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The purpose of this paper will be to identify the important financial aspects of a workers' compensation insurer and describe their incorporation into a dynamic financial model. The first section of the paper will identify and describe these financial aspects, e.g., claim frequency, claim severity, emergence patterns and investment returns. The second section of the paper will describe one or more approaches (e.g., regression on other variables, autoregression and/or distributions of random values) for incorporating these financial aspects into a dynamic financial model. The third and final section of the paper will identify the data elements needed to parameterize the models described in the second section. This final section will be presented in the format outlined in DFATFOV's Call of Papers: identification of variable, rationale for inclusion, possible source(s) of data and brief descriptions of the analytical methods presented in the second section.

DYNAMIC FINANCIAL ANALYSIS OF A WORKERS' COMPENSATION INSURER

The purpose of this paper is to identify the important financial aspects of a workers' compensation insurer and describe their incorporation into a dynamic financial model. The first section of the paper identifies and describes these financial aspects. That is, it will present brief descriptions of the drivers of workers' compensation financial results. The second section of the paper describes possible approaches for incorporating these drivers into a dynamic financial model. This section will discuss the approaches we have used in practice as well as possible alternatives and refinements to these approaches. The third and final section of the paper identifies the data elements needed to parameterize the models described in the second section. This section is meant to address the issues raised in the Call for Papers.

Key Financial Aspects

To evaluate the financial position of an insurer, it is necessary to keep track of the actual cash values as well as the booked values. The actual cash values will be dependent only on external values, whereas the booked values are dependent upon the various accounting rules that may be in effect (GAAP, statutory and tax), as well as the insurer's perception of the external environment. We have organized the key financial drivers of a workers' compensation insurer into the following categories:

- Premium - including rate level, exposure, payroll inflation, earning pattern and collectibility.
- Losses - including claim frequency, medical and indemnity severity, loss adjustment expenses, payment patterns and reserve adjustments.
- Operating expenses - including fixed and variable components.
- Reinsurance - including pricing and availability.
- Policyholder dividends.
- Investment returns.
- Residual market burdens and other assessments.

We observe that the list of drivers does not vary significantly across lines of insurance. Obviously there are some drivers, such as residual market burdens and separate analysis of medical and indemnity losses, that are significant to workers' compensation but not to all other lines. Nonetheless, an understanding of workers' compensation drivers, modeling approaches and data needs provides significant insight into the corresponding factors for models of other property-casualty lines of insurance.

Dynamic Financial Analysis

One of the refinements that dynamic financial analysis requires as compared to more traditional financial modeling is close attention to the timing of cash flows. Because many analyses focus on investment questions, more accurate projections of cash flows are required. Therefore, in each section below we reference the timing of the various revenue and cost drivers as well as their nominal and accounting values.

Premium

Obviously, premium volume is a key driver of insurer financial results. For all lines of insurance, the two primary components of premium volume are the rate level and the exposure base. For workers' compensation, the exposure base can be decomposed into changes in insured employee count and changes in wage inflation. The timing of the earning and collection of the premium, relative to when it is booked as written on the financial statements affect the income and cash flow statements, respectively.

For workers' compensation, the issues of retrospective premiums and audit premiums contribute to the complexity of modeling premium earning and collection patterns. A sophisticated model will allow premium from policies issued in one calendar year to be written, earned and collected over several years to reflect the timing of audit adjustments and retrospective premiums. We note that many models quite reasonably approximate these patterns by tracking premium booked as written in a year without tying it back to the year in which the policy was actually issued.

Losses and Loss Adjustment Expenses

The key components of workers' compensation losses and loss adjustment expenses are the frequency of claims (per whatever unit of exposure is used to project premium volume), the average cost of medical and indemnity per claim, the amount of allocated loss adjustment expenses, either per claim or per dollar of loss or indemnity, and the amount of unallocated loss adjustment expenses. As with premium, the projections from dynamic financial analyses are highly dependent upon the payment pattern assumption.

An important driver of calendar year results is of course the emergence of losses and loss adjustment expenses by calendar year for each accident or policy year. That is, analyses that are intended to provide insights regarding calendar year results must reflect not only projections of the ultimate cost of claims by accident year and their payment patterns, but also the initial amount reported by the insurer on its financial statements and the adjustments made thereto until the ultimate losses underlying the financial statements equal the actual ultimate results.

Operating Expenses

Operating expenses also affect both the income statement and cash flow results of a workers' compensation insurer. The key components of operating expenses that are often used in dynamic financial modeling include commissions, premium taxes, other acquisition expenses and general expenses. The timing of commission payments generally follows that of premium collection. Premium taxes and other acquisition expenses, on average, are incurred and paid when premium is written, whereas general expenses are usually incurred and paid over the term of the policy. Many expenses are treated differently under GAAP as compared to statutory accounting.

Reinsurance

The true profitability of a book of business can only be evaluated on a net of reinsurance basis. Whether business is modeled gross of reinsurance and netted down or modeled net of reinsurance often depends on the application and the importance of reinsurance to the book of business under review. In modeling reinsurance (or determining whether to model reinsurance separately), it is important to consider the timing and amount of reinsurance premium payments, the timing and amount of ceded loss payments and related recoveries, the impact of ceding commissions (particularly those with profit sharing features), and the collectibility of reinsurance.

