Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis

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Abstract

When people speak of parameterizing a model, whether it be for dynamic financial analysis or otherwise, they typically discuss the ranges of values that key model elements can assume. In our paper we have broadened the concept of parameterization to include the functionality a model needs to contain in order to perform the required task. Our concept of parameterization, therefore, encompasses both the narrower definition of defining ranges of possible values for key model elements and the broader definition of describing what needs to be included in the model's design in order for it to function properly. To that end, in Section I the paper describes a model currently being used to develop property/casualty insurance company pro-forma financial statements in a dynamic modeling framework. In Section II the paper lists the key elements of variability within the modeling framework, i.e. those parameters that need to be described through probability statements rather than fixed values. Section III returns to the narrower definition of parameterization and provides some commentary regarding our experiences in developing the specific ranges of values for each of the items listed in Section II.

Biographies

Gerald S. Kirschner, FCAS, MAAA has been a consulting actuary for Ernst & Young LLP's National Actuarial Services Group since 1994. He is based in Atlanta. His primary responsibility at Ernst & Young has been to jointly develop with SS&C Technologies, Inc. a dynamic financial analysis (DFA) model for property-casualty insurance companies. Prior to joining the firm, he was employed by Aetna Life & Casualty. He is a member of the CAS Dynamic Financial Analysis Subcommittee on Models.

William C. Scheel, Ph.D., AIAF, ARM, CEBS, CPCU is a software architect and has programmed many mathematical and actuarial systems. He has been with SS&C Technologies, Inc. for three years. During this time he designed *Finesse*, a DFA system. Prior to working for SS&C, he was employed by Price Waterhouse LLP.

Introduction

There are many facets to the concept of model parameterization. It is useful to begin with an analogy to the common actuarial problem of distinguishing between specification, parameter, and process risks. Specification risk relates to the questions "Are the model structure and the selected probability distributions correct?" Parameter risk narrows the question to "Assuming the specification is correct, are the distributional parameters correct?" Lastly, process risk is concerned with randomness, i.e. answering the question "Assuming everything else is correct, what can happen in my universe of possible outcomes?"

One might quibble between modeling loss severity with a Weibull distribution instead of a Lognormal distribution. Ferreting among the universe of possible probability distributions in model design is coping with *specification* risk. Even when this exercise is completed successfully, the student pursing this investigation still must deal with describing the parameters of the chosen process model. This second stage investigation is an exercise in *parameter* risk. The risk to the model designer is ending up choosing the wrong probability distribution or the wrong parameters. In an ideal world, the final risk, *process* risk, disappears under the weight of many, many recalculations of the model. In the real world, there could be overlooked correlations or unseen model overspecification, or combinations of the two that do not allow process risk to drop from the overall equation. Moreover, there is a corollary to this uncertainty. The specification risk may degenerate into subjective probability assessment—the knowledge set about the dynamic process may be so sparse that even a doctorate in statistics is no consolation.[1]

In financial modeling, there are many of these "risks", and the model designer should not be oblivious to them. Collectively, they constitute what we mean by parameterization problems associated with model design. The purpose of a dynamic financial model is to obtain and compare probability distributions for functions of random variables. Depending on point of view and the purpose attached to the modeling exercise, there are many risks associated with rendering these important goal or metric variables.

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[1] The mathematics describing the fitting of distributions with only sparse knowledge of the underlying risk characteristics is described in "Converting Experts' Knowledge into Dynamic Variable Distributions for Monte Carlo Simulation" by Euguene L. Filshtein in Contingencies, January/February 1996.

The model designer must leap many hurdles while formulating a corporate financial model, particularly one for dynamic financial analysis (DFA) Examples of hurdles to be overcome or pitfalls to be avoided include:

1. The model can use the wrong algebra when attempting to define causality or linkages among model constants and variables, i.e., the wrong model
2. Important components of the operational or economic environment might be omitted so that the model behavior is mischievous
3. Elements that more appropriately should be rendered in a dynamic manner are kept static
4. Model designers can be consumed by uncertainty regarding the dynamic behavior of those components deemed to be dynamic
5. The model's accounting framework may be inaccurate
6. The model could contain programming problems or other embedded and unknown deviant behavior
7. It might not be possible to achieve a consensus among decision makers about the metrics (i.e. output results) of comparison
8. Model results may not exhibit stochastic dominance[2] between different strategies under investigation
9. Model results cannot be implemented (i.e. the decision path that leads to the "best" long-term outcome is not feasible, either because it violates internal management operating constraints or regulatory boundaries).

In summary, the parameterization risks include functional mis-specification of the model, commission and omission errors in risk and process identification and failure of the accounting framework to adequately divulge the metrics needed for decision making.

In each of these looms a different dragon. Let us begin with a disclaimer to all readers who hope to find an easy recipe for defining the parameters. There is no magic bullet for alleviating either model, functional or dynamic variable mis-specification. Very often, there is not even a good place to start looking for a definition. With that in mind, we believe that (a) a definition that describes the event in question is better than no definition at all and (b) it is not worth quibbling over the finer points of parameter specification - in the overall perspective of what we are trying to model, the error introduced by using a Weibull instead of a Lognormal distribution to fit empirical claims severity data is not going to make or break our results.

[2] Stochastic dominance attempts to answer questions of choice among risky alternatives in a utility-theoretic framework – but one in which only certain limited information is known about the utility function of the decision maker. The idea of stochastic dominance is discussed in Exhibit 1. This Exhibit displays both an example of stochastic dominance and a user-defined "metric".

These thoughts are pursued in greater detail in Appendix A: Fundamentals of Dynamic Financial Analysis. We now turn to the two key concepts that form the basis for this paper:

- The model to be discussed is a *corporate financial model*, one that already has been deployed in the marketplace
- The model is *dynamic*.

By focusing on these key concepts, this paper will present:

1. An example of a model that has been built to perform dynamic financial analysis at a corporate level
2. What we have found to be some of the key parameters and model specifications that need to be described probabilistically
3. Approaches we have taken to develop specific ranges of possible values for the key parameters and model specifications.

Key Concepts

Corporate Financial Model

Day-to-day operations of a property-casualty insurance company include buying and selling assets, underwriting business, collecting premiums, administering claims and incurring the fixed costs related to running the insurance enterprise. A financial model of a property-casualty insurance enterprise needs to be able to model each of these operations separately and in conjunction with each other in order to produce realistic financial projections of the complete entity.

In order to perform a comprehensive dynamic financial analysis, a corporate financial model should have linkages and interrelationships between activity on the asset and liability sides of the business. For example, the model should:

- apply the same macroeconomic environmental conditions (i.e. interest rates, inflation rates, catastrophic events) across *all* aspects of the company
- allow investment decisions to be made after consideration of both operating needs and investment opportunities in the financial markets
- look at the risk/return tradeoffs generated by both investment and operating decisions in the context of the entire company's risk/return spectrum rather than in isolation
- provide a universal set of metrics or decision criteria by which multi-faceted company operations can be measured and managed.

These critical model components are couched in terms of one or more accounting frameworks (i.e. statutory, GAAP or economic). The accounting mechanisms serve to organize the model's projected results into a readily understood and consistent financial structure.

Dynamic vs. Static Corporate Financial Modeling

The purpose of a corporate financial model is to help company management understand how decisions made today affect the company's financial well-being tomorrow. Traditionally, corporate financial modeling has relied on static evaluations of current and future events and predetermined cause and effect relationships. Unfortunately, with static analysis, there is at best a limited ability to appreciate the sensitivity of bottom line results to changes in input variables, especially if the number of input variables is large and the interrelationships among them is complex. Yet it is critical that strategic decisions be made with the understanding of how each decision impacts the preceding ones, or how changes in the internal or external environment can alter the anticipated outcomes arising from each decision.

The essence of dynamic financial modeling is the ability to describe critical assumptions in terms of ranges of possible outcomes, rather than in terms of fixed values. Once each critical assumption is defined by a range of possible outcomes and the interrelationships among critical assumptions are mapped out, a series of model recalculations can be performed to develop ranges of results we can reasonably expect to see. The parameters used to model dynamic variables and the accounting interrelationships ultimately define the key criteria or metric variables that are of interest to management, regulators and stockholders. Differences in financial results arising from alternative strategic decisions can be evaluated by replacing one set of strategic decisions with another, re-running the modeling exercise and comparing the ranges of possible outcomes under each decision rule set.

SECTION I: MODEL STRUCTURE

The corporate financial model has been developed to include a minimum of one year of actual results and to produce pro-forma financial projections for an additional five years. For the purposes of simplification throughout the remainder of the article, it is assumed the actual results are valued as of December 31, 1996 and the projection period encompasses the years 1997-2001.

The corporate financial model has five distinct sections: invested assets, underwriting, accounting structure, tax calculations, and financial ratios.

Underwriting section

The underwriting section performs seven basic tasks:

1. It converts held loss and allocated loss adjustment expense (ALAE) reserves into calendar year payouts.
2. It converts indicated redundancies or deficiencies in held loss reserves into calendar year payouts and captures the accounting impacts of reserve redundancy or deficiency emergence. Reserve redundancies or deficiencies can arise either from variability in the held reserves (i.e. the held reserves represent the best estimate of ultimate losses, but actual loss emergence might vary in some range around the best estimate), or from deliberately holding reserves at a level other than the best estimate.
3. It calculates the inflationary impact on loss payments arising from differences between a simulated future level of inflation and a level of inflation that was implicitly (or explicitly) assumed when the held reserve level was established.
4. It allows the emergence of reserve redundancies or deficiencies into the model's accounting results to be scheduled at the same rate or faster than the redundancies or deficiencies emerge into the model's cash flows.
5. It calculates any additional premium inflows that might be derived from policies already written (i.e. audit premium, premium from retrospectively rated policies) and earns premiums on in-force and new business according to a user-defined premium earning pattern.
6. It calculates tax discounted loss reserve levels for federal income tax calculations.
7. It provides the vehicle for entering a five year underwriting plan, including future premium inflows and associated loss and variable expense outflows at a line of business level of detail. (Only variable expenses are included in the line of business section. Fixed expenses are addressed in a different section of the model.)

An example of the inter-relationship between the payout of held reserves, indicated reserve redundancy/deficiency emergence and inflationary impacts are shown in Exhibit 2.

Each line of business requires inputs—many of them, such as production volume, can be tied to economic activity that also is simulated within the model. Some of the important input items are:

- Premium volume projections for the 1997-2001 period on a direct/assumed/ceded basis
- Loss ratio projections associated with the premiums to be written between 1997 and 2001
- Variable expense projections for the future business writings
- Reserve payout patterns for loss and ALAE

- Premium earning patterns
- IRS tax discount factors applicable to held reserves, both historical and future
- Risk-based capital loss and premium factors.

Asset structure

Investment generation tracking

Assets are organized into investment year cohorts that correspond to the year in which the investments were purchased. All of the assets owned by the company at December 31, 1996 are combined into one investment year cohort. The investments purchased in 1997 will be a second cohort, the investments purchased in 1998 a third cohort, and so on. The changes in asset valuations of each investment year cohort reflect the interest rate environment projected to occur. The magnitude of a new investment year cohort is determined by many factors including asset allocation strategy, cash flow, and the operational and econometric environment at the time the investment year cohort is purchased.

