

*An Integrated Pricing and  
Reserving Process for Reinsurers*  
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## **AN INTEGRATED PRICING AND RESERVING PROCESS FOR REINSURERS**

### **ABSTRACT**

In today's market of increased competition, more complex reinsurance contracts and tightening (or should we say frightening) profit margins, actuaries are increasingly being called upon to improve their pricing and reserving practices concerning individual accounts as well as aggregate books of business. Increased understanding of that business is critical to continued success for both reinsurers and their clients. The purpose of this paper is to describe a framework for an integrated pricing and reserving process on an individual risk basis. Utilizing this framework, increasing levels of sophistication and knowledge can be brought to bear, risk by risk, on understanding a reinsurer's book of business.

# AN INTEGRATED PRICING AND RESERVING PROCESS FOR REINSURERS

## I. SUMMARY

The process described herein is dependent upon having significant actuarial and underwriting resources available to analyze the risk on every individual contract that is eventually written and put on the books. As well, this process relies heavily on a collaborative environment where underwriting, actuarial, claims, contracts, legal and accounting all have a significant role to play in understanding and evaluating risk. The concept of this paper is to explain a structure which a reinsurer (or a large accounting department of a primary company) can use to gain a thorough understanding of their book.<sup>1</sup> The focus is on the process, not techniques.<sup>2</sup>

Each contract is individually priced by a team that is centered around an underwriter and an actuary. The result of this analysis is an expected loss ratio, an expected loss development or lag pattern (note: a lag for a contract is the expected percentage of losses that have emerged. For example, a lag of 20% means that it is

<sup>1</sup> A second concept behind this paper, proposed by so-called friends of the authors, is to ensure a trip to Bermuda to present this paper. The authors eschew this as a basic concept of the paper.

<sup>2</sup> We will leave techniques to more capable actuaries. The reader may find, however, that these sophisticated techniques can be layered into the framework described herein.

expected that one fifth of the losses have emerged to date. The lag is also equal to the inverse of the cumulative loss development factor for the contract.), an expected payout pattern, an estimate of an aggregate distribution of result, a vector of committed capital over the lifetime of the deal and an estimated return on equity (ROE) for the deal. The pricing information that is developed is then used as the starting point of the reserving and risk analysis processes.

The reserving process begins by using the expected loss ratio, incurred lag and payout patterns developed from the pricing process. Every quarter, each contract is reserved either to its expected loss ratio, the Bornhuetter-Ferguson method, or other appropriate methods. After the data is assembled, staff from various professional departments meet and agree on expected ultimate loss ratios for each major contract. Over time, enough individual contract information is generated to provide feedback to the underwriters and pricing actuaries as part of their renewal process. Similarly, aggregate data is developed to help analyze future contracts.

The aggregate distribution that is used to price each contract is utilized in the reserving of each account. In particular, accounts with significant loss sensitive features are heavily dependent on the shape and variability of the aggregate distribution. Individual risk reserving also can provide consistent answers for accruals on contingent commissions and profit commissions.

The detailed information that is gathered from each contract also allows the company to assign capital to each contract for the current year, and for all years into the future where there is still risk as to the ultimate result. Using both actual data and simulation techniques, capital allocation formulas are continuously refined. Each quarter, a profitability study is produced showing profitability by contract, client company, line of business, and strategic business unit (SBU). In addition to "traditional" accounting data, the study aggregates vital statistics such as mean time to payment of losses, capital utilized, ROE, interest rate assumptions used, and performance vs. initial benchmarks (actual vs expected losses, ROE, etc).

As levels of sophistication continue to increase, more interesting analyses can be accomplished. These would include items such as estimating correlations between risks, estimating correlations between liability and asset accounts, defining drivers of economic results that affect the whole book (i.e.. interest rates), and determining an optimal debt to equity mix for the corporation.<sup>3</sup>

The most interesting fact is that the drivers of this type of analysis are not sophisticated mathematical techniques, but basic actuarial blocking and tackling. This includes good data from client companies, high levels of data quality for what is input into the reinsurer's systems, actuarial pricing software that allows for

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<sup>3</sup> In this case, there is almost perfect correlation between the importance of this work (very) and the impossibility of this work (very).

experience rating and exposure rating, and a generally agreed upon ROE methodology that is understandable by all parties involved in the process. Also necessary to the process are ways of linking pricing data to reserving and profitability data to provide continuous improvement in analysis and assumptions.<sup>4</sup>

## **II. CORPORATE PHILOSOPHY AND THE ANALYSIS OF RISK**

Not too many years ago, reinsurers operated largely by spreading risk. Shares of individual contracts tended to be relatively small, and actuarial involvement in the pricing of contracts was infrequent. As reinsurers and their clients have become more sophisticated, profit margins have been squeezed, and reinsurers have to work much harder to find structures that both satisfy client needs and provide an opportunity for adequate returns to capital. The concepts and techniques contained in this paper are contingent upon analyzing every risk in great detail. Therefore, these methodologies can only be well utilized by a company where the corporate philosophy matches up reasonably close to the following:

A. ***Be a lead reinsurer*** - While it is not necessary to always be a lead reinsurer, generally being the lead provides greater insight into a contract. There is more opportunity to talk to client company management about underwriting philosophy,

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<sup>4</sup> There are unlimited ways to do this, and there are always improvements needed. This plus the work on correlations (see Footnote Three) should keep the authors employed for thousands of years.

claims management philosophy, strategic direction and the like. Individual contract reserving and pricing doesn't do a lot of good if you can't really understand what the client is all about. Also, being the lead allows the reinsurer to work closely with the broker and client to create a structure that will maximize outcomes for both parties.

B. ***Underwrite Large Lines*** - The cost of understanding the risks in great detail are significant. These costs can only be justified if the reinsurer and the client are both willing to allow for large lines. This can be difficult, as often reinsurers are reluctant to take large lines as a major loss could seriously impact results. Similarly, clients are often unwilling to give a reinsurer a large line as sometimes they feel this means they may lose some control over the account. The best way to handle all of these issues is to develop a strong and trusting relationship between the reinsurer and the client.

C. ***Collaborative Environment*** - Individual contract analysis cannot be left to just the actuaries. There has to be a significant amount of input from all professional units of the company. Underwriting audits, claim audits and accounting audits need to be integrated into the pricing and reserving process. Contract language needs to be analyzed. Emerging issues of liability need to be explored. Each contract also has to be thought of as part of a relationship with the client, perhaps spanning many underwriting units and areas of expertise. There needs to be significant and varied client contact that is communicated to all members of the team. All the knowledge



gained in the collaboration of the various professional disciplines add to the value of both the pricing and the reserving of each individual contract. <sup>5</sup>

### III. PRICING INFORMATION

For every contract that is bound, a significant amount of information is collected through the pricing process. Even when contracts aren't bound, significant components of the following data are still available and can be added to the data warehouse. Basic information that is passed through into the corporate database on all contracts includes the following:

A. **Expected Loss Ratio (ELR)** - ELR's are developed using both experience and exposure rating. Data is obtained from the client company, and can be augmented with data from ZRC's proprietary database, or ISO, RAA, or the NCCI. For risks with property exposure, the ELR must have a catastrophe and non-catastrophe component.

B. **Aggregate Loss Distribution** - For each contract, an aggregate loss distribution needs to be established. The aggregate loss distribution describes the

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The amount of knowledge that is required to do this well is humbling. The authors are comforted in knowing that many others are responsible for adding to this body of knowledge. In addition, the authors acknowledge the huge value that is added by the others in the process.

probable loss scenarios that underlie the selected ELR. The aggregate distribution performs two functions. The first is to help estimate the riskiness of the contract and hence, the amount of capital required. The second is to help in the evaluation of different contract features such as loss corridors, contingent commissions or retrospective rating. These two issues are highly interactive. Starting with an aggregate loss distribution that describes the underlying loss process of a contract, the team can overlay different contract structures to analyze how the riskiness (required capital) and profitability changes. Aggregate distributions can be developed using a number of pre-packaged products. They also can be developed directly from the company's historic data, or selected by the team analyzing the contract.

C. **Loss Development Factors (Lags)** - Incurred lags (expected percentage of losses emerged over time) should also be developed as much as possible from client company data. Lags have multiple uses in this process. They are a critical element of the experience rating approach used to derive an expected loss ratio. They are also needed as part of the Bornhuetter-Ferguson process which will later be used as part of the reserving methodologies for the contract. Lags or loss development factors are often incorporated in a contract that has loss sensitive elements. They can also be an important determinant of the cash flow for the various features of a contract, such as when a contingent commission will be paid or when a retrospectively rated contract will generate additional premium (to the extent these items are measured from incurred losses).