Policyholder Dividends

Many workers' compensation insurers offer participating policies to insureds meeting certain size criteria. The resulting dividends are generally dependent upon the loss ratio incurred by each qualifying policyholder. Models need to incorporate the amount of such dividends as well as the timing with which they are incurred (for GAAP), declared and paid.

Investment Returns

Investment income and capital gains, both realized and unrealized, are significant contributors to workers' compensation insurers' financial results. As indicated previously, these values are dependent upon the amount and timing of cash flows from the company's underwriting operations, the economic environment in which the company operates as well as the company's investment strategy.

Residual Market Burdens and Other Assessments

Of lesser importance in the past couple of years, assessments (usually related to residual market mechanisms) can have a critical impact on the profitability of a book of workers' compensation business. As recently as the early 1990s, the residual market burden, on average, was 20% to 25% of voluntary market premium. Insurers also face other types of assessments, such as those from second injury funds and guaranty funds.

Modeling Approaches

The process used to perform a dynamic financial analysis is to first construct a base or expected value case. This base case will generally include the expectations regarding the dependencies among inputs, such as returns on various asset classes and the yield curve. The results are then tested under a relatively large number of different scenarios. For most applications, results are projected under varying economic environments. For workers' compensation, other scenarios to be tested (often simultaneously with changes in the economic environment) might include regulatory or legislative control of rates, changes in residual market burdens and significant changes to benefits.

Economic Scenarios

As indicated, most dynamic financial analyses include testing of a range of economic environments. At a minimum, the economic variables to be projected for each projection year for each scenario are:

- Change in gross domestic product.
- Consumer price inflation.
- Short-term treasury yield rate.
- Long-term treasury yield rate.
- Stock returns and dividend yields.

Assuming these economic variables are randomly generated, other asset returns are determined as functions of these variables. However, for applications focusing on analyses of asset strategies, refinements could be made to (a) include more asset classes, (b) project yield curves in more detail, allowing both convex and concave curves, (c) randomly generate the differentials between each of municipal and corporate bonds and government bonds, and (d) randomly generate the differentials between bonds of various qualities. For workers' compensation, another refinement of interest might be separate modeling of wage and medical inflation, rather than use of constants applied to consumer price inflation.

In the remainder of this section, we will discuss how each of the key drivers identified previously can be modeled in a dynamic financial analysis. For many of these drivers, the models will include formulas that incorporate components of the economic scenarios. We have assumed that randomly generated, internally consistent economic scenarios are available for use in modeling. The derivation of projections of economic variables is beyond the scope of this paper, as many such models have already been developed or can be developed based on methods and information available in economic and finance literature.

Losses and Loss Adjustment Expenses

In our work, we find it useful to first model losses and then project premium as losses divided by a modeled loss ratio. (The approach for modeling loss ratios will be discussed under the premium section.) We have applied models that have a varying range of detail for projecting future workers' compensation losses. In some instances, loss and loss adjustment expenses have been modeled in the aggregate using relatively simple equations, while in other applications we have modeled each major component of losses separately. In this paper, we describe a somewhat complex approach, thereby allowing readers to simplify the model as appropriate for their particular application.

Typically, our projections of workers' compensation losses rely on prior years' losses and changes in a number of variables:

> Changes in the number of workers insured for the total market - These changes can be approximated through use of the projected changes in real gross domestic product, adjusted for any significant changes in the percentage of the market that is self-insured. (We note that

changes in real gross domestic product include both changes in number of workers and changes in productivity and thereby only approximate the changes in the number of insured workers from year to year. The productivity change component of the change in real gross domestic product can be estimated using econometric methods. We are inclined to utilize a long-term average for this component.) The percentage of the market that is self-insured can be modeled based on rate adequacy; that is, when rates are high (i.e., loss ratios used in pricing are low), a greater proportion of the market is likely to be self-insured and vice versa. For a specialty insurer focusing on only a few industries, it may be appropriate to model insured exposure as a function of growth in those particular industries, rather than based on growth in total real gross domestic product.

Changes in the insurer's market share - Generally we assume that changes in market share are random with a serial correlation component. Changes in market share could be ignored entirely for relatively short projections periods. Theoretically, the insurer's market share could be modeled as a function of its expected loss ratio used in pricing relative to that underlying the pricing of the market on average. Although we have not generally done so, a sophisticated dynamic financial model could produce separate estimates of the insurer's expected loss ratio and the market expected loss ratio. Differences in these loss ratios in conjunction with evaluations of the elasticity of demand could drive models of the insurer's market share.

Alternately, insured exposure (i.e., the combination of the growth in the market place with the changes in the insurer's market share) could be based on the estimates implicit in the company's financial plan. The actuary must then identify and quantify the range of possible variations from the company's implicit projections.

