Comparing Reinsurance Programs—
A Practical Actuary’s System
by Robert A. Daino, FCAS, and
Charles A. Thayer
COMPARING REINSURANCE PROGRAMS

A PRACTICAL ACTUARY’S SYSTEM

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Biographies:

Mr. Daino has been in the industry since 1971. His experience includes actuarial and senior management positions at bureaus, primary insurers, and a reinsurance brokerage firm. Today, he is President of Cornerstone Consultants, Inc. in Ramsey, New Jersey and specializes in actuarial, management and related consulting, primarily in the reinsurance and specialty lines arenas. He has designed, prepared and delivered scores of reinsurance program analyses and presentations. He has a B.A in Mathematics from Fordham University, and has been a member of the CAS and AAA for over twenty years. He, his wife and two daughters reside in Upper Saddle River, NJ.

Mr. Thayer is a consultant with Cornerstone Consultants, Inc. He holds a Bachelor of Science degree in Mathematics from the University of Vermont. Before joining Cornerstone, he helped establish an actuarial function for a New York City reinsurance broker for six years. He began his insurance career in primary insurance, having served with two New England multi-line insurers where he was involved in pricing, product development, data quality and reporting systems. He resides in Mahwah, NJ.

Abstract:

This paper describes the elements of a simulation system used by the authors. A "user manual" approach is used to describe the elements of the system. A practical sample scenario is used to show how the system is used in practice.

It is not the authors' intent herein to discuss in any depth the technical issues involved in selecting the many parameters involved in a simulation. Rather, we try to show how a system can be used to control the parameters needed, and also help users analyze and communicate the results to others.
INTRODUCTION

Over the course of several years, the authors have been involved in many situations where reinsurance buyers were faced with making a decision between several competing recommendations for their reinsurance program, often with significantly different structures. We found early on that one of the tools we needed was a simulation system. This paper will describe the elements of our current system, and the steps we take in building a simulation and analyzing the results.

Having developed early versions of this system and presented the results to buyers, we learned much about what buyers, in general, consider important and what they are interested in seeing when making such a decision. To be sure, there is a wide variation in technical savvy among the many buyers of reinsurance, but most can understand the usefulness of comparing alternatives over many possible loss scenarios, the importance of getting a handle of some sort on the “odds” of favorable and unfavorable things happening, and almost all appreciate graphical representations of the results.

With reasonable assumptions about the variability of the number and size of claims by line of business, and with other necessary assumptions needed to mimic the insurance/reinsurance process, a more complete comparison between and evaluation of the several competing reinsurance programs can be made (versus single scenario comparisons). Most buyers understand this as well.

Furthermore, most buyers understand that, although the final simulation averages and aggregate distributions are only as good as the input assumptions (which are often very soft), the decision value lies primarily in the comparative analysis that results (i.e., the absolute values may be approximate/soft numbers, but the indications about whether Option A is better than Option B are much stronger).

Our original model was a Lotus 123™ spreadsheet, but within the last year, working in conjunction with a group of professional developers and a major reinsurer, we have brought this over and enhanced it greatly into a VB/Excel™ system with a graphical user interface, template libraries for maintaining and controlling simulations, histories, etc. In this paper, we will describe the elements of this system as well as a practical example of its use.
SYSTEM OVERVIEW

Our simulation system is a 32-bit Windows™ system designed to build and manage the components of large Excel™-based simulation models. The user is guided through the creation of:

- Simulation Lines of Business - up to 40 lines, based on up to 40 Input Lines of Business (when data is available in more detail than needed for simulation),

- Reinsurance Programs - up to three competing programs, made up of up to 15 treaties per program. A default gross = net = "no reinsurance" program is automatically included, and

- Other inputs, as needed, for beginning balance sheet values, investment and tax assumptions.

The system presents Excel™-like "patches" to the user in a Windows™ front-end graphical user interface (GUI) program which also maintains control over every patch within a given client's scenario. Multiple scenarios can be maintained for each client, and the system can control many clients. Patches can be saved for future use in similar scenarios for this or other clients. The program controls all of this in an Access™ database.

