Managing the Tax Liability of a Property-Liability Insurance Company

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A PROPERTY-LIABILITY INSURANCE COMPANY

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Abstract

The income tax burden placed upon a property-liability insurance company creates a variable liability with profound effects on the functioning of the enterprise. It directly affects product pricing and asset investment policies and, therefore, the potential profitability of the insurer. Recent research works have identified fuzzy sets theory as a potentially useful modeling paradigm for insurance uncertainty -- in claim cost forecasting, underwriting, rate classification, and premium determination. We view the insurance liabilities, properly priced, as a management tool of the short position in the government tax option. To implement that tool, we propose a new method of measuring uncertainty of taxes. Critical parameters of underwriting and investment are modeled as fuzzy numbers, leading to a model of uncertainty in the tax rate, rate of return and the asset-liability mix.

Keywords

Insurance, Taxes, Rate of Return, Fuzzy Sets, Investments, Swaps, Derivatives

INTRODUCTION

In this work[1], we analyze the tax management policy of a property-liability insurance company. Myers' Theorem (1984) implies that the present value of the expected tax liability , the government's tax option, is determined solely by the effective tax rate and the risk free rate. Therefore, controlling the effective tax rate of the firm is crucial in its financial management. A firm that can craft a lower effective tax rate than its competitors does enjoy a competitive advantage, but in competitive equilibrium this lowering of tax rates is achieved by all firms, and results in lower premium rates. We suggest some alternative methods of lowering the effective tax rate through the use of swaps with a life insurance firm. We also examine the uncertainty of the tax rate by proposing fuzzy sets methodology for modeling that uncertainty. Our analysis implies that uncertainty is indeed quite great, and may be underestimated under other methodologies.

MYERS' THEOREM AND ITS IMPLICATIONS

We assume that an insurance corporation holds an asset portfolio yielding a one-period investment return, and is subject to a tax liability on realized income. We also assume a simple Capital Asset Pricing Model market. Let T be the effective tax rate on the investment income, for now taken to be known with certainty.

Myers' Theorem (1984) says that the risk-adjusted present value of the tax liability on investment income from a risky investment portfolio held by a corporation is

$$\mathrm{PV}(\mathrm{T}\tilde{\mathrm{r}}_{\mathrm{A}}) = \frac{\mathrm{Tr}_{\mathrm{F}}}{1+\mathrm{r}_{\mathrm{F}}} \tag{1}$$

where $\tilde{r}_A$ is the rate of return on the risky portfolio, while r_F is the risk-free rate of return. In other words, the present value of the tax liability on the risky return is calculated as if that return were the risk free rate. The present value of the tax liability is independent of the investment strategy, and determined solely by the effective tax rate and the risk free rate.

Derrig (1994) notes that the tax liability itself is not risk free. In fact, the beta of the tax can be determined to be:

$$\beta_{TAX} = \beta_A \frac{1+r_F}{r_F}, \tag{2}$$

where β_A is the beta of the risky asset utilized by the company's investment strategy. Note that unless that asset is risk free, or the risk free rate equals zero, $\beta_{TAX} > \beta_A$.

[1] The second author received research funding from the College of Arts and Sciences of the University of Louisville. The authors also gratefully acknowledge the research assistance of Daniel Scala, the production assistance of Julie Jannuzzi and the comments of an anonymous referee.

The present value of the after-tax final investment holdings of the corporation equals

$$PV(1+(1-T)\tilde{r}_A) = \frac{1+(1-T)r_F}{1+r_F} \tag{3}$$

and the after-tax beta of the risky portfolio is:

$$\beta_{AFTER-TAX} = \beta_A \frac{(1-T)(1+r_F)}{1+(1-T)r_F}. \tag{4}$$

The implication of these results is that the effective tax rate and the risk free rate fully determine the present value of the expected investment tax liability, and when combined with the market riskiness of the investment portfolio, the after-tax, effective, riskiness of that portfolio.

Following Myers, we consider a one-period insurance company market value balance sheet at the time a policy is issued:

ASSETS	*LIABILITIES*
Asset Value	Present Value of Expected Losses and Expenses
(Premium + Equity Invested)	Present Value of Underwriting Tax
	Present Value of Investment Tax
	Present Value of Future Profits and Equity Returned

Any firm by virtue of its existence assumes a short position in a security producing cash flows of taxes payable by the firm. The government collecting the tax is long that security. One might naturally expect a firm to develop strategies to manage this short position.

In the case of tax on investment income, we see certain important implications for its management given by the Myers' Theorem. The present value of tax can be matched perfectly by investing a portion of assets given by the rate T at the risk free rate (e.g., if the effective tax rate is 35%, invest 35% of your portfolio in Treasury Bills maturing when taxes are due and use the interest earned to pay taxes). However, from the investor's perspective, the present value of the tax burden imposed on the investor's equity in the insurance firm is transferred to the policyholder through the premium charged (Myers and Cohn, 1987). An increase in the tax liability on the balance sheet, e.g., through a higher investment tax rate, results in an increase in the assets acquired from premiums.

The implication is that the effective tax rate on combined investment and underwriting income is an essential parameter in the implementation of theoretical underwriting profit models (Cummins (1990), Taylor (1994)). In this work, we will investigate two issues related to the management of the effective tax rate on investment income:

- Can a strategy of minimization of the tax liability through the use of derivative securities be rationally pursued, given the uncertainties of the firm's position, and
- Can fuzzy sets theory be used as a tool for management of uncertainty arising from forecasts of the effective tax rate and after-tax rate of return.