Changes in the frequency of each of medical-only, small and severe indemnity claims per injured worker - The frequency of workers' compensation claims per worker has generally decreased in recent years, at least in part as a result of changes in the mix of employees by class. Depending on the insurer's class mix and changes therein relative to the market as a whole, these trends could be extrapolated into the future or adjusted as appropriate. If losses are modeled separately for small and severe claims, the frequency of large claims will be expected to increase relative to that of all claims as the result of inflationary effects. We note that an alternative to adjusting the frequency trend for large claims is to index the threshold above which claims are considered large. (Consistency in trend rates among the retention, small and large claims is critical.)

Consideration should also be given to the impact of exposure growth on both frequency and severity trends. When companies are growing rapidly, both the frequency and severity of claims are more likely to increase faster than under more stable conditions. Further, it is generally believed that claim frequency increases as real gross domestic product increases. As the economy is coming out of a recession, workers generally lengthen their hours thereby increasing the time exposed to injury per worker. Employers then expand the work force with generally less experienced employees who tend to have more injuries.

Changes in average wages - Wage inflation can be modeled either directly by the economic scenario generator or as a function of projected consumer price inflation.

Changes in indemnity benefits for each of small and severe claims - Changes in indemnity benefits arise through two sources. One is the normal adjustments to maximum and minimum benefits that are frequently made in response to changes in wages. These changes can occur automatically or by the specific act of a state legislature. We prefer to model these changes through the indemnity trend rate. Note that this treatment may result in indemnity trends different from those produced by NCCI, as those values typically represent indemnity trends adjusted to constant law level.

The second source of indemnity benefit change is benefit reform. Benefit reforms occur randomly, generally when rates are perceived to be too high relative to the benefits. These types of changes can be expected to occur sporadically and would typically have a larger impact on losses than the normal adjustments discussed above. An approach to modeling this component is to assume that indemnity benefit reform always produces a savings and occurs with low frequency (probability increases after several years of both high rates and high loss ratios). The theory here is that workers' compensation involves three parties that significantly contribute to potential legislation: insurers, employers, and labor. We assume that two of these parties must support legislation for it to pass. This criterion will usually only be met when employers complain of high cost (rates) and insurers complain of low profitability (high loss ratios).

We acknowledge that labor can achieve benefit increases, but these increases tend to be smaller and more frequent minor changes to benefit structures or administrative or court decisions that increase benefits. For modeling, we include these changes in the underlying trend rate. The combined impact of the underlying trend rate and benefit reform adjustments then produce long-term averages consistent with observed experience.

Non-benefit changes in the cost of indemnity per claim - Historically, average costs of indemnity claims have increased slightly faster than wage inflation, even after adjustment for benefit changes. Thus, indemnity claim cost trend rates are generally modeled as a function of wage inflation.

Changes in medical benefits - In a similar manner to the indemnity benefit changes discussed above, we prefer to model medical benefit changes only for benefit reforms. Even in states with medical fee schedules, we find that adjustments are typically responsive to inflationary pressures and can therefore be modeled as part of the underlying trend rate. Medical benefit reforms may include changes in administration, broad changes to managed care provisions, choice of physician, etc. These changes would be expected to occur sporadically in response to the conditions identified above for indemnity benefit changes.

Non-benefit changes in the cost of medical per medical-only, small and severe claim - The changes in the average cost of medical on medical-only, small and severe claims is generally modeled as a function of medical inflation.

We have found it valuable to model each of medical and indemnity on each of medical-only claims and small indemnity claims in the aggregate and on severe indemnity claims individually. As such, in addition to incorporating the above relationships into our models, we also incorporate random error terms for each of medical-only and small indemnity losses in the aggregate and the number of large claims. Each of these error terms, as well as the size of large claims, will be randomly selected from a user-defined distribution for each projection year for each iteration. Modeling losses in this fashion can incorporate elements of both process risk and parameter risk. Note that the goal of the modeling process at this point is to determine the initial value to be booked as losses. This value may be allowed to change over time as future economic conditions affect the values that may have been initially booked. Approaches for modeling these adjustments are discussed later in this paper.

The model of losses for a single projection year for a single scenario might be as follows:

1. Generate the number of workers insured in the market using a formula such as:

$$NW_i = NW_{i-1}\ x\ [a + b\,\Delta GDP_i] + e_i \qquad [1]$$

where
- a and b are constants;
- i refers to the policy year;
- NW refers to number of workers;
- ΔGDP is the percentage change in the gross domestic product; and
- e is a randomly generated error term.

2. Generate the insurer's market share using a formula such as:

$$MKSH_i = a + b\,MKSH_{i-1} + e_i \qquad [2]$$

where
- a and b are constants;
- $MKSH$ is market share; and
- e is a randomly generated error term.

3. Generate the frequencies per insured worker of medical-only, small and large indemnity claims as:

$$FRMO_i = a + b\,FRMO_{i-1} + c\left(\frac{\Delta GDP_i}{\Delta GDP_{i-1}} - 1\right) + d\,\Delta MKSH_i + e_i \qquad [3]$$

$$FRSM_i = f + g\,FRSM_{i-1} + h\,(FRMO_i - FRMO_{i-1}) + j_i$$

$$FRLG_i = k + l\,FRLG_{i-1} + m\,(FRSM_i - FRSM_{i-1}) + n_i$$

where *a, b, c, d, f, g, h, k, l* and *m* are constants;
FRMO is frequency of medical only claims;
FRSM is frequency of small claims;
FRLG is frequency of large claims;
ΔGDP is the percentage change in the gross domestic product;
ΔMKSH is the percentage change in market share; and
e, j and *n* are randomly generated error terms.

