A Simulation Approach in Excess Reinsurance Pricing

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There are numerous papers in the actuarial literature dealing with the different aspects and applications of aggregate loss models. The great demand for research in this area stems from the increasing popularity of insurance and reinsurance arrangements involving aggregate limit and aggregate deductible provisions. The estimates of aggregate loss distributions are also important in the pricing of contracts containing retro adjustments, and profit and contingent commission features.

Some excellent practical methods are available to estimate aggregate loss distributions, including Heckman-Meyers [2] and Panjer [5]. The common assumption used in these methods is that all claims have the same loss size probability distribution. While this assumption is reasonable for many insurance contracts, there are situations where such an assumption becomes impractical.

As an example, one can consider the reinsurance program involving several layers of reinsurance coverage. Each of these layers may have both per occurrence and annual aggregate limits with a possibility to "drop down" if the underlying layers are exhausted, creating a quite difficult "two-dimensional" structure. This type of reinsurance program is quite common for large medical professional organizations. A specific example is considered later in the paper.

Pricing such programs can be challenging for reinsurance actuaries. From a theoretical standpoint, the major difficulty involved is that the reinsurer's loss severity distribution function is changing, depending on the exhaustion of the underlying layer coverage. This makes derived aggregate loss model techniques (Heckman - Meyers, Panjer) difficult to apply. One possible solution is to use stochastic simulation.

The simulation method can also be used successfully in place of Heckman - Meyers' or Panjer's method to build an aggregate loss distribution from estimated frequency and severity distributions. This paper systematically describes the stochastic simulation approach that involves the following steps:

1) Data preparation
2) Selection of frequency and severity distributions; goodness-of-fit tests
3) Estimation of the number of simulations required
4) Simulation of the excess losses
5) Pricing recommendations

This paper outlines some theoretical and practical considerations which may be useful in utilizing this approach. A pricing example will illustrate the application of the method.

1. Pricing Example.

1.1. Description of Coverage.

Our main example deals with the coalition of several hospitals (Alpha Hospital Union, AHU) which purchases a multi-layer reinsurance program to protect itself from catastrophic medical malpractice losses. AHU retains the first $3,000,000 per each and every occurrence, and wants to reinsure the excess. Coverage is claims made; the effective date for the coverage is January 1, 1997.

We will consider the pricing of the first two excess reinsurance layers. The first layer covers $3,000,000 in excess of $3,000,000 for each and every occurrence and is subject to an annual aggregate limit of $9,000,000. The second layer covers $3,000,000 for each and every occurrence in excess of the first layer coverage and is subject to annual aggregate limit of $12,000,000. In other words, the second layer covers $3 Mil xs $6 Mil before the first layer of excess coverage is exhausted, and $3 Mil xs $3 Mil after that.

Exhibit 1 shows the design of the coverage. After the first excess layer is exhausted, the second layer " drops down" to replace it. It makes the pricing of the second layer very difficult, because the severity distribution can change in the course of a year. We will demonstrate how to use simulation to estimate expected loss for the first and the second excess layers.

1.2. Data.

We assume that the following information is provided by the client:

- The complete list of all claims for report years 1983 through 1993 that exceed $1,000,000 at 12/31/95 evaluation date (see Exhibit 2);
- Incurred and paid loss development triangles by report year (see Exhibits 3-1 and 3-2);
- Paid claim count development triangle by report year (see Exhibit 4);
- Historical exposure (Basic class Full Time Equivalents) for years 1985 through 1993 and exposure projection for year 1997 (see Exhibit 5).

The loss and exposure data for report years 1994 and 1995 are also available but not used because of their immaturity.

1.3. Pricing Approach.

Our pricing approach is consistent with one described by Patrik [6]. The following main formula (a modification of Formula 6.2.1 from [6]) will be used:

$$RP = \frac{RLC \times DF}{(1 - CR - BF)(1 - IXL)(1 - TER)} \qquad (1.3.1)$$

Here RP = reinsurance premium (gross),
RLC = reinsurance loss cost,
DF = discount factor,
CR = reinsurance ceding commission rate,
BF = brokerage fee (if any),
IXL = reinsurer's internal expense loading,
TER = reinsurer's target economic return.

We will concentrate on the estimation of RLC; the other elements of the above formula are determined using other sources. Usually IXL is a function of the size of the account, and TER is a function of the level of risk (or potential volatility of account loss experience). While our methodology does provide a tool to measure potential account volatility, this topic is outside the scope of this paper.

The simulation method is used to estimate RLC. We model the loss severity and loss frequency distribution functions to simulate a statistically representative sample of loss experience in the reinsurance layers; the mean of this sample should give a good proxy for the expected loss in the layer. The details of the method follow.

1.4. Simulation Method - Step By Step.

When simulating loss experience one should be convinced that the severity and frequency loss distributions used in the simulations reflect reality to the greatest extent possible. To assure that, a good amount of meticulous work should be done.

First, historical individual losses should be trended and developed.

Second, loss frequency and loss severity distributions for the projected coverage period should be constructed based on adjusted loss data. Different types of loss severity curves (e.g., lognormal, Pareto, Weibull) fitted to the data should be examined. The Maximum Likelihood or the Least Squares methods may be used for curve fitting.

Next, a rigorous test of the goodness-of-fit needs to be performed. Percentile matching is probably the most important, but other tests (χ^2 - test, Kolmogorov - Smirnov) can also be performed.

Before starting the actual simulation process one needs to estimate the number of simulations required to achieve a certain precision depending on his goal. We recommend a relatively easy formula based on the application of the Central Limit Theorem.

When one is comfortable with the frequency and severity curves selected and the estimated number of simulations, one can run the simulation process.

The following sections explain in detail all the steps mentioned above.

2. Data Preparation.

2.1. Trending Individual Losses.

When trending the historical losses to the prospective experience period claim cost level it is important to select a proper severity trend factor. If underlying experience data is credible, it is better to select a trend factor using the account's own experience. One way of doing so involves the following steps:

- Develop the total incurred losses by year to ultimate;
- Develop the number of claims paid by year to ultimate;
- Calculate (untrended) average loss size by report year (divide the total ultimate loss by the ultimate number of claims);
- Fit an exponential regression to such averages.

This procedure is documented in Exhibit 6. The corresponding annual severity trend factor is 4.4%. Given the size of the account and regression characteristics we have decided to use this trend factor to bring individual losses to 7/1/97 level.

Alternatively, one can look at industrywide trend for Hospital Professional Liability from relevant sources. If necessary, one can adjust it for the difference in medical inflation for the state of the client's primary operations versus countrywide.

