
19	98,011.0	54.2
20	97,863.0	53.3
21	97,710.0	52.3
22	97,551.0	51.4
23	97,388.0	50.5
24	97,221.0	49.6
25	97,052.0	48.7
26	96,881.0	47.8
27	96,707.0	46.9
28	96,530.0	45.9
29	96,348.0	45.0
30	96,159.0	44.1
31	95,962.0	43.2
32	95,758.0	42.3
33	95,545.0	41.4
34	95,322.0	40.5
35	95,089.0	39.6
36	94,843.0	38.7
37	94,585.0	37.8
38	94,316.0	36.9
39	94,038.0	36.0
40	93,753.0	35.1
41	93,460.0	34.2
42	93,157.0	33.3
43	92,840.0	32.4
44	92,505.0	31.6
45	92,147.0	30.7
46	91,764.0	29.8
47	91,352.0	28.9
48	90,908.0	28.1
49	90,429.0	27.2
50	89,912.0	26.4
51	89,352.0	25.5
52	88,745.0	24.7
53	88,084.0	23.9
54	87,363.0	23.1
55	86,576.0	22.3
56	85,719.0	21.5
57	84,788.0	20.7
58	83,777.0	20.0
59	82,678.0	19.2
60	81,485.0	18.5
61	80,194.0	17.8
62	78,803.0	17.1
63	77,314.0	16.4
64	75,729.0	15.8
65	74,051.0	15.1
66	72,280.0	14.5
67	70,414.0	13.8
68	68,445.0	13.2
69	66,364.0	12.6
70	64,164.0	12.0
71	61,847.0	11.5
72	59,419.0	10.9
73	56,885.0	10.4
74	54,249.0	9.9
75	51,519.0	9.4
76	48,704.0	8.9
77	45,816.0	8.4
78	42,867.0	7.9
79	39,872.0	7.5
80	36,848.0	7.1
81	33,811.0	6.7
82	30,782.0	6.3
83	27,782.0	5.9
84	24,834.0	5.5
85	21,962.0	5.2
86	19,216.8	4.9
87	16,607.4	4.5
88	14,157.7	4.2
89	11,889.0	3.9
90	9,819.5	3.7
91	7,962.6	3.4
92	6,326.9	3.2
93	4,915.0	2.9
94	3,723.5	2.7
95	2,743.0	2.5
96	1,958.3	2.3
97	1,349.7	2.1
98	894.0	1.9
99	566.2	1.8
100	340.6	1.6
101	193.2	1.5
102	102.4	1.3
103	50.1	1.2
104	22.3	1.1
105	8.9	1.0
106	3.1	0.9
107	0.9	0.8
108	0.2	0.7
109	0.0	0.5
110	0.0	

Source: Vital Statistics of the United States, 1990 [US Department of Health and Human Services, 1994]
Note that the published tables extend only to age 85; beyond 85, the numbers are extrapolations.

Exhibit 2

Inflation:
Consumer Price Index and Medical Consumer Price Index

	Index at December		Annual Inflation			Index at December		Annual Inflation	
		Medical		Medical			Medical		Medical
Year	**CPI**	**CPI**	**CPI**	**CPI**	**Year**	**CPI**	**CPI**	**CPI**	**CPI**
1935	13.8	10.2			1966	32.9	27.2	3.5%	6.7%
1936	14.0	10.2	1.4%	0.0%	1967	33.9	28.9	3.0%	6.3%
1937	14.4	10.3	2.9%	1.0%	1968	35.5	30.7	4.7%	6.2%
1938	14.0	10.3	-2.8%	0.0%	1969	37.7	32.6	6.2%	6.2%
1939	14.0	10.4	0.0%	1.0%	1970	39.8	35.0	5.6%	7.4%
1940	14.1	10.4	0.7%	0.0%	1971	41.1	36.6	3.3%	4.6%
1941	15.5	10.5	9.9%	1.0%	1972	42.5	37.8	3.4%	3.3%
1942	16.9	10.9	9.0%	3.8%	1973	46.2	39.8	8.7%	5.3%
1943	17.4	11.4	3.0%	4.6%	1974	51.9	44.8	12.3%	12.6%
1944	17.8	11.7	2.3%	2.6%	1975	55.5	49.2	6.9%	9.8%
1945	18.2	12.0	2.2%	2.6%	1976	58.2	54.1	4.9%	10.0%
1946	21.5	13.0	18.1%	8.3%	1977	62.1	58.9	6.7%	8.9%
1947	23.4	13.9	8.8%	6.9%	1978	67.7	64.1	9.0%	8.8%
1948	24.1	14.7	3.0%	5.8%	1979	76.7	70.6	13.3%	10.1%
1949	23.6	14.9	-2.1%	1.4%	1980	86.3	77.6	12.5%	9.9%
1950	25.0	15.4	5.9%	3.4%	1981	94.0	87.3	8.9%	12.5%
1951	26.5	16.3	6.0%	5.8%	1982	97.6	96.9	3.8%	11.0%
1952	26.7	17.0	0.8%	4.3%	1983	101.3	103.1	3.8%	6.4%
1953	26.9	17.6	0.7%	3.5%	1984	105.3	109.4	3.9%	6.1%
1954	26.7	18.0	-0.7%	2.3%	1985	109.3	116.8	3.8%	6.8%
1955	26.8	18.6	0.4%	3.3%	1986	110.5	125.8	1.1%	7.7%
1956	27.6	19.2	3.0%	3.2%	1987	115.4	133.1	4.4%	5.8%
1957	28.4	20.1	2.9%	4.7%	1988	120.5	142.3	4.4%	6.9%
1958	28.9	21.0	1.8%	4.5%	1989	126.1	154.4	4.6%	8.5%
1959	29.4	21.8	1.7%	3.8%	1990	133.8	169.2	6.1%	9.6%
1960	29.8	22.5	1.4%	3.2%	1991	137.9	182.6	3.1%	7.9%
1961	30.0	23.2	0.7%	3.1%	1992	141.9	194.7	2.9%	6.6%
1962	30.4	23.7	1.3%	2.2%	1993	145.8	205.2	2.7%	5.4%
1963	30.9	24.3	1.6%	2.5%	1994	149.7	215.3	2.7%	4.9%
1964	31.2	24.8	1.0%	2.1%	1995	153.5	223.8	2.5%	3.9%
1965	31.8	25.5	1.9%	2.8%					
							Average	**4.2%**	**5.3%**

Source: US Department of Labor, Bureau of Labor Statistics

Completely Deterministic commutation calculation

Parameters:

(A)	Evaluation Date:	1/1/97
(B)	Age at evaluation date:	35
(C)	Annual indemnity payment	20,000
(D)	Annual medical payment: (at mid-1996 price levels)	70,000
(E)	Indemnity paid to date	70,000
(F)	Medical paid to date	300,000
(G)	Life expectancy:	39.6
(H)	Cost-of-Living Adjustment:	4.2%
(I)	Medical Inflation Rate:	5.36%
(J)	Annual Discount Rate:	4.2%

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cumulative of (5)
1996 and prior		*70,000*		*300,000*	*370,000*	*370,000*
1997	4.2%	20,840	5.36%	73,752	94,592	464,592
1998	4.2%	21,715	5.36%	77,705	99,420	564,012
1999	4.2%	22,627	5.36%	81,870	104,497	668,510
2000	4.2%	23,578	5.36%	86,258	109,836	778,346
2001	4.2%	24,568	5.36%	90,882	115,450	893,796
2002	4.2%	25,600	5.36%	95,753	121,353	1,015,148
2003	4.2%	26,675	5.36%	100,885	127,560	1,142,709
2004	4.2%	27,795	5.36%	106,293	134,088	1,276,797
2005	4.2%	28,963	5.36%	111,990	140,953	1,417,750
2006	4.2%	30,179	5.36%	117,993	148,172	1,565,922
2007	4.2%	31,447	5.36%	124,317	155,764	1,721,686
2008	4.2%	32,767	5.36%	130,981	163,748	1,885,434
2009	4.2%	34,144	5.36%	138,001	172,145	2,057,579
2010	4.2%	35,578	5.36%	145,398	180,976	2,238,555
2011	4.2%	37,072	5.36%	153,191	190,263	2,428,818
2012	4.2%	38,629	5.36%	161,402	200,031	2,628,850
2013	4.2%	40,251	5.36%	170,054	210,305	2,839,155
2014	4.2%	41,942	5.36%	179,169	221,111	3,060,265
2015	4.2%	43,704	5.36%	188,772	232,476	3,292,741
2016	4.2%	45,539	5.36%	198,890	244,429	3,537,170
2017	4.2%	47,452	5.36%	209,551	257,002	3,794,172
2018	4.2%	49,445	5.36%	220,783	270,227	4,064,400
2019	4.2%	51,521	5.36%	232,617	284,138	4,348,537
2020	4.2%	53,685	5.36%	245,085	298,770	4,647,308
2021	4.2%	55,940	5.36%	258,221	314,161	4,961,469
2022	4.2%	58,290	5.36%	272,062	330,352	5,291,820
2023	4.2%	60,738	5.36%	286,644	347,382	5,639,203
2024	4.2%	63,289	5.36%	302,009	365,297	6,004,500
2025	4.2%	65,947	5.36%	318,196	384,143	6,388,643
2026	4.2%	68,717	5.36%	335,252	403,968	6,792,611
2027	4.2%	71,603	5.36%	353,221	424,824	7,217,435

Exhibit 3, Page 2

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cumulative of (5)
2028	4.2%	74,610	5.36%	372,154	446,764	7,664,199
2029	4.2%	77,744	5.36%	392,101	469,845	8,134,044
2030	4.2%	81,009	5.36%	413,118	494,127	8,628,170
2031	4.2%	84,411	5.36%	435,261	519,672	9,147,843
2032	4.2%	87,956	5.36%	458,591	546,547	9,694,390
2033	4.2%	91,651	5.36%	483,171	574,822	10,269,212
2034	4.2%	95,500	5.36%	509,069	604,569	10,873,781
2035	4.2%	99,511	5.36%	536,356	635,867	11,509,648
2036	4.2%	62,214	5.36%	339,063	401,277	**11,910,925**
Total		2,104,844		9,806,081		

Future payments = 11,910,925 - 370,000 = 11,540,925

	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Cumulative	**Incremental Payments By Layer**					
	Total	**$500,000 xs**	**$500,000 xs**	**$1 million xs**	**$3 million xs**	**$5 million xs**	**$5 million xs**
Year	**Payment**	$370,000	$500,000	$1 million	$2 million	$5 million	$10 million
	Cumulative of (5)						
1996 and prior	370,000	0	0	0	0	0	0
1997	464,592	94,592	0	0	0	0	0
1998	564,012	35,408	64,012	0	0	0	0
1999	668,510	0	104,497	0	0	0	0
2000	778,346	0	109,836	0	0	0	0
2001	893,796	0	115,450	0	0	0	0
2002	1,015,148	0	106,204	15,148	0	0	0
2003	1,142,709	0	0	127,560	0	0	0
2004	1,276,797	0	0	134,088	0	0	0
2005	1,417,750	0	0	140,953	0	0	0
2006	1,565,922	0	0	148,172	0	0	0
2007	1,721,686	0	0	155,764	0	0	0
2008	1,885,434	0	0	163,748	0	0	0
2009	2,057,579	0	0	114,566	57,579	0	0
2010	2,238,555	0	0	0	180,976	0	0
2011	2,428,818	0	0	0	190,263	0	0
2012	2,628,850	0	0	0	200,031	0	0
2013	2,839,155	0	0	0	210,305	0	0
2014	3,060,265	0	0	0	221,111	0	0
2015	3,292,741	0	0	0	232,476	0	0
2016	3,537,170	0	0	0	244,429	0	0
2017	3,794,172	0	0	0	257,002	0	0
2018	4,064,400	0	0	0	270,227	0	0
2019	4,348,537	0	0	0	284,138	0	0
2020	4,647,308	0	0	0	298,770	0	0
2021	4,961,469	0	0	0	314,161	0	0
2022	5,291,820	0	0	0	38,531	291,820	0
2023	5,639,203	0	0	0	0	347,382	0
2024	6,004,500	0	0	0	0	365,297	0
2025	6,388,643	0	0	0	0	384,143	0
2026	6,792,611	0	0	0	0	403,968	0
2027	7,217,435	0	0	0	0	424,824	0
2028	7,664,199	0	0	0	0	446,764	0
2029	8,134,044	0	0	0	0	469,845	0
2030	8,628,170	0	0	0	0	494,127	0
2031	9,147,843	0	0	0	0	519,672	0
2032	9,694,390	0	0	0	0	546,547	0
2033	10,269,212	0	0	0	0	305,610	269,212
2034	10,873,781	0	0	0	0	0	604,569
2035	11,509,648	0	0	0	0	0	635,867
2036	**11,910,925**	0	0	0	0	0	401,277

	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	Present	Discounted Value by Layer						
	Value	$500,000 xs	$500,000 xs	$1 million xs	$3 million xs	$5 million xs	$5 million xs	All Layers
Year	Factor	$370,000	$500,000	$1 million	$2 million	$5 million	$10 million	Combined
1996 and prior								
1997	0.9796	92,666	0	0	0	0	0	92,666
1998	0.9402	33,289	60,181	0	0	0	0	93,470
1999	0.9023	0	94,284	0	0	0	0	94,284
2000	0.8659	0	95,106	0	0	0	0	95,106
2001	0.8310	0	95,937	0	0	0	0	95,937
2002	0.7975	0	84,697	12,081	0	0	0	96,778
2003	0.7653	0	0	97,628	0	0	0	97,628
2004	0.7345	0	0	98,488	0	0	0	98,488
2005	0.7049	0	0	99,357	0	0	0	99,357
2006	0.6765	0	0	100,236	0	0	0	100,236
2007	0.6492	0	0	101,124	0	0	0	101,124
2008	0.6230	0	0	102,023	0	0	0	102,023
2009	0.5979	0	0	68,503	34,428	0	0	102,931
2010	0.5738	0	0	0	103,850	0	0	103,850
2011	0.5507	0	0	0	104,779	0	0	104,779
2012	0.5285	0	0	0	105,718	0	0	105,718
2013	0.5072	0	0	0	106,668	0	0	106,668
2014	0.4868	0	0	0	107,628	0	0	107,628
2015	0.4671	0	0	0	108,599	0	0	108,599
2016	0.4483	0	0	0	109,580	0	0	109,580
2017	0.4302	0	0	0	110,573	0	0	110,573
2018	0.4129	0	0	0	111,577	0	0	111,577
2019	0.3963	0	0	0	112,591	0	0	112,591
2020	0.3803	0	0	0	113,618	0	0	113,618
2021	0.3650	0	0	0	114,655	0	0	114,655
2022	0.3502	0	0	0	13,495	102,209	0	115,704
2023	0.3361	0	0	0	0	116,765	0	116,765
2024	0.3226	0	0	0	0	117,838	0	117,838
2025	0.3096	0	0	0	0	118,922	0	118,922
2026	0.2971	0	0	0	0	120,019	0	120,019
2027	0.2851	0	0	0	0	121,128	0	121,128
2028	0.2736	0	0	0	0	122,249	0	122,249
2029	0.2626	0	0	0	0	123,383	0	123,383
2030	0.2520	0	0	0	0	124,529	0	124,529
2031	0.2419	0	0	0	0	125,688	0	125,688
2032	0.2321	0	0	0	0	126,860	0	126,860
2033	0.2228	0	0	0	0	68,076	59,968	128,045
2034	0.2138	0	0	0	0	0	129,243	129,243
2035	0.2052	0	0	0	0	0	130,454	130,454
2036	0.1969	0	0	0	0	0	79,008	79,008
Total		125,955	430,206	679,440	1,357,759	1,387,664	398,673	4,379,697

Method 2: Stochastic Mortality (Other inputs deterministic)

	Parameters:	
(A)	Evaluation Date:	1/1/97
(B)	Current Age:	35
(C)	Annual Indemnity Payment	20,000
(D)	Annual Medical Payment (at mid-1996 price levels)	70,000
(E)	Indemnity Paid to Date	70,000
(F)	Medical Paid to Date:	300,000
(G)	Cost-of-Living Adjustment	4.2%
(H)	Medical Inflation Rate:	5.36%
(I)	Annual Discount Rate:	4.2%

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant living to mid-year	(8) Present Value Factor	(9) Discount for mortality & investment income (7) x (8)
1996 and prior		70,000		300,000	370,000	370,000			
1997	4.2%	20,840	5.36%	73,752	94,592	464,592	0.999	0.9796	0.9784
1998	4.2%	21,715	5.36%	77,705	99,420	564,012	0.996	0.9402	0.9364
1999	4.2%	22,627	5.36%	81,870	104,497	668,510	0.993	0.9023	0.8962
2000	4.2%	23,578	5.36%	86,258	109,836	778,346	0.990	0.8659	0.8576
2001	4.2%	24,568	5.36%	90,882	115,450	893,796	0.987	0.8310	0.8206
2002	4.2%	25,600	5.36%	95,753	121,353	1,015,148	0.984	0.7975	0.7851
2003	4.2%	26,675	5.36%	100,885	127,560	1,142,709	0.981	0.7653	0.7510
2004	4.2%	27,795	5.36%	106,293	134,088	1,276,797	0.978	0.7345	0.7184
2005	4.2%	28,963	5.36%	111,990	140,953	1,417,750	0.975	0.7049	0.6870
2006	4.2%	30,179	5.36%	117,993	148,172	1,565,922	0.971	0.6765	0.6568
2007	4.2%	31,447	5.36%	124,317	155,764	1,721,686	0.967	0.6492	0.6278
2008	4.2%	32,767	5.36%	130,981	163,748	1,885,434	0.963	0.6230	0.5999
2009	4.2%	34,144	5.36%	138,001	172,145	2,057,579	0.958	0.5979	0.5730
2010	4.2%	35,578	5.36%	145,398	180,976	2,238,555	0.954	0.5738	0.5472
2011	4.2%	37,072	5.36%	153,191	190,263	2,428,818	0.948	0.5507	0.5222
2012	4.2%	38,629	5.36%	161,402	200,031	2,628,850	0.943	0.5285	0.4982
2013	4.2%	40,251	5.36%	170,054	210,305	2,839,155	0.936	0.5072	0.4750
2014	4.2%	41,942	5.36%	179,169	221,111	3,060,265	0.930	0.4868	0.4526
2015	4.2%	43,704	5.36%	188,772	232,476	3,292,741	0.923	0.4671	0.4310
2016	4.2%	45,539	5.36%	198,890	244,429	3,537,170	0.915	0.4483	0.4100
2017	4.2%	47,452	5.36%	209,551	257,002	3,794,172	0.906	0.4302	0.3898
2018	4.2%	49,445	5.36%	220,783	270,227	4,064,400	0.897	0.4129	0.3702
2019	4.2%	51,521	5.36%	232,617	284,138	4,348,537	0.886	0.3963	0.3512
2020	4.2%	53,685	5.36%	245,085	298,770	4,647,308	0.875	0.3803	0.3328
2021	4.2%	55,940	5.36%	258,221	314,161	4,961,469	0.863	0.3650	0.3150
2022	4.2%	58,290	5.36%	272,062	330,352	5,291,820	0.850	0.3502	0.2978
2023	4.2%	60,738	5.36%	286,644	347,382	5,639,203	0.836	0.3361	0.2810
2024	4.2%	63,289	5.36%	302,009	365,297	6,004,500	0.821	0.3226	0.2648
2025	4.2%	65,947	5.36%	318,196	384,143	6,388,643	0.805	0.3096	0.2491
2026	4.2%	68,717	5.36%	335,252	403,968	6,792,611	0.788	0.2971	0.2340
2027	4.2%	71,603	5.36%	353,221	424,824	7,217,435	0.769	0.2851	0.2194
2028	4.2%	74,610	5.36%	372,154	446,764	7,664,199	0.750	0.2736	0.2053
2029	4.2%	77,744	5.36%	392,101	469,845	8,134,044	0.730	0.2626	0.1917
2030	4.2%	81,009	5.36%	413,118	494,127	8,628,170	0.709	0.2520	0.1786
2031	4.2%	84,411	5.36%	435,261	519,672	9,147,843	0.686	0.2419	0.1660
2032	4.2%	87,956	5.36%	458,591	546,547	9,694,390	0.663	0.2321	0.1538
2033	4.2%	91,651	5.36%	483,171	574,822	10,269,212	0.638	0.2228	0.1420
2034	4.2%	95,500	5.36%	509,069	604,569	10,873,781	0.612	0.2138	0.1307
2035	4.2%	99,511	5.36%	536,356	635,867	11,509,648	0.584	0.2052	0.1199
2036	4.2%	103,690	5.36%	565,104	668,795	12,178,443	0.556	0.1969	0.1095
2037	4.2%	108,045	5.36%	595,394	703,439	12,881,882	0.527	0.1890	0.0996
2038	4.2%	112,583	5.36%	627,307	739,890	13,621,772	0.497	0.1813	0.0901
2039	4.2%	117,312	5.36%	660,931	778,242	14,400,014	0.466	0.1740	0.0812

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant living to mid-year	(8) Present Value Factor	(9) Discount for mortality & investment income (7) x (8)
2040	4.2%	122,239	5.36%	696,356	818,595	15,218,610	0.435	0.1670	0.0727
2041	4.2%	127,373	5.36%	733,681	861,054	16,079,664	0.403	0.1603	0.0647
2042	4.2%	132,723	5.36%	773,006	905,729	16,985,393	0.372	0.1538	0.0572
2043	4.2%	138,297	5.36%	814,440	952,737	17,938,129	0.340	0.1476	0.0501
2044	4.2%	144,105	5.36%	858,094	1,002,199	18,940,328	0.308	0.1417	0.0436
2045	4.2%	150,158	5.36%	904,087	1,054,245	19,994,574	0.277	0.1360	0.0376
2046	4.2%	156,465	5.36%	952,546	1,109,011	21,103,585	0.246	0.1305	0.0321
2047	4.2%	163,036	5.36%	1,003,603	1,166,639	22,270,224	0.217	0.1252	0.0271
2048	4.2%	169,884	5.36%	1,057,396	1,227,280	23,497,503	0.188	0.1202	0.0226
2049	4.2%	177,019	5.36%	1,114,072	1,291,091	24,788,594	0.162	0.1153	0.0187
2050	4.2%	184,453	5.36%	1,173,787	1,358,240	26,146,834	0.137	0.1107	0.0152
2051	4.2%	192,201	5.36%	1,236,702	1,428,902	27,575,737	0.114	0.1062	0.0121
2052	4.2%	200,273	5.36%	1,302,989	1,503,262	29,078,998	0.094	0.1019	0.0095
2053	4.2%	208,684	5.36%	1,372,829	1,581,513	30,660,512	0.075	0.0978	0.0074
2054	4.2%	217,449	5.36%	1,446,413	1,663,862	32,324,374	0.059	0.0939	0.0055
2055	4.2%	226,582	5.36%	1,523,940	1,750,522	34,074,896	0.045	0.0901	0.0041
2056	4.2%	236,098	5.36%	1,605,624	1,841,722	35,916,618	0.034	0.0865	0.0029
2057	4.2%	246,015	5.36%	1,691,685	1,937,700	37,854,318	0.025	0.0830	0.0021
2058	4.2%	256,347	5.36%	1,782,359	2,038,707	39,893,025	0.017	0.0796	0.0014
2059	4.2%	267,114	5.36%	1,877,894	2,145,008	42,038,032	0.012	0.0764	0.0009
2060	4.2%	278,333	5.36%	1,978,549	2,256,882	44,294,914	0.008	0.0733	0.0006
2061	4.2%	290,023	5.36%	2,084,599	2,374,622	46,669,535	0.005	0.0704	0.0003
2062	4.2%	302,203	5.36%	2,196,334	2,498,537	49,168,073	0.003	0.0676	0.0002
2063	4.2%	314,896	5.36%	2,314,057	2,628,953	51,797,026	0.002	0.0648	0.0001
2064	4.2%	328,122	5.36%	2,438,091	2,766,212	54,563,238	0.001	0.0622	0.0000
2065	4.2%	341,903	5.36%	2,568,772	2,910,675	57,473,913	0.0004	0.0597	0.0000
2066	4.2%	356,263	5.36%	2,706,459	3,062,721	60,536,634	0.0002	0.0573	0.0000
2067	4.2%	371,226	5.36%	2,851,525	3,222,750	63,759,385	0.0001	0.0550	0.0000
2068	4.2%	386,817	5.36%	3,004,366	3,391,184	67,150,568	0.00002	0.0528	0.0000
2069	4.2%	403,064	5.36%	3,165,400	3,568,464	70,719,032	0.00001	0.0507	0.0000
2070	4.2%	419,992	5.36%	3,335,066	3,755,058	74,474,091	0.000001	0.0486	0.0000
2071	4.2%	437,632	5.36%	3,513,825	3,951,457	78,425,548	0.0000002	0.0467	0.0000

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
	Incremental Payments by Layer												
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
1996 and prior													
1997	94,592	0	0	0	0	0	0	0	0	0	0	0	0
1998	35,408	64,012	0	0	0	0	0	0	0	0	0	0	0
1999	0	104,497	0	0	0	0	0	0	0	0	0	0	0
2000	0	109,836	0	0	0	0	0	0	0	0	0	0	0
2001	0	115,450	0	0	0	0	0	0	0	0	0	0	0
2002	0	106,204	15,148	0	0	0	0	0	0	0	0	0	0
2003	0	0	127,560	0	0	0	0	0	0	0	0	0	0
2004	0	0	134,088	0	0	0	0	0	0	0	0	0	0
2005	0	0	140,953	0	0	0	0	0	0	0	0	0	0
2006	0	0	148,172	0	0	0	0	0	0	0	0	0	0
2007	0	0	155,764	0	0	0	0	0	0	0	0	0	0
2008	0	0	163,748	0	0	0	0	0	0	0	0	0	0
2009	0	0	114,566	57,579	0	0	0	0	0	0	0	0	0
2010	0	0	0	180,976	0	0	0	0	0	0	0	0	0
2011	0	0	0	190,263	0	0	0	0	0	0	0	0	0
2012	0	0	0	200,031	0	0	0	0	0	0	0	0	0
2013	0	0	0	210,305	0	0	0	0	0	0	0	0	0
2014	0	0	0	221,111	0	0	0	0	0	0	0	0	0
2015	0	0	0	232,476	0	0	0	0	0	0	0	0	0
2016	0	0	0	244,429	0	0	0	0	0	0	0	0	0
2017	0	0	0	257,002	0	0	0	0	0	0	0	0	0
2018	0	0	0	270,227	0	0	0	0	0	0	0	0	0
2019	0	0	0	284,138	0	0	0	0	0	0	0	0	0
2020	0	0	0	298,770	0	0	0	0	0	0	0	0	0
2021	0	0	0	314,161	0	0	0	0	0	0	0	0	0
2022	0	0	0	38,531	291,820	0	0	0	0	0	0	0	0
2023	0	0	0	0	347,382	0	0	0	0	0	0	0	0
2024	0	0	0	0	365,297	0	0	0	0	0	0	0	0
2025	0	0	0	0	384,143	0	0	0	0	0	0	0	0
2026	0	0	0	0	403,968	0	0	0	0	0	0	0	0
2027	0	0	0	0	424,824	0	0	0	0	0	0	0	0
2028	0	0	0	0	446,764	0	0	0	0	0	0	0	0
2029	0	0	0	0	469,845	0	0	0	0	0	0	0	0
2030	0	0	0	0	494,127	0	0	0	0	0	0	0	0
2031	0	0	0	0	519,672	0	0	0	0	0	0	0	0
2032	0	0	0	0	546,547	0	0	0	0	0	0	0	0
2033	0	0	0	0	305,610	269,212	0	0	0	0	0	0	0

Exhibit 4, page 4

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
	Incremental Payments by Layer												
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
2034	0	0	0	0	0	604,569	0	0	0	0	0	0	0
2035	0	0	0	0	0	635,867	0	0	0	0	0	0	0
2036	0	0	0	0	0	668,795	0	0	0	0	0	0	0
2037	0	0	0	0	0	703,439	0	0	0	0	0	0	0
2038	0	0	0	0	0	739,890	0	0	0	0	0	0	0
2039	0	0	0	0	0	778,242	0	0	0	0	0	0	0
2040	0	0	0	0	0	599,986	218,610	0	0	0	0	0	0
2041	0	0	0	0	0	0	861,054	0	0	0	0	0	0
2042	0	0	0	0	0	0	905,729	0	0	0	0	0	0
2043	0	0	0	0	0	0	952,737	0	0	0	0	0	0
2044	0	0	0	0	0	0	1,002,199	0	0	0	0	0	0
2045	0	0	0	0	0	0	1,054,245	0	0	0	0	0	0
2046	0	0	0	0	0	0	5,426	1,103,585	0	0	0	0	0
2047	0	0	0	0	0	0	0	1,166,639	0	0	0	0	0
2048	0	0	0	0	0	0	0	1,227,280	0	0	0	0	0
2049	0	0	0	0	0	0	0	1,291,091	0	0	0	0	0
2050	0	0	0	0	0	0	0	1,358,240	0	0	0	0	0
2051	0	0	0	0	0	0	0	1,428,902	0	0	0	0	0
2052	0	0	0	0	0	0	0	1,503,262	0	0	0	0	0
2053	0	0	0	0	0	0	0	921,002	660,512	0	0	0	0
2054	0	0	0	0	0	0	0	0	1,663,862	0	0	0	0
2055	0	0	0	0	0	0	0	0	1,750,522	0	0	0	0
2056	0	0	0	0	0	0	0	0	1,841,722	0	0	0	0
2057	0	0	0	0	0	0	0	0	1,937,700	0	0	0	0
2058	0	0	0	0	0	0	0	0	2,038,707	0	0	0	0
2059	0	0	0	0	0	0	0	0	106,975	2,038,032	0	0	0
2060	0	0	0	0	0	0	0	0	0	2,256,882	0	0	0
2061	0	0	0	0	0	0	0	0	0	2,374,622	0	0	0
2062	0	0	0	0	0	0	0	0	0	2,498,537	0	0	0
2063	0	0	0	0	0	0	0	0	0	831,927	1,797,026	0	0
2064	0	0	0	0	0	0	0	0	0	0	2,766,212	0	0
2065	0	0	0	0	0	0	0	0	0	0	2,910,675	0	0
2066	0	0	0	0	0	0	0	0	0	0	2,526,087	536,634	0
2067	0	0	0	0	0	0	0	0	0	0	0	3,222,750	0
2068	0	0	0	0	0	0	0	0	0	0	0	3,391,184	0
2069	0	0	0	0	0	0	0	0	0	0	0	2,849,432	719,032
2070	0	0	0	0	0	0	0	0	0	0	0	0	3,755,058
2071	0	0	0	0	0	0	0	0	0	0	0	0	3,951,457
	130,000	500,000	1,000,000	3,000,000	5,000,000	5,000,000	5,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	8,425,548

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 x Column 9

	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
1996 and prior													
1997	92,546	0	0	0	0	0	0	0	0	0	0	0	0
1998	33,158	59,944	0	0	0	0	0	0	0	0	0	0	0
1999	0	93,651	0	0	0	0	0	0	0	0	0	0	0
2000	0	94,194	0	0	0	0	0	0	0	0	0	0	0
2001	0	94,733	0	0	0	0	0	0	0	0	0	0	0
2002	0	83,377	11,892	0	0	0	0	0	0	0	0	0	0
2003	0	0	95,800	0	0	0	0	0	0	0	0	0	0
2004	0	0	96,323	0	0	0	0	0	0	0	0	0	0
2005	0	0	96,832	0	0	0	0	0	0	0	0	0	0
2006	0	0	97,323	0	0	0	0	0	0	0	0	0	0
2007	0	0	97,792	0	0	0	0	0	0	0	0	0	0
2008	0	0	98,234	0	0	0	0	0	0	0	0	0	0
2009	0	0	65,651	32,995	0	0	0	0	0	0	0	0	0
2010	0	0	0	99,022	0	0	0	0	0	0	0	0	0
2011	0	0	0	99,359	0	0	0	0	0	0	0	0	0
2012	0	0	0	99,651	0	0	0	0	0	0	0	0	0
2013	0	0	0	99,892	0	0	0	0	0	0	0	0	0
2014	0	0	0	100,073	0	0	0	0	0	0	0	0	0
2015	0	0	0	100,187	0	0	0	0	0	0	0	0	0
2016	0	0	0	100,223	0	0	0	0	0	0	0	0	0
2017	0	0	0	100,175	0	0	0	0	0	0	0	0	0
2018	0	0	0	100,036	0	0	0	0	0	0	0	0	0
2019	0	0	0	99,796	0	0	0	0	0	0	0	0	0
2020	0	0	0	99,445	0	0	0	0	0	0	0	0	0
2021	0	0	0	98,971	0	0	0	0	0	0	0	0	0
2022	0	0	0	11,473	86,892	0	0	0	0	0	0	0	0
2023	0	0	0	0	97,621	0	0	0	0	0	0	0	0
2024	0	0	0	0	96,733	0	0	0	0	0	0	0	0
2025	0	0	0	0	95,701	0	0	0	0	0	0	0	0
2026	0	0	0	0	94,524	0	0	0	0	0	0	0	0
2027	0	0	0	0	93,201	0	0	0	0	0	0	0	0
2028	0	0	0	0	91,726	0	0	0	0	0	0	0	0
2029	0	0	0	0	90,088	0	0	0	0	0	0	0	0
2030	0	0	0	0	88,273	0	0	0	0	0	0	0	0
2031	0	0	0	0	86,265	0	0	0	0	0	0	0	0
2032	0	0	0	0	84,057	0	0	0	0	0	0	0	0
2033	0	0	0	0	43,408	38,239	0	0	0	0	0	0	0

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 x Column 9

	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
2034	0	0	0	0	0	79,039	0	0	0	0	0	0	0
2035	0	0	0	0	0	76,233	0	0	0	0	0	0	0
2036	0	0	0	0	0	73,234	0	0	0	0	0	0	0
2037	0	0	0	0	0	70,047	0	0	0	0	0	0	0
2038	0	0	0	0	0	66,684	0	0	0	0	0	0	0
2039	0	0	0	0	0	63,156	0	0	0	0	0	0	0
2040	0	0	0	0	0	43,596	15,885	0	0	0	0	0	0
2041	0	0	0	0	0	0	55,676	0	0	0	0	0	0
2042	0	0	0	0	0	0	51,764	0	0	0	0	0	0
2043	0	0	0	0	0	0	47,769	0	0	0	0	0	0
2044	0	0	0	0	0	0	43,723	0	0	0	0	0	0
2045	0	0	0	0	0	0	39,657	0	0	0	0	0	0
2046	0	0	0	0	0	0	174	35,433	0	0	0	0	0
2047	0	0	0	0	0	0	0	31,632	0	0	0	0	0
2048	0	0	0	0	0	0	0	27,783	0	0	0	0	0
2049	0	0	0	0	0	0	0	24,088	0	0	0	0	0
2050	0	0	0	0	0	0	0	20,590	0	0	0	0	0
2051	0	0	0	0	0	0	0	17,325	0	0	0	0	0
2052	0	0	0	0	0	0	0	14,328	0	0	0	0	0
2053	0	0	0	0	0	0	0	6,770	4,855	0	0	0	0
2054	0	0	0	0	0	0	0	0	9,234	0	0	0	0
2055	0	0	0	0	0	0	0	0	7,165	0	0	0	0
2056	0	0	0	0	0	0	0	0	5,415	0	0	0	0
2057	0	0	0	0	0	0	0	0	3,975	0	0	0	0
2058	0	0	0	0	0	0	0	0	2,824	0	0	0	0
2059	0	0	0	0	0	0	0	0	96	1,838	0	0	0
2060	0	0	0	0	0	0	0	0	0	1,271	0	0	0
2061	0	0	0	0	0	0	0	0	0	797	0	0	0
2062	0	0	0	0	0	0	0	0	0	474	0	0	0
2063	0	0	0	0	0	0	0	0	0	84	181	0	0
2064	0	0	0	0	0	0	0	0	0	0	138	0	0
2065	0	0	0	0	0	0	0	0	0	0	66	0	0
2066	0	0	0	0	0	0	0	0	0	0	24	5	0
2067	0	0	0	0	0	0	0	0	0	0	0	11	0
2068	0	0	0	0	0	0	0	0	0	0	0	4	0.00
2069	0	0	0	0	0	0	0	0	0	0	0	1	0.21
2070	0	0	0	0	0	0	0	0	0	0	0	0	0.22
2071	0	0	0	0	0	0	0	0	0	0	0	0	0.03
	125,704	425,899	659,848	1,241,298	1,048,489	510,228	254,647	177,949	33,565	4,463	409	21	0.47
		Overall Total =	**4,482,519**										

Fitting of Auto-regressive model for CPI

Model: *Inflation rate = average inflation + α (last year's inflation - average inflation) + error term where error term is represented by a shifted lognormal*

α = 0.5087

α is chosen to minimize the sum of the squared errors in Col. 4

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	**CPI at December**	**Annual % Increase in CPI**	**Least-Squares Fit of Inflation Model***	**Squared Error****	**Errors*****	**Error + .07**	**log(error + .07)**
1935	13.8						
1936	14.0	1.4%					
1937	14.4	2.9%	2.8%	0.00000	0.00074	0.07074	(2.64877)
1938	14.0	-2.8%	3.5%	0.00394	(0.06277)	0.00723	(4.93002)
1939	14.0	0.0%	0.6%	0.00004	(0.00633)	0.06367	(2.75402)
1940	14.1	0.7%	2.0%	0.00018	(0.01332)	0.05668	(2.87029)
1941	15.5	9.9%	2.4%	0.00565	0.07520	0.14520	(1.92967)
1942	16.9	9.0%	7.1%	0.00037	0.01935	0.08935	(2.41521)
1943	17.4	3.0%	6.6%	0.00136	(0.03683)	0.03317	(3.40598)
1944	17.8	2.3%	3.6%	0.00016	(0.01252)	0.05748	(2.85638)
1945	18.2	2.2%	3.2%	0.00009	(0.00968)	0.06032	(2.80815)
1946	21.5	18.1%	3.2%	0.02233	0.14943	0.21943	(1.51674)
1947	23.4	8.8%	11.3%	0.00059	(0.02433)	0.04567	(3.08639)
1948	24.1	3.0%	6.5%	0.00126	(0.03550)	0.03450	(3.36693)
1949	23.6	-2.1%	3.6%	0.00318	(0.05643)	0.01357	(4.29960)
1950	25.0	5.9%	1.0%	0.00244	0.04942	0.11942	(2.12514)
1951	26.5	6.0%	5.1%	0.00009	0.00936	0.07936	(2.53376)
1952	26.7	0.8%	5.1%	0.00189	(0.04344)	0.02656	(3.62827)
1953	26.9	0.7%	2.4%	0.00028	(0.01681)	0.05319	(2.93387)
1954	26.7	-0.7%	2.4%	0.00101	(0.03171)	0.03829	(3.26246)
1955	26.8	0.4%	1.7%	0.00017	(0.01293)	0.05707	(2.86352)
1956	27.6	3.0%	2.2%	0.00006	0.00749	0.07749	(2.55767)
1957	28.4	2.9%	3.6%	0.00004	(0.00666)	0.06334	(2.75926)
1958	28.9	1.8%	3.5%	0.00031	(0.01760)	0.05240	(2.94887)
1959	29.4	1.7%	2.9%	0.00015	(0.01212)	0.05788	(2.84931)
1960	29.8	1.4%	2.9%	0.00025	(0.01566)	0.05434	(2.91243)
1961	30.0	0.7%	2.7%	0.00043	(0.02067)	0.04933	(3.00923)
1962	30.4	1.3%	2.4%	0.00011	(0.01054)	0.05946	(2.82247)
1963	30.9	1.6%	2.7%	0.00012	(0.01080)	0.05920	(2.82677)
1964	31.2	1.0%	2.9%	0.00037	(0.01912)	0.05088	(2.97827)
1965	31.8	1.9%	2.5%	0.00004	(0.00617)	0.06383	(2.75151)
1966	32.9	3.5%	3.0%	0.00002	0.00435	0.07435	(2.59901)
1967	33.9	3.0%	3.8%	0.00006	(0.00766)	0.06234	(2.77520)
1968	35.5	4.7%	3.6%	0.00013	0.01127	0.08127	(2.50993)
1969	37.7	6.2%	4.4%	0.00031	0.01750	0.08750	(2.43612)

Exhibit 5, Page 2

Year	(1) CPI at December	(2) Annual % Increase in CPI	(3) Least-Squares Fit of Inflation Model*	(4) Squared Error**	(5) Errors***	(6) Error + .07	(7) log(error + .07)
1970	39.8	5.6%	5.2%	0.00001	0.00371	0.07371	(2.60755)
1971	41.1	3.3%	4.9%	0.00026	(0.01614)	0.05386	(2.92129)
1972	42.5	3.4%	3.7%	0.00001	(0.00301)	0.06699	(2.70328)
1973	46.2	8.7%	3.8%	0.00243	0.04927	0.11927	(2.12637)
1974	51.9	12.3%	6.5%	0.00344	0.05863	0.12863	(2.05085)
1975	55.5	6.9%	8.3%	0.00019	(0.01386)	0.05614	(2.87997)
1976	58.2	4.9%	5.6%	0.00005	(0.00710)	0.06290	(2.76621)
1977	62.1	6.7%	4.5%	0.00048	0.02180	0.09180	(2.38814)
1978	67.7	9.0%	5.5%	0.00127	0.03563	0.10563	(2.24785)
1979	76.7	13.3%	6.6%	0.00444	0.06660	0.13660	(1.99068)
1980	86.3	12.5%	8.8%	0.00137	0.03707	0.10707	(2.23427)
1981	94.0	8.9%	8.4%	0.00003	0.00509	0.07509	(2.58910)
1982	97.6	3.8%	6.6%	0.00076	(0.02755)	0.04245	(3.15954)
1983	101.3	3.8%	4.0%	0.00000	(0.00203)	0.06797	(2.68875)
1984	105.3	3.9%	4.0%	0.00000	(0.00026)	0.06974	(2.66298)
1985	109.3	3.8%	4.1%	0.00001	(0.00256)	0.06744	(2.69655)
1986	110.5	1.1%	4.0%	0.00083	(0.02881)	0.04119	(3.18948)
1987	115.4	4.4%	2.6%	0.00033	0.01830	0.08830	(2.42704)
1988	120.5	4.4%	4.3%	0.00000	0.00117	0.07117	(2.64263)
1989	126.1	4.6%	4.3%	0.00001	0.00353	0.07353	(2.61007)
1990	133.8	6.1%	4.4%	0.00029	0.01696	0.08696	(2.44231)
1991	137.9	3.1%	5.2%	0.00044	(0.02088)	0.04912	(3.01355)
1992	141.9	2.9%	3.6%	0.00005	(0.00704)	0.06296	(2.76531)
1993	145.8	2.7%	3.5%	0.00006	(0.00773)	0.06227	(2.77633)
1994	149.7	2.7%	3.4%	0.00006	(0.00769)	0.06231	(2.77569)
1995	153.5	2.5%	3.4%	0.00008	(0.00868)	0.06132	(2.79172)
Average		4.2%		0.00109	0.00032	0.07032	(2.76472)
Std. Dev.					0.03329	0.03329	0.51239

* Column 3 is calculated as: [Avg. of Col. 2] + α[Value of Col. 3 for previous yr - Avg. of Col. 2]
** Column 4 is calculated as: $\{\text{Col. 2 - Col. 3}\}^2$
*** Column 5 is calculated as {Col. 2 - Col. 3}

Shifted lognormal to model the error term is calculated by fitting a lognormal to Col. 6, the error term, plus a shift of .07, which ensures that all the error terms are positive. The lognormal is fitted using the method of moments where:

$\mu = -2.7647$

$\sigma = 0.5124$

Fitting of Model for Medical Inflation

Model: *Medical inflation $_t$ = inflation $_t$ + β(Medical inflation $_{t-1}$ - Inflation $_{t-1}$) + (Average medical inflation - average inflation) + error $_t$*

$\boxed{\beta = 0.382}$

ß is chosen to minimize the sum of the squared errors in column 6

Year	(1) Medical CPI at December	(2) Annual % Increase in Medical CPI	(3) Annual % Increase in Overall CPI	(4) Least-Squares Fit of Medical Inflation Model*	(5) Error**	(6) Squared Error***
1935	10.2					
1936	10.2	0.0%	1.4%			
1937	10.3	1.0%	2.9%	3.5%	-2.48%	0.00062
1938	10.3	0.0%	-2.8%	-2.3%	2.33%	0.00054
1939	10.4	1.0%	0.0%	2.2%	-1.25%	0.00016
1940	10.4	0.0%	0.7%	2.2%	-2.25%	0.00051
1941	10.5	1.0%	9.9%	10.8%	-9.86%	0.00972
1942	10.9	3.8%	9.0%	6.8%	-2.96%	0.00087
1943	11.4	4.6%	3.0%	2.1%	2.46%	0.00061
1944	11.7	2.6%	2.3%	4.1%	-1.45%	0.00021
1945	12.0	2.6%	2.2%	3.5%	-0.97%	0.00009
1946	13.0	8.3%	18.1%	19.4%	-11.08%	0.01228
1947	13.9	6.9%	8.8%	6.3%	0.67%	0.00004
1948	14.7	5.8%	3.0%	3.4%	2.33%	0.00054
1949	14.9	1.4%	-2.1%	0.1%	1.22%	0.00015
1950	15.4	3.4%	5.9%	8.4%	-5.05%	0.00255
1951	16.3	5.8%	6.0%	6.2%	-0.33%	0.00001
1952	17.0	4.3%	0.8%	1.9%	2.44%	0.00059
1953	17.6	3.5%	0.7%	3.3%	0.26%	0.00001
1954	18.0	2.3%	-0.7%	1.5%	0.79%	0.00006
1955	18.6	3.3%	0.4%	2.7%	0.64%	0.00004
1956	19.2	3.2%	3.0%	5.3%	-2.05%	0.00042
1957	20.1	4.7%	2.9%	4.2%	0.53%	0.00003
1958	21.0	4.5%	1.8%	3.6%	0.87%	0.00008
1959	21.8	3.8%	1.7%	3.9%	-0.12%	0.00000
1960	22.5	3.2%	1.4%	3.3%	-0.11%	0.00000
1961	23.2	3.1%	0.7%	2.5%	0.57%	0.00003
1962	23.7	2.2%	1.3%	3.4%	-1.27%	0.00016
1963	24.3	2.5%	1.6%	3.1%	-0.59%	0.00003
1964	24.8	2.1%	1.0%	2.5%	-0.41%	0.00002
1965	25.5	2.8%	1.9%	3.5%	-0.68%	0.00005
1966	27.2	6.7%	3.5%	5.0%	1.70%	0.00029
1967	28.9	6.3%	3.0%	5.4%	0.82%	0.00007
1968	30.7	6.2%	4.7%	7.1%	-0.88%	0.00008
1969	32.6	6.2%	6.2%	7.9%	-1.75%	0.00031
1970	35.0	7.4%	5.6%	6.7%	0.63%	0.00004
1971	36.6	4.6%	3.3%	5.1%	-0.54%	0.00003
1972	37.8	3.3%	3.4%	5.1%	-1.79%	0.00032
1973	39.8	5.3%	8.7%	9.8%	-4.53%	0.00205
1974	44.8	12.6%	12.3%	12.2%	0.37%	0.00001
1975	49.2	9.8%	6.9%	8.2%	1.64%	0.00027
1976	54.1	10.0%	4.9%	7.1%	2.83%	0.00080
1977	58.9	8.9%	6.7%	9.8%	-0.94%	0.00009

	(1)	(2)	(3)	(4)	(5)	(6)
Year	**Medical CPI at December**	**Annual % Increase in Medical CPI**	**Annual % Increase in Overall CPI**	**Least-Squares Fit of Medical Inflation Model***	**Error****	**Squared Error*****
1978	64.1	8.8%	9.0%	11.0%	-2.18%	0.00048
1979	70.6	10.1%	13.3%	14.4%	-4.24%	0.00180
1980	77.6	9.9%	12.5%	12.5%	-2.56%	0.00065
1981	87.3	12.5%	8.9%	9.1%	3.41%	0.00116
1982	96.9	11.0%	3.8%	6.4%	4.64%	0.00215
1983	103.1	6.4%	3.8%	7.7%	-1.29%	0.00017
1984	109.4	6.1%	3.9%	6.1%	0.00%	0.00000
1985	116.8	6.8%	3.8%	5.8%	0.98%	0.00010
1986	125.8	7.7%	1.1%	3.4%	4.31%	0.00186
1987	133.1	5.8%	4.4%	8.1%	-2.32%	0.00054
1988	142.3	6.9%	4.4%	6.1%	0.81%	0.00007
1989	154.4	8.5%	4.6%	6.8%	1.74%	0.00030
1990	169.2	9.6%	6.1%	8.7%	0.84%	0.00007
1991	182.6	7.9%	3.1%	5.6%	2.36%	0.00056
1992	194.7	6.6%	2.9%	5.9%	0.71%	0.00005
1993	205.2	5.4%	2.7%	5.3%	0.06%	0.00000
1994	215.3	4.9%	2.7%	4.8%	0.07%	0.00000
1995	223.8	3.9%	2.5%	4.6%	-0.61%	0.00004
Mean		5.3%	4.2%		-0.40%	0.00076
					2.75% = Std. Dev. of errors	*0.04477 = Sum of square errors*

Average difference between medical inflation and inflation (i.e., avg. of Col. 2 - avg. of Col. 3) = 1.16%

* Column 4 is calculated as Col. 3 for previous year + ß[Col. 2 for previous year - Col. 3 for previous year] + [Avg. of Col. 2 - Avg. of Col. 3]

** Column 5 = Column 2 - Column 4

*** Column 6 = $\{\text{Column 5}\}^2$

ß is fitted to minimize the sum of column 6.