4. Generate the average cost of medical on medical-only, small and large indemnity claims as:

$$MSVMO_i = a + b\,MSVMO_{i-1} \times (1 + c\,(emcpi_i)) \times (1 + d\,mb_i) + e_i \qquad [4]$$

$$MSVSM_i = f + g\,MSVSM_{i-1} \times (1 + h\,(emcpi_i)) \times [1 + dursm\,(emcpi_i - emcpi_{i-1})] \times (1 + j\,mb_i) + k(MSVMO_i - MSVMO_{i-1}) + l_i$$

$$MSVLG_i = m + n\,MSVLG_{i-1} \times (1 + o\,(emcpi_i)) \times [1 + durlg(emcpi_i - emcpi_{i-1})] + q\,(MSVMO_i - MSVSM_{i-1}) + r_i$$

where *a, b, c, d, f, g, h, j, k, m, n, o, p and q* are constants;
MSVMO is the average cost of medical only claims;
MSVSM is the average cost of medical on small indemnity claims;
MSVLG is the average cost of medical on large indemnity claims;
dursm is the duration of the medical payment pattern on small indemnity claims at the average medical inflation rate;
durlg is the duration of the medical payment pattern on large indemnity claims at the average medical inflation rate;
emcpi is the expected medical trend used in pricing and is determined as:

$$emcpi_i = a + b\,mcpi_{i-1} + c\,(emcpi_{i-1})$$

mcpi is the medical component of the consumer price index;
mb is the impact on medical of medical benefit reforms; and
e, l and *r* are randomly generated error terms;

5. Generate the average cost of indemnity on small and large indemnity claims as:

$$ISVSM_i = a + b\, ISVSM_{i-1}\, x\, (1 + c\,(ew_i))\, x\, (1 + d ibs_i) + f(MSVSM_i - MSVSM_{i-1}) + g_i \quad [5]$$

$$ISVLG_i = h + j\, ISVLG_{i-1}\, x\, (1 + k\,(ew_i))\, x\, (1 + l ibl_i) + m(ISVSM_i - ISVSM_{i-1}) + n(MSVLG_i - MSVLG_{i-1}) + o_i$$

where $a, b, c, d, e, f, h, j, k, l, m$ and n are constants;

$ISVSM$ is the average cost of indemnity on small indemnity claims;

$ISVLG$ is the average cost of indemnity on large indemnity claims;

ew is expected indemnity trend used in pricing and is estimated as:

$$ew_i = a + b\, w_{i-1} + c\,(ew_{i-1})$$

w is wage inflation;

ibs_i is the indemnity benefit change on small claims in year i;

ibl_i is the indemnity benefit change on large claims in year i;

$MSVSM$ is the average cost of medical on small indemnity claims;

$MSVLG$ is the average cost of medical on large indemnity claims; and

g and o are randomly generated error terms.

6. Generate the actual medical and indemnity cost of each large claim from a joint size of loss distribution with

$$mean = (MSVLG_i,\ ISVLG_i) \quad [6]$$

Generate $FRLG_i\ x\ NW_i\ x\ MKSH_i + e_i$ of these large claims.

7. Calculate medical and indemnity losses as:

$$INDLOSS = MKSH_i\, x\, NW_i\, x\, FRSM_i\, x\, ISVSM_i + \sum_{j=1}^{FRLG_i x NW_i x MKSH_i + e_i} LARGE_{j,ind} \quad [7]$$

$$MEDLOSS = MKSH_i\, x\, NW_i\, x\, (FRMO_i\, x\, MSVMO_i + FRSM_i\, x\, MSVSM_i) + \sum_{j=1}^{FRLG_i x NW_i x MKSH_i + e_i} LARGE_{j,med}$$

$$LOSS = INDLOSS + MEDLOSS$$

where $LARGE_{j,med}$ and $LARGE_{j,ind}$ refer to the medical and indemnity losses on the large claims generated in [6].

We usually model allocated loss adjustment expenses as a function of indemnity losses, although the recent introduction of managed care fees as part of allocated loss adjustment expenses suggests that total losses may be a more appropriate projection base in some instances. Unallocated loss adjustment expenses can be projected as a function of total losses or claim counts by type of claim (medical-only, small or large indemnity). More refined models projecting changes in average loss adjustment expenses per claim as functions of economic variables and mix of claims could also be developed.

The above process results in estimates of ultimate policy year losses and loss adjustment expenses as they are initially reported in the company's financial statements. To develop income statements, the underlying exposure must first be assigned to the year in which the related premium is earned. Premium earning patterns are discussed in a later section.

For each accident year, estimates of ultimate medical and indemnity should be adjusted over time to reflect two factors:

(1) The difference between the initially reported value and changes that are expected to occur due to economic conditions that subsequently become known.

(2) Any historically observed reporting biases that are expected to persist for future accident years.