The investment year cohort structure is needed to differentiate between assets purchased under different interest rate environments. The interest rate environment at the end of 1997 will most likely differ from that at the end of 1998, therefore, the characteristics of the assets purchased at the end of 1997 will most likely differ from those purchased at the end of 1998. For example, if interest rates are higher at December 31, 1997 than December 31, 1998, bonds purchased in 1997 will have higher coupon rates than those of the same time to maturity purchased in 1998. If the 1997 purchases were not maintained in a separate cohort from the 1998 purchases, the differences in their coupon rates would be lost.

Asset categories

Assets are subdivided into a number of homogenous groups for modeling. For simplicity, the structure displayed in this paper follows a statutory annual statement format. Bonds are divided into taxable and a tax-exempt groups, and further subdivided by maturity according to the divisions in Schedule D of the annual statement. Collateralized mortgage obligations can either be left in the standard bond groupings or separated into their own group. The other asset categories include preferred and common stocks, mortgage loans on real estate, real estate, cash, short term investments and other invested assets.

For each asset class and cohort, information about par, book and market values are retained. This allows the model to recalculate market values for each asset class/cohort combination based on changes in the interest rate environment from when it was purchased. The determination of unrealized gains and losses as well as various cash and accrual income effects also is enabled.

Asset rebalancing

The investment of operations and investment cash flow is done at year end in the model. Average cash balances from insurance operations are deemed to be invested at the short-term yield[3] until the end of the year when all sources of cash are combined with the market values of assets and tested for rebalancing. Depending on the rebalancing strategy, some existing assets may be sold and the pool of new money reinvested to produce approximately the proportions dictated by the strategy. The final allocations are subject to modification attributable to year-end closing transactions, primarily tax effects. The rebalancing can create capital gains or losses which are combined with operating results to determine the federal income tax liability for the year.

Asset - liability interrelationship

The model interrelates assets and liabilities in two ways. First, the amount of money available for reinvestment at any point in time is directly related to the underwriting cash flows. A severe underwriting shock such as a catastrophe will force much greater loss outflows than anticipated, with a corresponding need to liquidate assets. Second, the interest rate environment affecting asset market values is linked into the liability cash flow profiles.

This linkage is important because through it the model can stress-test the overall company financials in a variety of ways. For example, a scenario might evolve in which high interest rates with corresponding high inflation rates and an underwriting shock simultaneously occur. The high interest rates depress the market value of the bond portfolio at the same time the high inflation rate and underwriting shock are raising the calendar year loss outflows above the expected loss outflow level.

Accounting structure

The model includes both statutory and GAAP accounting structures. The accounting calculations begins with the statutory structure and applies a series of statutory to GAAP adjustments[4] to derive GAAP financials. The statutory to GAAP adjustments currently include:

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[3] By investing average cash balances at the short-term yield, the model calculates investment income earned on the average cash balance during the year.

[4] As statutory and GAAP accounting rules change, the model will need to be updated to reflect the changes.

- Restating bond valuations at market instead of amortized cost, based on the percentage of the bond portfolio that is either available for sale or available for trading
- Admitting assets that are not allowed by statutory accounting, but are for GAAP (premium receivables greater than 90 days past due, unbooked audit premium, furniture and equipment, non-admitted accounts receivable, prepaid expenses and travel advances)
- Goodwill
- Deferred acquisition costs
- Deferred federal income taxes
- Other miscellaneous statutory to GAAP adjustments (principally investments in affiliates)
- Reclassifying ceded unearned premium reserves and loss and loss adjustment expense reserves from a contra-liability to an asset.

Although the model produces a wide variety of financial reports spanning balance sheets, income statements, reconciliations, cash flows, tax and regulator and rating agency financial measures, for this paper, only the statutory accounting exhibits will be used. Exhibits 8 and 9 provide examples of the statutory accounting exhibits produced by the model. These exhibits display just one possible financial outcome that might occur. Exhibit 10 displays graphically the ranges of results that were generated for a few of the accounting metrics when the model was run.

Tax algorithms

The model calculates both current and deferred federal income taxes.

Current income taxes

Current income taxes are calculated in accordance with insurance company tax procedures, as described in Chapter 13 of Property-Casualty Insurance Accounting.[5] Current taxes are calculated by adjusting current year statutory net income as follows:

1. Increase or (decrease) current year net income by 20% of the change in the unearned premium reserve
2. Increase or (decrease) current year net income by the difference in the amount of tax discount in held reserves[6]

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[5] Property-Casualty Insurance Accounting, Sixth Edition, July 1994, by Insurance Accounting and Systems Association, Chapter 13.

[6] The model is seeded with historical tax discount factors, either industry, company-specific or a combination of the two, depending on what tax discount factor elections were made in 1987 and

3. Decrease current year net income by 85% of the amount of tax-exempt investment income earned during the year
4. Reduce current year net income by 59.5% of the amount of dividends received from common and preferred stock (the dividends received deduction is 70%, but 15% of the deduction must be added back into net income for tax purposes)
5. Apply a 35% tax rate to the resulting taxable net income amount.

Alternative minimum taxes also are calculated for the current year by increasing taxable net income by 75% of the amount of tax-exempt investment income and dividends received deduction excluded from regular taxable net income and multiplying the resulting alternative minimum taxable net income by the 20% AMT tax rate.

These calculations develop the preliminary current year tax position. If a projection year develops an operating loss, that loss is compared against the three prior calendar years to see if it can be used to offset prior years' operating gains. If not, it is retained for possible use as an operating loss carryforward, to be applied against operating gains in a later projection year.

Deferred income taxes for GAAP accounting

The major components of the deferred income tax calculation are the tax discount in held loss reserves, deferred taxes on deferred acquisition expenses, and deferred taxes on unrealized gains or losses on equities and bonds available for sale or trade. The GAAP income statement includes the calendar year change in the portion of the deferred tax asset arising from the tax discount in held loss reserves, the deferred taxes on deferred acquisition expenses and the deferred tax asset or liability arising from unrealized capital gains or losses on that portion of the bond portfolio available for trade.

Financial ratios

Based on the accounting results for each projection years , a series of financial ratios are developed. These include:

- projections of the National Association of Insurance Commissioners ("NAIC") Insurance Regulatory Information System ("IRIS") ratios
- a selection of A.M. Best's financial ratios (operating ratios, leverage ratios, and liquidity ratios)

1992. Projected future discount rates are developed using either pre-seeded industry payout patterns or company-specific payout patterns that evolve from the line of business underwriting structure and a rolling sixty month average interest rate that is linked to the model's projected risk-free interest rate projections.

- an approximate NAIC Risk Based Capital ("RBC") indication.

Model Mechanics

Figure 1 presents a schematic of the way the corporate model operates.

Figure 1 Corporate Model Flowchart

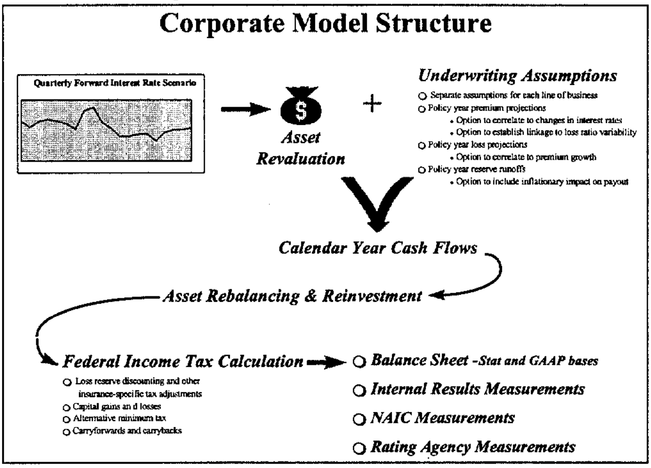

Figure 1 starts with initial conditions—the beginning balance sheet, including accident year modeling of liabilities, knowledge of accruals, tax carry backs and carry forwards, costs and valuations of assets, and so forth. The following sequence of steps is replicated many times for the entire planning horizon:

1. Stochastically generate an economic scenario (interest rates, inflation, competitive conditions, etc.) for the next period.
2. Apply the economic scenario to generate operations for the period.
3. Apply the economic scenario to value existing assets.
4. Apply endogenous effects on liabilities (e.g., correlated, random effects on loss volume or severity that are independent of economic effects).
5. Apply the economic scenario to value existing and new liabilities (e.g., inflationary impacts, shocks and other external effects).
6. Apply a reinsurance strategy based on currently liability and asset conditions or on functions of previously observed or future expected ones.
7. Apply an asset rebalancing strategy based on current liability and asset conditions or on functions of previously observed or future expected ones.
8. Rebalance the portfolio of assets (and/or liabilities), i.e., buy and sell.
9. Develop taxation effects and other fiscal period closing entries.

10. Tally assets and liabilities under the appropriate accounting scheme(s).
11. Create end-of-period financials, operating statistics and metrics.

The application of reinsurance and asset rebalancing strategies in steps six and seven presumes the model user has previously established the course or courses of action the company will take if different asset and/or liability results emerge from the model's projection horizon.

For example, reinsurance strategy options could include the purchase of less reinsurance in subsequent years if the loss ratio for previous years is better than expected, or the purchase of more quota share reinsurance if the company begins developing cash flow problems. The asset rebalancing strategy could be set up to take a higher position in tax-exempt bonds if the company is in a regular taxable income position rather than an alternative minimum taxable income position.

SECTION II: KEY DYNAMIC PARAMETERS

The table below defines what we have concluded are the most critical model elements to be modeled dynamically, both from the perspective of importance to the user and to model volatility.

Table 1: Parameters and Considerations

Parameters	Considerations
Interest rates	Is this a stochastic process and will forward yield curves be available as a by-product?
Inflation	This impact has implications for expenses, business production and retention, competitive conditions and liability payment levels.
Conversion of starting loss, loss adjustment expense and unearned premium reserves in cash outflows	Are the reserve levels on the December 31, 1996 financial a reasonable representation of the amounts that will actually be needed to meet these obligations? How volatile are the amounts actually needed? When will the obligations be paid?
Allocation of new money	What rule structure or structures should be established to tell the model how to use cash inflows for the purchase of new assets in each asset class?

Parameters	Considerations
Production of new business	Is production tied to interest rates, inflation, competitive environment, other influences? What are the correlations or functional relationships between business production and exogenous events such as interest rates or inflation? By what pattern are revenues collected and amortized? What loss ratio will the new business exhibit
Expenses	How do expenses depend on production volumes and exogenous factors? By what pattern are they paid?
Correlations	How do dynamic variables interact? Does the selection of a directional value in one dynamic variable predispose a second variable to take on different values than it otherwise might? Or are the variables independent of each other?
Risk factors in excess of risk-free interest rates that are applicable to different asset classes	What is the risk loading given by the free market to an average asset within each asset class? Is the risk loading stable or volatile? Does it change over time or with changes in the interest rate environment? How important is default risk and how should a model distinguish between it and other sources of general or specific financial risk?
Risk factors in excess of risk-free interest rates that are applicable to different liability classes	When calculating discounted reserves, what risk loading is appropriate for each liability class? Should it be a function of the volatility in the liability class payouts or a function of the length of the payout pattern?
Accounting accruals	When developing accounting entries for the model's financial statements, a substantial number of accrual items must be developed. How are these accrual items derived - are the accrual amounts based on fixed or variable relationships to other model elements?