2.2. Developing Individual Losses.

Some individual claims in excess of $1,000,000 from the database illustrated in Exhibit 2 are still open at 12/31/95. The ultimate values of these claims might be different from their reserved values which we observed. Generally, it is not easy to adjust individual claim values for possible development using aggregate development data only. The major complication stems from the fact that aggregate loss development is driven by two different forces - the appearance of new claims and the adjustment of values for already outstanding claims. Fortunately, for claims made coverage usually there are no new claims which appear after the first year, and all the development is attributable to the reserve adjustments for outstanding claims only. This makes it possible *for claims made coverage* to use aggregate loss development data to approximate the development of individual claims. A procedure similar to the one described below can be used to develop individual claims for *occurrence coverage*; however, more information would be necessary.

The following technique could be used to develop individual losses which are open at 12/31/95 at its n^{th} evaluation (n=1 for claims reported in 1995, n=2 for claims reported in 1994, etc.):

- For each report year and fixed n (n=1,2,...) create a development triangle for claims <u>open at n^{th} evaluation only</u>. This can be done by subtracting column n of Exhibit 3-2 (paid losses at n^{th} evaluation) from columns n and subsequent of Exhibit 3-1 (reported losses at n^{th} evaluation and subsequent);
- Select appropriate loss development factors;

- Apply selected n-to-ultimate development factor to open claims outstanding at n^{th} evaluation.

For claims that were reported in 1992 ($n = 4$) this procedure is illustrated in Exhibit 7; the corresponding factor to be applied to report year 1992 claims open at 12/31/95 is 1.075. Please note that no loss development adjustment is applied to closed claims.

Alternatively, one can fit a series of curves to claim values at 1[st], 2[nd], and subsequent evaluations, and investigate the movement of the parameters. This methodology is consistent with one currently used by ISO (Pareto soup).

3. Selection of Frequency and Severity Distributions.

To calculate the expected losses in both reinsurance layers (see Exhibit 1) we need to project the number of claims in excess of $3,000,000, and the claim severity for such claims. Because AHU retains the first $3,000,000 of each and every claim, we should concentrate on the portion of claims in excess of this amount.

3.1. Selection of Number of Claims Distribution.

For the Excess Claim (in excess of $3,000,000) Frequency distribution we use the Negative Binomial. This discrete distribution has been utilized extensively in actuarial work to represent the number of insurance claims. Since its variance is greater than its mean, the Negative Binomial distribution is especially useful in situations where the potential claim count can be subject to significant variability. As Exhibit 5 Column (5) illustrates, this is the case in our example.

To estimate parameters for the Negative Binomial distribution we start with the estimate of expected number of claims in excess of $3,000,000. Exhibit 5 summarizes our approach.

First, we select the total claim frequency based on the historical exposure information and our estimates of ultimate number of paid claims; this selected number is **0.40** claims per one Full Time Equivalent (FTE) of exposure and is shown at the bottom of column (4). Second, we select the probability that the paid claim exceeds $3 Mil; our selection of **1.50%** is shown at the bottom of column (6). Based on these two numbers and the estimation of **840** FTE exposure for year 1997 provided by AHU, we expect **5.00** claims in excess of $3,000,000 for the coming year.

In order to estimate both parameters of the Negative Binomial, we need to estimate the variance of the claim count distribution. One possible approach is to look at the sample of historical claims in excess of $3,000,000 *at a 1997 exposure level* and estimate the second moment of that distribution. This approach is documented in Exhibit 8; the estimated variance-to-mean ratio is 4.46.

The result of 4.46 would be appropriate to use had we estimated it from an *observed* statistical sample. However, since we manipulated the data (trending, loss development, etc.), there was a parameter risk involved. As a result, the actual variability of the number of excess claims from the estimated expectation may have been larger than predicted in Exhibit 8. Meyers [4] addressed this problem. He suggested considering the *mean* of the Number of Claims distribution to be a random variable. The principal effect of this assumption is to increase the potential variability of the number of claims distribution around its expected value. To attain the same effect, while avoiding unnecessary complications, one can judgementally increase the indicated variance-to-mean ratio.

Based on our evaluation of possible errors in the estimation procedure used to price medical malpractice accounts, we have judgementally chosen to increase the variance-to-mean ratio to 6.0.

In translating the results of our estimates of mean and variance-to-mean ratio to standard parameters (p,r) of the Negative Binomial distribution (see, for example, [3], p. 52), we have $p = 0.167$; $r = 1$.

3.2. Selection of Severity Distribution.

To select a loss severity distribution we apply the maximum likelihood method to fit a curve to individual claim data. Some caution is necessary in dealing with this particular data. The problem is that we do not have the complete set of historical information but only claims whose (untrended and undeveloped) values exceed \$1,000,000. This means that for different years we only have information about the incurred claims which exceed some threshold (equal to the trended and developed value of \$1,000,000). For example, for report year 1983, we only have information about claims whose values in 1997 dollars are greater than \$1,000,000 x 1.827 x 1.000 = 1,827,000 (see Exhibit 2). In this case our likelihood function can be written in the form:

$$L = \prod_i f(x_i,\Lambda) / [1 - F(t_i,\Lambda)] \qquad (3.2.1)$$

Here Λ is the set of parameters describing a member of particular family of distribution functions (for example, for the lognormal distribution, Λ consists of the two standard parameters, μ and σ), $f(x_i,\Lambda)$ - pdf of loss severity distribution given the set of parameters Λ, x_i - the value of (trended and developed) claim i, $F(t_i,\Lambda)$ - distribution function, t_i - corresponding threshold value (1,827,000 for 1983 claims, etc.). The maximum likelihood estimators are the set of parameters Λ_0 that maximizes the function (3.2.1).

It is recommended to try different types of loss distribution to fit the data and select the one that has the best fit. Also, one can fit the curve to the portion of the data in excess of different retention points, such as \$2 Mil, \$2.5 Mil, etc.; this approach is consistent with one suggested by Finger [1]. The next section describes our approach in comparing different distributions. Exhibit 9 contains the list of distribution functions fitted to different portions of the data we used in pricing the AHU account.

3.3. Goodness-of-Fit Tests for Severity Distributions.

To select which distribution to use one can use the percentile matching test. The idea is to compare the theoretical excess probabilities for the fitted loss distributions with the empirical excess probabilities. This approach is illustrated in Exhibit 10. Comparing the excess probabilities for five fitted curves with empirical data, we have selected the distribution **Lognormal-2** as having the best fit; this lognormal distribution was fitted to individual claims greater than \$2,000,000 (see Exhibit 9). Please note that only excess *conditional* probabilities should be considered; it is not that important how good the fit is for claim values below the retention of \$3,000,000.