D. **Payout Pattern** - Estimating a payout pattern is critical to understanding the risk of a reinsurance contract for many reasons. It provides the basis of the timing for loss cash flows which allow analysis of the present value of any contract. In aggregate, payout patterns are used to develop the duration of the book of liabilities and help set asset management policy. Payout patterns can also provide information on the sensitivity of a contract to inflation. Note that the payout pattern and the incurred lags need to be internally consistent for each contract. It is also important to think about the relationship of the payout pattern to the aggregate loss distribution for an individual risk. For some types of contracts, the "bad" end of the aggregate loss distribution may be more likely to arise due to a spate of early shock losses. Other contracts may be more influenced by long-emerging losses. All other things being equal, the former has a wider distribution of net present values than the later; they both have the same nominal aggregate loss distribution and expected payout, but the first contract has more downside on an economic or net present value basis.

E. **Analysis of ROE** - Of course, there needs to be some sort of metric that each company uses to determine whether they are generating appropriate returns from each contract. Rather than just using this metric as an underwriting decision tool, it is possible to capture information from this analysis in the corporate database. An example of this is our company's ROE (return on equity) methodology. The estimated expected return is the weighted average of the present values of all the estimated after tax cash flows from the contract over all of the points of the

aggregate distribution. Equity is then allocated to each contract based upon the downside risk of each contract (estimated from the aggregate loss distribution), the mean time to loss payment, the line of business underwritten as well as other factors. The estimated required capital for any contract is the present value of all future estimated capital commitments until the risk is extinguished on the contract.

A few more moments on ROE are necessary. The contracts that most reinsurers write have a varying degree of risk. And the causes of risk vary from contract to contract. Also, individual contracts can be correlated with other contracts such that potential for adverse results can increase dramatically. We believe that it is critical that any methodology for analyzing profitability contain a consistent way of measuring risk. We relate risk to capital need, wherein riskier deals require more capital and therefore a higher dollar return to preserve the ROE. There are other metrics that can be used effectively.

From the ROE process, we capture information on the present value of the cash flows under each scenario, the weighted average present value of the cash flows, the average interest rate used in the analysis, and a vector of required capital needed annually until the risk of the deal is extinguished. Creating this information and storing it in the corporate database allows for analysis of capital usage and expected vs. actual investment returns. It also allows for continual updating of capital allocation process assumptions.

We again stress that there are many metrics that can be used to help set pricing and profit targets, and there is nothing magical about how we define ROE. Two points are critical. First is that the methodology is widely understood by underwriters and actuaries. If there is no buy-in to the metric, it becomes another hurdle to be crossed rather than a value-added exercise. This argues for some simplicity of approach. The second point is that once the critical drivers of the metric are established, they should be captured and integrated into the databases that are used to manage the business. This information is at least as important as the accounting information that is collected and should be held (at least) to the same data quality standards.

#### **IV. THE RESERVING PROCESS USING INDIVIDUAL CONTRACT DATA**

With all of this data available, the reserving process by individual contract is relatively straightforward. Keep in mind that this reserving process is only one methodology and multiple methodologies can and should be used when arriving at a range of reasonable results. The individual contract method, with its intense focus on the "trees" sometimes can cause us to lose touch with the forest. Analysis of aggregate data is still the only way to view some over-arching trends such as a change in case reserve adequacy or a speed-up/slow down in claim payments.

A. **Initial Assumptions** - Generally, the reserving process starts by capturing the ELR on an ex-catastrophe basis and the appropriate incurred lag and payment patterns for each contract. All contracts are grouped into reserve "cells" for analysis. For larger contracts, a separate reserve cell is established to individually reserve the contract. For smaller contracts, multiple contracts with similar characteristics are combined into a reserve cell. Typically, such characteristics may be class of business (casualty vs. property, for example), line of business (auto liability, general liability, etc), quota share vs. excess of loss, high vs. low layer, etc.

We refer to these multiple contract groupings as aggregate cells. The initial ELR of an aggregate reserve cell will be the weighted average ELR of all contracts in the cell. The initial incurred lag and payment patterns for an aggregate cell will be selected from some subset of the contracts that enter into that cell, or sometimes by using other information (ISO, NCCI, RAA, ZRC's proprietary database).

As contracts are renewed in subsequent years, the corporate actuarial unit should review the ELR's and the lags for consistency with old years. Any major differences between contract years should result in further discussions among the reserving actuary, the pricing actuary and the underwriter to understand these differences.

Sometimes the contract terms change materially, resulting in a significant change in the ELR, the lags, or both. This could be due to a difference in price, layer or terms between years. In these cases, material differences between years can happen. Other times, new or updated information comes to light which materially changes the analysis, resulting in revised expectations for the current period. A common example is when more information (claim count triangles, pricing history, etc) is available in the renewal package than was available when the contract was initially priced. This additional information can greatly change expectations of a contract's profitability for both the current and prior years. When this is the case, the ELR's for old years are often updated to also reflect the new information.

As an example, let's say that we bound a new commercial multi-peril contract. The ceding company was not yet set up to supply data triangles, so the pricing analysis relied upon an exposure rating analysis and an ISO lag pattern. The overall pricing analysis came up with a 65% ELR. When the renewal package was received one year later, paid and incurred loss triangles were included. The analysis of the renewal contract resulted in an 85% ELR for the current year. After reviewing these results, and the company specific data that drove them, it was concluded that it was the historical data, not necessarily the latest twelve months activity, that drove the new loss ratio pick. If we would have had this data available when we priced the original contract one year ago, the ELR would have been a 75% after giving weight to both experience and exposure rating methodologies. In this case, we would change the prior year's ELR to reflect this new historical information.

In another example, suppose that a similar contract, originally priced at a 65% ELR, had run adversely over the first twelve months. Assume that lots of good information has been available on the contract since inception. Even though the estimated ultimate loss ratio, via the Bornhuetter-Ferguson method, was now 75%, we would not necessarily change the original 65% ELR that feeds that methodology. We have no reason to believe that the contract will not run a 65% loss ratio on a going forward basis.

As the information comes through the underwriting process, the corporate actuary also has responsibility to look for data quality and consistency. Is the payout pattern faster than the incurred lag pattern? Are the patterns very dissimilar to other treaties in the same line of business? Has the expected loss ratio dropped as industry pricing has weakened? In the pricing/reserving feedback process, the pricing actuaries each search very deeply into a smaller number of contracts, and the corporate actuaries spread their time over a larger number of contracts. The reserving actuary is usually in the best position to provide such reasonability checks.

**B. Quarterly Updating Process** - Each quarter, earned premium and case incurred losses are updated for each contract. Generally for our purposes, one of three methodologies (loss ratio, incurred Bornhuetter-Ferguson, paid Bornhuetter-Ferguson) are selected. For very green and for very long-tailed



contracts, it is often advisable to stick with the initial ELR as the estimated ultimate loss ratio for a period of time (12-24 months), rather than reacting too early to good or bad loss development news. For contracts that are more mature and for shorter tailed contracts, the incurred or paid Bornhuetter-Ferguson process using the initial ELR and pricing patterns are generally utilized. Information based on either of these methodologies, along with more detailed claims information, are provided to the SBU managers, underwriters, pricing actuaries, claims professionals, and accountants each quarter. As a group, these individuals along with the corporate actuarial staff will try to come to a more complete understanding of how each major contract and each aggregate reserve cell is performing. A large amount of time and effort is expended each quarter in this process.<sup>6</sup>

C. **Multiple Reserving Methodologies** - Individual contract reserving also allows us to experiment with different methodologies. These different methodologies can help in formulating a range of reasonable estimates. In addition to the standard methodologies (loss ratio, incurred Bornhuetter-Ferguson, paid Bornhuetter-Ferguson, incurred loss development, paid loss development) there is a bit more that can be done when reserving by individual contract. For example, we have calculated expected ultimate losses using a loss ratio methodology for lags less than 10% and an incurred Bornhuetter-Ferguson methodology for the rest. We

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Much of this time is expended creating user friendly reserving exhibits (an oxymoron?) that are comprehensible by those outside of the Actuarial profession. In addition, written summaries of the quarter's indications, trends, and oddities are very necessary icebreakers to the review process.

have used loss development capped and cuffed by plus or minus 25% of the Bornhuetter-Ferguson estimate.

