

*Loss Reserve Testing: Beyond Popular
Methods*

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LOSS RESERVE TESTING: BEYOND POPULAR METHODS

ABSTRACT

There are a number of popular actuarial methods in wide use which estimate ultimate claims costs from data in loss development triangle format. The typical actuarial reserve analysis shows the application of several methods to the data, with little other description of the nature of the world. The popular methods rely on assumptions that may not be consistent with the facts in any given case. In particular, the popular methods assume that most of the drivers of loss costs do not change from year to year, an assumption that more often than not is clearly violated.

Loss reserve estimates can be tested against the data they are designed to reflect. An actuarial report showing the results of these tests is sufficient for the purposes of the independent audit (of the insurance entity's reserves or the work of the individual who prepared the report). Loss reserve estimates which do not pass the tests can be revised as appropriate. Loss reserve estimates that pass all of the tests—or rather, strike a balance between the conflicting indications of various tests—are more robust than estimates made using the popular methods. They are less likely to be unreasonable because of incorrect assumptions. Also, at least in most cases, such estimates tend to be more stable from year to year than estimates based on the popular methods.

KEYWORDS

Loss reserves; IBNR; financial statements; audit; actuarial studies; insurance; reviewing the work of another professional.

LOSS RESERVE TESTING: BEYOND POPULAR METHODS

"Our lives teem with numbers, but we sometimes forget that numbers are only tools. They have no soul; they may indeed become fetishes. Many of our most critical decisions are made by computers, contraptions that devour numbers like voracious monsters and insist on being nourished with ever-greater quantities of digits to crunch, digest, and spew back."

- Peter L. Bernstein

I. Introduction

Who of us remembers the time when a "state-of-the-art" actuarial reserve analysis did not have paid and incurred loss development projections as its foundation. Occasionally, a student will seem to believe that if only we pick the correct development factors, everything will work out.

In fact, in a recent review of a client's reserves by a governmental agency, the reviewer used Coopers & Lybrand's ExhibitMaker software to generate paid and incurred loss development projections. This particular client had been affected by a large number of claims resulting from a catastrophe. Total claims costs for the year were about three times as large as other years. The reviewer, in selecting age-to-age link ratios, selected a figure well below unity for the first factors, both paid and incurred, to affect the year. The reviewer in turn selected large figures for the first link ratios to affect the following year. The selected age-to-age factors had no relationship to the observed historical factors, but the cumulative factors which resulted yielded the reviewer's perceived "correct answer". Among the readers of this article, we could no doubt find stories of similar abuses of our popular methods.

The typical actuarial reserve analysis starts with rote projections based on several actuarial methods. Based on those rote projections, a "best estimate" of the ultimate claims costs is selected. Section II of this article presents such a "typical" actuarial reserve analysis.

Section III explores the notion that the variation among the estimates of the different methods suggests a bias exists in each of the various methods. We suggest a framework for qualitatively studying changes in the underlying data which will affect the rote projections.

Section IV expands the framework into a series of "tests" for assessing the validity of a set of point estimates of ultimate loss by year (and the reserves derived from those estimates). Reserve analyses seem usually to "reinvent the wheel" in that new projections are made of the ultimate loss at each valuation. The estimates derived in the previous analysis are typically disregarded. The framework we suggest can be used to determine how the prior estimates might be revised, rather than "starting from scratch".

Loss Reserve Testing: Beyond Popular Methods

II. Popular Methods: A “Typical” Actuarial Reserve Analysis

This section illustrates a “typical” actuarial reserve analysis using simulated loss development triangles. The simulation was patterned after workers’ compensation insurance, but the following discussion should apply to any property-casualty line of insurance. The details of the data creation appear in Appendix B. The simulated data was presented to the authors as if a “client” had given data to a “consultant” to prepare an actuarial reserve analysis. The authors performed the analysis in the following three sections with no knowledge of the simulation parameters or the “true” ultimate claims costs.

A typical actuarial reserve analysis might consist of paid and incurred loss development projections and Bornhuetter-Ferguson projections (incurred, paid or both). If claim counts are available, frequency and severity projections might be included as well. The estimate presented as the “best estimate” is often a simple average of the various rote projections.

More recently, actuarial reserve reports have begun to include a “high estimate” and a “low estimate”, suggesting a range of estimates rather than a “best” point estimate. It has been suggested by some that the projections of the various methods, some higher than others, imply an appropriate range of estimates.

Others suggest a range by selecting “high” and “low” development factors—essentially performing separate “high” and “low” reserve analyses.

Often, expected severities (for frequency and severity projections), and expected loss ratios (for Bornhuetter-Ferguson projections), are based on the estimates of ultimate loss projected by paid and incurred development techniques on the more mature experience periods; thus, the methods are not independent.

The loss development methods require selection of a “tail factor”, or the loss development factor which projects emerged costs from the most mature review to their final value. The tail factor is in a sense the most important factor selection in that it is the only factor applied to all accident years, yet neither the paid method nor the incurred method includes a specific means by which the tail factor is to be determined.

Loss Reserve Testing: Beyond Popular Methods
II. Popular Methods: A “Typical” Actuarial Reserve Analysis

A reserve analysis might begin with the data shown in Exhibits 1, 2 and 3 below.

Exhibit 1

Accident Year	Cumulative Reported Incurred Loss (000 omitted)									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	94,034	118,649	111,701	105,055	101,381	99,628	98,960	98,965	99,123	99,278
1988	113,591	145,453	138,283	132,700	129,357	127,609	127,449	127,925	128,576	
1989	117,091	152,923	148,576	144,059	140,958	140,378	141,219	142,889		
1990	111,197	146,551	143,673	139,333	137,899	139,106	142,632			
1991	91,719	119,145	116,371	113,857	114,684	120,147				
1992	81,092	106,480	104,179	104,038	112,559					
1993	68,314	92,509	92,981	104,662						
1994	67,456	90,731	108,351							
1995	73,136	126,782								
1996	110,396									

Exhibit 2

Accident Year	Cumulative Paid Loss (000 omitted)									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	6,437	19,949	32,633	48,974	68,096	83,364	92,523	96,814	98,482	99,065
1988	8,081	24,876	40,878	62,619	87,681	107,425	119,293	125,093	127,502	
1989	9,490	28,283	46,947	72,330	100,318	121,443	133,644	139,423		
1990	9,363	27,336	46,166	72,392	101,230	122,542	134,511			
1991	8,643	24,363	40,767	63,575	87,978	105,458				
1992	8,100	23,239	39,327	61,408	84,217					
1993	7,517	21,695	37,316	58,693						
1994	7,807	22,314	38,410							
1995	9,033	26,319								
1996	10,542									

Exhibit 3

Accident Year	Case Reserves (000 omitted)									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	87,598	98,699	79,068	56,081	33,285	16,264	6,437	2,151	641	213
1988	105,510	120,577	97,405	70,081	41,675	20,184	8,156	2,832	1,074	
1989	107,600	124,640	101,629	71,729	40,640	18,935	7,574	3,466		
1990	101,834	119,215	97,507	66,941	36,670	16,564	8,121			
1991	83,076	94,782	75,604	50,282	26,706	14,689				
1992	72,991	83,241	64,852	42,630	28,341					
1993	60,797	70,814	55,664	45,969						
1994	59,649	68,418	69,941							
1995	64,103	100,463								
1996	99,854									

Loss Reserve Testing: Beyond Popular Methods

II. Popular Methods: A "Typical" Actuarial Reserve Analysis

Based on this data, incurred and paid loss development factors are calculated in Exhibits 4 and 5, respectively.

In this reserve analysis, we selected factors close to the three-year average. We assumed that case reserves are adequate by the tenth review, implying an incurred tail factor of 1.000. The paid tail factor was determined by dividing the 1987 reported incurred amount at the tenth review by the corresponding paid amount.