One Simulation from Method 3
Stochastic Mortality, Inflation, Medical Inflation, and Investment Yields

Parameters:

(A)	Evaluation Date:	1/1/97
(B)	Current Age:	35
(C)	Annual Indemnity Payment	20,000
(D)	Annual Medical Payment (at mid-1996 price levels)	Varies
(E)	Indemnity Paid to Date	70,000
(F)	Medical Paid to Date:	300,000
(G)	Cost-of-Living Adjustment	Varies
(H)	Medical Inflation Rate:	Varies
(I)	Annual Discount Rate:	Varies

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Cost of Living Adjustment	Indemnity Payment	Medical Inflation	Medical Payment	Total Payment	Cumulative Total Payment	Probability of claimant living to mid-year	Present Value Factor	Discount for mortality & investment income
					(2) + (4)	Cum. of (5)			(7) x (8)
1996 and prior		70,000		300,000	370,000	370,000			
1997	2.7%	20,541	2.69%	69,625	90,166	460,166	0.999	1.0000	0.9987
1998	0.9%	20,716	9.69%	116,357	137,073	597,239	0.996	0.9968	0.9929
1999	5.0%	21,752	7.73%	51,620	73,372	670,610	0.993	0.9813	0.9747
2000	2.4%	22,266	11.19%	43,111	65,377	735,988	0.990	0.9428	0.9337
2001	5.0%	23,380	10.32%	23,845	47,225	783,212	0.987	0.9010	0.8897
2002	5.0%	24,549	5.65%	43,978	68,527	851,739	0.984	0.8623	0.8489
2003	3.3%	25,369	5.17%	95,153	120,521	972,260	0.981	0.8264	0.8109
2004	3.1%	26,166	1.17%	250,254	276,419	1,248,680	0.978	0.8057	0.7880
2005	1.6%	26,587	6.55%	49,640	76,227	1,324,907	0.975	0.7822	0.7623
2006	5.0%	27,917	6.99%	81,635	109,552	1,434,459	0.971	0.7580	0.7360
2007	3.4%	28,875	10.27%	101,913	130,788	1,565,247	0.967	0.7420	0.7176
2008	5.0%	30,319	11.64%	99,335	129,655	1,694,902	0.963	0.7343	0.7070
2009	5.0%	31,835	5.11%	132,868	164,703	1,859,605	0.958	0.7267	0.6965
2010	4.8%	33,373	7.04%	110,591	143,964	2,003,569	0.954	0.7193	0.6858
2011	2.5%	34,193	7.38%	126,342	160,535	2,164,104	0.948	0.7029	0.6666
2012	4.3%	35,656	8.53%	75,493	111,149	2,275,253	0.943	0.6566	0.6189
2013	3.9%	37,063	12.24%	241,570	278,632	2,553,886	0.936	0.6054	0.5670
2014	5.0%	38,916	4.44%	391,743	430,658	2,984,544	0.930	0.5699	0.5299
2015	5.0%	40,861	-1.51%	239,565	280,426	3,264,970	0.923	0.5364	0.4949
2016	0.8%	41,182	-4.98%	117,385	158,568	3,423,538	0.915	0.5091	0.4657
2017	0.0%	41,182	-1.18%	151,238	192,421	3,615,959	0.906	0.4991	0.4522
2018	0.0%	41,182	4.60%	505,346	546,529	4,162,487	0.897	0.4969	0.4455
2019	0.0%	41,182	2.30%	321,015	362,198	4,524,685	0.886	0.4967	0.4402
2020	0.0%	41,182	7.33%	163,486	204,669	4,729,354	0.875	0.4949	0.4331
2021	5.0%	43,241	1.19%	193,421	236,663	4,966,016	0.863	0.4931	0.4256
2022	3.8%	44,882	4.18%	118,487	163,369	5,129,385	0.850	0.4931	0.4192
2023	5.0%	47,126	1.48%	156,834	203,960	5,333,345	0.836	0.4911	0.4106
2024	1.4%	47,775	2.19%	603,315	651,090	5,984,435	0.821	0.4454	0.3656
2025	0.1%	47,829	5.16%	150,581	198,410	6,182,845	0.805	0.3927	0.3160
2026	3.8%	49,643	3.11%	349,255	398,898	6,581,743	0.788	0.3458	0.2723
2027	1.7%	50,494	2.92%	149,743	200,237	6,781,980	0.769	0.2907	0.2237
2028	0.0%	50,505	4.66%	96,200	146,705	6,928,685	0.750	0.2520	0.1891
2029	1.4%	51,211	4.46%	337,926	389,137	7,317,822	0.730	0.2307	0.1684
2030	1.1%	51,779	2.90%	307,518	359,297	7,677,119	0.709	0.2232	0.1582
2031	0.0%	51,779	3.58%	156,003	207,782	7,884,901	0.686	0.2208	0.1515
2032	0.3%	51,960	7.39%	236,209	288,169	8,173,071	0.663	0.2192	0.1452
2033	5.0%	54,558	9.98%	236,796	291,354	8,464,425	0.638	0.2154	0.1373
2034	5.0%	57,286	12.47%	407,806	465,093	8,929,518	0.612	0.2116	0.1294
2035	5.0%	60,151	10.37%	533,333	593,483	9,523,001	0.584	0.2107	0.1231
2036	5.0%	63,158	10.32%	224,000	287,158	9,810,160	0.556	0.2086	0.1160
2037	5.0%	66,316	3.15%	567,911	634,227	10,444,386	0.527	0.1980	0.1043
2038	3.3%	68,476	7.85%	428,832	497,308	10,941,694	0.497	0.1868	0.0928

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant living to mid-year	(8) Present Value Factor	(9) Discount for mortality & investment income (7) x (8)
2039	4.0%	71,212	-3.11%	586,585	657,797	11,599,491	0.466	0.1787	0.0833
2040	0.0%	71,212	4.36%	159,131	230,343	11,829,835	0.435	0.1720	0.0748
2041	0.0%	71,212	8.06%	498,516	569,728	12,399,562	0.403	0.1669	0.0673
2042	4.6%	74,508	2.36%	436,885	511,393	12,910,956	0.372	0.1599	0.0594
2043	0.3%	74,714	4.09%	1,029,491	1,104,205	14,015,160	0.340	0.1517	0.0515
2044	2.3%	76,449	2.38%	523,272	599,722	14,614,882	0.308	0.1353	0.0417
2045	2.2%	78,156	7.11%	555,505	633,662	15,248,544	0.277	0.1169	0.0324
2046	2.7%	80,276	7.32%	1,182,773	1,263,049	16,511,592	0.246	0.1061	0.0261
2047	2.4%	82,185	3.30%	392,255	474,440	16,986,033	0.217	0.1011	0.0219
2048	0.9%	82,966	1.78%	274,463	357,428	17,343,461	0.188	0.0980	0.0185
2049	1.1%	83,851	-0.06%	436,779	520,629	17,864,090	0.162	0.0945	0.0153
2050	0.0%	83,851	1.54%	779,726	863,577	18,727,667	0.137	0.0911	0.0125
2051	0.0%	83,851	2.85%	239,547	323,398	19,051,066	0.114	0.0897	0.0102
2052	0.0%	83,851	3.63%	438,803	522,654	19,573,720	0.094	0.0888	0.0083
2053	0.3%	84,069	2.03%	980,719	1,064,789	20,638,509	0.075	0.0874	0.0066
2054	0.0%	84,069	11.94%	451,630	535,699	21,174,208	0.059	0.0843	0.0050
2055	4.3%	87,715	6.71%	843,104	930,819	22,105,027	0.045	0.0796	0.0036
2056	5.0%	92,101	14.17%	842,189	934,290	23,039,317	0.034	0.0756	0.0026
2057	5.0%	96,706	6.06%	823,588	920,294	23,959,611	0.025	0.0702	0.0017
2058	3.3%	99,852	-3.28%	400,213	500,065	24,459,676	0.017	0.0646	0.0011
2059	0.0%	99,852	24.39%	5,305,393	5,405,244	29,864,920	0.012	0.0599	0.0007
2060	5.0%	104,844	15.98%	1,891,811	1,996,656	31,861,576	0.008	0.0560	0.0004
2061	5.0%	110,087	5.35%	5,825,837	5,935,924	37,797,500	0.005	0.0535	0.0003
2062	2.5%	112,805	5.22%	1,102,848	1,215,652	39,013,153	0.003	0.0501	0.0001
2063	4.5%	117,903	3.14%	591,854	709,757	39,722,910	0.002	0.0470	0.0001
2064	0.8%	118,864	7.99%	1,406,116	1,524,980	41,247,889	0.001	0.0451	0.0000
2065	5.0%	124,807	10.89%	7,307,112	7,431,919	48,679,808	0.0004	0.0440	0.0000
2066	5.0%	131,047	9.24%	4,535,733	4,666,780	53,346,589	0.0002	0.0429	0.0000
2067	5.0%	137,600	16.37%	5,857,809	5,995,408	59,341,997	0.0001	0.0418	0.0000
2068	5.0%	144,480	16.02%	1,370,853	1,515,332	60,857,329	0.00002	0.0404	0.0000
2069	5.0%	151,704	12.40%	4,972,397	5,124,100	65,981,429	0.00001	0.0383	0.0000
2070	5.0%	159,289	9.96%	7,659,607	7,818,896	73,800,325	0.000001	0.0352	0.0000
2071	5.0%	167,253	11.63%	10,212,211	10,379,464	84,179,788	0.0000002	0.0320	0.0000

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
	Incremental Payments by Layer													
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million	$10 million xs $80 million
1996 and prior														
1997	90,166	0	0	0	0	0	0	0	0	0	0	0	0	0
1998	39,834	97,239	0	0	0	0	0	0	0	0	0	0	0	0
1999	0	73,372	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	65,377	0	0	0	0	0	0	0	0	0	0	0	0
2001	0	47,225	0	0	0	0	0	0	0	0	0	0	0	0
2002	0	68,527	0	0	0	0	0	0	0	0	0	0	0	0
2003	0	120,521	0	0	0	0	0	0	0	0	0	0	0	0
2004	0	27,740	248,680	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	76,227	0	0	0	0	0	0	0	0	0	0	0
2006	0	0	109,552	0	0	0	0	0	0	0	0	0	0	0
2007	0	0	130,788	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	129,655	0	0	0	0	0	0	0	0	0	0	0
2009	0	0	164,703	0	0	0	0	0	0	0	0	0	0	0
2010	0	0	140,395	3,569	0	0	0	0	0	0	0	0	0	0
2011	0	0	0	160,535	0	0	0	0	0	0	0	0	0	0
2012	0	0	0	111,149	0	0	0	0	0	0	0	0	0	0
2013	0	0	0	278,632	0	0	0	0	0	0	0	0	0	0
2014	0	0	0	430,658	0	0	0	0	0	0	0	0	0	0
2015	0	0	0	280,426	0	0	0	0	0	0	0	0	0	0
2016	0	0	0	158,568	0	0	0	0	0	0	0	0	0	0
2017	0	0	0	192,421	0	0	0	0	0	0	0	0	0	0
2018	0	0	0	546,529	0	0	0	0	0	0	0	0	0	0
2019	0	0	0	362,198	0	0	0	0	0	0	0	0	0	0
2020	0	0	0	204,669	0	0	0	0	0	0	0	0	0	0
2021	0	0	0	236,663	0	0	0	0	0	0	0	0	0	0
2022	0	0	0	33,984	129,385	0	0	0	0	0	0	0	0	0
2023	0	0	0	0	203,960	0	0	0	0	0	0	0	0	0
2024	0	0	0	0	651,090	0	0	0	0	0	0	0	0	0
2025	0	0	0	0	198,410	0	0	0	0	0	0	0	0	0
2026	0	0	0	0	398,898	0	0	0	0	0	0	0	0	0
2027	0	0	0	0	200,237	0	0	0	0	0	0	0	0	0
2028	0	0	0	0	146,705	0	0	0	0	0	0	0	0	0
2029	0	0	0	0	389,137	0	0	0	0	0	0	0	0	0
2030	0	0	0	0	359,297	0	0	0	0	0	0	0	0	0
2031	0	0	0	0	207,782	0	0	0	0	0	0	0	0	0
2032	0	0	0	0	288,169	0	0	0	0	0	0	0	0	0

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
	Incremental Payments by Layer													
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million	$10 million xs $80 million
2033	0	0	0	0	291,354	0	0	0	0	0	0	0	0	0
2034	0	0	0	0	465,093	0	0	0	0	0	0	0	0	0
2035	0	0	0	0	593,483	0	0	0	0	0	0	0	0	0
2036	0	0	0	0	287,158	0	0	0	0	0	0	0	0	0
2037	0	0	0	0	189,840	444,386	0	0	0	0	0	0	0	0
2038	0	0	0	0	0	497,308	0	0	0	0	0	0	0	0
2039	0	0	0	0	0	657,797	0	0	0	0	0	0	0	0
2040	0	0	0	0	0	230,343	0	0	0	0	0	0	0	0
2041	0	0	0	0	0	569,728	0	0	0	0	0	0	0	0
2042	0	0	0	0	0	511,393	0	0	0	0	0	0	0	0
2043	0	0	0	0	0	1,104,205	0	0	0	0	0	0	0	0
2044	0	0	0	0	0	599,722	0	0	0	0	0	0	0	0
2045	0	0	0	0	0	385,118	248,544	0	0	0	0	0	0	0
2046	0	0	0	0	0	0	1,263,049	0	0	0	0	0	0	0
2047	0	0	0	0	0	0	474,440	0	0	0	0	0	0	0
2048	0	0	0	0	0	0	357,428	0	0	0	0	0	0	0
2049	0	0	0	0	0	0	520,629	0	0	0	0	0	0	0
2050	0	0	0	0	0	0	863,577	0	0	0	0	0	0	0
2051	0	0	0	0	0	0	323,398	0	0	0	0	0	0	0
2052	0	0	0	0	0	0	522,654	0	0	0	0	0	0	0
2053	0	0	0	0	0	0	426,280	638,509	0	0	0	0	0	0
2054	0	0	0	0	0	0	0	535,699	0	0	0	0	0	0
2055	0	0	0	0	0	0	0	930,819	0	0	0	0	0	0
2056	0	0	0	0	0	0	0	934,290	0	0	0	0	0	0
2057	0	0	0	0	0	0	0	920,294	0	0	0	0	0	0
2058	0	0	0	0	0	0	0	500,065	0	0	0	0	0	0
2059	0	0	0	0	0	0	0	5,405,244	0	0	0	0	0	0
2060	0	0	0	0	0	0	0	135,080	1,861,576	0	0	0	0	0
2061	0	0	0	0	0	0	0	0	5,935,924	0	0	0	0	0
2062	0	0	0	0	0	0	0	0	1,215,652	0	0	0	0	0
2063	0	0	0	0	0	0	0	0	709,757	0	0	0	0	0
2064	0	0	0	0	0	0	0	0	277,090	1,247,889	0	0	0	0
2065	0	0	0	0	0	0	0	0	0	7,431,919	0	0	0	0
2066	0	0	0	0	0	0	0	0	0	1,320,192	3,346,589	0	0	0
2067	0	0	0	0	0	0	0	0	0	0	5,995,408	0	0	0
2068	0	0	0	0	0	0	0	0	0	0	658,003	857,329	0	0
2069	0	0	0	0	0	0	0	0	0	0	0	5,124,100	0	0
2070	0	0	0	0	0	0	0	0	0	0	0	4,018,571	3,800,325	0
2071	0	0	0	0	0	0	0	0	0	0	0	0	6,199,675	4,179,788
	130,000	500,000	1,000,000	3,000,000	5,000,000	5,000,000	5,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	4,179,788

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 24 = Column 10 x Column 9

	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million	$10 million xs $80 million
1996 and prior														
1997	90,049	0	0	0	0	0	0	0	0	0	0	0	0	0
1998	39,551	96,548	0	0	0	0	0	0	0	0	0	0	0	0
1999	0	71,517	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	61,045	0	0	0	0	0	0	0	0	0	0	0	0
2001	0	42,017	0	0	0	0	0	0	0	0	0	0	0	0
2002	0	58,170	0	0	0	0	0	0	0	0	0	0	0	0
2003	0	97,731	0	0	0	0	0	0	0	0	0	0	0	0
2004	0	21,858	195,953	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	58,110	0	0	0	0	0	0	0	0	0	0	0
2006	0	0	80,630	0	0	0	0	0	0	0	0	0	0	0
2007	0	0	93,851	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	91,665	0	0	0	0	0	0	0	0	0	0	0
2009	0	0	114,711	0	0	0	0	0	0	0	0	0	0	0
2010	0	0	96,288	2,448	0	0	0	0	0	0	0	0	0	0
2011	0	0	0	107,007	0	0	0	0	0	0	0	0	0	0
2012	0	0	0	68,794	0	0	0	0	0	0	0	0	0	0
2013	0	0	0	157,981	0	0	0	0	0	0	0	0	0	0
2014	0	0	0	228,222	0	0	0	0	0	0	0	0	0	0
2015	0	0	0	138,769	0	0	0	0	0	0	0	0	0	0
2016	0	0	0	73,838	0	0	0	0	0	0	0	0	0	0
2017	0	0	0	87,012	0	0	0	0	0	0	0	0	0	0
2018	0	0	0	243,501	0	0	0	0	0	0	0	0	0	0
2019	0	0	0	159,453	0	0	0	0	0	0	0	0	0	0
2020	0	0	0	88,650	0	0	0	0	0	0	0	0	0	0
2021	0	0	0	100,727	0	0	0	0	0	0	0	0	0	0
2022	0	0	0	14,245	54,235	0	0	0	0	0	0	0	0	0
2023	0	0	0	0	83,737	0	0	0	0	0	0	0	0	0
2024	0	0	0	0	238,064	0	0	0	0	0	0	0	0	0
2025	0	0	0	0	62,701	0	0	0	0	0	0	0	0	0
2026	0	0	0	0	108,634	0	0	0	0	0	0	0	0	0
2027	0	0	0	0	44,788	0	0	0	0	0	0	0	0	0
2028	0	0	0	0	27,744	0	0	0	0	0	0	0	0	0
2029	0	0	0	0	65,543	0	0	0	0	0	0	0	0	0
2030	0	0	0	0	56,851	0	0	0	0	0	0	0	0	0
2031	0	0	0	0	31,485	0	0	0	0	0	0	0	0	0
2032	0	0	0	0	41,853	0	0	0	0	0	0	0	0	0

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 24 = Column 10 x Column 9

	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million	$10 million xs $80 million
2033	0	0	0	0	40,012	0	0	0	0	0	0	0	0	0
2034	0	0	0	0	60,197	0	0	0	0	0	0	0	0	0
2035	0	0	0	0	73,083	0	0	0	0	0	0	0	0	0
2036	0	0	0	0	33,316	0	0	0	0	0	0	0	0	0
2037	0	0	0	0	19,808	46,367	0	0	0	0	0	0	0	0
2038	0	0	0	0	0	46,163	0	0	0	0	0	0	0	0
2039	0	0	0	0	0	54,806	0	0	0	0	0	0	0	0
2040	0	0	0	0	0	17,233	0	0	0	0	0	0	0	0
2041	0	0	0	0	0	38,354	0	0	0	0	0	0	0	0
2042	0	0	0	0	0	30,387	0	0	0	0	0	0	0	0
2043	0	0	0	0	0	56,878	0	0	0	0	0	0	0	0
2044	0	0	0	0	0	24,993	0	0	0	0	0	0	0	0
2045	0	0	0	0	0	12,459	8,041	0	0	0	0	0	0	0
2046	0	0	0	0	0	0	32,988	0	0	0	0	0	0	0
2047	0	0	0	0	0	0	10,388	0	0	0	0	0	0	0
2048	0	0	0	0	0	0	6,598	0	0	0	0	0	0	0
2049	0	0	0	0	0	0	7,956	0	0	0	0	0	0	0
2050	0	0	0	0	0	0	10,779	0	0	0	0	0	0	0
2051	0	0	0	0	0	0	3,310	0	0	0	0	0	0	0
2052	0	0	0	0	0	0	4,338	0	0	0	0	0	0	0
2053	0	0	0	0	0	0	2,798	4,192	0	0	0	0	0	0
2054	0	0	0	0	0	0	0	2,669	0	0	0	0	0	0
2055	0	0	0	0	0	0	0	3,366	0	0	0	0	0	0
2056	0	0	0	0	0	0	0	2,401	0	0	0	0	0	0
2057	0	0	0	0	0	0	0	1,597	0	0	0	0	0	0
2058	0	0	0	0	0	0	0	562	0	0	0	0	0	0
2059	0	0	0	0	0	0	0	3,817	0	0	0	0	0	0
2060	0	0	0	0	0	0	0	58	801	0	0	0	0	0
2061	0	0	0	0	0	0	0	0	1,514	0	0	0	0	0
2062	0	0	0	0	0	0	0	0	171	0	0	0	0	0
2063	0	0	0	0	0	0	0	0	52	0	0	0	0	0
2064	0	0	0	0	0	0	0	0	10	45	0	0	0	0
2065	0	0	0	0	0	0	0	0	0	124	0	0	0	0
2066	0	0	0	0	0	0	0	0	0	9	24	0	0	0
2067	0	0	0	0	0	0	0	0	0	0	16	0	0	0
2068	0	0	0	0	0	0	0	0	0	0	1	1	0.00	0
2069	0	0	0	0	0	0	0	0	0	0	0	1	0.00	0
2070	0	0	0	0	0	0	0	0	0	0	0	0	0.16	0
2071	0	0	0	0	0	0	0	0	0	0	0	0	0.04	0.02
	129,600	448,885	731,208	1,470,647	1,042,047	327,641	87,197	18,661	2,548	179	40	2	0.20	0.02
	Overall Total =	**4,258,655**												

Method 2, With Inflation and Investment Income "Capped"

	Parameters:	
(A)	Evaluation Date:	1/1/97
(B)	Current Age:	35
(C)	Annual Indemnity Payment	20,000
(D)	Annual Medical Payment (at mid-1996 price levels)	70,000
(E)	Indemnity Paid to Date	70,000
(F)	Medical Paid to Date:	300,000
(G)	***Cost-of-Living Adjustment***	***2.9785%***
(H)	Medical Inflation Rate:	5.36%
(I)	***Annual Discount Rate:***	***4.3887%***

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	**Cost of Living Adjustment**	**Indemnity Payment**	**Medical Inflation**	**Medical Payment**	**Total Payment**	**Cumulative Total Payment**	**Probability of claimant living to mid-year**	**Present Value Factor**	**Discount for mortality & investment income**
					(2) + (4)	Cum. of (5)			(7) x (8)
1996 and prior		70,000		300,000	370,000	370,000			
1997	3.0%	20,596	5.36%	73,752	94,348	464,348	0.999	0.9788	0.9775
1998	3.0%	21,209	5.36%	77,705	98,914	563,262	0.996	0.9376	0.9339
1999	3.0%	21,841	5.36%	81,870	103,711	666,973	0.993	0.8982	0.8922
2000	3.0%	22,491	5.36%	86,258	108,750	775,723	0.990	0.8604	0.8522
2001	3.0%	23,161	5.36%	90,882	114,043	889,766	0.987	0.8243	0.8139
2002	3.0%	23,851	5.36%	95,753	119,604	1,009,370	0.984	0.7896	0.7773
2003	3.0%	24,562	5.36%	100,885	125,447	1,134,817	0.981	0.7564	0.7422
2004	3.0%	25,293	5.36%	106,293	131,586	1,266,403	0.978	0.7246	0.7087
2005	3.0%	26,046	5.36%	111,990	138,037	1,404,440	0.975	0.6941	0.6765
2006	3.0%	26,822	5.36%	117,993	144,815	1,549,255	0.971	0.6650	0.6456
2007	3.0%	27,621	5.36%	124,317	151,938	1,701,193	0.967	0.6370	0.6160
2008	3.0%	28,444	5.36%	130,981	159,425	1,860,618	0.963	0.6102	0.5876
2009	3.0%	29,291	5.36%	138,001	167,292	2,027,910	0.958	0.5846	0.5602
2010	3.0%	30,164	5.36%	145,398	175,562	2,203,472	0.954	0.5600	0.5340
2011	3.0%	31,062	5.36%	153,191	184,253	2,387,725	0.948	0.5364	0.5087
2012	3.0%	31,987	5.36%	161,402	193,390	2,581,114	0.943	0.5139	0.4844
2013	3.0%	32,940	5.36%	170,054	202,994	2,784,108	0.936	0.4923	0.4610
2014	3.0%	33,921	5.36%	179,169	213,089	2,997,197	0.930	0.4716	0.4385
2015	3.0%	34,931	5.36%	188,772	223,703	3,220,901	0.923	0.4518	0.4168
2016	3.0%	35,972	5.36%	198,890	234,862	3,455,763	0.915	0.4328	0.3958
2017	3.0%	37,043	5.36%	209,551	246,594	3,702,356	0.906	0.4146	0.3756
2018	3.0%	38,146	5.36%	220,783	258,929	3,961,285	0.897	0.3971	0.3561
2019	3.0%	39,283	5.36%	232,617	271,899	4,233,185	0.886	0.3804	0.3372
2020	3.0%	40,453	5.36%	245,085	285,537	4,518,722	0.875	0.3645	0.3190
2021	3.0%	41,658	5.36%	258,221	299,879	4,818,601	0.863	0.3491	0.3014
2022	3.0%	42,898	5.36%	272,062	314,960	5,133,561	0.850	0.3345	0.2843
2023	3.0%	44,176	5.36%	286,644	330,821	5,464,382	0.836	0.3204	0.2679
2024	3.0%	45,492	5.36%	302,009	347,501	5,811,882	0.821	0.3069	0.2520
2025	3.0%	46,847	5.36%	318,196	365,043	6,176,926	0.805	0.2940	0.2366
2026	3.0%	48,242	5.36%	335,252	383,494	6,560,419	0.788	0.2817	0.2218
2027	3.0%	49,679	5.36%	353,221	402,900	6,963,320	0.769	0.2698	0.2076
2028	3.0%	51,159	5.36%	372,154	423,313	7,386,632	0.750	0.2585	0.1939
2029	3.0%	52,683	5.36%	392,101	444,784	7,831,416	0.730	0.2476	0.1808
2030	3.0%	54,252	5.36%	413,118	467,370	8,298,785	0.709	0.2372	0.1681
2031	3.0%	55,868	5.36%	435,261	491,129	8,789,914	0.686	0.2272	0.1560
2032	3.0%	57,532	5.36%	458,591	516,123	9,306,036	0.663	0.2177	0.1442
2033	3.0%	59,245	5.36%	483,171	542,417	9,848,453	0.638	0.2085	0.1330
2034	3.0%	61,010	5.36%	509,069	570,079	10,418,532	0.612	0.1998	0.1222
2035	3.0%	62,827	5.36%	536,356	599,182	11,017,715	0.584	0.1914	0.1118
2036	3.0%	64,698	5.36%	565,104	629,802	11,647,517	0.556	0.1833	0.1019
2037	3.0%	66,625	5.36%	595,394	662,019	12,309,536	0.527	0.1756	0.0925
2038	3.0%	68,610	5.36%	627,307	695,917	13,005,453	0.497	0.1682	0.0836
2039	3.0%	70,653	5.36%	660,931	731,584	13,737,036	0.466	0.1611	0.0751

Year	(1) Cost of Living Adjustment	(2) Indemnity Payment	(3) Medical Inflation	(4) Medical Payment	(5) Total Payment (2) + (4)	(6) Cumulative Total Payment Cum. of (5)	(7) Probability of claimant living to mid-year	(8) Present Value Factor	(9) Discount for mortality & investment income (7) x (8)
2040	3.0%	72,758	5.36%	696,356	769,114	14,506,150	0.435	0.1544	0.0672
2041	3.0%	74,925	5.36%	733,681	808,606	15,314,756	0.403	0.1479	0.0597
2042	3.0%	77,156	5.36%	773,006	850,163	16,164,919	0.372	0.1417	0.0526
2043	3.0%	79,454	5.36%	814,440	893,894	17,058,813	0.340	0.1357	0.0461
2044	3.0%	81,821	5.36%	858,094	939,915	17,998,728	0.308	0.1300	0.0400
2045	3.0%	84,258	5.36%	904,087	988,345	18,987,073	0.277	0.1245	0.0345
2046	3.0%	86,768	5.36%	952,546	1,039,314	20,026,387	0.246	0.1193	0.0294
2047	3.0%	89,352	5.36%	1,003,603	1,092,955	21,119,342	0.217	0.1143	0.0247
2048	3.0%	92,013	5.36%	1,057,396	1,149,409	22,268,751	0.188	0.1095	0.0206
2049	3.0%	94,754	5.36%	1,114,072	1,208,826	23,477,578	0.162	0.1049	0.0170
2050	3.0%	97,576	5.36%	1,173,787	1,271,363	24,748,941	0.137	0.1005	0.0138
2051	3.0%	100,483	5.36%	1,236,702	1,337,184	26,086,125	0.114	0.0962	0.0110
2052	3.0%	103,475	5.36%	1,302,989	1,406,464	27,492,589	0.094	0.0922	0.0086
2053	3.0%	106,557	5.36%	1,372,829	1,479,387	28,971,976	0.075	0.0883	0.0066
2054	3.0%	109,731	5.36%	1,446,413	1,556,144	30,528,120	0.059	0.0846	0.0050
2055	3.0%	113,000	5.36%	1,523,940	1,636,940	32,165,060	0.045	0.0811	0.0037
2056	3.0%	116,365	5.36%	1,605,624	1,721,989	33,887,049	0.034	0.0776	0.0026
2057	3.0%	119,831	5.36%	1,691,685	1,811,516	35,698,566	0.025	0.0744	0.0018
2058	3.0%	123,400	5.36%	1,782,359	1,905,760	37,604,325	0.017	0.0713	0.0012
2059	3.0%	127,076	5.36%	1,877,894	2,004,970	39,609,295	0.012	0.0683	0.0008
2060	3.0%	130,861	5.36%	1,978,549	2,109,410	41,718,705	0.008	0.0654	0.0005
2061	3.0%	134,759	5.36%	2,084,599	2,219,358	43,938,063	0.005	0.0626	0.0003
2062	3.0%	138,772	5.36%	2,196,334	2,335,106	46,273,169	0.003	0.0600	0.0002
2063	3.0%	142,906	5.36%	2,314,057	2,456,963	48,730,132	0.002	0.0575	0.0001
2064	3.0%	147,162	5.36%	2,438,091	2,585,253	51,315,385	0.001	0.0551	0.0000
2065	3.0%	151,545	5.36%	2,568,772	2,720,318	54,035,703	0.0004	0.0528	0.0000
2066	3.0%	156,059	5.36%	2,706,459	2,862,518	56,898,220	0.0002	0.0505	0.0000
2067	3.0%	160,707	5.36%	2,851,525	3,012,232	59,910,452	0.0001	0.0484	0.0000
2068	3.0%	165,494	5.36%	3,004,366	3,169,860	63,080,313	0.00002	0.0464	0.0000
2069	3.0%	170,423	5.36%	3,165,400	3,335,824	66,416,137	0.00001	0.0444	0.0000
2070	3.0%	175,499	5.36%	3,335,066	3,510,565	69,926,702	0.000001	0.0426	0.0000
2071	3.0%	180,727	5.36%	3,513,825	3,694,552	73,621,254	0.0000002	0.0408	0.0000

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
	Incremental Payments by Layer												
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
1996 and prior													
1997	94,348	0	0	0	0	0	0	0	0	0	0	0	0
1998	35,652	63,262	0	0	0	0	0	0	0	0	0	0	0
1999	0	103,711	0	0	0	0	0	0	0	0	0	0	0
2000	0	108,750	0	0	0	0	0	0	0	0	0	0	0
2001	0	114,043	0	0	0	0	0	0	0	0	0	0	0
2002	0	110,234	9,370	0	0	0	0	0	0	0	0	0	0
2003	0	0	125,447	0	0	0	0	0	0	0	0	0	0
2004	0	0	131,586	0	0	0	0	0	0	0	0	0	0
2005	0	0	138,037	0	0	0	0	0	0	0	0	0	0
2006	0	0	144,815	0	0	0	0	0	0	0	0	0	0
2007	0	0	151,938	0	0	0	0	0	0	0	0	0	0
2008	0	0	159,425	0	0	0	0	0	0	0	0	0	0
2009	0	0	139,382	27,910	0	0	0	0	0	0	0	0	0
2010	0	0	0	175,562	0	0	0	0	0	0	0	0	0
2011	0	0	0	184,253	0	0	0	0	0	0	0	0	0
2012	0	0	0	193,390	0	0	0	0	0	0	0	0	0
2013	0	0	0	202,994	0	0	0	0	0	0	0	0	0
2014	0	0	0	213,089	0	0	0	0	0	0	0	0	0
2015	0	0	0	223,703	0	0	0	0	0	0	0	0	0
2016	0	0	0	234,862	0	0	0	0	0	0	0	0	0
2017	0	0	0	246,594	0	0	0	0	0	0	0	0	0
2018	0	0	0	258,929	0	0	0	0	0	0	0	0	0
2019	0	0	0	271,899	0	0	0	0	0	0	0	0	0
2020	0	0	0	285,537	0	0	0	0	0	0	0	0	0
2021	0	0	0	299,879	0	0	0	0	0	0	0	0	0
2022	0	0	0	181,399	133,561	0	0	0	0	0	0	0	0
2023	0	0	0	0	330,821	0	0	0	0	0	0	0	0
2024	0	0	0	0	347,501	0	0	0	0	0	0	0	0
2025	0	0	0	0	365,043	0	0	0	0	0	0	0	0
2026	0	0	0	0	383,494	0	0	0	0	0	0	0	0
2027	0	0	0	0	402,900	0	0	0	0	0	0	0	0
2028	0	0	0	0	423,313	0	0	0	0	0	0	0	0
2029	0	0	0	0	444,784	0	0	0	0	0	0	0	0
2030	0	0	0	0	467,370	0	0	0	0	0	0	0	0
2031	0	0	0	0	491,129	0	0	0	0	0	0	0	0
2032	0	0	0	0	516,123	0	0	0	0	0	0	0	0
2033	0	0	0	0	542,417	0	0	0	0	0	0	0	0

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
	Incremental Payments by Layer												
Year	$130,000 xs $370,000	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
2034	0	0	0	0	151,547	418,532	0	0	0	0	0	0	0
2035	0	0	0	0	0	599,182	0	0	0	0	0	0	0
2036	0	0	0	0	0	629,802	0	0	0	0	0	0	0
2037	0	0	0	0	0	662,019	0	0	0	0	0	0	0
2038	0	0	0	0	0	695,917	0	0	0	0	0	0	0
2039	0	0	0	0	0	731,584	0	0	0	0	0	0	0
2040	0	0	0	0	0	769,114	0	0	0	0	0	0	0
2041	0	0	0	0	0	493,850	314,756	0	0	0	0	0	0
2042	0	0	0	0	0	0	850,163	0	0	0	0	0	0
2043	0	0	0	0	0	0	893,894	0	0	0	0	0	0
2044	0	0	0	0	0	0	939,915	0	0	0	0	0	0
2045	0	0	0	0	0	0	988,345	0	0	0	0	0	0
2046	0	0	0	0	0	0	1,012,927	26,387	0	0	0	0	0
2047	0	0	0	0	0	0	0	1,092,955	0	0	0	0	0
2048	0	0	0	0	0	0	0	1,149,409	0	0	0	0	0
2049	0	0	0	0	0	0	0	1,208,826	0	0	0	0	0
2050	0	0	0	0	0	0	0	1,271,363	0	0	0	0	0
2051	0	0	0	0	0	0	0	1,337,184	0	0	0	0	0
2052	0	0	0	0	0	0	0	1,406,464	0	0	0	0	0
2053	0	0	0	0	0	0	0	1,479,387	0	0	0	0	0
2054	0	0	0	0	0	0	0	1,028,024	528,120	0	0	0	0
2055	0	0	0	0	0	0	0	0	1,636,940	0	0	0	0
2056	0	0	0	0	0	0	0	0	1,721,989	0	0	0	0
2057	0	0	0	0	0	0	0	0	1,811,516	0	0	0	0
2058	0	0	0	0	0	0	0	0	1,905,760	0	0	0	0
2059	0	0	0	0	0	0	0	0	2,004,970	0	0	0	0
2060	0	0	0	0	0	0	0	0	390,705	1,718,705	0	0	0
2061	0	0	0	0	0	0	0	0	0	2,219,358	0	0	0
2062	0	0	0	0	0	0	0	0	0	2,335,106	0	0	0
2063	0	0	0	0	0	0	0	0	0	2,456,963	0	0	0
2064	0	0	0	0	0	0	0	0	0	1,269,868	1,315,385	0	0
2065	0	0	0	0	0	0	0	0	0	0	2,720,318	0	0
2066	0	0	0	0	0	0	0	0	0	0	2,862,518	0	0
2067	0	0	0	0	0	0	0	0	0	0	3,012,232	0	0
2068	0	0	0	0	0	0	0	0	0	0	89,548	3,080,313	0
2069	0	0	0	0	0	0	0	0	0	0	0	3,335,824	0
2070	0	0	0	0	0	0	0	0	0	0	0	3,510,565	0
2071	0	0	0	0	0	0	0	0	0	0	0	73,298	3,621,254
	130,000	500,000	1,000,000	3,000,000	5,000,000	5,000,000	5,000,000	10,000,000	10,000,000	10,000,000	10,000,000	10,000,000	3,621,254

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 x Column 9

	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
1996 and prior													
1997	92,224	0	0	0	0	0	0	0	0	0	0	0	0
1998	33,296	59,081	0	0	0	0	0	0	0	0	0	0	0
1999	0	92,526	0	0	0	0	0	0	0	0	0	0	0
2000	0	92,673	0	0	0	0	0	0	0	0	0	0	0
2001	0	92,820	0	0	0	0	0	0	0	0	0	0	0
2002	0	85,684	7,283	0	0	0	0	0	0	0	0	0	0
2003	0	0	93,112	0	0	0	0	0	0	0	0	0	0
2004	0	0	93,251	0	0	0	0	0	0	0	0	0	0
2005	0	0	93,381	0	0	0	0	0	0	0	0	0	0
2006	0	0	93,497	0	0	0	0	0	0	0	0	0	0
2007	0	0	93,595	0	0	0	0	0	0	0	0	0	0
2008	0	0	93,671	0	0	0	0	0	0	0	0	0	0
2009	0	0	78,085	15,636	0	0	0	0	0	0	0	0	0
2010	0	0	0	93,742	0	0	0	0	0	0	0	0	0
2011	0	0	0	93,729	0	0	0	0	0	0	0	0	0
2012	0	0	0	93,678	0	0	0	0	0	0	0	0	0
2013	0	0	0	93,583	0	0	0	0	0	0	0	0	0
2014	0	0	0	93,437	0	0	0	0	0	0	0	0	0
2015	0	0	0	93,233	0	0	0	0	0	0	0	0	0
2016	0	0	0	92,962	0	0	0	0	0	0	0	0	0
2017	0	0	0	92,619	0	0	0	0	0	0	0	0	0
2018	0	0	0	92,196	0	0	0	0	0	0	0	0	0
2019	0	0	0	91,688	0	0	0	0	0	0	0	0	0
2020	0	0	0	91,084	0	0	0	0	0	0	0	0	0
2021	0	0	0	90,375	0	0	0	0	0	0	0	0	0
2022	0	0	0	51,578	37,976	0	0	0	0	0	0	0	0
2023	0	0	0	0	88,614	0	0	0	0	0	0	0	0
2024	0	0	0	0	87,554	0	0	0	0	0	0	0	0
2025	0	0	0	0	86,372	0	0	0	0	0	0	0	0
2026	0	0	0	0	85,070	0	0	0	0	0	0	0	0
2027	0	0	0	0	83,645	0	0	0	0	0	0	0	0
2028	0	0	0	0	82,096	0	0	0	0	0	0	0	0
2029	0	0	0	0	80,413	0	0	0	0	0	0	0	0
2030	0	0	0	0	78,583	0	0	0	0	0	0	0	0
2031	0	0	0	0	76,594	0	0	0	0	0	0	0	0
2032	0	0	0	0	74,439	0	0	0	0	0	0	0	0
2033	0	0	0	0	72,121	0	0	0	0	0	0	0	0

Commutation Value by Layer, Discounted for Both Mortality and Investment Income

Columns are derived by multiplying the corresponding column from Exhibit 4, pages 3 and 4, by Column 9, from pages 1 and 2. For example, Column 23 = Column 10 x Column 9

	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)
Year	$500,000 xs $0	$500,000 xs $500,000	$1 million xs $1 million	$3 million xs $2 million	$5 million xs $5 million	$5 million xs $10 million	$5 million xs $15 million	$10 million xs $20 million	$10 million xs $30 million	$10 million xs $40 million	$10 million xs $50 million	$10 million xs $60 million	$10 million xs $70 million
2034	0	0	0	0	18,513	51,128	0	0	0	0	0	0	0
2035	0	0	0	0	0	67,002	0	0	0	0	0	0	0
2036	0	0	0	0	0	64,207	0	0	0	0	0	0	0
2037	0	0	0	0	0	61,265	0	0	0	0	0	0	0
2038	0	0	0	0	0	58,184	0	0	0	0	0	0	0
2039	0	0	0	0	0	54,976	0	0	0	0	0	0	0
2040	0	0	0	0	0	51,655	0	0	0	0	0	0	0
2041	0	0	0	0	0	29,462	18,778	0	0	0	0	0	0
2042	0	0	0	0	0	0	44,748	0	0	0	0	0	0
2043	0	0	0	0	0	0	41,203	0	0	0	0	0	0
2044	0	0	0	0	0	0	37,629	0	0	0	0	0	0
2045	0	0	0	0	0	0	34,054	0	0	0	0	0	0
2046	0	0	0	0	0	0	29,736	775	0	0	0	0	0
2047	0	0	0	0	0	0	0	27,047	0	0	0	0	0
2048	0	0	0	0	0	0	0	23,705	0	0	0	0	0
2049	0	0	0	0	0	0	0	20,509	0	0	0	0	0
2050	0	0	0	0	0	0	0	17,494	0	0	0	0	0
2051	0	0	0	0	0	0	0	14,691	0	0	0	0	0
2052	0	0	0	0	0	0	0	12,125	0	0	0	0	0
2053	0	0	0	0	0	0	0	9,818	0	0	0	0	0
2054	0	0	0	0	0	0	0	5,142	2,641	0	0	0	0
2055	0	0	0	0	0	0	0	0	6,027	0	0	0	0
2056	0	0	0	0	0	0	0	0	4,546	0	0	0	0
2057	0	0	0	0	0	0	0	0	3,331	0	0	0	0
2058	0	0	0	0	0	0	0	0	2,362	0	0	0	0
2059	0	0	0	0	0	0	0	0	1,615	0	0	0	0
2060	0	0	0	0	0	0	0	0	196	863	0	0	0
2061	0	0	0	0	0	0	0	0	0	663	0	0	0
2062	0	0	0	0	0	0	0	0	0	393	0	0	0
2063	0	0	0	0	0	0	0	0	0	220	0	0	0
2064	0	0	0	0	0	0	0	0	0	56	58	0	0
2065	0	0	0	0	0	0	0	0	0	0	55	0	0
2066	0	0	0	0	0	0	0	0	0	0	24	0	0
2067	0	0	0	0	0	0	0	0	0	0	9	0	0
2068	0	0	0	0	0	0	0	0	0	0	0	3	0.00
2069	0	0	0	0	0	0	0	0	0	0	0	1	0.00
2070	0	0	0	0	0	0	0	0	0	0	0	0	0.00
2071	0	0	0	0	0	0	0	0	0	0	0	0	0.03
	125,520	422,784	645,876	1,179,539	951,989	437,878	206,147	131,305	20,718	2,195	146	4	0.03
	Overall Total =	4,124,102											

Capital and Risk and their Relationship to Reinsurance Programmes

by Stewart M. Coutts and Timothy R.H. Thomas

Capital and risk and their relationship to reinsurance programmes

by Stewart M. Coutts & Timothy R.H. Thomas

Abstract
An earlier paper by the same authors developed the Daykin *et al.* (1994) asset/liability model to examine the effects of different reinsurance programmes on the capital of a direct property/casualty insurance company. By modelling the gross premiums and claims separately from the impact of reinsurance on them, it is possible to examine directly the effects of different reinsurance programmes on a company's expected performance just as easily as changes in asset mix or business volumes.

This paper goes on to discuss how such a model can be used to quantify capital at risk for management reporting purposes, both for the company as a whole, and within individual profit centres, and how this is affected by different reinsurance strategies. It therefore links closely to the Dynamic Financial Analysis project being sponsored by the Casualty Actuarial Society.

Biographies
Dr Stewart Coutts is a consulting actuary, who has specialised in property/casualty insurance for 25 years. He published papers on the rating of motor insurance in the early 1970's, and was a member of the British Solvency Working Party in the mid-1980's. The work done by this body was a forerunner of both the Daykin model and the NAIC Risk Based Capital model.

Tim Thomas is a Chartered Accountant, who has worked in various capacities in the insurance industry for over 20 years. He joined the reinsurance division of Willis Faber & Dumas as an executive director four years ago, and since then has been involved in various aspects of alternative risk transfer, as well as being involved in the Group's market security operations. He has a degree in Mathematics from Southampton University.

1. Introduction

1.1 Insurance companies have as their prime business the accepting of unwanted risk on behalf of others. They accept different types of risk in the expectation of being able to generate an adequate return on capital from the premiums charged. The management of the risk so assumed within the company is therefore of fundamental importance to the success of the operation.

1.2 Intuitively, an insurance company ought to be able to manage exposures of both liabilities and assets in such a way that it allocates its established "risk tolerance" between underwriting activities and investment strategy to maximise its expected overall return on capital. By this, we mean the management's willingness to live with unstable results in order to boost expected profitability. The "risk tolerance" level of an individual company is clearly a matter for its Board of Directors to establish, subject to regulatory minimum standards.

1.3 By expressing this "Company Standard" level in financial terms, it becomes possible to measure the amount of capital at risk in both the company as a whole, and in individual operating units. The company's performance can then be measured and managed, and different business strategies can be compared - a task ideally suited to stochastic modelling.

1.4 In order to improve return on capital, either in an individual profit centre or in the company as a whole, we can either increase profits or reduce capital employed. This paper addresses in particular the quantification of capital employed, and how this is affected by different reinsurance strategies.

1.5 Reinsurance has traditionally been bought to stabilise both profits and capital of an insurance company. It therefore has a major impact on the risk capital requirements of both the company as a whole, and each individual unit. If we want to manage risk capital, we have to be able to understand how reinsurance affects it.

1.6 We consider that a better understanding of the overall financial impact of reinsurance is of increasing importance because of the need to compare the relative merits of different reinsurance structures both with each other, and with the range of new capital market solutions being developed, which offer varying degrees of risk transfer.