One can also experiment with varying the parameters that feed the methodology of choice. We tried developing a range around the expected loss ratio, using the aggregate loss distribution. Another possibility is developing slow and fast lag patterns around the selected pattern, to develop a range of indications. The flexibility to try something new is a nice benefit of this process. You may find that different methodologies are necessary (produce more accurate results) for certain contracts or lines of business. You may also find that developing an indicated range of reserves helps validate (or call into question) your methodology of choice.

D. ***Feedback Loop for Renewals*** - The result of the quarterly process is to have a consensus-built estimate of how each major contract and many of the smaller contracts are performing. A major benefit of this type of process is that the IBNR should be appropriate for each contract (with some exceptions), and is not an allocation. In addition, the entire company has a buy-in to every IBNR number as each number was arrived at through a group process rather than by a corporate actuary sitting in his or her office. As experience matures across all contracts, it then also becomes possible to aggregate data to create pricing parameters such as loss development factors, trend factors and excess factors for your company's specific book of business.

E. **Mapping of Underlying Exposure** - It is extremely important to be able to map the exposure spread of every contract. This will affect how earned premium is allocated to accident year and how lag factors are interpolated (see next section). A simple drawing of a square or a parallelogram, is often sufficient to describe the loss occurring or risk attaching base case.<sup>7</sup> You then need to consider other pertinent facts in mapping the contract's exposure. Is there an unearned premium portfolio on the front end of this contract? At expiration, does the contract cut-off or run-off? Are the underlying policies written evenly throughout the life of the contract, or is there some seasonality to the ceding company writings? Also, are all underlying policies one year in length, of variable length, etc? Does the underlying exposure itself contain some type of seasonality? For example, the winter months may contain more than their proportional share of Homeowners' exposure. See Appendix A for practical examples of this process.

One should not ignore the premium earnings pattern that is implied by how the accounting department actually books the earned premium (which is mostly based upon how the ceding company reports written and unearned premium to the reinsurer). These bookings will map out the actual earnings and exposure patterns for the contract. We need to begin to tap this source of valuable information in our shop.

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<sup>7</sup> *Losses Occurring:* A contract which covers all accidents that occur (or are reported, in the claims made case) during the contract period. The exposure looks like an accident year box.

*Risks Attaching:* A contract which covers all accidents that result from underlying policies that incept during the contract period. The exposure looks like a policy year parallelogram.

F. ***Interpolation of Lag Factors*** - Another special topic that needs to be addressed is interpolation of lag factors. Most of the data that is received through the pricing-reserving loop is accident year in nature. For a January 1 losses occurring contract, usually no adjustment is needed. However, for contracts that are written on a risks attaching basis, or if there is a portfolio in, or if the effective date lands on other than a quarterly point, interpolations of existing lag factors will be required. Appendix B details a lag interpolation method and shows some of the situations and calculations in greater detail. Although interpolating the lags can be complex, this process adds significantly to the understanding of each contract's results.

There is a further special case for quota share business. In order to completely match reported loss and IBNR on an individual contract, the interpolation has to be to the date of the last bordereau report, not the date of the reserve study. Quota share contracts are generally reported 30, 60 or 90 days in arrears. So using the date of the reserve study would understate ultimate loss. Also, some loss reports are not received by the end of the quarter, and a further adjustment is needed. For aggregate quota share cells, we assume all contracts are 90 days in arrears (we have tested this assumption on occasion, and it pretty much holds true). For excess accounts which report individual losses, we assume reporting is current and therefore consistent with the reserve study date.

## V. RESERVING FOR CONTRACT FEATURES

One of the major benefits of analyzing each contract individually and creating an aggregate distribution for each contract individually is in valuing special contract features. We differentiate between the value of a feature at expected loss, versus the expected value of the feature. We believe the latter method is more accurate.

An example may help. Suppose a contract contains a profit commission feature such that we pay one-half point of commission for every point under a 65 loss ratio. Further, suppose our expectations of the contract's loss ratio is currently a 67. In this case, the profit commission at the expected loss ratio is zero. However, based on the distribution of potential ultimate losses around the 67, the expected profit commission may be 2 or 3 points (because within the aggregate distribution of results, there are possibilities that the loss ratio may fall below a 65, and some weight must be given to those possibilities). This distinction is important, especially when estimating profitability for individual contracts. We price the features based on their expectations and we should reserve for them on a similar basis. We currently do not follow this in our shop, as we book the contractual commission based upon the expected losses. The change is being discussed, though. The following are more detailed examples of contract features:

A. ***Retrospectively Rated Contracts*** - For almost all retrospectively rated reinsurance contracts, the aggregate loss distribution has a wider swing than the

minimum and the maximum loss in the premium calculation. As a simple example, suppose there is a contract with a contractual loss ratio of 80 (that is, premium is calculated as losses multiplied by 100/80), a minimum premium of 0, and a maximum premium of \$1m. If expected losses are \$400,000, based on that point estimate, one would expect earned premium of \$500,000. However, suppose the \$400,000 is the weighted average of an aggregate loss distribution with a high end of \$2m. If we were to take the weighted average of the retrospective premium under all of these scenarios, the resulting premium would be somewhat less than \$500,000, and the resulting loss ratio would be somewhat greater than 80 (weight is being given to the possibility that losses may exceed \$800,000, with no resulting increase in premium after that point).<sup>8</sup> See Appendix C for more details.

**B. Loss Corridors** - In many situations, a reinsured will agree to pay for losses occurring in a certain layer either defined by loss ratio or dollars of loss. Sometimes this loss corridor appears below the expected loss amount and sometimes well above the expected loss amount. By creating an aggregate loss distribution, each loss corridor can be priced and reserved for. This can be a very complex exercise on an individual contract basis. Suppose a contract had an initial expected gross loss ratio of 70 with a loss corridor from 65 to 75. Perhaps based on the aggregate loss distribution, the corridor was worth 3 points, bringing the net loss ratio to 67.

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<sup>8</sup> The hardest part of this process is convincing the accounting staff that there is a need to book a loss ratio in excess of the contractual loss ratio. The authors leave this as an exercise for the reader.

Now it's two years later and the estimate of the ultimate gross loss ratio has increased to 85. Clearly the value of the loss corridor is now more than 3 points, but its value is still somewhat unclear. In these situations, we would either have to rely on a subsequent study or be able to develop a re-estimated picture of the complete aggregate loss distribution after two years of emerged experience. Appendix D shows more details.<sup>9</sup>

C. **Loss Caps** - Often, contracts are capped either as to absolute dollar amount or to number of reinstatements. Given that the cap is lower than the high end of the expected aggregate loss distribution, the weighted average expected loss ratio net of the cap should be lower than the initial (uncapped) expected loss ratio.

## VI. PROFITABILITY AND CAPITAL USAGE

With all of this great data assembled and ready to use in one place, there are certainly many other types of analyses that can be done other than straight reserving. The following lists a few of the analyses that can now be accomplished using the compiled data:

A. **Profitability Studies** - The huge amount of work that individual contract pricing and reserving requires really bears fruit when looking at individual contract

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<sup>9</sup> The authors would prefer to say that they have developed a theoretically defensible process that re-estimates the aggregate loss distribution over time to facilitate the re-evaluation of loss sensitive features, but they'd be lying. However, this doesn't keep us from trying. All ideas and good wishes are appreciated.

profitability. Since all of the numbers are built bottom-up and built on a consensus basis, each contract's profitability can truly be considered a best estimate. One can then aggregate contracts by underwriting year, underwriter, pricing actuary, client company, client group, line of business, attachment point, etc, etc. The profitability of any slice or aggregation of the business is then also a consensus based estimate. Management should have more confidence in this approach as compared to a top down allocation.

B. **Basic Capital Analyses** - It is also now relatively easy to compare the amount of capital that has been allocated for all contracts underwritten to the total capital available in the corporation. If the capital allocation methodology is well accepted by management, then aggregating the capital numbers can give management an idea of whether capital is being under-utilized, fully utilized or over-utilized. Conversely, the comparison between allocated and actual capital can be used to help make refinements to the capital allocation procedures.

C. **Advanced Capital Analyses** - With aggregate loss and NPV distributions for each contract, there are many things that can be done to determine the aggregate capital need of a reinsurer. We have recently been playing around with different types of these analyses. Our basic technique is to run simulations by selecting from each aggregate loss distribution. When doing this type of analysis, there are at least two interesting questions that need to be answered, and probably a lot more than that. The first question is one of correlation. How much or how little correlation



is there between two accounts in the same line of business? How about two accounts for the same company? How about the same questions with different lines of business and different companies? It is also possible that correlation could vary across the distribution. For example, contracts might be highly correlated at the low end due to low inflation or some other factor, but act more independent at the high end of the aggregate loss distribution. The second question that needs to be answered is one of how the shape of the aggregate loss distribution changes as the contract matures. Generally, the risk of each contract should shrink over time, so one would expect the aggregate loss distribution to grow more narrow with time. But, does it narrow as losses are incurred, paid, or based on some combination of the two? Are there some contracts, such as workers compensation, where the risk is greatly reduced early on as the cases emerge while other types of business, such as excess umbrella, might remain a question mark for many years to come?