CRAFTING AN EFFECTIVE TAX RATE

Rational investors seek after-tax risk. In a world with taxes there is a question of whether true tax advantages exist, when all differences in risk are properly accounted for (Derrig, 1994). Stone[2] introduced the concept of a regulatory standard investment portfolio in the context of an insurance company -- that is a portfolio of zero-coupon Treasury securities whose maturities[3] are matched to the expected loss payment patterns. If this regulatory standard investment portfolio is used, computation of the effective investment tax rate is simple -- all income from Treasury securities is fully taxable at 35% corporate tax rate.[4] Further, the short position in the tax liability is fully covered by investing the portion of the policyholder premium equal to the expected tax liability in Treasury securities.

Myers (1984) posed the question whether some other investment portfolio with lower tax rates is actually superior in all relevant aspects to the regulatory standard portfolio, so that it brings about an additional value to the company holding such a portfolio. If such a portfolio exists, it must contain risky securities. In that case, the short position in the tax liability can be fully covered provided either (1) the effective tax rate of the portfolio is known with certainty, so the tax portion of the policyholder premium will exactly cover the option price of the tax liability, or (2) the uncertainty in the effective tax rate of the portfolio can be eliminated.

Cummins and Grace (1994) determined that insurers perceive a yield advantage for longer maturity tax exempt bonds, implying the existence of a portfolio with an effective tax rate lower than 35 percent. This can be justified only by a tax clientele effect -- a marginal buyer with a marginal tax rate of less that the insurers' 35% less, at a minimum, their 5.1% proration, alternative minimum tax, and capital gains income tax. Of course, the question of comparison of risk characteristics of longer maturity tax exempt bonds with the regulatory standard portfolio, or any other portfolio, remains a complicated issue to resolve.

An insurer, nevertheless, acts as a financial intermediary between, on one hand, the claimholders (policyholders, investors, government), and, on the other hand, the suppliers of securities. What Myers' Theorem implies is that:

- Claims of government (tax liabilities) are transferred to policyholders at the prevailing effective tax rates, so that an economic profit can be earned by crafting a lower effective tax rate (assuming of course this strategy is not available to, or employed by, the competitors of the firm, in which case a lower competitive premium develops);
- Investment tax liability acts to dampen the riskiness of the after-tax investment income of the insurer, so that higher expected profit can be earned by seeking higher level of risk if sufficient return compensation is available.

Traditionally, the pursuit of a lower effective tax rate has been performed by insurers through investments in tax exempt bonds, as indicated by Cummins and Grace (1994). Other tax-preferred

[2] See Derrig, 1990, pp. 7-9.
[3] The Regulatory Standard Company could also utilize immunization techniques, e.g., match durations rather than maturities, but this would not produce an exact match of expected cash flows i.e., a match of the net premium, loss, expense and equity flows.
[4] The marginal corporate tax rate in the US at the time of this writing is 35 percent

strategies have been employed as well, such as the corporate dividend exemption[5], or a capital gains preferred tax rate.

The perspective suggested above implies that insurers, through their financial intermediary status, act as issuer of derivative securities (i.e., insurance contracts). The pursuit of lower effective tax rate can be enhanced by augmenting the existing derivative position with other derivatives which exploit the nature of insurer's activities.[6] The notion that insurers issue derivative securities is not new. Smith (1982) discussed it in the context of a life insurance policy. Doherty and Garven (1986) modeled the insurance transaction as a bundle of long and short call options, thereby leading to the pricing of the transaction through options pricing theory. Ostaszewski (1995) presented a generalized perspective of that nature -- that all financial intermediaries are indeed derivative securities issuers.

It should be noted that tax implications of derivative securities do depend on whether the ownership of underlying assets is considered to have been transferred. The uncertainty created by Internal Revenue Service (IRS) interpretations of whether ownership has transferred for tax purposes contributes to the uncertainty of the effective tax rate when such swapping arrangements are employed. For the purpose of this work we only assume that certain parameters of underlying securities are traded in the derivative position, while ownership remains.

At this point we want to outline investment strategies for an insurer that pursues its goal of minimizing its effective investment tax rate while maximizing investment return. An insurer should exploit any clientele effect by using its comparative advantage. We give two examples here, which we will use to craft proposed derivative strategies for insurers, and leave other strategies to the creativeness of the reader.

SWAP OPPORTUNITIES: TAX EXEMPT PERPETUALS, PERPETUAL CMT

Unlike other financial institutions (life insurers, mutual funds, banks) property-liability insurers do not receive a portion of their investment income free of taxes (for other financial intermediaries deemed to be an expense). The tax shield of underwriting can be utilized by them, but only to a limited degree. Unlike other investors in the tax exempt market (individuals), insurers have very long "life expectancy." Finally, insurers enjoy some corporate tax preferences.

It would seem, therefore, natural for property-liability insurers to pursue the following derivative strategies to seek tax exempt income by swapping other forms of income for it. The tax exempt income most desired by property liability insurers is of a long term nature, and ideally the security should pay only the tax exempt income without any capital gains or losses, or capital returns. We proceed to describe tax exempt perpetuals. At this point we only look at the management of the tax liability, while other considerations such as the duration or convexity of the portfolio may deem perpetuals less desirable, and these issues would have to be balanced in practice -- as indeed they are, even with yield advantage in tax-exempt bonds perceived by property-liability companies. Alas, tax exempt perpetuals are not issued. They can, however, be crafted by a series of forward contracts for delivery of long term tax exempt bonds. Similar perpetual series of tax preferred items, such as corporate dividends and capital gains, can be

[5] The current stock dividend exemption available to property-liability insurers is nominally 70 percent. But through the proration provision of the tax code, at least 15 percent of the excluded 70 percent is taxed at the marginal rate of 35 percent yielding an overall effective tax rate of at least 14.2 percent. Alternate minimum tax provision can drive that effective rate higher than 14.2 percent.