Exhibit 4

Accident Year	Incurred Loss Development									
	Review									
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	1.262	0.941	0.941	0.965	0.983	0.993	1.000	1.002	1.002	
1988	1.280	0.951	0.960	0.975	0.986	0.999	1.004	1.005		
1989	1.306	0.972	0.970	0.978	0.996	1.006	1.012			
1990	1.318	0.980	0.970	0.990	1.009	1.025				
1991	1.299	0.977	0.978	1.007	1.048					
1992	1.313	0.978	0.999	1.082						
1993	1.354	1.005	1.126							
1994	1.345	1.194								
1995	1.734									
All-years Averages										
Arithmetic	1.357	1.000	0.992	1.000	1.004	1.006	1.005	1.003	1.002	
Volume	1.344	0.991	0.986	0.997	1.004	1.007	1.006	1.004	1.002	
Three-year Averages										
Arithmetic	1.478	1.059	1.034	1.026	1.017	1.010	1.005	1.003	1.002	
Volume	1.484	1.055	1.029	1.022	1.015	1.010	1.006	1.004	1.002	
Selected	1.400	1.050	1.030	1.020	1.015	1.010	1.005	1.003	1.002	1.000
Cumulative	1.599	1.142	1.088	1.056	1.035	1.020	1.010	1.005	1.002	1.000
Percent of Ultimate	62.53%	87.53%	91.93%	94.68%	96.58%	98.03%	99.01%	99.50%	99.80%	100.00%

Exhibit 5

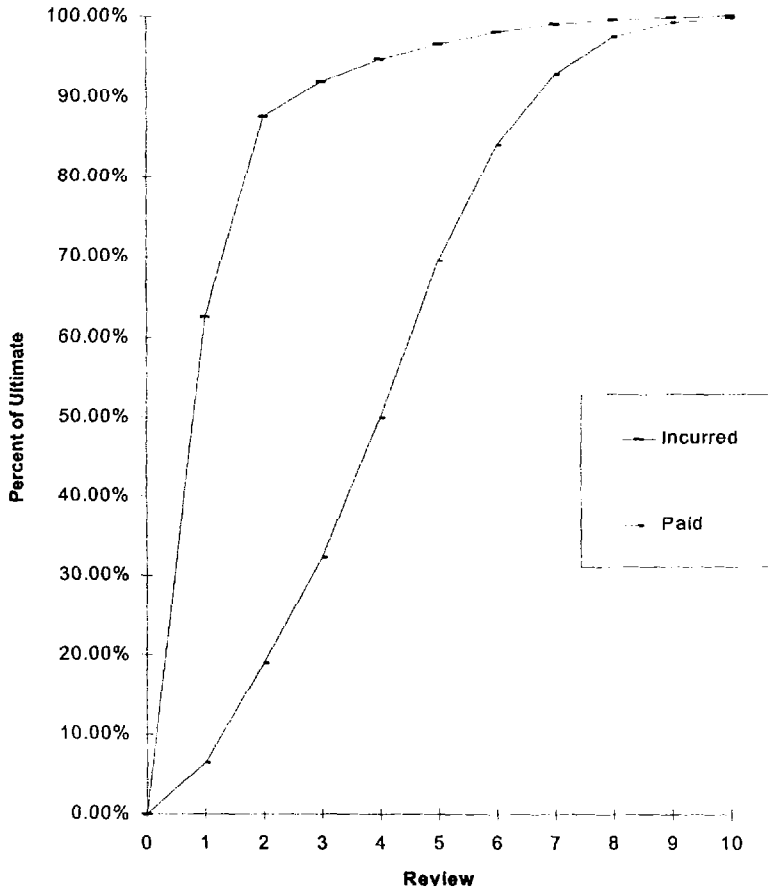
Accident Year	Paid Loss Development									
	Review									
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	3.099	1.636	1.501	1.390	1.224	1.110	1.046	1.017	1.006	
1988	3.078	1.643	1.532	1.400	1.225	1.110	1.049	1.019		
1989	2.980	1.660	1.541	1.387	1.211	1.100	1.043			
1990	2.920	1.689	1.568	1.398	1.211	1.098				
1991	2.819	1.673	1.559	1.384	1.199					
1992	2.869	1.692	1.561	1.371						
1993	2.886	1.720	1.573							
1994	2.858	1.721								
1995	2.914									
All-years Averages										
Arithmetic	2.936	1.679	1.548	1.389	1.214	1.105	1.046	1.018	1.006	
Volume	2.932	1.679	1.549	1.389	1.213	1.104	1.046	1.018	1.006	
Three-year Averages										
Arithmetic	2.886	1.711	1.565	1.385	1.207	1.103	1.046	1.018	1.006	
Volume	2.887	1.711	1.564	1.385	1.207	1.103	1.046	1.018	1.006	
Selected	2.900	1.700	1.550	1.390	1.210	1.105	1.050	1.018	1.006	1.002
Cumulative	15.304	5.277	3.104	2.003	1.441	1.191	1.078	1.026	1.008	1.002
Percent of Ultimate	6.53%	18.95%	32.21%	49.93%	69.40%	83.98%	92.80%	97.44%	99.19%	99.79%

Loss Reserve Testing: Beyond Popular Methods
II. Popular Methods: A "Typical" Actuarial Reserve Analysis

The popular loss development methods use the selected factors and, assuming that each selected average link ratio will repeat for each year into the future, project ultimate loss estimates. The development patterns derived from the selected factors in Exhibits 4 and 5 are displayed graphically in Exhibit 6.

Exhibit 6

Paid and Incurred Development Patterns



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The development patterns are applied to the most recent reported incurred loss and paid loss in Exhibits 7 and 8, respectively.

Incurred Loss Development					Exhibit 7
					(4)
					(1)
					(2)
					(3)
					(4)
Accident Year	Incurred to Date	LDF	Ultimate	Indicated Reserve	
1987	\$99,277,908	1.000	\$99,277,908	\$212,975	
1988	128,576,012	1.002	128,833,164	1,331,633	
1989	142,889,047	1.005	143,604,349	4,180,937	
1990	142,631,963	1.010	144,062,708	9,551,251	
1991	120,147,362	1.020	122,566,089	17,107,720	
1992	112,558,738	1.035	116,547,067	32,329,630	
1993	104,662,140	1.056	110,538,079	51,844,977	
1994	108,350,678	1.088	117,866,710	79,456,533	
1995	126,782,219	1.142	144,812,876	118,493,621	
1996	110,396,257	1.599	176,535,159	165,993,204	
Total	\$1,196,272,324		\$1,304,644,110	\$480,502,482	

Paid Loss Development					Exhibit 8
					(4)
					(1)
					(2)
					(3)
					(4)
Accident Year	Paid to Date	LDF	Ultimate	Indicated Reserve	
1987	\$99,064,933	1.002	\$99,277,908	\$212,975	
1988	127,501,531	1.008	128,542,295	1,040,764	
1989	139,423,412	1.026	143,091,598	3,668,186	
1990	134,511,457	1.078	144,952,932	10,441,474	
1991	105,458,369	1.191	125,577,275	20,118,906	
1992	84,217,437	1.441	121,343,745	37,126,308	
1993	58,693,102	2.003	117,548,540	58,855,438	
1994	38,410,177	3.104	119,236,215	80,826,038	
1995	26,319,255	5.277	138,894,286	112,575,031	
1996	10,541,955	15.304	161,335,504	150,793,549	
Total	\$824,141,628		\$1,299,800,296	\$475,658,668	

As in the typical reserve analysis, our factor selections did not consider information from other triangles of development data. We used some judgment in selecting the development factors, which is also typical. Such judgments affect the estimates of loss.

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In this case, we are also provided with the claim count data shown in Exhibits 9, 10 and 11.