Finally, one can perform the χ^2 - test to assure a good fit to empirical data for the selected distribution (see Hogg and Klugman [3], p. 103 for the description of the χ^2 - test). For **Lognormal-2** this test is documented in Exhibit 11; we tested the goodness-of-fit on the interval in excess of \$2,000,000. The test statistic value of 3.776 is smaller than 20% critical value of 9.803 for χ^2 - distribution with 7 degrees of freedom. This indicates an acceptable fit.

4. Estimation of the Number of Simulations Required.

Before starting the simulation process one should approximate the number of simulations to perform in order to achieve the intended goal. Different people may select different goals depending on their pricing philosophy. While we concentrated on the estimation of the expected reinsurer's loss cost only (the first moment of the aggregate loss distribution for both excess layers), one may want more information. For example, one may wish to price the account based on its expected variability (e.g., to select a profit load as a function of the variance of expected loss cost), or based on established expected deficit standards. Utilizing such approaches, one would need to perform enough simulations to approximate higher moments, or even percentiles of the aggregate loss distribution, with some reasonable degree of accuracy. The number of simulations required to achieve that is much larger than for an estimation of the first moment only. However, we focused on the simulation procedure and not on sophisticated pricing techniques. Thus we selected the number of simulations necessary to estimate the expected RLC with an acceptable degree of precision.

To describe our approach we first need to define some terms and values. "One simulation" is equivalent to the aggregate loss experience for a one year period in both reinsurance layers. Exhibit 12 shows the results of one simulation. First, we generate a random number n for claims in excess of \$3,000,000; this number is taken from the Negative Binomial distribution as specified in section 3.1. Secondly, we generate n claim values; all these values are taken from **Lognormal-2** distribution truncated at 3,000,000, as specified in section 3.2. Next, each claim value is apportioned to two reinsurance layers according to the terms described in section 1.1. Finally, the aggregate loss for each of reinsurance layers is calculated by adding the appropriate portions of n individual claim values.

We repeat N independent simulations resulting in samples of size N for the annual aggregate loss in both reinsurance layers; then we use the sample mean $\bar{X}$ as an estimate of the expected reinsurer's loss costs. If N is large enough, we can use the Central Limit Theorem to estimate the difference between $\bar{X}$ and the true expectation μ of the aggregate loss cost. Namely, according to the Central Limit Theorem, even though the aggregate loss distribution is skewed and not normal, for large N the distribution

$$(\bar{X} - \mu) / (\sigma / \sqrt{N}),$$

being derived from the sum of N independent aggregate loss distributions, converges to the standard normal distribution (σ is the standard deviation of the aggregate loss distribution). Therefore, at 95% confidence level,

$$|\bar{X} - \mu| \leq 1.96 * \sigma / \sqrt{N} \qquad (4.1.1)$$

Now, if we select T to be an acceptable tolerance for the difference $|\bar{X} - \mu|$, we can estimate the number N of simulations required to assure that this difference is less than T at the 95% confidence level:

$$N \geq (1.96 * \sigma / T)^2 \qquad (4.1.2)$$

For the practical use of the formula (4.1.2) σ and T need to be approximated.

When pricing a reinsurance contract, an actuary often knows a proposed price or existing terms for it. This knowledge can help to select T (5% of existing price, for example). Even if the actuary does not know an amount of premium anticipated for an account, he or she can easily approximate such an amount by running a relatively small number of simulations (say, 1000). The mean of the resulting sample could be used to reasonably select T. The same approach could be recommended to approximate the value of σ.

For our AHU example after 1000 simulation we have: for the 1-st Excess Layer $\bar{X}$ = \$4,532,000, σ = \$3,510,000; for the 2-nd Excess Layer $\bar{X}$ = \$1,788,000, σ = \$3,403,000. Selecting T = \$50,000 and approximating σ = \$3,500,000 we have by formula (4.1.2):

$$N \geq (1.96 * 3{,}500{,}000 / 50{,}000)^2 = 18{,}824.$$

Therefore, at a 95% confidence level, performing 20,000 simulations for an annual aggregate loss should assure that the sample mean differs from the true expected annual aggregate loss by less than \$50,000 (for either reinsurance layer).

Alternatively, one can monitor the convergence of the simulation process and stop it when the change in the sample mean (and, possibly, higher moments) in between simulations becomes reasonably small.

The third approach [1] is to use an upper bound for σ. For example, it can be proven that the standard deviation σ of any distribution whose values are concentrated on the finite segment [0;A] is less than A/2. For the 2-nd Excess layer, using T = \$50,000 and A = \$12,000,000, formula (4.1.2) implies that

$$N \geq (1.96 * 12{,}000{,}000 / 2 / 50{,}000)^2 = 55{,}320.$$

The indicated number of simulations for this method is usually significantly higher than it is really necessary to obtain a required tolerance level.

5. Simulation Results for the Excess Loss Distribution.

The simulation process has been described in Section 4; the results for one simulation are shown in Exhibit 12. Different software packages could be utilized for simulation. We use a package called *@RISK*; this one is designed to be used with standard spreadsheets, like *Lotus 1-2-3* or *Excel*. Exhibit 13 shows the settings for the simulation procedure; the number of simulations to run (20,000) has been specified in Section 4.

The simulation results are shown separately for the 1-st and the 2-nd reinsurance layers in Exhibit 14. Please note that the aggregate loss distributions for both reinsurance layers, although shown in detail (the four first moments and percentiles), should be used with great caution. The number of simulations we went through has been selected to achieve our goal, which is to obtain a reasonably accurate estimator for the expected aggregate loss. There is no warranty that the percentile statistics shown are accurate estimates of the true percentiles of the aggregate loss distribution; to achieve that, it might be necessary to run more simulations.

Using formula (4.1.1) we can refurbish our estimate of $| \bar{X} - \mu |$. Namely, using estimated results for the 1-st Excess layer, we can conclude that

$$| 4{,}481{,}577 - \mu_1 | \leq 1.96 * 3{,}498{,}020 / \sqrt{20{,}000} = 48{,}480,$$

where μ_1 is the expected annual aggregate loss for the 1-st Excess layer. For the 2-nd Excess layer the same approach leads to estimate

$$| 1{,}779{,}283 - \mu_2 | \leq 1.96 * 3{,}433{,}117 / \sqrt{20{,}000} = 47{,}580,$$

where μ_2 is the expected annual aggregate loss for the 2-nd Excess layer.

[1] The idea of this method has been suggested to the author by Marc Shamula.

To insure the quality of the results produced by the simulation method one could compare them to the results obtained by using another known technique if it is possible. To do such a comparison we estimated the annual aggregate loss for the 1-st Excess layer using the Panjer method. Using the Number of Claims and Severity distributions specified in Section 4, and the unit length of \$25,000 for discretization, we obtained the estimate of \$4,482,940. The difference of this result from the one produced by simulation method is about 0.03%.