2. Capital at risk *v* RBC

2.1 The risks to which the insurance company are subject can affect a company's balance sheet in different ways. The RBC model introduced recently in the USA is an attempt to quantify the overall effect of these risks, and set appropriate minimum capital standards.

2.2 In this paper, we differentiate between the values given by applying this model, and management's own internal quantification of capital at risk. To avoid confusion, we use the term "RBC" as the value determined by the NAIC RBC model, and "capital at risk" as the internal measure. In no way are we seeking to question the value of the RBC formula itself, both as a regulatory tool, and as a device for educating management as to the value of using quantative techniques to review the effectiveness of their strategies for maximising prudent returns.

2.3 The RBC model is designed to serve as a diagnostic tool for regulators, primarily as an early warning indicator of situations which may need regulatory attention. However, it is likely that companies with high scores will try to capitalise on them by encouraging the creation of "league tables", which in turn will trigger a flight to perceived quality.

2.4 It is therefore likely that companies, particularly those with lower than average RBC positions, will take action to improve their situations. Some of this action will undoubtedly be of a cosmetic nature, similar to the "financial reinsurance" abuses which FAS113 has tried to outlaw. Other action will undoubtedly be taken for sound business reasons. In any case, RBC implications will increasingly be taken into account by managements in formulating their strategic plans.

2.5 As managements become increasingly aware of the impact of RBC requirements on business, there will be an increasing realisation of the need to service capital. Thus managements now have a growing need for a tool for allocating capital to, and measurement of performance of, individual operating units.

3. RBC Formulae *v* Stochastic Asset/Liability Modelling

3.1 The authors see RBC formulae as as a regulatory tool, rather than for use inside a company, either for risk management or capital allocation purposes. From this perspective, there are a number of weaknesses, in particular

- they look back at where the company has come from, rather than attempting to factor in future business plans
- the company's exposure to catastrophic loss is considered neither gross nor net of reinsurance
- reinsurance factors are based on past average experience and no explicit allowance is made for changing future reinsurance programmes

3.2 Further, a model built along RBC lines involves the setting of various parameters for each class of business, which tend to be based on market average data. In theory, it would be possible to adjust these market figures for internal management purposes, and to assess the effect of different reinsurance arrangements. However, these adjusted parameters would need to be established and justified to management at both corporate and profit centre levels.

3.3 On the other hand, stochastic asset/liability modelling goes back to first principles to generate estimates of each individual cash flow for each line of business. By modelling the gross premiums and claims separately from the impact of reinsurance on them, it is possible to examine directly the effects of different reinsurance programmes on a company's expected performance just as easily as changes in asset mix or business volumes.

4. What is a Stochastic Model?

4.1 Our earlier paper to the Institute of Actuaries in February 1997(Coutts and Thomas (1997)) described the WISPR stochastic asset/liability model, able to simulate the major types of reinsurance treaty. This model is designed to simulate the development of both assets and liabilities of an insurance company which accepts new business for a period of three years, projecting forward until all outstanding claims have been paid. The three year planning horizon was set as a compromise between the desire to establish a medium term view of the company's development, and the difficulty of setting realistic input assumptions.

4.2 In this paper, we show how the output from this model can be developed as a means of allocating risk capital by profit centre, taking fully into account the different risk profiles of different classes of business, and how this process is influenced by different reinsurance structures. The model itself is described more fully in our first paper, but for convenience, the overall design is summarised in Appendix 1.

4.3 Stochastic model office systems, based on forecasting individual cash flows from each line of business, have been well-established in the Life Insurance industry for several years, and are still in their infancy in Property/Casualty insurance. They will grow in importance as their sophistication grows. They need to be driven from the top of an organisation as an integral part of the planning process, and require constant amendment and refinement. Their use gives a totally new dimension to management information, not a replacement for previous reports, but extra leverage from there.

4.4 By modelling each cash flow separately, the anticipated results arising from different strategies can be compared, and in particular the inter-relationship between investment risk and insurance risk can be managed. These models allow management to:-

- Establish the risk profile of the company in financial terms
- Understand and manage the volatility in earnings
- Compare alternative strategies on a level playing field
- Allocate risk capital by line of business, and set profit targets
- Examine the relative merits of different reinsurance structures

5. Why buy reinsurance?

5.1 Apart from certain non-financial considerations, such as the acquisition of technical assistance from reinsurers, the traditional reasons for buying reinsurance are:-

- To protect capital
- To stabilise earnings
- To release capital for alternative uses

5.2 These reasons translate easily into the new language of maximising return on capital at risk. What has happened is the growth of alternative risk transfer mechanisms, and the extra sophistication of capital markets. The range of options open to management now includes:-

- Traditional bond and equity finance
- "Act of God" bonds
- Reinsurance derivatives
- Financial or Finite Risk reinsurance
- Reinsurance captives
- Traditional reinsurance

5.3 Reinsurance has long been held to be a substitute for capital, but little work has been published as to how this can be measured. With the growing interest of capital markets in risk transfer products, this measurement will become critical, so that comparisons can be made into the cost-effectiveness of different instruments. In particular, for management to assess the effect of a particular reinsurance contract as compared to alternative strategies, management needs to measure:-

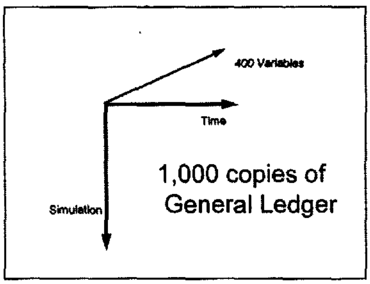
- how much capital is released by the reinsurance contract
- how much it costs to service
- over what time-scale the capital has to be repaid.

6. The main outputs from WISPR

6.1 When we built the model, we recognised that the outputs needed to be able to be interpreted by a wide range of people within the management team,including actuaries, underwriting managers, investment managers and non-executive directors. We therefore considered it essential to produce these outputs as easily understood graphs as far as possible, leaving the numeric values they summarised to be used for more detailed analysis by the appropriate specialists.

6.2 In order to generate all the cash flows, the model builds up for each simulation in each run, a summary of the company's general ledger from last year-end until the run off of the last claim from business accepted in three year's time.

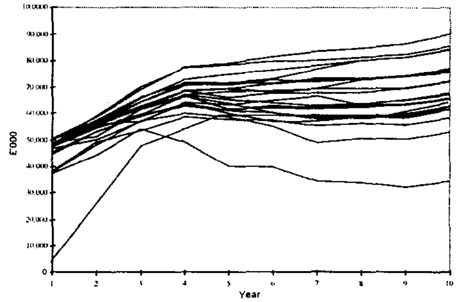
Figure 1. Outputs of each run



6.3 The output consists of values of a large number of variables (approx 400), each of which is indexed by a simulation number and projection year. This produces an enormous amount of data and we had to use a database package to manipulate it. The importance of keeping all the simulated data cannot be emphasised enough because this allows the database to be interrogated to identify which particular simulation run is giving odd results and why. Strategies can then be developed to overcome this problem.

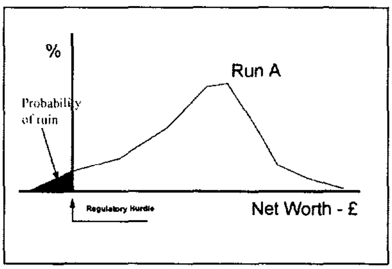
6.4 The graph below plots twenty simulations of the net worth (policyholder surplus) of the company over the ten year period from the last balance sheet until all claims from projection year three have been paid. Each line represents one simulation. There is considerable variation in result in the first four years, but results stabilise after the company enters runoff. In practice, of course, considerably more than twenty simulations would be made, but in this paper we have limited the number in order to produce clearer pictures!

Figure 2. Development of Net Worth



6.5 Alternatively, we can look at the net worth at a point in time. This is done by plotting the probability distribution of the simulation output at a fixed time, for example at the end of three years.

Figure 3. Probability Distribution of Net Worth at the end of 3 years



6.6 We believe this graph gives a very easily understood picture of the volatility of performance. Management should be trying to shift the graph as far to the right as possible, representing an increase in profits, whilst keeping it as peaked as possible, thus stabilising the profits. The left hand side shows the probability of failing to meet the chosen yardstick. The "regulatory hurdle" axis can be drawn in various places to indicate either internal or external requirements, whilst the "probability of ruin" is the probability of failing to meet this yardstick, at a fixed point of time.

6.7 Once this first run has been completed (a major task comparable to, and probably done in conjunction with, the annual budgeting process), other runs (different reinsurance programmes or different asset-mixes etc.) can be carried out, and the results compared, thus allowing a picture to be built up comparing the risks and returns expected from following different strategies. We can apply this technique to the particular case of looking at the impact of different reinsurance structures on capital at risk.

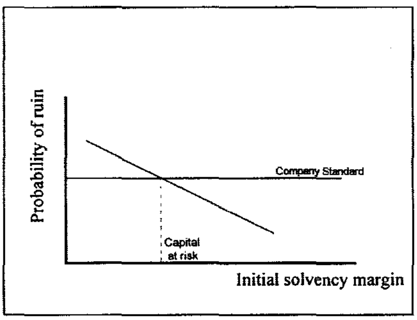
7. Capital Allocation by line of business

7.1Once each line of business has been fitted to the model, we can use this output to allocate risk capital to each line of business, and to assess how this is impacted by different reinsurance structures.

7.2 Management first needs to set its limit of risk tolerance, possibly as a maximum acceptable probability of ruin of, say, one in 100 years for the company as a whole, or, more likely, a probability of failure to meet a specifed multiple of regulatory requirement. A lower hurdle can then be established for an individual profit centre.

7.3 The model can now be run for a single profit centre within the company, to establish the capitalisation required to meet this ruin probability hurdle.

Figure 4. Set Company Standard for Probability of Ruin



7.4 Figure 3 showed the probability of ruin for a particular scenario. By altering the initial capital, leaving all other inputs unchanged, it is possible to build up a plot of the probability of ruin measured against the opening capital. Figure 4 shows this latter graph for a particular profit centre, and the capital at risk can be established by comparison against the company standard.

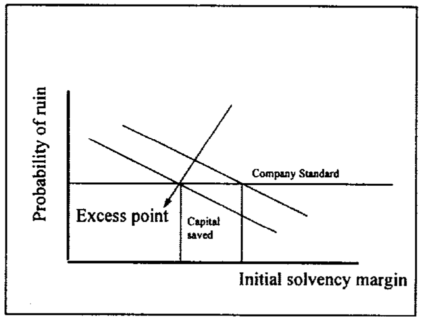
8. Comparison of different reinsurance structures

In paragraph 5.3 above, we identified three questions to address:

8.1 How much capital does a reinsurance contract release?

8.1.1 We can run the model twice, once with each programme, and plot the probability of ruin against initial solvency margin for both runs. Applying the company minimum standard to these gives the following pictorial results for two different excess of loss reinsurance programmes:-

Figure 5. Measure capital saved

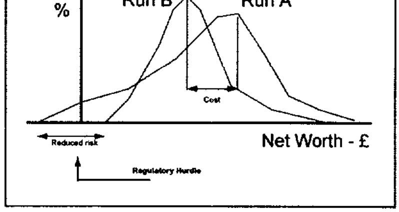


8.1.2 Figure 5 shows the risk capital saved by reducing the excess point at a predetermined probability of ruin. This capital saved can now be used for alternative purposes within the company.

8.2 How much does this cost to service?

8.2.1 The servicing cost of the extra reinsurance is the premium paid away, less the anticipated recoveries, taking into account any lost investment income. This can be examined by comparing the probability distributions.

Figure 6. Expected servicing cost



8.2.2 Figure 6 shows the net worth of a company at the end of the period. Run A is the present reinsurance arrangement and Run B is a different one. The difference between Run A and Run B is that Run B is safer but has a lower expected return than Run A. But the price the company pays under Run A is a higher probability of ruin.

8.3. Over what timescale does the capital have to be repaid?

8.3.1 This last point is the fundamental difference between reinsurance and borrowing. Traditional reinsurance tends to be renegotiated annually, but with the expectation of long term continuity. In particular, there is no contractual obligation for losses to be repaid, although a deficit usually leads to a price increase, and continued deficits to a cancellation of cover.

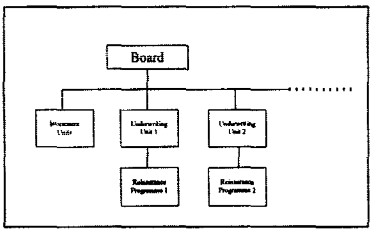
9. Company re-structuring

9.1 In spite of several weaknesses, the RBC formula approach has led to management having a far greater incentive to look at risk management and capital allocation. Perhaps, therefore, the greatest contribution comes from forcing management to impose proper controls on capital allocation.

9.2 It should be noted that in order to make this process fully effective, there will need to be much closer liaison between line insurance managers and the Treasury function than has often been the case, and indeed this trend has already started with Chief Financial Officers taking a growing interest in reinsurance purchasing decisions.

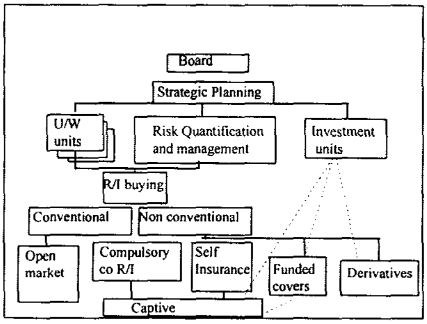
9.3 In order to achieve this, we believe the present management structure, as shown in Figure 7, has to be altered.

Figure 7. Present insurance company



9.4 In this structure, each underwriting unit has its own management team working independently, and having its own separate reinsurance arrangement. Taking company-wide decisions on risk management, or integrating investment policy with underwriting exposure is almost impossible to achieve. Therefore we believe a change is inevitable towards:-

Figure 8. New insurance company



9.5 Figure 8 shows what we believe will be the structure of the new insurance company. The reinsurance element has been promoted to be almost equal in importance to investments. Further, reinsurance requirements will be decided by looking at the corporate level as part of the overall risk management, rather then at a line of business level. Hence, the decision process between choosing reinsurance or capital becomes much closer in the management thinking.

9.6 Strategy is determined through a central "Risk Quantification and Management Committee" with individual companies expected to make say 10% after tax, and individual product lines 5% after tax return on risk adjusted capital - including risk-free investment return on reserves. The investment unit "borrows" from underwriting departments risk free, and has to earn the remaining5% after tax

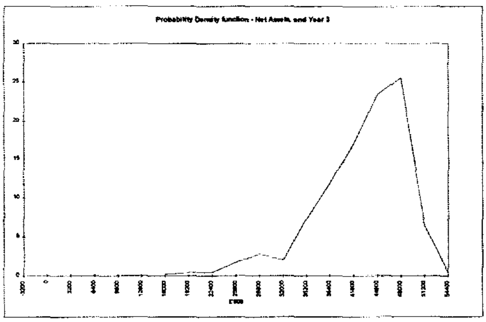
10. A comparison of Capital at Risk and RBC

10.1 This paper sets out a case for using the output from a fitted stochastic model to allocate capital by line of business, and to measure the impact of different reinsurance programmes on this capital requirement. But does it work in practice?

10.2 In our earlier paper, we gave a simple illustration of how the model could be used for a start-up monoline company, writing UK homeowners business. The company had an initial capital of £50 million, and writes an annual premium of £100 million. This line of business is exposed to catastrophe accumulations for both windstorm and flood, and therefore requires significant reinsurance protection. (Typically, a rerun of the 90A UK windstorm of January ,1990 would be expected to give a loss of around £40m, whilst the 1953 North Sea Tidal Surge floods would produce a loss of £90m plus.) The detailed assumptions for this illustration are shown in Appendix 2.

10.3 The first run of the model, with a catastrophe reinsurance programme of £80million excess of £10million, 95% placed, produced the individual plots of net worth previously shown in Figure 2. By taking a cross section through all 1,000 simulations at the end of Year 3, we produced the following distribution of net worth (Figure 9):-

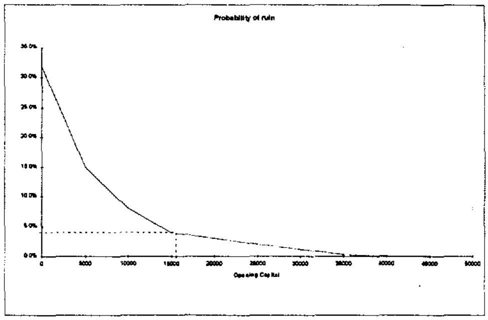
Figure 9. Demo Insurance Co. Net Worth - Run 1



10.4 It is worth noting in passing that although all the detailed assumptions used are for illustration purposes only, the answers being produced by the model reflect the unstable nature of the results of a mono-line company writing catastrophe-exposed business.

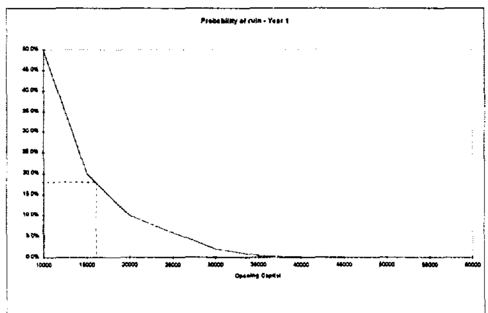
10.5 We now develop the output further to consider whether the initial capital is fully at risk, and indeed whether a lower figure could be justified (regulatory issues permitting!). We do this by plotting the probability of ruin, as explained in Section 7 above.

Figure 10 Demo Insurance Co - Probability of ruin -Run 1



10.6 Figure 10 shows that at the European Union solvency margin requirement of £16 million, there appears to be approximately a 4% probability of failure, defined as negative net worth at the end of Year 3. However, the start-up company has a very strong probability of making a loss in the first year, as can be seen from Figure 2, and at this £16 million initial capital, the probability of negative net worth at the end of the first year is approximately 19%. (Figure 11.)

Figure 11- Demo Insurance Co - Probability of ruin - Year 1

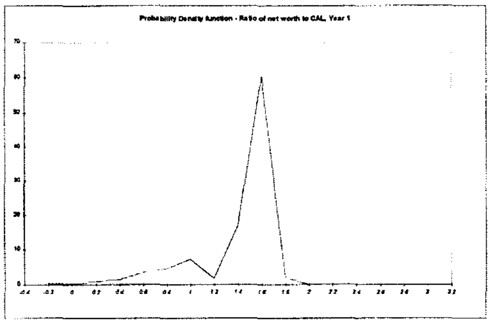


10.7 Not surprisingly, therefore, the UK Department of Trade & Industry (DTI) looks for a level significantly higher than this minimum figure when considering the business plan of a new company. At double the minimum figure, the probability of ruin drops to 1% at the end of Year 1, whilst there is an 18% chance the company will fail the minimum solvency test.

10.8 We can now assume that management's risk tolerance can be expressed as "not allowing the probability of failing the DTI solvency test to fall below 20%.", and re-run the model with an initial risk capital of £32 million. We can then estimate the

RBC requirement for the company at the end of both Years 1 and 3, assuming the same factors are appropriate for both US and UK homeowners business, and that UK Government securities require no risk factor. At the end of Year 1, this gives an RBC requirement of £16.4 million, against average total adjusted capital of £21.3 million. (see workings in Appendix 3), a ratio of 130%, with a standard deviation of 34%, but with a very skew distribution:-

Figure 12 Probability Distribution of RBC %

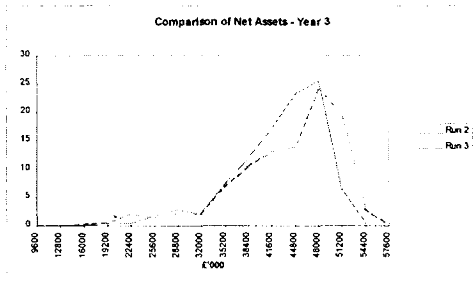


Interestingly, whether by design or coincidence, there is also a 20% chance of breaching the Company Action Level of the RBC rules. (By Year 3, that this ratio has improved to 247%, with a standard deviation of 47%.)

10.9 We can demonstrate from the detailed outputs that reducing the initial capital from £50 million to £32 million increases the 3-year average post tax return on capital from 7.6% to 9%. Not surprisingly, the standard deviation of this return also increases, from 4.6% to 7.8%.

10.10 A risk-averse owner may well be interested in reducing this volatility of earnings by reducing the catastrophe retention to around £6 million. Running the model on this assumption produces a post tax return on capital of 8.46%, with a standard deviation of 6.04%. Alternatively, buying this extra layer reduces the capital at risk from £32 million to £30 million whilst the average reduction in annual post tax profits is £ 0.3 million, equivalent to a 15% post tax servicing cost on the £2million saving.

Figure 13 Comparison of Net Assets at the end of Year 3.



10.11 Management can now decide whether they can use this £2 million released capital more effectivly elsewhere, bearing in mind its servicing cost, and the decrease in volatility in earnings.

11. Conclusion

11.1 We believe that Boards of Directors of insurance companies need a better understanding of the financial risks being assumed by their companies, and how reinsurance arrangements reduce these to manageable proportions. Although tools like WISPR take considerable effort to install, the benefits are substantial, and the timing is now right, with:-

- Increased attention on capital from rating agencies and regulators
- Lower profit margins
- Increase in interest sensitive products
- Increase in market volatility
- Increase in non-traditional competitors

11.2 Inevitably, such benefits can only be obtained at the cost of fitting far more complicated assumptions than are necessary to fit an RBC model.

11.3 By using stochastic modelling to establish estimates of means and variances, it is possible to take assumptions built up by underwriters, using concepts with which they are familiar, and translate these into the language of investment portfolio management. This reduces the gap in understanding across different disciplines at senior management level, and allows comparisons of reinsurance with other forms of risk transfer.

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A description of the WISPR stochastic model

Appendix 1

1. Overview

1.1. The model is designed to simulate the development of both assets and liabilities of an insurance company. This company is assumed to accept new business for a period of three years, and then projects until all outstanding claims have been paid. The three year planning horizon was set as a compromise between the desire to establish a medium term view of the company's development, and the difficulty of setting realistic input assumptions.

1.2. The assets are sub-divided by major categories such as Government stocks, Equities and Property. The models used project forward income cash flows until the claims have runoff or the company is ruined.

1.3. The liabilities work on a class-by-class basis (see section 2), modelling the claim payment cash flows of gross reinsurance results and their associated reinsurance recoveries and reinstatement premiums, after allowing for the effects of both financial and social inflation.

1.4. The reinsurance programme can comprise any combination of four main types, quota share, surplus, risk excess and catastrophe excess. The model can accommodate variable co-reinsurance of each cover, as well as factors such as event caps on proportional treaties. The catastrophe module allows information from GIS (Geographic Information Systems) models to be incorporated for storm, freeze, flood, earthquake and subsidence.

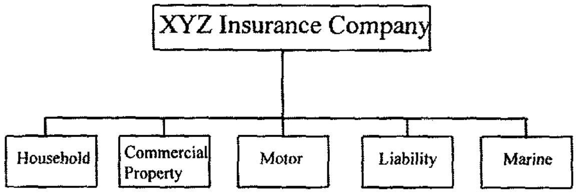
1.5. By combining the cash flows of assets and liabilities the model produces, the potential for profits or losses to emerge from the runoff of outstanding claims.

6. Each run consists of a user-specified number of Monte Carlo simulations, in each of which the variables are sampled from appropriate probability distributions, so that a probability distribution can be built up for the results of the company as a whole. The run can then be repeated with different assumptions, to examine the sensitivity of these results to changing circumstances.

2. Class and subclass structure

2.1 The model calculates gross of reinsurance transactions on a sub-class basis, whilst reinsurance transactions are at a class level (Figure A1).

Figure A1. Tree structure for group, company, class and subclass



2.2 Figure A1 shows a typical division of class and subclass of an insurance company .The main classes of business being household, motor, etc., with motor being split into subclasses Comprehensive and Third Party. The amount of detail at subclass level is company dependent, for example if a company is writing only two classes of business, homeowners and motor, it might be necessary to have three or four subclasses for each class.

2.3 We believe that in practice, the number of classes should be limited to six, and subclasses to no more than ten, so that the overall picture can still be seen without being lost in a mass of detail.

2.4 The class structure will vary from company, and it is essential to determine this before too much time is spent in trying to assemble input data.

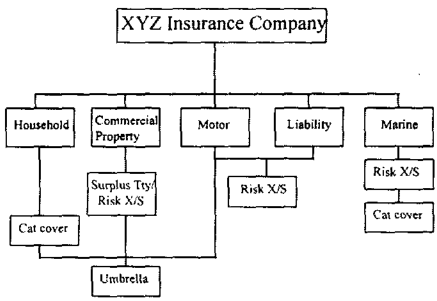
3. The main types of reinsurance

3.1 Reinsurance can be broken down into facultative (laying off parts of individual risks) and treaty (laying off risks aggregated over a block of business). Treaty reinsurance can be further analysed into proportional (principally quota share and surplus) and non-proportional (excess of loss on either a per risk or per event basis, or stop loss). To model reinsurances other than quota share treaties, it is necessary to generate both individual claims and event catastrophes (which is where claims aggregate across several policies to produce a potential recovery). Further, in the case of surplus treaties, commonly used to protect commercial property portfolios, it is necessary to determine the size of cession on each policy subject to a large claim, before a recovery can be calculated.

3.2 There is a bad debt risk involved in ceding business to any reinsurer, however much care is taken in selection, and this can never be entirely removed. Whilst management should not lose sight of this risk, we have ignored it in this paper for simplicity. The model itself can handle the failure of a fixed percentage of security, specified separately for each separate contract, but a more rigorous treatment is worthy of a detailed study.

3.3 We expect a reinsurance programme for the classes of business in Figure A2 to resemble:

Figure A2. Simple reinsurance programme

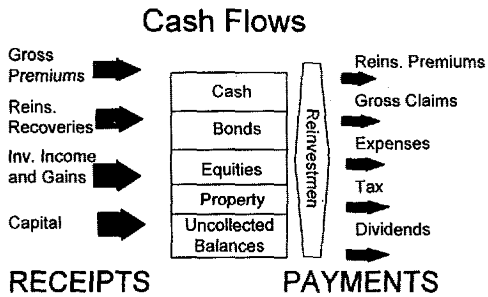


3.4 Figure A2 shows that the household business is protected by a catastrophe, whilst motor and liability are covered by risk excess of loss. Commercial property is protected by a combination of surplus treaty and risk excess. An umbrella whole account protection covers catastrophe accumulation over household, commercial property and motor.

4. Build up cash flows by class of business

4.1 The concept of cash flow modelling is now well documented, for example Daykin et al. (1994) (Chapter 1). In a simple diagram, Figure A3 illustrates the standard cash flows which have to be modelled.

Figure A3. Cash flow



4.2 Alternatively Figure A3 can be linked together in the Daykin et al. (1994) transition equation:-

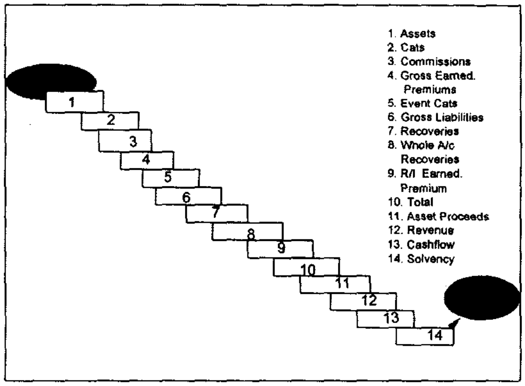
Assets(end of period) = Assets(beginning) +(Gross Premium - Claims - Expenses - Reinsurance Premiums + Reinsurance Recoveries) + Investment income & gains - Taxation - Dividends + New Capital [+ New borrowings].

4.3 With suitable adjustments for changes in provisions, or receivables, this equation can be interpreted on either a cash basis or on an accounting accruals basis.

5.Modular Approach gives flexibility

5.1 The cash flow computer programmes have to be designed very carefully, in particular the main problems relate to inter-relationships between transactions and that actuarial art in projecting forward is always improving. With this in mind, the model was built up in a modular fashion, see Figure A4.

Figure A4. Modular structure



5.2 There are five distinctive stages in building up the final output:

Stage 1 : the data base input

Stage 2: modules 1 and 2 which calculate inflation rates and investment returns and individual catastrophe losses

Stage 3: modules 3, 4, 5, 6 which are defined for each sub-class, calculate cashflows and technical reserves gross of reinsurance

Stage 4 : modules 7, 8, 9 which are reinsurance recovery calculations

Stage 5: modules 11,12,13,14 which are the basis for the outputs .

5.3 By building up the model in modules as shown above, we have attempted to create a flexible structure which will enable changes in the computer program to be made with the minimum of effort. For example, these changes could take the form of a more sophisticated asset model, advances in actuarial techniques, the specification of a different family of claims curves, etc. This flexible approach has also been adopted in relation to links to other models - for example, we have not attempted to duplicate packages for reserving, or for turning claims data into probability distributions.

Assumptions for the demonstration of a simple start-up company

Appendix 2

A detailed list of the parameters used in the simulations of the simple start-up company are given below:

- Initial capital £50 m.

- Opening investment portfolio:

Government Bonds	£45 m
Cash	£3 m
Working Capital	£2 m

- Positive cash flows invested 50% in Government Bonds, 50% in equities.

- Selling Rules

  There are two alternative strategies for how a negative cash flow will affect disinvestment. Firstly, to disinvest in proportion to asset holdings at the start of the year or secondly, the assets are ordered and the asset with the highest priority is sold first. For the start-up company we use the first method.

- The investment assumptions were as follows:-

	Cash	*Equities*	*Bonds*
Mean Real Return	1%	5%	3%
Running Yield	4.5%	3.0%	6.5%
Volatility of Capital Growth		20%	10%
Volatility of Income Growth		5%	1%

- The effective tax rate is 33%, and dividends will be at 50% of after-tax profits.

- Financial inflation was assumed to be normally distributed with a mean of 3.5% and a standard deviation of 0.5%.

- The average rate of financial inflation assumed in calculating the value of mean loss ratios was also assumed to be 3.5%.

- The business plan assumed that in each of the three years of the modelling period, the gross premium was £100 m, and that losses other than catastrophe ones were normally distributed with a mean loss ratio of 55% and standard deviation of 2%. This information could be estimated from competitors' published figures, or other sources. Claims reserves are not discounted.

- The unearned premium carried forward at the end of each year was assumed to be 40%

- Commissions and office expenses were assumed to be 28% of premiums, reducing to 1% of year 3 gross written premium once business is no longer being written.

- Social inflation can be applied at differential rates for attrition and large losses but was ignored in this case.

- Claims runoff patterns - the mean proportion and standard deviation of a claim paid in year *i* of development of the claim. These values are needed for past, future and catastrophe knock-on claims. These are all assumed to follow the same pattern:

Year	*Runoff Pattern*	*Standard Deviation*
1	64	5
2	28	3
3	4	3
4	2	2
5	1	2
6	1	2

- The catastrophe reinsurance programme was structured as follows:-

Layer	*Indemnity*	*Deductible*	*Rate on line*	*Co-reinsurance*
1	10 m	10 m	20%	5%
2	20 m	20 m	12%	5%
3	20 m	40 m	8%	5%
4	30 m	60 m	4%	5%

- The delay (in months) between making gross payments in respect of past and future claims and receiving the recovery payments. For the start-up company these values are taken as 3 months for quota share and 1 month for excess of loss.

- Because the account is not subject to any wide fluctuations in size of sum insured, no reinsurance of individual risks is necessary, and therefore this run of the model did not need to generate individual large losses other than for catastrophes.

- Natural perils catastrophe losses - these can be input either as a series of specific large losses or sampled by the model from a probability distribution. Under this, WISPR requires certain assumptions regarding the probability and potential size of each event for each peril to be input. These assumptions, obtained either from a GIS type model or from general management views, comprise the estimated maximum loss, the probability of an event of at least one tenth this size happening and a table setting out the relative probabilities of the size of the loss, given that one has happened. This table needs to be completed for each decimal of PML. For the start-up company the tables assumed are as follows:

	Claim Size	Probability
Storm	8000	0.42
PML £80,000	16000	0.20
Probability 20%	24000	0.12
	32000	0.07
	40000	0.05
	48000	0.04
	56000	0.03
	64000	0.03
	72000	0.02
	80000	0.02
Flood	10000	0.05
PML £100,000	20000	0.05
Probability 2%	30000	0.05
	40000	0.15
	50000	0.15
	60000	0.15
	70000	0.2
	80000	0.1
	90000	0.05
	100000	0.05

Estimate of Risk Based Capital requirement - Run 2

Appendix 3

This simplified calculation of the Company Action level RBC requirement at the end of Year 1 is based on the requirements as set out in the NAIC instructions for 31st December 1996. These are set out in detail for simulation 3 of run 2, and average figures are also shown for each heading. A revenue account for simulation 3 is included for reference.

R0 Asset Risk - Subsidiary Insurance Companies

Not applicable

R1 Asset Risk - Fixed Income

Only RBC amount is for cash working balance, £3 million at 0.3% = 9,000.

(average 9,000)

R2 Asset Risk - Equity

Not applicable

R3 Asset Risk - Credit

Outstanding reinsurance recoveries £5.7 million at 10% =	570,000
Unpaid reinsurances - nil	
	570,000
50%	285,000

(average 103,000)

R4 Underwriting Risk - Reserves

Gross outstanding loss reserves 19,233,000 at 18.3% =	3,523,000
50% of credit RBC	285,000
	3,808,000

((1,275*0.928) - 1 = 0.1832)

(average 2,553,000)

R5 Underwriting Risk - Net Written Premium

93,160,000 at 17.4% = 16,192,000

((0.917*0.942)+ 0.31 - 1 = 0.1738)

(average 16,176,000)

RBC (Company Action Level) = R0+SQRT(R1^2+R2^2+R3^2+R4^2+R5^2)

= SQRT(9000^2+285000^2+3808000^2+16192000^2)

=16,636,000

(average 16,414,000)

Comparing Reinsurance Programs—
A Practical Actuary's System

by Robert A. Daino, FCAS, and
Charles A. Thayer

COMPARING REINSURANCE PROGRAMS
A PRACTICAL ACTUARY'S SYSTEM
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Mr. Daino has been in the industry since 1971. His experience includes actuarial and senior management positions at bureaus, primary insurers, and a reinsurance brokerage firm. Today, he is President of Cornerstone Consultants, Inc. in Ramsey, New Jersey and specializes in actuarial, management and related consulting, primarily in the reinsurance and specialty lines arenas. He has designed, prepared and delivered scores of reinsurance program analyses and presentations. He has a B.A in Mathematics from Fordham University, and has been a member of the CAS and AAA for over twenty years. He, his wife and two daughters reside in Upper Saddle River, NJ.

Mr. Thayer is a consultant with Cornerstone Consultants, Inc. He holds a Bachelor of Science degree in Mathematics from the University of Vermont. Before joining Cornerstone, he helped establish an actuarial function for a New York City reinsurance broker for six years. He began his insurance career in primary insurance, having served with two New England multi-line insurers where he was involved in pricing, product development, data quality and reporting systems. He resides in Mahwah, NJ.

Abstract:

This paper describes the elements of a simulation system used by the authors. A "user manual" approach is used to describe the elements of the system. A practical sample scenario is used to show how the system is used in practice.

It is not the authors' intent herein to discuss in any depth the technical issues involved in selecting the many parameters involved in a simulation. Rather, we try to show how a system can be used to control the parameters needed, and also help users analyze and communicate the results to others.

INTRODUCTION

Over the course of several years, the authors have been involved in many situations where reinsurance buyers were faced with making a decision between several competing recommendations for their reinsurance program, often with significantly different structures. We found early on that one of the tools we needed was a simulation system. This paper will describe the elements of our current system, and the steps we take in building a simulation and analyzing the results.

Having developed early versions of this system and presented the results to buyers, we learned much about what buyers, in general, consider important and what they are interested in seeing when making such a decision. To be sure, there is a wide variation in technical savvy among the many buyers of reinsurance, but most can understand the usefulness of comparing alternatives over many possible loss scenarios, the importance of getting a handle of some sort on the "odds" of favorable and unfavorable things happening, and almost all appreciate graphical representations of the results.

With reasonable assumptions about the variability of the number and size of claims by line of business, and with other necessary assumptions needed to mimic the insurance/reinsurance process, a more complete comparison between and evaluation of the several competing reinsurance programs can be made (versus single scenario comparisons). Most buyers understand this as well.

Furthermore, most buyers understand that, although the final simulation averages and aggregate distributions are only as good as the input assumptions (which are often *very* soft), the decision value lies primarily in the comparative analysis that results (i.e., the absolute values may be approximate/soft numbers, but the indications about whether Option A is better than Option B are much stronger).

Our original model was a Lotus 123™ spreadsheet, but within the last year, working in conjunction with a group of professional developers and a major reinsurer, we have brought this over and enhanced it greatly into a VB/Excel™ system with a graphical user interface, template libraries for maintaining and controlling simulations, histories, etc. In this paper, we will describe the elements of this system as well as a practical example of its use.

SYSTEM OVERVIEW

Our simulation system is a 32-bit Windows™ system designed to build and manage the components of large Excel ™ - based simulation models. The user is guided through the creation of:

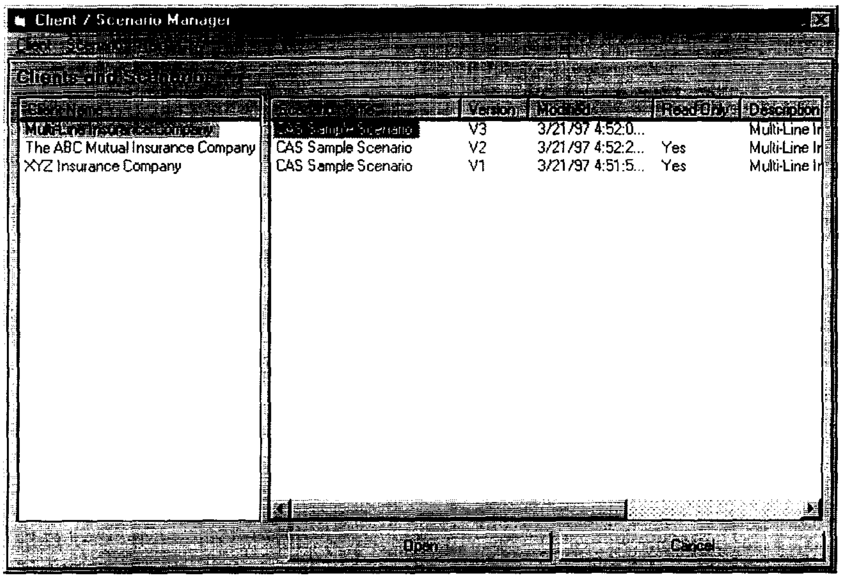
- Simulation Lines of Business - up to 40 lines, based on up to 40 Input Lines of Business (when data is available in more detail than needed for simulation),
- Reinsurance Programs - up to three competing programs, made up of up to 15 treaties per program. A default gross = net = "no reinsurance" program is automatically included, and
- Other inputs, as needed, for beginning balance sheet values, investment and tax assumptions.

The system presents Excel ™-like "patches" to the user in a Windows™ front-end graphical user interface (GUI) program which also maintains control over every patch within a given client's scenario. Multiple scenarios can be maintained for each client, and the system can control many clients. Patches can be saved for future use in similar scenarios for this or other clients. The program controls all of this in an Access™ database.

Once the user is satisfied that all inputs are ready, he instructs the program to compile the Excel™ simulation workbook. Once the workbook is built, the user can view it, run the simulation or detach the workbook for use outside the control and management of the system (for cases where the standard program is not sufficient to describe the alternatives being compared). The user specifies the outputs he would like to track from the simulation, the number of iterations, etc. The system then performs the simulations and records the requested data in another Access™ database. Standard format exhibits and charts can then be requested from the system, while custom exhibits and charts can easily be built by the user outside the system.

The Client/Scenario Manager

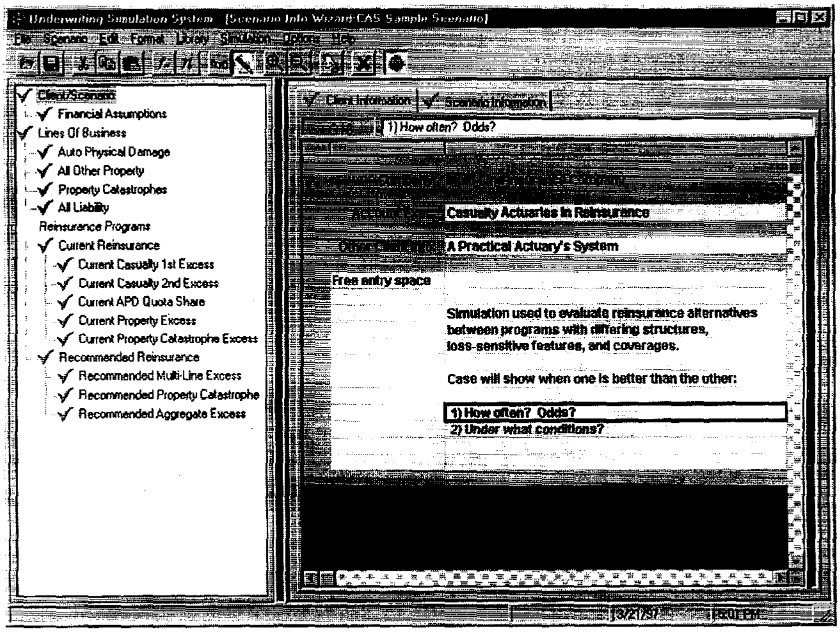
After an initial welcome screen, the system provides the following initial management window. Clients and their associated scenarios are managed from this window. All sample screens that follow are taken from the "CAS Sample Scenario" developed for the sample "Multi-Line Insurance Company" for the purpose of providing a working example for this paper.



A standard GUI interface for management of client and scenario properties allows users to build and control multiple scenarios for many clients.

Scenario Manager

This is the place most of the work is done. The major operations available from the scenario window are to Navigate and Change the Scenario Structure, View and Edit a Wizard Sheet, Recalculate the Scenario, Access the Template Library, Perform a Simulation, Print Reports, and Chart Results.



The major parts of the Scenario window follow:

- **Menu bar**
- **Toolbar Buttons**
- **Navigation Tree**

  The navigation tree is used to view and manipulate the entities of a scenario. With this, a user can add lines of business and reinsurance programs with their respective treaties.

- Navigation Tree Elastic

  The navigation tree area stretches to show more of the entity names or shrinks to show more of the data entry area (see below)

- **Wizard Tab**

  Clicking this Excel ™ - like tab displays the corresponding wizard sheet.

- Data Entry Area

  Gray background regions of this area cannot be modified. Only the white regions can be edited.

- **Formula Bar**

- **Scratchpad Elastic**

  The scratchpad elastic allows the user to resize the viewable area of the scratchpad, and in conjunction with the navigation elastic allows the user to customize his view of the workspace

- Scratchpad Area

  This is an Excel ™ - like spreadsheet on the right-hand side of a wizard sheet. This area is used to perform calculations which can be referenced from the Data Entry Area. A sample screen showing the scratchpad follows.

Navigate and Change the Scenario Structure

The structure of the current scenario can be viewed and altered from the navigation tree on the left-hand side of the Scenario window. This structure can be modified by manipulating its parts, which are also known as entities; the Client/Scenario entity, the Financial Assumptions entity, the Lines of Business entity, which in turn contains Line of Business entities, the Reinsurance Programs entity, which in turn contains Program entities, which in turn contain Treaty entities.

When an entity is added to the current scenario, a default set of worksheets is provided. The principal functions here are to Add a New Line of Business (LOB), Add a New Program, Add a New Treaty, and Show an Entity Wizard.

Risk Function Wizard

To allow the user to model a number of familiar frequency and severity models, we have built into this system the functions in the Excel ™ add-in called @Risk ™ which provides easy access to many of the functions used by property/casualty actuaries.

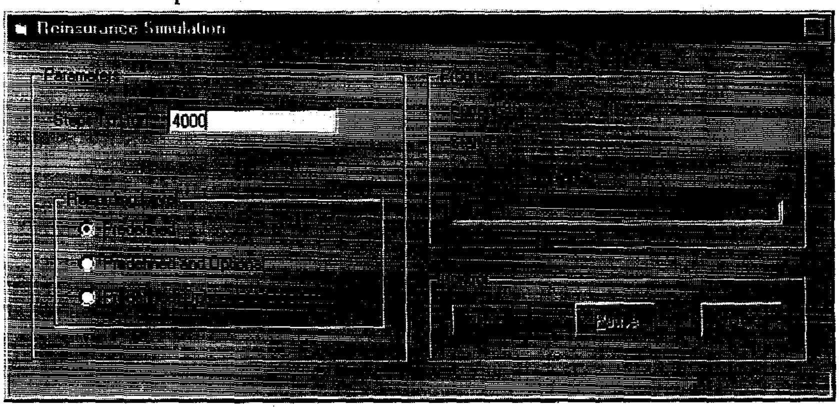
Since the spreadsheet-like features available in the front-end are not Excel ™ but rather an Excel-like VB component, the @Risk ™ add-in does not currently operate in this front-end. The default value displayed is the mean of the function. For more elaborate functions, the user can detach the workbook and work outside the system. (see below)

The Template Library

The application contains a storage area called the **Template Library**. The Template Library is used to store and retrieve wizard workbooks and wizard sheets. This facility allows the user to reuse wizard workbooks and sheets.

Perform a Simulation

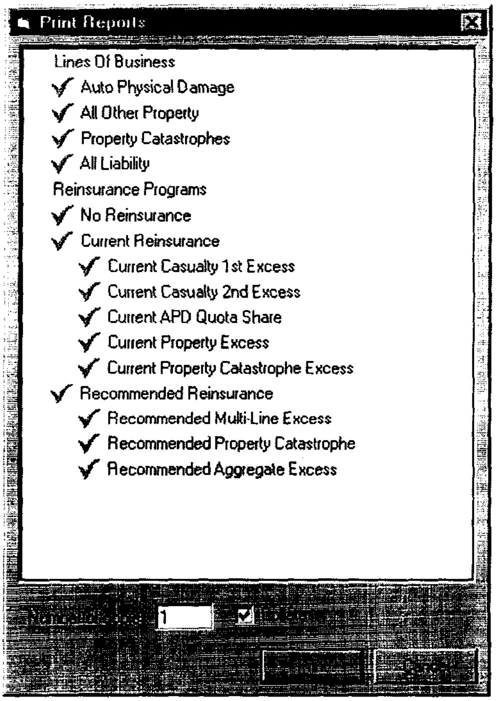
The user can Build and Display the Simulation Workbook, and Run Simulations from the Scenario Window. If the user is satisfied with the workbook built from his inputs, a simulation can be performed.



User-defined fields are fields that the user wants to capture but are not available in any of the built-in selections, yet are items of special interest in carrying out the current work. They are available only in "detached" workbooks. Workbooks are detached in cases where the system's standard features are not sufficient to completely describe what the user would like to test (such as very unique treaty terms).

Print Reports

Once simulation has been performed, the **Print Reports** dialog can be invoked. The user can print summary reports for each line of business, program, and treaty in the current scenario.



For each line of business and reinsurance treaty, there are two summary reports: a Statutory Underwriting Income report with the average underwriting results for the simulation, and a page of Highlights that shows averages, maximum and minimum values for several key variables of interest.

In addition, for the Overall Gross reporting level and the Net results for each reinsurance program, there are three reports: First, there is the Statutory Income Statement report, including investment and other income. Also, an extensive Highlights page gives averages, maximum and minimum values for key items at the Ceded and Net levels, and a Statutory Average Balance Sheet report that shows average levels of several asset and liability classes for the run.

Chart Results

Several standard charts are available. The system itself generates some default charts that can be modified, printed or deleted by the analyst. The user can also develop new custom charts of each type within the system to compare results of reinsurance programs, treaties or lines of business.

"Green is Good, Red is Bad" Charts:

"Red/Green" charts allow the user to compare the results of two reinsurance options over the whole range of outcomes for a simulation, and especially to determine the type of loss scenario in which one alternative does a better job of meeting the insurer's objectives than another. Each chart focuses on one particular variable, such as ceded or net underwriting profit, after-tax income, year-end surplus or any quantity that has importance in the evaluation process. The idea behind the Red/Green chart is simply to find the difference between the values of that variable for the two programs, determine whether a positive difference is better or worse for the cedant, and to plot green points when the first alternative is better and red ones when it is worse. The more green points that appear, the more often the first program comes out better.