Exhibit 9

Accident Year	Reported Claim Count										
	Review										
	1	2	3	4	5	6	7	8	9	10	
1987	7,543	8,515	8,525	8,525	8,525	8,525	8,525	8,525	8,525	8,525	8,525
1988	8,790	10,009	10,031	10,032	10,032	10,032	10,032	10,032	10,032	10,032	8,525
1989	9,417	10,582	10,609	10,610	10,610	10,610	10,610	10,610	10,610		
1990	8,637	9,705	9,724	9,726	9,726	9,726	9,726				
1991	7,438	8,372	8,387	8,387	8,387	8,387	8,387				
1992	6,406	7,205	7,220	7,221	7,221						
1993	5,894	6,615	6,634	6,636							
1994	5,840	6,546	6,568								
1995	6,448	7,301									
1996	7,259										

Exhibit 10

Accident Year	Closed Claim Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	2,007	5,166	5,422	5,444	5,782	6,490	7,340	8,019	8,346	8,468
1988	2,472	6,039	6,299	6,330	6,740	7,650	8,636	9,418	9,797	
1989	2,695	6,612	6,756	6,798	7,305	8,307	9,359	10,083		
1990	2,680	6,147	6,215	6,262	6,791	7,720	8,641			
1991	2,387	5,416	5,476	5,549	6,005	6,796				
1992	2,103	4,674	4,702	4,780	5,211					
1993	1,992	4,343	4,355	4,441						
1994	2,041	4,363	4,367							
1995	2,385	4,941								
1996	2,835									

Exhibit 11

Accident Year	Open Claim Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	5,536	3,349	3,103	3,081	2,743	2,035	1,185	506	179	57
1988	6,318	3,970	3,732	3,702	3,292	2,382	1,396	614	235	
1989	6,722	3,970	3,853	3,812	3,305	2,303	1,251	527		
1990	5,957	3,558	3,509	3,464	2,935	2,006	1,085			
1991	5,051	2,956	2,911	2,838	2,382	1,591				
1992	4,303	2,531	2,518	2,441	2,010					
1993	3,902	2,272	2,279	2,195						
1994	3,799	2,183	2,201							
1995	4,063	2,360								
1996	4,424									

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Claim count development factors appear in Exhibits 12 and 13.

Exhibit 12

Reported Claim Count Development										
Accident Year	Review									
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	1.129	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1988	1.139	1.002	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
1989	1.124	1.003	1.000	1.000	1.000	1.000	1.000	1.000		
1990	1.124	1.002	1.000	1.000	1.000	1.000				
1991	1.126	1.002	1.000	1.000	1.000					
1992	1.125	1.002	1.000	1.000						
1993	1.122	1.003	1.000							
1994	1.121	1.003								
1995	1.132									
All-years Averages										
Volume	1.127	1.002	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
Arithmetic	1.127	1.002	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
Three-year Averages										
Volume	1.125	1.003	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
Arithmetic	1.125	1.003	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
Selected	1.125	1.003	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Cumulative	1.128	1.003	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Percent of Ultimate	88.62%	99.70%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Exhibit 13

Closed Claim Count Development										
Accident Year	Review									
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	2.574	1.050	1.004	1.062	1.122	1.131	1.093	1.041	1.015	
1988	2.443	1.043	1.005	1.065	1.135	1.129	1.091	1.040		
1989	2.453	1.022	1.006	1.075	1.137	1.127	1.077			
1990	2.294	1.011	1.008	1.084	1.137	1.119				
1991	2.269	1.011	1.013	1.082	1.132					
1992	2.223	1.006	1.017	1.090						
1993	2.180	1.003	1.020							
1994	2.138	1.001								
1995	2.072									
All-years Averages										
Volume	2.294	1.018	1.010	1.076	1.133	1.126	1.087	1.041	1.015	
Arithmetic	2.298	1.019	1.010	1.076	1.133	1.126	1.086	1.040	1.015	
Three-year Averages										
Volume	2.130	1.003	1.017	1.086	1.135	1.125	1.087	1.041	1.015	
Arithmetic	2.126	1.003	1.016	1.085	1.135	1.125	1.086	1.040	1.015	
Selected	2.130	1.010	1.010	1.080	1.135	1.125	1.090	1.040	1.015	1.007
Cumulative	3.471	1.629	1.613	1.597	1.479	1.303	1.158	1.063	1.022	1.007
Percent of Ultimate	28.81%	61.37%	61.98%	62.60%	67.61%	76.74%	86.33%	94.10%	97.86%	99.33%

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The reported counts are developed to ultimate in Exhibit 14. Based on the projected ultimate counts and the average of the development projections, severities are calculated. The frequency and severity projections are performed for accident years 1991 and later. The severities are based on the older years' values and the average annual increase. Exhibit 14 shows the indicated reserve based on the frequency and severity assumptions.

										Exhibit 14
										Frequency Severity Method
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
				Average						
Accident Year	Reported Count	LDL	Ultimate Count	Development Ultimate	Indicated Severity	Percent Change	Selected Severity	Ultimate	Indicated Reserve	
1987	8,525	1.000	8,525	\$99,277,908	\$11,646					
1988	10,032	1.000	10,032	128,687,729	12,828	10.2%				
1989	10,610	1.000	10,610	143,347,974	13,511	5.3%				
1990	9,726	1.000	9,726	144,507,820	14,858	10.0%				
1991	8,387	1.000	8,387	124,071,682	14,793	-0.4%	\$15,000	\$125,805,000	\$20,346,631	
1992	7,221	1.000	7,221	118,945,406	16,472	11.3%	15,990	115,463,790	31,246,353	
1993	6,636	1.000	6,636	114,043,310	17,186	4.3%	17,045	113,112,876	54,419,774	
1994	6,568	1.000	6,568	118,551,463	18,050	5.0%	18,170	119,342,743	80,932,566	
1995	7,301	1.003	7,323	141,853,581	19,371	7.3%	19,370	141,841,514	115,522,260	
1996	7,259	1.128	8,191	168,935,331	20,625	6.5%	20,648	169,124,893	158,582,938	
Total	82,265		83,219	\$1,302,222,203		6.6%				
							Total 1991-1996	\$784,690,817	\$461,050,522	

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To round out the methods, incurred and paid Bornhuetter-Ferguson projections appear in Exhibits 15 and 16, respectively.

Our "client" tells us that premium rates were increased at the beginning of 1991. The selected expected loss ratio of 80% is based on the loss ratios on the older years and the knowledge that rate levels increased.

Exhibit 15

Incurred Bornhuetter-Ferguson									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Accident Year	Earned Premium	Average Development Ultimate	Indicated Loss Ratio	Selected Expected Loss Ratio	Expected Loss	Expected Percent Unreported	Expected Unreported	Case Reserve	Indicated Reserve
1987	\$110,537,942	\$99,277,908	89.8%						
1988	139,638,242	128,687,729	92.2%						
1989	161,016,927	143,347,974	89.0%						
1990	159,468,213	144,507,820	90.6%						
1991	153,300,139	124,071,682	80.9%	80.0%	\$122,640,111	1.97%	\$2,420,188	\$14,688,993	\$17,109,181
1992	142,966,702	118,945,406	83.2%	80.0%	114,373,362	3.42%	3,913,943	28,341,302	32,255,245
1993	141,499,363	114,043,310	80.6%	80.0%	113,199,491	5.32%	6,017,414	45,969,037	51,986,451
1994	151,634,759	118,551,463	78.2%	80.0%	121,307,807	8.07%	9,793,851	69,940,501	79,734,352
1995	178,663,700	141,853,581	79.4%	80.0%	142,930,960	12.45%	17,796,339	100,462,965	118,259,304
1996	210,691,772	168,935,331	80.2%	80.0%	168,553,418	37.47%	63,148,542	99,854,302	163,002,844
Total	\$1,549,417,760	\$1,302,222,203	84.0%						
Total 1991-1996					\$783,005,148		\$103,090,277	\$359,257,100	\$462,347,376

Exhibit 16

Paid Bornhuetter-Ferguson							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Earned Premium	Average Development Ultimate	Indicated Loss Ratio	Selected Expected Loss Ratio	Expected Loss	Expected Percent Unpaid	Indicated Reserve
1987	\$110,537,942	\$99,277,908	89.8%				
1988	139,638,242	128,687,729	92.2%				
1989	161,016,927	143,347,974	89.0%				
1990	159,468,213	144,507,820	90.6%				
1991	153,300,139	124,071,682	80.9%	80.0%	\$122,640,111	16.02%	\$19,648,339
1992	142,966,702	118,945,406	83.2%	80.0%	114,373,362	30.60%	34,993,651
1993	141,499,363	114,043,310	80.6%	80.0%	113,199,491	50.07%	56,677,910
1994	151,634,759	118,551,463	78.2%	80.0%	121,307,807	67.79%	82,230,298
1995	178,663,700	141,853,581	79.4%	80.0%	142,930,960	81.05%	115,846,791
1996	210,691,772	168,935,331	80.2%	80.0%	168,553,418	93.47%	157,539,831
Total	\$1,549,417,760	\$1,302,222,203	84.0%				
Total 1991-1996					\$783,005,148		\$466,936,819

Loss Reserve Testing: Beyond Popular Methods
II. Popular Methods: A "Typical" Actuarial Reserve Analysis

The results of the various methods are summarized in Exhibit 17. The selected point estimate for each year is essentially a rounded average of the projections of the various methods.