[6] While derivatives have received adverse publicity, such as the billion dollar losses in the Orange County/Robert Citron affair (NY Times, December 2, 1994, page D1), the value of derivatives as hedging securities, as opposed to speculative positions, remains valid.

created. In fact, preferred stocks exist precisely for that purpose -- they provide predictable dividend income enjoying corporate tax preference. Tax-exempt perpetuals proposed here could be viewed as special synthetic adjustable-rate preferred stocks (created with the use of forward contracts).

Let us then discuss one implication -- although the comparative advantage of insurers for the existing tax free bonds remains a debatable issue, one can hardly argue the fact that the theoretical clientele effect does exist for tax exempt perpetuals.

There are numerous ways of trading fully taxable income for tax preferred items. The simplest trade is designed in the following example: company A trades to company B the current capital gains on its tax exempt bond portfolio, which would be taxed (currently) at the full corporate rate, for a forward commitment to purchase new issue tax exempt bonds of the same quality as the current portfolio matures and of equal tax exempt income to company A. This is illustrated below:

COMPANY POSITION	
Company A ***Property Insurer***	***Company B*** ***Life Insurance Company***
Asset = Long Term Tax Exempt Bond purchased at a discount	Asset = Cash

Assume that investment income of B qualifies for reserve deduction. Let now A enter with B into the following swap:

COMPANY SWAP POSITION	
Company A ***Property Insurer***	***Company B*** ***Life Insurance Company***
Asset = Long Term Tax Exempt Bond purchased at a discount	Asset = Cash
A pays B annual amortization of tax exempt bond discount.	B pays A a forward commitment to purchase same amount of tax exempt income as A is now receiving beyond A's bond maturity.

This swap converts the fully taxable capital gain income to the property insurer A into (future) tax exempt coupon income. Thus, the capital gain portion of the government's tax claim short position is covered. In a more general sense, property and casualty insurance companies form a natural clientele for long term forward contracts for tax exempt income, and they should be willing to pay out of current taxable income for those forwards. It should be stressed that the actual portion of capital gains which would be traded this way would depend on the risk profiles of the companies involved, and we do not imply that all taxable gains should be traded.

The more promising trade can be devised by utilizing another source of taxable income of a property liability insurance firm -- premium revenue. A typical insurer has an underwriting loss which is balanced by an investment gain. Thus the operating income is already swapped for investment income for a typical company. However, capital requirements pose an additional hidden tax which penalizes an insurer with large liabilities and asset base. Disregarding for the moment reasonable solvency concerns we must admit that in view of this powerful combination of incentives, securitization of insurer's premium receivables creates a natural opportunity to trade a portion of premium receivables (possibly equal to the capital requirements) for forward delivery of long term tax exempt bonds. The remaining premium and capital requirement would be invested in taxables to offset the loss and expense payouts.

Note that annuity companies are a natural clientele for perpetual assets yielding five year constant maturity Treasuries (CMT). The natural trade here is to exchange premium cash flows crafted to match perpetual five year CMT income for tax exempt perpetuals. The trade could, of course, be settled at any point by matching to market. We should note that to the degree that underwriting losses provide a tax shield, this trade may need to be examined by comparing the value of the tax shield possibly lost and capital requirement tax released (so that, for example, only a partial swap may be desirable). The strategy also must be viewed in the context of all tax management, and asset-liability management strategies utilized. Tax management strategies are indeed prevalent in current practices of property-liability companies, and they generally rest on the two pilars brought forth above: tax shield of underwriting and the use of tax-preferred investments (Almagro and Ghezzi (1988)). We propose that a derivative of the structured note type can enhance existing practices.

Our final proposal addresses the degree of risk assumed by property liability insurers. Since insurance firm's beta is "dampened" by the investment tax, it would appear appropriate that insurance firms leverage up their investments to higher beta, in pursuit of higher returns. One such strategy would be for A to issue floating (e.g., LIBOR, or 5 year Constant Maturity Treasury) notes, to be purchased by B, a life insurance company with such floating liabilities, while A uses proceeds to purchase long term bonds. The resulting leverage ratio (from equation (4) with asset beta of one) should be

$$\frac{1+(1-T)r_F}{(1-T)(1+r_F)}. \qquad (5)$$

In this case the property insurer holds a tax exempt portfolio with beta equal to that of the market, while lowering its investment tax rate by the use of the interest expense exemption. Clearly, this strategy not only increases expected return, but also the risk of the firm, and it actually exploits higher expected return for higher risk accepted. Let us add here, that in case of most floating assets there is a significant clientele which in fact pursues floating income of perpetual securities: money market funds (LIBOR), life and annuity insurance companies (five year CMT). Our main conclusion is that if the so common among property liability insurers belief in their comparative advantage in the tax exempt securities markets is valid, one should expect it to be fully utilized in swaps of floating taxable income for long term tax exempt income. In either case, the caveat of IRS interpretations of ownership remains. On the other hand, holdings of nonconvertible preferreds by corporate clientele indicate that such opportunities are perceived and utilized, albeit to a limited degree, and can be enhanced by various forms of synthetic structured notes.

It is up to the securities marketplace to determine if our proposals are valid. We must admit that the limited size of the asset base of property liability insurers, in relation to e.g., household mortgages, puts them second in line in financial engineering creativity. Time will tell.

FUZZY PARAMETERS

As we have stated above, Myers' Theorem implies that calculation of the effective investment tax rate becomes an essential part of both the ratemaking and portfolio management process. However, that calculation is not only affected by the composition of the insurer's investment portfolio, with varying rates of investment tax on tax exempt bonds, taxable bonds, preferred stock, and common stock, and insurance liabilities but also by future changes in the tax code and IRS interpretations of that code. Derrig (1994) shows how the 1986 Tax Reform Act sharply increased effective tax rates of U.S. property-liability insurers.