We certainly have more questions than answers, but we have attempted some interesting things with the data we have collected. One example is our work on Umbrella. We started with the aggregate loss distribution for each contract that we wrote in a given year. From our ROE model, we extracted the net present value profit from each loss scenario (we "discretize" our aggregate distributions into scenarios). We knew that each contract was somewhat independent, but also correlated due to things like inflation and tort law. We even figured that some losses would be fully correlated as there are sometimes more than one client writing

different layers of the same risk. So here's what we did -- we randomly selected pairs of contracts and fully correlated them (i.e. if Contract A was at the 80 percentile of the distribution, then so was Contract B). Then we randomly selected from the fully correlated pairs. We then re-selected the correlated pairs and went through the process again. We ran one million scenarios in this fashion and compared the "bad" end of the distribution with the capital allocated to the contracts. Then we did the same thing with groups of three contracts, four contracts and five contracts to see how much the shape of the aggregate distribution would change.

While we are not sure if we accomplished anything important, or theoretically defensible, we did get comfortable with our capital allocation and we had fun. If we can scrape more time together, having all of this information available should yield more interesting things in the future.

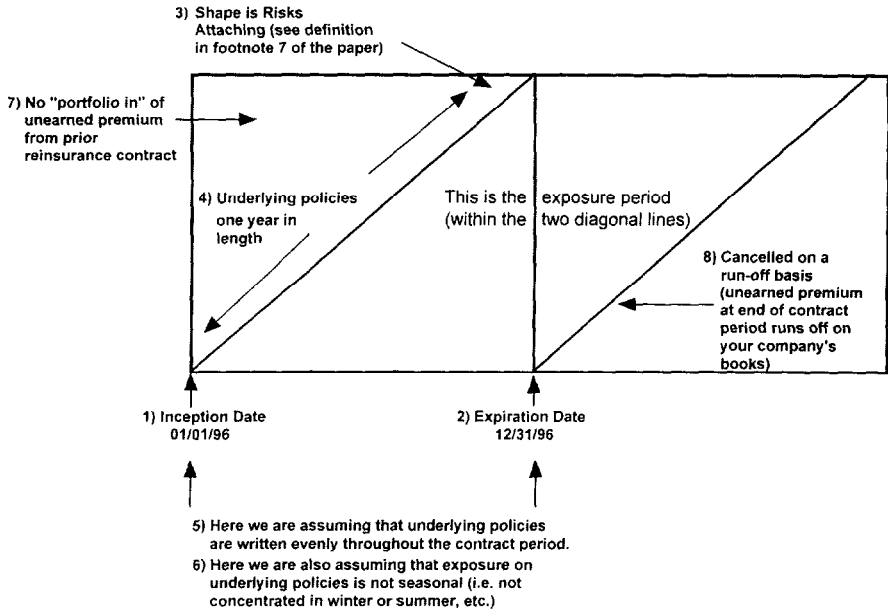
## **VII. Conclusion**

At the core of this paper is the belief that there is real value in an integrated pricing and reserving process on an individual contract basis. Over time, this type of process can lead to a greater in-depth knowledge of clients, the marketplace and profitability. This knowledge should create value for both the client and the reinsurer in jointly understanding the risks of their businesses and in establishing an appropriate price per unit of risk. The process requires everyone's commitment and much hard work.

Call us in ten years, and we'll let you know how (and if) it worked.

APPENDIX A

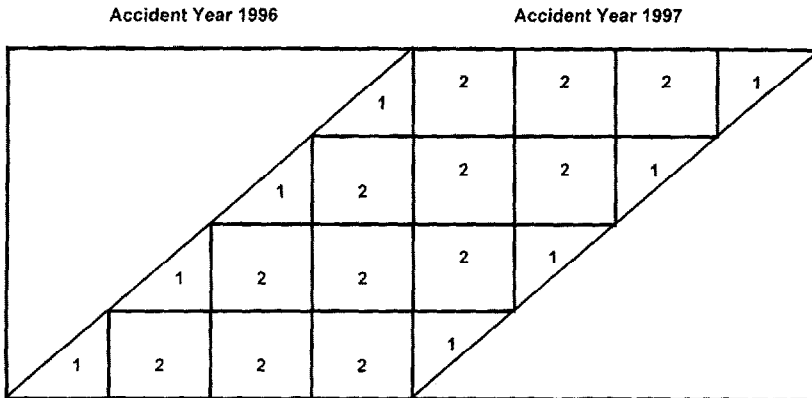
EXHIBIT 1



PARAMETER	SPECIFICS
1) Inception Date	01/01/96
2) Expiration Date	12/31/96
3) Claims Basis (risks attaching or losses occurring)	R.A.
4) Length of underlying policies	12 months
5) Seasonality in writings of underlying policies	no
6) Seasonality in exposure	no
7) Unearned Premium portfolio at the beginning of the contract	no
8) Cancelled on a cut-off or run-off basis	run-off

## APPENDIX A

### EXHIBIT 2



- 1) Once the exposure has been sketched, block-off the area within the exposure period.
- 2) Enter relative weights within the various blocks of exposure (here each full block has a weight of two, and therefore half a block has a weight of one).
- 3) Count the weights within each quarterly period to determine the exposure within that quarter (for this example, quarters one through four for AY 1996 would be 1, 3, 5, 7).
- 4) Also take a cumulative count of the weights, quarter to quarter, to determine the percent exposed over time (for this example, quarters one through four for AY 1996 would be 1, 4, 9, 16 or 6%, 25%, 56%, 100%).
- 5) All of the exposure information necessary to perform the lag interpolation process is now present (see Appendix B for that process).

**APPENDIX A**

**EXHIBIT 3**

Accident Year 1996				Accident Year 1997			
2	2	2	2				
2	2	2	2				
2	2	2	2				
2	2	2	2				

<u>PARAMETER</u>		<u>SPECIFICS</u>
1)	Inception Date	01/01/96
2)	Expiration Date	12/31/96
3)	Claims Basis (risks attaching or losses occurring)	L.O.
4)	Length of underlying policies	12 months
5)	Seasonality in writings of underlying policies	no
6)	Seasonality in exposure	no
7)	Unearned Premium portfolio at the beginning of the contract	yes
8)	Cancelled on a cut-off or run-off basis	cut-off

**APPENDIX A**

**EXHIBIT 4**

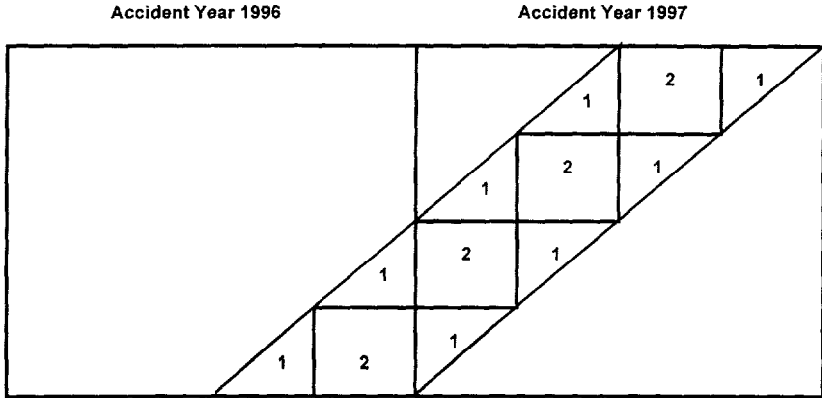
Accident Year 1996		Accident Year 1997		
2	6	2	2	
2	6	2	2	
2	6	2	2	
2	6	2	2	

<b>PARAMETER</b>	<b>SPECIFICS</b>
1) Inception Date	10/01/96
2) Expiration Date	09/30/97
3) Claims Basis (risks attaching or losses occurring)	L.O.
4) Length of underlying policies	12 months
5) Seasonality in writings of underlying policies	no
6) Seasonality in exposure	yes *
7) Unearned Premium portfolio at the beginning of the contract	yes
8) Cancelled on a cut-off or run-off basis	cut-off

\* Note that half of this contract's exposure falls during the winter months (first quarter of 1997).