To calculate the first adjustment, it is necessary to determine the medical and indemnity payment patterns. We find it convenient to represent the payment pattern by a zero inflation rate, then modify it by subsequently observed inflation for the components of losses that may be sensitive to inflation (medical and, if weekly benefits escalate, indemnity). Variability in the zero inflation rate payment pattern can be reflected by either randomly selecting from a set of pre-determined payment patterns with associated probabilities (say slow, medium, fast) or utilizing a distribution with randomly selected parameters. Under the latter approach, a theoretical distribution is used to model claim payments, such as a Poisson distribution with mean equal to the average lag between accident year and calendar year of payment. For a Poisson with mean 3, the payment pattern would be:

Paid in AY +	0	1	2	3	4	5	6	7	8	9
Percent	5%	15%	22%	22%	17%	10%	5%	2%	1%	1%

Obviously, a longer tailed distribution, such as Lognormal, Weibull, or Pareto, might be more appropriate for modeling unlimited workers' compensation loss payment patterns. Refinements to random selection of the parameters of the distribution include making the mean parameter dependent upon one or more of the actual loss ratio, the percentage of losses emanating from large claims or the mix of losses between medical and indemnity. We generally expect that the payment pattern will lengthen when losses are higher than expected or there are more large claims than average. Alternatively, separate payment patterns could be modeled for each of medical-only, small and large indemnity claims.

Once the zero inflation rate payment patterns have been derived, the formula for the booked reserves (for medical) at the kth evaluation date for accident year i would be:

$$BOOKED_{i,k} = MEDLOSS_i \; x \qquad [8]$$

$$\frac{\sum_{j=1}^{k} NOINFPP_{i,j} \prod_{l=1}^{j}(1+mtrend_{i+l-1}) + \left(\prod_{l=1}^{k}(1+mtrend_{i+l-1})\right)\left(\sum_{j=k+1}^{\infty} NOINFPP_{i,j}(1+emcpi_{i+k})^j\right)}{\sum_{j=1}^{\infty} NOINFPP_{i,j}\,(1+emcpi_i)^j}$$

where $MEDLOSS_i$	is the initial estimate of ultimate medical losses for Accident Year i;
$NOINFPP_i$	is the randomly selected zero inflation rate payment pattern for medical for Accident Year i;
$mtrend$	is the observed medical claim cost trend rate which we model as a function of the medical component of the consumer price index; and
$emcpi$	is the estimated medical inflation rate used in pricing.

Any historical bias could then be incorporated, as desired. For example, if a company has had the tendency to book initial reserves that lead to a redundancy of 5% of ultimate losses and reduce that redundancy over five years, booked ultimate medical losses for Accident Year i at evaluation date k could be modified as:

$$ADJBOOKED_{i,k} = \begin{cases} BOOKED_{i,k} + (0.05 - 0.01k)\,ULTMEDLOSS_i & k < 5 \\ BOOKED_{i,k} & k \geq 5 \end{cases} \qquad [9]$$

where $BOOKED_{i,k}$	is as derived in [8] above; and
$ULTMEDLOSS_i$	is the ultimate medical losses as defined below in [10].

The actual dollar amount of ultimate medical losses for any given accident year would be calculated as:

$$ULTMEDLOSS_i = MEDLOSS_i \frac{\sum_{k=1}^{\infty} NOINFFP_{i,k} \prod_{j=1}^{k} (1 + mtrend_{i+j-1})}{\sum_{k=1}^{\infty} NOINFFP_{i,k} (1 + emcpi_i)^k} \quad [10]$$

where $MEDLOSS_i$ is the initial estimate of ultimate medical losses for Accident Year i;

$NOINFPP_i$ is the randomly selected zero inflation rate payment pattern for medical for Accident Year i;

mtrend is the observed medical claim cost rate; and

emcpi is the estimated medical inflation rate used in pricing.

Ultimate indemnity losses would be calculated similarly, using indemnity claim cost trend, wage inflation and indemnity trend used in pricing instead of the corresponding values for medical. The analyst must exercise care, however, to distinguish between the accident year component and the calendar year component on indemnity losses. Only the calendar year component of these values (if any) should be used to adjust the ultimate losses, booked reserves, and payment pattern.

Last, models of loss and loss adjustment expense payments need to be developed. As discussed above, we consider a two-step approach to modeling payment patterns. In the first step, parameter risk is addressed either through random selection of a pre-determined payment pattern or through random selection of the parameters of a selected distribution. The result of this first step is the zero inflation rate payment pattern. The second step incorporates adjustments for the actual inflation observed during the payment period. For medical, the kth increment of the payment pattern for the ith accident year to be applied to the actual ultimate medical losses (ULTMEDLOSS) is:

$$PAYPATT_{i,k} = \frac{NOINFPP_{i,k} \prod_{j=1}^{k} (1 + mtrend_{i+j-1})}{\sum_{l=1}^{\infty} NOINFPP_{i,l} \prod_{j=1}^{l} (1 + mtrend_{i+j-1})} \quad [11]$$

where $NOINFPP_i$ is the randomly selected zero inflation rate payment pattern for medical for Accident Year i; and

mtrend is the observed medical claim cost rate.

A similar calculation would be made for indemnity payments.