6. Pricing Recommendations.

The final step in the process is to convert the estimated loss cost to a recommended price for reinsurance coverage by using formula (1.3.1). We will not attempt to give a recipe on how to select corresponding factors. However, we will briefly discuss their relationship with the simulation pricing approach.

CR and BF are external variables suggested by a broker or client and often are not under the control of the reinsurer; we will not discuss them.

IXL reflects the reinsurer's expenses; it might be a separate load or it might be combined with the TER under the concept of "risk based capital". If a reinsurance company uses a separate load for IXL in its pricing formula, it is usually expressed as a function of the size of account (reinsurance premium net of commission and brokerage fees).

TER for the contract should, at least theoretically, reflect the level of risk that the reinsurer is taking by writing a particular contract. Usually the risk of the contract is measured by the potential variability of its loss experience. If a reinsurance company utilizes some unified approach to reflect risk in the pricing formula (e.g., use risk load proportional to the variance of the expected loss cost), the simulation method is an ideal provider of information. Exhibit 14 shows various characteristics of the expected aggregate loss distributions (higher moments, mode, and percentiles) one can use to measure the risk. However, as discussed earlier, one must make sure to run enough simulations to obtain reliable estimates for these characteristics.

DF is a function of the expected payout pattern for the account's losses and interest rates. While some information can be extracted from the historical loss emergence pattern for the account (see Exhibit 3.2), the estimated payout pattern may not be a good predictor for the high attaching reinsurance layers. For example, one can anticipate a significant delay in payments for the 2-nd Excess layer, because the payments in this layer would intensify considerably after the coverage of the 1-st Excess layer is exhausted. According to Exhibit 14, the probability that the coverage of the 1-st Excess layer will be depleted is about 25%. An alternative way to deal with this problem is to simulate the payment date of each excess loss in addition to its value. Then calculate the present value of such payments in 1997 dollars while applying the corresponding discount factor to the simulated claim value. Using this approach one can omit the DF multiplier in formula (1.3.1) because the produced RLC is already discounted.

Exhibit 15 displays the recommended reinsurance premiums derived by application of formula (1.3.1) for both reinsurance layers of coverage. The loading factors used in this exhibit are for illustrative purposes only, and are not actual factors used for pricing.

7. Final Remarks and Conclusions.

This paper illustrates the application of a simulation method in excess reinsurance pricing. Our considerations were intentionally limited by the data described in Section 1.2; having more detailed information one can achieve much more accurate results. For example, getting the individual development information for large claims, one can use it to estimate the development factor more accurately. There are countless variations of the types of data which reinsurance actuaries can find available for a pricing analysis. We have not even tried to reflect these variations. Rather, we attempted to show the application scheme of the simulation method in reinsurance pricing emphasizing its critical points.

We have considered the simulation approach in computing aggregate loss distributions. As we demonstrated, the scope of the applicability of the simulation method is more broad than for other aggregate loss distribution techniques. It combines easy programming with highly accurate results. Although it currently requires a substantial amount of computer resources, this will become less of an issue with further advancements of computer technology. With the development of efficient simulation software and increasing speed of modern computers, simulation methods promise to become one of the leading tools in actuarial practice.

8. Acknowledgments.

I would like to express my deep appreciation to Marc Shamula for his outstanding effort in reviewing the drafts of this paper. He suggested numerous improvements and provided the author with valuable insights.

I am also very thankful to Gary Patrik for his continuous support and encouragement.

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Exhibit 1

Pricing Example:
Alpha Hospital Union

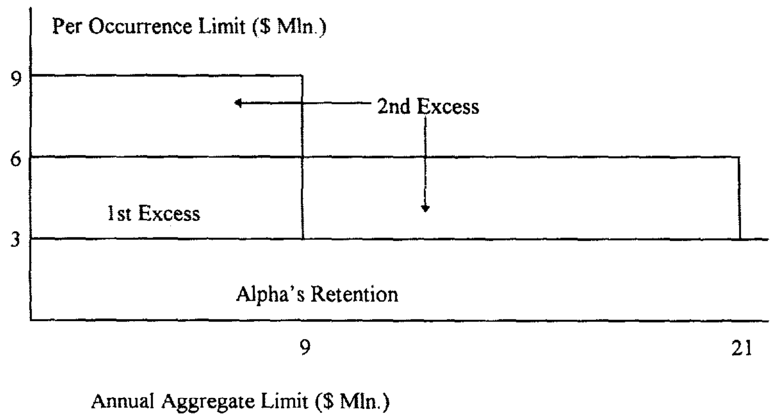

Exhibit 2

ALPHA HOSPITAL UNION

Incurred Cases Over $1,000,000 @ 12/31/95 - Extract

Trended to 07/01/97 Trend = 4.4%

Case #	Total Incurred Loss	Trend Factor	Trended Loss	Develop-ment Factor	Trended & Developed Loss	1-st Excess Layer Loss	2-nd Excess Layer Loss
Report Year 1983							
C83-0988	7,454,310	1.827	13,621,170	1.000	13,621,170	3,000,000	3,000,000
C83-0518	5,854,006	1.827	10,696,954	1.000	10,696,954	3,000,000	3,000,000
C83-0832	4,800,106	1.827	8,771,177	1.000	8,771,177	3,000,000	2,771,177
C83-0021	3,228,345	1.827	5,899,115	1.000	5,899,115		2,899,115
C83-0656	3,157,378	1.827	5,769,438	1.000	5,769,438		329,708
C83-0305	2,093,321	1.827	3,825,099	1.000	3,825,099		0
C83-0441	2,131,311	1.827	3,894,519	1.000	3,894,519		0
C83-0209	2,106,704	1.827	3,849,554	1.000	3,849,554		0
C83-0767	1,911,213	1.827	3,492,337	1.000	3,492,337		0
C83-0008	1,641,695	1.827	2,999,849	1.000	2,999,849		
C83-0390	1,500,234	1.827	2,741,360	1.000	2,741,360		
C83-0962	1,300,452	1.827	2,376,300	1.000	2,376,300		
C83-0481	1,198,792	1.827	2,190,538	1.000	2,190,538		
C83-0190	1,187,056	1.827	2,169,094	1.000	2,169,094		
C83-0271	1,137,370	1.827	2,078,303	1.000	2,078,303		
C83-0450	1,141,698	1.827	2,086,210	1.000	2,086,210		
C83-0393	1,103,989	1.827	2,017,306	1.000	2,017,306		
C83-0468	1,095,040	1.827	2,000,954	1.000	2,000,954		
Total Report Year 1983						**9,000,000**	**12,000,000**
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Report Year 1992							
C92-0921	3,720,867	1.240	4,614,734	1.000	4,614,734	1,614,734	
C92-0691	3,032,036	1.240	3,760,424	1.075	4,042,456	1,042,456	
C92-0423	2,877,629	1.240	3,568,924	1.075	3,836,594	836,594	
C92-0802	2,376,103	1.240	2,946,916	1.075	3,167,934	167,934	
C92-0331	2,309,169	1.240	2,863,902	1.000	2,863,902		
C92-0669	2,240,742	1.240	2,779,038	1.075	2,987,465		
C92-0473	2,281,805	1.240	2,829,964	1.000	2,829,964		
C92-0698	2,217,662	1.240	2,750,413	1.075	2,956,694		
C92-0721	2,134,174	1.240	2,646,869	1.075	2,845,384		
C92-0205	2,074,380	1.240	2,572,710	1.075	2,765,663		
C92-0075	1,673,136	1.240	2,075,074	1.075	2,230,705		
Total Report Year 1992						**3,661,718**	**0**