**APPENDIX A**

**EXHIBIT 5**



PARAMETER	SPECIFICS
1) Inception Date	07/01/96
2) Expiration Date	12/31/96
3) Claims Basis (risks attaching or losses occurring)	R.A.
4) Length of underlying policies	12 months
5) Seasonality in writings of underlying policies	no
6) Seasonality in exposure	no
7) Unearned Premium portfolio at the beginning of the contract	no
8) Cancelled on a cut-off or run-off basis	run-off



## APPENDIX B

### Lag Factor Interpolation

To set the correct mood for this process, let's start with a little joke. How many actuaries does it take to interpolate a lag factor? And the answer is -- However many you want. Not a very funny joke, but a very pointed statement. Every actuary seems to have their own interpolation method. None of them are correct, but they're all pretty good estimates. What's being presented here is one of those methods.<sup>10</sup>

We'll begin with an incurred accident year lag pattern at twelve month evaluations (twelve months, twenty-four months, etc.). Next, we'll need to create factors at each quarter point. For evaluations after twelve months, linearly interpolate between twelve month points. Granted this is not exactly correct, since any given loss development pattern is not linear between annual points. But, for this particular method, it's close enough.

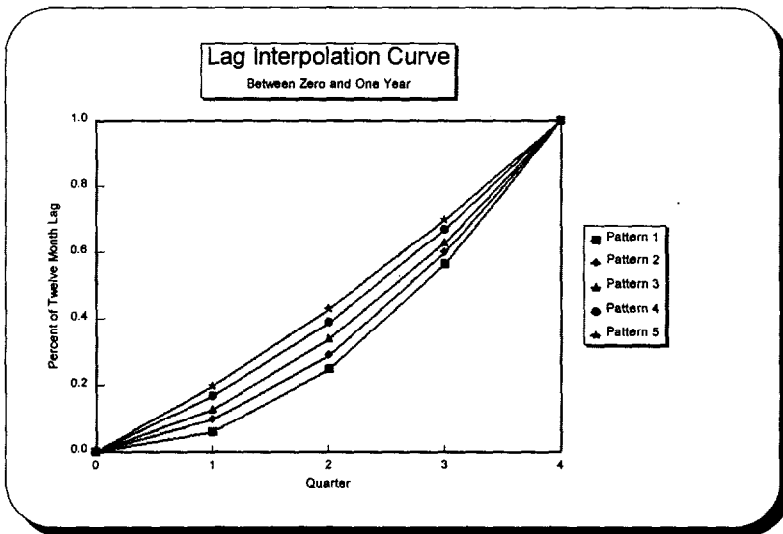
For interpolated factors at the first three quarters, we'll be a bit more careful. The loss development curve between zero and one year definitely has a ramping up which we cannot ignore. Think of it as accidents just beginning to happen and loss

---

<sup>10</sup> The general ideas behind the interpolation methods described herein were taught to one of the authors by Malcolm Handte, FCAS, MAAA. Interpretations of this interpolation method have changed some over time. Resulting lags are usually close to other actuaries' interpolated lags in our shop.

reporting beginning to get into the swing of things. Table 1 contains a graph of five curves that represent more to less severe ramp-ups between zero and one. The most severe is roughly  $(1/4)^2$ ,  $(2/4)^2$ ,  $(3/4)^2$ ,  $(4/4)^2$ , more parabolic in nature. The least severe is very close to linear. The factors in Table 2 correspond to the points in the five graphs, and represent factors to apply to the twelve month lag point, to create lags at the first three quarters. We tend to choose pattern one for long tailed casualty patterns (where twelve month lag points are, say, 15% or less). Conversely, pattern five goes well with quicker property patterns (with twelve month lags of 65% or greater). Anything between those two can use patterns two, three or four, based upon one's particular judgment.

**TABLE 1**



**TABLE 2**

<b>Quarter</b>	<b>Pattern 1</b>	<b>Pattern 2</b>	<b>Pattern 3</b>	<b>Pattern 4</b>	<b>Pattern 5</b>
1	6%	10%	13%	17%	20%
2	25%	29%	34%	39%	43%
3	56%	60%	63%	67%	70%
4	100%	100%	100%	100%	100%

We now have an accident year lag pattern at quarterly evaluations. Graph it if you like and, if you feel so inclined, smooth some points more to your liking. Now, if all reinsurance contracts were January 1 incepting losses occurring contracts, the task at hand would be complete. Unfortunately, a more general method is necessary to estimate accident year lags for things like a September 20 incepting risks attaching contract.

In order to accurately interpolate lag factors of any given point in time, we must be able to sketch the exposure of the given contract. Refer to Appendix A for this process. Recall, gathered information must include the inception and expiration dates of the contract, the length of the underlying primary policies, any seasonality imbedded in the exposure, whether the contract is losses occurring or risks attaching, whether or not there is a portfolio of unearned premium at the beginning of the contract, and whether the contract is canceled runoff or cutoff.

Once the exposure has been sketched, more necessary information must be gathered in order to complete the interpolation calculation. Table 3 contains the interpolation formula as well as a list of the necessary pieces of information within that equation. For example, if the Evaluation Date (ED) is 9/30/96 and earned premium has been booked (received or accrued) by the reinsurer through this date, then the Premium Information Date (PID) will be 9/30/96. But, if a quota share contract has a one quarter reporting delay (at 9/30/96, the reinsurer has just received the primary company premium statement through 6/30/96), and the reinsurer does not accrue for the missing premium, then the PID will be 6/30/96. If you are dealing with an excess of loss contract, and no loss reporting delay is apparent, then the Loss Information Date (LID) will be 9/30/96. In the case of a quota share contract, the LID will equal the last date through which primary company loss statements have been received.

**TABLE 3**

$$\text{Equation: } \text{LAG (PID)} = \text{LAG [LID - MED + AF]} \times \text{MF}$$

<b>Parameter</b>	<b>Description</b>
PID	Premium Information Date (usually equal to the Evaluation Date - ED)
LID	Loss Information Date
MED	Mean Exposure Date
AF	Additive Factor
MF	Multiplicative Factor

The Mean Exposure Date (MED) is the average accident date for the premium earned so far. It can usually be determined by viewing the exposure parallelogram and drawing a line through the apparent mean of exposure. For more complicated risks attaching shapes, once the exposure diagram has been properly drawn and weighted, as in Appendix A, we can add up the weights (area under the curve) and divide by two to get the mean. Then we can determine (usually by eye) where this mean falls on the exposure parallelogram.

Let's look at Exhibit 2 in Appendix A and calculate the MED. The contract is 1/1/96 incepting and is risks attaching. If the ED (and PID) is 12/31/96, we need to find the mean area under the triangle between 1/1/96 and 12/31/96. Note that the weights (area under the curve) are equal to sixteen. Half of this is eight. By counting back from the 12/31/96 point, we can see that the MED falls slightly to the left of 10/1/96. Here we can estimate and call the MED 9/15/96 (10/1/96 would also be a fairly good, and easier to handle, estimate).

We can now calculate the relative "age" of the given exposure, as the loss information date minus the mean exposure date. Note that this is the key expression in our search for the appropriate lag factor. Whether we are dealing with a risks attaching or losses occurring contract, and regardless of any other parameters, this relative age of exposure will determine how much time has passed since the mean date of exposure (or the average accident date), and thus how "developed" this exposure is.

A twelve month accident year lag factor assumes a 7/1 average date of loss, or six months of average loss emergence, not twelve months. The loss information date minus the mean exposure date must be increased to reflect this accident year assumption, or else the lag would be understated, yielding expected losses to date that are too low. Therefore, the Additive Factor (AF) is determined based on symmetry -- six months of emerged loss needs a six month AF, three months of emerged loss needs a three month AF, etc. The examples in Exhibit 1 display this.

Table 4 contains the AF values at the first four quarterly evaluations. The quarter four factor is also the factor for all quarters greater than four, and for quarters where the year of your evaluation date is greater than the accident year you are choosing a factor for (hence the exposure in that AY has ended).

**TABLE 4**

<b>Additive Factor (AF)</b>		<b>Multiplicative Factor (MF)</b>	
<b>Quarter</b>	<b>Factor (months)</b>	<b>Quarter</b>	<b>Factor</b>
1	1.5	1	$4/1 = 4.00$
2	3	2	$4/2 = 2.00$
3	4.5	3	$4/3 = 1.33$
4+	6	4+	$4/4 = 1.00$

The Multiplicative Factor (MF) is necessary to gross up the full exposure AY lag for the portion of exposure "earned" to date. The MF values are listed in Table 4. In the second example on Exhibit 1, the six month factor pulled from the AY lag pattern must be multiplied by  $4/2 = 2$  since only half of the full AY's exposure (and premium) has been earned as of 6/30/96. Note that the six month lag of 14% (in the footnote on Exhibit 1) means that six months after the inception date of this contract, 14% of the total estimated ultimate losses are estimated to have been reported. In our example, since half of the full exposure has been earned as of 6/30/96,  $(.14 \times 2) = 28\%$  of the six month exposure period's ultimate losses are estimated to have been reported as of 6/30/96.