Exhibit 17

Comparison of Reserve Estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Incurred Loss Development	Paid Loss Development	Frequency Severity	Incurred Bornhuetter Ferguson	Paid Bornhuetter Ferguson	Average	Selected Reserve Estimate
1987	\$212,975	\$212,975	\$212,975	\$212,975	\$212,975	\$212,975	\$212,975
1988	1,331,633	1,040,764	1,186,198	1,186,198	1,186,198	1,186,198	1,200,000
1989	4,180,937	3,668,186	3,924,562	3,924,562	3,924,562	3,924,562	3,900,000
1990	9,551,251	10,441,474	9,996,363	9,996,363	9,996,363	9,996,363	10,000,000
1991	17,107,720	20,118,906	20,346,631	17,109,181	19,648,339	18,866,155	18,900,000
1992	32,329,630	37,126,308	31,246,353	32,255,245	34,993,651	33,590,237	33,000,000
1993	51,844,977	58,855,438	54,419,774	51,986,451	56,677,910	54,756,910	54,750,000
1994	79,456,533	80,826,038	80,932,566	79,734,352	82,230,298	80,635,957	80,000,000
1995	118,493,621	112,575,031	115,522,260	118,259,304	115,846,791	116,139,401	115,000,000
1996	165,993,204	150,793,549	158,582,938	163,002,844	157,539,831	159,182,473	160,000,000
Total	\$480,502,482	\$475,658,668	\$476,370,620	\$477,667,474	\$482,256,917	\$478,491,232	\$476,962,975

A summary of the loss experience as of December 31, 1996, including our selected "best estimates", appear in Exhibit 18.

Exhibit 18

Summary

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Earned Premium	Paid Loss	Case Reserve	Incurred Loss	Selected Total Reserve	Selected Ultimate	Ultimate Loss Ratio
1987	\$110,537,942	\$99,064,933	\$212,975	\$99,277,908	\$212,975	\$99,277,908	89.81%
1988	139,638,242	127,501,531	1,074,481	128,576,012	1,200,000	128,701,531	92.17%
1989	161,016,927	139,423,412	3,465,635	142,889,047	3,900,000	143,323,412	89.01%
1990	159,468,213	134,511,457	8,120,505	142,631,963	10,000,000	144,511,457	90.62%
1991	153,300,139	105,458,369	14,688,993	120,147,362	18,900,000	124,358,369	81.12%
1992	142,966,702	84,217,437	28,341,302	112,558,738	33,000,000	117,217,437	81.99%
1993	141,499,363	58,693,102	45,969,037	104,662,140	54,750,000	113,443,102	80.17%
1994	151,634,759	38,410,177	69,940,501	108,350,678	80,000,000	118,410,177	78.09%
1995	178,663,700	26,319,255	100,462,965	126,782,219	115,000,000	141,319,255	79.10%
1996	210,691,772	10,541,955	99,854,302	110,396,257	160,000,000	170,541,955	80.94%
Total	\$1,549,417,760	\$824,141,628	\$372,130,696	\$1,196,272,324	\$476,962,975	\$1,301,104,603	83.97%

Loss Reserve Testing: Beyond Popular Methods

II. Popular Methods: A “Typical” Actuarial Reserve Analysis

Thus ends a “typical” actuarial reserve analysis. The stable ultimate loss ratios and the relative consistency of the estimates promulgated by the various methods might lead us to believe that the estimates are reasonable. As often happens, however, the expected severities selected for the frequency and severity approach, and the expected loss ratios used for the Bornhuetter-Ferguson calculations, were derived from the results of the development methods. It should be no surprise, then that the latter projections are between the paid and incurred development projections. Our final estimate is thus very sensitive to, and dependent on, the selected development factors and the assumption that the selected average link ratios will repeat indefinitely into the future.

Our Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves mandates that our estimates be “derived from reasonable assumptions and appropriate actuarial methods”. The many assumptions implicit in these popular methods are discussed in Appendix A.

In this section and the next two, we explore whether or not the estimates made using the popular methods in the way done in the typical reserve analysis actually provide a scientific explanation of the loss development process, including loss reserve estimates. In this section, we have performed a “typical” reserve analysis. In the next section, we record a number of observations about the “client’s” loss development process—findings and conclusions that define the causes of changes observed in the experience. Then in Section IV we test the estimates derived in this section to see if they are consistent with the observations we discuss in Section III.

Loss Reserve Testing: Beyond Popular Methods

III. Analysis of Historical Development Data

In this section we record a number of observations from the loss experience. While the patterns in some exhibits have two or more potential explanations, often we can eliminate one or more of the potential explanations by considering other exhibits. This is rather like the children's game of Clue^{®1}, in which possibilities are eliminated as the game proceeds.

We supplement the observations with information obtained from the "client". As Berquist and Sherman² point out, estimates of loss should reflect information beyond the loss development triangles and exposure histories alone.

We do not mean to imply that the observations we make in this paper are the only observations that could have been made, or are the best that could have been made. The process of drawing findings and conclusions from complex sets of data is a highly personal one. Nonetheless, we do mean to suggest that the data have a story to tell, and that different actuaries would find very similar stories in a given set of data. Loss reserve estimates that are inconsistent with the evidence in the development history are in an important sense inferior to estimates that are consistent with that evidence.

One reason to test the estimates made by the popular methods is that those methods rely on assumptions that are often not appropriate for the problem at hand. One important set of assumptions is that the loss reserving and payment processes are not changing, even in a steady way. An important set of observations we can make from the loss development data is that the loss development processes are changing; in some problems, we can even observe something about the nature of the changes over time.

We begin by looking at the incurred and paid development factors (link ratios) in Exhibits 4 and 5. The more mature the settlement process, the closer the factors are to 1.000. This can be seen by looking left and right along any row. A change in the adequacy of reported losses or a change in the timing at which claims are paid will show up as a change from year to year (down the columns) in the incurred or paid age-to-age factors. In Exhibit 4, we observe that either the settlement process has been speeding up or the case reserves have been strengthened (or both). In Exhibit 5, we observe that the settlement pattern of immature claims sped up in 1988 to 1991, but slowed from 1991 to 1996 (first column). In addition, the settlement pattern of more mature claims slowed from year to year (second and third columns).

The changes in the settlement pattern do not appear to be due to changes in the reporting pattern. It appears that the reporting of claims has been relatively consistent, and that virtually all claims are reported within three to four years (Exhibit 12).

¹ Clue[®] is a trademark of Hasbro, Inc. for its detective game equipment.

² James R. Berquist and Richard E. Sherman, "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach", PCAS LXIV, 1977, p. 123.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Closed claim count development factors (Exhibit 13) suggest a slow down between the 1st and 2nd reviews and between the 2nd and 3rd reviews, then a speed up between the 3rd and 4th reviews and between the 4th and 5th reviews, and finally a slow down at the older reviews.

When asked to explain the apparent shift in closing patterns, our "client" tells us of a "fascinating new program instituted at the company in the late 1980's. This program reduced the time to closing for all types of claims, except Medical Only. The most dramatic decrease has been with Temporary Disability claims. Internal studies have shown the average time from reporting to closure for these claims has dropped from 350 days in 1987 to 220 days by 1994. These claims dominate the 3 to 4 and 4 to 5 development periods."