Premium

We model premium based on a modeled expected loss ratio implicit in the rates actually charged. Expected losses (based on the insured exposure, prior year losses, and recent inflation and interest rates) are divided by the expected loss ratio to derive premium. We note that the expected losses used in the premium determination differ from the actual modeled policy year results because these expected losses are those expected at the time that the policy is priced.

Expected losses based on information available at the time of pricing first need to be estimated. Expected losses could be calculated using the algorithm laid out in Formulas [1] through [7] with the exceptions that the current year error terms would be excluded and:

$$\sum_{j=1}^{FRLG_i \times NW_i \times MKSH_i + e_i} LARGE_{j,med} + LARGE_{j,ind} \qquad [12]$$

would be replaced by

$$MKSH_i \times NW_i \times FRLG_i \times (MSVLG_i + ISVLG_i)$$

in Formula [7].

These expected losses would then be divided by an expected loss ratio underlying the insurer's market rates. This loss ratio is influenced by interest rates (as they determine the discount factor used to calculate the underwriting profit margin), prior year(s') expected loss ratios, changes in desired market share, and a random error term. If more than one autoregressive term (i.e. more than one prior year's expected loss ratio) is included in the formula, underwriting cycles can be modeled. The model for the expected loss ratio used in pricing would then take the following form:

$$ELR_i = a + \sum_{j=1}^{n} b_j ELR_{i-j} + c\,(int_{i-1} - int_{i-2}) + d\,\Delta MKSH_i + e_i \qquad [13]$$

where a, b, c and d are constants;
ELR is the expected loss ratio;
n is the number of autoregressive terms in the model;
int is the short-term government yield;
$\Delta MKSH$ is the percentage change in market share; and
e is a randomly generated error term.

In theory, we expect that the c coefficient will approximate the duration of the loss payment pattern.

The next step in modeling premium is to allocate the policy year premium across calendar years through the use of an earning pattern. In theory, a multi-year earning pattern should be used to reflect the earning of audit and retrospective premiums. Further, the actual booking of premium as written could be lagged to reflect many insurers' practice of recording written premium on a monthly basis. In practice, we generally look at the company's statutory financial statements and compare the unearned premium reserve with calendar year written premium to derive an estimate of the percentage of a calendar year's written premium that is earned during that calendar year and the portion remaining to be earned in the subsequent calendar year. As a result of the booking of premiums as written as monthly installments are made, we generally find that 80% to 90% of workers' compensation premium written in a calendar year is also earned in that calendar year. By comparison, for other lines of business with annual policies, we generally find that approximately 50% of premium is earned in the year it is written.

Models must consider the timing and amount of premium collected. Similar to the approach used for premium earning patterns, we generally review agents' balances as a percentage of calendar year written premium to estimate the percentage of premium written in a calendar year that is also collected in the year. We generally assume that the remainder of the premium is collected in the subsequent calendar year.

For many insurers, the percentage of premium that is never collected is immaterial. For other insurers, uncollectible premium is of significant concern. For insurers in the latter category, models of the expected value of the percentage of premium that is uncollectible and variability therein can be constructed. We expect that the percentage of uncollectible premium is negatively correlated with changes in gross domestic product and positively correlated with interest rates. When the gross domestic product increases at a lower rate than average, more insureds would be expected to experience financial difficulties and therefore default on premium payments. When interest rates are high, insureds are more likely to purchase paid loss retro policies, thereby increasing insurers' credit risk. As such, the formula for the percentage of premium that is uncollectible might take the form of:

$$PU_i = a + b\,PU_{i-1} + c(\Delta gdp_i - \Delta gdp_{i-1}) + d(int_i - int_{i-1}) + e_i \qquad [14]$$

where a, b, c and d are constants;
PU_i is the percent of premium from policy year i that is uncollectible;
Δgdp is the percentage change in real gross domestic product; and
int is the short-term interest rate.

Operating Expenses

We find it practical to model the fixed and variable components of expenses separately. Occasionally, we model commissions and premium taxes separate from all other variable expenses. We have found that it is reasonable to model commission and premium tax rates as constants over time. One possible refinement is to have commission rates vary with the expected loss ratio or whatever other measure of the competitive marketplace is used. That is, in very competitive markets, insurers may pay higher than usual commission rates to maintain growth targets.

We model all other variable expenses as a constant percentage possibly with a random error term. The distribution and standard deviation of the error term can be derived from historical company experience or the experience of other insurers with similar characteristics.

Fixed expenses (actually, all expenses that are not charged as a percentage of premium) are usually dependent upon inflation rates and, to a limited extent, changes in exposure. Our fixed expense models take the form of:

$$FE_i = FE_{i-1} \ x \ (1 + cpi_i) \ x \left(a + b \frac{NW_i \ x \ MKSH_i}{NW_{i-1} \ x \ MKSH_{i-1}} \right) + e_i \qquad [15]$$

where a and b are constants;
FE is fixed expenses;
cpi is consumer price inflation;
NW is the number of workers;
$MKSH$ is market share; and
e is a randomly generated error term.