Exhibit 2 deals with a 10/1/96 incepting losses occurring contract. Note that from year-end to the following first quarter (examples one and two), the AF and MF values jump from the first quarter values in Table 4 to the fourth quarter values. The resulting lags appear to be smooth and quite reasonable to the authors.

Risks attaching cases are explored in Exhibits 3 and 4. Note that we use the same AF and MF factors for the risks attaching cases as we do with the losses occurring cases. This has been challenged by other actuaries in our shop. The MF is easy to question since it is meant to gross up the full exposure AY lag for the portion of exposure "earned" to date. If the losses occurring MF at 6/30/96 is  $4/2 = 2$  since half of the AY exposure has been earned, then why isn't the risks attaching MF at

6/30/96 equal to  $16/4 = 4$  (since only 25% of a risks attaching contract's AY exposure has been earned as of 6/30)? We believe that if the MF for the risks attaching cases were increased in this fashion, then the AF would necessarily have to be decreased or else your answers would be too large. We experimented with a few different sets of factors and really could not get any to work as reasonably well as the current set.

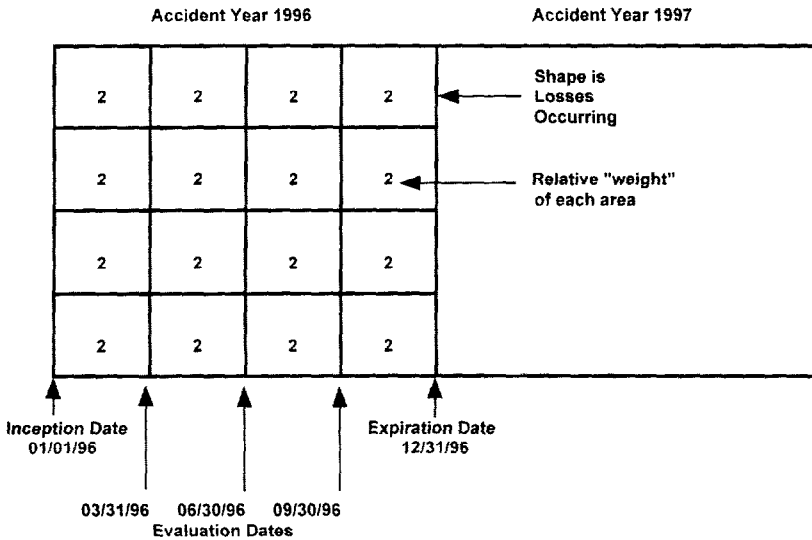
It is a premise of this methodology that the (LID - MED) expression creates an AY type measure of exposure. Whether the contract being considered is losses occurring or risks attaching in nature, the relative "age" of exposure we have calculated is now a general measure of development that can then be used in the overall interpolation formula (which uses AF and MF values that are losses occurring or AY in nature).

The basic "check" of this process is whether or not the resulting interpolated lags appear reasonable, especially when compared to the interpolated lags in the quarters surrounding your evaluation quarter. The risks attaching and losses occurring lags of the same "age" should also appear reasonable (and relatively close together) when compared to each other. It also helps to compare results to those from other actuaries' interpolation methods. This interpolation method should produce smooth and reasonable results.



APPENDIX B

EXHIBIT 1\*



$$AY 19\#\# \text{ Lag @ PID} = \text{Lag} [ (LID - MED) + AF ] \times (MF)$$

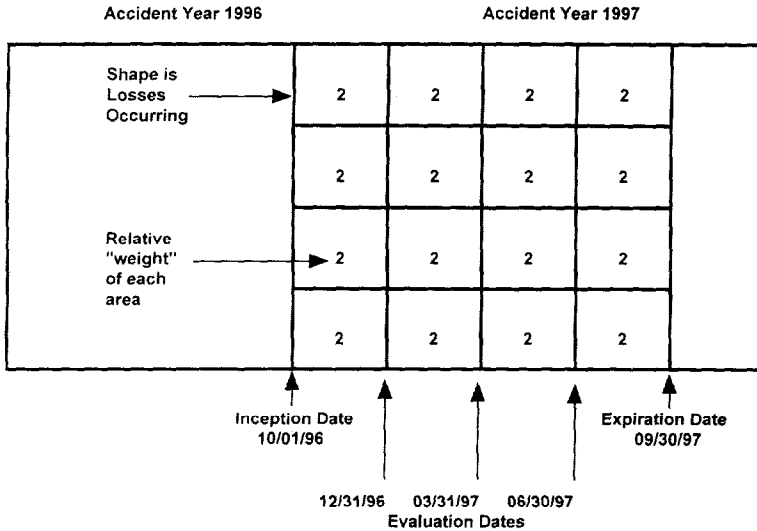
- 1) AY 1996 Lag @ 03/31/96
  - = Lag [ (03/31/96 - 2/15/96) + 1.5 ] x (4/1)
  - = Lag [ 1.5 + 1.5 ] x (4/1)
  - = Lag [ 3.0 ] x (4/1)
  - = [ 3 mo. lag ] x (4/1)
  - = [ 0.06 ] x (4/1)
  - = 0.240
  
- 2) AY 1996 Lag @ 06/30/96
  - = Lag [ (06/30/96 - 4/01/96) + 3.0 ] x (4/2)
  - = Lag [ 3.0 + 3.0 ] x (4/2)
  - = Lag [ 6.0 ] x (4/2)
  - = [ 0.14 ] x (4/2)
  - = 0.280
  
- 3) AY 1996 Lag @ 09/30/96
  - = Lag [ (09/30/96 - 5/15/96) + 4.5 ] x (4/3)
  - = Lag [ 4.5 + 4.5 ] x (4/3)
  - = Lag [ 9.0 ] x (4/3)
  - = [ 0.26 ] x (4/3)
  - = 0.347

\* Note that the lags for Exhibits 1 through 4 are as follows:

Months	3	6	9	12	15	18	21	24
Lag	6.0%	14.0%	26.0%	40.0%	48.0%	56.0%	64.0%	72.0%

APPENDIX B

EXHIBIT 2



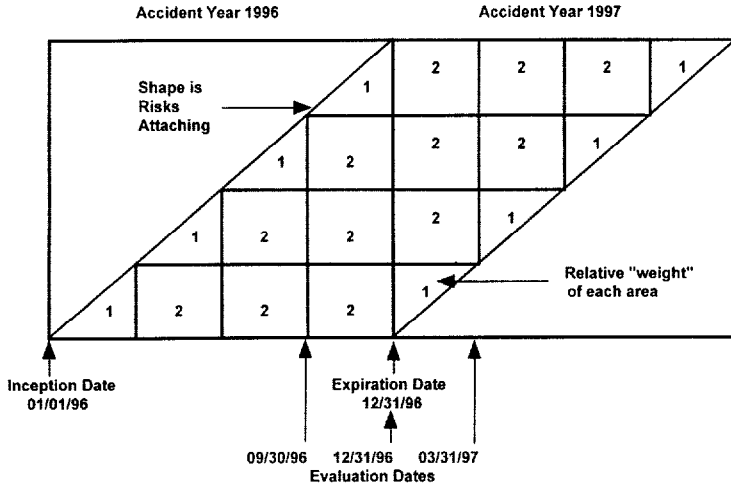
$$\text{AY 19## Lag @ PID} = \text{Lag} [ (\text{LID} - \text{MED}) + \text{AF} ] \times (\text{MF})$$

- 1) AY 1996 Lag @ 12/31/96
  - = Lag [ (12/31/96 - 11/15/96) + 1.5 ] x (4/1)
  - = Lag [ 1.5 + 1.5 ] x (4/1)
  - = Lag [ 3.0 ] x (4/1)
  - = [ 3 mo. lag ] x (4/1)
  - = [ 0.06 ] x (4/1)
  - = 0.240
  