Once the user is satisfied that all inputs are ready, he instructs the program to compile the Excel™ simulation workbook. Once the workbook is built, the user can view it, run the simulation or detach the workbook for use outside the control and management of the system (for cases where the standard program is not sufficient to describe the alternatives being compared). The user specifies the outputs he would like to track from the simulation, the number of iterations, etc. The system then performs the simulations and records the requested data in another Access™ database. Standard format exhibits and charts can then be requested from the system, while custom exhibits and charts can easily be built by the user outside the system.
The Client/Scenario Manager

After an initial welcome screen, the system provides the following initial management window. Clients and their associated scenarios are managed from this window. All sample screens that follow are taken from the "CAS Sample Scenario" developed for the sample "Multi-Line Insurance Company" for the purpose of providing a working example for this paper.

A standard GUI interface for management of client and scenario properties allows users to build and control multiple scenarios for many clients.
Scenario Manager

This is the place most of the work is done. The major operations available from the scenario window are to Navigate and Change the Scenario Structure, View and Edit a Wizard Sheet, Recalculate the Scenario, Access the Template Library, Perform a Simulation, Print Reports, and Chart Results.

The major parts of the Scenario window follow:

- **Menu bar**
- **Toolbar Buttons**
- **Navigation Tree**

The navigation tree is used to view and manipulate the entities of a scenario. With this, a user can add lines of business and reinsurance programs with their respective treaties.
Navigate and Change the Scenario Structure

The structure of the current scenario can be viewed and altered from the navigation tree on the left-hand side of the Scenario window. This structure can be modified by manipulating its parts, which are also known as entities; the Client/Scenario entity, the Financial Assumptions entity, the Lines of Business entity, which in turn contains Line of Business entities, the Reinsurance Programs entity, which in turn contains Program entities, which in turn contain Treaty entities.

When an entity is added to the current scenario, a default set of worksheets is provided. The principal functions here are to Add a New Line of Business (LOB), Add a New Program, Add a New Treaty, and Show an Entity Wizard.
Risk Function Wizard

To allow the user to model a number of familiar frequency and severity models, we have built into this system the functions in the Excel™ add-in called @Risk™ which provides easy access to many of the functions used by property/casualty actuaries.

Since the spreadsheet-like features available in the front-end are not Excel™ but rather an Excel-like VB component, the @Risk™ add-in does not currently operate in this front-end. The default value displayed is the mean of the function. For more elaborate functions, the user can detach the workbook and work outside the system. (see below)

The Template Library

The application contains a storage area called the Template Library. The Template Library is used to store and retrieve wizard workbooks and wizard sheets. This facility allows the user to reuse wizard workbooks and sheets.

Perform a Simulation

The user can Build and Display the Simulation Workbook, and Run Simulations from the Scenario Window. If the user is satisfied with the workbook built from his inputs, a simulation can be performed.

User-defined fields are fields that the user wants to capture but are not available in any of the built-in selections, yet are items of special interest in carrying out the current work. They are available only in “detached” workbooks. Workbooks are detached in cases where the system's standard features are not sufficient to completely describe what the user would like to test (such as very unique treaty terms).
Print Reports

Once simulation has been performed, the Print Reports dialog can be invoked. The user can print summary reports for each line of business, program, and treaty in the current scenario.

For each line of business and reinsurance treaty, there are two summary reports: a Statutory Underwriting Income report with the average underwriting results for the simulation, and a page of Highlights that shows averages, maximum and minimum values for several key variables of interest.

In addition, for the Overall Gross reporting level and the Net results for each reinsurance program, there are three reports: First, there is the Statutory Income Statement report, including investment and other income. Also, an extensive Highlights page gives averages, maximum and minimum values for key items at the Ceded and Net levels, and a Statutory Average Balance Sheet report that shows average levels of several asset and liability classes for the run.
Chart Results

Several standard charts are available. The system itself generates some default charts that can be modified, printed or deleted by the analyst. The user can also develop new custom charts of each type within the system to compare results of reinsurance programs, treaties or lines of business.