Histograms:

Histograms show the non-cumulative distribution of some variable of interest for the given reinsurance programs, or for selected lines of business or treaties, either as frequencies or counts. In some cases, these will be the roughly bell-shaped curves that are often encountered in insurance statistics. The definition and interpretation of these charts is fairly easy to grasp. Cumulative versions are also available.

Other Variability Charts:

Charts showing the year-to-year change in key percentile values are also available. These charts help the user see the variability of any selected output item in another fashion, which some users find very helpful.

Other Charts:

Other charts are produced by the system or are in development, which help the user express himself, analyze the output more easily, explain results and communicate to others more conveniently.

HIGHLIGHTS OF THE INPUTS

This section is organized by input wizard tab much the way the system inputs would be entered. Highlights only are provided in this paper.

CAS Sample Scenario

To illustrate the use of the system, we have included a scenario called the CAS Sample Scenario for a client called the Multi-Line Insurance Company. See more details about this sample scenario in a separate section below.

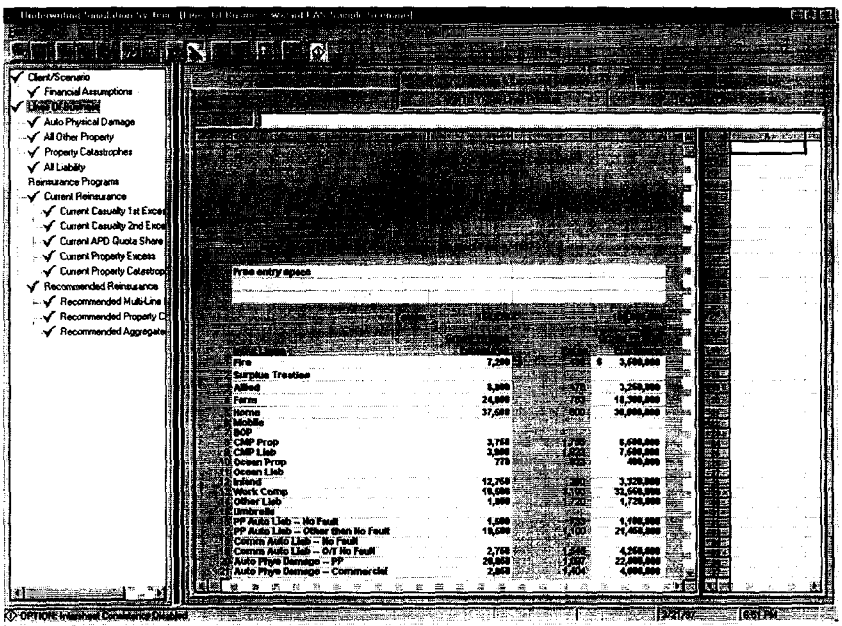
The system provides a Test Layer capability to allow the user to test a particular excess of loss reinsurance layer to find out how many claims and loss dollars to that layer are expected from each of the subject lines of business. This helps in checking the aggregate effect on reinsurance layers of size of loss distribution selections for the various lines of business as well as other aspects of the excess layer, all while still in the front-end system, before any simulations have been attempted.

Financial Assumptions are entered in two tabs: Balance Sheet Assumptions, and Cash Flow Assumptions. The system is focused on the underwriting side of the business. It has extensive inputs for product lines and treaties - the driving assumptions for gross, ceded and net results. However, the system builds complete income statements, including investment income, and balance sheets for each reinsurance program. The investment and cash flow assumptions are intentionally simplified, so they will not be a major source of questions/issues for clients, and thereby cloud the insurance/reinsurance underwriting result discussions. As a result of client requests for additional functionality, enhancements to the system in the asset and tax areas will be made.

Lines of Business

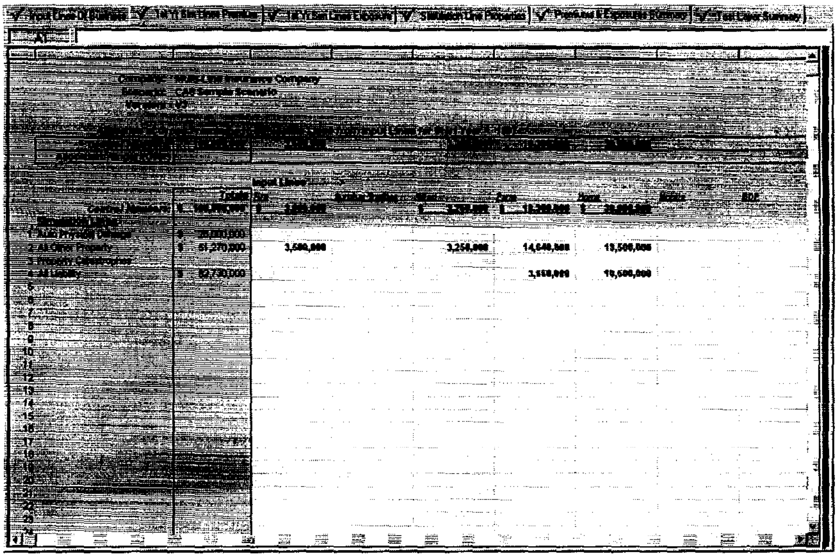
There are six tabs in this section: Input Lines of Business, 1st Year Sim Lines Premium, 1st Year Sim Lines Exposure, Simulation Line Properties, Premium and Exposures Summary, and Test Layer Summary. We will only highlight key tabs for this paper.

From our experience, the data we obtain is usually in more detail than we need for simulation purposes. The user can enter the detail available, and then map the detail to Simulation Lines. If the user has already combined data outside this system into the desired Simulation Lines, then the mapping will simply be one-to-one. Of course, there are also cases when we want to split an input line into more than one simulation line (e.g., Homeowners to property versus casualty). This facility is helpful in such cases.



Once the user has completed the Input Lines sheet, he begins to enter Simulation Lines. New lines are entered by right clicking the mouse when on the Lines of Business navigation tree item and responding to the dialogs presented.

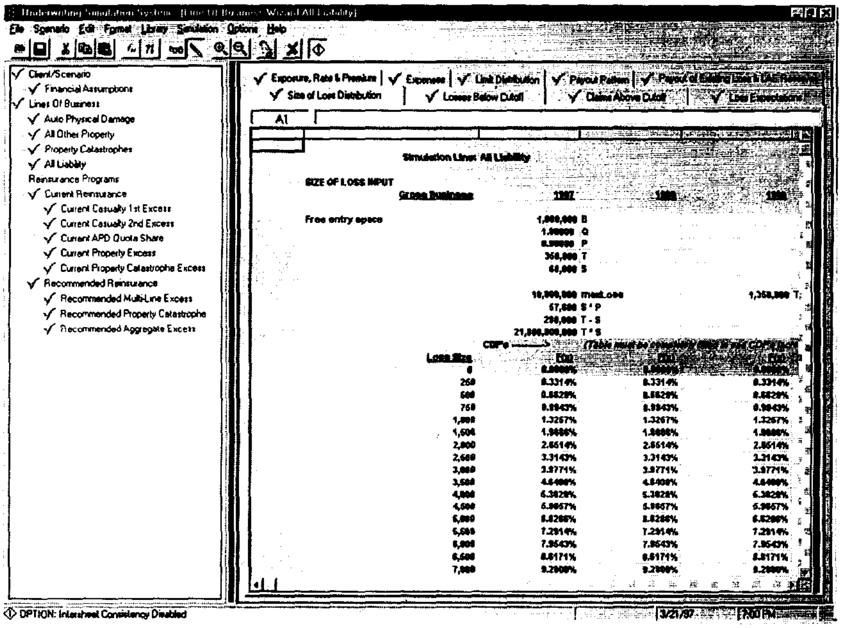
Once all Simulation Lines are entered, the user visits the 1st Year Sim Lines Premium Tab and the 1st Year Sim Lines Exposure Tab. These two tabs are identical in structure and have the role of allowing the user to allocate the Input Lines' premium and exposure to the selected Simulation Lines, keeping track of the total allocations, and posting the results to the appropriate Simulation Lines.



Individual Simulation LOBs

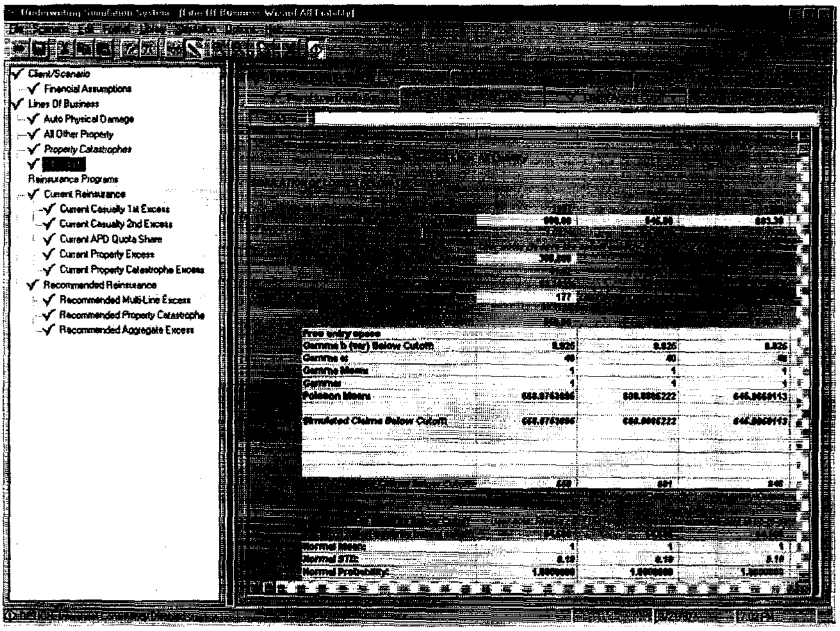
There are nine tabs in this section: Exposure, Rate and Premium, Expenses, Limit Distribution, Payout Pattern, Payout of Existing Loss and LAE Reserves, Size of Loss Distribution, Losses Below Cutoff, Claims Above Cutoff, and Loss Expectations. We will show here only the last four relating to key loss assumptions.

Size of Loss Distribution Tab



Here the user enters the loss sizes and CDF by year for the given LOB. While this example seems to mimic the ISO 5 parameter Pareto model, it is not correct to consider it a continuous model. The size of loss distribution is entered as a discrete distribution, and all of the possible loss sizes in the simulation run will come from the list of values in the first column of the table. There is no interpolation in sampling and no attempt to determine an interval mean in computing the key expectations used elsewhere in the model. The averages and variances of the simulation results depend on the loss joints that are selected in the size of loss distribution, so these must be chosen carefully.

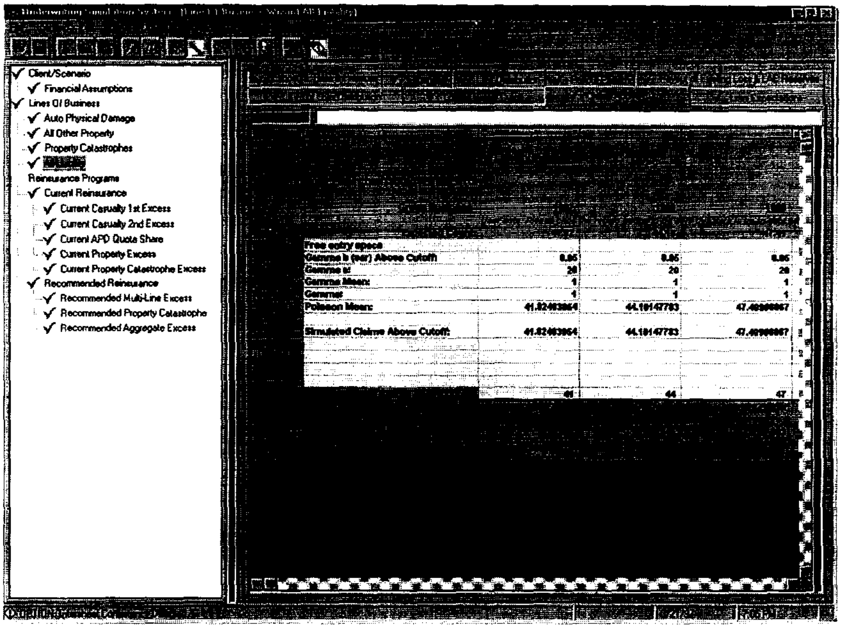
Losses Below Cutoff Tab



The cutoff value is the point in the size of loss distribution at which the model will begin to simulate individual claims for the given LOB. The lower the cutoff, the larger the number of claims the model will individually simulate.

On this tab, we enter the cutoff point, the expected number of claims for each year, and the risk model assumptions we wish to use to simulate the number and average severity of claims below the cutoff. The sample uses a mixed Gamma/Poisson to simulate the number of claims below the cutoff. This procedure is discussed in the Heckman-Meyers paper, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" (PCAS LXX, 1983, page 22ff). In addition, a censored and truncated Normal random variable with a mean of 1.0 is used to modify the severity below the cutoff that is used to compute the aggregate loss level. The user can easily specify other claims processes using other Excel ™ functions, @Risk ™ functions, or even a constant set of claims with fixed amounts.

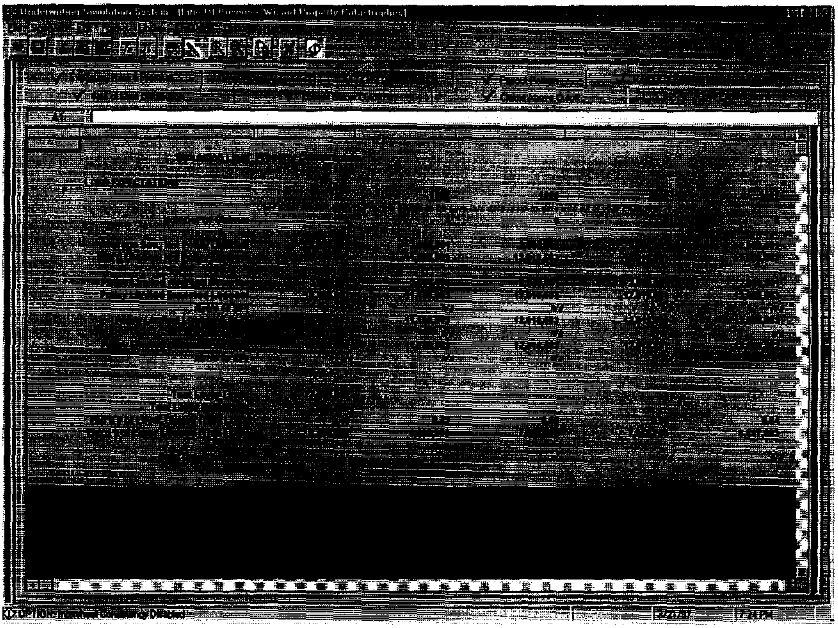
Claims Above Cutoff Tab



This tab is analogous to the previous tab, except that it regulates the simulation of claims above the cutoff point. When the simulation workbook is compiled and the simulation is run , the actual size of a simulated claim is chosen from the table in the Size of Loss Distribution tab. The sample uses a mixed Gamma/Poisson distribution to simulate the number of claims above the cutoff, but again, the user can specify other processes.

The parameters and claims process chosen here form the heart of the individual large claim simulation done by the model.

Loss Expectations Tab



The Loss Expectations Tab for each LOB brings together the inputs from the previous tabs and allows the user to make a preliminary review of the results that can be expected from the user's premium, loss frequency and severity assumptions. If care has been taken to preserve the integrity of the means from the severity distribution in the Size of Loss Tab, the results shown here should be a reasonable benchmark for checking the outcomes of the simulation runs. The averages that appear on the final reports from a simulation should come fairly close to the values shown on this tab.

Reinsurance Programs

In this system, a program consists of up to 15 treaties. Up to three programs can be compared simultaneously. For example, the user might be comparing the Current Program to a Recommended Program and to a Competitor's Program. The final output would have results of the simulation for each of these three programs plus the default "no reinsurance" or gross = net program.

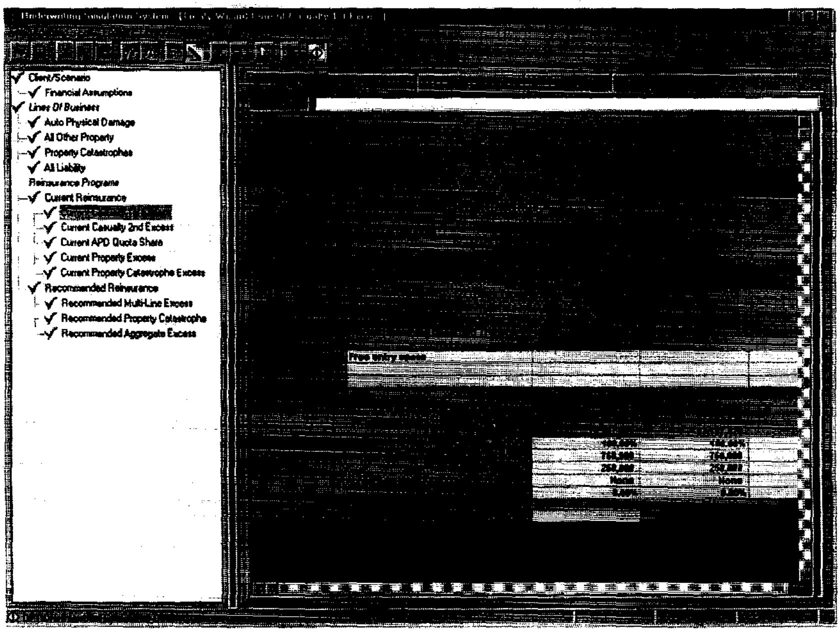
The system does not accommodate facultative reinsurance. However, to reflect broad assumptions about "fac", a user could enter facultative covers in bulk as a treaty, with pertinent, broad assumptions.

A treaty is created with a special dialog. In this dialog the user selects the generic treaty type that best matches the treaty. This affects the way that losses and premium are accounted for in the treaty. If the subject premium base is earned premium, the user simply selects that option and moves on. When the subject base is written premium, there is an additional check box the user may select to specify whether there will be a transfer of the subject beginning unearned premium reserve at the start of the first simulation year. The method by which the treaty subject premium is calculated (which treaties inure to this treaty's benefit, etc.) can be specified by checking the inuring treaties in an extension of the standard dialog. Another dialog is used to specify the LOBs that are subject to each of the treaties.

Treaties

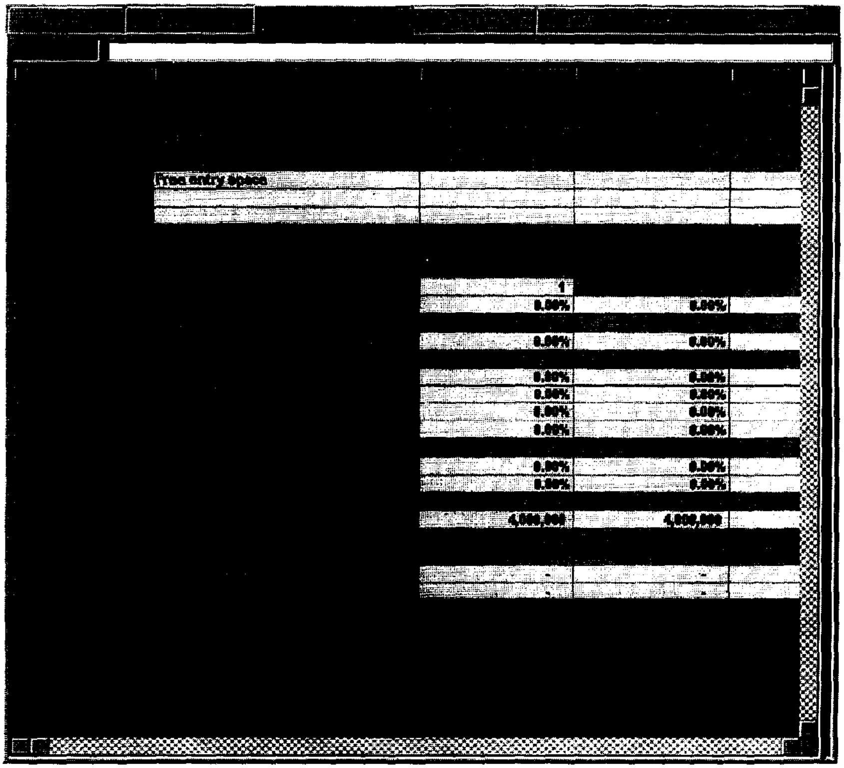
At the treaty level there are five tabs: Treaty Terms, Ceding Commissions, Excess Treaties, Payout Pattern, and Payout of Existing Loss & LAE Reserves. Sample tabs are highlighted below.

Treaty Terms Tab



The Treaty Terms tab gives the user the context in which all of the rest of the calculations for the treaty operate. The heading area tells the user the program and treaty names, the type of coverage provided by the treaty, the subject base, and the subject calculation, which tells the user if there are inuring treaties.

Ceding Commissions Tab



This is the location for entering data and/or formulas that spell out the terms of any applicable ceding commission or profit commission, whether it is implemented as a flat commission, a sliding scale or custom-designed plan. We will not go into this in any detail here.

CAS SAMPLE SCENARIO

Input areas covered in the previous sections give the reader a broad overview of the types of inputs and manner in which the many inputs needed to perform a complete simulation are entered. In this section we go into a bit more detail concerning a particular sample.

Note: Rather than populate an appendix with dozens of reported results from our system, we will send a copy of a section showing the individual line of business assumptions, output exhibits, treaty results, etc., to any reader who calls or writes us for a copy. These give the reader all the basic assumptions needed by line, treaty, etc., as well as key output.

Lines of Business in Sample Scenario

In the CAS Sample Scenario, we show a multi-line insurance carrier that is involved in over a dozen lines of business. We have segregated the overall book of business into four simulation lines, to simplify our sample. In a real-world case, we would separate lines based largely on their loss characteristics and reinsurance treatment (lines must map into treaties properly, and significant size of loss differences would be recognized).

The simulation lines in our sample are Automobile Physical Damage, All Other Property, Property Catastrophes and All Liability. All of the individual input lines were included on the Input Lines of Business tab, but the premium and exposures were allocated to the simulation lines (in the 1st Year Sim Lines Premium and 1st Year Sim Lines Exposure tabs). See the section above titled "Lines of Business."

The Property Catastrophes line serves a unique purpose in this scenario. It's there to generate losses for the Property Catastrophe treaties in the reinsurance programs. No premium is assigned to this line of business. The Property Catastrophe treaties will pick up losses from this line of business, while their subject premium comes from the other lines that are specified as subject to the treaties in the Treaty Subject Lines tab for each Program. There can be several such "lines" if needed. In the sample scenario we have assumed that recoveries from the current risk excess would not materially affect the SOL distribution for property cats. If we knew that the distributions differed, we could reflect these differences in the system. Size of loss distributions obtained through a portfolio analysis using one of the several commercially available catastrophe models can be accommodated by the system using one or more Property Catastrophe lines of business.

Reinsurance Programs Used for Sample Scenario

The Multi-Line Insurance Company has a Current Reinsurance Program that consists of five treaties: 1st and 2nd Casualty Excess treaties, a Property Risk Excess treaty, an Automobile Physical Damage Quota Share, and a Property Catastrophe Treaty.

We will assume that there is a proposal for an alternative structure under consideration which we call the Recommended Reinsurance Program, and that other programs may have been considered and rejected, leaving these two alternatives. The Recommended program consists of only three treaties: a Multi-line Excess treaty, a Property Catastrophe treaty and an Aggregate Excess treaty.

The key terms used for the Current Program were:

1st Casualty Excess: 100% of $750,000 xs $250,000; ALE included, Swing rated 4%/15% loss load 100/85ths

2nd Casualty Excess: 100% of $4 mill. xs $1 mill.; ALE included, Flat rated 6.23% with 35% ceding commission

APD Quota Share: 40% with 30% ceding commission

Property Excess: 100% of $4,800,000. xs $200,000; ALE included, Flat rated 3.41%

Property Catastrophe: 95% of $47 mill. xs $3 mill., Flat rated 15.7%. No reinstatements. The property excess inures to the benefit of the cat treaty, but we assume that the benefit to the catastrophe excess of the inuring treaty is negligible.

The key terms used for the Recommended Program were:

Multi-Line Excess: 100% of $4,650,000 xs $350,000; ALE included, Flat rated 12.3% with 35% ceding commission. Also features a profit commission of 50% after 20% reinsurer expenses, adjusted at the end of 3 years with a deficit carryforward.

Property Catastrophe: 95% of $77 mill. xs $3 mill., Flat rate 18.1% One free reinstatement.

Aggregate Excess of Loss Ratio: 95% of 20% xs 77%, Flat rate 1.20% with a 35% ceding commission and 25% profit commission after 20% reinsurer expenses, adjusted annually.

As you can see, there are significant structural differences between the two programs. The first provides coverage closer to the ground while the second provides larger and broader catastrophe and "worst case" coverage.

Setting up the Scenario

There are a variety of reasons that buyers seek a change in their reinsurance: saving money (which means different things to different people), adapting coverage to their current mix of business, changes in management, problems with current reinsurer(s) or broker, or changes in strategic direction, goals, or appetite for risk. In today's reinsurance marketplace, many buyers want to redesign their reinsurance to reduce cessions, simplify administration, broaden protection, and protect earnings.

In our example, Multi-Line Insurance chose to assume more risk down low to pay for more property catastrophe limit and an aggregate stop loss. They increased their working cover retentions and canceled the quota share. Under the current program, the company cedes about $33 million, while the new program calls for a cession of $26.5 million.

After all of these changes, is the new structure a better way to manage risk than the current program? Is either better than no reinsurance at all? When is it better? How often? These questions led to the use of simulations to provide better information to evaluate the alternatives.

Which Program is "Better?"

Buyers come from many different points of view when deciding between competing programs. Some focus on the amount of ceded premium, some base decisions on historical "what-if's" (running competing proposals through several actual prior years of losses), some rely on estimates of average ceded underwriting profits, others on estimates of total ceded profit (including investment income), and some focus on worst case loss scenarios. Simulation models can provide this information and enhance it by providing insights into the effects of variability, giving estimated odds for profit and loss levels.

The key issues buyers usually focus on initially are ceded premium, expected ceded profits, and the variability and magnitude of worst cases in their net results. We will highlight these items for the comparison at hand.

One straightforward approach is to look at the average, maximum and minimum levels of ceded underwriting profit, total net income and surplus for each of the alternatives: No Reinsurance, the Current Program and the Recommended Program. All of these values are displayed in the Simulation Highlights report produced by the system for each of the programs. Copies of these exhibits follow.

The first year averages are fairly typical of the differences between the programs under review. Average net income and ending surplus were highest for the No Reinsurance option, of course, followed by the Recommended and Current Programs, respectively.

Under the Recommended program, the ceded profit is $850,000 better and total net income $500,000 better than in the Current program.

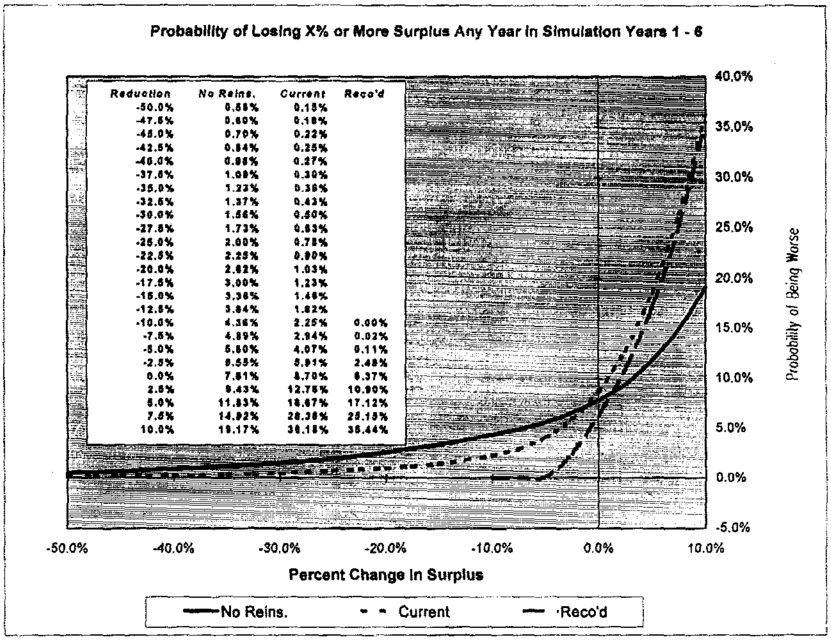
Next, we note the minimum and maximum values for each of the alternatives. For surplus and net income, the maximum values are seen in the No Reinsurance alternative, as we would expect The minimum values from the simulation for net income are negative for all options, but the Recommended program has much smaller worst case losses due to the stop-loss. In both the No Reinsurance and the Current program, the worst case net losses exhaust the company's surplus, leaving a negative balance in all but Year 6 (2002).

Note: We "allow" negative values in surplus and invested assets (which behave like loans) in standard set-ups, but the system can easily handle defined constraints on behavior. In more elaborate runs we would define the decision rules appropriate to the case.

Before either of these alternatives is selected, the buyer needs to determine the frequency of losses of that magnitude to be sure that the probability of such huge losses is sufficiently remote to assume the risk.

What is the return time for a loss that would cause a given level of reduction in surplus? Equivalently, are the odds of such a loss equal to 1 in 10 , 1 in 20, or more like 1 in 1000? This will help determine the level of risk involved in keeping these losses net. The chart on the next page illustrates the probabilities of losing XX% of surplus in any year in the Simulation Years 1-6 for each of the reinsurance alternatives.

As an example of the useful nature of this chart/table combination, consider the probability of having a net loss in surplus: 7.8%, 8.7% and 6.4% (1 in 13, 12 and 16) for the No Reinsurance, Current and Recommended cases, respectively. At a 5% reduction, the odds are 1 in 18, 25 and 909. There are no surplus reductions worse than 10% for the Recommended Program. Coming from another direction, the 1-in-100 loss (1.0% probability) would result in a loss of surplus of 40%, 21%, and 3.5%, respectively. Note that these probabilities are not just single year probabilities. If the reduction in surplus occurs in any one of the years, it is counted in the totals in computing the probabilities. This is a practical result, since regulatory concerns would be triggered immediately.

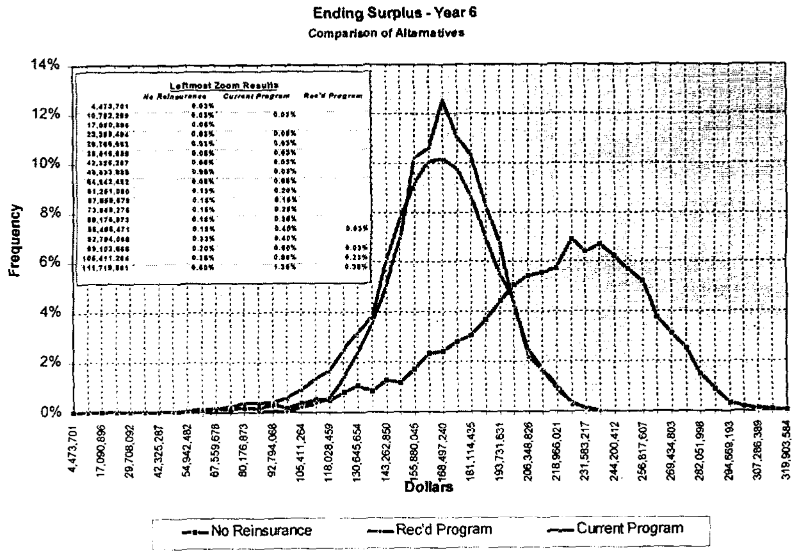


Reduction	No Reins.	Current	Reco'd
-50.0%	0.58%	0.15%	
-47.5%	0.60%	0.18%	
-45.0%	0.70%	0.22%	
-42.5%	0.84%	0.25%	
-40.0%	0.96%	0.27%	
-37.5%	1.08%	0.30%	
-35.0%	1.22%	0.36%	
-32.5%	1.37%	0.43%	
-30.0%	1.56%	0.50%	
-27.5%	1.73%	0.63%	
-25.0%	2.00%	0.75%	
-22.5%	2.25%	0.90%	
-20.0%	2.62%	1.03%	
-17.5%	3.00%	1.23%	
-15.0%	3.36%	1.46%	
-12.5%	3.84%	1.82%	
-10.0%	4.36%	2.25%	0.00%
-7.5%	4.89%	2.94%	0.02%
-5.0%	5.60%	4.07%	0.11%
-2.5%	6.55%	5.91%	2.48%
0.0%	7.81%	8.70%	6.37%
2.5%	9.43%	12.75%	10.90%
5.0%	11.83%	18.67%	17.12%
7.5%	14.92%	28.38%	25.15%
10.0%	19.17%	38.15%	36.44%

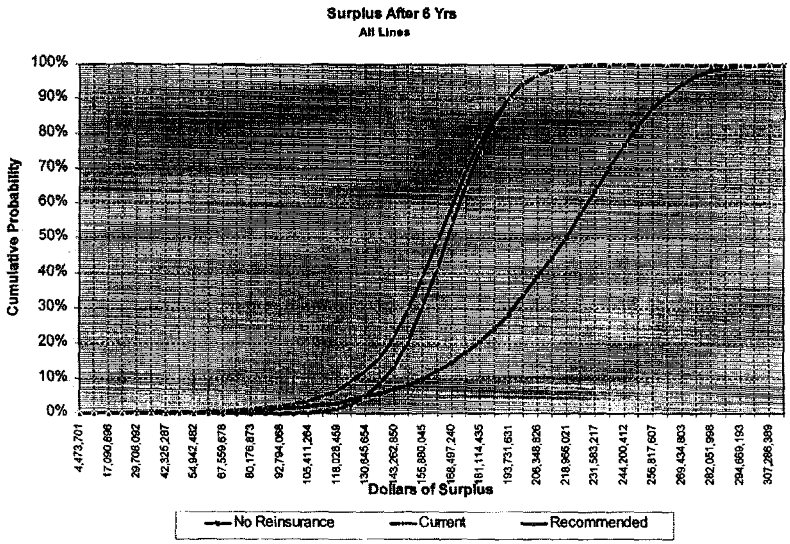
From this chart it is clear that the Recommended Program is the best at arresting runaway losses from catastrophes and other sources of high aggregate loss levels. We did not model other high severity issues such as Clash, ECO/XPL or runaway ALE in this analysis. That would have made matters somewhat worse in the Recommended program, but far worse for the Current program, which has little or no provision for these exposures. Most of the losses would go right to the net. Even without these, there are significant probabilities of disastrous depletion of surplus under the Current program. If the full spectrum of results from the simulation had been displayed on this chart, the No Reinsurance line would stretch to -140%, and the Current line would tail out to about the -120% level. Investors and regulators would be dismayed by far smaller losses of capital than these. This is why we stop the chart scale at a -50% reduction, even though greater degrees of impairment may be possible.

While we are considering the odds of observing certain values of interest to the reinsurance buyer, we can look at their distributions using charts available in the system. First, we will consider the non-cumulative distribution of surplus in Simulation Year 6. The non-cumulative "histogram" view gives another dramatic illustration of the reduction in variability achieved. The nearly bell-shaped curves just stop dead at a certain point for the Recommended program, while the other options have rather pronounced tails that stretch well into undesirable values.

At the low end of the scale, the text box provides a "zoom in" look at the frequencies.



The fact that the two reinsurance program net results reach their modal and mean values at lower surplus levels accentuates the reinsurance buyer's dilemma: Buying reinsurance over time causes a reduction in net worth due to the net costs involved (accumulated ceded profits) in exchange for this "tightening" of the tail(s) of the curve. The same idea is expressed by the differing slopes of the curves in the cumulative distribution graph.

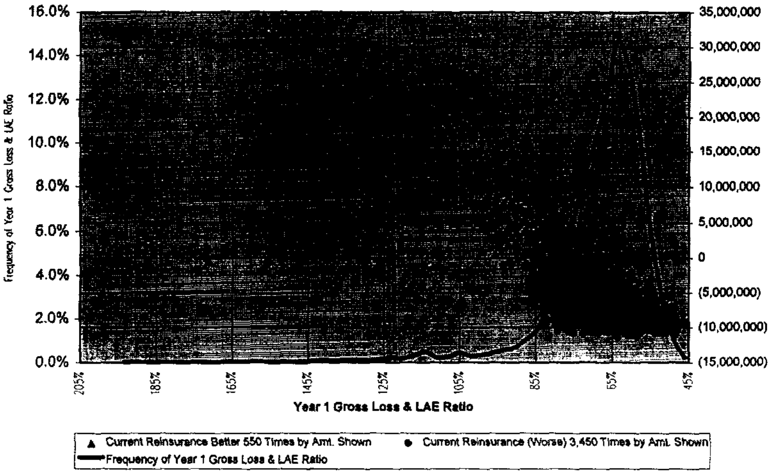


Having looked at the best and worst cases, and evaluated some empirical probabilities, we now will want to compare the programs head to head, as if each trial were a horse race. This is the concept behind Red/Green charts, which compare pairs of programs based on the sheer number of trials for which one program has a better result than the other. "Better" is defined by the user - in this case a program is better when the difference between net after-tax income is positive. All green markers are above the x-axis, all red markers are below.

Evaluating a Red/Green chart is easy when it is almost entirely filled with Green markers, especially when they are in the places where they should be Green (i.e. – where the need for reinsurance protection is greatest). We refer to the decision between the alternatives in such a case as a "slam dunk." When the decision between two alternatives is a "slam dunk," one of the alternatives is better than the other in almost every case. As long as the Red dots appear in relatively low-impact areas, the decision between the two programs is fairly easy. We have seen such comparisons in practice. It makes the decision between the two alternatives simpler. The decision in this sample is by no means a "slam dunk," but it is helpful to evaluate it using Red/Green charts nonetheless.

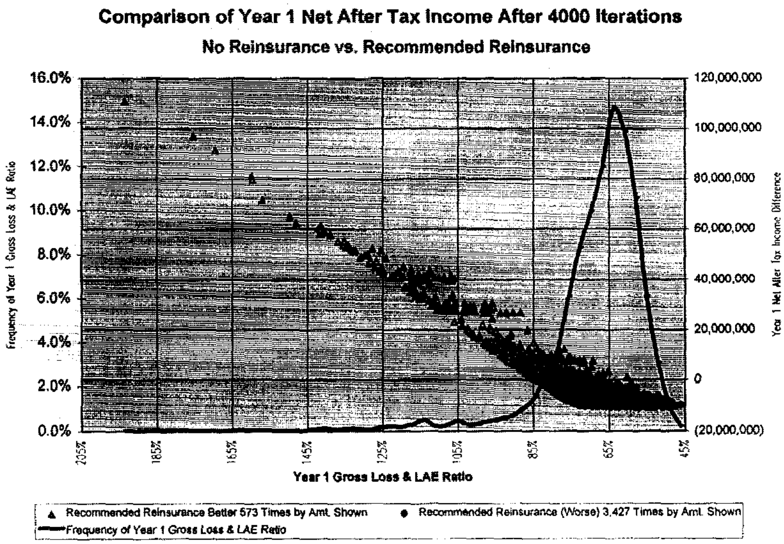
Comparison of Year 1 Net After Tax Income After 4000 Iterations

No Reinsurance vs. Current Reinsurance



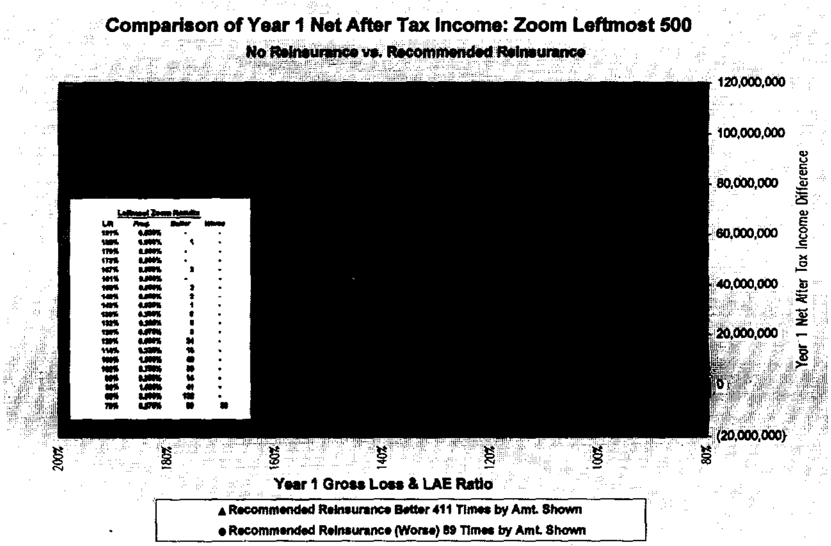
Note that this chart is filled with Red dots, yet the company saw fit to buy this program for years, primarily because the Green markers are "in the right places." That is, when the total gross loss ratio was greater than 66%, and especially as the loss ratio breaks into the 90% range and above. In the worst years of gross loss experience, the Current program can have a net income benefit of \$20-\$25 million. At the lower end, the premium is weighing down the program, so that total income is as much as \$10-\$12 million worse than it would have been had there been no reinsurance at all.

Now let's see how the Recommended program compares with No Reinsurance.



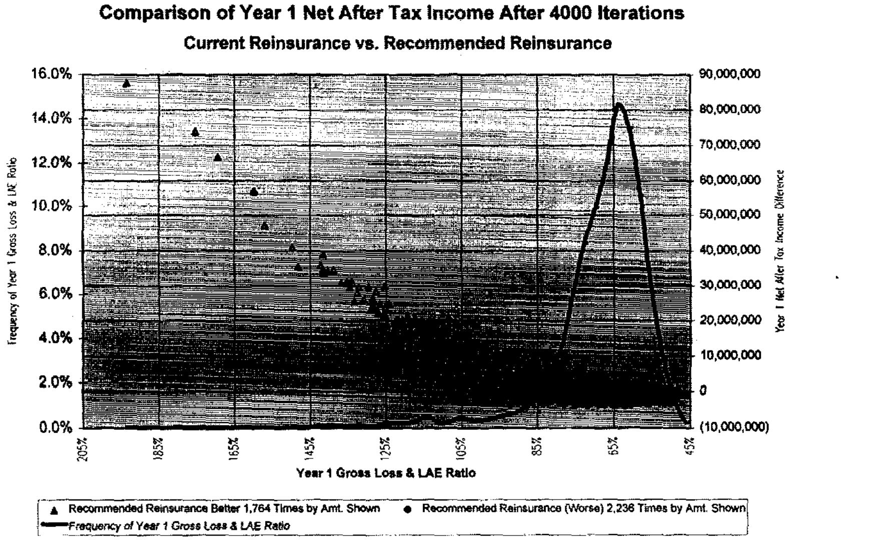
The Recommended program fares better, but it still comes out worse in 3,427 trials out of 4,000. Note, however, that the size of the potential benefit to total income in the worst years has a far wider range than in the Current program, up to $110 million in the most extreme case. Any reinsurance program may look unattractive in the good years, in which there is a large amount of ceded profit. But, this program is a good example of what happens with high risk cessions - when reinsurance is needed, it delivers.

Let's now zoom in on the 500 trials with the worst gross loss ratios to get a better look at what is happening in that range.



Here, we find the lion's share (411 of the 573 trials) of the Green markers from the full-spectrum Red/Green chart above. Most of the cases in which the Recommended program "wins" are in years when the gross loss ratios are above 80%, as you would expect given the stop-loss. One might be tempted to suggest that the company buy only the stop-loss, but the realities of the market are such that stop-loss reinsurers require acceptable working cover and property cat protections underneath them.

Next, we compare the Current and Recommended programs (Green means Recommended is better).

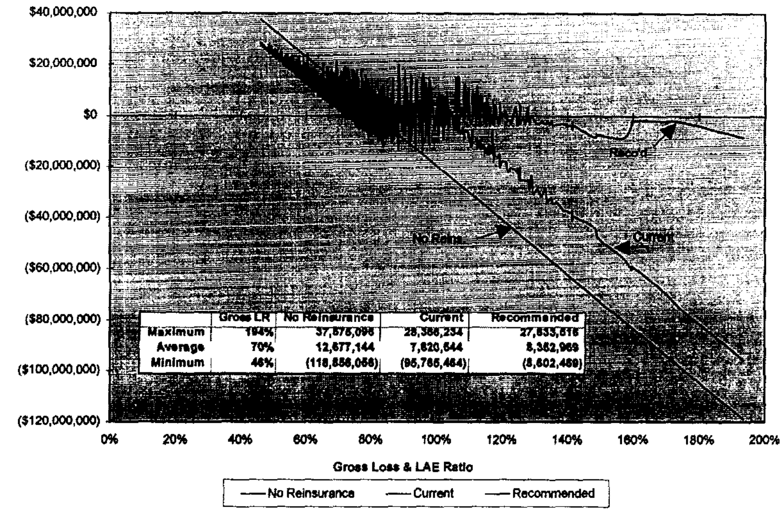


It appears that the Current program wins a majority of the times in head-to-head comparisons with the Recommended structure, but when the gross loss ratio breaks the 95% level, the Recommended program begins to shine. In the most extreme cases, there are clumps of cases where the benefit is $20-$40 million, from the extra property cat coverage mostly, and a handful of very large differences of between $40-$90 million when the extra cat cover plus reinstatement cover plus the aggregate all come to bear.

While this situation does not result in a simple "slam dunk" superiority between any of the alternatives, we believe the buyer will continue to buy reinsurance as a result of this analysis. The direct underwriting assumptions here are fairly optimistic, but we still have enough loss potential from property catastrophe shock losses and/or high loss frequency in the various lines of business to create very unfavorable experience, as we have seen.

Another means of looking at the variability of total income (say) from the reinsurance alternatives at various gross loss ratio levels is a chart we call the "wiggle chart." This is not yet another probability chart per se, but a graphical display of the range of values of after tax income for each alternative at every observed value of the gross loss ratio. The picture really highlights the benefits of the Recommended program, and speaks for itself.

Variability of After Tax Net Income by Gross Loss Ratio



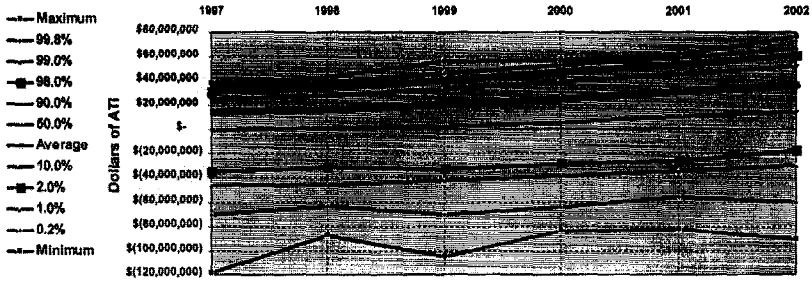
As can be seen from the wiggles for the No Reinsurance and Current options, in the years with the worst gross loss experience, the bottom seems to drop out from under both of these alternatives. While the Recommended program also experiences some bumpiness at the top end, the combination of the extra property cat limit and reinstatement plus the stop loss have limited the damage to the company's balance sheet to a far greater degree. This picture depicts the impacts to the buyer very clearly.

The final chart series of interest to us allows us to look at year by year variability of a single quantity, after tax income in this case, over the whole six-year time span of the simulation. Note that this is a series of three charts - all with the same scale - which depict a number of user-selected percentiles that allow a side by side comparison of the variability.

This display allows a buyer to see how reinsurance alternatives work to control variability over the 6 simulation years by watching the spreading arms of the pairs of "1 in N" percentile lines (the pairs of lines representing the 100(1/Nth) and 100(1-1/Nth)-percentile values for each year), which resemble confidence intervals for the mean at various confidence levels. Again, this is easier to see in the pictures. By now, we can see that the Recommended program is the best choice for controlling the variability of net income, so we would expect the differences between its "1 in N" values at each end of the range of values to be the narrowest. The Current program is anticipated to come in second, and the No Reinsurance case will establish the maximum and minimum points on the scale for the whole series.