Interest rates

The introduction of volatility in the model's projected interest rate environment has a many-tiered impact on the financial model. For example, changing interest rates are the primary driver of changes in the market value of previously purchased assets. Additionally, the model also relies on changes in interest rates to signal changes in the overall level of inflation in the insurance environment. Specific areas within the model that are directly affected by changes in interest rates include bond pricing, equity pricing, and loss reserve discounting. Other areas that can contain linkages to changes in interest rates include future premium, loss and expense levels.

Interest rates and bond pricing

A risk factor that is reflective of actual bond return over risk-free yields is specified for each bond category being modeled. The different risk factors are added to the arbitrage free interest rates to develop interest rate curves that are specifically applicable to each bond category. Since the future cash flows from each bond category are known[7] (coupon amounts and timing and principal repayment amount and timing), we can use traditional bond valuation methods to calculate changes in market values for the holdings in each bond category arising from changes in interest rates.

Interest rates and equity pricing

The model employs two alternatives for pricing equities. The choice between them is dependent on one's view of interrelationships between interest rate movements and equity prices.

One alternative bases the rate of return on equities on a normally distributed random variable with a mean market return and standard deviation based on investor expectations. This alternative uncouples equity pricing from changes in interest rates and is a conventional random walk model.

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[7] The future cash flows of bonds held at December 31, 1996 are known because the bonds themselves are known quantities. We know their coupon rate and timing, their maturity date, and their par, book, and market values. This is sufficient information to project future cash flows arising from the December 31, 1996 bond portfolio.

The future cash flows of bonds purchased during 1997-2001 are known because (a) we know the risk-free interest rate environment at the time the bonds are (will be) purchased, (b) the risk factor that is added to the risk-free interest rate for each bond category, (c) the time to maturity of the bonds that are purchased, and (d) the total dollar amount of new investments in each bond category. With this information, we can calculate an appropriate coupon rate for each dollar of investment in each bond category. We make a simplifying assumption that new bonds are purchased at par, so the new bonds' market values at the time of purchase equal their book, par and statement values. We now have sufficient information to project future cash flows arising from new bond purchases.

The second alternative relates equity returns to the projected interest rate environment through the Capital Asset Pricing Model (CAPM). Readers might recall that the CAPM formula is $R = R_f + \beta(R_m - R_f)$, where

R = the expected return on a given stock,
R_f = the risk free rate, such as the rate on Treasury bills
R_m = the overall market return
β quantifies the undiversifiable or systematic risk associated with the stock in question

In the Capital Assets Pricing Model, three hypotheses are made:

1. The expected return from a common stock is related only to the stock's systematic risk
2. The difference between the expected return from a common stock and the return on a risk-free rate is proportional to the firm's systematic risk
3. The systematic risk and the factor of proportionality are relatively constant over time.[8]

In our use of CAPM to link equity returns with interest rate movements, we have made some slight modifications to the basic CAPM formula. Our revised formula is $R = R_f + \beta_P\,(\alpha) + e_P$, where the terms are as follows:

R_f = short term risk free rate of return
β_P = the beta of the stock portfolio being held
α = average excess return of the market portfolio over the risk free rate[9]
e_P = stock price volatility (the random variable)

With $\beta_P = 1$, this simplifies to $R = R_f + \alpha + e_P$, which is the conventional random walk model noted above.[10]

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[8] "Pricing Insurance Policies: The Internal Rate of Return Model" by Sholom Feldblum, May 1992, page 31. Mr. Feldblum also references Portfolio Theory and Capital Markets by William F. Sharpe, New York, McGraw-Hill, 1970 and "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, February 1965, pages 13 ff.

[9] Underlying the assertion of a constant excess return of the market portfolio over the short term risk free rate are assumptions that average risk aversion and average stock price volatility are constants. α can also be thought of as the difference between R_m and R_f, where R_m is the expected return of an index portfolio, such as the S&P 500.

[10] The use of CAPM to model equity returns is described in more detail in "Using CAPM to Generate Scenarios for an Equity Portfolio" by Vladimir Fishman and William C. Scheel in The Chalke Perspective, Second Quarter 1996, Volume 7, Issue 2.

Interest rates and loss reserve discounting

The model allows loss reserves to be discounted for accounting purposes. The discount rate can be fixed, or it can be made a function of the simulated interest rate environment. If the latter approach is used, risk loadings can be specified separately for reserve discounting by line of business.[11]

Inflation

Inflationary impacts can affect one or more model components, including loss payout amounts, future premium volumes, and future expense levels. As such, the implications of inflation as a model parameter will be discussed in the context of each affected component.

Inflation and loss payouts

The model assumes three basic factors can affect loss payouts:

1. The timing with which loss reserves will be paid out
2. Reserve redundancies or deficiencies, excluding those arising from changes in the level of inflation affecting loss payment levels[12]
3. The impact of changes in inflation on future loss payments

Changes in the inflationary environment affecting each line of business are translated into changes in calendar year loss payouts. If, for example, the stochastically generated future inflation rate is substantially greater than what existed in the past and was expected to exist in the future, future loss payouts will be greater than anticipated. Conversely, if the stochastically generated future in-

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[11] For a thorough discussion of the issues surrounding the selection of an interest rate for discounting reserves, we recommend the reader refer to "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach" by Robert Butsic in Evaluating Insurance Company Liabilities, Casualty Actuarial Society Discussion Paper Program, 1988, pp. 147-188.

[12] There is typically some element of claims inflation implicit in held loss reserves. Consider reserves that are developed from a traditional actuarial analysis of a paid loss triangle. Unless the paid loss triangle is specifically detrended prior to the analysis, the loss payment amounts in that paid loss triangle include some level of inflation in the payment amounts. The assumption in component 2 is that the levels of inflation in the future will be consistent with those in the historical payment triangles, so that when I assume I will pay $100 five years from now, the inflationary pressures on loss costs will not cause me to actually pay anything other than $100. This concept, along with the concept of inflationary impacts on future loss payments, are described in much greater detail in "The Effect of Inflation of Losses and Premiums for Property-Liability Insurers" by Robert Butsic, Inflation Implications for Property-Casualty Insurance, Casualty Actuarial Society Discussion Paper Program, 1981, pp. 58-102.

flation rate is lower than what was expected to exist in the future, future loss payouts will be lower than anticipated.

By including the inflationary impacts on loss payouts, we incorporate a linkage between the macroeconomic environment affecting assets and the macroeconomic environment affecting losses. We are also in a position to examine the financial statement implications of unanticipated inflationary pressures on loss payouts.

Inflation and future premium volumes

If premium for a line of business is dependent upon an exposure base that is inflation sensitive (such as workers' compensation), a formula for projecting future premium volumes can be utilized that incorporates a linkage to changes in inflation. Volatility in future inflation rates will have a direct effect on future premium volumes.

Inflation and future loss ratios

Future loss ratio projections can also be made dependent on changes in inflation. The model has been developed with the concept in mind that there exists a "force of loss" that is independent of inflationary impacts. This force of loss describes the loss ratio that would arise if there were no other changes occurring that have an impact on the final loss ratio. Other changes might include premium rate changes, inflationary increases in the premium exposure base, or inflationary impacts on loss costs. The final projected loss ratio is developed by first randomly sampling from the probability distribution that describes this force of loss, then modifying the random sample to reflect the other changes.

Exhibit 3 provides an example of the interrelationships between premium development and loss ratio development, including inflationary and rate impact influences.

Inflation and future expense levels

Future expense levels can either be assumed to be stable with current expense levels, or the model can apply an expense growth factor. The growth factor can be predetermined by the user; it can be made partly or completely random; it can be tied to changes in interest and inflation rates; or it can be a combination of the three. As with the inclusion of an inflationary linkage in the loss reserves, by adjusting the year-to-year expense levels for inflation, we link together the assets macroeconomic environment with the macroeconomic environment affecting company operations.

Conversion of loss reserves into cash outflows

As described in the section "Inflation and loss payouts" on page 15, there are three components that can affect loss payouts. The inflationary component has already been addressed, but the other two have not.

Variability in loss payout patterns

The model is structured so that variability in the timing of loss payments does not affect the overall amount that ultimately will be paid out, but it does affect when the payments occur. We liken the imposition of variability in this area of the model to the induction of an "accordion effect" in cash flow patterns, i.e. either stretching or compressing the basic patterns. Imposing variability on the loss payout pattern stress tests the company's asset liquidity. Exhibit 4, "Accordion effect in payout patterns", provides an example of the imposition of variability in loss payout patterns.

Variability in indicated reserve levels

As sometimes occurs in actuarial analysis, it turns out that, in hindsight, held reserves were either redundant or inadequate to meet the claims obligations. Allowing for variability in the indicated (as opposed to the held) reserve levels allows the model to quantify the income statement, cash flow, and tax implications of loss reserve redundancy or deficiency. Exhibit 2, which was first described in the section "Underwriting section" starting on page 5, demonstrates the model structure used to model variability in held reserves.

Allocation of new money

How assets are rebalanced at the end of each projection period is a critical model input. This determines what assets are to be bought and sold at any point in time, and it defines a risk profile that the company is willing to assume in addition to the risk profile being determined by the company's underwriting activity. The allocation algorithm can be as simple as "maintain the same relative mix of assets next year as we had this year", or it can be a complex algorithm that adjusts next year's asset mix to better match the projected liability duration. The dynamic nature of this element is not so much in creating randomness within the rebalancing algorithm, but in crafting an algorithm that is sensitive to the changing financial projections that are emerging from the model.

Production of new business

New business – premium volumes

New business production is a function of many different inputs. This can include the relative amount of business that is expected to be retained each year, a company's internal growth objectives, the overall insurance market conditions, and company reactions to prior year underwriting results. The interrelationship of inflation and new business volume has already been addressed. In general, it would seem that the more linkages that are established between new business production and other events being played out in the model, the better the model will be. The model then should be more reactive; it should do what the company itself might do when faced with similar circumstances. However, in some cases, the inclusion of additional dynamic elements in these linkages could lead to greater confusion in what the model is doing than is warranted by the additional realism that is gained. Each additional component has model building costs associated with it so, one must carefully reviewing both the model's objectives and the goals of company management for the model before expanding it. As Rodney Kreps and Michael Steel noted in their 1996 DFA paper, "For any of these models, a salient requirement is parsimony...there is no point in trying to model a detail whose behavior is masked by the random noise created by other terms."[13]

New business – loss ratio projections

Accompanying the mechanics for developing future premium volume projections are the processes for creating the associated loss ratios. Loss ratio projections need to consider, among other things:

- the underlying risk exposure taken on by the company
- the macroeconomic forces acting on the underlying risk exposure (i.e. inflation)
- the actions being taken by the company that may have an impact on the underlying risk exposure (such as a loss ratio's deterioration arising from reduced underwriting standards)
- and the actions being taken by the company's competitors.

An example of how these varied forces can be incorporated into the projection of future premium volumes and loss ratios is given on Exhibit 3.

