Loss Development and Annual Aggregate Deductibles
by Vincent P. Connor, ACAS
Loss Development and Annual Aggregate Deductibles

Vincent P. Connor

Abstract

The use of an Annual Aggregate deductible by a reinsurer can cause inconsistencies in loss development and incorrect IBNR reserves. This paper describes how AAD business can be added to non AAD business with the combined used to select loss development factors and estimate IBNR reserves when using a chain ladder or Bornhuetter/Ferguson method. The inclusion of similar AAD and non AAD business in loss development triangles increases the credibility of the loss development factors.

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LOSS DEVELOPMENT AND ANNUAL AGGREGATE DEDUCTIBLES

The reinsurer that uses Annual Aggregate Deductibles (AADs) needs to make some adjustments for reserving if it is using a Loss or Chainladder method (reported loss x (to ultimate factor minus one)) or an Expected Loss or Bornhuetter/Ferguson\(^1\) (B/F) method (premium x loss ratio x percent of loss unreported) to develop IBNR reserves.

This paper will describe how, using certain modifications, AAD business can be added to non AAD business with the combined used to select loss development factors and estimate IBNR reserves. The topic will be covered in four parts:

1. AAD/Reinsurance background.
2. AAD and Chainladder IBNR.
3. AAD and B/F IBNR.
4. AAD and Indicated Loss Ratios.

AAD/REINSURANCE BACKGROUND

Quota share reinsurance provides the benefits of reinsurance on all risks. Since there is a cost to reinsurance, and most ceding companies would like to minimize costs, some insurance companies look for other types of reinsurance that will meet their needs but lower their costs.

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One approach is to use surplus share reinsurance. For policies under a certain retention or line the company keeps 100%. For policies over the retention the company cedes on a share basis the amount over the retention or the surplus amount. For example, if the retention is $100,000 and the policy limit is $300,000 the company cedes 66 2/3% (($300,000 - $100,000)/$300,000) of this policy and recovers 66 2/3% of every loss. Small policies with limits of $100,000 or less cede 0% (($100,000 - 100,000)/100,000). The result is that the company has share reinsurance but just on the larger policies.

Another approach is to use excess reinsurance, which applies only to the larger claims. A $150,000 retention means the insurer pays for claims under $150,000 and also the first $150,000 of larger claims. Generally one pays less for excess reinsurance than for surplus share or share reinsurance.

The reinsurance premium can be further lowered if the ceding company has an annual aggregate deductible with its excess reinsurance. The ceding company might be willing to keep the first million dollars of excess losses per accident (or fiscal) year. This will then be a one million dollar annual aggregate deductible and the premium will be lower with an AAD because fewer losses are paid by the reinsurer. Including an AAD will normally result in increased volatility as the more predictable losses are being excluded.

There is a difference between an AAD and the usual deductible that is applied to an individual claim. A deductible, for example a $250 Auto Physical Damage deductible, applies to each claim. An AAD applies to all claims above the retention
until the aggregate deductible is reached. Table 1 shows the effect of an AAD on a series of five cases in chronological order.

<table>
<thead>
<tr>
<th>Loss #</th>
<th>Loss</th>
<th>Retains</th>
<th>Contribution</th>
<th>AAD to Date</th>
<th>Reinsurance</th>
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<tr>
<td>1</td>
<td>$500,000</td>
<td>$100,000</td>
<td>$400,000</td>
<td>$400,000</td>
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</tr>
<tr>
<td>2</td>
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<td>0</td>
<td>400,000</td>
<td>0</td>
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<td>3</td>
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<td>100,000</td>
<td>100,000</td>
<td>500,000</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>900,000</td>
<td>100,000</td>
<td>500,000</td>
<td>1,000,000</td>
<td>300,000</td>
</tr>
<tr>
<td>5</td>
<td>400,000</td>
<td>100,000</td>
<td>0</td>
<td>1,000,000</td>
<td>300,000</td>
</tr>
</tbody>
</table>

**Total**  $2,050,000  $450,000  $1,000,000  $1,000,000  $600,000
AAD AND CHAINLADDER IBNR

The AAD can cause an inconsistency in the loss development triangle for the reinsurer because no case losses are incurred until the AAD is eroded. The AAD business may contribute no losses in, say, the first two years, followed by a sudden increase in activity in the third year. How can this inconsistency be addressed?