Aside from the basic data triangles and development factors, other exhibits are useful to observe the effect of changes in the claims reporting and settlement process on the data.

First, consider Exhibits 19, 20 and 21, the incremental changes in the reported incurred loss, paid loss and case reserves from Exhibits 1, 2 and 3, respectively.

Exhibit 19

Accident Year	Incremental Reported Incurred Loss (000 omitted)									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	94,034	24,614	-6,948	-6,645	-3,674	-1,753	-668	4	158	155
1988	113,591	31,862	-7,170	-5,583	-3,343	-1,747	-161	476	651	
1989	117,091	35,832	-4,347	-4,517	-3,101	-580	840	1,670		
1990	111,197	35,354	-2,878	-4,340	-1,434	1,206	3,526			
1991	91,719	27,426	-2,775	-2,514	828	5,463				
1992	81,092	25,389	-2,301	-142	8,521					
1993	68,314	24,195	472	11,681						
1994	67,456	23,275	17,619							
1995	73,136	53,647								
1996	110,396									

The incremental reported incurred loss shows significant shifts in calendar years 1995 and 1996. Where aggregate reported incurred had typically decreased over time, presumably either from favorable case reserve development or from case reserve deterioration, the decreases stopped in calendar year 1995 and reported incurred for all accident years actually increased substantially in 1996.

Loss Reserve Testing: Beyond Popular Methods III. Analysis of Historical Development Data

Exhibit 20

Accident Year	Incremental Paid Loss (000 omitted)									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	6,437	13,513	12,683	16,342	19,122	15,268	9,159	4,291	1,668	583
1988	8,081	16,795	16,002	21,741	25,062	19,744	11,868	5,800	2,408	
1989	9,490	18,792	18,664	25,383	27,988	21,126	12,201	5,779		
1990	9,363	17,973	18,830	26,226	28,837	21,312	11,970			
1991	8,643	15,719	16,404	22,808	24,404	17,480				
1992	8,100	15,138	16,088	22,081	22,810					
1993	7,517	14,177	15,622	21,377						
1994	7,807	14,506	16,096							
1995	9,033	17,286								
1996	10,542									

On the other hand, incremental paid losses do not appear to have a calendar year shift. It appears payments peaked on accident years 1989 or 1990, decreased through 1993 or 1994, then increased in 1995 and 1996.

Exhibit 21

Accident Year	Change in Case Reserves (000 omitted)									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	87,598	11,102	-19,631	-22,987	-22,796	-17,022	-9,826	-4,287	-1,510	-428
1988	105,510	15,067	-23,172	-27,324	-28,405	-21,491	-12,028	-5,324	-1,757	
1989	107,600	17,040	-23,011	-29,899	-31,089	-21,706	-11,361	-4,109		
1990	101,834	17,381	-21,708	-30,566	-30,271	-20,106	-8,443			
1991	83,076	11,706	-19,179	-25,322	-23,576	-12,017				
1992	72,991	10,250	-18,389	-22,222	-14,288					
1993	66,797	10,018	-15,150	-9,695						
1994	59,649	8,769	1,523							
1995	64,103	36,360								
1996	99,854									

Shifts are also apparent in the incremental aggregate case reserve changes. The changes are consistent with the changes in incremental incurred, and also reflect the shift in open counts observed in Exhibit 24 below.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Incremental claim counts are shown in Exhibits 22, 23 and 24.

Exhibit 22

Accident Year	Incremental Reported Claim Count									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	7,543	972	10	0	0	0	0	0	0	0
1988	8,790	1,219	22	1	0	0	0	0	0	0
1989	9,417	1,165	27	1	0	0	0	0	0	0
1990	8,637	1,068	19	2	0	0	0	0	0	0
1991	7,438	934	15	0	0	0	0	0	0	0
1992	6,406	799	15	1	0	0	0	0	0	0
1993	5,894	721	19	2	0	0	0	0	0	0
1994	5,840	706	22	0	0	0	0	0	0	0
1995	6,448	853	0	0	0	0	0	0	0	0
1996	7,259	0	0	0	0	0	0	0	0	0

Reported counts appear consistent from year to year. Most claims are reported in the year incurred, with very few reported more than two years late, and none reported historically more than four years late.

Exhibit 23

Accident Year	Incremental Closed Claim Count									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	2,007	3,159	256	22	338	708	850	679	327	122
1988	2,472	3,567	260	31	410	910	986	782	379	0
1989	2,695	3,917	144	42	507	1,002	1,052	724	0	0
1990	2,680	3,467	68	47	529	929	921	0	0	0
1991	2,387	3,029	60	73	456	791	0	0	0	0
1992	2,103	2,571	28	78	431	0	0	0	0	0
1993	1,992	2,351	12	86	0	0	0	0	0	0
1994	2,041	2,322	4	0	0	0	0	0	0	0
1995	2,385	2,556	0	0	0	0	0	0	0	0
1996	2,835	0	0	0	0	0	0	0	0	0

The incremental closed counts are consistent with the changes noted in the closed count development factors, particularly in the "2 to 3" and "3 to 4" columns. The incremental closed counts suggest that once claims have been open more than a couple of years, they are likely to remain open for several more years.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Exhibit 24

Accident Year	Change in Open Count									
	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	5,536	-2,187	-246	-22	-338	-708	-850	-679	-327	-122
1988	6,318	-2,348	-238	-30	-410	-910	-986	-782	-379	
1989	6,722	-2,752	-117	-41	-507	-1,002	-1,052	-724		
1990	5,957	-2,399	-49	-45	-529	-929	-921			
1991	5,051	-2,095	-45	-73	-456	-791				
1992	4,303	-1,772	-13	-77	-431					
1993	3,902	-1,630	7	-84						
1994	3,799	-1,616	18							
1995	4,063	-1,703								
1996	4,424									

There are many shifts in the change in open counts, consistent with shifts observed in closed claims (Exhibit 23) and shifts observed in case reserves (Exhibit 21).

Exhibit 25 shows the ratios of cumulative closed claim count to cumulative reported claim count.

Exhibit 25

Accident Year	Ratio of Cumulative Closed to Cumulative Reported Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.266	0.607	0.636	0.639	0.678	0.761	0.861	0.941	0.979	0.993
1988	0.281	0.603	0.628	0.631	0.672	0.763	0.861	0.939	0.977	
1989	0.286	0.625	0.637	0.641	0.689	0.783	0.882	0.950		
1990	0.310	0.633	0.639	0.644	0.698	0.794	0.888			
1991	0.321	0.647	0.653	0.662	0.716	0.810				
1992	0.328	0.649	0.651	0.662	0.722					
1993	0.338	0.657	0.656	0.669						
1994	0.349	0.667	0.665							
1995	0.370	0.677								
1996	0.391									

The ratios increase consistently over time, suggesting a speed up in the closure of claims.

Loss Reserve Testing: Beyond Popular Methods III. Analysis of Historical Development Data

Exhibit 26 shows a similar ratio, the ratio of cumulative paid loss to cumulative reported incurred loss.

Exhibit 26

Ratio of Cumulative Paid Loss to Cumulative Reported Incurred Loss

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.068	0.168	0.292	0.466	0.672	0.837	0.935	0.978	0.994	0.998
1988	0.071	0.171	0.296	0.472	0.678	0.842	0.936	0.978	0.992	
1989	0.081	0.185	0.316	0.502	0.712	0.865	0.946	0.976		
1990	0.084	0.187	0.321	0.520	0.734	0.881	0.943			
1991	0.094	0.204	0.350	0.558	0.767	0.878				
1992	0.100	0.218	0.377	0.590	0.748					
1993	0.110	0.235	0.401	0.561						
1994	0.116	0.246	0.354							
1995	0.124	0.208								
1996	0.095									

All things being equal, we would expect the ratios to behave in a manner consistent with the ratios in Exhibit 25. Through calendar year 1995 the ratios of Exhibit 26 increase steadily, but the ratios along the last diagonal decrease, suggesting case reserve strengthening.