"Green is Good, Red is Bad" Charts:

"Red/Green" charts allow the user to compare the results of two reinsurance options over the whole range of outcomes for a simulation, and especially to determine the type of loss scenario in which one alternative does a better job of meeting the insurer's objectives than another. Each chart focuses on one particular variable, such as ceded or net underwriting profit, after-tax income, year-end surplus or any quantity that has importance in the evaluation process. The idea behind the Red/Green chart is simply to find the difference between the values of that variable for the two programs, determine whether a positive difference is better or worse for the cedant, and to plot green points when the first alternative is better and red ones when it is worse. The more green points that appear, the more often the first program comes out better.

Histograms:

Histograms show the non-cumulative distribution of some variable of interest for the given reinsurance programs, or for selected lines of business or treaties, either as frequencies or counts. In some cases, these will be the roughly bell-shaped curves that are often encountered in insurance statistics. The definition and interpretation of these charts is fairly easy to grasp. Cumulative versions are also available.
Other Variability Charts:
Charts showing the year-to-year change in key percentile values are also available. These charts help the user see the variability of any selected output item in another fashion, which some users find very helpful.

Other Charts:
Other charts are produced by the system or are in development, which help the user express himself, analyze the output more easily, explain results and communicate to others more conveniently.
HIGHLIGHTS OF THE INPUTS

This section is organized by input wizard tab much the way the system inputs would be entered. Highlights only are provided in this paper.

CAS Sample Scenario

To illustrate the use of the system, we have included a scenario called the CAS Sample Scenario for a client called the Multi-Line Insurance Company. See more details about this sample scenario in a separate section below.

The system provides a Test Layer capability to allow the user to test a particular excess of loss reinsurance layer to find out how many claims and loss dollars to that layer are expected from each of the subject lines of business. This helps in checking the aggregate effect on reinsurance layers of size of loss distribution selections for the various lines of business as well as other aspects of the excess layer, all while still in the front-end system, before any simulations have been attempted.

Financial Assumptions are entered in two tabs: Balance Sheet Assumptions, and Cash Flow Assumptions. The system is focused on the underwriting side of the business. It has extensive inputs for product lines and treaties - the driving assumptions for gross, ceded and net results. However, the system builds complete income statements, including investment income, and balance sheets for each reinsurance program. The investment and cash flow assumptions are intentionally simplified, so they will not be a major source of questions/issues for clients, and thereby cloud the insurance/reinsurance underwriting result discussions. As a result of client requests for additional functionality, enhancements to the system in the asset and tax areas will be made.
Lines of Business

There are six tabs in this section: Input Lines of Business, 1st Year Sim Lines Premium, 1st Year Sim Lines Exposure, Simulation Line Properties, Premium and Exposures Summary, and Test Layer Summary. We will only highlight key tabs for this paper.

From our experience, the data we obtain is usually in more detail than we need for simulation purposes. The user can enter the detail available, and then map the detail to Simulation Lines. If the user has already combined data outside this system into the desired Simulation Lines, then the mapping will simply be one-to-one. Of course, there are also cases when we want to split an input line into more than one simulation line (e.g., Homeowners to property versus casualty). This facility is helpful in such cases.

Once the user has completed the Input Lines sheet, he begins to enter Simulation Lines. New lines are entered by right clicking the mouse when on the Lines of Business navigation tree item and responding to the dialogs presented.
Once all Simulation Lines are entered, the user visits the 1st Year Sim Lines Premium Tab and the 1st Year Sim Lines Exposure Tab. These two tabs are identical in structure and have the role of allowing the user to allocate the Input Lines’ premium and exposure to the selected Simulation Lines, keeping track of the total allocations, and posting the results to the appropriate Simulation Lines.
Individual Simulation LOBs

There are nine tabs in this section: Exposure, Rate and Premium, Expenses, Limit Distribution, Payout Pattern, Payout of Existing Loss and LAE Reserves, Size of Loss Distribution, Losses Below Cutoff, Claims Above Cutoff, and Loss Expectations. We will show here only the last four relating to key loss assumptions.