- 2) AY 1996 Lag @ 03/31/97
  - = Lag [ (03/31/97 - 11/15/96) + 6.0 ] x (4/4)
  - = Lag [ 4.5 + 6.0 ] x (4/4)
  - = Lag [ 10.5 ] x (4/4)
  - = [(9 mo. lag) + (12 mo. - 9 mo. lags) x (1/2)] x (4/4)
  - = [ 0.26 + (0.40 - 0.26) x (1/2) ] x (4/4)
  - = [ 0.26 + (0.14) x (1/2) ] x (4/4)
  - = [ 0.26 + 0.07 ] x (4/4)
  - = [ 0.33 ] x (4/4)
  - = 0.330
  
- 3) AY 1996 Lag @ 06/30/97
  - = Lag [ (06/30/97 - 11/15/96) + 6.0 ] x (4/4)
  - = Lag [ 7.5 + 6.0 ] x (4/4)
  - = Lag [ 13.5 ] x (4/4)
  - = [ 0.40 + (0.48 - 0.40) x (1/2) ] x (4/4)
  - = 0.440

Linearly interpolating between 9 and 12 month points

APPENDIX B

EXHIBIT 3

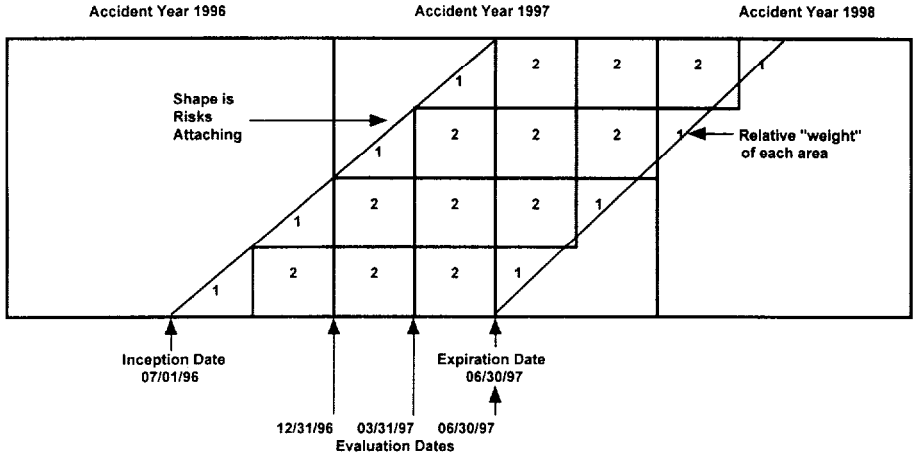


AY 19## Lag @ PID = Lag [ (LID - MED) + AF ] x (MF)

- 1) AY 1996 Lag @ 09/30/96
  - = Lag [ (09/30/96 - 7/15/96) + 4.5 ] x (4/3)
  - = Lag [ 2.5 + 4.5 ] x (4/3)
  - = Lag [ 7.0 ] x (4/3)
  - = [(6 mo. lag) + (9 mo. - 6 mo. lags) x (1/3) ] x (4/3) ← Linearly interpolating between 6 and 9 month points
  - = [ 0.14 + (0.26 - 0.14) x (1/3) ] x (4/3)
  - = [ 0.14 + (0.12) x (1/3) ] x (4/3)
  - = [ 0.14 + 0.04 ] x (4/3)
  - = [ 0.18 ] x (4/3)
  - = 0.240
  
- 2) AY 1996 Lag @ 12/31/96
  - = Lag [ (12/31/96 - 9/15/96) + 6.0 ] x (4/4)
  - = Lag [ 3.5 + 6.0 ] x (4/4)
  - = Lag [ 9.5 ] x (4/4)
  - = [ 0.26 + (0.40 - 0.26) x (1/6) ] x (4/4)
  - = 0.283
  
- 3) AY 1996 Lag @ 03/31/97
  - = Lag [ (03/31/97 - 9/15/96) + 6.0 ] x (4/4)
  - = Lag [ 6.5 + 6.0 ] x (4/4)
  - = Lag [ 12.5 ] x (4/4)
  - = [ 0.40 + (0.48 - 0.40) x (1/6) ] x (4/4)
  - = 0.413

APPENDIX B

EXHIBIT 4



$$\text{AY 19## Lag @ PID} = \text{Lag} [ (\text{LID} - \text{MED}) + \text{AF} ] \times (\text{MF})$$

- 1) AY 1996 Lag @ 12/31/96
  - = Lag [ (12/31/96 - 11/01/96) + 3.0 ] x (4/2)
  - = Lag [ 2.0 + 3.0 ] x (4/2)
  - = Lag [ 5.0 ] x (4/2)
  - = [ (3 mo. lag) + (6 mo. - 3 mo. lags) x (2/3) ] x (4/2) ← Linearly interpolating between 3 and 6 month points
  - = [ 0.06 + (0.14 - 0.06) x (2/3) ] x (4/2)
  - = [ 0.06 + (0.08) x (2/3) ] x (4/2)
  - = [ 0.06 + 0.053 ] x (4/2)
  - = [ 0.113 ] x (4/2)
  - = 0.226
  
- 2) AY 1996 Lag @ 03/31/97
  - = Lag [ (03/31/97 - 11/01/96) + 6.0 ] x (4/4)
  - = Lag [ 5.0 + 6.0 ] x (4/4)
  - = Lag [ 11.0 ] x (4/4)
  - = [ 0.26 + (0.40 - 0.26) x (2/3) ] x (4/4)
  - = 0.353
  
- 3) AY 1996 Lag @ 06/30/97
  - = Lag [ (06/30/97 - 11/01/96) + 6.0 ] x (4/4)
  - = Lag [ 8.0 + 6.0 ] x (4/4)
  - = Lag [ 14.0 ] x (4/4)
  - = [ 0.40 + (0.48 - 0.40) x (2/3) ] x (4/4)
  - = 0.453

## APPENDIX C

### Reserving For Account Features Contracts With Loss Corridors

**Terms of Contract:**

Premium Calculated as Losses Multiplied by 100/80  
(Note: Results in "Contractual Loss Ratio" of 80)

Minimum Premium = \$0

Maximum Premium = \$1,000,000

**Expected Losses: \$400,000**

**Expected Aggregate Loss Distribution:**

(Note: Displayed on an Incremental Basis)

Scenario	Probability	Losses
1	10.0%	\$100,000
2	20.0%	\$200,000
3	26.0%	\$300,000
4	15.0%	\$400,000
5	10.0%	\$500,000
6	8.0%	\$600,000
7	5.0%	\$800,000
8	3.0%	\$1,000,000
9	2.0%	\$1,200,000
10	1.0%	\$2,000,000
Expected	100.0%	\$400,000

## APPENDIX C

### Reserving For Account Features Contracts With Loss Corridors

#### Calculation of Retro Premium and Expected Loss Ratio

Probability	Losses	Premium
10.0%	\$100,000	\$125,000
20.0%	\$200,000	\$250,000
26.0%	\$300,000	\$375,000
15.0%	\$400,000	\$500,000
10.0%	\$500,000	\$625,000
8.0%	\$600,000	\$750,000
5.0%	\$800,000	\$1,000,000
3.0%	\$1,000,000	\$1,000,000
2.0%	\$1,200,000	\$1,000,000
1.0%	\$2,000,000	\$1,000,000
<b>Expected Amounts</b>	<b>\$400,000</b>	<b>\$467,500</b>
<b>Expected Loss Ratio</b>	<b>85.6%</b>	

## APPENDIX C

### Reserving For Account Features Contracts With Loss Corridors

#### Accounting For Premium and Losses Over Time

Start with the Expected Loss and Premium

Expected Loss = \$400,000

Expected Premium = \$467,500

Expected Loss Ratio = 85.6%

Theory: As time elapses, the aggregate distribution of loss collapses around a single point. If loss emerge as expected (\$400,000), premium will eventually reach \$500,000. We need a process that recognizes this but is simple to implement. Our solution was to create an "Insurance Charge" (IC) equal to Contractual Premium (\$500,000) less Expected Premium (\$467,500). The IC is multiplied by (1 - Lag), or the percent of loss expected to be unemerged at each point in time. As all losses are reported, the ultimate premium converges to the contractual premium. This stuff is not rocket science.....

(1)	(2)	(3)	(4)	(5)	(6)
End of Year	Estimated Ultimate Loss	Lag Factor	Estimated Insurance Charge	IC X (1 - Lag)	Estimated Ultimate Premium
0	\$400,000	0.00	\$32,500	\$32,500	467,500
1	\$400,000	0.25	\$32,500	\$24,375	475,625
2	\$400,000	0.50	\$32,500	\$16,250	483,750
3	\$400,000	0.70	\$32,500	\$9,750	490,250
4	\$400,000	0.85	\$32,500	\$4,875	495,125
5	\$400,000	0.95	\$32,500	\$1,625	498,375
6	\$400,000	1.00	\$32,500	\$0	500,000

Note: There are simpler ways of creating this process, but the above seems to be a good way of generically describing the retrospective premium process. Note that the IC could be calculated for contracts with additive loads, or a combination of additive and multiplicative loads.