Clearly, the investment tax rate will vary within the range between zero percent (assuming a tax exempt bond portfolio issued completely before 1986) and 35 percent. In practice, the calculation of the effective tax rate, including the implicit tax embedded in the lower yields of tax-exempt bonds, becomes immensely complicated, especially when projecting future income and taxes, where the returns also become uncertain. We believe that we have made a case for estimation of the effective tax rate as an important tool of asset-liability management. However, we also believe that the traditional probabilistic approach may not be appropriate in this context. Uncertainty of taxes goes beyond the standard probability model, in which all outcomes of experiments are clearly defined, and future states of the world are mutually exclusive. Even legislated taxes are subject to interpretations, both in the regulatory context of the Internal Revenue Code, and in the practical terms of how the firms perceive them. Thus we propose that the management of the tax liabilities should be undertaken with the use of an alternative uncertainty model. Likewise, the choice among estimates for the expected after-tax returns on risky assets is not amenable to purely probabilistic models.[7]

We propose the use of fuzzy sets theory for estimation of the uncertainty in the tax rate and after-tax rate of return of a property-liability insurer. Lotfi Zadeh (1965) suggested a methodology for uncertainty radically different from traditional probabilistic models, including that uncertainty caused by vagueness and imprecision of human perception, or other human factors.

There may be several reasons for wanting to search for models of a form of uncertainty other than randomness. One is that vagueness is unavoidable. It is caused by the imprecision of natural language, or human perception of the phenomena observed. But also when the phenomena observed become so complex that exact measurement involving all features considered significant would be next to impossible, mathematical precision is often abandoned in favor of more workable simple, but vague, "common sense" models. Complexity of the problem may be another cause of vagueness.

These reasons were the motivation behind the development of the fuzzy sets theory (FST). This area has become a dynamic research and applications field, with success stories ranging from a fuzzy logic rice cooker to an artificial intelligence in control of the Sendai subway system in Japan.

Let us define the basic concepts of FST. Recall that a *characteristic function* of a subset E of a universe of discourse U is

$$\chi_E(x) = \begin{cases} 1 \text{ } if \text{ } x \in E \\ 0 \text{ } if \text{ } x \notin E \end{cases} \tag{6}$$

In other words, the characteristic function describes the membership of an element x in a set E. It equals 1 if x is a member of E, and 0 otherwise.

Zadeh (1965) suggested that there are sets whose membership should be described differently. One example would be the set of "good drivers." This is an important concept in auto insurance, yet its inescapable vagueness is obvious.

[7] Good discussions of what has become known as the equity risk premium puzzle can be found in Mehra and Prescott (1985), Ibbotson (1996), p. 151-161 and Abel (1996).

In the fuzzy sets theory, an element's membership in a set is described by the *membership function* of the set. If U is the universe of discourse, and $\widetilde{E}$ is a fuzzy subset of U, the membership function $\mu_E : U \to [0,1]$ assigns to every element x its degree of membership $\mu_E(x)$ in the set $\widetilde{E}$. We write either (E, μ_E) or $\widetilde{E}$ for that fuzzy set, to distinguish from the standard set notation E. The membership function is a generalization of the characteristic function of an ordinary set. Ordinary sets are termed *crisp sets* in fuzzy sets theory. They are considered a special case -- a fuzzy set is crisp if, and only if, its membership function does not have fractional values.

On the base of this definition, one then develops such concepts as set theoretic operations on fuzzy sets (union, intersection, etc.), as well as the notions of fuzzy numbers, fuzzy relations, fuzzy arithmetic, and approximate reasoning (known popularly as "fuzzy logic"). Pattern recognition, or the search for structure in data, provided an early impetus for developing FST because of the fundamental involvement of human perception (Dubois and Prade, 1980), and the inadequacy of standard mathematics to deal with complex and ill-defined systems (Bezdek and Pal, 1992). A complete presentation of all aspects of FST is available in Zimmerman (1991). Numerical manipulations of FST are amply described in Kaufmann and Gupta (1991).

A *fuzzy number* is a fuzzy subset of the real line such that its membership function has a value of one for at least one point, is zero outside a certain closed interval (finite support), and has a convex area under its graph. If two fuzzy numbers are given, $\tilde{A}$ with membership function μ_A, and $\tilde{B}$ with membership function μ_B, then fuzzy addition is performed by defining the membership function of $\widetilde{C} = \widetilde{A} + \widetilde{B}$ as μ_C with $\mu_c(z) = \max \{\min (\mu_A(x), \mu_B(y)) : x+y=z\}$ (Kaufmann and Gupta, 1991). Similar application of the so called maximin principle (Zadeh, 1965) allows for the creation of other fuzzy arithmetic operations. We will utilize them in the illustrations that follow.

The first recognition of FST applicability to the problem of insurance underwriting is due to DeWit (1982). Lemaire (1990) set out a more extensive agenda for FST in insurance theory, most notably in the financial aspects of the business. Under the auspices of the Society of Actuaries, Ostaszewski (1993) assembled a large number of possible applications of fuzzy sets theory in actuarial science. Cummins and Derrig (1993, 1996) complemented that work by exploring applications of fuzzy sets to property-casualty insurance forecasting and pricing problems. Derrig and Ostaszewski (1995) applied fuzzy clustering algorithms to problems of auto rating territories and fraud detection. Young (1996) modeled the rate changing decision problem in fuzzy logic terms.