One approach would be to group together in a triangle similar AAD business. This will work if there is enough similar business to be credible and if enough years are available to select loss development factors. Given that AADs are written on a fiscal or policy year as opposed to accident year basis, the varying sizes of AADs, the number of lines that might be covered, etc., this is not usually a very practical solution. If this approach is taken, data must be grouped so that the AAD is effective the first day of the year e.g. you should not look at an AAD that covers fiscal year on a calendar year basis.

Another approach is to include the business subject to the AAD with the non AAD business. We would handle the loss as if it were just excess, that is, include the ground up or eroded AAD loss in the triangle with non AAD business and make adjustments as appropriate. If losses gross of or before the AAD are consistent (as to loss development) with those without an annual aggregate deductible, then loss development factors can be selected in the usual manner from the combined data. This approach assumes that the computer system capture losses gross or before the AAD.
How the company structures its reserve segments can influence how AADs are handled. If all of a contract is in one segment it is easier to handle the AAD than if lines are in different segments and the AAD covers multiple lines.

Let's assume that we are using a Chainladder method to develop IBNR. That is, we are taking the reported losses by accident year and multiplying by the to ultimate development factor less one to determine IBNR. Let's also assume that the reported losses gross of the AAD for a particular accident year are $4 million, the ultimate gross of AAD losses are $8 million (the to ultimate development factor is then 2.0), and the AAD is $2 million. This is displayed on Figure 1.

If we follow the Chainladder method formula, we will develop an IBNR (assuming no AAD) of $4 million. That would be the reported losses of $4 million multiplied by the to ultimate factor minus one of 1.0 (2.0 - 1.0 = 1.0). If there is an AAD of $2 million and we use the net of AAD reported losses the IBNR calculated would be $2 million x (2.0-1.0) or $2 million. This is wrong because we are applying factors developed from losses gross of the AAD to losses that are net of the AAD. Since we normally would not have factors net of the AAD, the approach is to make the calculation gross of the AAD and then adjust, if necessary, for the AAD. In this case no adjustment is necessary for the AAD. The IBNR is $4 million.

Figure 2. shows a chart of the same accident year, only evaluated earlier, i.e. there are fewer reported losses. Gross of the AAD, there are $1 million of reported losses, the to ultimate factor is 8.0 and the Chainladder method IBNR is $7 million (gross of AAD). We can see that if we have ultimate losses gross of the AAD of $8 million, the most the reinsurer is going to pay is the $8 million of ultimate loss less
the $2 million of AAD or $6 million. In this case the correct IBNR net of the AAD, would be $6 million not the formula reserve of $7 million.

Figure 1

Ultimate Loss

$8 Million

$6 Million

IBNR

$4 Million

$2 Million

0

AAD

Reported

Ultimate Loss $8,000,000
Reported Loss $4,000,000 (Before AAD)
AAD $2,000,000
To Ultimate Factor 2.0 ($8,000,000 ÷ $4,000,000)
Ultimate Loss $8,000,000
Reported Loss $1,000,000 (Before AAD)
AAD $2,000,000
To Ultimate Factor 8.0 ($8,000,000 ÷ $1,000,000)
This example assumes that the ultimate losses are known and larger than the AAD. There actually might be a distribution of possible ultimate loss results. In theory, we should be subtracting the expected value of the AAD from the expected value of the ultimate loss. The expected value of the AAD will be less than the full AAD if there is the possibility that the AAD would not be fully used. This paper deals with a fixed ultimate loss rather than an expected value.