Exhibit 3-1

ALPHA HOSPITAL UNION

Reported Losses ($ 000)

Report Year	1	2	3	4	5	6	7	8	9	10	11	Ult.
	Evaluation Year											
1983				88,420	91,350	93,593	93,723	93,277	93,914	93,888	93,848	93,848
1984			66,200	69,814	70,282	72,664	72,591	72,397	71,077	71,213	71,575	71,575
1985		73,094	77,151	80,754	82,720	83,984	84,278	84,452	85,566	85,405	85,470	85,470
1986	43,357	48,147	51,946	54,388	55,248	56,209	57,079	57,239	56,747	56,859		56,859
1987	60,455	66,167	70,353	74,966	75,122	76,016	76,213	76,032	76,202			76,202
1988	62,839	65,756	79,543	73,818	76,470	76,333	75,521	75,700				75,700
1989	80,524	85,021	90,377	93,878	98,685	100,593	100,794					100,794
1990	60,507	66,776	71,690	73,010	76,054	77,195						77,349
1991	62,216	66,810	69,397	73,249	75,525							76,660
1992	57,860	63,610	69,004	71,596								75,288
1993	59,360	65,386	70,250									76,458

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983				1.033	1.025	1.001	0.995	1.007	1.000	1.000	1.000
1984			1.055	1.007	1.034	0.999	0.997	0.982	1.002	1.005	1.000
1985		1.056	1.047	1.024	1.015	1.004	1.002	1.013	0.998	1.001	
1986	1.110	1.079	1.047	1.016	1.017	1.015	1.003	0.991	1.002		
1987	1.094	1.063	1.066	1.002	1.012	1.003	0.998	1.002			
1988	1.046	1.210	0.928	1.036	0.998	0.989	1.002				
1989	1.056	1.063	1.039	1.051	1.019	1.002					
1990	1.104	1.074	1.018	1.042	1.015						
1991	1.074	1.039	1.056	1.031							
1992	1.099	1.085	1.038								
1993	1.102	1.074									
Last 3	1.092	1.066	1.037	1.041	1.011	0.998	1.001	1.002	1.000	1.002	
Last 5	1.087	1.067	1.016	1.032	1.012	1.003	1.000	0.999	N/A	N/A	
Best 3 of 5	1.092	1.070	1.032	1.036	1.015	1.003	1.001	1.000	N/A	N/A	
Selected	1.092	1.070	1.035	1.036	1.013	1.002	1.000	1.000	1.000	1.000	1.000
Cumulative	1.272	1.165	1.088	1.052	1.015	1.002	1.000	1.000	1.000	1.000	1.000
Percentage Reported	78.6%	7.2%	6.0%	3.2%	3.4%	1.3%	0.2%	0.0%	0.0%	0.0%	0.0%

Exhibit 3-2

ALPHA HOSPITAL UNION

Paid Losses

Report Year	Evaluation Year 1	2	3	4	5	6	7	8	9	10	11	12
1983				30,563	37,828	46,321	52,139	60,068	68,701	75,438	81,548	86,053
1984			10,393	17,962	24,807	30,667	34,428	46,015	50,036	56,452	60,326	65,303
1985		2,234	17,436	25,322	42,617	47,263	54,168	57,949	66,211	70,764	75,231	
1986	211	802	11,621	12,137	18,960	27,538	33,747	37,267	41,887	45,711		
1987	166	830	13,212	19,768	27,687	36,160	41,669	50,569				
1988	390	2,269	19,637	24,774	34,030	45,975	51,405					
1989	726	2,448	24,402	30,784	39,690	62,815						
1990	507	1,818	24,083	27,515	33,241							
1991	381	1,408	19,560	25,554								
1992	466	1,212	21,501									
1993	430	1,510										

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.	After 11
1983				1.238	1.225	1.126	1.152	1.144	1.098	1.081	1.151	
1984			1.728	1.381	1.236	1.123	1.337	1.087	1.128	1.069	1.186	
1985		7.805	1.452	1.683	1.109	1.146	1.070	1.143	1.069	1.063	1.136	
1986	3.801	14.490	1.044	1.562	1.452	1.225	1.104	1.124	1.091			
1987	5.000	15.918	1.496	1.401	1.306	1.152	1.214	0.000				
1988	5.818	8.654	1.262	1.374	1.351	1.118	0.000					
1989	3.372	9.969	1.262	1.289	1.583	0.000						
1990	3.586	13.247	1.143	1.208	0.000							
1991	3.696	13.892	1.306	0.000								
1992	2.601	17.740	0.000									
1993	3.512	0.000										
Last 3	3.269	14.960	1.237	1.290	1.413	1.165	1.129	1.118	1.096	1.071		
Last 5	3.353	12.701	1.294	1.367	1.360	1.153	1.175	1.124	N/A	N/A		
Best 3 of 5	3.490	12.369	1.277	1.355	1.370	1.140	1.157	1.133	N/A	N/A		
Selected	3.500	12.500	1.280	1.350	1.370	1.150	1.160	1.120	1.100	1.070	1.170	
Cumulative	213.098	60.885	4.871	3.805	2.819	2.057	1.789	1.542	1.377	1.252	1.170	
Percentage Paid During Prior Period	0.5%	1.2%	18.9%	5.7%	9.2%	13.1%	7.3%	8.9%	7.8%	7.3%	5.6%	14.5%

Exhibit 4

ALPHA HOSPITAL UNION

Paid Claim Count

Report Year	Evaluation Year 1	2	3	4	5	6	7	8	9	10	11	Ult.
1983					354	401	433	466	488	503	511	542
1984				212	265	311	341	379	403	417	443	470
1985			116	176	230	375	313	347	385	390	397	421
1986		51	85	116	145	182	222	260	274	285		307
1987	32	67	105	164	217	251	281	306	324			359
1988	28	59	110	162	208	249	269	296				348
1989	46	101	163	227	269	312	343					443
1990	26	79	125	166	195	234						333
1991	20	63	104	142	175							298
1992	25	56	111	156								330
1993	29	60	102									304