In this work, we will illustrate how FST can be useful in estimation of the effective tax rate and after-tax rate of return on an insurance firm's asset and liability portfolio. Let us begin with a simple model of an insurance firm's expected investment income and tax position. Table 1 displays the expected CAPM results for a simple one period investment portfolio. We assume a bond/stock allocation of 80/20, approximately the allocation of the US property-liability industry in 1994.[8] We assume only US government bond holdings and diversified (beta=1) stock holdings. Using corporate bonds, which are taxed at the same rate as Treasuries, would only increase the expected yield (and uncertainty) and, therefore, the bond assessment weight in the tax rate calculation. Using tax-exempt bonds with implicit tax rates equal to the effective property-liability rate of less than 30 percent would be the equivalent of using Treasury securities but with a slightly higher beta than we assume here. The estimation of the effective tax rate of

[8] The actual proportion of P-L company portfolios on an annual statement (amortized bonds, market stocks) basis for 1994 QIII is 18.2 (stocks), 75.3 (bonds), 0.7 (mortgages), 4.8 (miscellaneous) and 0.9 (cash) according to the Board of Governors of the Federal Reserve System Flow of Funds Report.

tax-exempt securities with a positive tax-advantage to property-liability insurers, such as perceived by the US portfolio managers (Cummins and Grace, (1994)) is beyond the scope of this paper.

We use CAPM expected yields with a bond beta of 0.049 and stock beta of one. We use an expected market risk premium (MRP), excess of Treasury Bills, of 8.6 percent, the 1926-1993 average MRP for the US stock market (Ibbotson Associates, 1994). The expected tax rates reflect the dividend exclusion available to US property-liability companies. The capital gain marginal rate, currently equal to the marginal corporate rate, is adjusted downward to reflect the effective tax advantage of annually deferring 50 percent of the unrealized capital gains. With this set of assumptions the nominal tax rate is 32.4 percent, lower than the marginal rate of 35 percent because of the tax preferences available to stock income. Note that none of the uncertainty of the expected income or tax assumptions is reflected in Table 1.

Table 1
Non-Fuzzy Investment Tax Rate Example

Categories	(1) Assets	(2) Expected Return on Assets	(3) Expected Pre-Tax Income (1) × (2)	(4) Tax Rate	(5) Taxes (3) × (4)
US Government Bonds	800.0	5.70%	45.60	35.0%	15.96
Stocks:	200.0	13.88%			
Dividends		3.81%	7.62	14.2%	1.08
Capital Gains		10.07%	20.14	33.3%	6.71
Total	1000.0	7.34%	73.36	32.4%	23.75

Notes: Asset mix approximates US property liability company holdings (Federal Flow of Funds, 1994 QIII), Risk-Free Return of 5.28% is Cash-Flow weighted Treasury Bill and Note average yields, November 1993-October 1994. Bond and Stock Returns are CAPM with Bond Beta of .049, stock beta of 1.0, and Market Risk Premium of 8.6%; Dividend Yield is 10-Year S&P Average Yield 1984-1993; Corporate Tax Rate is 35; Dividend and Capital Gains Tax Rates reflect P-L dividend exclusions and deferral of unrealized Capital Gains of 50% per period

Fuzzy set theory gives us a way to rework Table 1 into a display that reveals the uncertainty in the various input parameters and, hence, in the tax results themselves. Table 2 portrays a version of Table 1 where the tax rates and investment income expectations are suitably uncertain. Admittedly, there are many ways to portray the parameters as fuzzy numbers by incorporating as much or as little of the random and non-random uncertainty into the membership function. Generally, we choose to illustrate the FST effect by using triangular (i.e., the shape of the graph of the membership function is triangular) fuzzy numbers, with the uncertainty pegged at plus or minus a value dependent on the uncertainty illustrated.[9] Each fuzzy member is identified by four variables (m_1, m_2, m_3, m_4) representing the left axis, left top, right top and right axis points.[10] The tax rate outcome is the fuzzy number (31.3%, 32.4%, 32.4%, 33.4%) portraying an uncertain range of about 2 percent on the tax rate, arising directly from an assumed 2 percent uncertainty range in the marginal tax rate.

[9] The "fuzziness" of stock returns in this example represents the uncertainty in the estimation of the CAPM expected, rather than actual, return. Uncertainty in the expected equity risk premium could arise for example in choosing, contrary to Ibbotson's advice, some shorter more recent time period to average equity returns excess of the risk free rate (Ibbotson, (1996) Table A16). Random variation could be illustrated by fuzzy numbers with support equal to one standard deviation about the mean.

[10] Although we do not use the illustration here, $m_2 < m_3$ describes a uniform range of uncertainty for the expected or middle values. This situation may often be the case for non-random uncertainty (Berliner and Babad, (1994)).

Table 2
Fuzzy Investment Tax Rate Example
Corporate Tax Rates and Returns

		Fuzzy Number	Investment Categories: US Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Investments		800.0	200.0			1000.0
(2)	Expected Return	m_1	4.42%	13.08%	3.59%	9.49%	6.15%
		m_2	5.70%	13.88%	3.81%	10.07%	7.34%
		m_3	5.70%	13.88%	3.81%	10.07%	7.34%
		m_4	6.98%	14.68%	4.03%	10.65%	8.52%
(3)	(1) × (2) Expected Pre-Tax Income	m_1	35.36		7.18	18.98	61.52
		m_2	45.60		7.62	20.14	73.36
		m_3	45.60		7.62	20.14	73.36
		m_4	55.84		8.06	21.30	85.20
(4)	Tax Rate	m_1	34.0%		13.8%	32.0%	31.0%
		m_2	35.0%		14.2%	33.3%	32.4%
		m_3	35.0%		14.2%	33.3%	32.4%
		m_4	36.0%		14.6%	34.7%	33.6%
(5)	(3) × (4) Taxes	m_1	12.02		0.99	6.08	19.09
		m_2	15.96		1.08	6.71	23.75
		m_3	15.96		1.08	6.71	23.75
		m_4	20.10		1.18	7.38	28.66

Notes: Investment Returns are CAPM Table 1 returns with Fuzzy Risk-Free Rates, Market Risk Premiums, and crisp Betas of .049 (Bonds) and 1 (Stocks).