If the AAD only applies to one line of business in a contract that covers multiple lines of business just the AAD line can be handled separately. A multi-year contract that has an AAD that spans a number of years can be included for loss development purposes, but handled separately to develop the IBNR.

The possible relative sizes of AAD, reported loss gross of the AAD and Ultimate loss before AAD are depicted on the line graphs on Figure 3. There are six ways to order three variables by size assuming none of the three are equal to another.

The first two situations discussed assumed the ultimate loss is larger than both the AAD and reported loss. It is possible for the AAD to be larger than the reported and ultimate loss. This is shown in situations 3 & 4 on the line graphs on Figure 3. In this case the IBNR would be zero as the AAD eliminated all losses.

It is also possible for the reported loss to be larger than the ultimate loss and AAD (situations 5 & 6 on Figure 3). This means that there will be negative development. In situation 5, the company has reported losses of R which have been partially offset by the AAD of A. The net of AAD reported losses are R minus A. Since ultimately there will be no losses as the AAD is larger than the ultimate loss, the IBNR should bring the booked loss to zero and this negative IBNR amount would be A - R.
<table>
<thead>
<tr>
<th>Situation</th>
<th>IBNR</th>
</tr>
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<tbody>
<tr>
<td>1. A&lt;R&lt;U</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A R U</td>
</tr>
<tr>
<td></td>
<td>U - R</td>
</tr>
<tr>
<td>2. R&lt;A&lt;U</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>R A U</td>
</tr>
<tr>
<td></td>
<td>U - A</td>
</tr>
<tr>
<td>3. R&lt;U&lt;A</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>R U A</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4. U&lt;R&lt;A</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>U R A</td>
</tr>
<tr>
<td></td>
<td>0</td>
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<tr>
<td>5. U&lt;A&lt;R</td>
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</tr>
<tr>
<td></td>
<td>U A R</td>
</tr>
<tr>
<td></td>
<td>A - R</td>
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<tr>
<td>6. A&lt;U&lt;R</td>
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<tr>
<td></td>
<td>A U R</td>
</tr>
<tr>
<td></td>
<td>U - R</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

\[\text{ Dollars}\]

\[
\begin{align*}
A & = \text{AAD} \\
R & = \text{Reported Loss} \\
U & = \text{Ultimate Loss before AAD}
\end{align*}
\]
In situation 6 when the AAD is less than the ultimate loss, and the ultimate loss is less than the reported loss, the net of AAD reported loss is the amount between A and R or R - A. Since the ultimate loss is less than reported loss the IBNR is the amount between U and R or U - R.

As the situation with the ultimate loss less than the reported loss (negative development) is unusual, I will not consider it further (situations 4, 5 and 6).

In general assuming positive development, one approach to develop net of AAD IBNR is to make two calculations, and use the smaller IBNR of the two but not less than zero. One calculation is to develop the formula IBNR gross of the AAD. This gives the correct (and smaller) IBNR in situation 1 (the formula IBNR is equivalent to U-R).

The second calculation is to develop the ultimate loss gross of the AAD and subtract the AAD (minimum IBNR of zero). This gives the correct (and smaller) IBNR in situation 2. There is a minimum IBNR value of zero because in situation 3 the ultimate loss minus the AAD is negative, but the true IBNR is zero. The two calculations can be expressed as:

\[ \min (U-R, \max (0, U-A)) \]

When the AAD equals the case reported, both calculations produce the same IBNR.
AAD AND BORNHUIETTER/FERGUSON IBNR

If a B/F loss method IBNR with premiums (as a measure of exposure) is being used (premium x loss ratio x percent of loss unreported), the same general approach will apply, i.e. develop the IBNR gross of the AAD and make adjustments as appropriate by making a second calculation. As the reinsurer collects premium to pay losses net of the AAD, the net of AAD premium must be increased in order to develop gross of AAD IBNR. If the business is being written at an 80% loss ratio, we can add the AAD divided by .8 to the premium. This approach is for loss reserving. For pricing the probability of the AAD being completely used, the risk load, etc. would be considered.