The emerging paid and reported incurred loss ratios appear in Exhibits 27 and 28, respectively.

Exhibit 27

Ratio of Cumulative Paid Loss to Earned Premium

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.058	0.180	0.295	0.443	0.616	0.754	0.837	0.876	0.891	0.896
1988	0.058	0.178	0.293	0.448	0.628	0.769	0.854	0.896	0.913	
1989	0.059	0.176	0.292	0.449	0.623	0.754	0.830	0.866		
1990	0.059	0.171	0.289	0.454	0.635	0.768	0.844			
1991	0.056	0.159	0.266	0.415	0.574	0.688				
1992	0.057	0.163	0.275	0.430	0.589					
1993	0.053	0.153	0.264	0.415						
1994	0.051	0.147	0.253							
1995	0.051	0.147								
1996	0.050									

Paid loss ratios appear fairly consistent and reflect the 1991 rate activity. Paid loss ratios improve each year starting with 1993.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Exhibit 28

Ratio of Cumulative Reported Incurred Loss to Earned Premium

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.851	1.073	1.011	0.950	0.917	0.901	0.895	0.895	0.897	0.898
1988	0.813	1.042	0.990	0.950	0.926	0.914	0.913	0.916	0.921	
1989	0.727	0.950	0.923	0.895	0.875	0.872	0.877	0.887		
1990	0.697	0.919	0.901	0.874	0.865	0.872	0.894			
1991	0.598	0.777	0.759	0.743	0.748	0.784				
1992	0.567	0.745	0.729	0.728	0.787					
1993	0.483	0.654	0.657	0.740						
1994	0.445	0.598	0.715							
1995	0.409	0.710								
1996	0.524									

The incurred loss ratios also reflect the 1991 rate activity but not the improvement since 1993 noted in the paid loss ratios. The historical incurred loss ratios fluctuate in a manner consistent with case reserve strengthening.

Note that improvement was initially observed through accident year 1990 (see for example the 3rd review column) but that improvement has subsequently proved illusory (see the 7th review column).

Emerging frequencies appear in Exhibits 29 and 30.

Exhibit 29

Ratio of Cumulative Closed Count to Earned Exposure

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.019	0.049	0.052	0.052	0.055	0.062	0.070	0.076	0.079	0.081
1988	0.020	0.049	0.051	0.051	0.055	0.062	0.070	0.077	0.080	
1989	0.020	0.050	0.051	0.052	0.055	0.063	0.071	0.077		
1990	0.022	0.051	0.051	0.052	0.056	0.064	0.071			
1991	0.022	0.050	0.051	0.052	0.056	0.063				
1992	0.023	0.050	0.050	0.051	0.056					
1993	0.023	0.051	0.051	0.052						
1994	0.024	0.052	0.052							
1995	0.026	0.054								
1996	0.028									

Loss Reserve Testing: Beyond Popular Methods III. Analysis of Historical Development Data

Exhibit 30

Ratio of Cumulative Reported Count to Earned Exposure

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.072	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081
1988	0.071	0.081	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082
1989	0.072	0.080	0.081	0.081	0.081	0.081	0.081	0.081	0.081	0.081
1990	0.071	0.080	0.080	0.080	0.080	0.080	0.080	0.080		
1991	0.069	0.078	0.078	0.078	0.078	0.078	0.078			
1992	0.069	0.077	0.078	0.078	0.078	0.078				
1993	0.069	0.078	0.078	0.078						
1994	0.069	0.078	0.078							
1995	0.070	0.079								
1996	0.072									

Both closed and reported frequencies appear quite consistent from year to year, with a slight worsening each of the last two or three accident years.

Emerging severities appear in Exhibits 31, 32 and 33.

Exhibit 31

Reported Incurred Loss per Reported Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	12,466	13,934	13,103	12,323	11,892	11,687	11,608	11,609	11,627	11,646
1988	12,923	14,532	13,786	13,228	12,894	12,720	12,704	12,752	12,817	
1989	12,434	14,451	14,005	13,578	13,285	13,231	13,310	13,467		
1990	12,874	15,101	14,775	14,326	14,178	14,302	14,665			
1991	12,331	14,231	13,875	13,575	13,674	14,325				
1992	12,659	14,779	14,429	14,408	15,588					
1993	11,590	13,985	14,016	15,772						
1994	11,551	13,861	16,497							
1995	11,342	17,365								
1996	15,208									

The case reserve strengthening is reflected in the last diagonal.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Exhibit 32

Accident Year	Paid Loss per Closed Claim									
	1	2	3	4	5	6	7	8	9	10
1987	3,207	3,862	6,019	8,996	11,777	12,845	12,605	12,073	11,800	11,699
1988	3,269	4,119	6,490	9,892	13,009	14,043	13,813	13,282	13,014	
1989	3,522	4,277	6,949	10,640	13,733	14,619	14,280	13,828		
1990	3,494	4,447	7,428	11,561	14,906	15,873	15,567			
1991	3,621	4,498	7,445	11,457	14,651	15,518				
1992	3,852	4,972	8,364	12,847	16,161					
1993	3,774	4,995	8,569	13,216						
1994	3,825	5,114	8,796							
1995	3,787	5,327								
1996	3,719									

It appears from the paid and incurred severities that severities improve for accident year 1991. The recent increased severities are presumably affected by the shift in the settlement process.

Exhibit 33

Accident Year	Case Reserves per Open Claim									
	1	2	3	4	5	6	7	8	9	10
1987	15,823	29,471	25,481	18,202	12,135	7,992	5,432	4,250	3,579	3,736
1988	16,700	30,372	26,100	18,930	12,660	8,474	5,842	4,612	4,572	
1989	16,007	31,396	26,377	18,817	12,297	8,222	6,055	6,576		
1990	17,095	33,506	27,786	19,325	12,494	8,257	7,484			
1991	16,447	32,064	25,972	17,717	11,212	9,233				
1992	16,963	32,889	25,755	17,464	14,100					
1993	15,581	31,168	24,425	20,943						
1994	15,701	31,341	31,777							
1995	15,777	42,569								
1996	22,571									

Case reserves appear consistent with the strengthening along the last diagonal and with improved costs in 1991.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

The growth in severities from year to year at each maturity are shown in Exhibits 34, 35 and 36.

Exhibit 34

Growth in Average Incurred per Reported Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.037	1.043	1.052	1.073	1.084	1.088	1.094	1.098	1.102	
1989	0.962	0.994	1.016	1.026	1.030	1.040	1.048	1.056		
1990	1.035	1.045	1.055	1.055	1.067	1.081	1.102			
1991	0.958	0.942	0.939	0.948	0.964	1.002				
1992	1.027	1.038	1.040	1.061	1.140					
1993	0.916	0.946	0.971	1.095						
1994	0.997	0.991	1.177							
1995	0.982	1.253								
1996	1.341									

Incurred severity growth rates appear consistent with the strengthening along the last diagonal and with improved costs in 1991.

Exhibit 35

Growth in Average Paid per Closed Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.019	1.067	1.078	1.100	1.105	1.093	1.096	1.100	1.103	
1989	1.077	1.038	1.071	1.076	1.056	1.041	1.034	1.041		
1990	0.992	1.040	1.069	1.087	1.085	1.086	1.090			
1991	1.036	1.012	1.002	0.991	0.983	0.978				
1992	1.064	1.105	1.123	1.121	1.103					
1993	0.980	1.005	1.024	1.029						
1994	1.014	1.024	1.026							
1995	0.990	1.042								
1996	0.982									

Paid severities show accident year changes consistent with the incurred severities. As stated previously, the paid severities are affected by the change in settlement patterns.