	Fuzzy Parameter: Risk-Free	MRP
m_1	4.00%	0.061
m_2	5.28%	0.086
m_3	5.28%	0.086
m_4	6.56%	0.111

1. A Fuzzy Number is Identified by the Left Axis, Left Top, Right Top, and Right Axis points (m_1, m_2, m_3, m_4).

INCLUDING THE INSURANCE POLICY TAX HEDGE

The illustrations in Tables 1 and 2 focused on the uncertainty in insurer's investment portfolio. But tax considerations involve the interplay, and uncertainty, of the insurance or liability part of the company's entire portfolio of assets. Table 3 reworks the simple investment illustration of Table 1 to show the interaction with writing insurance liabilities and using the tax shield of those liabilities to offset some of the tax liabilities from investments. This situation, of course, assumes that property-liability insurers are writing to a nominal underwriting loss, a recent historical fact. We assume, in addition to all investment assumptions of Table 1, liabilities written at 2:1 to the surplus (net worth) of the company. We assume an expected underwriting loss of 4.07 percent, a recent value for Massachusetts private passenger automobile insurance rates. The tax rate for liability returns will be assumed to be 34.5 percent, a value lower than the marginal rate reflecting the discounting of loss reserves for tax purposes. The expected tax rate for the pre-tax income on the insurers portfolio drops to 31.1 percent from 32.4 percent because of the effect of the tax shield.

Table 3 Non-Fuzzy Portfolio Tax Rate Example					
	(1)	(2)	(3)	(4)	(5)
Categories	**Portfolio Weights**	**Expected Return**	**Expected Pre-Tax Income (1) × (2)**	**Tax Rate**	**Taxes (3) × (4)**
Liabilities	-667.0	4.07%	-27.15	34.5%	- 9.36
US Government Bonds	800.0	5.70%	45.60	35.0%	15.96
Stocks:	200.0	13.88%			
Dividends		3.81%	7.62	14.2%	1.08
Capital Gains		10.07%	20.14	33.3%	6.71
Surplus/Totals	333.0	13.88%	46.21	31.1%	14.39

Notes: Investment Returns and Tax Rates as in Table 1; Expected Return on Liabilities as in expected underwriting profit margin for Massachusetts private passenger automobile liabilities, Tax Rate for Liabilities reflects discounting of Loss Reserves.

The effects of making the entire insurer portfolio fuzzy, investments and liabilities, are shown in Table 4. In addition to the fuzzy tax rate and investment returns of Table 2, we use a fuzzy underwriting return of plus or minus 10 percent of the expected.

Table 4
Fuzzy Portfolio Tax Rate Example
Corporate Tax Rates and Returns

		Fuzzy Number[1]	Liabilities	US Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Portfolio Weights		-667.0	800.0	200.0			333.0
(2)	Expected Pre-Tax Return	m_1	3.65%	4.42%	13.08%	3.59%	9.49%	9.48%
		m_2	4.07%	5.70%	13.88%	3.81%	10.07%	13.88%
		m_3	4.07%	5.70%	13.88%	3.81%	10.07%	13.88%
		m_4	4.49%	6.98%	14.68%	4.03%	10.65%	18.27%
(3)	(1) × (2) Expected Pre-Tax Income	m_1	-29.95	35.36	26.16	7.18	18.98	31.57
		m_2	-27.15	45.60	27.76	7.62	20.14	46.21
		m_3	-27.15	45.60	27.76	7.62	20.14	46.21
		m_4	-24.35	55.84	29.36	8.06	21.30	60.85
(4)	Tax Rate	m_1	33.6%	34.0%		13.8%	32.0%	28.6%
		m_2	34.5%	35.0%		14.2%	33.3%	31.1%
		m_3	34.5%	35.0%		14.2%	33.3%	31.1%
		m_4	35.4%	36.0%		14.6%	34.7%	33.0%
(5)	(3) × (4) Taxes Paid	m_1	-10.06	12.02	7.07	0.99	6.08	9.03
		m_2	- 9.36	15.96	7.79	1.08	6.71	14.39
		m_3	- 9.36	15.96	7.79	1.08	6.71	14.39
		m_4	- 8.61	20.10	8.56	1.18	7.38	20.05
(6)	(3) - (5) Expected After-Tax Income	m_1	-19.89	23.34	19.09	6.19	12.90	22.54
		m_2	-17.79	29.64	19.97	6.54	13.43	31.82
		m_3	-17.79	29.64	19.97	6.54	13.43	31.82
		m_4	-15.74	35.74	20.80	6.88	13.92	40.80
(5)	(6) ÷ (1) Expected After-Tax Return	m_1	2.36%	2.92%	9.55%	3.10%	6.45%	6.77%
		m_2	2.67%	3.71%	9.98%	3.27%	6.71%	9.56%
		m_3	2.67%	3.71%	9.98%	3.27%	6.71%	9.56%
		m_4	2.98%	4.47%	10.40%	3.44%	6.96%	12.25%

Note: Investment Returns are CAPM with Fuzzy Risk-Free Rates, Market Risk Premiums, and crisp Betas of .049 (Bonds) and 1 (Stocks).