In both examples (Figures 1 and 2), at an 80% loss ratio, the reinsurer would have received $7,500,000 of premium ($6,000,000 ÷ .8) and expected to pay losses net of the AAD of $6 million ($7,500,000 x .8). Since our calculations are gross of the AAD of $2 million, the premium must be adjusted. The increase is the AAD of $2 million divided by .8 or $2,500,000 for a total premium of $10,000,000 ($7,500,000 + $2,500,000).

In the first situation discussed (reported loss greater than AAD - Figure 1) the first IBNR calculation would be:

\[ \text{premium} \times \text{loss ratio} \times \% \text{ of loss unreported} \]

or $4 million where 50% = (2-1)/2 and 2 is the LDF.
The second calculation is ultimate loss of $8 million minus the AAD of $2 million which equals $6 million. The IBNR is the lower value of $4 million or
\[
\min (U-R, (\max(0, U-A))) =
\min (8-4, (\max(0, 8-2))) =
\min (4,6) = $4 million
\]

Under the second situation discussed (reported loss less than AAD - Figure 2) the first IBNR calculation is
\[
\frac{10,000,000 \times 80\% \times 7/8}{10,000,000 \times 80\% \times 7/8}
\]
or $7 million where $7/8 = (8-1)/8$ and 8 is the LDF.

The second calculation is ultimate loss of $8 million minus the AAD of $2 million, or $6 million. The IBNR therefore is the lower figure of $6 million or
\[
\min (U-R, (\max(0, U-A))) =
\min (8-1, (\max(0, 8-2))) =
\min (7,6) = $6 million
\]

We can express the two calculation AAD adjustment rule a different way. We can just use the formula IBNR, however, when the reported loss is less than the AAD we will subtract the unused AAD from the formula IBNR. This can be seen by looking at the Figure 3 line graphs. In our example (Figure 2) $10,000,000 \times .8 \times (7/8)=7,000,000$ minus (AAD of $2,000,000$ minus reported loss of $1,000,000) = $6,000,000. Due to situation 3 the minimum IBNR should be zero. The formula is
\[
\max (0, U-R-\max(0, A-R))
\]
In the first situation discussed (reported loss greater than the AAD - Figure 1) no adjustment is necessary as reported losses are greater than the AAD.

AAD AND INDICATED LOSS RATIOS

When using a B/F loss method, one might initially use a loss ratio based on conversations with the Pricing Actuary or the Underwriter or based on previous accident year indications. As data becomes available, it is appropriate to develop an indicated ultimate or burned loss ratio that incorporates loss development factors to assist in selecting the loss ratio for the B/F method.

In the second situation discussed (reported loss less than AAD - Figure 2) we can develop an indicated ultimate loss ratio of 80% ($1,000,000 of reported loss x 8.0 the to ultimate factor + $10,000,000 of AAD adjusted premium). The indicated loss ratio for the first situation (reported loss more than AAD - Figure 1) is the same 80% or $4,000,000 of reported losses x 2.0 (the to ultimate factor) + $10,000,000 AAD adjusted premium.

Just as we are able to add the AAD business in the triangle to the non AAD experience, we can also develop indicated loss ratios by including the AAD business. One problem with mixing AAD with non AAD business in the indicated loss ratio is that there can be an inconsistency between the loss ratios for the AAD contract and the non AAD business. If the two do not have the same loss ratios, doing the IBNR combined and separately for AAD and non AAD can produce different results. Because of this doing a separate loss ratio calculation for the AAD contracts is preferred.
In general, if losses gross of the AAD are captured, the appropriate IBNR reserves can be developed net of the AAD by the use of two calculations and by following certain simple rules. The combination of similar AAD and non AAD business in loss development triangles increases the credibility of the loss development and IBNR indications.