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[13] "A Stochastic Planning Model for the Insurance Corporation of British Columbia" by Rodney E. Kreps and Michael M. Steel, in The Casualty Actuarial Society Forum, Spring 1996, pp. 156-157.

Expenses

Like new business production, expense projections are functions of many different inputs. Some expense projections can be tied directly to new business production, such as commission amounts. Other ones are tied directly to loss calculations. It is the year-to-year projection of fixed expenses that gives the greatest opportunities for directly including a dynamic component. For example, it can be assumed that salaries will grow five percent plus or minus one percent next year. Alternatively, including a dynamic component in expense projections must be weighed in relation to the relative benefits to be gained from the added complexity.

Correlations

This may be the most difficult set of model parameters to develop. A significant portion of the time there is no readily available information source that correlates different model components. In these situations, it falls back on the model developer's judgment to establish correlations that intuitively seem reasonable but for which there might be little or no empirical support. The area of correlation is one that current actuarial literature does not seem to address well, at least not in terms of providing statistical analysis of correlations. We would expect the amount and quality of correlation data to improve as dynamic financial modeling becomes more widespread, but that does not help current model builders. In our opinion, the best that can be done is to work judgmentally with the company to develop correlations that seem reasonable to both model developers and the party or parties for whom the model is being developed.

One data source that we have found particularly useful, at least in providing a mathematical foundation for the correlation of non-normal random variables is "A Distribution-Free Approach To Inducing Rank Correlation among Input Variables," by Ronald L. Iman and W.J. Conover.[14]. Using the mathematics described in this article, we have been able to implement an algorithm for the using pairwise rank correlations among dynamic variables. The model requires the correlation matrix to be positive definite[15]. In order to achieve this objective, the

[14] "A Distribution-Free Approach To Inducing Rank Correlation among Input Variables," by Ronald L. Iman and W.J. Conover.[14], *Commun. Statist.-Simula. Computa.*, 11(3), 1982, pp. 311-334.

[15] Given an equation $f = ax^2 + 2bxy + cy^2$, the equation is said to be **positive definite** if for all points other than $x = y = 0$, the equation is positive. In terms of matrix mathematics and linear algebra, given a symmetric matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

model automatically rescales the user-created (subjective) correlation matrix until a positive definite one is found. The resulting correlations may not be as strong as those initially established by the user, but they will retain the directional relationships that were established.

Accounting entries

The final category of parameters relates to the development of accounting entries from cash flow projections. It is our belief that a model must begin by quantifying cash flows— if cash can not be developed in a reasonably accurate manner, it does not matter how accurately the accounting accruals are developed. In keeping with this belief, we have concentrated on getting correct the details of the asset and liability cash flows, and have built up the balance sheet and income statement structure around the cash flows. To go from cash basis accounting to accrual accounting, we have employed a number of ratios that relate accounting accruals to annual company operations. An example of one such ratio is the relative level of premium written during the year that is due and not yet collected, which is used to quantify the agents' balance asset. These ratios can remain stable over time or can be allowed to vary, as the model user desires.

SECTION III: DEFINING PARAMETERS OF DYNAMIC RANDOM VARIABLES

Identification of model structure is arguably the most important task in dynamic financial analysis. Once model specification is laid to bed, the DFA investigator needs to establish process and parameters for the dynamic components that are modeled. The question most often dealt with in this context is, "What are the data sources?" This section of the paper addresses this question by revisiting parts of Section II and discussing how we have selected relevant parameters in some of our own models. We have omitted from this section certain items that were in Section II because either:

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satisfying any of the following tests insures the matrix A is positive definite:

- $x^TAx > 0$ for all nonzero vectors x.
- All the eigenvalues of A satisfy $\lambda_i > 0$.
- All the submatrices A_k have positive determinants.
- All the pivots (without row exchanges) satisfy $d_i > 0$.

from Linear Algebra and its Applications, 2nd edition, by Gilbert Strang, Academic Press, Inc., 1976, pp. 245-250.

a) the parameters are more appropriately defined by examination of company-specific data and discussion with company management (e.g. expense growth), or
b) the variability in the parameters is best developed in conjunction with company management, either because there is not much existing information on which to base the variability, or because the model's accuracy will not be significantly enhanced by an exhaustive analysis of the parameters' potential variability (e.g. accounting accrual percentages).

For those items that we are including in this section, a number of them can be appropriately parameterized with information from the either the DFA web site (http://dfa.risknet.com) or other Internet locations, but some can not.

There seems to be a clear dividing line between those items that can be analyzed with data available from the web site and those that can not. If a key dynamic element relates to economic issues that are not under the company's direct control, such as interest rates, inflation rates, and asset values, then the DFA web site can provide useful supporting data. On the other hand, if the control of the key dynamic element rests with the company being modeled, such as volume projections or expense growth, it is better to rely on the expectations of company management than the data from the DFA web site. With this distinction in mind, let us turn to some of the key dynamic elements from Section II.

Parameterizing an interest rate model

There has been considerable literature about the projection of interest rates. In fact, this may be the most well-documented of all DFA model parameters. The one-factor model we have implemented is closely based on the first of two interest rate generation algorithms described in a paper by James Tilley in the late 1980s[16]. It is a one-factor lognormal model that reverts interest rates to short-term expectations. In other words, projected interest rates have a tendency to move from an initial seeding (the actual December 31, 1996 interest rate level) to an equilibrium that represents historic interest rates expectations in the short-term spectrum of the yield curve. The input values needed to parameterize the Tilley model were derived from investigations using historical data and so-called stylized comparisons between model results and conventional expectations for such a model.

[16] "An Actuarial Layman's Guide to Building Stochastic Interest Rate Generators" by James A. Tilley, Transactions of the Society of Actuaries, Volume XLIV.

Asset risk parameters

Parameters describing the additional returns over the risk-free rate must be specified for each asset class. We have used information from Bloomberg databases that are available on a subscription basis to develop our selected asset class returns in excess of risk-free rates.[17] In addition, we developed the equity model volatility parameters from studies using data from the Ibbotson web site (http://ibbotson.com). Bond and equity price behavior is holding period-specific, so depending on the particular historical period examined, one can obtain wildly different indications of rate of return and volatility from the information in these studies. Judgment plays an important role in developing these parameters, however, one should choose historical values over extended periods rather than being influenced by short period of time. We have, however, modified historical information to reflect what is believed to be atypical periods of monetary authority involvement—behavior that is not likely to be repeat by the Federal Reserve Board.

Loss reserve discounting

We do not feel there is any one correct answer to the selection of an interest rate for loss reserve discounting. In the absence of strong preferences, we chose to use the short term risk-free interest rate being produced by the model's interest rate generator. This at least links liabilities to the interest rate environment that is impacting the asset side of the balance sheet.

Inflationary impacts

Impact on loss reserves

The level of inflation implicitly embedded in held loss reserves is often a difficult value for companies to quantify. It can be estimated by examining trends in claims payment patterns using a variety of loss reserving methods.

Another alternative is to use industry data to quantify historical changes in prices for different commodities. For example, the model underlying this paper includes information on the private passenger auto line of business. Exhibit 5 shows information collected from the Bureau of Labor Statistics web site (http://stats.bls.gov) on changes in the cost of automobiles and medical care. The changes in cost in the two indices were averaged to produce a 4.4% annual inflation rate, which was included in the simulation parameters.

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[17] Further information on Bloomberg services is available at their web site, http://www.bloomberg.com.

Impact on future premium volumes

For those lines whose premium base is inflation sensitive, such as workers' compensation (sensitive to payroll inflation) or homeowners (sensitive to increases in construction labor and materials costs), a component of future premium estimates can be inflation.

Historical relationships can be determined between interest rates and price indices that are applicable to the specific line of business in question. The DFA web site has links to information sources for historical interest rates. The Bureau of Labor Statistics can be used to gather information on payroll inflation or other price index changes. Once data has been collected for each, a regression equation can be established that links each line's inflationary component to an underlying interest rate level. The regression equation can then be used to relate projected future interest rates with projections of future inflationary pressures on premium volume. We leave it up to the model user to decide if he/she wants to specify the regression's error term as a random variable or to ignore it.

Impact on future expense levels

It can be argued that a company's *a priori* expectation of changes in expense levels will be more of a expense driver than will external inflationary pressures. Additionally, in this context, the issue of incremental value added must be examined. Will the model results be that much better for the inclusion of an inflation-linked expense component? Or will the added volatility just add to the noise that a dynamic model inevitably captures?

The simplest way to parameterize this component, in our opinion, is to ignore inflationary impacts all together. Instead, concentrate on the company's historical expense growth as it relates to changes in company operations. Has the company grown considerably in recent years? How have expenses changed in the same time frame? What are the company's operational expectations for the next five years? Equally rapid growth, or slower growth? We believe in this area simpler is better—link the expense changes to operational projections and (at least at first) do not confuse the issue with additional linkages to inflationary factors.

Conversion of loss reserves into cash outflows

Variability in loss payout patterns

We approach the creation of a dynamic payout pattern with the basic idea that incremental variability in a payout pattern should decrease the closer the cumulative pattern is to 100%.

When parameterizing a loss payout pattern to include variability, we have generally elected to ignore the possibility of correlations between successive incremental payout percentages. From a very unscientific sampling of data, we have found no conclusive evidence to link a higher-than-expected incremental payout in time T with either a higher-than-expected or lower-than-expected incremental payout in time T+1.

We typically try to use company-specific data as the starting point for developing the mean and ranges of variability around each incremental payout percentage. We may add in some judgment as to whether the data has sufficient variability or too much variability. For the industry-wide example in this paper, we developed the baseline payout patterns from an analysis of the industry paid loss development patterns and we based the variability on the actual observed variability in the incremental payout percentages. The development of one such variability parameter for private passenger auto is displayed in Exhibit 6.

Variability in indicated reserve levels

We address this dynamic element in a similar fashion to the way we address variability in the payout pattern. We begin by assuming that reserve variability decreases as accident years age. We do, however, expect there to be some correlation between accident year reserve redundancies or deficiencies, both within lines of business and across lines of business. The strength of the correlations is often based on expert judgment.

Exhibit 7 provides an example of a data format for determining reserve variability parameters. From this information, with some actuarial judgment thrown in, we develop final reserve variability parameters.

Allocation of new money

While the allocation of new money is a critical variable in the overall model dynamics, it is not one that we try to parameterize as a random variable. We see the allocation of new money as a management guideline, one that should be compared among competing allocation strategies. For example, we pose the question "What is the impact on our year-to-year and multi-year financial performance if we allocate more funds to taxable bonds as opposed to equities?" In this context, variable definition involves discussing investment philosophy with company management and developing the three or five or ten different broad asset allocation strategies to be evaluated.

Production of new business

Premium volumes

The amount of parameterization that is required for this variable is dependent upon the complexity with which each year's new business projection is established. If the new business projection is quantified by an analysis of retention ratios on current business in combination with expectations of writing certain volumes of completely new business, the number of variables to parameterize might be large. If, on the other hand, the new business projection is quantified by a simple growth factor applied to the prior year's level of writings, the parameterization of the growth factor could entail much less work. Either way, the data that should be used to develop the parameters needs to be specific to the company being modeled.