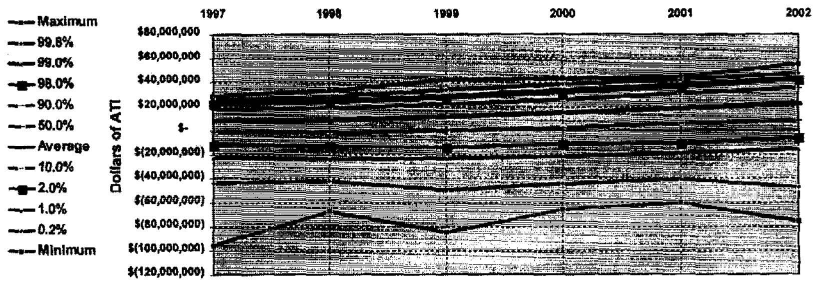
That is exactly what we see in this chart series:

All Lines; After Tax Net Income ; No Reinsurance

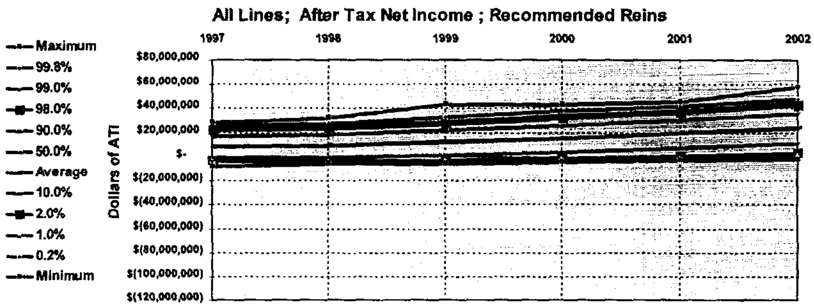


	Percentiles	1997	1998	1999	2000	2001	2002
1 in 4,000	Maximum	$ 37,575,096	$ 43,337,268	$ 55,553,480	$ 56,417,780	$ 61,990,838	$ 76,223,376
1 in 500	99.8%	35,075,164	38,812,744	46,060,556	52,600,164	58,915,108	66,156,844
1 in 100	99.0%	32,384,098	35,207,976	41,304,300	48,068,160	54,145,920	62,186,236
1 in 50	98.0%	30,367,580	33,582,408	39,008,796	45,611,936	52,334,308	60,058,832
1 in 10	90.0%	25,236,834	28,286,200	33,411,644	38,939,232	45,832,700	52,128,852
	50.0%	15,461,841	17,073,690	22,154,604	26,341,286	32,150,468	38,010,616
	Average	12,677,144	14,645,312	18,913,924	23,252,894	29,189,177	34,766,396
1 in 10	10.0%	(796,647)	937,030	2,507,565	5,197,078	10,899,460	14,711,638
1 in 50	2.0%	(35,406,672)	(31,383,978)	(33,376,068)	(26,980,272)	(28,044,230)	(18,363,620)
1 in 100	1.0%	(46,285,472)	(45,859,820)	(42,364,084)	(38,557,940)	(34,996,400)	(30,335,032)
1 in 500	0.2%	(70,553,232)	(63,463,860)	(70,011,432)	(63,792,598)	(55,980,996)	(60,079,312)
1 in 4,000	Minimum	(118,858,058)	(87,632,008)	(105,060,928)	(83,558,720)	(81,914,260)	(90,321,360)
	Skewness	(2.57)	(2.18)	(2.23)	(1.87)	(1.88)	(1.81)
	Std. Dev.	14,044,933	14,226,519	15,718,618	16,428,297	16,883,034	17,673,332
	Std. Dev./Avg	110.79%	97.14%	83.11%	70.65%	57.84%	50.83%

All Lines; After Tax Net Income ; Current Reins



	Percentiles	1997	1998	1999	2000	2001	2002
1 in 4,000	Maximum	$ 28,366,234	$ 33,335,330	$ 42,415,284	$ 42,811,084	$ 44,652,244	$ 55,283,380
1 in 500	99.8%	28,007,758	28,182,636	33,759,584	38,153,800	41,958,944	47,687,676
1 in 100	99.0%	25,380,168	24,784,088	28,748,182	33,904,368	38,218,396	43,713,684
1 in 50	98.0%	21,744,742	23,447,490	27,372,986	31,888,590	36,282,296	41,934,440
1 in 10	90.0%	17,228,368	18,670,728	22,078,884	25,900,860	30,597,364	35,239,136
	50.0%	8,771,879	9,331,227	12,391,943	15,186,545	19,171,222	23,183,634
	Average	7,820,544	8,462,720	11,310,949	14,242,822	18,141,581	22,016,971
1 in 10	10.0%	(1,548,508)	(1,488,883)	281,190	2,297,153	5,763,025	8,464,964
1 in 50	2.0%	(12,420,137)	(12,505,101)	(14,281,181)	(11,495,577)	(10,850,339)	(8,326,981)
1 in 100	1.0%	(22,411,686)	(24,129,824)	(23,405,864)	(21,543,082)	(18,805,954)	(15,856,465)
1 in 500	0.2%	(43,853,192)	(42,805,920)	(50,173,524)	(44,358,016)	(40,515,568)	(47,824,896)
1 in 4,000	Minimum	(95,765,484)	(67,274,400)	(84,238,432)	(65,705,976)	(60,074,476)	(75,737,312)
	Skewness	(2.16)	(1.51)	(1.83)	(1.36)	(1.31)	(1.43)
	Std. Dev.	8,740,983	8,934,769	9,340,803	10,452,513	10,982,648	11,764,887
	Std. Dev./Avg	111.77%	105.56%	87.89%	73.39%	60.54%	53.44%



	Percentiles		1997		1998		1999		2000		2001		2002
1 in 4,000	Maximum	$	27,633,516	$	32,062,692	$	43,107,820	$	43,203,732	$	45,460,764	$	57,364,356
1 in 500	99.8%		25,296,330		27,318,636		32,787,066		37,676,596		41,682,216		47,764,172
1 in 100	99.0%		22,763,914		24,666,310		28,621,636		33,614,776		38,112,188		44,495,926
1 in 50	98.0%		21,510,152		22,867,436		26,734,368		31,694,380		36,540,960		42,546,932
1 in 10	90.0%		16,725,276		18,105,240		21,793,220		25,884,160		30,543,768		35,352,368
	50.0%		8,526,356		9,279,699		12,466,077		15,418,545		19,564,648		24,025,120
	Average		8,352,969		9,010,675		12,138,869		15,263,199		19,380,942		23,647,200
1 in 10	10.0%		(1,033,749)		(1,085,808)		1,443,586		4,018,943		7,770,430		11,037,958
1 in 50	2.0%		(2,703,228)		(3,758,272)		(2,663,831)		(1,150,059)		432,804		2,726,049
1 in 100	1.0%		(2,862,124)		(4,024,101)		(3,205,080)		(1,864,642)		(516,728)		1,246,898
1 in 500	0.2%		(3,610,169)		(4,712,233)		(4,239,173)		(3,136,190)		(2,118,529)		(283,369)
1 in 4,000	Minimum		(8,602,469)		(6,025,347)		(6,092,125)		(4,538,569)		(2,867,988)		(2,330,983)
	Skewness		0.08		0.05		(0.05)		(0.02)		(0.11)		(0.16)
	Std. Dev.		6,402,349		6,977,120		7,540,079		8,254,772		8,773,351		9,397,463
	Std. Dev./Avg		76.65%		77.43%		62.12%		54.08%		45.27%		39.74%

Other Analyses

Since there is a database of results for every trial from each line, each treaty and each program, there are literally dozens of useful analyses one can perform, "drilling down" into issues that arise between the parties involved.

Since the system uses Excel™ as its calculator, any function or relationship that can be defined in Excel™ can be used in a simulation. Of course, the more one puts "in motion" the "blacker" the box gets. Sometimes that is necessary and sometimes it is not. To keep the focus on underwriting risk, we leave certain issues out of most of our analyses, like interest rate and asset risk, but since they can be described fairly easily in Excel™ the system can handle them.

Another frequent analysis involves taking the net present value of future cash flows, which the system handles easily. We usually do this as a custom calculation since there are different definitions of what is and is not included in NPV analysis from company to company.

As an example of how to use the database of results, we answer the questions: "How often do we trigger the aggregate? Total the aggregate?" In just a few minutes a user can extract the ceded losses and limits by year for the stop loss and produce an analysis such as the following:

Aggregate Excess Treaty in Recommended Program

Number of Trials out of 4,000 with a Loss to the Treaty

	1997	1998	1999	2000	2001	2002
Losses	342	256	193	143	98	63
Frequency	8.550%	6.400%	4.825%	3.575%	2.450%	1.575%
Total Losses	5	0	2	0	0	0
Frequency	0.125%	0.000%	0.050%	0.000%	0.000%	0.000%

Conclusion

Perhaps one of the most important conclusions to note is that this is a *system* - where users can build models quicker, under better control - and where there is a "memory," letting the user re-use prior elements that "worked" for their needed purposes. This is a very practical result, too, since simulation has been around for some time in the hard-to-control spreadsheet world. The more comfort and control, the more often and more consistently analyses can be done.

We have seen how users / buyers react to this kind of information, and know that it has been a critical factor in their decisions. We all benefit when actuaries communicate well, and when the best information we have is brought together in meaningful and instructive analyses. We think simulation and the tabular and graphic representation of its results is an excellent vehicle for such communication in the reinsurance arena.

This paper has not been a technical actuarial article, but rather an exposition of our systematic approach to building and using simulations to analyze reinsurance alternatives. Our hope is that it might spur discussion of the strengths and weaknesses of this approach.

Pricing Extra-Contractual Obligations and Excess of Policy Limits Exposures in Clash Reinsurance Treaties

by Paul Braithwaite, FCAS, and
Bryan C. Ware, FCAS

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Abstract: The authors examine the loss exposures due to extra-contractual obligations and excess of policy limit third party actions. They discuss how these components of clash cover reinsurance can be priced.

PRICING EXTRA-CONTRACTUAL OBLIGATIONS AND EXCESS OF POLICY LIMITS EXPOSURES IN CLASH REINSURANCE TREATIES

By Paul Braithwaite and Bryan Ware

"Bad Faith Award Shocks Insurer."[1] As headlines such as this become more common in the trade press, reinsurers need to pay more attention to the prices charged for Extra-Contractual Obligations (ECO) and Excess of Policy Limits (XPL) coverage provided in clash treaties.

Reinsurance casualty clash treaties provide coverage for exposures including multiple policy occurrences, multiple claimant workers' compensation (WC) occurrences, runaway allocated loss adjustment expenses (ALAE), ECO/XPL judgments, and a few other less visible exposures. In years past, multiclaimant WC losses and runaway ALAE have been the most frequent kinds of losses we have seen reported to clash covers. Multi-policy occurrences are more common in some lines of business, such as professional liability lines, or in treaties structured to cover on a per-coverage-part basis (for instance, an auto accident involving auto liability and workers' compensation). But ECO/XPL coverage is becoming much more important for reinsurers to consider due to fears of enormous jury awards, particularly punitive damages against corporate defendants.

Traditionally in clash pricing, a judgmental rate-on-line approach has been the norm. However, there is a significant amount of data available on which to base pricing models for some of the events mentioned. By using such an approach, it is possible, at first, to simply add consistency to judgment. Over time, more data can be gathered, increasing the accuracy of the model projections.

The purpose of this paper is to take an in-depth look at the ECO/XPL component of clash reinsurance coverage, first at the definitions and some background of the coverages, then at the underwriting considerations which come into play, and finally at the framework of a pricing model which can be used to incorporate the specific characteristics of the cedant's book of business into a price for this component of the clash treaty.

DEFINITIONS and BACKGROUND

XPL

The Excess of Policy Limits (XPL) component covers judgments in excess of the original policy limits against the *insured* for claims brought by a third party. Consider an insured who buys a policy with limits of $1 million. The insured loses a lawsuit brought by a third party for a verdict of $10 million. The $9 million above the policy limit may be an XPL judgment. An XPL judgment normally involves alleged questionable claims handling or defense of a lawsuit by the insurer. This often takes the form of the cedant failing to settle a claim within the policy limits when the opportunity was presented. It is due to the handling of the claim that the insurer is considered liable for the excess verdict. The claim for which the damages were awarded must otherwise be covered by the primary policy.

An XPL judgment may include compensatory and punitive damages, but these are not always covered by the primary policy. Compensatory damages can include infliction of emotional distress or loss of business opportunity or business reputation. Punitive damages are usually reserved for situations involving conduct that was exceptionally malicious, egregious, or outrageous.

One example of an XPL claim, affirmed on appeal, can be found in the case of *Fortman v. Hemco*, 211 Cal. App. 3d 241 (App. Div., California 1989). This case involves a three-year-old plaintiff who was injured in 1981 when she fell out of a jeep her mother was driving and was run over by a following vehicle. The plaintiff sued Hemco, who manufactured the mold used to form the fiberglass top and doors on the jeep. The doors on the jeep were designed to be rear-hinged and had exposed interior handles. The plaintiff hooked a sleeve on the door handle, opening the latch. The wind caught the door and threw it open, pulling the plaintiff out of the jeep.

Hemco's insurance company had the opportunity to settle the case for $1 million in 1984. They chose to go to court, primarily because Hemco neither designed the jeep top nor manufactured the actual top on the Fortman jeep. In 1986 a jury awarded Ms. Fortman $17.7 million in economic damages for the personal injury claim and $6 million for pain and suffering. The court concluded that Hemco provided expertise and could have corrected the design. Hemco's insurer was required to pay the entire loss, including the amounts in excess of the policy limit. Under the clash reinsurance program that the insurer purchases today, this would have been a sizeable reported loss to the clash layer.

ECO

The Extra-Contractual Obligations (ECO) component of a casualty clash reinsurance treaty covers judgments against the *reinsured* which are "extra" or outside the policy. The plaintiff in the lawsuit is normally the original insured. The major cost in the judgment is often punitive damages. By nature, these are liabilities not covered under any provision of the policy. ECO claims normally involve wrongful or negligent claims handling by the reinsured. If, in addition, it is found that the cedant dealt with the claim in "bad faith," punitive damages may be awarded. These clauses first began to appear in reinsurance treaties in the late 1970s; XPL clauses have been around since the 1960s.

A fairly recent example of an ECO loss is the case of *Hedrick v. Sentry Insurance Co.*, 96-128100-90 (Dist. Ct., Tarrant Co., Texas). This case began with an auto accident on an interstate in May 1986. A truck rear-ended a northbound car and knocked it and a second car over the median into the southbound lanes. One of the cars struck head-on another vehicle containing Virgie Poston and her two adult daughters. All three were killed.

Ms. Poston's grandchildren filed suit against the drivers of both cars and the truck for damages in the deaths of their mothers and grandmother. The insurance companies for the three drivers settled, and the money was split among the grandchildren, the husband of one of the daughters, the driver of the car that struck Ms. Poston's, and Ms. Poston's husband.

Mr. Poston felt that he had been inadequately compensated for the death of his wife. In 1989, he filed suit against his own insurer, State Farm. According to Mr. Poston's attorney, the insurer failed to investigate, lost the claim file, and denied the $20,000 settlement Mr. Poston sought.

Mr. Poston and his son sued State Farm for bad faith arising out of their handling of the claim. On December 10, 1993 the jury awarded them $2.17 million in compensatory damages and

$100 million in punitive damages. In March of 1994 while the appeal was pending, this case was settled out of court for an undisclosed amount.

There are a few items of note in this example. The first is the obvious one. It would have been easy for the insurer to settle the claim for a small amount of money early on in the case, and thereby avoid the shock verdict. The settled amount was probably substantially less than the verdict. On the other hand, with the negotiating leverage provided by the shock verdict, the settlement was probably very large. From the reinsurer's perspective, this claim is also likely to have been a substantial loss for any clash reinsurance program the insurer chose to purchase.

The second point is the elapsed time involved here. The accident was in May 1986 and the jury award was late 1993. It is entirely possible that a clash reinsurer would see no reported losses to the layer from a claim such as this for seven years. If appealed, the final value of the claim may not be known for several years after that. This can be a fairly long-tailed coverage.

UNDERWRITING CONSIDERATIONS

We have identified five general areas of underwriting considerations which we take into account when determining an expected loss for the ECO/XPL component of a clash program: 1) the states in which the reinsured writes business, 2) other reinsurance coverages the company purchases, 3) the size of the ceding company, 4) the quality of and approach to claims management by the reinsured, and 5) the policy limits which the reinsured sells. We will look at each of these in more detail.

Before we dive into these considerations it is worthwhile to briefly discuss our goal in this exercise. The steps we will go through in any clash pricing exercise are to first determine what types of events (ECO/XPL, WC multiclaimant, etc.) represent significant exposure to loss, then to determine an expected severity and frequency of loss for each type of event. This may take the form of, for example, an expectation of a $5 million loss every 25 years. Thus our task is separated into determining both what size of loss is reasonable to expect in the layer (the severity) and how often we expect to incur that loss (the frequency).

States

The states in which the company writes business can be particularly important for several reasons. Different states have different rules about how to handle damages. For instance, in Texas punitive damages are limited to four times the actual damages, except in cases of gross negligence or malice, in which case this limit does not apply. However, in a few other states, punitive damages cannot be covered by insurance at all. The states in which a cedant writes can have an effect on both the frequency and the severity of losses.

The level of litigation in the states is of utmost importance when determining an expected frequency of claims. Research has shown that California and Texas tend to have the highest incidence of punitive damage claims.[2,3] New York, Florida, and Illinois are also higher than the norm.

This raises some interesting coverage issues. First, are punitive damages insurable? Punitive damages can be assessed either directly or vicariously against the insured. An example of a case where punitive damages might be assessed directly is a case where a doctor inadvertently leaves an instrument inside a patient after surgery. The damages are assessed against the doctor. However, punitive damages assessed against the hospital in the same case are vicarious.

Currently, 46 states allow recovery, at least in part, of punitive damages, depending on whether the damages are vicarious or direct. Michigan, New Hampshire, Nebraska, and Washington do not permit recovery of punitive damages. However, even these states allow recovery of non-economic damages (usually pain and suffering). Coverage of vicariously assessed punitive damages is more common than of directly assessed damages.

The next coverage issue is whether the punitive damages are recoverable under reinsurance in the states in question. In the case of punitives assessed against the insured (XPL), the indemnity payment is made by the reinsurer on a contractual basis where the original policy sold by the insurer provided coverage for punitive damages. This assumes that the original damages were insurable to begin with.

On the other hand, some insurance departments view ECO coverage of punitive damages similarly to direct errors & omissions insurance. This may then revert to the state laws on insurability; however, the situation is unclear. Even if state laws mandate that punitive damages are uninsurable (and thus not reinsurable in this sense), a court may rule that in a given situation it is in the public's best interest to require a reinsurer to cover the punitive damages. For example, New York and California (among other states) prohibit insurance coverage of directly assessed punitive damages (vicariously assessed punitives are insurable). Regulators in these two states have voiced concerns with respect to reinsurance recoverables for these uninsurable damages, due to public policy concerns. However, this has not been thoroughly tested in the courts.

Other Coverage

The main question to answer here is whether the cedant purchases any insurance or reinsurance which inures to the benefit of the clash program, specifically errors & omissions coverage. An E&O policy may cover alleged faulty claims handling, but not always. If the company's reinsurance program is structured such that this is inuring coverage, then the clash reinsurer is further removed from loss. An E&O policy may provide some protection against ECO claims, but will probably be of little value on XPL claims. If the answers to these questions are not contained in the pricing submission material, it is worth asking.

For instance, say a company buys clash protection for a layer of $15 million excess $10 million, and sells only policy limits of $5 million. A judgment of $20 million, including $15 million ECO, will be a $10 million loss to the clash layer. However, if the reinsured buys an E&O policy with limits of $10 million that inures to the clash protection, then there is no loss to the clash layer ($5 million inside the policy limit, $10 million to the E&O, and $5 million company retention).

The second coverage point to note is the coinsurance percentages allowed. The current standard is 90%/90%, meaning the company retains a 10% coinsurance share of each ECO and XPL judgment. The most common variations on these shares are 80%/80%, 80%/100%, and 100%/100%. (It often simplifies matters to have the ceding company retain an equal percentage for ECO and XPL. This usually circumvents having to differentiate between the two, which is sometimes difficult.) Another variation is that the indemnity loss may be required to attach the layer before ECO or XPL will be covered. From a reinsurer's standpoint, it is wise to have the cedant retaining a non-zero percentage. Since the cedant's actions normally determine the incidence of ECO claims and usually of XPL claims, they should have a direct stake in the outcome.

In applying coinsurance, the cedant's share is typically subtracted from the ground-up loss. For instance, use the same $15 million xs $10 million clash layer mentioned above and assume a $12 million ground-up ECO loss with a 90% reinsurance coverage share. (For simplicity, we are assuming that the entire loss is ECO. Normally a portion will be a covered loss within the policy, and thus not subject to the coinsurance percentage.) Using this application, the coinsurance share is 10% of $12 million, or $1.2 million. The remaining loss is $10.8 million, resulting in an $800,000 loss to the clash layer.

Other variations of this calculation are used in other reinsurance covers. For instance, in quota share reinsurance, the ECO or XPL loss may be subject to an additional limit or recovered pro rata in addition to the limit of the contract. Although these methods of calculation are not meaningful in the clash context, they are mentioned as another source of inuring protection. It is also worthwhile to find out if the cedant's excess of loss reinsurance (if any) provides additional coverage for ECO/XPL.

Size of the Ceding Companies

All else equal, a larger insurer will have a higher ECO judgment from a given incident than a smaller company. One reason for this is the actions of juries and judges in determining damages. Punitive damages are by definition intended to be painful. Consider two companies, one with $10 million in premium, the other with $1 billion. A $1 million verdict against the first company will likely have a definite effect on the way they do business. They will feel the loss, whereas the second company would not. Proportionally, it would take a verdict of $100 million against the larger company to have the same effect. This sometimes has the effect of making larger companies targets of these lawsuits.

This relationship is not as clear for XPL, where the verdict is officially against the insured. A proportionately larger punitive damage award is probably more likely against a large insured than a small insured, regardless of the size of the insurer. However, since the claims handling of the insurer is also responsible for the judgment, its size may become a factor as well.

The deep-pocket effect also comes into play. People are more likely to sue a larger company because there is a potential for much bigger rewards. Thus, the frequency of losses will also be higher.

Overall, one would expect less total loss on ECO/XPL covers on 100 insurance companies of $50 million each in premium than on one company of $5 billion.

Claims Management Practices

Probably the single most important piece of qualitative information we seek when determining a price for the ECO/XPL portion of a clash cover is an honest, unbiased evaluation of the quality of the cedant's claims management practices. There are several topics which are important for us to cover when evaluating the cedant's claims staff.

What is the average workload and experience level of the staff? The lower the workload, in general, the less likely a claim is to fall through the cracks.

What are the cedant's views on settling claims? An early settlement in the *Hedrick v. Sentry* case mentioned above could have saved potentially $100 million. A company which chooses to settle potential ECO/XPL claims early on may develop a frequency problem on their own books, but this probably won't translate into a problem for high clash layers since the severity is kept low.

Some companies will choose to go to court on a declaratory judgment action instead of outright refusing coverage and taking the chance of suffering an ECO/XPL loss. In a DJ case, the facts of the case are not disputed by either side. They choose to go to court for a ruling only on the applicable laws or coverage issues. This approach may avoid the danger of a large punitive damage award for disputing coverage or failing to provide a defense.

How stable is the claims staff? Do they have trouble hiring and keeping good people? A stable claims staff tends to better manage its claims inventory.

Another important indicator of at least the company's past claims practices is the level of reported ECO/XPL claims over the past ten to twenty years. Large claims will normally be included in a pricing submission, although they may not be identified as ECO/XPL. Claims descriptions will often identify them.

Policy Limits

The policy limits sold by the cedant are also important. It is important to look at the size of the policies as well as the relationship between the policy limits and the attachment point of the clash layer. Limits profiles are normally available from the cedant. Clearly, the lower the policy limit, the more likely a judgment is to be in excess of the limit. An insured buying too little coverage can be a problem. On the other hand, it is desirable to have a buffer zone between the most frequent policy limits sold and the attachment of the clash layer.

A PRICING MODEL

The pricing model described in detail below is an exposure rating model. It is based on the characteristics of the specific cedant being priced. It does not, however, depend on the actual loss history of the cedant. Before using the model, one should examine the historic ECO/XPL losses of the cedant. Ideally, there will be none of any consequence. A lack of historic ECO/XPL claims can be significant, depending on the expected number of losses and the number of years of data available. In practice, there often are one or more. If there are, a quick experience rating is normally done. A look at the experience can at least serve as a reasonability test of the exposure rate.

Experience Rating

To perform an experience rating, any historic ECO/XPL losses are trended to the average accident date of the clash treaty being priced. They are also developed to ultimate. The development is an unusually tricky process, due to the nature of the claims. Closed claims are not usually liable to change. Open claims often change, but the amount of change depends on the status of the lawsuit. If a claim has received a jury-awarded shock verdict and appeals are pending, then the value may very well be expected to drop after appeal. On the other hand, if the suit has not gone to trial at all, it may not even be reported in the submission material and may skyrocket upon verdict. A provision is also included for unreported claims which will arise out of the historic exposures. Our approach when evaluating a cedant who has historic ECO/XPL claims is to estimate the future reportings (IBNR) for open and unreported claims. If the cedant has experienced rapid growth or other changes in the relative level of exposure, an adjustment is made to reflect this, as well.

After trending and developing, these claims are slotted into the layer (adjusting for coinsurance) and are used to determine an expected frequency of claims and average severity. These results are compared to the exposure rate, determined below. If a cedant has a high historic frequency of ECO/XPL losses in the layer, the reinsurer should, of course, re-evaluate whether this risk is acceptable before continuing.

Exposure Rating Model

The exposure rating model described here is essentially a market share approach. The main input is the premium of the company being rated. Several other factors modify the results of the main calculation. The basic formula is:

$$E[L] = \text{Premium} * \text{Constant} * \int_{\text{Attachment}}^{\text{Limit}} f(x)dx.$$

The result of the equation, E[L], is the expected losses for the ECO/XPL portion of the clash cover. Each of the three components of this formula, which are multiplied together to get this result, can be split into frequency and severity components. This allows E[L] to be stated as a layer severity and a return period. For example, say the formula shows E[L] to be $100,000. Splitting the formula into component pieces may show that this corresponds to a $1 million loss every 10 years (or a $10 million loss every 100 years).

Premium

The first portion of the formula is the Premium. To be specific, this is the premium which gives rise to ECO/XPL exposure. A few aspects must be considered.

1) The first is what *lines of business* are exposed. Most clash covers are limited to casualty business and don't include property in the subject premium base. Property business does not tend to have XPL claims, and ECO claims arising from property are extremely rare. Any ECO claim resulting entirely from the handling of a property loss is likely to be of little consequence.

2) *Workers' Compensation* business presents a lower hazard from ECO/XPL than other casualty lines. Because of the nature of WC business, it just doesn't give rise to anywhere near the number of losses as general liability and auto. However, there is a risk. ECO losses from compensation claims do happen and should not be ignored. Our solution is to include a small portion of the WC premium.

3) The final aspect of the premium component is in the *attachment points* that the company writes. Clearly a company which writes $50 million of GL and AL business excess of an average SIR of $500,000 presents a very different risk than one that writes $50 million in ground-up GL and AL policies. Increasing the premium for the former company can capture some of this added risk.

Constant

The second component of the formula is the Constant. Originally, the base constant is set such that the formula produces the desired frequency of losses for the market. Once calibrated, this constant is modified multiplicatively on a case-by-case basis in order to model the frequency of losses for the individual company being examined. We have identified four separate factors for which the constant may be adjusted.

1) The frequency is adjusted based on the *states* in which the company writes business. For example, we have identified California and to a lesser extent Texas as having a particularly high frequency of ECO/XPL claims. Several other states have also been singled out as deserving an increase in the frequency constant.

2) The relative *hazard level* of the cedant's book can also affect the frequency. A hazard adjustment should consider the line of business mix as well as the types of risks insured. A more hazardous book is inherently more likely to produce injuries which could result in ECO/XPL claims.

3) The cedant's *claims management* practices and philosophies are important determinants of the expected frequency of losses. Subjective opinions such as "better than average" or"terrible" must be quantified. For instance, an "average" company gets a claims handling multiplier of 1; an "above average" company may get .75; a "below average" company may get 1.5.

4) The final adjustment to the frequency is to increase the frequency constant for *"target" insurance companies*. The larger the company, the more likely they are to be the recipient of

lawsuits. This effect is more than simply proportional. The constant needs to be increased to compensate for the deep-pocket effect. One way to measure this is by the overall premium size of the company. The premium component mentioned earlier is for exposed lines only.

It can also be argued that this target company effect will apply to larger underlying insureds, as well. If larger insureds are more likely to be sued for larger amounts, this can be factored in here.

Expected Average Severity

The final component of the formula is the severity. Similarly to the frequency component, we start with a basic severity curve, then modify it to reflect the specific characteristics of the cedant. We will examine the basic curve and the three categories of modifiers we have identified.

The data we started with was a collection of punitive damage claims. These were gathered primarily from two sources. The first is underwriting submissions. Any identified ECO/XPL claims are pulled and collected into a single source. The second group consists of published studies of large verdicts and settlements, such as are contained in *The National Law Journal*.

To this data, we fit a Pareto curve. This is one of several standard curves used in actuarial work to represent severity distributions. An algebraic formula can then be used to calculate the expected severity given any attachment and limit combination. We have selected the two parameter Pareto curve. The first is a shape parameter, which is entirely determined by the fit to the data. The second is the scale parameter, which can be adjusted to reflect differing characteristics of the cedant.

1) The first severity modifier is for the *states* in which the company writes. Care must be taken to ensure that this modifier does not overlap with the frequency adjustment for the states. The adjustment discussed before was done to reflect solely the relative number of lawsuits filed. This severity adjustment is intended to capture state differences in the amount of the judgments, given that the suit is filed. Clearly, there is a cause-and-effect relationship here. The larger the verdicts in a state, the more people are going to decide to file suit. At the same time, once a precedent is set for large judgments in a state, future large jury awards are more likely. (This effect is somewhat similar to the lottery. More people buy tickets when the jackpot gets high, even though no single person has any greater chance of winning. The increased sales cause the jackpot to go even higher.) This modifier can be applied as an adjustment to the scale parameter.

2) The second severity modifier is for the *coinsurance* factor, discussed above. This can be accomplished by modifying the scale parameter or by adjusting the limit and attachment upwards (i.e., with a 90% coinsurance factor, it takes a loss greater than $10 million to attach a layer excess of $9 million). If the latter method is used, the layer severity must be adjusted, as well.

3) The third severity modifier is for *inuring protection*, such as Errors & Omissions coverage purchased by the cedant. This does not change the number of suits filed against the company, but does lower the chance of penetration of the reinsurance layer. The reinsurer is covering a higher layer for the cases where inuring protection has an effect.

Clearly, the second and third severity modifiers also have an effect on the frequency of claims in the excess layer being priced. They do not, however, have any effect on the pure number of cases being pursued. The frequency component is intended to capture only the number of cases. The severity component then adjusts for the likelihood of the loss reaching the clash layer.

SUMMARY

As clash reinsurance becomes a larger portion of many reinsurers' books of business, it is important to have logical methods of evaluating the associated exposures. The pricing of this business has typically been done using judgmental methods. The underwriting considerations shown above detail what we have identified as the most significant characteristics affecting the ECO/XPL portion of these exposures. The exposure rating model described presents a method of translating this information into expected losses for the clash treaty. By using a model such as this, the reinsurer can add logic and consistency to the pricing approach as well as compare clash programs.

1 Joanne Wojcik, "Bad Faith Award Shocks Insurers," *Business Insurance*, December 13, 1993.

2 Margaret Cronin Fisk, "The Year's Largest Verdicts," *The National Law Journal*, January 17, 1994.

3 Margaret Cronin Fisk, "Verdicts. The Big Numbers of 1994," *The National Law Journal*, February 6, 1995.

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Evaluating Variations in Contract Terms for Casualty Clash Reinsurance Treaties

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Abstract: The authors examine variations in event definitions and commutation clauses which are commonly encountered in casualty catastrophe reinsurance contracts in the market today. Changes in these aspects of the contract may affect the exposures the reinsurer is asked to cover. In this article, the variations are contrasted with special emphasis given to the effects this may have on the pricing/underwriting process.

Evaluating Variations In Contract Terms For Casualty Clash Reinsurance Treaties

I. Introduction

Look across our industry and you will find among the most critical issues one factor that is that huge casualty catastrophe losses do happen. They occur in many forms ranging from a large extra-contractual obligations (ECO) award under a single policy to workers' compensation multiclaimant losses to multipolicy losses (true clash) or runaway allocated loss adjustment expense (ALAE). In light of this, casualty catastrophe reinsurance remains an integral part of most companies' reinsurance programs.

Over the years, the clash product has become more sophisticated and more tailored to the specific cedent's needs. Therefore, any reinsurer selling clash protection must carefully evaluate the various contract terms that have also evolved for their effect on the exposures for which protection is being sought. Reinsurers also must be able to model and compare the different coverage that different contract terms will produce.

Our purpose here is to analyze event definitions and commutation clauses and to examine how changing contract clause provisions can affect both the exposures and the modeling needs. We also will look at the modeling process itself. We will briefly discuss the different ways in which these covers attach and then examine commutation clauses.

For each variation in the clauses under discussion, we will consider related potential changes in the reinsurer's exposure to loss. We will provide examples of the types of loss covered under one definition which are excluded from others. We will then look at the types of information needed for the reinsurer to price these added exposures.

II. The Modeling Process

Before jumping into the various examples, it will be helpful to lay out the structure of our modeling/pricing process. The three general steps are: (i) determining the expected losses; (ii) building a discrete aggregate distribution; and (iii) calculating the return on equity (ROE) for each point of the aggregate distribution.

To determine the expected losses one must first identify the type of event that can cause a loss. Such an event can include ECO and excess of policy limits (XPL) judgments. It may also include such things as workers' compensation multiclaimant losses, multipolicy losses, the stacking of uninsured motorist limits or runaway allocated loss adjustment expenses. Once the causes of loss have been identified, we determine a frequency and average layer severity for each cause.

Models can be constructed to estimate the frequencies and severities for each cause based on exposure.[1,2] Certainly, experience can be used, where available. If neither of these are available, the frequencies and severities can be selected judgmentally.

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These selections, of course, will vary with each cedent and each reinsurance program. As an example, consider the runaway ALAE exposure on a clash layer attaching at $5 million for two hypothetical cedents. Cedent A writes policies with ALAE payable in addition to the policy limit of $1 million. Clearly, the ALAE exposure will come about from paying very large ALAE amounts on comparatively small insured indemnity losses.

Cedent B uses the same policy form, but writes maximum policy limits of $5 million. All else being equal, Cedent B has about the same exposure to large ALAE losses that Cedent A has. In addition, Cedent B has exposure to relatively small ALAE losses from insured indemnity losses which are at, or near, the $5 million policy limit. Not only does Cedent B have the runaway exposure exhibited by Cedent A, but it also has a "trickle" exposure. The runaway exposure will be characterized by comparatively high severity and low frequency losses. In comparison, the trickle exposure will be characterized by higher frequency, but lower severity, and could result in higher expected losses than the runaway exposure. Both of these exposures should be considered when determining the expected losses.

The next step is to develop an aggregate distribution for the various causes of loss. This can be accomplished by methods such as described by Panjer[3] or other methods.[4] For example, a clash aggregate distribution may indicate the following: 90% chance of no loss; 4% chance of expected losses equaling half the layer; 3% chance of expected losses equaling the full layer; 2% chance of two full layers of losses; and 1% chance of three full layers of losses.

The final step is to calculate return on equity using the aggregate distribution as input. This means modeling cash flows for each point of the aggregate distribution, calculating a return on equity (net present value return, in dollars, divided by the surplus allocated to the specific layer) for each point, and weighting these ROEs together using the probabilities of achieving each expected loss outcome.

This process provides a structure for assessing the implications of the pertinent contract terms. If a change in a contract term can affect expected losses, the reinsurer needs to modify the inputs used in determining the aggregate distribution, produce a new aggregate distribution and recalculate the ROE. If a change affects cash flow only, this can be modeled in the ROE calculation, without change to the aggregate distribution.

III. Event Definitions

The event definition is the linchpin in underwriting and pricing clash covers. It should reflect both parties' expectations as to the scope of coverage provided. Although the event definition has become somewhat standardized, increasing litigiousness, unpredictable jury awards, emerging toxic torts and new theories of liability also are defining the type of catastrophic events casualty insurers can expect to face. In light of the uncertainty that these emerging trends impose on clash pricing, it is critical that all parties to the reinsurance contract agree on the intent and construction of an event definition.

For example, if a cedent is expecting clash coverage for losses that are of a continuous or latent nature, rather than the result of a spontaneous occurrence, the event definition should include language that refers to repeated exposures. If the cedent expects the cover to respond to economic losses arising out of E&O and D&O policies it has issued to financial institutions, the event definition should specifically include wrongful acts and errors and omissions in addition to the standard language appearing in element 4 below. In simple terms, the crafting of an event definition should encompass a careful review of the underlying business and the scope and extent of catastrophe coverage desired. Failure to do so could result in unnecessary contract disputes.

An event definition typically includes the following elements:

1. Damage, injury or loss arising out of one or more than one policy; that is the

2. direct consequence of one particular accident, disaster or casualty; that

3. takes place in its entirety at a specific time and place; and

4. is traceable to the same single accident, disaster or casualty.

A. "Damage, injury or loss arising out of one or more than one policy." The terms "damage, injury or loss" typically relate to the terms used in the insuring agreement of the underlying policies being covered, i.e., the CGL, umbrella policies etc. If both clash and contingency protection are being purchased to protect against loss arising under a single policy involving, for example, runaway ALAE and a clash of two policies, this intent is captured in element 1 above. However, if the catastrophe protection purchased is strictly clash, then this element should be tailored to read "arising out of more than one policy."

Another variation on this theme occurs with definitions using "more than one insured," rather than "more than one policy." For instance, if an insurer writes commercial auto and workers' compensation using separate policies, an auto accident involving one insured car driven by a worker where the worker's employer is also the car owner, could be a clash loss under the "more than one policy" scenario, but not the "more than one insured" wording.

This part of the event definition comes into play when the pricing actuary or underwriter is selecting the types of occurrences which are intended to be covered by the clash layer. Runaway ALAE and ECO/XPL are not nearly as great if the coverage is limited to events arising out of two or more policies. On the other hand, if two or more policies must be involved, one might expect the selected severity to be higher and the payout somewhat quicker than would otherwise be the case. Ideally, the cedent should be able to provide historic losses accumulated using the appropriate event definition.

B. "Direct consequence of one particular accident, disaster or casualty." This element sets forth the requirement that recoverable losses be caused directly by a single event and that there be an appreciable degree of causation between the single event and the loss. An example of the difficulty with the common factor requirement is illustrated by the following example. Loss attributable to an explosion in a factory gives rise to claims under an insured's liability and workers' compensation policies and would be treated as caused by one event. However, if the cedent's loss was due to two separate explosions occurring at different times during the year could the cedent lump the two incidents together and consider them one event if both incidents resulted from defective made boilers from the same manufacturer? In other words, if the cause of each explosion proved to be the defective made boilers, would that common factor satisfy the criterion that the losses were the result of one particular manufacturing accident? These are, admittedly, difficult questions without clear cut answers as indicated by court decisions in the US and England[5].

Another example illuminating the difficulty in causation analysis is the determination of proximate cause. Proximate cause in a chain of several events refers to the nearest cause preceding the final event. Thus, in a causal sequence of events resulting in damage to a California condominium complex, would a subsidence problem be a direct or intervening cause of damage where the builder already had a judgment against it for construction defects?

Pricing for these exposures depends on how well both parties identify and address the problem areas during the underwriting process. Given the uncertainty in judicial outcomes, it is reasonable to assume that half the decisions will favor the cedent and half the reinsurer. Thus, setting up an additional occurrence type in the expected losses for this exposure may be appropriate in the reinsurance analysis.

C. "Taking place in its entirety at a specific time and place." This element requires that the event must commence and end within a specific time period and occur in its entirety at an identifiable site. Explosions such as occurred in Bhopal, India, or fires like the Puerto Rican Dupont Plaza Hotel are concrete examples of how this element is traditionally interpreted. However, workers' compensation catastrophe covers often are intended to provide protection against occupational disease or cumulative injury, which, by definition, are gradually occurring injuries. Hence, specific wording should be added to the event definition to encompass this different criterion. Consider, for example: "As respects occupational disease or cumulative injury under workers' compensation policies suffered by an employee for which an insured is liable, such occupational disease or cumulative injury shall also be deemed to be an "event" within the meaning of this contract."

Loss aggregation in a clash cover is another problem, particularly with respect to products liability losses and occupational diseases or cumulative injuries. For example, reinsurers should be clear on whether all carpal tunnel claims incurred by the cedent for workers doing essentially the same functions can be accumulated across insureds. This can be the case if the clash cover is worded to provide aggregate extraction coverage. Aggregate extraction allows a cedent to extract an original insured's policy loss that is related to a specific clash event and combine it with the losses from other insureds involved in that same event.

Again, from a pricing standpoint, the first step is to make sure the desired coverage is understood. It is always worthwhile to discuss intent with the cedent and/or broker to ensure all parties share a common understanding of the coverage sought. Model parameters can then be selected and matched accordingly.

D. "Is traceable to the same single accident, disaster or casualty." This element requires that losses under a catastrophe cover be connected to each other in the sense that they arose out of the same single event. This is the essence of a catastrophe cover. It is not intended to give protection for losses which do not arise out of the same event and so are unrelated. An example of ambiguous wording sometimes used in this context is the term "causative agency." The ambiguity arises because that term could be used as a basis on which to allow losses from unrelated lead claims to be aggregated. For example, a cedent could accumulate all its lead paint claims based on a "causative agency" theory that lead is the causative agent of all lead paint claims. Unless this "batch" type of exposure is explicitly understood, priced for (a very difficult exercise due to not knowing with certainty what types of events can lead to a batch loss) and expressed contractually, catastrophe covers would not be expected to respond to losses that while alike in nature are not connected to each other. Simply put, clash covers provide protection for a clash of policies or insureds, but not a clash of events.

Disputes over whether an event loss is directly traceable to a single cause or is really the result of multiple unrelated occurrences are becoming more common in casualty insurance and reinsurance. This is particularly true for pollution, toxic torts and more recently property construction defect claims. Consider a case similar to our earlier case where a cedent writes a large book of California contractors business, and a major earthquake hits. Insurers writing homeowners business in the state look for subrogation possibilities and tie losses from many of the individual homes back to specific construction defect claims brought against the cedents' insureds. A cedent might argue that the single or proximate cause is the earthquake, allowing all its construction defect claims to be aggregated into one clash loss to meet the contract's retention. In contrast, a reinsurer might contend that each construction project claim is a separate event, that the earthquake was merely an intervening cause, and therefore the losses cannot be aggregated. Further difficulties arise when trying to assign these losses to policies and underwriting or accident years.

IV. "Business Disaster" Event Definitions

Over the years, buyers have been exploring other definitional options for obtaining broader clash coverage. The coverage provided by the event definition we discussed above has left uncovered an entire complement of losses often referred to generically as "business disasters." An example of a "business disaster" cause of loss would be the Savings and Loan crisis, which resulted in multi-policy losses for insurers under both D&O and E&O policies. Where such coverage is contemplated, it is necessary that the event definition clearly reflect that coverage is being provided for all wrongful acts, offenses, omissions or errors committed by professionals acting in their professional capacity in connection with losses sustained by a financial or commercial institution. An example of a business disaster event definition appears at the end of this article in Exhibit A.

One of the most difficult aspects of the "business disaster" event definition is clarifying what business risk is being covered. The cedent will often have specific exposures in mind, but the wording is nonspecific. The reinsurer usually wants the coverage defined as narrowly as possible. Both parties to the contract negotiations should strive for the same understanding of what is and is not covered.

V. Attachment Basis

Generally, casualty catastrophe covers respond to losses provided that the event occurred during the term of the reinsurance contract. This is the typical "losses occurring during " (LOD) basis. The exposure to loss can be measured by looking at the projected makeup of the book for the coming year. Compared to claims-made catastrophe covers, the LOD catastrophe structure has drawbacks for the reinsurer that are similar to those occurrence policies compared to claims-made policies have for insurers. Casualty losses covered by an occurrence structure arise from later calendar periods (are projected further into the future), adding greater uncertainty in expected losses by virtue of the longer tail.

Moving along the attachment spectrum towards claims-made, some clash covers are written on a "losses discovered" basis. In this case, coverage depends on whether the cedent has established a reserve of a specified dollar amount for an event that occurs after the inception of the catastrophe contract. The difficulty in pricing for this exposure depends, to a certain extent, on the adequacy of the reserves for potentially covered claims immediately before the inception of the reinsurance. A claims audit prior to binding the reinsurance can be helpful.

Finally, there is the pure claims-made clash cover where several requirements must be met to qualify for coverage: (i) the event must occur during the term of the reinsurance contract; (ii) notice from the original insured to the insurer has to be given during the term of the reinsurance contract; and (iii) the cedent must provide notice of all claims arising from the same event within a specific period of time (e.g., 24 months) from the date of the first notice.

For the losses discovered and pure claims-made structures, the exposure can be estimated by looking at reporting patterns for the various types of possible losses. Usually premium brought to current level is used as a proxy for comparing historic exposure to current exposure. When doing this, an additional area of concern is changes in the claims adjusting practices of the cedent. The occurrence of ECO/XPL claims tends to be related to the claims management practices of the cedent. Historic problems in this area for a cedent can indicate increased exposure to ECO/XPL claims in a claims-made clash structure.

VI. Commutation Clauses

Most typically, commutation clauses allow the parties to extinguish the reinsurer's known and unknown -- but predictable -- liabilities under the reinsurance contract by the reinsurer's payment to the cedent of a sum of money that is discounted to reflect the time value of money. In exchange, the cedent gives the reinsurer a full and final release of all its past, present and future obligations under the contract.

The items that determine the settlement amount of a proposed commutation include: (i) the value of paid losses and ALAE recoverable; (ii) the estimated value of unpaid losses and ALAE, which includes reserves for outstanding losses, ALAE and IBNR; (iii) the value of disputed items; (iv) the value of present balances due; (v) the value of return premiums and future premiums due; and (vi) the value of credits, such as cash, letters of credit, funds withheld, etc. Calculations done to determine the present value as respects items (ii) through (vi) above should include assumptions for payout patterns, current interest rates, reinvestment and tax considerations. The pricing procedures should also include trend analysis, benefit escalation analysis, reserving analysis, in-depth pricing/reserving by treaty or by claim including an analysis of the value and adequacy of the commuting party's IBNR.

The determination of the commutation values include a stochastic analysis of the claims. This includes, at the least, an analysis of the ultimate claim value, and, if possible can also include escalation rates, discount rates, mortality and any other necessary variables.[6] Above all, the commutation clause may limit the reinsurer's options on how the calculations are performed to determine commutation values. Appropriate attention to these issues should be addressed in the contract negotiation process and the original pricing of the reinsurance deal.

Commutation clauses in casualty catastrophe covers, particularly workers' compensation clash covers, are usually desirable to reinsurers for two key reasons. First, reinsurers can reduce the volatility in financial results that occurs when a reinsurer experiences an unanticipated escalation in frequency or severity of covered losses or ALAE by capping adverse loss experience. Second, a commutation can enable a reinsurer to minimize or eliminate the ultimate liabilities on its books at an early date by making a cash payment that reflects the net present value of the losses in return for a full and final release. Where claims involve long-term periodic payouts which can be affected by the escalation of indemnity benefits, inflation of medical costs and increased life expectancies These goals are accomplished, however, by shifting the potential volatility back to the cedent, who must be comfortable with the reassumption of this exposure. If the cedent is uncomfortable with this, a clash product without the price benefit of the commutation clause may be more appropriate.

In a commutation, the cedent receives a cash payment from the reinsurer and reassumes the ceded reserves for outstanding losses, allocated loss adjustment expense and any premium reserves. The balance sheet effect of the commutation for the cedent depends on the answers to a number of questions: (i) does the reinsurer carry its reserves on an undiscounted (most typical) or discounted basis?; (ii) will the commutation payment be discounted (typical) or not?; and (iii) has the cedent written off the reinsurance as uncollectible?