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Exhibit 36

Growth in Average Case Reserve

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.055	1.031	1.024	1.040	1.043	1.060	1.075	1.085	1.277	
1989	0.959	1.034	1.011	0.994	0.971	0.970	1.036	1.426		
1990	1.068	1.067	1.054	1.027	1.016	1.004	1.236			
1991	0.962	0.957	0.935	0.917	0.897	1.118				
1992	1.031	1.026	0.992	0.986	1.258					
1993	0.919	0.948	0.948	1.199						
1994	1.008	1.006	1.301							
1995	1.005	1.358								
1996	1.431									

Average case reserves clearly show the strengthening in calendar year 1996. Also, the growth rates suggest a slight deterioration in case reserve adequacy during calendar years 1994 and 1995.

Loss Reserve Testing: Beyond Popular Methods III. Analysis of Historical Development Data

Similar growth rates in the total dollars from accident year to accident year appear in Exhibits 37, 38 and 39.

Exhibit 37

Accident Year	Growth in Reported Incurred Losses									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.208	1.226	1.238	1.263	1.276	1.281	1.288	1.293	1.297	
1989	1.031	1.051	1.074	1.086	1.090	1.100	1.108	1.117		
1990	0.950	0.958	0.967	0.967	0.978	0.991	1.010			
1991	0.825	0.813	0.810	0.817	0.832	0.864				
1992	0.884	0.894	0.895	0.914	0.981					
1993	0.842	0.869	0.893	1.006						
1994	0.987	0.981	1.165							
1995	1.084	1.397								
1996	1.509									

Exhibit 38

Accident Year	Growth in Paid Losses									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.256	1.247	1.253	1.279	1.288	1.289	1.289	1.292	1.295	
1989	1.174	1.137	1.148	1.155	1.144	1.130	1.120	1.115		
1990	0.987	0.967	0.983	1.001	1.009	1.009	1.006			
1991	0.923	0.891	0.883	0.878	0.860	0.861				
1992	0.937	0.954	0.965	0.966	0.957					
1993	0.928	0.934	0.949	0.956						
1994	1.039	1.029	1.029							
1995	1.157	1.180								
1996	1.167									

Exhibit 39

Accident Year	Growth in Case Reserves									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.204	1.222	1.232	1.250	1.252	1.241	1.267	1.317	1.677	
1989	1.020	1.034	1.043	1.024	0.975	0.938	0.929	1.224		
1990	0.946	0.956	0.959	0.933	0.902	0.875	1.072			
1991	0.816	0.795	0.775	0.751	0.728	0.887				
1992	0.879	0.878	0.858	0.848	1.061					
1993	0.833	0.851	0.858	1.078						
1994	0.981	0.966	1.256							
1995	1.075	1.468								
1996	1.558									

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

Growth rates in claim counts appear in Exhibits 40, 41 and 42.

Exhibit 40

Accident Year	Growth in Reported Claim Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.165	1.175	1.177	1.177	1.177	1.177	1.177	1.177	1.177	1.177
1989	1.071	1.057	1.058	1.058	1.058	1.058	1.058	1.058		
1990	0.917	0.917	0.917	0.917	0.917	0.917	0.917			
1991	0.861	0.863	0.863	0.862	0.862	0.862				
1992	0.861	0.861	0.861	0.861	0.861					
1993	0.920	0.918	0.919	0.919						
1994	0.991	0.990	0.990							
1995	1.104	1.115								
1996	1.126									

Exhibit 41

Accident Year	Growth in Closed Claim Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.232	1.169	1.162	1.163	1.166	1.179	1.177	1.174	1.174	1.174
1989	1.090	1.095	1.073	1.074	1.084	1.086	1.084	1.071		
1990	0.994	0.930	0.920	0.921	0.930	0.929	0.923			
1991	0.891	0.881	0.881	0.886	0.884	0.880				
1992	0.881	0.863	0.859	0.861	0.868					
1993	0.947	0.929	0.926	0.929						
1994	1.025	1.005	1.003							
1995	1.169	1.132								
1996	1.189									

Exhibit 42

Accident Year	Growth in Open Claim Count									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.141	1.185	1.203	1.202	1.200	1.171	1.178	1.213	1.313	
1989	1.064	1.000	1.032	1.030	1.004	0.967	0.896	0.858		
1990	0.886	0.896	0.911	0.909	0.888	0.871	0.867			
1991	0.848	0.831	0.830	0.819	0.812	0.793				
1992	0.852	0.856	0.865	0.860	0.844					
1993	0.907	0.898	0.905	0.899						
1994	0.974	0.961	0.966							
1995	1.069	1.081								
1996	1.089									

Loss Reserve Testing: Beyond Popular Methods
III. Analysis of Historical Development Data

The incremental paid loss per incremental closed claim appears in Exhibit 43.

Exhibit 43

Incremental Paid Loss per Incremental Closed Claim

Accident Year	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	3,207	4,278	49,544	742,801	56,575	21,565	10,775	6,320	5,100	4,779
1988	3,269	4,708	61,547	701,313	61,128	21,697	12,036	7,417	6,355	
1989	3,522	4,798	129,614	604,349	55,203	21,084	11,598	7,982		
1990	3,494	5,184	276,911	558,007	54,513	22,941	12,996			
1991	3,621	5,190	273,402	312,436	53,517	22,099				
1992	3,852	5,888	574,585	283,085	52,923					
1993	3,774	6,030	1,301,816	248,565						
1994	3,825	6,247	4,024,121							
1995	3,787	6,763								
1996	3,719									

The significant increase in the 2nd to 3rd review column highlights the apparent paradox in the speed up in closure suggested by Exhibit 25 contrasted with the low incremental closed claim count along the last diagonal of Exhibit 23. Partial payments on open claims are also affecting this exhibit.

Exhibit 44 shows the incremental paid loss per ending open claim.

Exhibit 44

Incremental Paid Loss per Ending Open Claim

Accident Year	Review									
	1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1987	1,163	4,035	4,087	5,304	6,971	7,503	7,729	8,480	9,317	10,228
1988	1,279	4,230	4,288	5,873	7,613	8,289	8,501	9,446	10,248	
1989	1,412	4,734	4,844	6,659	8,468	9,173	9,753	10,966		
1990	1,572	5,051	5,366	7,571	9,825	10,624	11,032			
1991	1,711	5,318	5,635	8,037	10,245	10,987				
1992	1,883	5,981	6,389	9,046	11,348					
1993	1,927	6,240	6,855	9,739						
1994	2,055	6,645	7,313							
1995	2,223	7,325								
1996	2,383									

The increases down each column appear to reflect either a higher paid or a lower ending open count, or some combination. Both are consistent with the noted shift in settlement patterns.

Loss Reserve Testing: Beyond Popular Methods

III. Analysis of Historical Development Data

We have made the following observations:

- Claim reporting pattern: Claims have historically been reported in a consistent manner, with nearly 90% of all claims being reported within two years, over 99% being reported within three years, and no claims historically reported after four years.
- Claim closing pattern: At the first review, the claim closing pattern sped up until 1991 and slowed after 1991. In general, as the years passed, temporary disability claims and other small claims were closed more quickly after the first review, and more serious claims increasingly tended to close within a few years of the fourth review.
- Claim payment pattern: Paid losses are driven more by the number of reported claims than by the number of closed claims. After the second review, most payments are on open claims. Paid loss ratios are fairly consistent, except for an improvement in 1991.
- Case reserve adequacy: It appears that case reserves deteriorated gradually through 1995, then were dramatically strengthened during calendar year 1996.
- Rate level: Rates were made more adequate in 1991. In other years, premium is a good measure of exposure to loss.
- Claim frequency: Frequencies have been quite consistent, with a slight reduction noted in 1991.
- Claim severity: The average cost per claim declined in 1991. Perhaps there is a slight increase from year to year, but observed annual inflation rates are distorted by the shifting settlement rate and case reserve strengthening.

To summarize, we observed in the underlying data a speed-up in closure which would tend to cause paid loss development to overstate reserve estimates and we also observed a shift in case reserve adequacy on the last diagonal of incurred development factors which might tend to cause incurred loss development to overstate reserve estimates. Thus, popular projection methods will tend to overstate reserve need in this case. A better point estimate or "best" estimate in this case might be less than the \$477 million total reserve estimate suggested by the methods. The \$160 million estimate for the 1996 year seems most in conflict with the development history, and a lower estimate is indicated.