Loss ratios

Parameterization of future loss ratios can be a very simplistic or sophisticated analysis. The level of research and number of considerations should be commensurate with (a) the purpose for which the model is being used, (b) the amount of data available, both company-specific and industry-wide, and (c) the level of uncertainty in other key model parameters. In our experience, we have based loss ratio volatility parameters on a combination of discussions with companies, examinations of historical volatility, and historical and projected future rate adequacy.

Correlation (Revisited)

As noted in the preceding section on correlation (beginning on page 19), dynamic financial modeling must allow for correlated variates. This is particularly true for the relationship among lines of business. Correlation and causality are both important to modeling. The business environment will affect new business production—the relationship is largely causal and must be functionally built into the model. But, there also are company-specific effects such as agent activities, persistency of existing business and other phenomena that may be understood as correlated with one another. It is important, however, to distinguish between the modeling of causality and these correlated, random phenomena. Model specification must define relationships between, say, an economic or business environment scenario and production of new business. Omission of this causality consideration is an example of specification risk in model design. However, given a business scenario there still needs to be considerations of correlation among, say, loss ratios for the lines of business.

CONCLUSION

Ultimately, whether a model succeeds or fails depends on the level of trust that is placed in its parameterization. It always is preferable to build model parameters from actual data, but actuarial judgment can provide acceptable surrogates so long as the assumptions underlying the judgment are reasonable to those parties placing reliance on the model. In many cases, the data is just not available, or at least not readily available for use in model specification. In other cases, the marginal improvements that can be gained by more accurate parameter specifications are not significant enough to warrant either the work that would be needed to improve the parameterization or the model's additional complexity.

Whenever data are sparse, the need for models that reflect *subjective* reasoning or understanding will dominate choices in parameterization. With only limited, subjective information, a good choice of a distribution for process risk is one with parameters that make sense within the context of the sparse information. The Weibull distribution has merit because its parameters can be chosen with information about central tendency and chance-constrained probability estimates of the extreme tails—no other information is required to fully specify a distribution within this rich family of distributions.

It is our view that all decisions regarding model specification and model complexity should only be made after a review of:

- the additional value the parameter brings to the model results
- the ease or difficulty with which the parameter's dynamic specifications can be developed
- the amount of work that will be needed to maintain and otherwise update the parameter's specifications and
- the additional complexity engendered by the parameter's inclusion.

If, after reviewing these elements, the decision is to include the parameter in the model, there are a number of alternatives a model developer can take. Some information is readily available on the Internet, much of it already included in the DFA web site. Other data sites, including the Bureau of Labor Statistics, are also easily accessible. Still other data sites are available for a charge, such as the Insurance Services Office, AM Best and National Council on Compensation Insurance sites. Undoubtedly there are many more sites that as yet remain undiscovered by the actuarial community at large. As property-casualty insurance company dynamic financial modeling moves out of its infancy, more and more sources of information will become known.

Dynamic financial analysis is a slowly evolving area of actuarial knowledge. If the actuarial profession is patient, it will either find or develop the data sources needed to more fully address DFA issues. Until then, the best we can do is work with the limited information at our disposal, and where none exists, use our judgment to fill the gaps. If we expend our energy bemoaning what we do not have, we will not be in a position to take advantage of what we do have. Imperfect knowledge in and of itself is not a good excuse for abandoning the cause of DFA. Rather, the imperfections should be understood for what they are, and the model building should continue and purse alternatives in design that are consistent with subjective understanding. Later in time better data will become available and the imperfections can be reduced or eliminated entirely.

Appendix A: Fundamentals of Dynamic Financial Analysis

The Tenets of DFA

DFA methods are helping return and risk analysis, but the methods are still in their infancy. The computation techniques evolve with every new generation of software and computers. But, within this state of flux we find some well-anchored principles. First, DFA is very *ad hoc* and relies entirely on repetitive simulation of business events and the accounting of those events using a virtual general ledger. Second, the primary purpose of DFA is to understand with some measure of confidence the range in which general ledger-based metrics will fall. Third, DFA is interested in answering both narrow and broad questions. Narrowly, we ask: "How does ***a scenario*** measure up." Broadly, we ask: "How do we measure up in the presence of many scenarios—what is the business impact of the ***virtual scenario***?"

Accounting Frameworks

Most aspects of dynamic financial analysis ultimately are dependent on an accounting system. The primary purpose of DFA is to provide decision makers with ranges instead of point estimates.

The point estimate approach has been the bulwark of forecasting. But, static analysis leading to the point estimate is not very useful if one needs to allocate capital, choose among competing strategies with different risk profiles, or identify alternatives that optimize some goal function.

Business performance ultimately is measured by one or more accounting measurements, or metrics. These appear throughout financial statements. They range widely in their complexity and component parts:

- Balance sheet or income statement accounts
- Financial ratios,
- Complex functions of cash flows,
- Regulatory criteria, and
- Operational measurements such as business volume.

A common element to any of these metrics is the underlying accounting system from which they are derived.

It is difficult to envision DFA without an accounting system. There are many dynamic analyses that could skirt around accounting; but, if the analysis is truly financial, it will require an accounting system. For example, one might want the aggregate loss distributions for all parties to a risk transfer agreement so that each layer can be appropriately priced. The purpose of the analysis is to set prices, and it may be separated from the more interesting question of what impact a particular portfolio of reinsurance has on the solidity of an enterprise. The latter is dynamic ***financial*** analysis and certainly would require understanding how surplus and cash flows are affected by the reinsurance. Surplus is a creature of an accounting system—most objects of DFA are accounting-based metrics. However, DFA can be used in cash-based accounting exercises. Actuaries might use DFA, for example, to analyze cash flows for pricing purposes, and then use the same DFA methods in a broader context and different model. The latter usage for asset management would lead to understanding the broader implications of such pricing on surplus generation or other important metrics.

To the extent that an accounting framework is used to measure the magnitude and force of variables, its focus will greatly affect the end-product of DFA. In addition to cash-based accounting, there are statutory and GAAP bases and tax and management accounting ledgers. Each framework will be important to a particular constituency. While managerial accounting could work well for management, statutory accounting might be preferred by a regulator and GAAP accounting preferred by stockholders.

In summary, physical and financial processes are responsible for asset changes from period to period. But, the stock measurement or asset volume observed at the end of a period will be greatly influenced by the method of bookkeeping. It is important to recognize that both the magnitude and its probabilistic dispersion can be affected by the system of measurement.

Principle 1: DFA Requires an Accounting Framework

The First Principle of Financial Return and Risk measurement is to understand that DFA is conditional upon one or more accounting frameworks. Most business decisions will be made based on how they are perceived to affect results measured by an accounting system. DFA measurement results will almost always be complex accounting functions of dynamic input variables.

Static and Dynamic Financial Analysis

Traditional financial forecasting has relied on essentially static evaluations of current and future events. A traditional financial forecasting model might include a single set of assumptions (or maybe three sets of assumptions: best case, base case, worst case) about future operating results from various operating divisions or business units, an expectation of investment returns from the investment division and a projection of fixed expenses from a corporate planning division. From these inputs a financial plan is developed and critical business decisions are made.

What is missing from this picture? To begin, there is no sense of how likely it is that the base case will be achieved, or the worst case avoided. In a static forecasting environment, there is no way to quantify the variability of possible outcomes. Yet this is a critical factor is strategic decision making. It is very difficult to know which of a series of strategic options to pursue without being able to appreciate differences in both the range of possible outcomes and the most likely result to arise from each option. The essence of dynamic financial modeling is the ability to describe critical assumptions in terms of ranges of possible outcomes, rather than in terms of fixed values. Once each critical assumption is defined by a range of possible outcomes, a sophisticated modeling environment takes over, recalculating the integrated financial model again and again, returning different values each time. At the conclusion of the modeling exercise, we are left with a range of results we can reasonably expect to see, given the parameters and interrelationships that have been defined for the key variables. Differences in financial results arising from alternative strategic decisions can be evaluated by replacing one set of strategic decisions with another, re-running the modeling exercise and comparing the ranges of possible outcomes under each decision path.

Principle 2: DFA Is Communicated in the Form of Confidence Statements

The second DFA principle is how the work-product of the analysis is communicated to management. The boundaries of a metric are declared with an attached probability. This approach is not classical statistical inference regarding the probability of chance explanations of phenomena; it is the substitution of range estimates for point estimates. DFA implicitly places greater value on confidence bands than on the expected value of a distribution.

The Demeanor of Model, Process and Parameter Risk Assumptions

The demeanor of DFA for asset and liability valuation is very ugly. There are at least three faces to the problem of setting up a DFA experiment:

1. The functional relationships yielding changes in asset and liability value may be obscure and consist of both physical phenomena and accounting relationships. This inability to adequately understand and specify important functional relationships is ***DFA specification risk***.
2. The joint probability distribution of the model variables is almost always unknown. We may have some fuzzy understanding of the marginal distributions, but it may be limited to certain beliefs about central tendency and extreme behavior. We posit some degree of correlation

among variables. But, this inability to really define the joint probability distribution is a manifestation of ***DFA process risk***.

3. Model designers often are lured into the belief that specification and process risk are non-existent. If only they knew the parameters of the (analytic) distribution(s), all would be well. These poor souls suffer only ***DFA parameter risk***.

Collectively, we will refer to these various risks of mis-specification of the DFA experiment as ***model risk***. But, it is clear that mis-specification of a model for asset valuation almost always will occur to one degree or another.

Principle 3: DFA Has Its Own Risk Profile

> ***Although one explains DFA as a confidence measure, it fails to explain the underlying uncertainty both in the DFA model, process and parameter risk. While one might render a DFA confidence statement, there must be caveats. A well-defined accounting framework will not eliminate the subjectivity involved in the assessment of causality.***

Simulation Will Always Be Preferred for DFA Work

Simulation usually is the most tractable approach; analytic solutions usually evade us. Simulation is particularly powerful under these circumstances:

- Rule-based reasoning,
- Transformation of variables in a complex way (such as with an accounting system),
- Node events with probability distributions that are substantially empirical or subjective.

A node event is one where a probability distribution defines two or more possible outcomes, and depending on the outcome a different set of events follows.

Principle 4 DFA Must Deal with Subjectivity

> ***The elicitation of probability distributions for key variables is not like the contrivance of bets in a casino. The physical processes are rarely known. Sometimes, only a sense of central tendency and confidence in some tail point will be acknowledged as "known." This fuzzy understanding means that the choice among probability distributions should be dictated as much by the intuitiveness of their parameters as by their ability to define physical processes. Is an actuary who fits data to parameterize a lognormal distribution and declares it to be a severity distribution wiser than another who uses a Weibull distribution fit with the same data? The latter can successfully argue that he has chosen to augment his subjective understanding of central tendency and a cutoff point, and that he has chosen the Weibull because it* a priori *is a rational way to handle (and acknowledge) subjectivity.***

Spreadsheet-Centricity for DFA Work

A spreadsheet is an excellent programming venue for rules, accounting systems and scenario representation with dynamic variables. DFA models can be segmented into components—the nodes serve as logical breakpoints among strains of causality. This chained quality of the models, even with feedback and dependency relationships, is easily expressed in spreadsheet components. The components may be as small as a single cell. Or, they may be aggregations of cells or sheets within a workbook. In any case, the components can be easily modified when they are expressed as elements of a spreadsheet.