Link-ratios

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983					1.133	1.080	1.076	1.047	1.031	1.016	1.000
1984				1.250	1.174	1.096	1.111	1.063	1.035	1.062	1.060
1985			1.517	1.307	1.630	0.835	1.109	1.110	1.013	1.018	
1986		*1.667*	*1.365*	*1.250*	*1.255*	*1.220*	1.171	1.054	1.040		
1987	2.094	1.567	1.562	1.323	1.157	1.120	1.089	1.059			
1988	2.107	1.864	1.473	1.284	1.197	1.080	1.100				
1989	2.196	1.614	1.393	1.185	1.160	1.099					
1990	3.038	1.582	1.328	1.175	1.200						
1991	3.150	1.651	1.365	1.232							
1992	2.240	1.982	1.405								
1993	2.069	1.700									
Last 3	2.486	1.778	1.366	1.197	1.186	1.100	1.120	1.074	1.026	N/A	
Last 5	2.539	1.706	1.393	1.240	1.194	1.071	1.116	1.067	N/A	N/A	
Best 3 of 5	2.491	1.655	1.388	1.234	1.186	1.100	1.107	1.059	N/A	N/A	
Selected	2.500	1.710	1.410	1.240	1.200	1.100	1.100	1.060	1.030	1.015	1.060
Cumulative	12.748	5.099	2.982	2.115	1.706	1.421	1.292	1.175	1.108	1.076	1.060
Percentage Paid During Prior Period	7.8%	11.8%	13.9%	13.7%	11.3%	11.7%	7.0%	7.7%	5.1%	2.7%	7.1%

Exhibit 5

ALPHA HOSPITAL UNION

Statistical Data

Claim Trend = 4.4%

(1) Report Year	(2) FTE Exposure	(3) Ultimate # of Claims Paid	(4) Claim Frequency	(5) Ult. Number of Trended and Developed Claims > $3M	(6) Probability {Claim > $3M}	(7) Trended and Devel. Loss in 2-nd Excess Layer @12/95
1983		542		9	1.66%	12,000,000
1984		470		7	1.49%	0
1985	762.14	421	0.552	13	3.09%	4,082,847
1986	798.19	307	0.384	7	2.28%	0
1987	773.70	359	0.464	5	1.39%	0
1988	834.66	348	0.417	1	0.29%	0
1989	861.21	443	0.515	6	1.35%	3,914,229
1990	836.91	333	0.397	3	0.90%	0
1991	859.55	298	0.347	0	0.00%	0
1992	834.09	330	0.396	4	1.21%	0
1993	813.45	304	0.374	0	0.00%	0
	All Year Average		0.427	1983-93 Avg.	1.24%	
				1983-89 Avg.	1.65%	
Selected			**0.40**		**1.50%**	
1997-est.	840.00	**333**		**5.00**		

Notes. (2) is Full Time Equivalents for AHU
(3) is from Exhibit 4
(4) = (3) / (2)
(5) and (7) are from Exhibit 2
(6) = (5) / (3)

Exhibit 6

ALPHA HOSPITAL UNION

Trend Estimation

(1) Report Year	(2) Ultimate # of Claims	(3) Ultimate Loss	(4) Ultimate Average Claim Size	(5) Log	(6) Predicted Average Claim Size
1983	542	93,848	173.26	5.1548	169.94
1984	470	71,575	152.42	5.0267	177.36
1985	421	85,470	203.10	5.3137	185.10
1986	307	56,859	185.43	5.2227	193.18
1987	359	76,202	212.23	5.3577	201.62
1988	348	75,700	217.72	5.3832	210.42
1989	443	100,794	227.42	5.4268	219.61
1990	333	77,349	232.56	5.4492	229.20
1991	298	76,660	256.83	5.5484	239.20
1992	330	75,288	228.19	5.4302	249.65
1993	304	76,458	251.36	5.5269	260.55

Regression Output:

(7) Constant	-79.6070
Std Err of Y Est	0.0769
R Squared	0.7904
No. of Observations	11
Degrees of Freedom	9
(8) X Coefficient(s)	0.0427
Std Err of Coef.	0.0073
(9) Annual Trend Indicate	**4.4%**

Notes. (2) is from Exhibit 4
(3) is from Exhibit 3-1
(4) = (3) / (2)
(5) = ln{ (4) }
(6) = exp{ (7) + (1) * (8) }
(9) = exp{ (8) } - 1

Exhibit 7

ALPHA HOSPITAL UNION

The Development of Losses That Were Open At Fourth Evaluation ($ 000)

Evaluation Year

Report Year	4	5	6	7	8	9	10	11
1983	57,857	60,787	63,030	63,160	62,714	63,351	63,325	63,285
1984	51,852	52,320	54,702	54,629	54,435	53,115	53,251	53,613
1985	55,432	57,398	58,662	58,956	59,130	60,244	60,083	60,148
1986	42,251	43,111	44,072	44,942	45,102	44,610	44,722	
1987	55,198	55,354	56,248	56,445	56,264	56,434		
1988	49,044	51,696	51,559	50,747	50,926			
1989	63,094	67,901	69,809	70,010				
1990	45,495	48,539	49,680					

	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-ult.
1983	1.051	1.037	1.002	0.993	1.010	1.000	0.999	1.000
1984	1.009	1.046	0.999	0.996	0.976	1.003	1.007	1.000
1985	1.035	1.022	1.005	1.003	1.019	0.997	1.001	
1986	1.020	1.022	1.020	1.004	0.989	1.003		
1987	1.003	1.016	1.004	0.997	1.003			
1988	1.054	0.997	0.984	1.004				
1989	1.076	1.028	1.003					
1990	1.067	1.024						
Last 3	1.066	1.016	0.997	1.001	1.004	1.001	1.004	
Last 5	1.044	1.017	1.003	1.001	0.999	N/A	N/A	
Best 3 of 5	1.047	1.021	1.004	1.001	0.996	N/A	N/A	
Selected	1.050	1.020	1.003	1.001	1.000	1.000	1.000	1.000
Cumulative	**1.075**	1.024	1.004	1.001	1.000	1.000	1.000	1.000

Exhibit 8

ALPHA HOSPITAL UNION

Number of Claims Distribution Analysis

(1)	(2)	(3)	(4)
Report Year	FTE Exposure	Ult. Number of Trended and Developed Claims > $3M	Number of Claims > $3M @ 1997 Exposure
1985	762.14	13	14.328
1986	798.19	7	7.367
1987	773.70	5	5.428
1988	834.66	1	1.006
1989	861.21	6	5.852
1990	836.91	3	3.011
1991	859.55	0	0.000
1992	834.09	4	4.028
1993	813.45	0	0.000
1997-est.	840.00	6.00	