Generally, U.S. reinsurers carry loss reserves on their books on an undiscounted or minimally discounted basis. Where the commutation is effected on a discounted basis, a reinsurer will usually increase its underwriting income and its surplus. If the reinsurer carries its reserves on a discounted basis, surplus will either increase or decrease depending on the assumptions used in determining the discount rate applied in the financial statement and the discount rate that applies to the commutation. Normally, if the discount applied to commuted losses is greater than the financial statement discount, the reinsurer will sustain a statutory increase in surplus. The cedent will reduce its surplus if it reflects the undiscounted value of the reserves being reassumed but records the cash payment on a discounted basis. For workers' compensation reserves, if any applicable statutory benefit escalation is not reflected in the reserves, the reserves are effectively discounted.

The tax implications of a commutation are also important to by both parties. Some factors for reinsurers to consider are: (i) U.S. reinsurers carry their loss reserves on their financial books on an undiscounted basis, while for tax reporting purposes reserves are discounted. Where the reinsurer may carry its reserves at slightly less than their full future value, a commutation may not increase underwriting income, and so, for tax purposes, there may be a decrease in taxable income; (ii) the impact on IBNR reserves following a commutation may have significant tax consequences for the reinsurer. Factors to consider from the cedent's perspective include: (i) the effect of reassuming loss reserves that are greater than the cash payment received. This could result in a statutory reduction in surplus and thus have tax implications; and (ii) the deductibility of any decrease or increase in IBNR.

A. Contract Options

There are a range of contract variations that the parties to a commutation can negotiate.

1. Mandatory v. Optional. From the reinsurer's point of view a forced or mandatory commutation is often the more desirable. This depends in large part on the line or class of business covered by the reinsurance contract. Many workers' compensation clash covers include mandatory commutation wording. A mandatory commutation clause provides that after a specified period of time, usually from the inception date of the contract, both parties must come to an agreement on the commutation payment and terms to discharge the reinsurer of its liabilities under the contract. A mandatory commutation can require that the parties appoint one or more actuaries (or other qualified parties) to determine the net present value ("capitalized value") of the claims in an arbitration-like proceeding or specify other methods upon which the parties can, by a formula, reach an agreed value.

Where the commutation clause sets forth the specific basis for calculating the final value of the claim or claims being commuted, this type of mandatory commutation is commonly referred to as an "Agreed Value Commutation." In such a clause, various escalation and discount percentages are agreed to (or can be simulated) for index-linked benefits, un-indexed or fixed benefits and future medical costs. Tables are also identified for use in calculating impaired life expectancies, survivors' life expectancies and remarriage probabilities. At the end of a predetermined period, the final agreed value is calculated based on the above factors. An additional alternative sometimes included in an Agreed Value clause is that: (i) the parties may mutually agree to use another method; (ii) an annuity may be purchased or a quote obtained which will determine the agreed value; or (iii) reinsurance may be purchased or a quote obtained which will determine the agreed value. For a smaller cedent, purchasing an annuity or reinsurance provides a way to avoid having the reassumed reserves show up on its books and can thus stabilize its financial results.

In a mandatory commutation, the issues arising from the Financial Accounting Standards Board (FASB) Statement No. 113, should be reviewed to ensure that the catastrophe cover qualifies for risk transfer accounting treatment. In this regard, the cash flow analysis should include the contemplated commutation settlement amount. A mandatory commutation should be carefully evaluated by the reinsurer to determine its effect on the price, depending on the exact structure and application of the commutation, as discussed below.

Optional commutation wording entitles either party to request commutation a certain period of time after the effective date of the contract. If the parties do not agree to commute or fail to agree to the commutation settlement, there is no legal requirement to proceed. A variation on this theme is where either party after a specific period of time may ask to commute, and if agreed to by the other party, the commutation then becomes enforceable. Optional commutations normally have little or no value from a pricing standpoint. Theoretically, any reinsurance contract can be commuted at any time, so long as both parties agree. All the optional commutation clause can do is predetermine some of the parameters to be used in case of commutation, which can, in fact, be more limiting than helpful.

2. Known and Unknown Liabilities. It is usually desirable from a reinsurer's perspective that when agreement is reached on the value for a claim or group of claims subject to commutation, the final value should include IBNR. Commutation wording such as this the identification of the liabilities being commuted should specifically identify IBNR in addition to paid losses and allocated loss adjustment expense recoverable and reserves for losses and ALAE. In the event the commutation is for known liabilities only, the wording should reference that the adverse development on the known claims is included in the commutation amount.

The value of the nominal losses to be commuted can be estimated by a variety of processes. All known losses in the layer and open potential losses below the layer should be examined. These can be evaluated using the parameters set out in the commutation clause. By examining as many potential losses (losses which may develop into the layer) as possible, the uncertainty surrounding the unknown liabilities should be minimized. As mentioned earlier, a stochastic process of evaluating the claims and parameters can be helpful. Claim reporting patterns for each of the types of occurrences which can cause losses can also be helpful in estimating the remaining liability from unknown losses.

3. Discount Rates and Escalation Rates. To reflect the net present value of the ultimate losses being commuted, the discount and escalation rates may be selected at the time of commutation, based on agreed objective measures. Often in commutation clauses applicable to workers' compensation clash covers, separate escalation rates for indemnity and medical benefits are considered where applicable. (Escalation rates are typically unnecessary for other lines of business or types of loss.)

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When selecting escalation and discount rates, it is important to remember that these are normally variables. As discussed in *Levels of Determinism in Workers' Compensation Reinsurance Commutations*, by Gary Blumsohn[7], these can be simulated as a measure of the variability in the rates. The importance of selecting proper escalation rates can be magnified when combined with certain commutation mechanics, such as commuting between the layers, discussed below. When applying escalation rates, it is also important to determine whether the losses are reserved by the cedent on a nonescalated basis, to prevent escalating the losses twice.

The bottom line is that the pricing actuary should be aware of the values to be used for the escalation and discount rates, and form an opinion on the adequacy of these rates. Any perceived differences should be considered in the cash flow modeling.

4. Commuting Ground-Up v. Commuting by Layer. The more standard commutation clauses in the market today work by first determining the discounted value of the covered loss. At that point, the retention and reinsurance layers are applied which generally have the effect of collapsing the losses into the retention and the lower layers of a reinsurance structure. On the other hand, commuting by layer means that the ultimate covered loss would first be apportioned to the layers before any discounting occurred. If this is contemplated, the pricing assumptions for all layers should reflect this.

The interplay between this mechanism and the discounting/escalation form the heart of the commutation provision. The simplest case is where the loss does not escalate and discounts from the ground up before layering. Say the nominal loss is $11 million ground up and the reinsurance layer is $10 million excess $5 million. Further assume the discounted value of the $11 million loss is $7 million. Then under the ground up commutation, the reinsurer will pay $2 million, but that isn't the whole story. Looking at the original $11 million as the sum of the $5 million retention and the $6 million excess loss, suppose the retention discounts to $4 million and the excess to $3 million. In this case, the reinsurer saves $1 million ($3m - $2m) off of the discounted value of the reinsurer's payments had the loss not been commuted. This $1 million has discounted out of the layer, and will revert to the cedent. From a modeling point of view, this approach affects the expected losses as well as the cash flow and investment income.

On the other hand, using the same example, assume the clash contract has a commutation provision where losses are commuted between the layers. Then, the original $11 million is divided into the $5 million retention and $6 million excess loss before discounting. Using the above figures, the reinsurer will pay the discounted value of the $6 million, or $3 million. In this case, the savings to the reinsurer (and thus on the price the cedent pays), result mainly from a reduction in loss adjustment expenses which would have been paid had the claim not been commuted. The present value of the expected losses at treaty inception is unchanged; only the cash flow and realization of investment income are really affected. (There is no real economic savings on the loss.)

For a third example, assume the same reinsurance layer of $10 million xs $5 million, but a nominal loss of $30 million. Further assume the $5 million retention still discounts to $4 million, and the $25 million excess loss discounts to $12 million. Thus, the discounted ground up loss is $16 million, and using the method discussed above of discounting ground up and then layering, the reinsurer suffers a full loss to the $10 million xs $5 million layer. The loss has actually collapsed into the reinsurer's layer at commutation. The reinsurer pays the full $10 million, as it would have without commuting, but pays it much earlier than the payments would have come without the commutation. Note that using the method of commuting between the layers, the reinsurer would only have paid the discounted value of the $10 million nominal loss in the layer.

Thus it is possible for the ground up method of discounting to be worse for the reinsurer than the layer discounting method, but it isn't likely. Given the shape of most loss distributions and the size of losses affecting most clash programs, smaller losses are usually more frequent. Thus, losses tend to discount out of clash layers more than collapse down into them, particularly for higher layers. An exception to this might be a low excess layer on an exposure suffering from very high severity casualty losses.

An additional difficulty may arise in the case of commuting workers' compensation losses which have escalating benefits. When valuing a commutation clause which discounts between the layers, losses will be escalated, layered and then discounted. If the escalation rate is consistently overestimated, losses can be overinflated into the reinsurer's layer, then trapped there by the commutation calculation. Regardless of the discount rate used, the losses aren't allowed to discount out of the layer. (Conversely, a consistent underestimation of the escalation rate, such as using 0%, will reduce the reinsurer's liability at commutation using this method.)

5. Mechanisms for handling disputes concerning valuation. To facilitate agreement on the value of the claim or claims to be commuted, the parties can agree to submit any disputes to a panel of arbitrators who are actuaries, either members of the American Academy of Actuaries or Fellows of the Casualty Actuarial Society or both.

6. Full and Final Release. The reinsurer will want to be assured that its payment of the commutation amount where it covers both known and unknown liabilities will completely and finally release it from all past, existing, and future obligations with respect to the liabilities commuted, including any contingent liabilities. This acts to commute the entire contract as the release of the reinsurer of liability for future loss development acts as a sunset provision.
If only known liabilities are covered, the full and final release applies to the known liabilities and the adverse development on the known liabilities. In essence, the parties are essentially commuting losses within the contract. After the commutation of the original losses occurs, subsequent losses are also subject to commutation.

B. Additional contractual terms.

1. Offset. The decision to commute may be affected by the existence in the contract of offset provisions that allow the parties to offset debts and credits under the contract in the ordinary course of business and in the event of a party's insolvency. Of course in the latter situation offsetting may be subject to legal challenge by regulators or the debtor party on the basis that policyholders, claimants and all other general creditors have priority over reinsurers claims under reinsurance contracts. In the absence of an offset clause, commutation may be the only reasonable alternative for a cedent to secure large recoverables.

2. Loss Caps and/or Sunset provisions. Contractual terms that place limits on the amount of money a reinsurer can lose may mitigate the need for the parties to enter into a commutation. For example, a loss ratio cap will limit reinsurance coverage when paid losses exceed some multiple of reinsurance premiums earned over the course of the treaty term. A sunset provision will likewise end the reinsurer's payment obligations after a specified number of years from the inception date of the contract. Claims that are not notified to the reinsurer before the sunset date are not recoverable.

C. Other Considerations.

Beyond the above, there is an additional exposure presented by commutation clauses of which the reinsurer should be aware.

Consider the situation where a cedent has significant workers' compensation exposure. One approach for reinsuring large workers' compensation exposures used frequently in today's market is to buy "carve-out" coverage, typically from Accident and Health (A&H) markets. These products will usually be sold with commutation and sunset clauses. Complementary coverage can then be purchased from property and casualty markets to reinsure other exposures excluded from the carve-out cover. These will typically include EXO/XPL, Employers Liability (EL), possibly multiclaimant losses (depending on the carve-out product) and non-workers' compensation losses.

As an example, assume a cedent buys carve-out protection for workers' compensation single claimant losses with a layer of $10 million excess of $5 million. Then, the cedent buys traditional P&C protection for $10 million excess $5 million, as well. Further assume that the carve-out cover has a commutation clause but the P&C cover does not.

The P&C cover can be worded any number of ways in order to have it apply only after the carve-out cover, and so avoid double coverage or coverage gaps. At times, more than one method to achieve this will be used in a single contract. One method is to have the P&C cover contain a "maximum any one life" (MAOL) warranty or representation which, for our example, would of $5 million or less. This will cause the loss amount from any single claimant to be limited to the MAOL and so under this cover there would be no recovery for a single claimant. A second method is to exclude workers' compensation in the "Business Covered" clause of the contract. Since this method could also exclude losses otherwise covered resulting from workers' compensation occurrences, such as ECO or EL, care should be taken to clarify the scope of contractual coverage. A third method is to specifically list in the Business Covered article only the types of loss which will be covered, such as occupational disease, cumulative injuries, employers' liability and ECO. This type of P&C cover is often called a Difference in Conditions (DIC) cover.

A fourth method is to state that the carve-out cover inures to the benefit of the P&C cover (assuming it is not used in combination with any of the foregoing). There are two exposures worthy of mention with respect to this approach. The first is fairly obvious. In our example, if the cedent has a workers compensation loss which is greater than $15 million (ground-up), then the excess amount above $15 million can attach the P&C cover. For example, say the cedent has a $16 million loss. The carve-out will pay $10 million, leaving a net loss to the cedent of $6 million. The P&C cover attaches at $5 million of ultimate net loss to the cedent after inuring reinsurance, so it provides an additional $1 million in protection. Excluding complications caused by the carve-out's commutation or sunset clause, this exposure is fairly easy to price.

The second potential exposure caused by the inuring reinsurance wording relates directly to the commutation clause on the carve-out cover. Consider the following example. Say the carve-out has a commutation clause which is mandatory after five years from expiration with the losses discounted and then layered. The P&C cover has no commutation clause. Assume the cedent has a ground-up loss of $14 million, and at the time of commutation this discounts to $7 million. Thus, the carve-out cover pays $2 million at commutation, and the cedent has its $5 million retention. However, these are discounted amounts. Say the $2 million carve-out portion represents $6 million undiscounted. Thus, the $5 million cedent retention represents $8 million undiscounted ultimate net loss. Herein lies the problem.

Consider the retention, first. By the time this pays out, the inuring carve-out has long been commuted. The P&C cover attaches based on the cedent's ultimate net loss. The cedent has very good arguments for claiming a $3 million recovery from the P&C cover.

Taken a step further, the ultimate gross loss is our original $14 million. The recovery from inuring reinsurance is $2 million. Unless the P&C cover has a provision which takes credit for the implied future investment income determined in the carve-out commutation calculation or for investment income from annuities purchased with the proceeds of the commutation, the cedent has an argument that its ultimate net loss to the P&C cover is $12 million ($14 million gross - $2 million ceded). This would mean the P&C cover potentially responds for $7 million.

This coverage is not what the P&C reinsurer is normally intending to do. It also isn't necessarily the coverage the cedent is trying to purchase to begin with, but could be worth pursuing in the event of a large loss. The reinsurer should therefore be aware of the ambiguity presented here and structure the contract such that this exposure is either excluded or paid for.

One way the reinsurer may avoid this problem is if the contract is worded such that it is warranted or deemed that the cedent will "maintain" the inuring coverage. This causes the P&C cover to apply as if the carve-out were never commuted. (The alternative to this is that the contract may be worded such that the cedent "is allowed to purchase" or "agrees to purchase" inuring coverage, which doesn't necessarily require its existence throughout the term of the P&C cover.)

An additional consideration is that a judge or arbiter could find that this interpretation goes against the custom and practice in the insurance and reinsurance industry. Yet bear in mind that this type of reinsurance structure--a workers compensation carve-out cover followed by a P&C wrap-around cover--has not been in general use long enough to have acquired a customary interpretation.

From the reinsurer's point of view, the desirable outcome may be to exclude this exposure, because accurately measuring it is virtually impossible. The exposure to the reinsurers on the P&C cover depends on intricate negotiations in which only the carve-out market and cedent participate, thus precluding any meaningful input from the P&C reinsurers in the determination of the ultimate value or discounting of the claim being commuted, which values determine, in part, the P&C markets losses.

VI. Conclusion

In this paper, we have considered some of the more pivotal sections in a clash reinsurance contract, and some of the variations in these clauses, to show how they can affect the reinsurer's exposures and the modeling process. Overall, the key is to understand what exposures the cedent has and what exposure it is seeking protection for. If the pricing actuary and underwriter know what coverage is desired and why, the modeling process becomes more enlightened.

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Exhibit A

Business Disaster Event Definition

1. "Event" shall mean all damage, injury or loss covered by one or more policies of insurance issued by the company, which is a direct consequence of one particular accident, disaster or casualty which takes place in its entirety at a specific time and place and is traceable to the same single act, omission, mistake, error or series or acts, omissions, mistakes or errors.

   As respects coverage provided under policies classified by the company as Professional Liability, Directors and Officers Liability, Public Officials Liability, Educators Legal Liability or other liability coverages written by the company on a claims made, losses reported or losses discovered basis, the term Event shall also mean, all damage, injury or loss covered by one or more such policies which arises out of a claim(s) against more than one original insured of the company by:

a) the same allegedly injured third-party or parties and/or

b) other original insureds of the company that have had a claim(s) against them as in paragraph (a) above and, the alleged act, omission, mistake, error or series of acts, omissions, mistakes or errors are traceable to the same Central Loss.

   "Central Loss" shall mean the failure (including but not limited to liquidation) or impairment (including but not limited to severe financial loss and/or the need to seek or receive protection under State or Federal statute or regulatory authority) of one or more nonprofit institutions, public entities, or commercial enterprises, without whose failure or impairment there would have been no claim(s) against the original insured(s).

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Footnotes:

1. Pricing Extra-Contractual Obligations and Excess of Policy Limits Exposures in Clash Reinsurance Treaties, Paul Braithwaite and Bryan Ware, Journal of Reinsurance, Spring 1995.

2. On Pricing Multiple-Claimant Occurrences for Worker's Compensation Per-Occurrence Excess of Loss Reinsurance Contracts, Gregory Graves, Casualty Actuarial Society Discussion Paper Program, 1990.

3. Recursive Evaluation of a Family of Compound Distributions, H.H. Panjer, ASTIN Bulletin. 12, Part 1.

4. The methodology for accomplishing this is beyond the scope of this paper.

5. The "Event" Debate in Asbestos Related Excess of Loss Reinsurance Disputes. Tort and Insurance Law Journal, Vol. 31, Number 3, Spring 1996.

6. The CAS literature includes a number of good articles on this topic: See e.g., Conner, Vincent and Olsen, Richard (1991): Commutation Pricing in the Post Tax-Reform Era, PCAS, vol. LXXVIII; Ferguson, Ronald (1971): Actuarial Note on Workmen's Compensation Loss Reserves, PCAS, vol. LVIII; Steeneck, Lee (1996): Actuarial Note on Workmen's Compensation Loss Reserves - 25 Years Later, Casualty Actuarial Society Forum, Summer 1996.

7. Blumsohn, Gary (1997): Levels of Determinism in Worker's Compensation Reinsurance Commutations.

Additional Reading:

1. The Decision Making Process For A Commutation. Terrence M. O'Brien, Coopers & Lybrand. April 2, 1989.
2. Commutation is A Useful Alternative To Solving An Insurance Dispute. Peter Matthews. Post Magazine. November, 1987.
3. As The Reinsurance Industry gradually Becomes More Aware Of the Benefits To Be Gained From the Commutation Of Losses. Ken Louw. Reuter Textline. September 23, 1987.
4. Commute or Not To Commute. Phillip Wolf. Best's Review. April, 1987.

Loss Development and Annual Aggregate Deductibles

by Vincent P. Connor, ACAS

Loss Development and Annual Aggregate Deductibles

Vincent P. Connor

Abstract

The use of an Annual Aggregate deductible by a reinsurer can cause inconsistencies in loss development and incorrect IBNR reserves. This paper describes how AAD business can be added to non AAD business with the combined used to select loss development factors and estimate IBNR reserves when using a chain ladder or Bornhuetter/Ferguson method. The inclusion of similar AAD and non AAD business in loss development triangles increases the credibility of the loss development factors.

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LOSS DEVELOPMENT AND ANNUAL AGGREGATE DEDUCTIBLES

The reinsurer that uses Annual Aggregate Deductibles (AADs) needs to make some adjustments for reserving if it is using a Loss or Chainladder method (reported loss x (to ultimate factor minus one)) or an Expected Loss or Bornhuetter/Ferguson[1] (B/F) method (premium x loss ratio x percent of loss unreported) to develop IBNR reserves.

This paper will describe how, using certain modifications, AAD business can be added to non AAD business with the combined used to select loss development factors and estimate IBNR reserves. The topic will be covered in four parts:

1. AAD/Reinsurance background.
2. AAD and Chainladder IBNR.
3. AAD and B/F IBNR.
4. AAD and Indicated Loss Ratios.

AAD/REINSURANCE BACKGROUND

Quota share reinsurance provides the benefits of reinsurance on all risks. Since there is a cost to reinsurance, and most ceding companies would like to minimize costs, some insurance companies look for other types of reinsurance that will meet their needs but lower their costs.

1. Bornhuetter, Ronald L, and Ferguson, Ronald E. "The Actuary and IBNR", PCAS Vol. LIX 1972, p.181.

One approach is to use surplus share reinsurance. For policies under a certain retention or line the company keeps 100%. For policies over the retention the company cedes on a share basis the amount over the retention or the surplus amount. For example, if the retention is $100,000 and the policy limit is $300,000 the company cedes 66 2/3% (($300,000 - $100,000)/$300,000) of this policy and recovers 66 2/3% of every loss. Small policies with limits of $100,000 or less cede 0% (($100,000 - 100,000)/100,000). The result is that the company has share reinsurance but just on the larger policies.

Another approach is to use excess reinsurance, which applies only to the larger claims. A $150,000 retention means the insurer pays for claims under $150,000 and also the first $150,000 of larger claims. Generally one pays less for excess reinsurance than for surplus share or share reinsurance.

The reinsurance premium can be further lowered if the ceding company has an annual aggregate deductible with its excess reinsurance. The ceding company might be willing to keep the first million dollars of excess losses per accident (or fiscal) year. This will then be a one million dollar annual aggregate deductible and the premium will be lower with an AAD because fewer losses are paid by the reinsurer. Including an AAD will normally result in increased volatility as the more predictable losses are being excluded.

There is a difference between an AAD and the usual deductible that is applied to an individual claim. A deductible, for example a $250 Auto Physical Damage deductible, applies to each claim. An AAD applies to all claims above the retention until the aggregate deductible is reached. Table 1 shows the effect of an AAD on a series of five cases in chronological order.

Table 1

Cover $900,000 Xs $100,000
AAD of $1,000,000

Loss #	Loss	Ceding Company Retains	Ceding Company AAD Contribution	Eroded AAD to Date	Reinsurance
1	$500,000	$100,000	$ 400,000	$ 400,000	0
2	50,000	50,000	0	400,000	0
3	200,000	100,000	100,000	500,000	0
4	900,000	100,000	500,000	1,000,000	300,000
5	400,000	100,000	0	1,000,000	300,000
Total	$2,050,000	$ 450,000	$1,000,000	$1,000,000	$ 600,000

AAD AND CHAINLADDER IBNR

The AAD can cause an inconsistency in the loss development triangle for the reinsurer because no case losses are incurred until the AAD is eroded. The AAD business may contribute no losses in, say, the first two years, followed by a sudden increase in activity in the third year. How can this inconsistency be addressed?

One approach would be to group together in a triangle similar AAD business. This will work if there is enough similar business to be credible and if enough years are available to select loss development factors. Given that AADs are written on a fiscal or policy year as opposed to accident year basis, the varying sizes of AADs, the number of lines that might be covered, etc., this is not usually a very practical solution. If this approach is taken, data must be grouped so that the AAD is effective the first day of the year e.g. you should not look at an AAD that covers fiscal year on a calendar year basis.

Another approach is to include the business subject to the AAD with the non AAD business. We would handle the loss as if it were just excess, that is, include the ground up or eroded AAD loss in the triangle with non AAD business and make adjustments as appropriate. If losses gross of or before the AAD are consistent (as to loss development) with those without an annual aggregate deductible, then loss development factors can be selected in the usual manner from the combined data. This approach assumes that the computer system capture losses gross or before the AAD.

How the company structures its reserve segments can influence how AADs are handled. If all of a contract is in one segment it is easier to handle the AAD than if lines are in different segments and the AAD covers multiple lines.

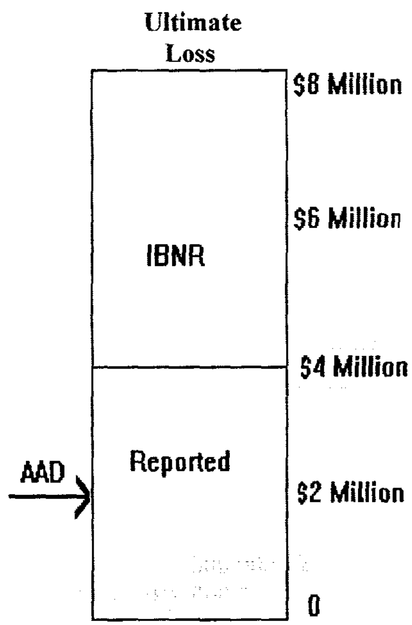
Let's assume that we are using a Chainladder method to develop IBNR. That is, we are taking the reported losses by accident year and multiplying by the to ultimate development factor less one to determine IBNR. Let's also assume that the reported losses gross of the AAD for a particular accident year are $4 million, the ultimate gross of AAD losses are $8 million (the to ultimate development factor is then 2.0), and the AAD is $2 million. This is displayed on Figure 1.

If we follow the Chainladder method formula, we will develop an IBNR (assuming no AAD) of $4 million. That would be the reported losses of $4 million multiplied by the to ultimate factor minus one of 1.0 (2.0 - 1.0 = 1.0). If there is an AAD of $2 million and we use the net of AAD reported losses the IBNR calculated would be $2 million x (2.0-1.0) or $2 million. This is wrong because we are applying factors developed from losses gross of the AAD to losses that are net of the AAD. Since we normally would not have factors net of the AAD, the approach is to make the calculation gross of the AAD and then adjust, if necessary, for the AAD. In this case no adjustment is necessary for the AAD. The IBNR is $4 million.

Figure 2. shows a chart of the same accident year, only evaluated earlier, i.e. there are fewer reported losses. Gross of the AAD, there are $1 million of reported losses, the to ultimate factor is 8.0 and the Chainladder method IBNR is $7 million (gross of AAD). We can see that if we have ultimate losses gross of the AAD of $8 million, the most the reinsurer is going to pay is the $8 million of ultimate loss less

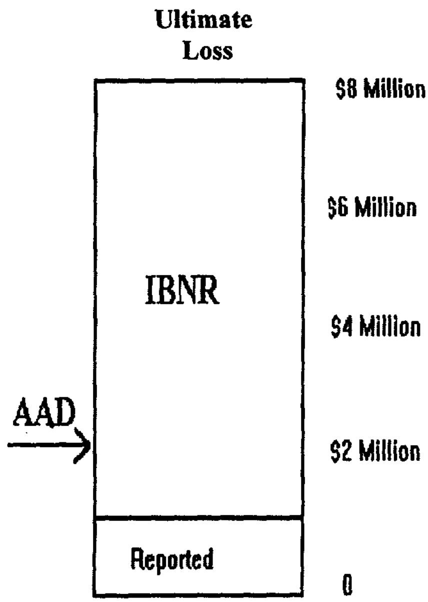
the $2 million of AAD or $6 million. In this case the correct IBNR net of the AAD, would be $6 million not the formula reserve of $7 million.

Figure 1



Ultimate Loss	$8,000,000
Reported Loss	$4,000,000 (Before AAD)
AAD	$2,000,000
To Ultimate Factor	2.0 ($8,000,000 ÷ $4,000,000)

Figure 2.



Ultimate Loss	$8,000,000
Reported Loss	$1,000,000 (Before AAD)
AAD	$2,000,000
To Ultimate Factor	8.0 ($8,000,000 ÷ $1,000,000)

This example assumes that the ultimate losses are known and larger than the AAD. There actually might be a distribution of possible ultimate loss results. In theory, we should be subtracting the expected value of the AAD from the expected value of the ultimate loss. The expected value of the AAD will be less than the full AAD if there is the possibility that the AAD would not be fully used. This paper deals with a fixed ultimate loss rather than an expected value.

If the AAD only applies to one line of business in a contract that covers multiple lines of business just the AAD line can be handled separately. A multi-year contract that has an AAD that spans a number of years can be included for loss development purposes, but handled separately to develop the IBNR.

The possible relative sizes of AAD, reported loss gross of the AAD and Ultimate loss before AAD are depicted on the line graphs on Figure 3. There are six ways to order three variables by size assuming none of the three are equal to another.

The first two situations discussed assumed the ultimate loss is larger than both the AAD and reported loss. It is possible for the AAD to be larger than the reported and ultimate loss. This is shown in situations 3 & 4 on the line graphs on Figure 3. In this case the IBNR would be zero as the AAD eliminated all losses.

It is also possible for the reported loss to be larger than the ultimate loss and AAD (situations 5 & 6 on Figure 3). This means that there will be negative development. In situation 5, the company has reported losses of R which have been partially offset by the AAD of A. The net of AAD reported losses are R minus A. Since ultimately there will be no losses as the AAD is larger than the ultimate loss, the IBNR should bring the booked loss to zero and this negative IBNR amount would be A - R.

Figure 3.

Situation		IBNR
1. A<R<U	0 A R U	U - R
2. R<A<U	0 R A U	U - A
3. R<U<A	0 R U A	0
4. U<R<A	0 U R A	0
5. U<A<R	0 U A R	A - R
6. A<U<R	0 A U R	U - R
	0 Dollars	

A = AAD

R = Reported Loss

U = Ultimate Loss before AAD

In situation 6 when the AAD is less than the ultimate loss, and the ultimate loss is less than the reported loss, the net of AAD reported loss is the amount between A and R or R - A. Since the ultimate loss is less than reported loss the IBNR is the amount between U and R or U - R.

As the situation with the ultimate loss less than the reported loss (negative development) is unusual, I will not consider it further (situations 4, 5 and 6).

In general assuming positive development, one approach to develop net of AAD IBNR is to make two calculations, and use the smaller IBNR of the two but not less than zero. One calculation is to develop the formula IBNR gross of the AAD. This gives the correct (and smaller) IBNR in situation 1 (the formula IBNR is equivalent to U-R).

The second calculation is to develop the ultimate loss gross of the AAD and subtract the AAD (minimum IBNR of zero). This gives the correct (and smaller) IBNR in situation 2. There is a minimum IBNR value of zero because in situation 3 the ultimate loss minus the AAD is negative, but the true IBNR is zero. The two calculations can be expressed as:

$$\min (U-R, (\max (0, U-A))).$$

When the AAD equals the case reported, both calculations produce the same IBNR.

AAD AND BORNHUETTER/FERGUSON IBNR

If a B/F loss method IBNR with premiums (as a measure of exposure) is being used (premium x loss ratio x percent of loss unreported), the same general approach will apply, i.e. develop the IBNR gross of the AAD and make adjustments as appropriate by making a second calculation. As the reinsurer collects premium to pay losses net of the AAD, the net of AAD premium must be increased in order to develop gross of AAD IBNR. If the business is being written at an 80% loss ratio, we can add the AAD divided by .8 to the premium. This approach is for loss reserving. For pricing the probability of the AAD being completely used, the risk load, etc. would be considered.

In both examples (Figures 1 and 2), at an 80% loss ratio, the reinsurer would have received \$7,500,000 of premium (\$6,000,000 ÷ .8) and expected to pay losses net of the AAD of \$6 million (\$7,500,000 x .8). Since our calculations are gross of the AAD of \$2 million, the premium must be adjusted. The increase is the AAD of \$2 million divided by .8 or \$2,500,000 for a total premium of \$10,000,000 (\$7,500,000 + \$2,500,000).

In the first situation discussed (reported loss greater than AAD - Figure 1) the first IBNR calculation would be:

\$10,000,000	x	80%	x	50%
premium	x	loss ratio	x	% of loss unreported

or \$4 million where 50% = (2-1)/2 and 2 is the LDF.

The second calculation is ultimate loss of \$8 million minus the AAD of \$2 million which equals \$6 million. The IBNR is the lower value of \$4 million or

min (U-R, (max(0, U-A))) =

min (8-4, (max(0, 8-2))) =

min (4,6) = \$4 million

Under the second situation discussed (reported loss less than AAD - Figure 2) the first IBNR calculation is

\$10,000,000 x 80% x 7/8

or \$7 million where 7/8 = (8-1)/8 and 8 is the LDF.

The second calculation is ultimate loss of \$8 million minus the AAD of \$2 million, or \$6 million. The IBNR therefore is the lower figure of \$6 million or

min (U-R, (max (0,U-A))) =

min (8-1, (max (0, 8-2))) =

min (7,6) = \$6 million

We can express the two calculation AAD adjustment rule a different way. We can just use the formula IBNR, however, when the reported loss is less than the AAD we will subtract the unused AAD from the formula IBNR. This can be seen by looking at the Figure 3 line graphs. In our example (Figure 2) \$10,000,000 x .8 x (7/8)=\$7,000,000 minus (AAD of \$2,000,000 minus reported loss of \$1,000,000) = \$6,000,000. Due to situation 3 the minimum IBNR should be zero. The formula is

max (0, U-R-max(0, A-R))

In the first situation discussed (reported loss greater than the AAD - Figure 1) no adjustment is necessary as reported losses are greater than the AAD.

AAD AND INDICATED LOSS RATIOS

When using a B/F loss method, one might initially use a loss ratio based on conversations with the Pricing Actuary or the Underwriter or based on previous accident year indications. As data becomes available, it is appropriate to develop an indicated ultimate or burned loss ratio that incorporates loss development factors to assist in selecting the loss ratio for the B/F method.

In the second situation discussed (reported loss less than AAD - Figure 2) we can develop an indicated ultimate loss ratio of 80% ($1,000,000 of reported loss x 8.0 the to ultimate factor ÷ $10,000,000 of AAD adjusted premium). The indicated loss ratio for the first situation (reported loss more than AAD - Figure 1) is the same 80% or $4,000,000 of reported losses x 2.0 (the to ultimate factor) ÷ $10,000,000 AAD adjusted premium.

Just as we are able to add the AAD business in the triangle to the non AAD experience, we can also develop indicated loss ratios by including the AAD business. One problem with mixing AAD with non AAD business in the indicated loss ratio is that there can be an inconsistency between the loss ratios for the AAD contract and the non AAD business. If the two do not have the same loss ratios, doing the IBNR combined and separately for AAD and non AAD can produce different results. Because of this doing a separate loss ratio calculation for the AAD contracts is preferred.

In general, if losses gross of the AAD are captured, the appropriate IBNR reserves can be developed net of the AAD by the use of two calculations and by following certain simple rules. The combination of similar AAD and non AAD business in loss development triangles increases the credibility of the loss development and IBNR indications.

An Integrated Pricing and Reserving Process for Reinsurers

by Leonard R. Goldberg, FCAS, and Joseph LaBella

AN INTEGRATED PRICING AND RESERVING PROCESS FOR REINSURERS

ABSTRACT

In today's market of increased competition, more complex reinsurance contracts and tightening (or should we say frightening) profit margins, actuaries are increasingly being called upon to improve their pricing and reserving practices concerning individual accounts as well as aggregate books of business. Increased understanding of that business is critical to continued success for both reinsurers and their clients. The purpose of this paper is to describe a framework for an integrated pricing and reserving process on an individual risk basis. Utilizing this framework, increasing levels of sophistication and knowledge can be brought to bear, risk by risk, on understanding a reinsurer's book of business.

AN INTEGRATED PRICING AND RESERVING PROCESS FOR REINSURERS

I. SUMMARY

The process described herein is dependent upon having significant actuarial and underwriting resources available to analyze the risk on every individual contract that is eventually written and put on the books. As well, this process relies heavily on a collaborative environment where underwriting, actuarial, claims, contracts, legal and accounting all have a significant role to play in understanding and evaluating risk. The concept of this paper is to explain a structure which a reinsurer (or a large accounting department of a primary company) can use to gain a thorough understanding of their book. [1] The focus is on the process, not techniques.[2]

Each contract is individually priced by a team that is centered around an underwriter and an actuary. The result of this analysis is an expected loss ratio, an expected loss development or lag pattern (note: a lag for a contract is the expected percentage of losses that have emerged. For example, a lag of 20% means that it is

1 A second concept behind this paper, proposed by so-called friends of the authors, is to ensure a trip to Bermuda to present this paper. The authors eschew this as a basic concept of the paper.

2 We will leave techniques to more capable actuaries. The reader may find, however, that these sophisticated techniques can be layered into the framework described herein.

expected that one fifth of the losses have emerged to date. The lag is also equal to the inverse of the cumulative loss development factor for the contract.), an expected payout pattern, an estimate of an aggregate distribution of result, a vector of committed capital over the lifetime of the deal and an estimated return on equity (ROE) for the deal. The pricing information that is developed is then used as the starting point of the reserving and risk analysis processes.

The reserving process begins by using the expected loss ratio, incurred lag and payout patterns developed from the pricing process. Every quarter, each contract is reserved either to its expected loss ratio, the Bornhuetter-Ferguson method, or other appropriate methods. After the data is assembled, staff from various professional departments meet and agree on expected ultimate loss ratios for each major contract. Over time, enough individual contract information is generated to provide feedback to the underwriters and pricing actuaries as part of their renewal process. Similarly, aggregate data is developed to help analyze future contracts.

The aggregate distribution that is used to price each contract is utilized in the reserving of each account. In particular, accounts with significant loss sensitive features are heavily dependent on the shape and variability of the aggregate distribution. Individual risk reserving also can provide consistent answers for accruals on contingent commissions and profit commissions.

The detailed information that is gathered from each contract also allows the company to assign capital to each contract for the current year, and for all years into the future where there is still risk as to the ultimate result. Using both actual data and simulation techniques, capital allocation formulas are continuously refined. Each quarter, a profitability study is produced showing profitability by contract, client company, line of business, and strategic business unit (SBU). In addition to "traditional" accounting data, the study aggregates vital statistics such as mean time to payment of losses, capital utilized, ROE, interest rate assumptions used, and performance vs. initial benchmarks (actual vs expected losses, ROE, etc).

As levels of sophistication continue to increase, more interesting analyses can be accomplished. These would include items such as estimating correlations between risks, estimating correlations between liability and asset accounts, defining drivers of economic results that affect the whole book (i.e.. interest rates), and determining an optimal debt to equity mix for the corporation.[3]

The most interesting fact is that the drivers of this type of analysis are not sophisticated mathematical techniques, but basic actuarial blocking and tackling. This includes good data from client companies, high levels of data quality for what is input into the reinsurer's systems, actuarial pricing software that allows for

[3] In this case, there is almost perfect correlation between the importance of this work (very) and the impossibility of this work (very).

experience rating and exposure rating, and a generally agreed upon ROE methodology that is understandable by all parties involved in the process. Also necessary to the process are ways of linking pricing data to reserving and profitability data to provide continuous improvement in analysis and assumptions. [4]

II. CORPORATE PHILOSOPHY AND THE ANALYSIS OF RISK

Not too many years ago, reinsurers operated largely by spreading risk. Shares of individual contracts tended to be relatively small, and actuarial involvement in the pricing of contracts was infrequent. As reinsurers and their clients have become more sophisticated, profit margins have been squeezed, and reinsurers have to work much harder to find structures that both satisfy client needs and provide an opportunity for adequate returns to capital. The concepts and techniques contained in this paper are contingent upon analyzing every risk in great detail. Therefore, these methodologies can only be well utilized by a company where the corporate philosophy matches up reasonably close to the following:

A. ***Be a lead reinsurer*** - While it is not necessary to always be a lead reinsurer, generally being the lead provides greater insight into a contract. There is more opportunity to talk to client company management about underwriting philosophy,

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[4] There are unlimited ways to do this, and there are always improvements needed. This plus the work on correlations (see Footnote Three) should keep the authors employed for thousands of years.

claims management philosophy, strategic direction and the like. Individual contract reserving and pricing doesn't do a lot of good if you can't really understand what the client is all about. Also, being the lead allows the reinsurer to work closely with the broker and client to create a structure that will maximize outcomes for both parties.

B. ***Underwrite Large Lines*** - The cost of understanding the risks in great detail are significant. These costs can only be justified if the reinsurer and the client are both willing to allow for large lines. This can be difficult, as often reinsurers are reluctant to take large lines as a major loss could seriously impact results. Similarly, clients are often unwilling to give a reinsurer a large line as sometimes they feel this means they may lose some control over the account. The best way to handle all of these issues is to develop a strong and trusting relationship between the reinsurer and the client.

C. ***Collaborative Environment*** - Individual contract analysis cannot be left to just the actuaries. There has to be a significant amount of input from all professional units of the company. Underwriting audits, claim audits and accounting audits need to be integrated into the pricing and reserving process. Contract language needs to be analyzed. Emerging issues of liability need to be explored. Each contract also has to be thought of as part of a relationship with the client, perhaps spanning many underwriting units and areas of expertise. There needs to be significant and varied client contact that is communicated to all members of the team. All the knowledge gained in the collaboration of the various professional disciplines add to the value of both the pricing and the reserving of each individual contract. [5]

III. PRICING INFORMATION

For every contract that is bound, a significant amount of information is collected through the pricing process. Even when contracts aren't bound, significant components of the following data are still available and can be added to the data warehouse. Basic information that is passed through into the corporate database on all contracts includes the following:

A. ***Expected Loss Ratio (ELR)*** - ELR's are developed using both experience and exposure rating. Data is obtained from the client company, and can be augmented with data from ZRC's proprietary database, or ISO, RAA, or the NCCI. For risks with property exposure, the ELR must have a catastrophe and non-catastrophe component.

B. ***Aggregate Loss Distribution*** - For each contract, an aggregate loss distribution needs to be established. The aggregate loss distribution describes the

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[5] The amount of knowledge that is required to do this well is humbling. The authors are comforted in knowing that many others are responsible for adding to this body of knowledge. In addition, the authors acknowledge the huge value that is added by the others in the process.

probable loss scenarios that underlie the selected ELR. The aggregate distribution performs two functions. The first is to help estimate the riskiness of the contract and hence, the amount of capital required. The second is to help in the evaluation of different contract features such as loss corridors, contingent commissions or retrospective rating. These two issues are highly interactive. Starting with an aggregate loss distribution that describes the underlying loss process of a contract, the team can overlay different contract structures to analyze how the riskiness (required capital) and profitability changes. Aggregate distributions can be developed using a number of pre-packaged products. They also can be developed directly from the company's historic data, or selected by the team analyzing the contract.

C. ***Loss Development Factors (Lags)*** - Incurred lags (expected percentage of losses emerged over time) should also be developed as much as possible from client company data. Lags have multiple uses in this process. They are a critical element of the experience rating approach used to derive an expected loss ratio. They are also needed as part of the Bornhuetter-Ferguson process which will later be used as part of the reserving methodologies for the contract. Lags or loss development factors are often incorporated in a contract that has loss sensitive elements. They can also be an important determinant of the cash flow for the various features of a contract, such as when a contingent commission will be paid or when a retrospectively rated contract will generate additional premium (to the extent these items are measured from incurred losses).

D. ***Payout Pattern*** - Estimating a payout pattern is critical to understanding the risk of a reinsurance contract for many reasons. It provides the basis of the timing for loss cash flows which allow analysis of the present value of any contract. In aggregate, payout patterns are used to develop the duration of the book of liabilities and help set asset management policy. Payout patterns can also provide information on the sensitivity of a contract to inflation. Note that the payout pattern and the incurred lags need to be internally consistent for each contract. It is also important to think about the relationship of the payout pattern to the aggregate loss distribution for an individual risk. For some types of contracts, the "bad" end of the aggregate loss distribution may be more likely to arise due to a spate of early shock losses. Other contracts may be more influenced by long-emerging losses. All other things being equal, the former has a wider distribution of net present values than the later; they both have the same nominal aggregate loss distribution and expected payout, but the first contract has more downside on an economic or net present value basis.

E. ***Analysis of ROE*** - Of course, there needs to be some sort of metric that each company uses to determine whether they are generating appropriate returns from each contract. Rather than just using this metric as an underwriting decision tool, it is possible to capture information from this analysis in the corporate database. An example of this is our company's ROE (return on equity) methodology. The estimated expected return is the weighted average of the present values of all the estimated after tax cash flows from the contract over all of the points of the aggregate distribution. Equity is then allocated to each contract based upon the downside risk of each contract (estimated from the aggregate loss distribution), the mean time to loss payment, the line of business underwritten as well as other factors. The estimated required capital for any contract is the present value of all future estimated capital commitments until the risk is extinguished on the contract.

A few more moments on ROE are necessary. The contracts that most reinsurers write have a varying degree of risk. And the causes of risk vary from contract to contract. Also, individual contracts can be correlated with other contracts such that potential for adverse results can increase dramatically. We believe that it is critical that any methodology for analyzing profitability contain a consistent way of measuring risk. We relate risk to capital need, wherein riskier deals require more capital and therefore a higher dollar return to preserve the ROE. There are other metrics that can be used effectively.

From the ROE process, we capture information on the present value of the cash flows under each scenario, the weighted average present value of the cash flows, the average interest rate used in the analysis, and a vector of required capital needed annually until the risk of the deal is extinguished. Creating this information and storing it in the corporate database allows for analysis of capital usage and expected vs. actual investment returns. It also allows for continual updating of capital allocation process assumptions.

We again stress that there are many metrics that can be used to help set pricing and profit targets, and there is nothing magical about how we define ROE. Two points are critical. First is that the methodology is widely understood by underwriters and actuaries. If there is no buy-in to the metric, it becomes another hurdle to be crossed rather than a value-added exercise. This argues for some simplicity of approach. The second point is that once the critical drivers of the metric are established, they should be captured and integrated into the databases that are used to manage the business. This information is at least as important as the accounting information that is collected and should be held (at least) to the same data quality standards.

IV. THE RESERVING PROCESS USING INDIVIDUAL CONTRACT DATA

With all of this data available, the reserving process by individual contract is relatively straightforward. Keep in mind that this reserving process is only one methodology and multiple methodologies can and should be used when arriving at a range of reasonable results. The individual contract method, with its intense focus on the "trees" sometimes can cause us to lose touch with the forest. Analysis of aggregate data is still the only way to view some over-arching trends such as a change in case reserve adequacy or a speed-up/slow down in claim payments.

A. ***Initial Assumptions*** - Generally, the reserving process starts by capturing the ELR on an ex-catastrophe basis and the appropriate incurred lag and payment patterns for each contract. All contracts are grouped into reserve "cells" for analysis. For larger contracts, a separate reserve cell is established to individually reserve the contract. For smaller contracts, multiple contracts with similar characteristics are combined into a reserve cell. Typically, such characteristics may be class of business (casualty vs. property, for example), line of business (auto liability, general liability, etc), quota share vs. excess of loss, high vs. low layer, etc.

We refer to these multiple contract groupings as aggregate cells. The initial ELR of an aggregate reserve cell will be the weighted average ELR of all contracts in the cell. The initial incurred lag and payment patterns for an aggregate cell will be selected from some subset of the contracts that enter into that cell, or sometimes by using other information (ISO, NCCI, RAA, ZRC's proprietary database).

As contracts are renewed in subsequent years, the corporate actuarial unit should review the ELR's and the lags for consistency with old years. Any major differences between contract years should result in further discussions among the reserving actuary, the pricing actuary and the underwriter to understand these differences.

Sometimes the contract terms change materially, resulting in a significant change in the ELR, the lags, or both. This could be due to a difference in price, layer or terms between years. In these cases, material differences between years can happen. Other times, new or updated information comes to light which materially changes the analysis, resulting in revised expectations for the current period. A common example is when more information (claim count triangles, pricing history, etc) is available in the renewal package than was available when the contract was initially priced. This additional information can greatly change expectations of a contract's profitability for both the current and prior years. When this is the case, the ELR's for old years are often updated to also reflect the new information.

As an example, let's say that we bound a new commercial multi-peril contract. The ceding company was not yet set up to supply data triangles, so the pricing analysis relied upon an exposure rating analysis and an ISO lag pattern. The overall pricing analysis came up with a 65% ELR. When the renewal package was received one year later, paid and incurred loss triangles were included. The analysis of the renewal contract resulted in an 85% ELR for the current year. After reviewing these results, and the company specific data that drove them, it was concluded that it was the historical data, not necessarily the latest twelve months activity, that drove the new loss ratio pick. If we would have had this data available when we priced the original contract one year ago, the ELR would have been a 75% after giving weight to both experience and exposure rating methodologies. In this case, we would change the prior year's ELR to reflect this new historical information.