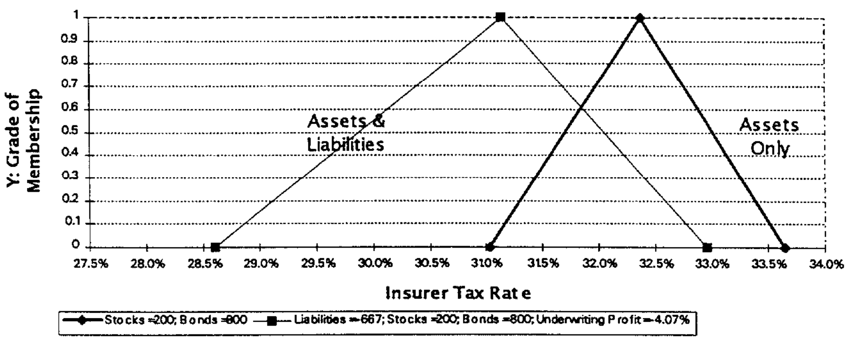
Fuzzy Parameter	Risk-Free	MRP
m_1	4.00%	0.061
m_2	5.28%	0.086
m_3	5.28%	0.086
m_4	6.56%	0.111

1. A Fuzzy Number is Identified by the Left Axis, Left Top, Right Top, and Right Axis points (m_1,m_2,m_3,m_4).

In addition to showing the effect of these fuzzy numbers on the tax rate, we list the fuzzy expected after-tax returns. The fuzzy tax rate spans 28.6 percent to 33.0 percent, a 4.4 percent gap. While the overall expected tax rate has been reduced by the effect of the tax shield (and policyholder tax hedge), the uncertainty has increased! Likewise, the after-tax rate of return, expected to be 9.56 percent, obtains a wide fuzzy range from 6.77 percent to 12.25 percent - a gap of about 5.5 percent.

Figure 1 displays the effect of a fuzzy tax shield on the fuzzy expected tax rate.

Figure 1
Fuzzy Investment Tax Rates
Effect of Liability Tax Shield

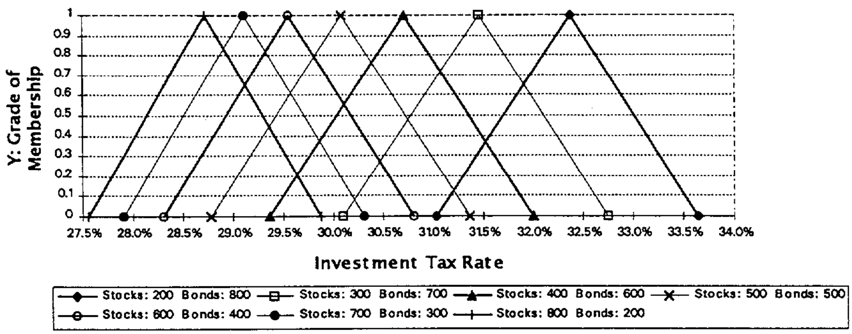


ASSET ALLOCATION

A common method of tax management in property-liability companies is to balance the trade-off of increased risk from a larger stock allocation with the decreased tax rate that emanates from the stock income preferences. Figure 2 shows the fuzzy range of tax rates as the asset allocation changes from 80/20 bond/stock to 20/80. If we measure the uncertainty of the *difference* between two fuzzy expected tax rates by the height of their intersection (the point at which they cross), one can observe the increasing uncertainty in distinguishing tax outcomes as the asset allocation moves to a larger stock position. Thus, while 80/20 and 20/80 are clearly distinct, even in the fuzzy sense, 50/50 and 40/60 retain a high degree (0.7 to 0.8) of uncertainty in differentiation of results.

Figure 2

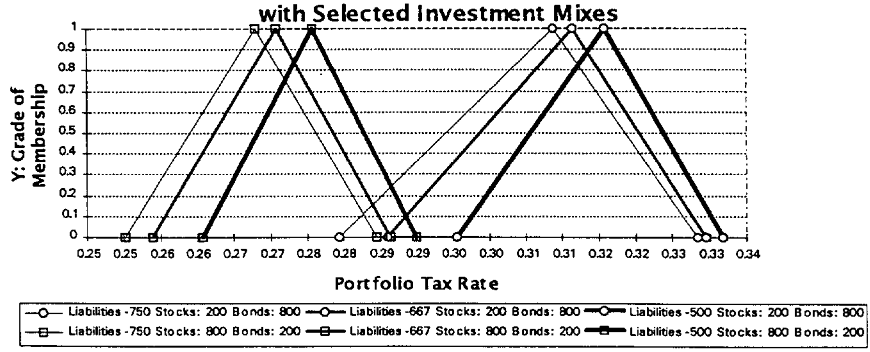
Fuzzy Investment Tax Rates with Selected Asset Mixes



The fuzzy tax effect of adding the insurance liabilities to the invested asset portfolio is demonstrated in Figure 3. Leverage ratios of 1:1 to 3:1, liabilities to surplus, provide for lower crisp expected tax rates. But those lower rates have little to distinguish them from one another on a fuzzy (uncertain) basis on either end of the assets allocation spectrum.