Reinsurance

Our models of ceded reinsurance have been relatively simple. We first assume that the reinsurance terms (i.e., premium rate, attachment point, participation, and commission schedule) are constant over time and across scenarios. When determining ceded losses, we apply the reinsurance terms to each of the large claims individually and, for quota share treaties, to small claims in the aggregate. We also model the payment pattern of cessions separate from the payment pattern of direct losses. Once ceded losses have been modeled, we can then calculate any sliding scale premium or commission adjustments.

Refinements to our simple models might include the incorporation of a pricing cycle for reinsurance (i.e., increases and declines in the price of reinsurance relative to ceded losses and commissions), and changes in the retention and in reinsurer quality that are sensitive to that pricing cycle. That is, as reinsurance rates increase, an insurer might increase its retention to reduce ceded premium or it might purchase reinsurance from less expensive and, presumably, lower quality reinsurers. In the latter case, the issue of collectibility of reinsurance must be addressed.

Policyholder Dividends

We model policyholder dividends as functions of the premium volume and loss ratio on policies written in each year. That is, policyholder dividends can be calculated using a formula such as:

$$PD_i = GEP_i \; x \; (a + bLR_i) + e_i \qquad [16]$$

where a and b are constants;
PD is policyholder dividends incurred;
GEP is gross earned premium;
LR_i is the estimate of the accident year i loss ratio at the end of calendar year $i+1$; and
e is a randomly generated error term.

We incur policyholder dividends in the year premium is earned, declare them in the following year and pay them in the year after that. The timing of declaration and payment will vary across insurers. For participating or retrospectively rated policies, the above formula could be reevaluated at successive evaluation dates to more closely follow the actual flow of dividend or retrospective premium payments between insurers and insureds.

Investment Yields

We model investment yields based on the output of our economic scenario generator. The output of the economic scenario generator includes short and long term interest rates and S&P 500 returns and dividend yields. We interpolate between the short and long term rates to model yields on government bonds and apply factors for municipal and corporate bonds. We usually assume that insurers' stock portfolios are sufficiently diversified to use the S&P 500 total return and dividend yields and we control for bond defaults by using a default function that is conditional on gross domestic product. Other considerations in modeling assets depend on the mix of assets held. For example, the sensitivity of prepayment rates to interest rates is important for an insurer with a significant mortgage-backed security holding. As there is significant literature available regarding assets and the economy, we will not expand upon these relationships further in this paper.

Residual Markets

The driving factor for residual market assessments is the perception by the market place of overall rate adequacy. If it is believed that rates are inadequate, particularly when the cause is regulatory rate suppression, the size of the residual market will increase. If rates for the residual market are low (we expect that rate adequacy in the competitive and residual market are positively correlated), the loss ratio in the residual market will also grow. However, the increase in loss ratios may be offset to some extent because, as the residual market expands, the quality of insureds may improve and the loss ratio may be lower than if the residual market were smaller. Nonetheless, we generally expect that the deficit as a percentage of residual market premium will grow. There will be a compounding effect when the deficit is compared to the premium in the competitive market, because, as indicated, it is expected that the residual market will have a greater market share when these conditions are present.

We model residual market burdens as being positively correlated with both the expected loss ratio in pricing (which affects residual market size) and the difference between the actual loss ratio experienced by the insurer and the expected loss ratio from pricing (which, if assumed to also indicate that the residual market experience was better or worse than average, would be expected to be indicative of the residual market burden as a percentage of residual market premium). A further refinement is possible when insurer market share is explicitly modeled in projecting losses. In this situation, the insurer's participation in the residual market can be modeled as being positively correlated with the size of its residual market burden. The formula for the residual market burden will take the form of:

$$RM_i = MKSH_i \ x \ RMMS_i \ x \ NW_i \ x \ (a\,PPI_i + b) \ x \ (c + d\,LR_i) + e_i \qquad [17]$$

where a, b, c and d are constants;

RM_i	is the residual market burden emanating from premium earned by the residual market in year i;
$MKSH$	is the insurer's share of the insurer;
$RMMS$	is the market share of the residual market;
NW	is the number of insured workers in the market;
PPI_i	is the insurer's premium per insured worker which is presumed to bear some relationship to the residual market's average premium per insured worker;
LR_i	is the insurer's ultimate loss ratio on premium earned in year i; and
e	is a randomly generated error term

To model the timing of the cash flow impact of residual market burdens, we occasionally model residual market participation (such as business assumed from the National Pool) as a separate line of business. This approach allows for more refined projections of the timing of payment of residual market burdens.

Data Requirements

The types of data needed to develop the models underlying the dynamic financial analysis depend heavily on the level of detail used in the dynamic financial analysis. In this section of the paper, we present an inventory of the data that would be valuable in developing the parameters for a highly detailed model that incorporates all of the features described in the previous section. To the extent that a simpler model is being applied, less data are needed.

The data listed in this section would be used primarily to develop the parameters (constants) in the formulas presented and/or described in the previous section and to develop the distributions of losses by size and the distributions of the error terms. The "Formulas" column in the tables that follow provide references to the previously presented formulas whose parameters might be dependent on each data element. The "Possible Source" column indicates whether we found the desired information on the CAS DFA web site. If not, alternative sources are suggested. Of course, to the extent that the data for a specific insurer were available and credible, we would rely upon them before looking to these industry sources.