Principle 5 DFA Is A Natural Application for Spreadsheets

This principle of DFA acknowledges the evolutionary nature of DFA modeling. A DFA model is never really "finished"; there are always more detailed questions that can be asked of the model and more information that can be extracted from it. A spreadsheet-based environment acknowledged the inevitable expansion of DFA models and provides a framework for implementing changes that is programmer-free.

Stochastic Dominance: Stochastic dominance looks at the range of possible outcomes in terms of the risk/reward tradeoffs that are indicated by each. Strictly defined, stochastic dominance is an approach for choosing among risky alternatives based on certain knowledge about their cumulative probability distributions and about utility that is derived by the decision maker.[18] An evaluation of two (or more) alternative strategic directions for stochastic dominance provides the information needed to answer the question, "Which alternative has higher expected utility?" The answer to this question can be the basis for a management decisions. Graphically, it sometimes is possible to identify stochastic dominance and begin to answer the question, "Does greater reward justify greater risk," directly within a utility framework.

The picture on the following page displays the relationship between three different asset allocation strategies. The measurement, an example of a user defined metric, is the internal rate of return on the change in book value of all invested assets over the five year projection horizon plus investment income, realized and unrealized capital gains, less the difference between the market value of assets maturing and sold and those purchased during the five years.

While there is no clear cut stochastic dominance evident in this picture, it does illustrate that higher return is only achieved at the price of higher risk. The ultimate choice is a business decision; there is no alternative in this decision set that stochastically dominates the other. This finding may seem to be a bane of dynamic financial analysis – there is no mechanically driven choice within a loosely defined utility framework. However, it points out the reality underlying strategic business decisions - it is not very often that one strategic direction is clearly superior to all others.

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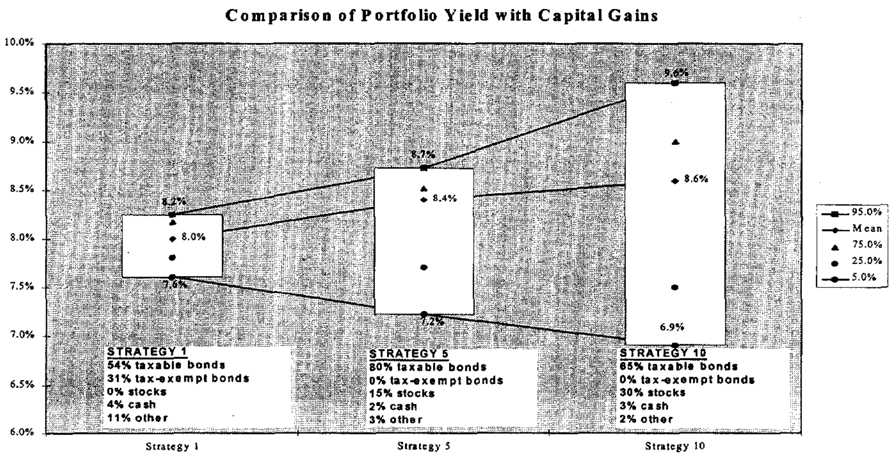
[18] Stochastic dominance also has been called a general efficiency criterion. It provides a framework for decision making under uncertainty based on sparse understanding of the decision-maker's utility function. For example, it can be shown that under the loose assumption that one prefers more to less, if $F(x) \leq G(x)$ for all x and at least one point in the domain of x is such that the strong inequality holds, one should prefer the alternative with cumulative distribution F(x). This is a typical "risk averse" profile. The reader will find a discussion of the general efficiency criterion (first degree stochastic dominance) in Haim Levy and Marshall Sarnat, *Investment and Portfolio Analysis*, John Wiley & Sons, Inc., 1972, pp. 264 ff.

Exhibit 1 continued



Example of portfolio yield with capital gains calculation:

		1996	1997	1998	1999	2000	2001
(1)	**Investment Income**		40,000	38,000	44,000	43,000	45,000
(2)	**Realized capital gains**		2,000	-1,000	1,000	8,000	-3,000
(3)	**Unrealized capital gains**		10,000	-4,000	-10,000	26,000	-1,000
(4)	**Assets maturing or sold**		100,000	110,000	120,000	130,000	140,000
(5)	**Assets purchased**		120,000	135,000	150,000	165,000	180,000
(6) = (4) - (5)	**Net Sales**		-20,000	-25,000	-30,000	-35,000	-40,000
(7)	**Book Value**	-500,000					650,000
(8)= (1) + (2) + (3) + (6) + (7)	**Cash Flows**	-500,000	32,000	8,000	5,000	42,000	651,000
	Internal rate of return	8.66%					

The interaction of loss reserve payouts, reserve redundancy/deficiency emergence into cash flows and accounting and unanticipated inflation:

Suppose a company had one year of loss reserves on its books with the following additional information:

Held reserves: $100,000, based on a reserve range of $80,000 to $105,000
Inflation level implicit in held reserves: 5%
Reserve payout pattern: 25% over each of the next four years

The model parameters incorporate the $100,000 held reserve into the first loss triangle (payout of held reserves), and the potential for reserve variability in the second and third triangles (payout and accounting impacts of reserve redundancy/deficiency emergence). Any impacts arising from changes in the inflationary environment are captured in the calendar year financial statements only.

Let us assume the model randomly selects parameters such that the reserve redundancy is $10,000, excluding a change in inflation, and that inflation increases to 8% in years three and four. We decide to reflect the $10,000 reserve redundancy by weakening reserves $5,000 in years one and two.

The model results look as follows:

	Year 0	**Year 1**	**Year 2**	**Year 3**	**Year 4**
Payout percentage		25%	25%	25%	25%
1. Expected inflation rate		5%	5%	5%	5%
2. Actual inflation rate		5%	5%	8%	8%
3. Held reserve cash flow	n/a	25,000	25,000	25,000	25,000
4. Redundancy cashflow	n/a	-2,500	-2,500	-2,500	-2,500
5. Inflationary impact	n/a	0	0	643	1,304
6. Reserve weakening	n/a	-5,000	-5,000	0	0
7. Held reserves	100,000	72,500	45,000	22,500	0
8. Net cash flow		22,500	22,500	23,592	23,804
9. Income statement impact (- gain/ + loss)		-5,000	-5,000	+643	+1,304

Row 3 formula: $\left[\left(\prod_{i=1}^{4}(1+Row2_{Year\,i}) \Big/ \prod_{i=1}^{4}(1+Row1_{Year\,i})\right)-1\right]*(Row3_{Year\,i}+Row4_{Year\,i})$

Example: Year 3 inflationary impact = $\left[\frac{(1.05)(1.05)(1.08)}{(1.05)(1.05)(1.05)}-1\right]*(25{,}000-2{,}500)$

Row 7 formula: $Row\ 7_{Year\,i-1} - Row\ 3_{Year\,i} + Row\ 6_{Year\,i} - Row\ 4_{Year\,i}$

Row 8 formula: $Row\ 3_{Year\,i} + Row\ 4_{Year\,i} + Row\ 5_{Year\,i}$

Row 9 formula: $Row\ 5_{Year\,i} + Row\ 6_{Year\,i}$

[Page revised: 5 March 2001]

Exhibit 3

	Actuals 1996	*Projected 1997*	*Projected 1998*	*Projected 1999*	*Projected 2000*	*Projected 2001*
Insurance Industry Composite — Workers' Compensation						
INCOME STATEMENT RELATED ASSUMPTIONS						
Premiums						
1. Net written premiums prior to rate, competitive impacts:		*18,656*	*18,656*	*17,441*	*17,842*	*19,648*
2. Rate impact on net written premiums:		*n/a*	*-10.0%*	*-2.5%*	*6.7%*	*-10.0%*
3. Competitive impact on net written premiums:		*n/a*	*n/a*	*n/a*	*n/a*	*n/a*
4. Other impact on net written premiums:		*n/a*	*3.9%*	*4.9%*	*3.2%*	*2.7%*
5. Final expected net written premiums = (1) * [1+ (2)] * [1 + (4)]	*18,656*	*18,656*	*17,441*	*17,842*	*19,648*	*18,161*
Net Loss & ALAE Ratio, net of subrogation/salvage						
6. Net loss & ALAE ratio, prior to rate, inflation impacts		*71.5%*	*71.6%*	*82.4%*	*75.5%*	*79.0%*
7. Impact due to premium rate changes		*0.0%*	*9.5%*	*3.6%*	*-5.2%*	*8.4%*
8. Inflationary impact		*n/a*	*2.3%*	*2.3%*	*1.2%*	*1.2%*
9. Other impact		*n/a*	*n/a*	*n/a*	*n/a*	*n/a*
10. Final expected loss & ALAE ratio = (6) * [1+(7)] * [1 + (8)]		*71.5%*	*80.2%*	*87.1%*	*72.6%*	*86.7%*
Inflation and interest rate information						
11. Inflation rate that is implicitly embedded in premium growth levels		*3.2%*	*3.2%*	*3.2%*	*3.2%*	*3.2%*
12. Inflation rate that is implicitly embedded in loss payout pattern		*4.7%*	*4.7%*	*4.7%*	*4.7%*	*4.7%*
13. Risk free interest rate underlying asset valuations in current scenario		*7.3%*	*9.2%*	*6.0%*	*5.1%*	*3.9%*
Premium earning information						
14. Percent of written premium that is earned during first twelve months		*86.5%*	*86.5%*	*86.5%*	*86.5%*	*86.5%*
15. Percent of written premium that is earned during second twelve months		*13.5%*	*13.5%*	*13.5%*	*13.5%*	*13.5%*

Exhibit 3 Formula explanations:

2.	**Rate impact on net written premiums:** Model is assuming an expected loss ratio of 81.9%. If the prior year's loss ratio is less than 81.9%, a rate decrease is implemented. The rate decrease is the lesser of a 10% decrease and the difference between the prior year's loss ratio and 81.9%. For example, the projected 1997 loss ratio equals 71.5%, so the rate change in 1998 is the smaller of -10% and (71.5% / 81.9%), or -12.7%. A similar formula exists for rate increases if the prior year's loss ratio exceeds 81.9%. The 81.9% expected loss ratio was derived from the ten year average industry loss and ALAE ratio. The 10% cap was implemented based on judgment.
3.	**Competitive impact on net written premiums:** this premium adjustment element is not being used in this example. It could be used to incorporate an underwriting cycle element in pricing.
4.	**Other impact on net written premiums:** this is being used to quantify the impact of wage inflation on premium levels. Based on Bureau of Labor Statistics data, wage inflation has averaged 3.2% over the past ten years. For example purposes, it was assumed that the risk-free interest rate over the past ten years has averaged 6.0%. The projected wage inflation impact is equal to [(3.2% / 6.0%) * prior year projected risk free interest rates in (13)]. Example: 1999 premium inflation impact = (3.2% / 6.0%) * 9.2% = 4.9%.
6.	**Net loss and ALAE ratio, prior to rate, inflation impacts:** this is a stochastically generated loss ratio. The distributional parameters were developed from historical industry loss ratios. This is the underlying "force of loss" that is associated with the policies being earned during the year. This is the loss ratio that would develop, absent any other influences on the loss ratio, such as premium rate changes, premium inflation, and loss inflation.
7.	**Impact on loss ratio from premium rate changes:** $$\frac{1}{\left\{\left[\left(1+\text{current year rate change \% from (2)}\right)*(14)\right]+\left[\left(1+\text{prior year rate change \% from(2)}\right)*(15)\right]\right\}}-1$$ Example: 1998 impact = 1 / { [(1 - 2.5%)*86.5%] + [(1 - 10.0%) * 13.5%] } - 1 = 3.6%.
8.	**Inflationary impact on loss ratio:** This is the result of inflationary pressure on loss costs, partly offset by inflationary increases in premium volume. Based on Bureau of Labor Statistics data, loss inflation has averaged 4.6% over the past ten years. Again, for example purposes, it was assumed that the historical risk free interest rate over the past ten years has averaged 6.0%. Formula: $$\frac{\left\{\left[\left(4.7\%/6.0\%\right)*\text{prior year risk free interest rate in (13)}\right]+1\right\}}{\left\{\left[\left(1+\text{current year prem inflation \% from (4)}\right)*(14)\right]+\left[\left(1+\text{prior year prem inflation \% from (4)}\right)*(15)\right]\right\}}-1$$ Example: 1999 impact = $\frac{\left\{\left[\left(4.7\%/6.0\%\right)*9.2\%\right]+1\right\}}{\left\{\left[\left(1+4.9\%\right)*(86.5\%)\right]+\left[\left(1+3.9\%\right)*(13.5\%)\right]\right\}}-1=2.3\%$
9.	**Other impact on net written premiums:** this element is not being used in this example. It could be used as the counterpart to item (3) in the premium development calculation.
11.	**Inflation rate that is implicitly embedded in the premium growth levels:** this is the ten year average wage inflation statistic from the Bureau of Labor Statistics Producer Price Index.
12.	**Inflation rate that is implicitly embedded in the loss payout pattern:** this is an average of the ten year average wage inflation statistic from the Bureau of Labor Statistics Producer Price Index and the medical care inflation index from the Bureau of Labor Statistics Consumer Price Index.
13.	**Risk free interest rate underlying asset valuations in current scenario:** this is a stochastically generated future interest rate path, based on a one factor mean-reverting interest rate model.
14.	**Percent of written premium that is earned in first twelve months:** this was calculated from industry statistics in Best's Aggregates and Averages, 1996 edition.
15.	**Percent of written premium that is earned in second twelve months:** the complement of (14).

Accordion effect in payout patterns

The basic payout pattern is the expected pattern for a line of business. For example the basic pattern for homeowners as set forth by the Internal Revenue Service in their 1996 publication of industry tax discount factors was:

Incr:	66.9%	23.6%	2.9%	2.3%	1.7%	1.2%	0.5%	0.4%	0.2%	0.1%	0.1%	0.1%
Cum:	66.9%	90.5%	93..4%	95.7%	97.4%	98.6%	99.1%	99.5%	99.7%	99.8%	99.9%	100.0%

The accordion effect arises when volatility is allowed to occur within the incremental payout percentages. For example, one iteration of the model used in this paper gave rise to the following incremental volatility amounts for the first five homeowners payout increments:

3.8%	-3.6%	0.8%	-0.1%	-0.1%

Combining the basic pattern with the incremental volatility, we derive a new payout pattern, one that may or may not add up to 100%.

Incr:	70.7%	20.0%	3.7%	2.2%	1.6%	1.2%	0.5%	0.4%	0.2%	0.1%	0.1%	0.1%
Cum:	70.7%	90.7%	94.4%	96.6%	98.2%	99.4%	99.9%	100.3%	100.5%	100.6%	100.7%	100.8%

The following table displays the difference in the projected calendar year 1996 payouts for homeowners. The reserve levels were taken from Best's Aggregates and Averages, 1996 edition.

Accident Year	Reserve	1996 payout, based on baseline payout pattern	1996 payout, based on revised payout pattern
1991	287	132	132
1992	470	186	179
1993	628	219	216
1994	1,367	417	501
1995	4,862	3,467	3,230

The formulas for the accident year 1995 payouts are as follows:

Baseline: 4,862 * 23.6% / (100% - 66.9%) = 3,467
Revised: 4,862 * 20.0% / (100.8% - 70.7%) = 3,230

The accordion effect does not change the overall amount that will ultimately be paid out. It does, however, shift when the payouts will occur.

Producer Price Index and Consumer Price Index economic inflationary data

	Cars	Change	Med Care	Change	Average Change
Dec-87	112.2		130.1		
Dec-88	116.5	3.8%	138.6	6.5%	5.2%
Dec-89	119.0	2.1%	149.3	7.7%	4.9%
Dec-90	124.1	4.3%	162.8	9.0%	6.7%
Dec-91	127.9	3.1%	177.0	8.7%	5.9%
Dec-92	128.7	0.6%	190.1	7.4%	4.0%
Dec-93	132.9	3.3%	201.4	5.9%	4.6%
Dec-94	135.7	2.1%	211.0	4.8%	3.4%
Dec-95	138.0	1.7%	220.5	4.5%	3.1%
Dec-96	137.0	-0.7%	228.2	3.5%	1.4%
				Average:	**4.4%**

Example of data retrieval from Bureau of Labor Statistics Web site

Consumer Price Index-All Urban Consumers

Series Catalog:

Series ID : CUUR0000SA5

Not Seasonally Adjusted
Area : U.S. City Average
Item : Medical care
Base Period : 1982-84=100

Data:

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
1987	126.6	127.4	128.1	128.7	129.2	129.9	130.7	131.2	131.7	132.3	132.8	133.1	130.1
1988	134.4	135.5	136.3	136.9	137.5	138.2	139.3	139.9	140.4	141.2	141.8	142.3	138.6
1989	143.8	145.2	146.1	146.8	147.5	148.5	149.7	150.7	151.7	152.7	153.9	154.4	149.3
1990	155.9	157.5	158.7	159.8	160.8	161.9	163.5	165.0	165.8	167.1	168.4	169.2	162.8
1991	171.0	172.5	173.7	174.4	175.2	176.2	177.5	178.9	179.7	180.2	181.8	182.6	177.0
1992	184.3	186.2	187.3	188.1	188.7	189.4	190.7	191.5	192.3	193.3	194.3	194.7	190.1
1993	196.4	198.0	198.6	199.4	200.5	201.1	202.2	202.9	203.3	204.4	204.9	205.2	201.4
1994	206.4	207.7	208.3	209.2	209.7	210.4	211.5	212.2	212.8	214.0	214.7	215.3	211.0
1995	216.6	217.9	218.4	218.9	219.3	219.8	220.8	221.6	222.1	222.9	223.5	223.8	220.5
1996	225.2	226.2	226.6	227.0	227.4	227.8	228.7	229.2	229.4	230.1	230.5	230.6	228.2
1997	231.8	232.7	233.4										

Industry Private Passenger Automobile Paid Loss & ALAE data from 1996 Best's Aggregates & Averages

PAID Percent of Ultimate Loss:

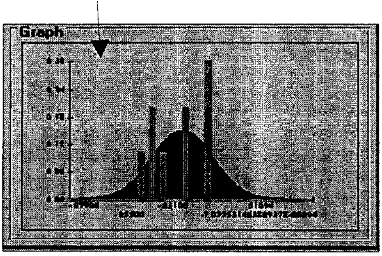
	Age in Months									
AY	12	24	36	48	60	72	84	96	108	120
1986	32.8%	65.1%	81.2%	90.2%	95.0%	97.4%	98.6%	99.2%	99.5%	99.7%
1987	32.6%	65.3%	81.3%	90.2%	95.0%	97.3%	98.6%	99.2%	99.5%	
1988	33.3%	66.3%	82.2%	90.9%	95.4%	97.7%	98.7%	99.2%		
1989	33.4%	66.7%	82.6%	91.1%	95.6%	97.7%	98.6%			
1990	34.6%	67.6%	83.3%	91.6%	95.7%	97.5%				
1991	35.0%	67.9%	83.7%	91.6%	95.5%					
1992	35.8%	68.9%	83.8%	91.3%						
1993	36.3%	69.0%	83.4%							
1994	37.0%	68.4%								
1995	36.2%									
Selected	36.2%	68.4%	83.4%	91.3%	95.5%	97.5%	98.6%	99.2%	99.5%	99.7%
Incremental										
Selected	36.2%	32.2%	15.0%	7.9%	4.2%	2.0%	1.1%	0.6%	0.3%	0.2%

Incremental Volatility

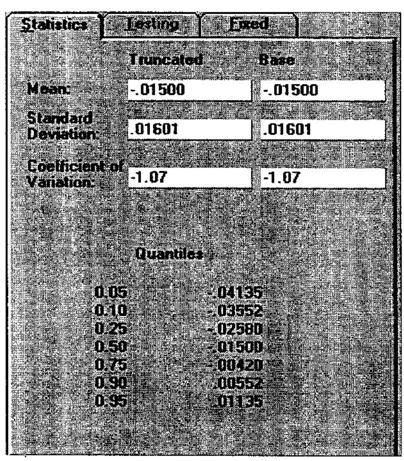
(actual incremental percentage paid amount minus selected incremental payout percentage)

AY	12	24	36	48	60	72	84	96	108	120
1986	-0.034	0.001	0.011	0.011	0.006	0.004	0.001	0	0	0
1987	-0.036	0.005	0.01	0.01	0.006	0.003	0.002	0	0	
1988	-0.029	0.008	0.009	0.008	0.003	0.003	-0.001	-0.001		
1989	-0.028	0.011	0.009	0.006	0.003	0.001	-0.002			
1990	-0.016	0.008	0.007	0.004	-0.001	-0.002				
1991	-0.012	0.007	0.008	0	-0.003					
1992	-0.004	0.009	-0.001	-0.004						
1993	0.001	0.005	-0.006							
1994	0.008	-0.008								
1995	0									

Fitting a Normal distribution to the empirical data set.