(5)	*All Year Average*	4.558
(6)	All Year Variance	**20.327**
(7)	Variance-to-Mean Ratio	**4.460**

Notes. (2) is Full Time Equivalents for AHU
(3) is form Exhibit 5
(4) = (3) * 840 / (2), where 840 is estimated FTE exposure for 1997
(5) and (6) are based on column (4)

Exhibit 9

ALPHA HOSPITAL UNION

Severity Curve Fitting Results

Name of Distribution	Lognormal	Pareto
Type of Distribution	Lognormal	Pareto
Data Fitted To	All Claims	All Claims
Parameter Estimators	Mu = 13.580 Sigma = 0.861	B = 4,978,593 Q = 6.313

Name of Distribution	Lognormal - 2	Pareto - 2
Type of Distribution	Lognormal	Pareto
Data Fitted To	Claims in Excess of $ 2 Mil	Claims in Excess of $ 2 Mil
Parameter Estimators	Mu = 14.979 Sigma = 0.371	B = 4,625,321 Q = 6.524

Name of Distribution	Lognormal - 2.5
Type of Distribution	Lognormal
Data Fitted To	Claims in Excess of $ 2.5 Mil
Parameter Estimators	Mu = 15.059 Sigma = 0.356

Exhibit 10

ALPHA HOSPITAL UNION

Severity Curve Fitting Analysis

	Empirical	Pareto	Lognormal	Pareto-2	**Lognorm-2**	Lognorm-2.5
				Prob{X>x}		
x						
2,000,000		11.86%	14.04%	9.59%	**89.76%**	93.92%
2,500,000		7.66%	9.05%	5.97%	**74.74%**	82.13%
3,000,000		5.09%	6.06%	3.83%	**56.93%**	65.83%
3,500,000		3.47%	4.19%	2.53%	**40.48%**	48.98%
4,000,000		2.42%	2.98%	1.72%	**27.39%**	34.42%
4,500,000		1.72%	2.17%	1.19%	**17.91%**	23.21%
5,000,000		1.24%	1.61%	0.84%	**11.45%**	15.19%
6,000,000		0.68%	0.93%	0.44%	**4.51%**	6.17%
7,000,000		0.39%	0.56%	0.24%	**1.74%**	2.42%
				Prob{X>x|X>2,000,000}		
2,000,000	**100.00%**	100.00%	100.00%	100.00%	**100.00%**	100.00%
2,500,000	**79.25%**	64.61%	64.46%	62.21%	**83.27%**	87.45%
3,000,000	**63.21%**	42.94%	43.19%	39.97%	**63.43%**	70.10%
3,500,000	**45.29%**	29.25%	29.88%	26.41%	**45.09%**	52.15%
4,000,000	**27.36%**	20.37%	21.23%	17.89%	**30.51%**	36.65%
4,500,000	**16.04%**	14.47%	15.44%	12.38%	**19.95%**	24.71%
5,000,000	**11.32%**	10.46%	11.44%	8.74%	**12.76%**	16.17%
6,000,000	**4.72%**	5.72%	6.60%	4.59%	**5.02%**	6.57%
7,000,000	**1.96%**	3.30%	4.02%	2.55%	**1.94%**	2.57%
				Prob{X>x|X>2,500,000}		
2,500,000	**100.00%**	100.00%	100.00%	100.00%	**100.00%**	100.00%
3,000,000	**79.77%**	66.46%	67.00%	64.25%	**76.17%**	80.15%
3,500,000	**57.15%**	45.28%	46.35%	42.45%	**54.16%**	59.64%
4,000,000	**34.53%**	31.54%	32.94%	28.75%	**36.64%**	41.91%
4,500,000	**20.24%**	22.40%	23.95%	19.91%	**23.96%**	28.25%
5,000,000	**14.28%**	16.19%	17.75%	14.06%	**15.32%**	18.49%
6,000,000	**5.95%**	8.86%	10.25%	7.38%	**6.03%**	7.51%
7,000,000	**2.48%**	5.11%	6.24%	4.10%	**2.33%**	2.94%

Exhibit 11

ALPHA HOSPITAL UNION

Goodness-of-Fit Test for Lognormal-2 Distribution

Range		Number of Claims		$\chi 2$
From	To	Empirical	Lognorm-2	
2,000,000	2,500,000	22.00	17.74	1.024
2,500,000	3,000,000	17.00	21.03	0.774
3,000,000	3,500,000	19.00	19.43	0.010
3,500,000	4,000,000	19.00	15.46	0.812
4,000,000	4,500,000	12.00	11.19	0.058
4,500,000	5,000,000	5.00	7.63	0.904
5,000,000	6,000,000	7.00	8.20	0.176
6,000,000	Infinity	5.00	5.32	0.020
		106	106	**3.776**

Degrees of Freedom	7
$\chi 2$ (7) 10% Critical Value	12.017
$\chi 2$ (7) 20% Critical Value	9.803

Stochastic Simulation Worksheet

Number of Claims Distribution:		Negative Binomial	**Severity Distribution:**	Lognormal		
Parameters:	p	0.167	Parameters:			
	r	1.000	Mu	15.059	Mu-1	3,694,545
			Sigma	0.356	Sigma-1	1,358,052
Number of Claims	14					

Claim #	1	2	3	4	5	6	7	8	9	10
Ground Up	3,220,292	7,365,376	3,324,321	4,977,541	3,079,357	6,009,490	3,117,650	4,010,786	4,590,674	4,480,066
Retained	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000	3,000,000
1-st Excess	220,292	3,000,000	324,321	1,977,541	79,357	3,000,000	117,650	280,839	0	0
2-nd Excess	0	1,365,376	0	0	0	9,490	0	729,947	1,590,674	1,480,066

Claim #	11	12	13	14	15	16	17	18	19	20
Ground Up	3,674,992	3,346,734	5,064,726	3,929,901	0	0	0	0	0	0
Retained	3,000,000	3,000,000	3,000,000	3,000,000	0	0	0	0	0	0
1-st Excess	0	0	0	0	0	0	0	0	0	0
2-nd Excess	674,992	346,734	2,064,726	929,901	0	0	0	0	0	0