In another example, suppose that a similar contract, originally priced at a 65% ELR, had run adversely over the first twelve months. Assume that lots of good information has been available on the contract since inception. Even though the estimated ultimate loss ratio, via the Bornhuetter-Ferguson method, was now 75%, we would not necessarily change the original 65% ELR that feeds that methodology. We have no reason to believe that the contract will not run a 65% loss ratio on a going forward basis.

As the information comes through the underwriting process, the corporate actuary also has responsibility to look for data quality and consistency. Is the payout pattern faster than the incurred lag pattern? Are the patterns very dissimilar to other treaties in the same line of business? Has the expected loss ratio dropped as industry pricing has weakened? In the pricing/reserving feedback process, the pricing actuaries each search very deeply into a smaller number of contracts, and the corporate actuaries spread their time over a larger number of contracts. The reserving actuary is usually in the best position to provide such reasonability checks.

B. ***Quarterly Updating Process*** - Each quarter, earned premium and case incurred losses are updated for each contract. Generally for our purposes, one of three methodologies (loss ratio, incurred Bornhuetter-Ferguson, paid Bornhuetter-Ferguson) are selected. For very green and for very long-tailed contracts, it is often advisable to stick with the initial ELR as the estimated ultimate loss ratio for a period of time (12-24 months), rather than reacting too early to good or bad loss development news. For contracts that are more mature and for shorter tailed contracts, the incurred or paid Bornhuetter-Ferguson process using the initial ELR and pricing patterns are generally utilized. Information based on either of these methodologies, along with more detailed claims information, are provided to the SBU managers, underwriters, pricing actuaries, claims professionals, and accountants each quarter. As a group, these individuals along with the corporate actuarial staff will try to come to a more complete understanding of how each major contract and each aggregate reserve cell is performing. A large amount of time and effort is expended each quarter in this process. [6]

C. ***Multiple Reserving Methodologies*** - Individual contract reserving also allows us to experiment with different methodologies. These different methodologies can help in formulating a range of reasonable estimates. In addition to the standard methodologies (loss ratio, incurred Bornhuetter-Ferguson, paid Bornhuetter-Ferguson, incurred loss development, paid loss development) there is a bit more that can be done when reserving by individual contract. For example, we have calculated expected ultimate losses using a loss ratio methodology for lags less than 10% and an incurred Bornhuetter-Ferguson methodology for the rest. We

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Much of this time is expended creating user friendly reserving exhibits (an oxymoron?) that are comprehensible by those outside of the Actuarial profession. In addition, written summaries of the quarter's indications, trends, and oddities are very necessary icebreakers to the review process.

have used loss development capped and cuffed by plus or minus 25% of the Bornhuetter-Ferguson estimate.

One can also experiment with varying the parameters that feed the methodology of choice. We tried developing a range around the expected loss ratio, using the aggregate loss distribution. Another possibility is developing slow and fast lag patterns around the selected pattern, to develop a range of indications. The flexibility to try something new is a nice benefit of this process. You may find that different methodologies are necessary (produce more accurate results) for certain contracts or lines of business. You may also find that developing an indicated range of reserves helps validate (or call into question) your methodology of choice.

D. ***Feedback Loop for Renewals*** - The result of the quarterly process is to have a concensus-built estimate of how each major contract and many of the smaller contracts are performing. A major benefit of this type of process is that the IBNR should be appropriate for each contract (with some exceptions), and is not an allocation. In addition, the entire company has a buy-in to every IBNR number as each number was arrived at through a group process rather than by a corporate actuary sitting in his or her office. As experience matures across all contracts, it then also becomes possible to aggregate data to create pricing parameters such as loss development factors, trend factors and excess factors for your company's specific book of business.

E. ***Mapping of Underlying Exposure*** - It is extremely important to be able to map the exposure spread of every contract. This will affect how earned premium is allocated to accident year and how lag factors are interpolated (see next section). A simple drawing of a square or a parallelogram, is often sufficient to describe the loss occurring or risk attaching base case.[7] You then need to consider other pertinent facts in mapping the contract's exposure. Is there an unearned premium portfolio on the front end of this contract? At expiration, does the contract cut-off or run-off? Are the underlying policies written evenly throughout the life of the contract, or is there some seasonality to the ceding company writings? Also, are all underlying policies one year in length, of variable length, etc? Does the underlying exposure itself contain some type of seasonality? For example, the winter months may contain more than their proportional share of Homeowners' exposure. See Appendix A for practical examples of this process.

One should not ignore the premium earnings pattern that is implied by how the accounting department actually books the earned premium (which is mostly based upon how the ceding company reports written and unearned premium to the reinsurer). These bookings will map out the actual earnings and exposure patterns for the contract. We need to begin to tap this source of valuable information in our shop.

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7 *Losses Occurring:* A contract which covers all accidents that occur (or are reported, in the claims made case) during the contract period. The exposure looks like an accident year box.
Risks Attaching: A contract which covers all accidents that result from underlying policies that incept during the contract period. The exposure looks like a policy year parallelogram.

F. ***Interpolation of Lag Factors*** - Another special topic that needs to be addressed is interpolation of lag factors. Most of the data that is received through the pricing-reserving loop is accident year in nature. For a January 1 losses occurring contract, usually no adjustment is needed. However, for contracts that are written on a risks attaching basis, or if there is a portfolio in, or if the effective date lands on other than a quarterly point, interpolations of existing lag factors will be required. Appendix B details a lag interpolation method and shows some of the situations and calculations in greater detail. Although interpolating the lags can be complex, this process adds significantly to the understanding of each contract's results.

There is a further special case for quota share business. In order to completely match reported loss and IBNR on an individual contract, the interpolation has to be to the date of the last bordereau report, not the date of the reserve study. Quota share contracts are generally reported 30, 60 or 90 days in arrears. So using the date of the reserve study would understate ultimate loss. Also, some loss reports are not received by the end of the quarter, and a further adjustment is needed. For aggregate quota share cells, we assume all contracts are 90 days in arrears (we have tested this assumption on occasion, and it pretty much holds true). For excess accounts which report individual losses, we assume reporting is current and therefore consistent with the reserve study date.

V. RESERVING FOR CONTRACT FEATURES

One of the major benefits of analyzing each contract individually and creating an aggregate distribution for each contract individually is in valuing special contract features. We differentiate between the value of a feature at expected loss, versus the expected value of the feature. We believe the latter method is more accurate.

An example may help. Suppose a contract contains a profit commission feature such that we pay one-half point of commission for every point under a 65 loss ratio. Further, suppose our expectations of the contract's loss ratio is currently a 67. In this case, the profit commission at the expected loss ratio is zero. However, based on the distribution of potential ultimate losses around the 67, the expected profit commission may be 2 or 3 points (because within the aggregate distribution of results, there are possibilities that the loss ratio may fall below a 65, and some weight must be given to those possibilities). This distinction is important, especially when estimating profitability for individual contracts. We price the features based on their expectations and we should reserve for them on a similar basis. We currently do not follow this in our shop, as we book the contractual commission based upon the expected losses. The change is being discussed, though. The following are more detailed examples of contract features:

A. ***Retrospectively Rated Contracts*** - For almost all retrospectively rated reinsurance contracts, the aggregate loss distribution has a wider swing than the minimum and the maximum loss in the premium calculation. As a simple example, suppose there is a contract with a contractual loss ratio of 80 (that is, premium is calculated as losses multiplied by 100/80), a minimum premium of 0, and a maximum premium of $1m. If expected losses are $400,000, based on that point estimate, one would expect earned premium of $500,000. However, suppose the $400,000 is the weighted average of an aggregate loss distribution with a high end of $2m. If we were to take the weighted average of the retrospective premium under all of these scenarios, the resulting premium would be somewhat less than $500,000, and the resulting loss ratio would be somewhat greater than 80 (weight is being given to the possibility that losses may exceed $800,000, with no resulting increase in premium after that point).[8] See Appendix C for more details.

B. ***Loss Corridors*** - In many situations, a reinsured will agree to pay for losses occurring in a certain layer either defined by loss ratio or dollars of loss. Sometimes this loss corridor appears below the expected loss amount and sometimes well above the expected loss amount. By creating an aggregate loss distribution, each loss corridor can be priced and reserved for. This can be a very complex exercise on an individual contract basis. Suppose a contract had an initial expected gross loss ratio of 70 with a loss corridor from 65 to 75. Perhaps based on the aggregate loss distribution, the corridor was worth 3 points, bringing the net loss ratio to 67.

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8 The hardest part of this process is convincing the accounting staff that there is a need to book a loss ratio in excess of the contractual loss ratio. The authors leave this as an exercise for the reader.

Now it's two years later and the estimate of the ultimate gross loss ratio has increased to 85. Clearly the value of the loss corridor is now more than 3 points, but its value is still somewhat unclear. In these situations, we would either have to rely on a subsequent study or be able to develop a re-estimated picture of the complete aggregate loss distribution after two years of emerged experience. Appendix D shows more details.[9]

C. ***Loss Caps*** - Often, contracts are capped either as to absolute dollar amount or to number of reinstatements. Given that the cap is lower than the high end of the expected aggregate loss distribution, the weighted average expected loss ratio net of the cap should be lower than the initial (uncapped) expected loss ratio.

VI. PROFITABILITY AND CAPITAL USAGE

With all of this great data assembled and ready to use in one place, there are certainly many other types of analyses that can be done other than straight reserving. The following lists a few of the analyses that can now be accomplished using the compiled data:

A. ***Profitability Studies*** - The huge amount of work that individual contract pricing and reserving requires really bears fruit when looking at individual contract

[9] The authors would prefer to say that they have developed a theoretically defensible process that re-estimates the aggregate loss distribution over time to facilitate the re-evaluation of loss sensitive features, but they'd be lying. However, this doesn't keep us from trying. All ideas and good wishes are appreciated.

profitability. Since all of the numbers are built bottom-up and built on a consensus basis, each contract's profitability can truly be considered a best estimate. One can then aggregate contracts by underwriting year, underwriter, pricing actuary, client company, client group, line of business, attachment point, etc, etc. The profitability of any slice or aggregation of the business is then also a consensus based estimate. Management should have more confidence in this approach as compared to a top down allocation.

B. ***Basic Capital Analyses*** - It is also now relatively easy to compare the amount of capital that has been allocated for all contracts underwritten to the total capital available in the corporation. If the capital allocation methodology is well accepted by management, then aggregating the capital numbers can give management an idea of whether capital is being under-utilized, fully utilized or over-utilized. Conversely, the comparison between allocated and actual capital can be used to help make refinements to the capital allocation procedures.

C. ***Advanced Capital Analyses*** - With aggregate loss and NPV distributions for each contract, there are many things that can be done to determine the aggregate capital need of a reinsurer. We have recently been playing around with different types of these analyses. Our basic technique is to run simulations by selecting from each aggregate loss distribution. When doing this type of analysis, there are at least two interesting questions that need to be answered, and probably a lot more than that. The first question is one of correlation. How much or how little correlation is there between two accounts in the same line of business? How about two accounts for the same company? How about the same questions with different lines of business and different companies? It is also possible that correlation could vary across the distribution. For example, contracts might be highly correlated at the low end due to low inflation or some other factor, but act more independent at the high end of the aggregate loss distribution. The second question that needs to be answered is one of how the shape of the aggregate loss distribution changes as the contract matures. Generally, the risk of each contract should shrink over time, so one would expect the aggregate loss distribution to grow more narrow with time. But, does it narrow as losses are incurred, paid, or based on some combination of the two? Are there some contracts, such as workers compensation, where the risk is greatly reduced early on as the cases emerge while other types of business, such as excess umbrella, might remain a question mark for many years to come?

We certainly have more questions than answers, but we have attempted some interesting things with the data we have collected. One example is our work on Umbrella. We started with the aggregate loss distribution for each contract that we wrote in a given year. From our ROE model, we extracted the net present value profit from each loss scenario (we "discretize" our aggregate distributions into scenarios). We knew that each contract was somewhat independent, but also correlated due to things like inflation and tort law. We even figured that some losses would be fully correlated as there are sometimes more than one client writing different layers of the same risk. So here's what we did -- we randomly selected pairs of contracts and fully correlated them (i.e. if Contract A was at the 80 percentile of the distribution, then so was Contract B). Then we randomly selected from the fully correlated pairs. We then re-selected the correlated pairs and went through the process again. We ran one million scenarios in this fashion and compared the "bad" end of the distribution with the capital allocated to the contracts. Then we did the same thing with groups of three contracts, four contracts and five contracts to see how much the shape of the aggregate distribution would change.

While we are not sure if we accomplished anything important, or theoretically defensible, we did get comfortable with our capital allocation and we had fun. If we can scrape more time together, having all of this information available should yield more interesting things in the future.

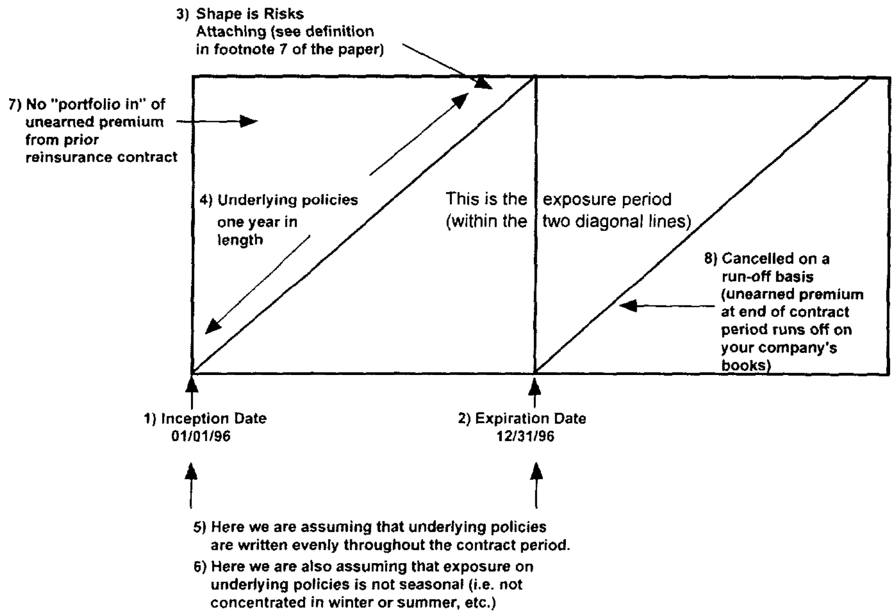
VII. Conclusion

At the core of this paper is the belief that there is real value in an integrated pricing and reserving process on an individual contract basis. Over time, this type of process can lead to a greater in-depth knowledge of clients, the marketplace and profitability. This knowledge should create value for both the client and the reinsurer in jointly understanding the risks of their businesses and in establishing an appropriate price per unit of risk. The process requires everyone's commitment and much hard work.

Call us in ten years, and we'll let you know how (and if) it worked.

APPENDIX A

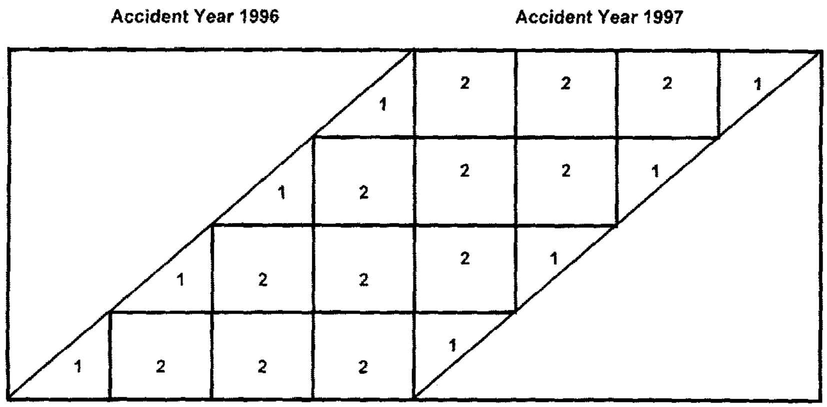
EXHIBIT 1



	PARAMETER	SPECIFICS
1)	Inception Date	01/01/96
2)	Expiration Date	12/31/96
3)	Claims Basis (risks attaching or losses occurring)	R.A.
4)	Length of underlying policies	12 months
5)	Seasonality in writings of underlying policies	no
6)	Seasonality in exposure	no
7)	Unearned Premium portfolio at the beginning of the contract	no
8)	Cancelled on a cut-off or run-off basis	run-off

APPENDIX A

EXHIBIT 2



1) Once the exposure has been sketched, block-off the area within the exposure period.

2) Enter relative weights within the various blocks of exposure (here each full block has a weight of two, and therefore half a block has a weight of one).

3) Count the weights within each quarterly period to determine the exposure within that quarter (for this example, quarters one through four for AY 1996 would be 1, 3, 5, 7).

4) Also take a cumulative count of the weights, quarter to quarter, to determine the percent exposed over time (for this example, quarters one through four for AY 1996 would be 1, 4, 9, 16 or 6%, 25%, 56%, 100%)).

5) All of the exposure information necessary to perform the lag interpolation process is now present (see Appendix B for that process).

APPENDIX A

EXHIBIT 3

Accident Year 1996				Accident Year 1997
2	2	2	2	
2	2	2	2	
2	2	2	2	
2	2	2	2	

	PARAMETER	SPECIFICS
1)	Inception Date	01/01/96
2)	Expiration Date	12/31/96
3)	Claims Basis (risks attaching or losses occurring)	L.O.
4)	Length of underlying policies	12 months
5)	Seasonality in writings of underlying policies	no
6)	Seasonality in exposure	no
7)	Unearned Premium portfolio at the beginning of the contract	yes
8)	Cancelled on a cut-off or run-off basis	cut-off

APPENDIX A

EXHIBIT 4

Accident Year 1996 — Accident Year 1997

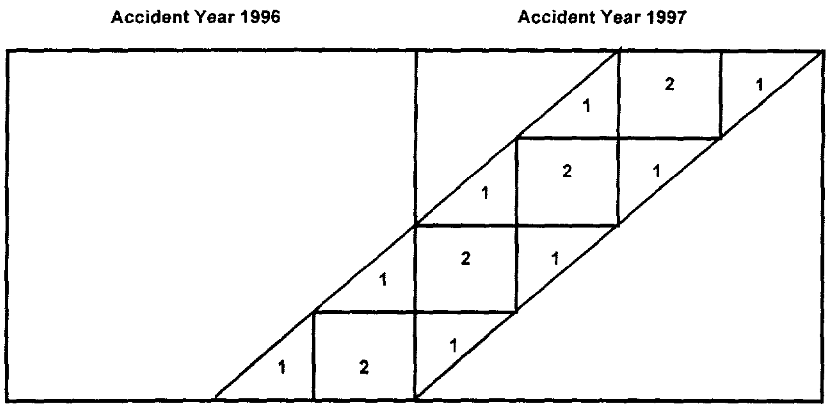
	2	6	2	2	
	2	6	2	2	
	2	6	2	2	
	2	6	2	2	

	PARAMETER	SPECIFICS
1)	Inception Date	10/01/96
2)	Expiration Date	09/30/97
3)	Claims Basis (risks attaching or losses occurring)	L.O.
4)	Length of underlying policies	12 months
5)	Seasonality in writings of underlying policies	no
6)	Seasonality in exposure	yes *
7)	Unearned Premium portfolio at the beginning of the contract	yes
8)	Cancelled on a cut-off or run-off basis	cut-off

* Note that half of this contract's exposure falls during the winter months (first quarter of 1997).

APPENDIX A

EXHIBIT 5



	PARAMETER	SPECIFICS
1)	Inception Date	07/01/96
2)	Expiration Date	12/31/96
3)	Claims Basis (risks attaching or losses occurring)	R.A.
4)	Length of underlying policies	12 months
5)	Seasonality in writings of underlying policies	no
6)	Seasonality in exposure	no
7)	Unearned Premium portfolio at the beginning of the contract	no
8)	Cancelled on a cut-off or run-off basis	run-off

APPENDIX B

Lag Factor Interpolation

To set the correct mood for this process, let's start with a little joke. How many actuaries does it take to interpolate a lag factor? And the answer is -- However many you want. Not a very funny joke, but a very pointed statement. Every actuary seems to have their own interpolation method. None of them are correct, but they're all pretty good estimates. What's being presented here is one of those methods.[10]

We'll begin with an incurred accident year lag pattern at twelve month evaluations (twelve months, twenty-four months, etc.). Next, we'll need to create factors at each quarter point. For evaluations after twelve months, linearly interpolate between twelve month points. Granted this is not exactly correct, since any given loss development pattern is not linear between annual points. But, for this particular method, it's close enough.

For interpolated factors at the first three quarters, we'll be a bit more careful. The loss development curve between zero and one year definitely has a ramping up which we cannot ignore. Think of it as accidents just beginning to happen and loss

[10] The general ideas behind the interpolation methods described herein were taught to one of the authors by Malcolm Handte, FCAS, MAAA. Interpretations of this interpolation method have changed some over time. Resulting lags are usually close to other actuaries' interpolated lags in our shop.

reporting beginning to get into the swing of things. Table 1 contains a graph of five curves that represent more to less severe ramp-ups between zero and one. The most severe is roughly $(1/4)^2$, $(2/4)^2$, $(3/4)^2$, $(4/4)^2$, more parabolic in nature. The least severe is very close to linear. The factors in Table 2 correspond to the points in the five graphs, and represent factors to apply to the twelve month lag point, to create lags at the first three quarters. We tend to choose pattern one for long tailed casualty patterns (where twelve month lag points are, say, 15% or less). Conversely, pattern five goes well with quicker property patterns (with twelve month lags of 65% or greater). Anything between those two can use patterns two, three or four, based upon one's particular judgment.

TABLE 1

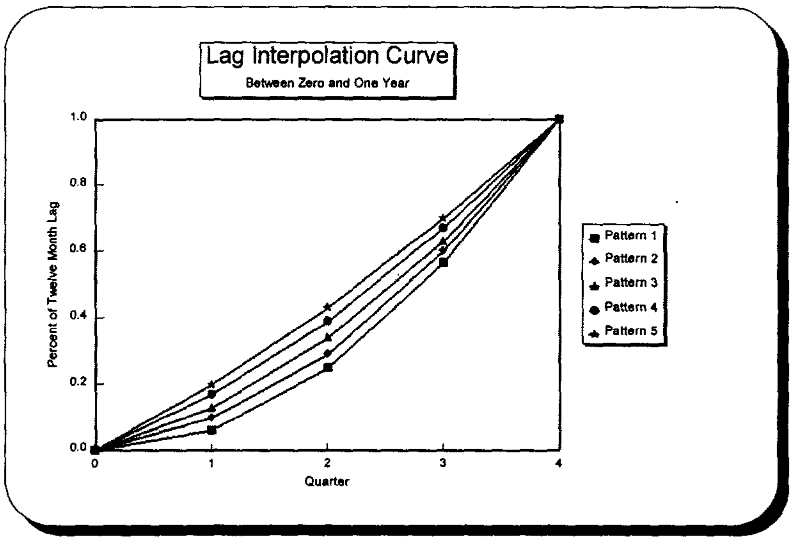


TABLE 2

Quarter	Pattern 1	Pattern 2	Pattern 3	Pattern 4	Pattern 5
1	6%	10%	13%	17%	20%
2	25%	29%	34%	39%	43%
3	56%	60%	63%	67%	70%
4	100%	100%	100%	100%	100%

We now have an accident year lag pattern at quarterly evaluations. Graph it if you like and, if you feel so inclined, smooth some points more to your liking. Now, if all reinsurance contracts were January 1 incepting losses occurring contracts, the task at hand would be complete. Unfortunately, a more general method is necessary to estimate accident year lags for things like a September 20 incepting risks attaching contract.

In order to accurately interpolate lag factors of any given point in time, we must be able to sketch the exposure of the given contract. Refer to Appendix A for this process. Recall, gathered information must include the inception and expiration dates of the contract, the length of the underlying primary policies, any seasonality imbedded in the exposure, whether the contract is losses occurring or risks attaching, whether or not there is a portfolio of unearned premium at the beginning of the contract, and whether the contract is canceled runoff or cutoff.

Once the exposure has been sketched, more necessary information must be gathered in order to complete the interpolation calculation. Table 3 contains the interpolation formula as well as a list of the necessary pieces of information within that equation. For example, if the Evaluation Date (ED) is 9/30/96 and earned premium has been booked (received or accrued) by the reinsurer through this date, then the Premium Information Date (PID) will be 9/30/96. But, if a quota share contract has a one quarter reporting delay (at 9/30/96, the reinsurer has just received the primary company premium statement through 6/30/96), and the reinsurer does not accrue for the missing premium, then the PID will be 6/30/96. If you are dealing with an excess of loss contract, and no loss reporting delay is apparent, then the Loss Information Date (LID) will be 9/30/96. In the case of a quota share contract, the LID will equal the last date through which primary company loss statements have been received.

TABLE 3

Equation: LAG (PID) = LAG [LID - MED + AF] x MF

Parameter	Description
PID	Premium Information Date (usually equal to the Evaluation Date - ED)
LID	Loss Information Date
MED	Mean Exposure Date
AF	Additive Factor
MF	Multiplicative Factor

The Mean Exposure Date (MED) is the average accident date for the premium earned so far. It can usually be determined by viewing the exposure parallelogram and drawing a line through the apparent mean of exposure. For more complicated risks attaching shapes, once the exposure diagram has been properly drawn and weighted, as in Appendix A, we can add up the weights (area under the curve) and divide by two to get the mean. Then we can determine (usually by eye) where this mean falls on the exposure parallelogram.

Let's look at Exhibit 2 in Appendix A and calculate the MED. The contract is 1/1/96 incepting and is risks attaching. If the ED (and PID) is 12/31/96, we need to find the mean area under the triangle between 1/1/96 and 12/31/96. Note that the weights (area under the curve) are equal to sixteen. Half of this is eight. By counting back from the 12/31/96 point, we can see that the MED falls slightly to the left of 10/1/96. Here we can estimate and call the MED 9/15/96 (10/1/96 would also be a fairly good, and easier to handle, estimate).

We can now calculate the relative "age" of the given exposure, as the loss information date minus the mean exposure date. Note that this is the key expression in our search for the appropriate lag factor. Whether we are dealing with a risks attaching or losses occurring contract, and regardless of any other parameters, this relative age of exposure will determine how much time has passed since the mean date of exposure (or the average accident date), and thus how "developed" this exposure is.

A twelve month accident year lag factor assumes a 7/1 average date of loss, or six months of average loss emergence, not twelve months. The loss information date minus the mean exposure date must be increased to reflect this accident year assumption, or else the lag would be understated, yielding expected losses to date that are too low. Therefore, the Additive Factor (AF) is determined based on symmetry -- six months of emerged loss needs a six month AF, three months of emerged loss needs a three month AF, etc. The examples in Exhibit 1 display this.

Table 4 contains the AF values at the first four quarterly evaluations. The quarter four factor is also the factor for all quarters greater than four, and for quarters where the year of your evaluation date is greater than the accident year you are choosing a factor for (hence the exposure in that AY has ended).

TABLE 4

Additive Factor (AF)		**Multiplicative Factor (MF)**	
Quarter	**Factor (months)**	**Quarter**	**Factor**
1	1.5	1	4/1 = 4.00
2	3	2	4/2 = 2.00
3	4.5	3	4/3 = 1.33
4+	6	4+	4/4 = 1.00

The Multiplicative Factor (MF) is necessary to gross up the full exposure AY lag for the portion of exposure "earned" to date. The MF values are listed in Table 4. In the second example on Exhibit 1, the six month factor pulled from the AY lag pattern must be multiplied by 4/2 = 2 since only half of the full AY's exposure (and premium) has been earned as of 6/30/96. Note that the six month lag of 14% (in the footnote on Exhibit 1) means that six months after the inception date of this contract, 14% of the total estimated ultimate losses are estimated to have been reported. In our example, since half of the full exposure has been earned as of 6/30/96, (.14 x 2) = 28% of the six month exposure period's ultimate losses are estimated to have been reported as of 6/30/96.

Exhibit 2 deals with a 10/1/96 incepting losses occurring contract. Note that from year-end to the following first quarter (examples one and two), the AF and MF values jump from the first quarter values in Table 4 to the fourth quarter values. The resulting lags appear to be smooth and quite reasonable to the authors.

Risks attaching cases are explored in Exhibits 3 and 4. Note that we use the same AF and MF factors for the risks attaching cases as we do with the losses occurring cases. This has been challenged by other actuaries in our shop. The MF is easy to question since it is meant to gross up the full exposure AY lag for the portion of exposure "earned" to date. If the losses occurring MF at 6/30/96 is 4/2 = 2 since half of the AY exposure has been earned, then why isn't the risks attaching MF at

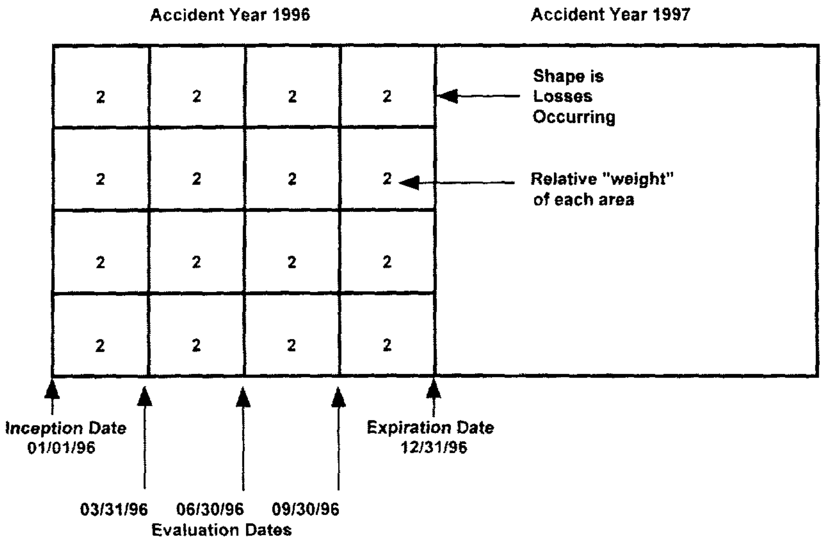
6/30/96 equal to 16/4 = 4 (since only 25% of a risks attaching contract's AY exposure has been earned as of 6/30)? We believe that if the MF for the risks attaching cases were increased in this fashion, then the AF would necessarily have to be decreased or else your answers would be too large. We experimented with a few different sets of factors and really could not get any to work as reasonably well as the current set.

It is a premise of this methodology that the (LID - MED) expression creates an AY type measure of exposure. Whether the contract being considered is losses occurring or risks attaching in nature, the relative "age" of exposure we have calculated is now a general measure of development that can then be used in the overall interpolation formula (which uses AF and MF values that are losses occurring or AY in nature).

The basic "check" of this process is whether or not the resulting interpolated lags appear reasonable, especially when compared to the interpolated lags in the quarters surrounding your evaluation quarter. The risks attaching and losses occurring lags of the same "age" should also appear reasonable (and relatively close together) when compared to each other. It also helps to compare results to those from other actuaries' interpolation methods. This interpolation method should produce smooth and reasonable results.

APPENDIX B

EXHIBIT 1 *



AY 19## Lag @ PID = Lag [(LID - MED) + AF] x (MF)

1) AY 1996 Lag @ 03/31/96
= Lag [(03/31/96 - 2/15/96) + 1.5] x (4/1)
= Lag [1.5 + 1.5] x (4/1)
= Lag [3.0] x (4/1)
= [3 mo. lag] x (4/1)
= [0.06] x (4/1)
= 0.240

2) AY 1996 Lag @ 06/30/96
= Lag [(06/30/96 - 4/01/96) + 3.0] x (4/2)
= Lag [3.0 + 3.0] x (4/2)
= Lag [6.0] x (4/2)
= [0.14] x (4/2)
= 0.280

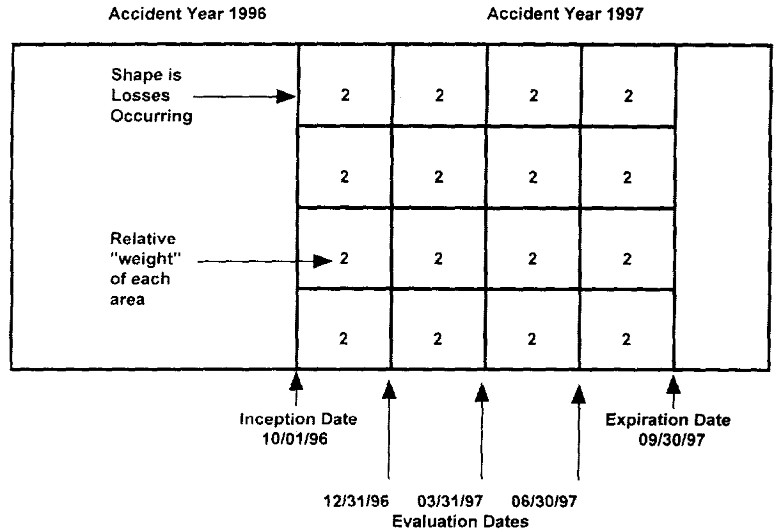
3) AY 1996 Lag @ 09/30/96
= Lag [(09/30/96 - 5/15/96) + 4.5] x (4/3)
= Lag [4.5 + 4.5] x (4/3)
= Lag [9.0] x (4/3)
= [0.26] x (4/3)
= 0.347

*** Note that the lags for Exhibits 1 through 4 are as follows:**

Months	3	6	9	12	15	18	21	24
Lag	6.0%	14.0%	26.0%	40.0%	48.0%	56.0%	64.0%	72.0%

APPENDIX B

EXHIBIT 2



AY 19## Lag @ PID = Lag [(LID - MED) + AF] x (MF)

1) AY 1996 Lag @ 12/31/96
= Lag [(12/31/96 - 11/15/96) + 1.5] x (4/1)
= Lag [1.5 + 1.5] x (4/1)
= Lag [3.0] x (4/1)
= [3 mo. lag] x (4/1)
= [0.06] x (4/1)
= 0.240

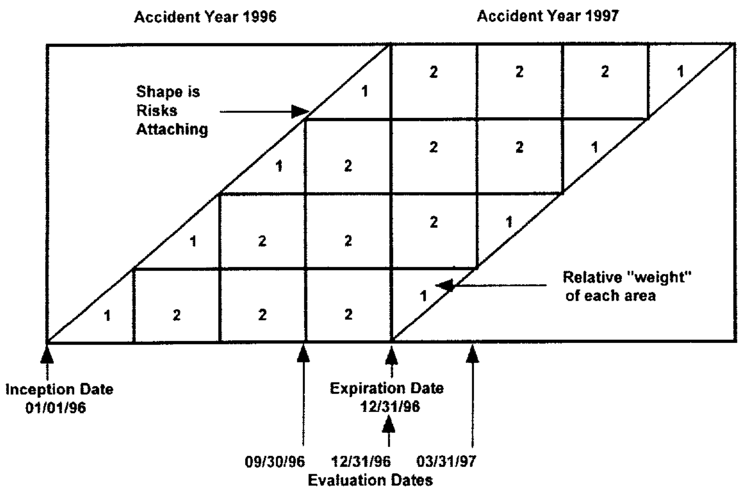
2) AY 1996 Lag @ 03/31/97
= Lag [(03/31/97 - 11/15/96) + 6.0] x (4/4)
= Lag [4.5 + 6.0] x (4/4)
= Lag [10.5] x (4/4)
= [(9 mo. lag) + (12 mo. - 9 mo. lags) x (1/2)] x (4/4)
= [0.26 + (0.40 - 0.26) x (1/2)] x (4/4)
= [0.26 + (0.14) x (1/2)] x (4/4)
= [0.26 + 0.07] x (4/4)
= [0.33] x (4/4)
= 0.330

Linearly interpolating between 9 and 12 month points

3) AY 1996 Lag @ 06/30/97
= Lag [(06/30/97 - 11/15/96) + 6.0] x (4/4)
= Lag [7.5 + 6.0] x (4/4)
= Lag [13.5] x (4/4)
= [0.40 + (0.48 - 0.40) x (1/2)] x (4/4)
= 0.440

APPENDIX B

EXHIBIT 3



AY 19## Lag @ PID = Lag [(LID - MED) + AF] x (MF)

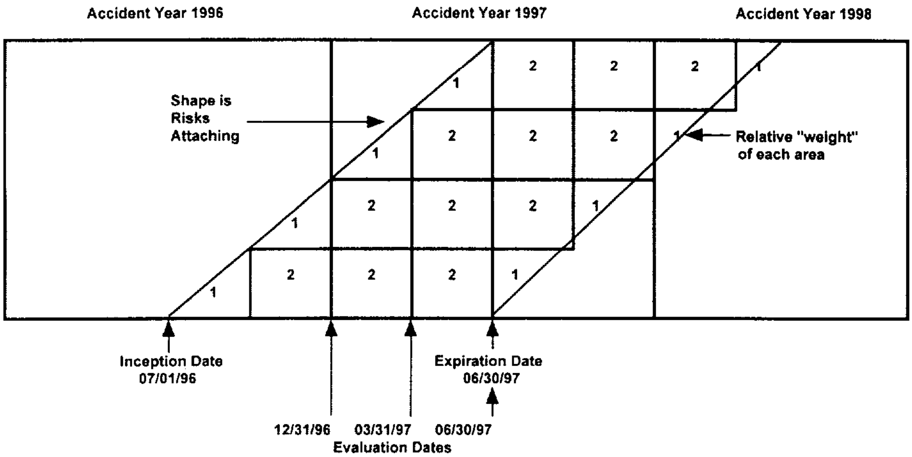
1) AY 1996 Lag @ 09/30/96
= Lag [(09/30/96 - 7/15/96) + 4.5] x (4/3)
= Lag [2.5 + 4.5] x (4/3)
= Lag [7.0] x (4/3)
= [(6 mo. lag) + (9 mo. - 6 mo. lags) x (1/3)] x (4/3) Linearly interpolating between 6 and 9 month points
= [0.14 + (0.26 - 0.14) x (1/3)] x (4/3)
= [0.14 + (0.12) x (1/3)] x (4/3)
= [0.14 + 0.04] x (4/3)
= [0.18] x (4/3)
= 0.240

2) AY 1996 Lag @ 12/31/96
= Lag [(12/31/96 - 9/15/96) + 6.0] x (4/4)
= Lag [3.5 + 6.0] x (4/4)
= Lag [9.5] x (4/4)
= [0.26 + (0.40 - 0.26) x (1/6)] x (4/4)
= 0.283

3) AY 1996 Lag @ 03/31/97
= Lag [(03/31/97 - 9/15/96) + 6.0] x (4/4)
= Lag [6.5 + 6.0] x (4/4)
= Lag [12.5] x (4/4)
= [0.40 + (0.48 - 0.40) x (1/6)] x (4/4)
= 0.413

APPENDIX B

EXHIBIT 4



AY 19## Lag @ PID = Lag [(LID - MED) + AF] x (MF)

1) AY 1996 Lag @ 12/31/96
= Lag [(12/31/96 - 11/01/96) + 3.0] x (4/2)
= Lag [2.0 + 3.0] x (4/2)
= Lag [5.0] x (4/2)
= [(3 mo. lag) + (6 mo. - 3 mo. lags) x (2/3)] x (4/2) ← Linearly interpolating between 3 and 6 month points
= [0.06 + (0.14 - 0.06) x (2/3)] x (4/2)
= [0.06 + (0.08) x (2/3)] x (4/2)
= [0.06 + 0.053] x (4/2)
= [0.113] x (4/2)
= 0.226

2) AY 1996 Lag @ 03/31/97
= Lag [(03/31/97 - 11/01/96) + 6.0] x (4/4)
= Lag [5.0 + 6.0] x (4/4)
= Lag [11.0] x (4/4)
= [0.26 + (0.40 - 0.26) x (2/3)] x (4/4)
= 0.353

3) AY 1996 Lag @ 06/30/97
= Lag [(06/30/97 - 11/01/96) + 6.0] x (4/4)
= Lag [8.0 + 6.0] x (4/4)
= Lag [14.0] x (4/4)
= [0.40 + (0.48 - 0.40) x (2/3)] x (4/4)
= 0.453

APPENDIX C

Reserving For Account Features
Contracts With Loss Corridors

Terms of Contract:

Premium Calculated as Losses Multiplied by 100/80
(Note: Results in "Contractual Loss Ratio" of 80)

Minimum Premium = $0
Maximum Premium = $1,000,000

Expected Losses: **$400,000**

Expected Aggregate Loss Distribution:
(Note: Displayed on an Incremental Basis)

Scenario	Probability	Losses
1	10.0%	$100,000
2	20.0%	$200,000
3	26.0%	$300,000
4	15.0%	$400,000
5	10.0%	$500,000
6	8.0%	$600,000
7	5.0%	$800,000
8	3.0%	$1,000,000
9	2.0%	$1,200,000
10	1.0%	$2,000,000
Expected	100.0%	$400,000

APPENDIX C

Reserving For Account Features
Contracts With Loss Corridors

Calculation of Retro Premium and Expected Loss Ratio

	Probability	Losses	Premium
	10.0%	$100,000	$125,000
	20.0%	$200,000	$250,000
	26.0%	$300,000	$375,000
	15.0%	$400,000	$500,000
	10.0%	$500,000	$625,000
	8.0%	$600,000	$750,000
	5.0%	$800,000	$1,000,000
	3.0%	$1,000,000	$1,000,000
	2.0%	$1,200,000	$1,000,000
	1.0%	$2,000,000	$1,000,000
Expected Amounts		$400,000	$467,500
Expected Loss Ratio		**85.6%**	

APPENDIX C

Reserving For Account Features
Contracts With Loss Corridors

Accounting For Premium and Losses Over Time

Start with the Expected Loss and Premium

Expected Loss = $400,000

Expected Premium = $467,500

Expected Loss Ratio = 85.6%

Theory: As time elapses, the aggregate distribution of loss collapses areound a single point. If loss emerge as expected ($400,000), premium will eventually reach $500,000. We need a process that recognizes this but is simple to implement. Our solution was to create an "Insurance Charge" (IC) equal to Contractual Premium ($500,000) less Expected Premium ($467,500). The IC is multiplied by (1 - Lag), or the percent of loss expected to be unemerged at each point in time. As all losses are reported, the ultimate premium converges to the contractual premium. This stuff is not rocket science.....

(1)	(2)	(3)	(4)	(5)	(6)
End of Year	Estimated Ultimate Loss	Lag Factor	Estimated Insurance Charge	IC X (1 - Lag)	Estimated Ultimate Premium
0	$400,000	0.00	$32,500	$32,500	467,500
1	$400,000	0.25	$32,500	$24,375	475,625
2	$400,000	0.50	$32,500	$16,250	483,750
3	$400,000	0.70	$32,500	$9,750	490,250
4	$400,000	0.85	$32,500	$4,875	495,125
5	$400,000	0.95	$32,500	$1,625	498,375
6	$400,000	1.00	$32,500	$0	500,000

Note: There are simpler ways of creating this process, but the above seems to be a good way of generically describing the retrospective premium process. Note that the IC could be calculated for contracts with additive loads, or a combination of additive and multiplicative loads.

APPENDIX D

Reserving For Account Features
Contracts With Loss Corridors

Terms of Contract:

Quota Share contract with a loss corridor between a 65% and 75% loss ratio.

Expected Premium: **$1,000,000**

Expected Losses: **$700,000**
(Ground up - excluding corridor)

Expected Aggregate Loss Distribution:
(Note: Displayed on an Incremental Basis)

Scenario	Probability	Losses
1	6.0%	$200,000
2	12.0%	$400,000
3	20.0%	$500,000
4	25.0%	$600,000
5	14.0%	$700,000
6	9.0%	$800,000
7	6.0%	$1,000,000
8	4.0%	$1,500,000
9	2.0%	$2,000,000
10	2.0%	$3,000,000
Expected	100.0%	$700,000

APPENDIX D

Reserving For Account Features
Contracts With Loss Corridors

Calculation of Value of Loss Corridor and Expected Loss Ratio

Probability	Ground Up Losses	Loss Corridor	Net Losses
6.0%	$200,000	$0	$200,000
12.0%	$400,000	$0	$400,000
20.0%	$500,000	$0	$500,000
25.0%	$600,000	$0	$600,000
14.0%	$700,000	$50,000	$650,000
9.0%	$800,000	$100,000	$700,000
6.0%	$1,000,000	$100,000	$900,000
4.0%	$1,500,000	$100,000	$1,400,000
2.0%	$2,000,000	$100,000	$1,900,000
2.0%	$3,000,000	$100,000	$2,900,000
Expected	$700,000	$30,000	$670,000

Expected Premium:	**$1,000,000**
Expected Loss Ratio:	**67.0%**
Expected Value of Corridor:	**3.0%**

Thus the initial reserves will be set to equal a 67% loss ratio

APPENDIX D

Reserving For Account Features
Contracts With Loss Corridors

Aggregate Loss Distribution of the Same Account at 24 Months

Note: After time goes by, the aggregate loss distrbution begins to collapse upon the point estimate. For purposes of illustration, we will assume that the aggregate distribution has collapsed by half (perhaps the account has a lag of .50 after 24 months). In the real world, the collapse of the aggregate distribution is often referred to as "non-trivial" which means pretty hard to do.

We will state the distribution as a percent of expected so we can apply to different evaluations of ultimate loss and see what the answers are.

Scenario	Probability	Initial Ground Up Losses	As % Of Expected	Collapse Factor	Agg Distrib @24 Months
1	6.0%	$200,000	28.6%	0.5	64.3%
2	12.0%	$400,000	57.1%	0.5	78.6%
3	20.0%	$500,000	71.4%	0.5	85.7%
4	25.0%	$600,000	85.7%	0.5	92.9%
5	14.0%	$700,000	100.0%	0.5	100.0%
6	9.0%	$800,000	114.3%	0.5	107.1%
7	6.0%	$1,000,000	142.9%	0.5	121.4%
8	4.0%	$1,500,000	214.3%	0.5	157.1%
9	2.0%	$2,000,000	285.7%	0.5	192.9%
10	2.0%	$3,000,000	428.6%	0.5	264.3%
Expected		$700,000	100.0%		100.0%

The aggregate distribution as of 24 months is calculated by taking .5 of the difference between the initial losses as a percent of expected and unity and adding/subtracting this number to the initial losses as a percent of expected.

APPENDIX D

Reserving For Account Features
Contracts With Loss Corridors

Calculation of Value of Loss Corridor and Expected Loss Ratio
At 24 Months Using Collapsed Aggregate Distribution
Examples Using Better and Worse Than Expected Results

Current Evaluation of Ultimate Loss: **$500,000**
Expected Premium: **$1,000,000**

Scenario	Probability	Agg Distrib @24 Months	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	$321,429	$0	$321,429
2	12.0%	78.6%	$392,857	$0	$392,857
3	20.0%	85.7%	$428,571	$0	$428,571
4	25.0%	92.9%	$464,286	$0	$464,286
5	14.0%	100.0%	$500,000	$0	$500,000
6	9.0%	107.1%	$535,714	$0	$535,714
7	6.0%	121.4%	$607,143	$0	$607,143
8	4.0%	157.1%	$785,714	$100,000	$685,714
9	2.0%	192.9%	$964,286	$100,000	$864,286
10	2.0%	264.3%	$1,321,429	$100,000	$1,221,429
Expected			**$500,000**	**$8,000**	**$492,000**

Expected Loss Ratio: **49.2%**
Expected Value of Corridor: **0.8%**

Current Evaluation of Ultimate Loss: **$850,000**
Expected Premium: **$1,000,000**

Scenario	Probability	Agg Distrib @24 Months	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	$546,429	$0	$546,429
2	12.0%	78.6%	$667,857	$17,857	$650,000
3	20.0%	85.7%	$728,571	$78,571	$650,000
4	25.0%	92.9%	$789,286	$100,000	$689,286
5	14.0%	100.0%	$850,000	$100,000	$750,000
6	9.0%	107.1%	$910,714	$100,000	$810,714
7	6.0%	121.4%	$1,032,143	$100,000	$932,143
8	4.0%	157.1%	$1,335,714	$100,000	$1,235,714
9	2.0%	192.9%	$1,639,286	$100,000	$1,539,286
10	2.0%	264.3%	$2,246,429	$100,000	$2,146,429
Expected			**$850,000**	**$79,857**	**$770,143**

Expected Loss Ratio: **77.0%**
Expected Value of Corridor: **8.0%**

APPENDIX D

Reserving For Account Features
Contracts With Loss Corridors

Calculation of Value of Loss Corridor and Expected Loss Ratio
At 24 Months Using Collapsed Aggregate Distribution

First let us assume that the estimate of ultimate losses has been unchanged at the 24 month evaluation.