## APPENDIX D

### Reserving For Account Features Contracts With Loss Corridors

**Terms of Contract:**

Quota Share contract with a loss corridor  
between a 65% and 75% loss ratio.

**Expected Premium:** **\$1,000,000**

**Expected Losses:** **\$700,000**  
(Ground up - excluding corridor)

**Expected Aggregate Loss Distribution:**  
(Note: Displayed on an Incremental Basis)

Scenario	Probability	Losses
1	6.0%	\$200,000
2	12.0%	\$400,000
3	20.0%	\$500,000
4	25.0%	\$600,000
5	14.0%	\$700,000
6	9.0%	\$800,000
7	6.0%	\$1,000,000
8	4.0%	\$1,500,000
9	2.0%	\$2,000,000
10	2.0%	\$3,000,000
Expected	100.0%	\$700,000



## APPENDIX D

### Reserving For Account Features Contracts With Loss Corridors

#### Calculation of Value of Loss Corridor and Expected Loss Ratio

Probability	Ground Up Losses	Loss Corridor	Net Losses
6.0%	\$200,000	\$0	\$200,000
12.0%	\$400,000	\$0	\$400,000
20.0%	\$500,000	\$0	\$500,000
25.0%	\$600,000	\$0	\$600,000
14.0%	\$700,000	\$50,000	\$650,000
9.0%	\$800,000	\$100,000	\$700,000
6.0%	\$1,000,000	\$100,000	\$900,000
4.0%	\$1,500,000	\$100,000	\$1,400,000
2.0%	\$2,000,000	\$100,000	\$1,900,000
2.0%	\$3,000,000	\$100,000	\$2,900,000
Expected	\$700,000	\$30,000	\$670,000

**Expected Premium:** **\$1,000,000**

**Expected Loss Ratio:** **67.0%**

**Expected Value of Corridor:** **3.0%**

Thus the initial reserves will be set to equal a 67% loss ratio

## APPENDIX D

### Reserving For Account Features Contracts With Loss Corridors

#### Aggregate Loss Distribution of the Same Account at 24 Months

Note: After time goes by, the aggregate loss distribution begins to collapse upon the point estimate. For purposes of illustration, we will assume that the aggregate distribution has collapsed by half (perhaps the account has a lag of .50 after 24 months). In the real world, the collapse of the aggregate distribution is often referred to as "non-trivial" which means pretty hard to do.

We will state the distribution as a percent of expected so we can apply to different evaluations of ultimate loss and see what the answers are.

Scenario	Probability	Initial Ground Up Losses	As % Of Expected	Collapse Factor	Agg Distrib @24 Months
1	6.0%	\$200,000	28.6%	0.5	64.3%
2	12.0%	\$400,000	57.1%	0.5	78.6%
3	20.0%	\$500,000	71.4%	0.5	85.7%
4	25.0%	\$600,000	85.7%	0.5	92.9%
5	14.0%	\$700,000	100.0%	0.5	100.0%
6	9.0%	\$800,000	114.3%	0.5	107.1%
7	6.0%	\$1,000,000	142.9%	0.5	121.4%
8	4.0%	\$1,500,000	214.3%	0.5	157.1%
9	2.0%	\$2,000,000	285.7%	0.5	192.9%
10	2.0%	\$3,000,000	428.6%	0.5	264.3%
Expected		\$700,000	100.0%		100.0%

The aggregate distribution as of 24 months is calculated by taking .5 of the difference between the initial losses as a percent of expected and unity and adding/subtracting this number to the initial losses as a percent of expected.

## APPENDIX D

### Reserving For Account Features Contracts With Loss Corridors

#### Calculation of Value of Loss Corridor and Expected Loss Ratio At 24 Months Using Collapsed Aggregate Distribution Examples Using Better and Worse Than Expected Results

Current Evaluation of Ultimate Loss: **\$500,000**  
 Expected Premium: **\$1,000,000**

Scenario	Probability	Agg Distrib @24 Months	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	\$321,429	\$0	\$321,429
2	12.0%	78.6%	\$392,857	\$0	\$392,857
3	20.0%	85.7%	\$428,571	\$0	\$428,571
4	25.0%	92.9%	\$464,286	\$0	\$464,286
5	14.0%	100.0%	\$500,000	\$0	\$500,000
6	9.0%	107.1%	\$535,714	\$0	\$535,714
7	6.0%	121.4%	\$607,143	\$0	\$607,143
8	4.0%	157.1%	\$785,714	\$100,000	\$685,714
9	2.0%	192.9%	\$964,286	\$100,000	\$864,286
10	2.0%	264.3%	\$1,321,429	\$100,000	\$1,221,429
<b>Expected</b>			<b>\$500,000</b>	<b>\$8,000</b>	<b>\$492,000</b>
<b>Expected Loss Ratio:</b>				<b>49.2%</b>	
<b>Expected Value of Corridor:</b>				<b>0.8%</b>	

Current Evaluation of Ultimate Loss: **\$850,000**  
 Expected Premium: **\$1,000,000**

Scenario	Probability	Agg Distrib @24 Months	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	\$546,429	\$0	\$546,429
2	12.0%	78.6%	\$667,857	\$17,857	\$650,000
3	20.0%	85.7%	\$728,571	\$78,571	\$650,000
4	25.0%	92.9%	\$789,286	\$100,000	\$689,286
5	14.0%	100.0%	\$850,000	\$100,000	\$750,000
6	9.0%	107.1%	\$910,714	\$100,000	\$810,714
7	6.0%	121.4%	\$1,032,143	\$100,000	\$932,143
8	4.0%	157.1%	\$1,335,714	\$100,000	\$1,235,714
9	2.0%	192.9%	\$1,639,286	\$100,000	\$1,539,286
10	2.0%	264.3%	\$2,246,429	\$100,000	\$2,146,429
<b>Expected</b>			<b>\$850,000</b>	<b>\$79,857</b>	<b>\$770,143</b>
<b>Expected Loss Ratio:</b>				<b>77.0%</b>	
<b>Expected Value of Corridor:</b>				<b>8.0%</b>	

## APPENDIX D

### Reserving For Account Features Contracts With Loss Corridors

#### Calculation of Value of Loss Corridor and Expected Loss Ratio At 24 Months Using Collapsed Aggregate Distribution

First let us assume that the estimate of ultimate losses has been unchanged at the 24 month evaluation.

Current Evaluation of Ultimate Loss: **\$700,000**  
 Expected Premium: **\$1,000,000**

Scenario	Probability	Agg Distrib @24 Month	Ground Up Losses	Loss Corridor	Net Losses
1	6.0%	64.3%	\$450,000	\$0	\$450,000
2	12.0%	78.6%	\$550,000	\$0	\$550,000
3	20.0%	85.7%	\$600,000	\$0	\$600,000
4	25.0%	92.9%	\$650,000	\$0	\$650,000
5	14.0%	100.0%	\$700,000	\$50,000	\$650,000
6	9.0%	107.1%	\$750,000	\$100,000	\$650,000
7	6.0%	121.4%	\$850,000	\$100,000	\$750,000
8	4.0%	157.1%	\$1,100,000	\$100,000	\$1,000,000
9	2.0%	192.9%	\$1,350,000	\$100,000	\$1,250,000
10	2.0%	264.3%	\$1,850,000	\$100,000	\$1,750,000
Expected			<b>\$700,000</b>	<b>\$30,000</b>	<b>\$670,000</b>
<b>Expected Loss Ratio:</b>				<b>67.0%</b>	
<b>Expected Value of Corridor:</b>				<b>3.0%</b>	

Note that the value of the corridor has not changed in this example. If the loss corridor is about the expected value of the distribution this is often the case. If the corridor had been well above the expected loss amount, the 24 month value of the corridor would have been reduced substantially. For example, if the loss corridor was 10 points excess of 140 LR (\$1,400,000) the value at 24 months would have been  $\$100,000 \times .02 = \$2,000$ , compared to an initial value of  $\$100,000 \times (.04 + .02 + .02) = \$8,000$ . (Note on Appendix D Page 3 that initial scenarios 8, 9, and 10 are all greater than \$1,400,000 with probabilities of .04, .02 and .02 respectively)