1-st Excess	9,000,000
2-nd Excess	9,191,906

Exhibit 13

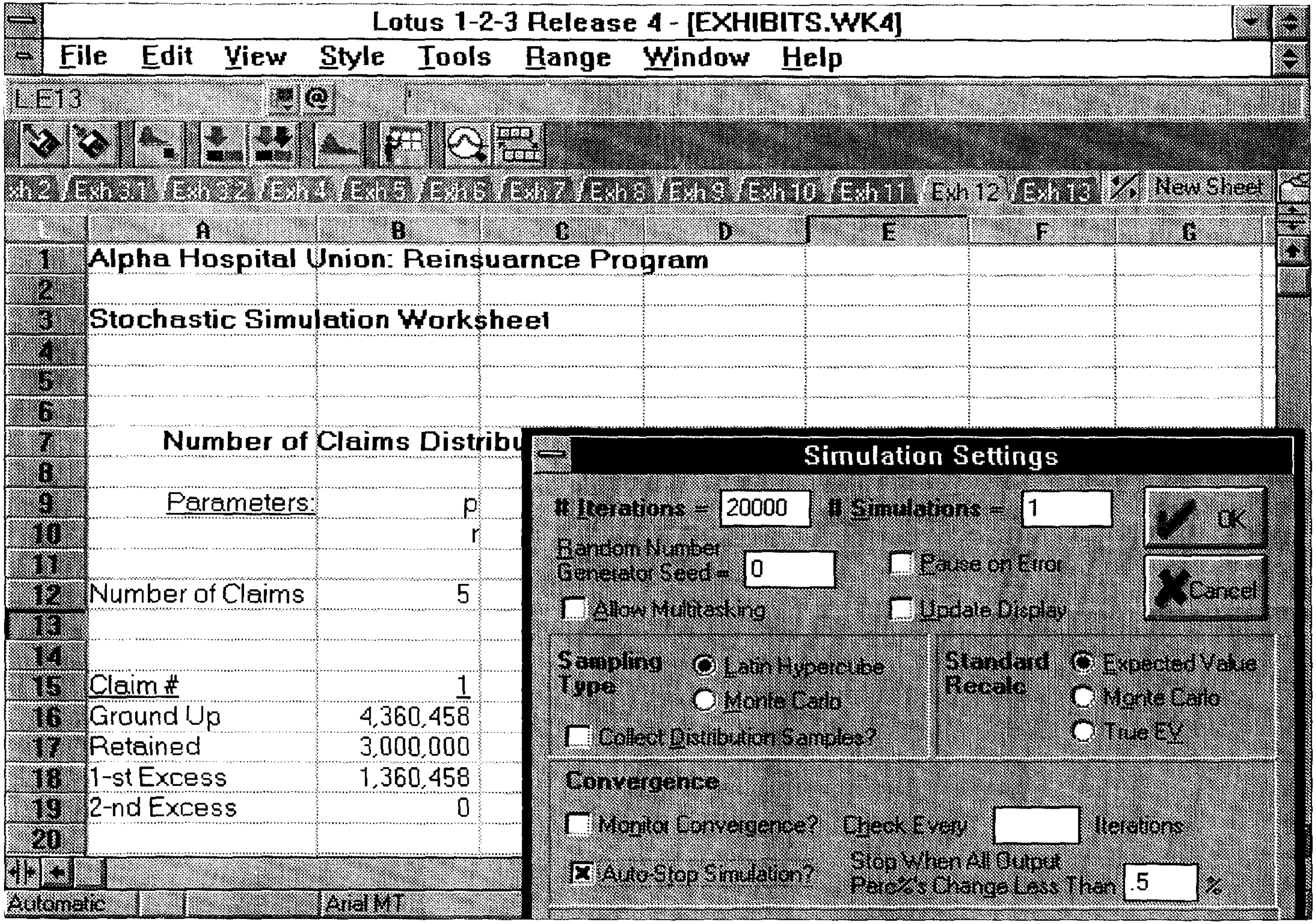
Lotus 1-2-3 Release 4 - [EXHIBITS.WK4]
File Edit View Style Tools Range Window Help
L:E13
Exh 3.1 Exh 3.2 Exh 4 Exh 5 Exh 6 Exh 7 Exh 8 Exh 9 Exh 10 Exh 11 Exh 12 Exh 13 New Sheet
A B C D E F G
Alpha Hospital Union: Reinsuarnce Program
Stochastic Simulation Worksheet
Number of Claims Distribu
Parameters: p
r
Number of Claims 5
Claim # 1
Ground Up 4,360,458
Retained 3,000,000
1-st Excess 1,360,458
2-nd Excess 0
Automatic
Arial MT
Simulation Settings
Iterations = 20000
Simulations = 1
OK
Random Number Generator Seed = 0
Pause on Error
Cancel
Allow Multitasking
Update Display
Sampling Type
Latin Hypercube
Monte Carlo
Collect Distribution Samples?
Standard Recalc
Expected Value
Monte Carlo
True EV
Convergence
Monitor Convergence?
Check Every
Iterations
Auto-Stop Simulation?
Stop When All Output Perc%'s Change Less Than .5 %

Exhibit 14

ALPHA HOSPITAL UNION

Simulation Statistics

Iterations = 20,000

Name	1-st Excess	2-nd Excess
Cell	L:B28	L:B30
Minimum =	0	0
Maximum =	9,000,000	12,000,000
Mean =	4,481,577	1,779,283
Std Deviation =	3,498,020	3,433,117
Variance =	1.224E+13	1.179E+13
Skewness =	0.092	2.017
Kurtosis =	1.444	5.803
Mode =	9,000,000	0
5% Perc =	0	0
10% Perc =	0	0
15% Perc =	0	0
20% Perc =	417,546	0
25% Perc =	1,029,013	0
30% Perc =	1,591,121	0
35% Perc =	2,168,108	0
40% Perc =	2,805,473	0
45% Perc =	3,334,980	0
50% Perc =	4,088,441	0
55% Perc =	4,837,891	0
60% Perc =	5,682,205	0
65% Perc =	6,615,973	269,680
70% Perc =	7,713,470	813,716
75% Perc =	9,000,000	1,671,010
80% Perc =	9,000,000	2,967,957
85% Perc =	9,000,000	4,741,905
90% Perc =	9,000,000	7,617,267
95% Perc =	9,000,000	12,000,000
Target #1 (Value)=	0	0
Target #1 (Perc%)=	16.67%	62.06%
Target #2 (Value)=	9,000,000	12,000,000
Target #2 (Perc%)=	74.91%	94.70%

Exhibit 15

ALPHA HOSPITAL UNION

Pricing Recommendations

		1-st Excess Layer	2-nd Exces Layer
(1)	ESTIMATED LOSS COST FOR THE LAYER	4,481,577	1,779,283
(2)	COMMISSION	0.00%	0.00%
(3)	BROKERAGE	5.00%	5.00%
(4)	IXL AS % OF RISK PREM	3.50%	5.00%
(5)	TER AS % OF PURE PREM	15.00%	25.00%
(6)	LOSS DISCOUNT FACTOR	0.750	0.550
(7)	RECOMMENDED REINSURANCE PREMIUM	**4,313,425**	**1,445,770**