Size of Loss Distribution Tab

Here the user enters the loss sizes and CDF by year for the given LOB. While this example seems to mimic the ISO 5 parameter Pareto model, it is not correct to consider it a continuous model. The size of loss distribution is entered as a discrete distribution, and all of the possible loss sizes in the simulation run will come from the list of values in the first column of the table. There is no interpolation in sampling and no attempt to determine an interval mean in computing the key expectations used elsewhere in the model. The averages and variances of the simulation results depend on the loss joints that are selected in the size of loss distribution, so these must be chosen carefully.
The cutoff value is the point in the size of loss distribution at which the model will begin to simulate individual claims for the given LOB. The lower the cutoff, the larger the number of claims the model will individually simulate.

On this tab, we enter the cutoff point, the expected number of claims for each year, and the risk model assumptions we wish to use to simulate the number and average severity of claims below the cutoff. The sample uses a mixed Gamma/Poisson to simulate the number of claims below the cutoff. This procedure is discussed in the Heckman-Meyers paper, “The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions” (PCAS LXX, 1983, page 22ff). In addition, a censored and truncated Normal random variable with a mean of 1.0 is used to modify the severity below the cutoff that is used to compute the aggregate loss level. The user can easily specify other claims processes using other Excel™ functions, @Risk™ functions, or even a constant set of claims with fixed amounts.
Claims Above Cutoff Tab

This tab is analogous to the previous tab, except that it regulates the simulation of claims above the cutoff point. When the simulation workbook is compiled and the simulation is run, the actual size of a simulated claim is chosen from the table in the Size of Loss Distribution tab. The sample uses a mixed Gamma/Poisson distribution to simulate the number of claims above the cutoff, but again, the user can specify other processes.

The parameters and claims process chosen here form the heart of the individual large claim simulation done by the model.
Loss Expectations Tab

The Loss Expectations Tab for each LOB brings together the inputs from the previous tabs and allows the user to make a preliminary review of the results that can be expected from the user's premium, loss frequency and severity assumptions. If care has been taken to preserve the integrity of the means from the severity distribution in the Size of Loss Tab, the results shown here should be a reasonable benchmark for checking the outcomes of the simulation runs. The averages that appear on the final reports from a simulation should come fairly close to the values shown on this tab.
Reinsurance Programs

In this system, a program consists of up to 15 treaties. Up to three programs can be compared simultaneously. For example, the user might be comparing the Current Program to a Recommended Program and to a Competitor's Program. The final output would have results of the simulation for each of these three programs plus the default "no reinsurance" or gross = net program.

The system does not accommodate facultative reinsurance. However, to reflect broad assumptions about "fac", a user could enter facultative covers in bulk as a treaty, with pertinent, broad assumptions.

A treaty is created with a special dialog. In this dialog the user selects the generic treaty type that best matches the treaty. This affects the way that losses and premium are accounted for in the treaty. If the subject premium base is earned premium, the user simply selects that option and moves on. When the subject base is written premium, there is an additional check box the user may select to specify whether there will be a transfer of the subject beginning unearned premium reserve at the start of the first simulation year. The method by which the treaty subject premium is calculated (which treaties inure to this treaty's benefit, etc.) can be specified by checking the inuring treaties in an extension of the standard dialog. Another dialog is used to specify the LOBs that are subject to each of the treaties.
Treaties

At the treaty level there are five tabs: Treaty Terms, Ceding Commissions, Excess Treaties, Payout Pattern, and Payout of Existing Loss & LAE Reserves. Sample tabs are highlighted below.

**Treaty Terms Tab**

The Treaty Terms tab gives the user the context in which all of the rest of the calculations for the treaty operate. The heading area tells the user the program and treaty names, the type of coverage provided by the treaty, the subject base, and the subject calculation, which tells the user if there are inuring treaties.
Ceding Commissions Tab

This is the location for entering data and/or formulas that spell out the terms of any applicable ceding commission or profit commission, whether it is implemented as a flat commission, a sliding scale or custom-designed plan. We will not go into this in any detail here.
CAS SAMPLE SCENARIO

Input areas covered in the previous sections give the reader a broad overview of the types of inputs and manner in which the many inputs needed to perform a complete simulation are entered. In this section we go into a bit more detail concerning a particular sample.

Note: Rather than populate an appendix with dozens of reported results from our system, we will send a copy of a section showing the individual line of business assumptions, output exhibits, treaty results, etc., to any reader who calls or writes us for a copy. These give the reader all the basic assumptions needed by line, treaty, etc., as well as key output.