Fitting Parameters



Example of the parameter development for Private Passenger Automobile Loss Reserve Volatility

① E&Y Projection of Industry Ultimates based on historical paid loss and ALAE

AY	@12/92	@12/93	@12/94	@12/95	Industry Held Ult at 12/95
1986	26,406,610	26,427,683	26,429,159	26,413,038	26,500,698
1987	29,548,320	29,616,034	29,619,906	29,615,379	29,704,772
1988	32,656,743	32,729,085	32,711,916	32,686,971	32,774,500
1989	35,843,963	35,983,377	36,002,069	35,951,523	36,055,351
1990	38,330,949	38,498,661	38,479,624	38,388,591	38,612,934
1991	37,670,396	38,082,996	38,086,500	37,966,123	38,419,836
1992	39,615,561	40,370,102	40,291,377	40,120,162	41,011,278
1993		43,200,096	43,431,530	43,097,655	44,613,169
1994			47,556,950	46,570,975	48,197,320
1995				49,454,854	50,571,974

② E&Y Projection of Industry Ultimates based on historical incurred loss and ALAE

AY	@12/92	@12/93	@12/94	@12/95	Industry Held Ult at 12/95
1986	26,435,231	26,446,182	26,446,393	26,435,063	26,500,698
1987	29,585,085	29,629,415	29,642,891	29,631,640	29,704,772
1988	32,684,503	32,713,484	32,710,846	32,686,419	32,774,500
1989	35,848,550	35,939,792	35,949,356	35,907,663	36,055,351
1990	38,250,686	38,379,424	38,369,401	38,330,857	38,612,934
1991	37,747,473	37,976,174	37,937,228	37,856,009	38,419,836
1992	39,716,251	40,143,201	40,067,955	39,978,163	41,011,278
1993		43,058,315	43,013,254	42,859,799	44,613,169
1994			46,089,101	45,707,212	48,197,320
1995				47,001,834	50,571,974

③ Retrospective Reserve Levels (12/31/95 Held Ultimate - Paid at 12/31/XX)

AY	@12/92	@12/93	@12/94	@12/95
1986	453,854	290,590	208,933	175,515
1987	883,751	492,229	328,722	243,350
1988	1,595,112	851,005	508,254	356,636
1989	3,321,067	1,699,822	939,433	593,635
1990	6,643,782	3,454,263	1,874,086	1,169,235
1991	12,636,618	6,657,484	3,637,568	2,171,259
1992	26,667,636	13,380,267	7,407,071	4,371,781
1993		28,971,671	14,886,787	8,668,443
1994			30,978,332	16,322,168
1995				32,665,810

④ (E&Y Projection of Industry Ultimates - 12/31/95 Industry Held Ultimate) / Retrospective Reserve Level

	12	24	36	48	60	72	84	96	108	120
1995 paid	-3.4%									
1995 inc	-10.9%									
1994 paid	-2.1%	-10.0%								
1994 inc	-6.8%	-15.3%								
1993 paid	-4.9%	-7.9%	-17.5%							
1993 inc	-5.4%	-10.7%	-20.2%							
1992 paid	-5.2%	-4.8%	-9.7%	-20.4%						
1992 inc	-4.9%	-6.5%	-12.7%	-23.6%						
1991 paid		-5.9%	-5.1%	-9.2%	-20.9%					
1991 inc		-5.3%	-6.7%	-13.3%	-26.0%					
1990 paid			-4.2%	-3.3%	-7.1%	-19.2%				
1990 inc			-5.5%	-6.8%	-13.0%	-24.1%				
1989 paid				-6.4%	-4.2%	-5.7%	-17.5%			
1989 inc				-6.2%	-6.8%	-11.3%	-24.9%			
1988 paid					-7.4%	-5.3%	-12.3%	-24.5%		
1988 inc					-5.6%	-7.2%	-12.5%	-24.7%		
1987 paid						-17.7%	-18.0%	-25.8%	-36.7%	
1987 inc						-13.5%	-15.3%	-18.8%	-30.1%	
1986 paid							-20.7%	-25.1%	-34.2%	-49.9%
1986 inc							-14.4%	-18.8%	-26.0%	-37.4%

⑤ Loss Reserve Volatility

	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72	Age 84	Age 96	Age 108	Age 120
Paid Obs 1	-3.4%	-10.0%	-17.5%	-20.4%	-20.9%	-19.2%	-17.5%	-24.5%	-36.7%	-49.9%
Inc Obs 1	-10.9%	-15.3%	-20.2%	-23.6%	-26.0%	-24.1%	-24.9%	-24.7%	-30.1%	-37.4%
Paid Obs 2	-2.1%	-7.9%	-9.7%	-9.2%	-7.1%	-5.7%	-12.3%	-25.8%	-34.2%	
Inc Obs 2	-6.8%	-10.7%	-12.7%	-13.3%	-13.0%	-11.3%	-12.5%	-18.8%	-26.0%	
Paid Obs 3	-4.9%	-4.8%	-5.1%	-3.3%	-4.2%	-5.3%	-18.0%	-25.1%		
Inc Obs 3	-5.4%	-6.5%	-6.7%	-6.8%	-6.8%	-7.2%	-15.3%	-18.8%		

Fitting a Normal distribution to the Age 24 reserve volatility

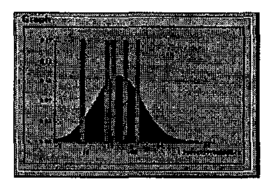


Exhibit 8

Sample Model Output: Beginning Balance Sheet

Insurance Industry Composite
Calendar year 1996 - Actual Results

Balance Sheet Assets (000):	
Bonds	*430,860*
Taxable bonds maturing < 1 year	*15,091*
Taxable bonds maturing 1-5 years	*96,036*
Taxable bonds maturing 5-10 years	*76,423*
Taxable bonds maturing 10-20 years	*26,808*
Taxable bonds maturing 20+ years	*22,964*
Tax-exempt bonds maturing < 1 year	*7,279*
Tax-exempt bonds maturing 1-5 years	*35,285*
Tax-exempt bonds maturing 5-10 years	*55,779*
Tax-exempt bonds maturing 10-20 years	*73,482*
Tax-exempt bonds maturing 20+ years	*21,714*
Stocks	
Preferred stocks	*11,694*
Common stocks	*122,377*
Mortgage loans on real estate	*2,818*
Real estate	
Properties occupied by the company	*7,201*
Other properties	*1,688*
Collateralized Mortgage Obligations	*32,243*
Cash on hand and on deposit	*4,851*
Short-term investments	*37,534*
Other invested assets	*9,885*
Aggregate write-ins for other invested assets	*498*
Subtotal, cash and invested assets	**661,648**
Agents balances < 90 days past due	*16,570*
Premiums booked but not yet due	*32,494*
Accrued retrospective premiums	*6,264*
Funds held by reinsured companies	*3,856*
Bills receivable, taken for premiums	*935*
Reinsurance recoverable on paid losses	*10,711*
Federal income tax recoverable	*0*
Computer equipment	*2,130*
Interest receivable	*8,546*
Receivable from parent, subsidiary, affiliate	*8,065*
Equities and deposits in pools and assoc.	*2,571*
Receivables relating to uninsured A&H plans	*45*
Aggregate write-ins for other assets	*7,635*
Total Net Admitted Assets	**761,473**

Balance Sheet Liabilities (000):	
Loss reserves net of subrogation and salvage	
Reserves gross of discount	*310,240*
Less tabular and nontabular discount	*12,559*
Allocated loss adjustment expense reserves	
Reserves gross of discount	*48,468*
Less tabular and nontabular discount	*278*
Reins. payable on paid Loss & LAE	*2,352*
Unallocated loss adjustment expenses	*13,740*
Contingent commissions	*2,279*
Other expenses (excl. taxes, licenses, fees)	*6,319*
Taxes, licenses, and fees	*2,560*
Federal and foreign income taxes	*716*
Borrowed money	*1,812*
Interest	*49*
Unearned premium	*103,852*
Dividends declared and unpaid	
To stockholders	*296*
To policyholders	*1,546*
Funds held under reinsurance treaties	*8,914*
Amounts retained for account of others	*4,529*
Provision for reinsurance	*3,453*
XS of statutory over statement reserves	*1,434*
Net adjust. due to foreign exchange rates	*623*
Drafts outstanding	*4,914*
Payable to parent, subsidiary, affiliates	*6,278*
Payable for securities	*1,935*
Liability for $ held under uninsured A&H	*0*
Other liabilities	*17,999*
Total liabilities	**531,472**
Write-ins for special surplus funds	*18,277*
Common capital stock	*7,367*
Preferred capital stock	*1,682*
Write-ins for other than special surplus funds	*508*
Surplus notes	*3,087*
Gross paid in and contributed surplus	*91,883*
Unassigned funds (surplus)	*107,506*
Less treasury stock, at cost	
Shares common	*282*
Shares preferred	*28*
Policyholders Surplus	**230,001**
Total Liability + Surplus	**761,473**

Exhibit 9

Sample Model Output: Base Case Balance Sheet at end of Five Years

Insurance Industry Composite
Calendar year 2001 Baseline Projection

Balance Sheet ***Assets:***	
Bonds	1,215,673
Taxable bonds maturing < 1 year	175,744
Taxable bonds maturing 1-5 years	167,062
Taxable bonds maturing 5-10 years	194,129
Taxable bonds maturing 10-20 years	71,244
Taxable bonds maturing 20+ years	60,640
Tax-exempt bonds maturing < 1 year	57,238
Tax-exempt bonds maturing 1-5 years	81,347
Tax-exempt bonds maturing 5-10 years	141,321
Tax-exempt bonds maturing 10-20 years	205,661
Tax-exempt bonds maturing 20+ years	61,286
Stocks	
Preferred stocks	33,720
Common stocks	352,869
Mortgage loans on real estate	8,124
Real estate	
Properties occupied by the company	7,201
Other properties	4,867
Collateralized mortgage obligations	19,651
Cash on hand and on deposit	(43,331)
Short-term investments	108,227
Other invested assets	28,501
Aggregate write-ins for invested assets	498
Subtotal, cash and invested assets	**1,736,002**
Agents' balances or uncollected premium	
Agents balances < 90 days past due	9,830
Premiums booked but not yet due	19,275
Accrued retrospective premiums	3,716
Funds held by reinsured companies	3,856
Bills receivable, taken for premiums	915
Reinsurance recoverable on paid losses	10,791
Federal income tax recoverable	0
Computer equipment	2,130
Interest receivable	18,942
Receivable from parent, subsidiary, affiliate	8,065
Equities and deposits in pools and assoc.	2,571
Receivables relating to uninsured A&H plans	45
Aggregate write-ins for other assets	7,635
Total Net Admitted Assets	**1,823,775**

Balance Sheet ***Liabilities:***	
Loss reserves net of subrogation and salvage	
Reserves gross of discount	469,824
Less tabular and nontabular discount	19,018
Allocated loss adjustment expense reserves	
Reserves gross of discount	60,995
Less tabular and nontabular discount	365
Reins. payable on paid Loss & LAE	2,352
Unallocated loss adjustment expenses	19,123
Contingent commissions	865
Other expenses (excl. taxes, licenses, fees)	4,803
Taxes, licenses, and fees	1,289
Federal and foreign income taxes	9,865
Borrowed money	1,812
Interest	49
Unearned premium	125,308
Dividends declared and unpaid	
To stockholders	(296)
To policyholders	2,559
Funds held under reinsurance treaties	8,914
Amounts retained for account of others	4,529
Provision for reinsurance	3,491
XS of statutory over statement reserves	736
Net adjust. due to foreign exchange rates	623
Drafts outstanding	4,914
Payable to parent, subsidiary, affiliates	6,278
Payable for securities	1,935
Liability for $ held under uninsured A&H	0
Other liabilities	17,999
Total liabilities	**728,582**
Write-ins for special surplus funds	18,277
Common capital stock	7,367
Preferred capital stock	1,682
Write-ins for other than special surplus funds	508
Surplus notes	3,087
Gross paid in and contributed surplus	91,883
Unassigned funds (surplus)	972,698
Less treasury stock, at cost	
Shares common	282
Shares preferred	28
Policyholders Surplus	**1,095,193**
Total Liability + Surplus	**1,823,775**

Exhibit 10

Graphical displays of some output metrics produced by the corporate financial model, based on 250 iterations using dynamic variable parameters as described in the paper.