Figure 3

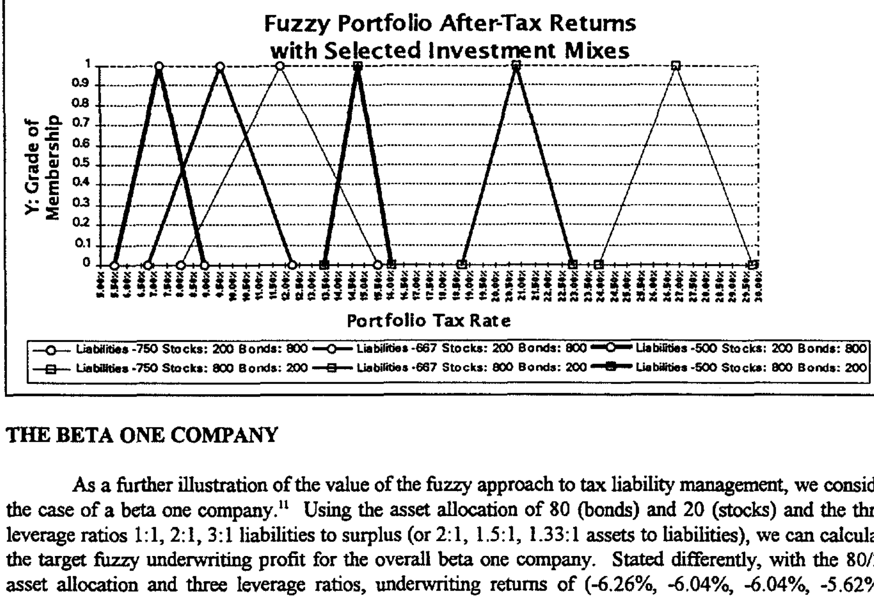
Fuzzy Portfolio Tax Rates
with Selected Investment Mixes



AFTER-TAX RATES OF RETURN

The fuzzy after-tax rates of return were displayed in Table 2. They reflected, of course, the uncertainty in the tax rates, expected investment yields and in the liabilities. Figure 4 shows the portfolio effect on after-tax rates of return for different leverage ratios and the extremes of the asset allocation illustrations (80/20, 20/80). Note that the ability to distinguish the fuzzy outcomes at the low investment risk level (80/20) for different leverage ratios but not to distinguish at the high investment risk level (20/80) lends the interpretation that the fuzzy after-tax rates of return reflect *total* uncertainty.

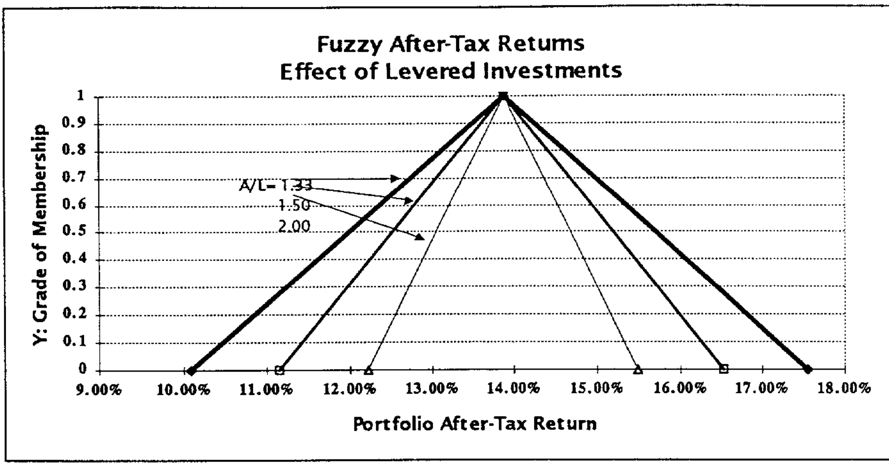
Figure 4



THE BETA ONE COMPANY

As a further illustration of the value of the fuzzy approach to tax liability management, we consider the case of a beta one company.[11] Using the asset allocation of 80 (bonds) and 20 (stocks) and the three leverage ratios 1:1, 2:1, 3:1 liabilities to surplus (or 2:1, 1.5:1, 1.33:1 assets to liabilities), we can calculate the target fuzzy underwriting profit for the overall beta one company. Stated differently, with the 80/20 asset allocation and three leverage ratios, underwriting returns of (-6.26%, -6.04%, -6.04%, -5.62%), (0.36%, 0.78%, 0.78%, 1.20%) and (2.62%, 3.04%, 3.04%, 3.46%) will result in three fuzzy after-tax returns, all "centered" on 13.88 percent - the beta one expected return. Figure 5 shows those fuzzy after-tax returns and their ranges of uncertainty. Note that the intuitive result of more uncertainty in the higher leveraged firm obtains even when the target after-tax return is the same.

[11] US property-liability companies are often thought of as being of average (beta) risk. Unfortunately, this view does not necessarily take into account the vast distribution of the capitalization of those companies. Our simplifying assumption is used regardless of leverage of the firm.



CONCLUSION

This paper has explored the management of the government's short position for tax liabilities in the context of a property-liability insurance firm. We viewed the writing of the insurance liability as covering that short position under certain circumstances. Alternative derivative (swap) positions were suggested as the beginning of possible elements in a tax hedging portfolio.

By virtue of the Myers Theorem, the tax management focus falls upon the effective tax rate of the investment portfolio. We show the ability of fuzzy set theory to illustrate not only the parametric interactions, but also the uncertainty, random and non-random, in the key parameters and outcomes. The advantages of the underwriting tax shield and the effects of parametric uncertainty on tax rate and after-tax return uncertainty were illustrated. Outcomes generally follow intuitive results; the benefit is the quantification, and graphic display, of the uncertainty of those results.

A good next step would be to expand and integrate the derivative security selection into the fuzzy set context. Better levels of uncertainty for primary and derivative assets combined may be shown through the fuzzy set paradigm. Finally, someone might undertake the formidable task of making the foregoing ideas rigorous (e.g., fuzzy partial derivatives on leverage). The richness of the fuzzy approach can only help to illuminate the problems of uncertainty.

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