Lines of Business in Sample Scenario

In the CAS Sample Scenario, we show a multi-line insurance carrier that is involved in over a dozen lines of business. We have segregated the overall book of business into four simulation lines, to simplify our sample. In a real-world case, we would separate lines based largely on their loss characteristics and reinsurance treatment (lines must map into treaties properly, and significant size of loss differences would be recognized).

The simulation lines in our sample are Automobile Physical Damage, All Other Property, Property Catastrophes and All Liability. All of the individual input lines were included on the Input Lines of Business tab, but the premium and exposures were allocated to the simulation lines (in the 1st Year Sim Lines Premium and 1st Year Sim Lines Exposure tabs). See the section above titled “Lines of Business.”

The Property Catastrophes line serves a unique purpose in this scenario. It's there to generate losses for the Property Catastrophe treaties in the reinsurance programs. No premium is assigned to this line of business. The Property Catastrophe treaties will pick up losses from this line of business, while their subject premium comes from the other lines that are specified as subject to the treaties in the Treaty Subject Lines tab for each Program. There can be several such “lines” if needed. In the sample scenario we have assumed that recoveries from the current risk excess would not materially affect the SOL distribution for property cats. If we knew that the distributions differed, we could reflect these differences in the system. Size of loss distributions obtained through a portfolio analysis using one of the several commercially available catastrophe models can be accommodated by the system using one or more Property Catastrophe lines of business.
Reinsurance Programs Used for Sample Scenario

The Multi-Line Insurance Company has a Current Reinsurance Program that consists of five treaties: 1st and 2nd Casualty Excess treaties, a Property Risk Excess treaty, an Automobile Physical Damage Quota Share, and a Property Catastrophe Treaty.

We will assume that there is a proposal for an alternative structure under consideration which we call the Recommended Reinsurance Program, and that other programs may have been considered and rejected, leaving these two alternatives. The Recommended program consists of only three treaties: a Multi-line Excess treaty, a Property Catastrophe treaty and an Aggregate Excess treaty.

The key terms used for the Current Program were:

1st Casualty Excess: 100% of $750,000 xs $250,000; ALE included, Swing rated 4%/15% loss load 100/85ths

2nd Casualty Excess: 100% of $4 mill. xs $1 mill.; ALE included, Flat rated 6.23% with 35% ceding commission

APD Quota Share: 40% with 30% ceding commission

Property Excess: 100% of $4,800,000. xs $200,000; ALE included, Flat rated 3.41%

Property Catastrophe: 95% of $47 mill. xs $3 mill., Flat rated 15.7%. No reinstatements. The property excess inures to the benefit of the cat treaty, but we assume that the benefit to the catastrophe excess of the inuring treaty is negligible.

The key terms used for the Recommended Program were:

Multi-Line Excess: 100% of $4,650,000 xs $350,000; ALE included, Flat rated 12.3% with 35% ceding commission. Also features a profit commission of 50% after 20% reinsurer expenses, adjusted at the end of 3 years with a deficit carryforward.

Property Catastrophe: 95% of $77 mill. xs $3 mill., Flat rate 18.1% One free reinstatement.

Aggregate Excess of Loss Ratio: 95% of 20% xs 77%, Flat rate 1.20% with a 35% ceding commission and 25% profit commission after 20% reinsurer expenses, adjusted annually.

As you can see, there are significant structural differences between the two programs. The first provides coverage closer to the ground while the second provides larger and broader catastrophe and “worst case” coverage.
Setting up the Scenario

There are a variety of reasons that buyers seek a change in their reinsurance: saving money (which means different things to different people), adapting coverage to their current mix of business, changes in management, problems with current reinsurer(s) or broker, or changes in strategic direction, goals, or appetite for risk. In today's reinsurance marketplace, many buyers want to redesign their reinsurance to reduce cessions, simplify administration, broaden protection, and protect earnings.

In our example, Multi-Line Insurance chose to assume more risk down low to pay for more property catastrophe limit and an aggregate stop loss. They increased their working cover retentions and canceled the quota share. Under the current program, the company cedes about $33 million, while the new program calls for a cession of $26.5 million.