Economic Indices

	Frequency	Formulas	Possible Source
Interest rates for each annual maturity on government bonds	Annual	ESG*, [13], [14]	CAS DFA Web Site
Interest rates at each annual maturity on corporate bonds of each quality of interest for modeling	Annual	ESG	CAS DFA Web Site
Interest rates at various maturities on municipal bonds	Annual	ESG	Moody's
Annual default rates for corporate bonds of each quality	Annual	ESG	Prof. Altman-NYU Moody's/ S&P
Annual default rates for municipal bonds	Annual	ESG	Moody's/ S&P
S&P 500 total return	Annual	ESG	S&P/ Ibbotson Associates
S&P 500 dividend yield	Annual	ESG	S&P/ Ibbotson Associates
Consumer price index	Annual	ESG, [15]	CAS DFA Web Site
Wage inflation countrywide and by state	Annual	ESG, [5], [8], [10], [11]	NCCI Annual Statistical Bulletin

	Frequency	Formulas	Possible Source
Medical component of the consumer price index countrywide and by region	Annual	ESG, [4], [8], [10], [11]	M&R Health Cost Index Database
Gross domestic product	Annual	ESG, [1], [3], [14]	CAS DFA Web Site
Number of workers covered by commercially insured workers' compensation programs by state	Annual	ESG, [1]	

* ESG = Economic Scenario Generator

Losses and Loss Adjustment Expenses

	Frequency	Formulas	Possible Source
Market share of each insurer in each state and nationwide	Annual	[2], [3], [13], [15], [17]	Statutory Annual Statement Page 14 **
Number of medical-only claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin
Number of small indemnity claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin
Number of large indemnity claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin

** Possibly distorted for servicing carriers by residual market premium.

	Frequency	Formulas	Possible Source
Average cost of medical-only claims by state	Policy or accident year	[4]	NCCI Annual Statistical Bulletin
Average cost of medical on small indemnity claims by state	Policy or accident year	[4], [5]	NCCI Annual Statistical Bulletin
Average cost of indemnity on small indemnity claims by state	Policy or accident year	[5]	NCCI Annual Statistical Bulletin
Average cost of medical on large indemnity claims by state	Policy or accident year	[4], [5]	NCCI Annual Statistical Bulletin
Average cost of indemnity on large indemnity claims by state	Policy or accident year	[5]	NCCI Annual Statistical Bulletin
Distribution of combined medical and indemnity losses on large indemnity claims by state	Policy or accident year	[6]	NCCI Excess Loss Premium Factor Calculation
Changes in medical benefits by state	Annual	[4]	NCCI Annual Statistical Bulletin
Changes in indemnity benefits on small indemnity claims by state	Annual	[5]	NCCI Annual Statistical Bulletin

	Frequency	Formulas	Possible Source
Changes in indemnity benefits on large indemnity claims by state	Annual	[5]	NCCI Annual Statistical Bulletin
Allocated loss adjustment expenses by state	Policy or accident year	ALAE model	Rate filings
Unallocated loss adjustment expenses by state	Calendar year	ULAE model	Rate filings
Reserve adjustments by calendar year	Accident Year	[8], [9]	Statutory Annual Statement Schedule P Part 2D
Paid medical, indemnity and allocated loss adjustment expense triangles by state	Accident Year	[8], [10]	Rate filings or NCCI Annual Statistical Bulletin

Premium

	Frequency	Formulas	Possible Source
Expected loss ratio underlying rates by state	Policy year	[13]	Rate filings
Medical and indemnity trend rates underlying pricing by state	Policy year	[8], [10], [11]	Rate filings
Direct premium uncollectible by state	Policy year	[14]	

Expense

	Frequency	Formulas	Possible Source
General expenses by insurer	Calendar year	[15]	Rate filings

	Frequency	Formulas	Possible Source
Commissions by insurer by state	Calendar year	Variable expense model	Rate filings
Premium taxes, licenses and fees by state	Calendar year	Variable expense model	Rate filings, NCCI Annual Statistical Bulletin, State agencies
Other acquisition expenses by insurer by state	Calendar year	Variable expense model	Rate filings

Reinsurance

	Frequency	Formulas	Possible Source
Projected ultimate workers' compensation ceded loss ratios for each of quota share and excess insurance (preferably for ranges of attachment points)	Accident or policy year	Ceded premium and loss models	
Development of workers' compensation ceded paid losses for excess insurance (preferably for ranges of attachment points)	Accident or policy year	Ceded loss model	
Summary of reinsurers not meeting obligations and credit rating in each of five years prior to defaulting on obligations		Ceded loss model	

Policyholder Dividends

	Frequency	Formulas	Possible Source
Policyholder dividends with corresponding policy year loss ratios for workers' compensation by insurer	Policy year	[16]	

Residual market

	Frequency	Formulas	Possible Source
Emergence of residual market burdens by state	Calendar year by policy year	[17]	National and State Pool Mgmt. Reports
Residual market premium by state	Policy year	[17]	National and State Pool Mgmt. Reports
Residual market direct loss ratio by state	Policy year	[17]	National and State Pool Mgmt. Reports