Current Evaluation of Ultimate Loss:	**$700,000**
Expected Premium:	**$1,000,000**

Scenario	Probability	Agg Distrib @24 Month	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	$450,000	$0	$450,000
2	12.0%	78.6%	$550,000	$0	$550,000
3	20.0%	85.7%	$600,000	$0	$600,000
4	25.0%	92.9%	$650,000	$0	$650,000
5	14.0%	100.0%	$700,000	$50,000	$650,000
6	9.0%	107.1%	$750,000	$100,000	$650,000
7	6.0%	121.4%	$850,000	$100,000	$750,000
8	4.0%	157.1%	$1,100,000	$100,000	$1,000,000
9	2.0%	192.9%	$1,350,000	$100,000	$1,250,000
10	2.0%	264.3%	$1,850,000	$100,000	$1,750,000
Expected			**$700,000**	**$30,000**	**$670,000**

Expected Loss Ratio:	**67.0%**
Expected Value of Corridor:	**3.0%**

Note that the value of the corridor has not changed in this example. If the loss corridor is about the expected value of the distribution this is often the case. If the corridor had been well above the expected loss amount, the 24 month value of the corridor would have been reduced substatially. For example, if the loss corridor was 10 points excess of 140 LR ($1,400,000) the value at 24 months would have been $100,000 X .02 = $2,000, compared to an initial value of $100,000 X (.04 + .02 + .02) = $8,000. (Note on Appendix D Page 3 that initial scenarios 8, 9, and 10 are all greater than $1,400,000 with probabilities of .04, .02 and .02 respectively)

Reinsurance Contracts with a Multi-Year Aggregate Limit

by Regina M. Berens, FCAS

REINSURANCE CONTRACTS WITH A MULTI-YEAR AGGREGATE LIMIT

Regina M. Berens

ABSTRACT

Excess of Loss reinsurance contracts commonly include an aggregate limit which specifies the maximum amount the reinsurer will pay under the contract. This paper discusses pricing implications of an aggregate limit which applies over multiple years. Monte Carlo simulations are used to test the sensitivity of the pricing to relationships between the average ground-up loss, the per-claim limit and the aggregate limit under the contract. A pricing example using historic data is also included. Risk charges and applications to clash covers are explored. Underwriting and reserving considerations of a contract with a multi-year aggregate are discussed.

THE AUTHOR

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INTRODUCTION

A typical excess of loss contract protects the purchaser (the ceding insurance company) for a fixed amount per claim in excess of a per-claim retention, up to an aggregate limit. As an example, a ceding company may purchase reinsurance for $500,000 in excess of the first $1 million per claim, with the reinsurer's total liability under the contract limited to $10 million.

This has the advantage (to the reinsurer) of capping the liability under the contract. The ceding company, of course, wants to purchase reinsurance which will provide the maximum stability in its year-to-year loss experience. If the losses to the contract are less than the aggregate, the ceding company may reap part of the savings through loss-sensitive provisions in the contract or more favorable pricing at renewal, but any other savings goes to the reinsurer. If, in the second year if the contract, the losses are in excess of the aggregate, the reinsurer's liability is still capped at the agreed-upon limit and the ceding company must pay any additional losses.

One way to stabilize the results of the ceding company using an Excess of Loss contract is to provide a multi-year aggregate. This contract would, for example, cover the layer described above but include an aggregate limit of $30 million over three years. The purpose of this paper is to explore the pricing, underwriting and reserving implications of this concept. Although it applies to a contract between a primary insurance company and a reinsurer, it could apply with appropriate adjustments to excess coverage purchased by a self-insured entity. A three-year period was selected to simplify assumptions with regard to changes in exposures, inflation and other factors which change with each contract period; the model could obviously be generalized beyond three years.

SIMPLIFYING ASSUMPTIONS

1. The ceding company's exposure is relatively stable from year to year.
2. The treatment of Allocated Loss Adjustment Expenses (ALAE) will not be considered. This is equivalent to including ALAE in the loss portion when calculating the reinsurer's liability.
3. Inflation is minimal. Under the double-digit inflation conditions prevalent in the late 1970s, some indexing of the attachment points, layer and aggregate might be needed from year to year.
4. Losses have been adjusted to ultimate settlement value, including IBNR.

THE MODEL

A model was developed which would generate ground-up claims using a specified distribution. Using a Monte Carlo simulation, three years' worth of excess claims were generated, and the reinsured losses compared using a single-year aggregate and a three-year aggregate equal to three times the annual aggregate. A 6% annual rate of inflation was applied to claims in the second and third years.

The model was run for a variety of sample contracts in order to test the impact of a multi-year aggregate for various combinations of the following:

1. Per-claim retention as a function of the average claim size.
2. The reinsured layer in relation to the retention and as a function of average claim size.
3. The aggregate as a function of expected losses in the excess layer.

A PRIORI EXPECTATIONS

Direction of the pricing impact of a multi-year aggregate.

The impact of this contract provision cannot be a decrease in the pure premium. The reinsurer's liability in any given year is either what it would have been with a single-year aggregate limit (if losses under the contract are greater than or equal to one-third of the three-year aggregate) or greater (if the ceding company can carry over "credit" from a prior year in which losses were less than one-third of the three-year aggregate).

Retention as a function of average claim size.

If the ceding company retains a minimal portion of each claim, the frequency and severity on the excess cover are likely to be close to the ground-up frequency and severity. Conversely, if the excess cover is written with a high-level attachment point, claims will reach the excess layer less often. The advantage to the ceding company (and, thus, the percentage surcharge for a multi-year aggregate) will be greater if its retention is a substantial portion of the average claim size and thus losses to the excess layer are more volatile.

The reinsured layer as a function of the retention and the average claim size.

Given a particular retention, the larger the layer, the larger the average excess claim size. This means that the aggregate can be "used up" by fewer claims. If the coverage is written at a relatively high attachment point, the excess cover becomes low-frequency and high severity. The lack of predictability in this layer would again make a multi-year aggregate more desirable from the ceding company's point of view but also proportionately more expensive.

The aggregate as a function of expected losses in the layer.

If the aggregate is so large that it would cover all claims in the excess layer except under the most extreme circumstances, a multi-year aggregate has little effect. This would imply that a ceding company would be willing to accept a contract with a lower aggregate limit, if it applied to a series of years so that experience of favorable years might be recouped in the future.

FIRST MODEL: POISSON/ LOGNORMAL

The first simulation was a "vanilla casualty" book with a relatively simple loss distribution. A portfolio with a Poisson frequency with 30 expected claims was constructed, using a Lognormal severity with an average claim value of $150,000 and a Standard Deviation of $37,500. The distribution of the severity was thus $\Lambda(11.92, .25)$.

The top section of Exhibit I shows the results of the first model. Surcharges are shown by layer as well as by ratio of the annual aggregate to the expected losses. The results of four iterations are shown (each with 100 trials) to provide a gauge of variability.

As would be expected, the impact of a three-year aggregate on the $150,000 XS $150,000 layer is minimal if the annual aggregate is set at twice the expected losses in the layer. This is because the losses are relatively predictable, particularly with the selected Poisson/Lognormal functions. The most that could be said about providing a 3-year aggregate for a low-level excess contract on this type of business is that it could be offered to make a prospective deal more attractive at little or no extra cost. The surcharge for the $150,000 XS $300,000 layer is substantial because so few claims penetrate the layer that there is a large percentage increase to the expected losses when a three-year aggregate applies.

The second section of Exhibit I shows the results of the model when the distribution is changed to Poisson with 15 expected claims per year and the severity is Lognormal with an average claim size of $300,000. Again, the surcharge for a three-year aggregate increases as the underlying business becomes lower-frequency and higher-severity.

Exhibit II shows the results of this first model in graphic form. The indicated *percentage* surcharge increases as:

(a) The attachment point increases.

(b) The ratio of the annual aggregate to the expected losses decreases.
(c) The parameters change from 15 expected claims, $300,000 average severity to 30 expected claims, $150,000 average severity.

The last result appeared counter-intuitive, since the high-frequency, low-severity example was expected to be less volatile. An examination of the raw data, however, revealed the reason. For the layers in question (excess of $300,000), results in the excess layer are actually more volatile for the low-severity example since so few claims penetrate the layer.

It should be pointed out, however, that the Poisson-Lognormal model is probably not meaningful for some types of business, so the process was repeated with another frequency/severity distribution.

SECOND MODEL- NEGATIVE BINOMIAL/ SINGLE-PARAMETER PARETO

Negative Binomial claim frequency and Single-Parameter Pareto severity distributions have been used by various authors to model claim distributions for casualty business. [1, 2] The model was re-run with two distributions:

Negative Binomial frequency distribution with 30 expected claims, p =.10 and a Single Parameter Pareto Severity with q = 1.5 and average claim value of $150,000.

Negative Binomial frequency distribution with 15 expected claims, p =.05 and a Single-Parameter Pareto Severity with q = 1.5 and average claim value of $300,000.

The results are shown on Exhibit III in a format similar to Exhibit I. They are shown graphically in Exhibit IV. At the lower layers, the indicated percentage surcharge is higher for the new distribution than for the Poisson/Lognormal model. In the higher layers, however, the percentage surcharge is higher for the Negative Binomial/Pareto model. The reason becomes apparent from Exhibit V, which is a graph of percentiles of claim size from Monte Carlo runs of 3000 claims from a Lognormal and Pareto distribution, each with an expected claim value of $150,000. At a $150,000 attachment point, far more claims fall under the retention with the Pareto model. As the attachment point increases, more claims exceeding the attachment point are expected in the Pareto model, but many claims are so large that the expected payment in the layer is $150,000 (the maximum). This actually reduces volatility in the layer.

MEASURING VARIABILITY; RISK CHARGES

It becomes apparent from examining the detailed results of 100 iterations (which are not reproduced with this paper in order to conserve trees and tedium) that most of the time the three-year aggregate provides no benefit. Some examples are shown in graphic form on Exhibits VI and VII. The graphs were created as follows:

1. The losses covered by the reinsurance contract over the three year period, with single-year aggregates and with a three-year aggregate, were sorted based on the value of losses with single-year aggregates.

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[1] P. E. Heckman and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions", PCAS LXX, P. 22.

[2] S.W. Philbrick, "A Practical Guide to the Single-Parameter Pareto", PCAS LXXII, p. 44.

2. Two lines were graphed: the sorted (ascending) losses for each iteration with single-year aggregates and the corresponding value of losses with a three-year aggregate. A "spike" thus represents an instance where the reinsurer covered more losses because of the existence of a multi-year aggregate.

Exhibit VI shows the results of the 100 iterations which make up the 7% surcharge shown on Exhibit III, the second iteration in the sixth line of data. This corresponds to $150,000 Excess of $300,000 coverage, with an annual aggregate equal to 2.5 times the expected losses in the layer. The expected (Negative binomial) number of ground-up losses is 30; average severity (Pareto) is $150,000.

For the iterations where incurred losses in the layer were low, the three-year aggregate had no benefit. Where the simulated losses were high, however, the number of cases where the three-year aggregate provided a benefit increased. There are 19 "spikes" in the graph, i.e. instances where the reinsurer would have paid more with a three-year aggregate. The average surcharge is $43,253, which is 6.50% of expected losses in the layer ($665,321). The largest actual benefit in the simulations was $758,752. The standard deviation of the differences between losses under annual and three-year aggregates was $123,175.

Exhibit VII shows the results of the 100 iterations which make up the 22% surcharge shown on Exhibit III, the second iteration in the seventh line of the second section of data. This corresponds to $150,000 Excess of $450,000 coverage, with an annual aggregate equal to 2.5 times the expected losses in the layer. The expected (Negative binomial) number of ground-up losses is 15; average severity (Pareto) is $300,000.

As would be expected, the losses in this example showed considerably more variation; so did the impact of the three-year aggregate. There are 32 "spikes" in the graph, where the reinsurer would have paid more with a three-year aggregate. The average surcharge is $108,117, which is 22.16% of expected losses in the layer ($487,968). The largest actual benefit in the simulations was $1,357,126. The standard deviation of the differences between losses under annual and three-year aggregates was $244,294.

With apologies to Feldblum[3], who has pointed out that a risk charge should be a function not only of the particular portfolio but of the insurer's entire book of business, a simplified risk charge will be calculated as a function of variance for the two graphed examples. If the risk charge is set at 0.5% of the standard deviation of the additional losses under the three-year aggregate, for the first example the risk-adjusted surcharge is 6.6% rather than the original 6.5%; for the second it is 22.4% rather than the original 22.16%.

CLASH COVERS

A clash cover example was constructed with the same set of random numbers used in the last section to produce the 22.16% (non-risk adjusted) surcharge. First, the individual claim severities were capped at $300,000, to simulate a case where the ceding company issues policies with limits no higher than $300,000 (or, alternatively, other reinsurance is available excess of that limit). A value of 5% was selected as the probability that another insured was involved in the same occurrence. (The possibility of occurrences with more than two claimants was ignored). Five percent of the claims, at random, were then increased by the amount of a second randomly-chosen claim value two simulate a two-claimant occurrence.

The indicated surcharge for $150,000 excess $450,000 was then calculated. The surcharge was $29,221, which was 160.9% of the $18,159 losses expected with annual aggregates. For 36 out of 100 iterations, the reinsurer would have paid more with a three-year aggregate. The largest

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[3] S. Feldblum, "Risk Loads for Insurers", PCAS LXXVII, page 160.

difference was $244,000. The standard deviation of the difference between losses incurred under an annual aggregate and a three-year aggregate was $52,712. In this case, an adjustment for risk calculated as described in the earlier section would have a more substantial impact: the risk-adjusted surcharge would be [($29,221 + .05*52,712)/18,159], or 175.4%. Decreasing the probability of clash to 2.5% increases the surcharge to 317.6% (355% risk-adjusted) on expected losses of $5,419.

A REAL DATA EXAMPLE

Finally, a pricing example was developed using suitably disguised ground-up casualty claim statistics from an insured entity, in an attempt to see what happens when this coverage provision is priced using actual experience. The losses were mostly Workers' Comp, with a few Auto and GL claims. The following factors needed to be taken into account:

Loss Development

Loss development, so easily ignored in the Monte Carlo model, needs to be addressed when working with actual data. In the reinsured layer (or the layer excess of a Self-Insured Retention), there are three sources of development: (1) losses which have been reported to the ceding company but appear to be below the retention (and thus may not have been reported to the reinsurer), (2) reported losses above the attachment point but less than the layer, and (3) losses which have been incurred but not yet reported to the ceding company (also known as "pure IBNR"). An evaluation of the amount of aggregate left at the end of a contract period should include an estimate for this development.

In this example, losses with 9 or more years of development are in order to minimize distortion from undeveloped data.

Bringing Prior Year Claims to Present Level

In this example, an annual rate of 6% was used to bring all claims to current level.

Low Credibility of Experience

The number of reported claims per year for this entity ranged from 102 to 147. The results from this example should not be expected to be as smooth as the model would imply. A ceding company or self-insured operating under these conditions, however, is probably the perfect candidate for purchasing a multi-year aggregate because it has less tolerance for fluctuations in year-to-year results than a larger entity.

Changes in Exposure

This may or may not be a significant factor, but it should be investigated. A self-insured which varies the extent to which it uses "leased" employees on projects from year to year, for example, is not a good candidate for a multi-year aggregate. (This is because the number of employees actually on the self-insured's payroll, as opposed to the leasing company's payroll, will fluctuate.) Similarly, a ceding company increasing or decreasing its market presence in a given line of business will be more likely to prefer an annual aggregate which is adjusted based upon the facts in a given year..

Exhibit VIII shows the results of a three-year vs. annual aggregate for the self-insured entity.

Losses in the $6,000,000 XS $2,000,000 are shown for two subsequent three-year periods. This layer was selected because the entity generally had a few catastrophic claims of $3 to $8 million (ground-up) in each year, and it is realistic to expect that it would be interested in protecting its bottom line from losses of this magnitude. The annual aggregate in each case was set equal to

1.25 times the average incurred losses in the layer for the three-year period; the three-year aggregate was set equal to triple that value.

This example illustrates some of the perils of applying the model to real life. We must first assume that a reasonable estimate of prospective losses in the layer can be made, using either some averaging process on past results, or a selected theoretical distribution. In this example, using the results for Years 1 through 3 to set an aggregate limit for Years 4 through 6 would have produced an aggregate higher than the company needed in the later years. If an appropriate distribution were found to approximate results and select the aggregate shown in the Exhibit, a 13% surcharge would result. In Years 1 through 3, the entity would recover about $3 million more from its insurer under a three-year aggregate; in Years 4 through 6 it would have collected nothing extra, despite having paid the 13% surcharge.

An interesting complication is shown in the results of Years 7 and 8. The organization increased its emphasis on loss control and loss-reduction programs beginning in Year 6. Although results are shown for Years 7 and 8, it is clear that the underlying loss distribution has changed- both frequency and severity. Renewal decisions made after the implementation of this program would have to take these changes into account on a prospective basis.

ADDITIONAL CONSIDERATIONS

Indexation

The leveraged impact of inflation on excess claims has been well-documented in the actuarial literature and will not be reviewed here. [4] For an excess-of-loss contract with a multi-year aggregate, claim inflation will erode the amount of reinsurance coverage available. One solution, of course, is to index the attachment point, the layer covered and the aggregate every year by an agreed-upon inflation rate.[5] In the absence of indexation, the two parties should be aware of the impact of inflation on the coverage provided.

Renewal/Runoff Adjustments and Decisions

Since the model is based on a contract which is renewed annually (as opposed to a contract which runs from 1/1/YY to 12/31/YY+2), it is possible that either the ceding company or the reinsurer may decide not to renew. Two examples, using the contract described in the introduction:

1. The ceding company, having incurred $20 million of losses within the reinsurance layer in the first year, decides not to renew.
2. Losses to the contract in the first year are $8 million but the reinsurer decides to tighten pricing at renewal. The ceding company is faced with the decision of accepting renewal terms it finds unpalatable, or walking away from $2 million of coverage which could be carried forward to the next accident year (one-third of the $30 million aggregate, minus $8 million incurred) for which it has paid a surcharge. It is unlikely, of course, that renewal terms would be tightened with such favorable results, but it is probably not impossible.

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[4] J. T. Lange, "The Interpretation of Liability Increased Limits Statistics", PCAS LVI, Page 170.

[5] R. E. Ferguson, "Non-Proportional Reinsurance and the Index Clause", PCAS LXI, Page 141.

These contingencies are best handled at the inception of the contract. One simple solution would be a stipulation that, in the event of non-renewal, a pro-rated portion of the aggregate applies. These renewal or rate change rights are valuable options which can be priced as such. They may or may not offset each other.

In the event of losses which are more in line with expectations, some decisions can still be made on an annual basis.

First, the experience should be analyzed to determine whether the exposures are at a level anticipated under the contract. This should include an estimate of IBNR in order to determine whether the assumptions about expected losses under the contract are still valid. If the experience is different from that assumed at the inception of the contract, the rate, annual aggregate and/or multi-year aggregate should be adjusted. This could take the form of a re-pricing of the entire three-year period based on new information, with the adjustments applied to the future contract period. This, again is a possibility which must be agreed upon in advance between the ceding company and the reinsurer. If it is not, the likelihood exists that changes in the contract provisions for experience worse than expected could cause the ceding company to "walk away" from the contract at renewal.

Reinstatement

Considering Item 1 in the above section, what if the ceding company wishes to purchase additional limits so that $20 million (not $10 million) is available to cover losses in the layer over the next two years? The models used in this paper would probably imply a smaller reinstatement premium than is needed, if the worse-than-average experience in the first year is due to factors not recognized when the aggregate was determined. While the technique is still applicable, it is important to determine the reason for the adverse development. Was the exposure greater than expected? Has the limits profile changed? Do the initial assumptions about ground-up claim frequency and claim severity still hold true? Any changes should be taken into account in pricing the reinstatement, i.e. the extension of the three-year aggregate from $30 million to $40 million.

Changes in Exposures

It is generally not prudent to assume that anything in the reinsurance market will remain static. Pricing of a multi-year aggregate could be enhanced by including an estimate of the potential growth of the ceding company's book over the next three years, including changes in limits profiles if any are anticipated.

The author has done some preliminary analysis using the Negative Binomial/Pareto model with an assumption of a 6% growth in exposure (corresponding to a 6% increase in claim count from the first to the second year and from the second to the third year). If the calculation of expected losses and aggregate limits was based on an assumption of stable exposures, the ceding company has obviously purchased too little protection and will find itself responsible for more losses over the aggregate limit than it would had the proper aggregate limit been negotiated. The percentage surcharge for the 3-year aggregate on the portfolio with the expected severity of $300,000 would also be about half of what it should have been if exposure growth had been anticipated in the pricing.

Aggregates Covering More than Three Years

While an aggregate covering a lengthier period is possible, it has not been explored in the model. The property-casualty industry has occasionally provided coverage on a 3-year basis (an unfortunate example being three-year reinsurance contracts which could be found in the London market in the 1970s), and in the current market examples of longer-term contracts do exist. As a practical matter, however, many factors can change over the long run which would complicate pricing a longer-term aggregate.

Continuous Coverage

The possibility of the unused (or over-used) portion of an annual aggregate being rolled forward into the next year at renewal would be conducive to long-term relationships between a ceding company and its reinsurer, and such "roller" contracts exist in the current market. This is easiest to visualize in a zero-inflation, stable-exposure situation, with appropriate adjustments then made to reflect reality. The cumulative aggregate for year *n* would be *n* times the expected losses in the layer. The coverage available to the ceding company for losses in the excess layer in year *n* would be the cumulative aggregate, minus losses incurred on all preceding years.

With the volatile examples studied earlier in this paper, multi-year aggregates which accumulate each year would be a very attractive contract feature. Since, as observed earlier, the actual number of years where a multi-year aggregate provides additional coverage are infrequent, the extension of the multi-year aggregate beyond three years gives the ceding company a better chance of being able to take advantage of the additional coverage at some point. The author ran some extremely simplified simulations of contracts in which the aggregate was extended each year and found that the ceding company could nearly always be assured of collecting all losses in the layer each year, unless losses were particularly disastrous in the first year.

Varying exposures and claim inflation could be handled by increasing the aggregate each year by the agreed-upon multiple of expected losses in the layer, calculated using appropriate exposures and inflation assumptions. (The aggregate could even be adjusted after the fact based on actual exposures.)

It should be noted that, as the number of years covered by the aggregate increases at each renewal, the adequacy of reserve estimates can be extremely important. This is because the aggregate for the current year can be eroded by adverse development on old years.

Contracts with Annual Aggregate Deductibles

The concepts in this paper could be extended to contracts in which the ceding company or self-insured absorbs losses in the layer up to a certain level before the reinsurance coverage applies. As an example, a large self-insured might purchase coverage for $500,000 excess $500,000 per occurrence, with the agreement that the self-insured pays the first $1.5 million in claims in the layer. The insurer agrees to pay up to $5 million in losses in the layer *after* the first $1.5 million. The insured layer now exhibits more volatility because of the aggregate deductible, making it an ideal candidate for a multi-year aggregate limit approach.

Reserving Implications

As mentioned earlier, an IBNR provision is necessary to evaluate the amount of cover remaining at each evaluation point. Let us return again to the initial example. If, after the first year, the ultimate settlement value (including IBNR) of claims in the layer is $8 million, the reinsurer's liability under the contract for the second year could be anywhere from zero to $22 million. If, in fact, the losses exceeded 12 million, bringing total losses above 2/3 of the $30 million aggregate, the excess would be a "credit" against potential losses during the third year. Any method used to evaluate IBNR on contracts with single-year aggregate limits could be used for this type of contract, with the additional complication that the company's liability for the current year is a function of prior years' losses (including an appropriate IBNR provision) on the contract.

IBNR (both case development and pure IBNR) evaluation is particularly important in multi-year aggregate contracts because it is a critical part of the year-end decision-making. An understated estimate of ultimate losses gives the ceding company false assurance of how much of the three-year aggregate remains for subsequent years. It can also leave the ceding company blissfully unaware of the need to purchase reinstatement coverage, meaning that it will have less protection than expected for later years.

Could a Multi-Year Aggregate be provided at some level with no surcharge?

It was noted earlier that at low attachment points, the indicated surcharge is minimal. For higher attachment points, there should still be some multiple of the annual aggregate which could be provided as a three-year aggregate without increasing the expected losses. The answer, interestingly enough, is probably unmarketable. For the Negative Binomial/Pareto model, this level was determined from one of the simulations for both $150,000 excess of $150,000 where the average claim is $150,000 and for $150,000 excess $300,000 where the average claim is $300,000. The annual aggregate limit was set at 2.5 times the annual expected losses. The multi-year aggregate which could be supplied with no increase to the expected losses was about 65% of the annual aggregate in the first case and 55% in the second! This would be 1.625 and 1.375, respectively, times expected annual losses in the layer.

The reason for this result is that there are actually very few cases where the multi-year aggregate limit provided greater protection (about 5 out of 100 simulations in the $300,000 average claim case) but the impact when it did was substantial; generally 30% more losses were paid by the reinsurer. Thus, the multi-year aggregate had to be cut significantly in order to affect these cases.

CONCLUSIONS

The objective of this paper was to explore the implications of applying a multi-year aggregate limit to Excess of Loss reinsurance contracts rather than a single year limit. It is meant to generate additional thought and dialogue on how best to provide insurance and reinsurance products which fit the needs of the customer and are appropriately priced and reserved. Some conclusions can be drawn from the points covered in this paper are the following:

1. A multi-year aggregate can be a useful tool to provide coverage when and where the ceding company (or self-insured) needs it, while still limiting the liability of the reinsurer. It should, however, carry an appropriate price.

2. Pricing is highly dependent upon the loss distribution which is selected.

4. In general, the greater the volatility of the business, the higher the percentage surcharge and the indicated risk charge.

5. Prospective changes in exposures should be quantified in the pricing process.

6. The consequences of non-renewal during the 3-year period should be specified in the contract.

7. The validity of assumptions made at the inception of the contract should be checked at each renewal and adjustments made, if necessary.

8. The contract wording issues involved in offering a multi-year aggregate can be more complicated than the pricing issues.

Exhibit I

Percentage Surcharge for Three-Year Aggregate

Poisson Frequency; Lognormal Severity

Expected No. of Claims	Ave Claim		Iteration Number: 1	2	3	4	Average	Annual Agg/ Expected Losses In Layer
30	$150K	150 X 150	0%	0%	0%	0%	0%	2
30	$150K	150 X 300	182%	252%	194%	206%	208%	2
30	$150K	150 X 450	800%	N/A	N/A	N/A	800%	2
30	$150K	150 X 600	N/A	N/A	N/A	N/A	N/A	2
30	$150K	150 X 150	0%	0%	0%	0%	0%	2.5
30	$150K	150 X 300	146%	206%	151%	161%	166%	2.5
30	$150K	150 X 450	800%	N/A	N/A	N/A	800%	2.5
30	$150K	150 X 600	N/A	N/A	N/A	N/A		2.5
30	$150K	150 X 150	0%	0%	0%	0%	0%	3
30	$150K	150 X 300	123%	172%	122%	N/A	139%	3
30	$150K	150 X 450	739%	N/A	N/A	N/A	739%	3
30	$150K	150 X 600	N/A	N/A	N/A	N/A	N/A	3
15	$300K	150 X 150	0%	0%	0%	0%	0%	2
15	$300K	150 X 300	0%	0%	0%	0%	0%	2
15	$300K	150 X 450	10%	10%	12%	11%	11%	2
15	$300K	150 X 600	203%	177%	258%	224%	216%	2
15	$300K	150 X 150	0%	0%	0%	0%	0%	2.5
15	$300K	150 X 300	0%	0%	0%	0%	0%	2.5
15	$300K	150 X 450	4%	4%	6%	5%	5%	2.5
15	$300K	150 X 600	167%	130%	209%	171%	169%	2.5
15	$300K	150 X 150	0%	0%	0%	0%	0%	3
15	$300K	150 X 300	0%	0%	0%	0%	0%	3
15	$300K	150 X 450	2%	2%	3%	2%	2%	3
15	$300K	150 X 600	144%	98%	173%	171%	147%	3

Percentage surcharge is expressed as a function of expected losses within the layer.

Exhibit II

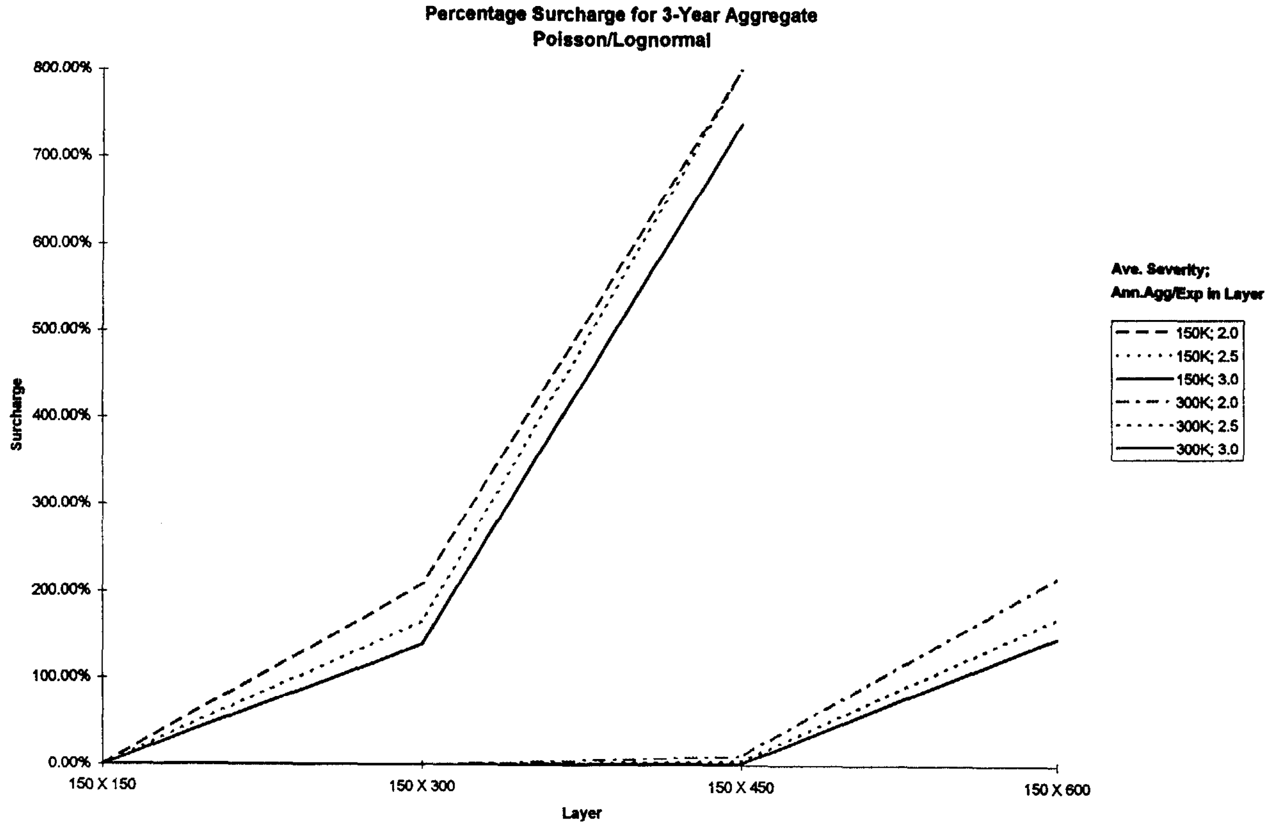

Percentage Surcharge for 3-Year Aggregate
Poisson/Lognormal
800.00%
700.00%
600.00%
500.00%
400.00%
300.00%
200.00%
100.00%
0.00%
Surcharge
150 X 150
150 X 300
150 X 450
150 X 600
Layer
Ave. Severity;
Ann.Agg/Exp in Layer
150K; 2.0
150K; 2.5
150K; 3.0
300K; 2.0
300K; 2.5
300K; 3.0

Exhibit III

Percentage Surcharge for Three-Year Aggregate

Negative Binomial Frequency; Pareto Severity

Expected No. of Claims	Ave Claim		Iteration Number: 1	2	3	4	Average	Annual Agg./ Expected Losses in Layer
30	$150K	150 X 150	6%	5%	7%	7%	6%	2
30	$150K	150 X 300	15%	13%	14%	13%	13%	2
30	$150K	150 X 450	23%	21%	26%	17%	22%	2
30	$150K	150 X 600	27%	32%	38%	29%	32%	2
30	$150K	150 X 150	2%	2%	3%	3%	3%	2.5
30	$150K	150 X 300	7%	7%	7%	7%	7%	2.5
30	$150K	150 X 450	12%	12%	14%	10%	12%	2.5
30	$150K	150 X 600	18%	20%	24%	16%	20%	2.5
30	$150K	150 X 150	1%	1%	1%	1%	1%	3
30	$150K	150 X 300	4%	4%	4%	4%	4%	3
30	$150K	150 X 450	7%	6%	8%	6%	7%	3
30	$150K	150 X 600	11%	11%	14%	8%	11%	3
15	$300K	150 X 150	23%	21%	24%	19%	22%	2
15	$300K	150 X 300	32%	28%	27%	26%	27%	2
15	$300K	150 X 450	38%	34%	32%	39%	35%	2
15	$300K	150 X 600	52%	43%	43%	45%	44%	2
15	$300K	150 X 150	13%	12%	16%	11%	13%	2.5
15	$300K	150 X 300	21%	18%	18%	16%	17%	2.5
15	$300K	150 X 450	26%	22%	21%	25%	23%	2.5
15	$300K	150 X 600	35%	29%	30%	30%	30%	2.5
15	$300K	150 X 150	8%	7%	12%	7%	9%	3
15	$300K	150 X 300	13%	10%	13%	10%	11%	3
15	$300K	150 X 450	18%	15%	14%	18%	16%	3
15	$300K	150 X 600	25%	20%	21%	21%	21%	3

Percentage surcharge is expressed as a function of expected losses within the layer.

Exhibit IV

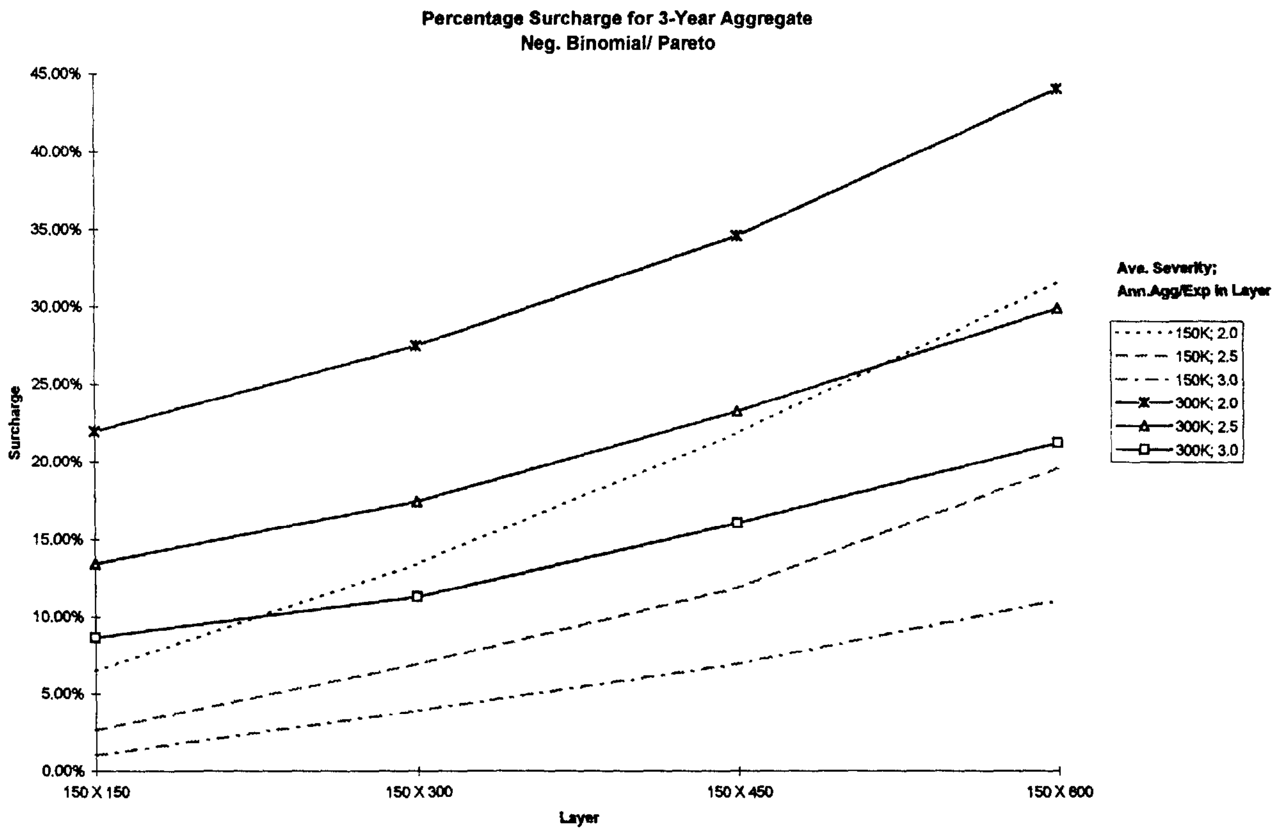
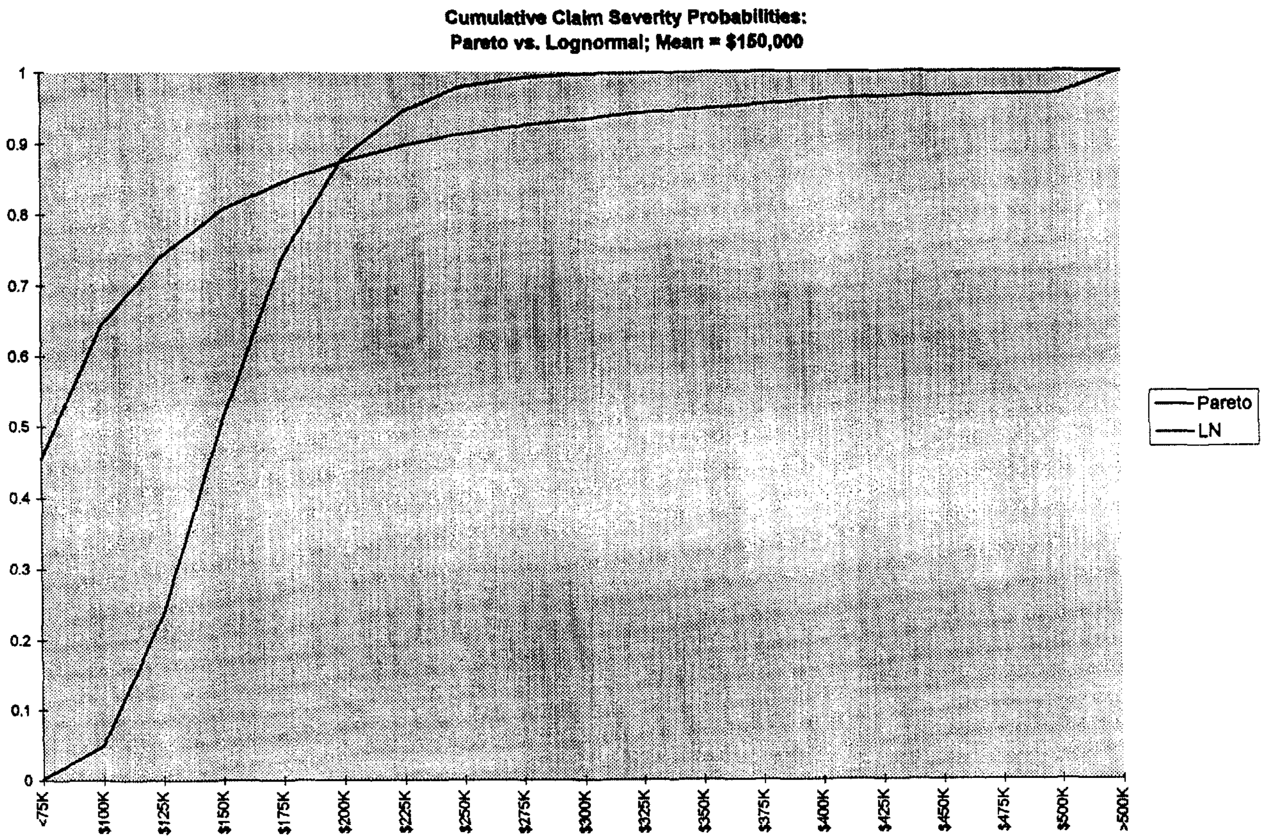

Percentage Surcharge for 3-Year Aggregate
Neg. Binomial/ Pareto
45.00%
40.00%
35.00%
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
Surcharge
150 X 150
150 X 300
150 X 450
150 X 600
Layer
Ave. Severity;
Ann.Agg/Exp in Layer
150K; 2.0
150K; 2.5
150K; 3.0
300K; 2.0
300K; 2.5
300K; 3.0

Exhibit V



Cumulative Claim Severity Probabilities:
Pareto vs. Lognormal; Mean = $150,000
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
<75K
$100K
$125K
$150K
$175K
$200K
$225K
$250K
$275K
$300K
$325K
$350K
$375K
$400K
$425K
$450K
$475K
$500K
>500K
Pareto
LN

Exhibit VI

Negative Binomial/Pareto
Simulated Losses, $150K X $300K;
Annual = 2.5 X Expected; Ave Sev. =$150K

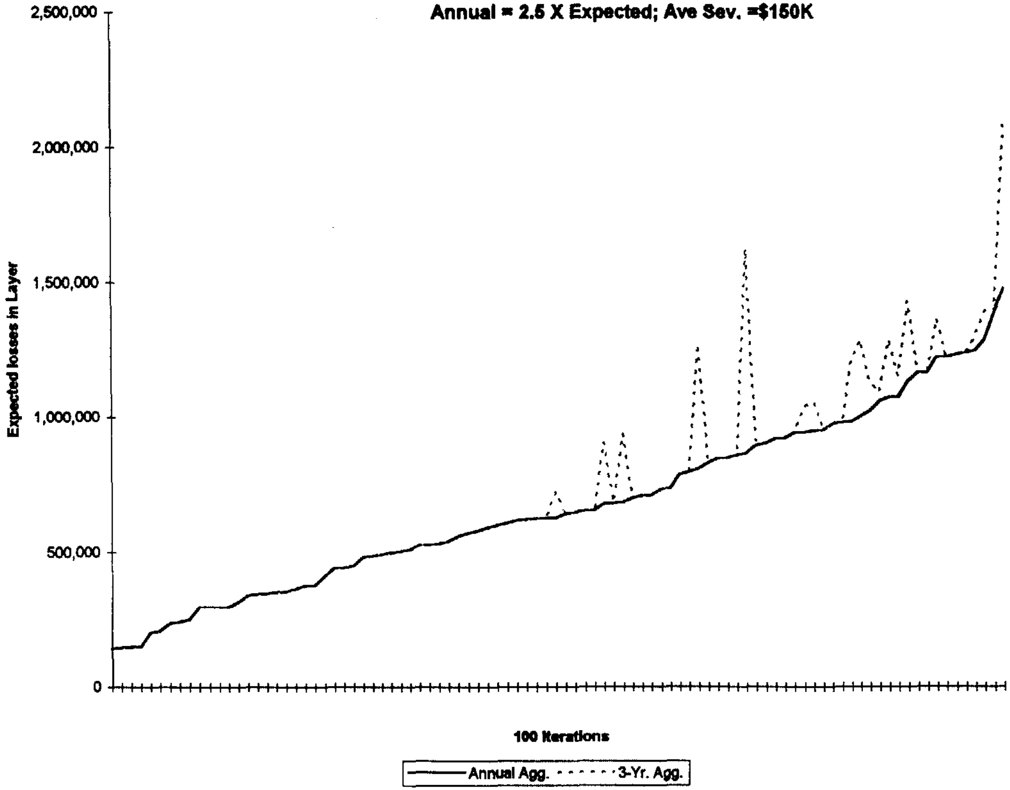


Exhibit VII

Negative Binomial/Pareto
Simulated Losses, $150K X $450K;
Annual = 2.5 X Expected; Ave Sev. =$300K

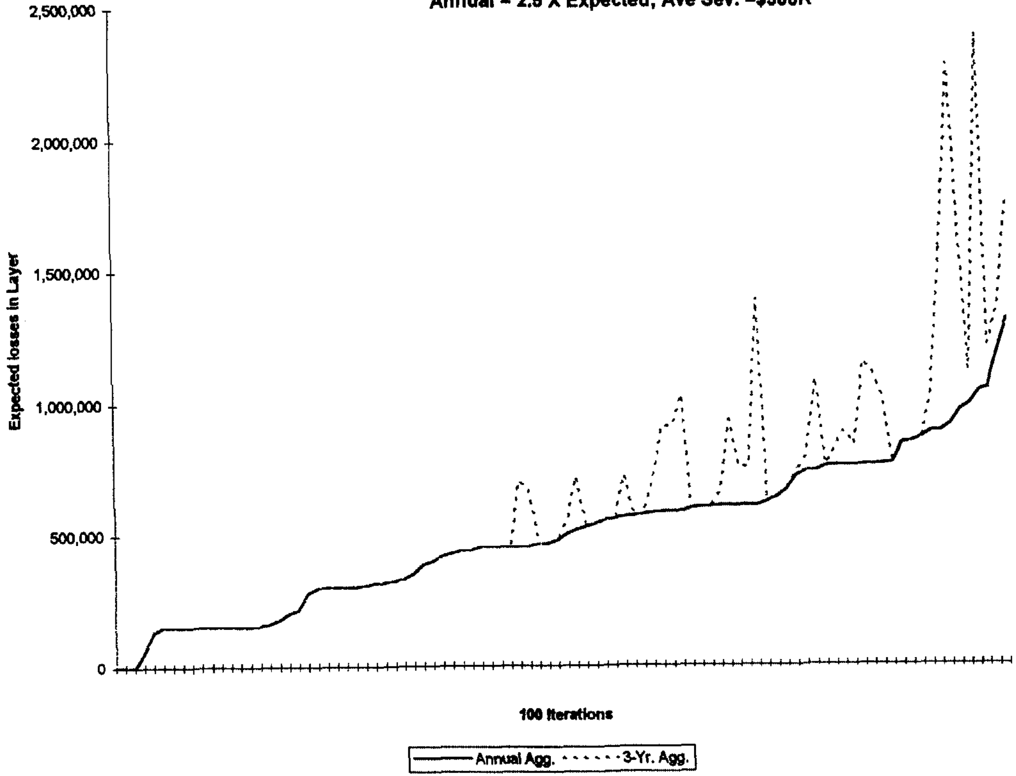


Exhibit VIII

Self-Insured Example

$6,000,000 Excess $2,000,000

Accident Year	(1) Number of Claims	(2) Average Ground-U Claim	(3) Claims in Layer	(4) Selected Annual Aggregate	(5) Losses Subj. to (4)	(6) Losses Subj. to 3-Yr Agg.	(7) Percentage Surcharge (6)/(5)-1
1	131	329,649	4,128,326	5,599,305	4,128,326	4,128,326	
2	105	355,307	4,097,040	5,599,305	4,097,040	4,097,040	
3	131	455,805	10,050,103	5,599,305	5,599,305	8,572,549	
			6,091,823		13,824,671	16,797,915	
4	124	335,791	4,800,232	5,599,305	4,800,232	4,800,232	
5	147	273,075	0	5,599,305	0	0	
6	102	308,162	3,800,963	5,599,305	3,800,963	3,800,963	
			2,867,065		8,601,194	8,601,194	
Yrs 1-6	740	342,456	4,479,444		22,425,865	25,399,109	13.3%

THE SEQUEL: A Loss Reduction Program was Implemented in Year 6.

Accident Year	(1) Number of Claims	(2) Average Ground-U Claim	(3) Claims in Layer
7	92	196,828	0
8	75	198,553	1,120,884
Total	907	315,128	

NOTES:

Column (4) is 125% of the average of Column (3) for the current and next two years. This annual aggregate is then used for a three-year contract period.

Losses in Column (6) are losses in the layer, limited to a cumulative value over 3 years of 3 times Column (4).