After all of these changes, is the new structure a better way to manage risk than the current program? Is either better than no reinsurance at all? When is it better? How often? These questions led to the use of simulations to provide better information to evaluate the alternatives.

Which Program is “Better?”

Buyers come from many different points of view when deciding between competing programs. Some focus on the amount of ceded premium, some base decisions on historical “what-if’s” (runnning competing proposals through several actual prior years of losses), some rely on estimates of average ceded underwriting profits, others on estimates of total ceded profit (including investment income), and some focus on worst case loss scenarios. Simulation models can provide this information and enhance it by providing insights into the effects of variability, giving estimated odds for profit and loss levels.

The key issues buyers usually focus on initially are ceded premium, expected ceded profits, and the variability and magnitude of worst cases in their net results. We will highlight these items for the comparison at hand.

One straightforward approach is to look at the average, maximum and minimum levels of ceded underwriting profit, total net income and surplus for each of the alternatives: No Reinsurance, the Current Program and the Recommended Program. All of these values are displayed in the Simulation Highlights report produced by the system for each of the programs. Copies of these exhibits follow.

The first year averages are fairly typical of the differences between the programs under review. Average net income and ending surplus were highest for the No Reinsurance option, of course, followed by the Recommended and Current Programs, respectively.
Under the Recommended program, the ceded profit is $850,000 better and total net income $500,000 better than in the Current program.

Next, we note the minimum and maximum values for each of the alternatives. For surplus and net income, the maximum values are seen in the No Reinsurance alternative, as we would expect. The minimum values from the simulation for net income are negative for all options, but the Recommended program has much smaller worst case losses due to the stop-loss. In both the No Reinsurance and the Current program, the worst case net losses exhaust the company’s surplus, leaving a negative balance in all but Year 6 (2002).

Note: We “allow” negative values in surplus and invested assets (which behave like loans) in standard set-ups, but the system can easily handle defined constraints on behavior. In more elaborate runs we would define the decision rules appropriate to the case.

Before either of these alternatives is selected, the buyer needs to determine the frequency of losses of that magnitude to be sure that the probability of such huge losses is sufficiently remote to assume the risk.

What is the return time for a loss that would cause a given level of reduction in surplus? Equivalently, are the odds of such a loss equal to 1 in 10, 1 in 20, or more like 1 in 1000? This will help determine the level of risk involved in keeping these losses net. The chart on the next page illustrates the probabilities of losing XX% of surplus in any year in the Simulation Years 1-6 for each of the reinsurance alternatives.

As an example of the useful nature of this chart/table combination, consider the probability of having a net loss in surplus: 7.8%, 8.7% and 6.4% (1 in 13, 12 and 16) for the No Reinsurance, Current and Recommended cases, respectively. At a 5% reduction, the odds are 1 in 18, 25 and 909. There are no surplus reductions worse than 10% for the Recommended Program. Coming from another direction, the 1-in-100 loss (1.0% probability) would result in a loss of surplus of 40%, 21%, and 3.5%, respectively. Note that these probabilities are not just single year probabilities. If the reduction in surplus occurs in any one of the years, it is counted in the totals in computing the probabilities. This is a practical result, since regulatory concerns would be triggered immediately.
From this chart it is clear that the Recommended Program is the best at arresting runaway losses from catastrophes and other sources of high aggregate loss levels. We did not model other high severity issues such as Clash, ECO/XPL or runaway ALE in this analysis. That would have made matters somewhat worse in the Recommended program, but far worse for the Current program, which has little or no provision for these exposures. Most of the losses would go right to the net. Even without these, there are significant probabilities of disastrous depletion of surplus under the Current program. If the full spectrum of results from the simulation had been displayed on this chart, the No Reinsurance line would stretch to -140%, and the Current line would tail out to about the -120% level. Investors and regulators would be dismayed by far smaller losses of capital than these. This is why we stop the chart scale at a -50% reduction, even though greater degrees of impairment may be possible.

While we are considering the odds of observing certain values of interest to the reinsurance buyer, we can look at their distributions using charts available in the system. First, we will consider the non-cumulative distribution of surplus in Simulation Year 6. The non-cumulative “histogram” view gives another dramatic illustration of the reduction in variability achieved. The nearly bell-shaped curves just stop dead at a certain point for the Recommended program, while the other options have rather pronounced tails that stretch well into undesirable values.
At the low end of the scale, the text box provides a “zoom in” look at the frequencies.

The fact that the two reinsurance program net results reach their modal and mean values at lower surplus levels accentuates the reinsurance buyer’s dilemma: Buying reinsurance over time causes a reduction in net worth due to the net costs involved (accumulated ceded profits) in exchange for this “tightening” of the tail(s) of the curve. The same idea is expressed by the differing slopes of the curves in the cumulative distribution graph.
Having looked at the best and worst cases, and evaluated some empirical probabilities, we now will want to compare the programs head to head, as if each trial were a horse race. This is the concept behind Red/Green charts, which compare pairs of programs based on the sheer number of trials for which one program has a better result than the other. "Better" is defined by the user - in this case a program is better when the difference between net after-tax income is positive. All green markers are above the x-axis, all red markers are below.

Evaluating a Red/Green chart is easy when it is almost entirely filled with Green markers, especially when they are in the places where they should be Green (i.e. where the need for reinsurance protection is greatest). We refer to the decision between the alternatives in such a case as a "slam dunk." When the decision between two alternatives is a "slam dunk," one of the alternatives is better than the other in almost every case. As long as the Red dots appear in relatively low-impact areas, the decision between the two programs is fairly easy. We have seen such comparisons in practice. It makes the decision between the two alternatives simpler. The decision in this sample is by no means a "slam dunk," but it is helpful to evaluate it using Red/Green charts nonetheless.
Note that this chart is filled with Red dots, yet the company saw fit to buy this program for years, primarily because the Green markers are "in the right places." That is, when the total gross loss ratio was greater than 66%, and especially as the loss ratio breaks into the 90% range and above. In the worst years of gross loss experience, the Current program can have a net income benefit of $20-$25 million. At the lower end, the premium is weighing down the program, so that total income is as much as $10-$12 million worse than it would have been had there been no reinsurance at all.
Now let's see how the Recommended program compares with No Reinsurance.

The Recommended program fares better, but it still comes out worse in 3,427 trials out of 4,000. Note, however, that the size of the potential benefit to total income in the worst years has a far wider range than in the Current program, up to $110 million in the most extreme case. Any reinsurance program may look unattractive in the good years, in which there is a large amount of ceded profit. But, this program is a good example of what happens with high risk cessions - when reinsurance is needed, it delivers.
Let's now zoom in on the 500 trials with the worst gross loss ratios to get a better look at what is happening in that range.

Here, we find the lion's share (411 of the 573 trials) of the Green markers from the full-spectrum Red/Green chart above. Most of the cases in which the Recommended program "wins" are in years when the gross loss ratios are above 80%, as you would expect given the stop-loss. One might be tempted to suggest that the company buy only the stop-loss, but the realities of the market are such that stop-loss reinsurers require acceptable working cover and property cat protections underneath them.
Next, we compare the Current and Recommended programs (Green means Recommended is better).

Comparison of Year 1 Net After Tax Income After 4000 Iterations

Current Reinsurance vs. Recommended Reinsurance

It appears that the Current program wins a majority of the times in head-to-head comparisons with the Recommended structure, but when the gross loss ratio breaks the 95% level, the Recommended program begins to shine. In the most extreme cases, there are clumps of cases where the benefit is $20-$40 million, from the extra property cat coverage mostly, and a handful of very large differences of between $40-$90 million when the extra cat cover plus reinstatement cover plus the aggregate all come to bear.

While this situation does not result in a simple "slam dunk" superiority between any of the alternatives, we believe the buyer will continue to buy reinsurance as a result of this analysis. The direct underwriting assumptions here are fairly optimistic, but we still have enough loss potential from property catastrophe shock losses and/or high loss frequency in the various lines of business to create very unfavorable experience, as we have seen.

Another means of looking at the variability of total income (say) from the reinsurance alternatives at various gross loss ratio levels is a chart we call the "wiggle chart." This is not yet another probability chart per se, but a graphical display of the range of values of after tax income for each alternative at every observed value of the gross loss ratio. The picture really highlights the benefits of the Recommended program, and speaks for itself.
As can be seen from the wiggles for the No Reinsurance and Current options, in the years with the worst gross loss experience, the bottom seems to drop out from under both of these alternatives. While the Recommended program also experiences some bumpiness at the top end, the combination of the extra property cat limit and reinstatement plus the stop loss have limited the damage to the company's balance sheet to a far greater degree. This picture depicts the impacts to the buyer very clearly.

The final chart series of interest to us allows us to look at year by year variability of a single quantity, after tax income in this case, over the whole six-year time span of the simulation. Note that this is a series of three charts — all with the same scale — which depict a number of user-selected percentiles that allow a side by side comparison of the variability.

This display allows a buyer to see how reinsurance alternatives work to control variability over the 6 simulation years by watching the spreading arms of the pairs of “1 in N” percentile lines (the pairs of lines representing the 100(1/Nth) and 100(1-1/Nth)-percentile values for each year), which resemble confidence intervals for the mean at various confidence levels. Again, this is easier to see in the pictures. By now, we can see that the Recommended program is the best choice for controlling the variability of net income, so we would expect the differences between its “1 in N” values at each end of the range of values to be the narrowest. The Current program is anticipated to come in second, and the No Reinsurance case will establish the maximum and minimum points on the scale for the whole series.
That is exactly what we see in this chart series:

### All Lines; After Tax Net Income; No Reinsurance

<table>
<thead>
<tr>
<th>Year</th>
<th>Maximum</th>
<th>98.9%</th>
<th>98.0%</th>
<th>96.9%</th>
<th>96.0%</th>
<th>90.0%</th>
<th>Average</th>
<th>10.0%</th>
<th>2.0%</th>
<th>Minimum</th>
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<tbody>
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<td>$3,400,000</td>
<td>$1,700,000</td>
<td>$850,000</td>
<td>$170,000</td>
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</tbody>
</table>

### All Lines; After Tax Net Income; Current Reins

<table>
<thead>
<tr>
<th>Year</th>
<th>Maximum</th>
<th>98.9%</th>
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<th>96.9%</th>
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<td>$170,000</td>
</tr>
</tbody>
</table>

### Example Chart

![Example Chart](chart.jpg)
Since there is a database of results for every trial from each line, each treaty and each program, there are literally dozens of useful analyses one can perform, “drilling down” into issues that arise between the parties involved.

Since the system uses Excel™ as its calculator, any function or relationship that can be defined in Excel™ can be used in a simulation. Of course, the more one puts “in motion” the “blackler” the box gets. Sometimes that is necessary and sometimes it is not. To keep the focus on underwriting risk, we leave certain issues out of most of our analyses, like interest rate and asset risk, but since they can be described fairly easily in Excel™ the system can handle them.

Another frequent analysis involves taking the net present value of future cash flows, which the system handles easily. We usually do this as a custom calculation since there are different definitions of what is and is not included in NPV analysis from company to company.
As an example of how to use the database of results, we answer the questions: "How often do we trigger the aggregate? Total the aggregate?" In just a few minutes a user can extract the ceded losses and limits by year for the stop loss and produce an analysis such as the following:

<table>
<thead>
<tr>
<th>Aggregate Excess Treaty in Recommended Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Trials out of 4,000 with a Loss to the Treaty</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Losses</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>Total Losses</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
</tbody>
</table>

**Conclusion**

Perhaps one of the most important conclusions to note is that this is a system - where users can build models quicker, under better control - and where there is a "memory," letting the user re-use prior elements that "worked" for their needed purposes. This is a very practical result, too, since simulation has been around for some time in the hard-to-control spreadsheet world. The more comfort and control, the more often and more consistently analyses can be done.

We have seen how users / buyers react to this kind of information, and know that it has been a critical factor in their decisions. We all benefit when actuaries communicate well, and when the best information we have is brought together in meaningful and instructive analyses. We think simulation and the tabular and graphic representation of its results is an excellent vehicle for such communication in the reinsurance arena.

This paper has not been a technical actuarial article, but rather an exposition of our systematic approach to building and using simulations to analyze reinsurance alternatives. Our hope is that it might spur discussion of the strengths and weaknesses of this approach.