Loss Reserve Testing: Beyond Popular Methods

IV. Tests of Reserve Estimates

There are many instances where estimates of loss reserves have already been made, and the task at hand is not necessarily to determine independent estimates but to ascertain whether the estimates already derived are reasonable. Examples include:

- When an auditing firm is testing the reserves carried by a client.
- When one actuary is reviewing another's work product.
- When a regulator is testing the sufficiency of reserves carried by a company.
- When a regulator is reviewing the loss costs used in a rate filing to assure they are not inadequate or excessive.

Often, when independent estimates are promulgated there is a tendency for the parties to argue over who's estimate is "correct". Let's face it, it is virtually certain that neither estimate will be "correct". The following reserve test framework will allow focus to be directed at the more appropriate question, "Is the company making reasonable estimates of its unknown loss costs?"

A "typical" loss reserve study estimating ultimate losses by accident year was presented in Section II. It suggested the following conclusions:

- Estimated reserve for accident years 1987 to 1996: The estimate is about \$477 million.
- Estimated reserve for accident year 1996: The estimate is about \$160 million.

The next several exhibits outline our proposed framework for testing the reasonableness of a given set of reserve estimates. To illustrate, we analyze the ultimate loss estimates shown for each accident year in Section II, Exhibit 17, Column 7.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

First, we test the ultimate claim count estimates from Exhibit 14, Column 3. Since claims are reported consistently and relatively promptly, the ultimate claim counts should be relatively easy to estimate.

Exhibit 45 shows the ratio of cumulative reported claim counts to the ultimate claim count estimates.

Exhibit 45

Ratio of Reported Count to Ultimate Count

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.885	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1988	0.876	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1989	0.888	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1990	0.888	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1991	0.887	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1992	0.887	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1993	0.888	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1994	0.889	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1995	0.881	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1996	0.886	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Each column of Exhibit 45 shows consistency.

Exhibit 46 is a similar test, showing the ratio of cumulative closed claims to ultimate claims.

Exhibit 46

Ratio of Closed Count to Ultimate Count

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.235	0.606	0.636	0.639	0.678	0.761	0.861	0.941	0.979	0.993
1988	0.246	0.602	0.628	0.631	0.672	0.763	0.861	0.939	0.977	0.993
1989	0.254	0.623	0.637	0.641	0.689	0.783	0.882	0.950	0.977	0.993
1990	0.276	0.632	0.639	0.644	0.698	0.794	0.888	0.950	0.977	0.993
1991	0.285	0.646	0.653	0.662	0.716	0.810	0.888	0.950	0.977	0.993
1992	0.291	0.647	0.651	0.662	0.722	0.810	0.888	0.950	0.977	0.993
1993	0.300	0.654	0.656	0.669	0.722	0.810	0.888	0.950	0.977	0.993
1994	0.311	0.664	0.665	0.669	0.722	0.810	0.888	0.950	0.977	0.993
1995	0.326	0.675	0.665	0.669	0.722	0.810	0.888	0.950	0.977	0.993
1996	0.346	0.675	0.665	0.669	0.722	0.810	0.888	0.950	0.977	0.993

The ratios of closed to ultimate are consistent with the ratios of closed to reported.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Exhibit 47

Accident Year	Indicated Number of IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	982	10	0	0	0	0	0	0	0	0
1988	1,242	23	1	0	0	0	0	0	0	0
1989	1,193	28	1	0	0	0	0	0		
1990	1,089	21	2	0	0	0	0			
1991	949	15	0	0	0	0				
1992	815	16	1	0	0					
1993	742	21	2	0						
1994	728	22	0							
1995	875	22								
1996	932									

The hindsight IBNR claim counts in Exhibit 47 show consistency. The total open plus IBNR claim counts are shown in Exhibit 48.

Exhibit 48

Accident Year	Indicated Number of Open + IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	6,518	3,359	3,103	3,081	2,743	2,035	1,185	506	179	57
1988	7,560	3,993	3,733	3,702	3,292	2,382	1,396	614	235	
1989	7,915	3,998	3,854	3,812	3,305	2,303	1,251	527		
1990	7,046	3,579	3,511	3,464	2,935	2,006	1,085			
1991	6,000	2,971	2,911	2,838	2,382	1,591				
1992	5,118	2,547	2,519	2,441	2,010					
1993	4,644	2,293	2,281	2,195						
1994	4,527	2,205	2,201							
1995	4,938	2,382								
1996	5,356									

Exhibit 49

Accident Year	Ratio of Open to Open + IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.849	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1988	0.836	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1989	0.849	0.993	1.000	1.000	1.000	1.000	1.000	1.000		
1990	0.845	0.994	0.999	1.000	1.000	1.000	1.000			
1991	0.842	0.995	1.000	1.000	1.000	1.000				
1992	0.841	0.994	1.000	1.000	1.000					
1993	0.840	0.991	0.999	1.000						
1994	0.839	0.990	1.000							
1995	0.823	0.991								
1996	0.826									

Loss Reserve Testing: Beyond Popular Methods

IV. Tests of Reserve Estimates

Tests of the selected ultimate loss estimates from Section II begin with Exhibit 50.

The upper portion of Exhibit 50 is reproduced from Exhibit 4. Rather than select a single set of age-to-age factors, optimistic and pessimistic factors are selected. This suggests a range of development curves which lie between the optimistic and pessimistic curves. The lower portion of Exhibit 50 shows the ratios of the historical cumulative reported incurred loss to the ultimate loss estimates.

Exhibit 50										
Incurred Loss Development										
Review										
Accident Year	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	1.262	0.941	0.941	0.965	0.983	0.993	1.000	1.002	1.002	
1988	1.280	0.951	0.960	0.975	0.986	0.999	1.004	1.005		
1989	1.306	0.972	0.970	0.978	0.996	1.006	1.012			
1990	1.318	0.980	0.970	0.990	1.009	1.025				
1991	1.299	0.977	0.978	1.007	1.048					
1992	1.313	0.978	0.999	1.082						
1993	1.354	1.005	1.126							
1994	1.345	1.194								
1995	1.734									
All-years Averages										
Arithmetic	1.357	1.000	0.992	1.000	1.004	1.006	1.005	1.003	1.002	
Volume	1.344	0.991	0.986	0.997	1.004	1.007	1.006	1.004	1.002	
Three-year Averages										
Arithmetic	1.478	1.059	1.034	1.026	1.017	1.010	1.005	1.003	1.002	
Volume	1.484	1.055	1.029	1.022	1.015	1.010	1.006	1.004	1.002	
Selected										
Optimistic	1.300	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000	
Pessimistic	1.500	1.060	1.050	1.030	1.020	1.015	1.010	1.005	1.002	
Cumulative										
Optimistic	1.287	0.990	0.990	1.000	1.000	1.000	1.000	1.000	1.000	
Pessimistic	1.816	1.211	1.142	1.088	1.056	1.035	1.020	1.010	1.005	
Percent of Ultimate										
Optimistic	77.70%	101.01%	101.01%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
Pessimistic	55.06%	82.59%	87.55%	91.93%	94.68%	96.58%	98.03%	99.01%	99.50%	
99.80%										
Ratio of Reported Incurred Loss to Ultimate Loss										
Review										
Accident Year	1	2	3	4	5	6	7	8	9	10
1987	0.947	1.195	1.125	1.058	1.021	1.004	0.997	0.997	0.998	1.000
1988	0.883	1.130	1.074	1.031	1.005	0.992	0.990	0.994	0.999	
1989	0.817	1.067	1.037	1.005	0.983	0.979	0.985	0.997		
1990	0.769	1.014	0.994	0.964	0.954	0.963	0.987			
1991	0.738	0.958	0.936	0.916	0.922	0.966				
1992	0.692	0.908	0.889	0.888	0.960					
1993	0.602	0.815	0.820	0.923						
1994	0.570	0.766	0.915							
1995	0.518	0.897								
1996	0.647									

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Exhibit 51 is created for paid losses, based on Exhibit 5, in a similar manner.

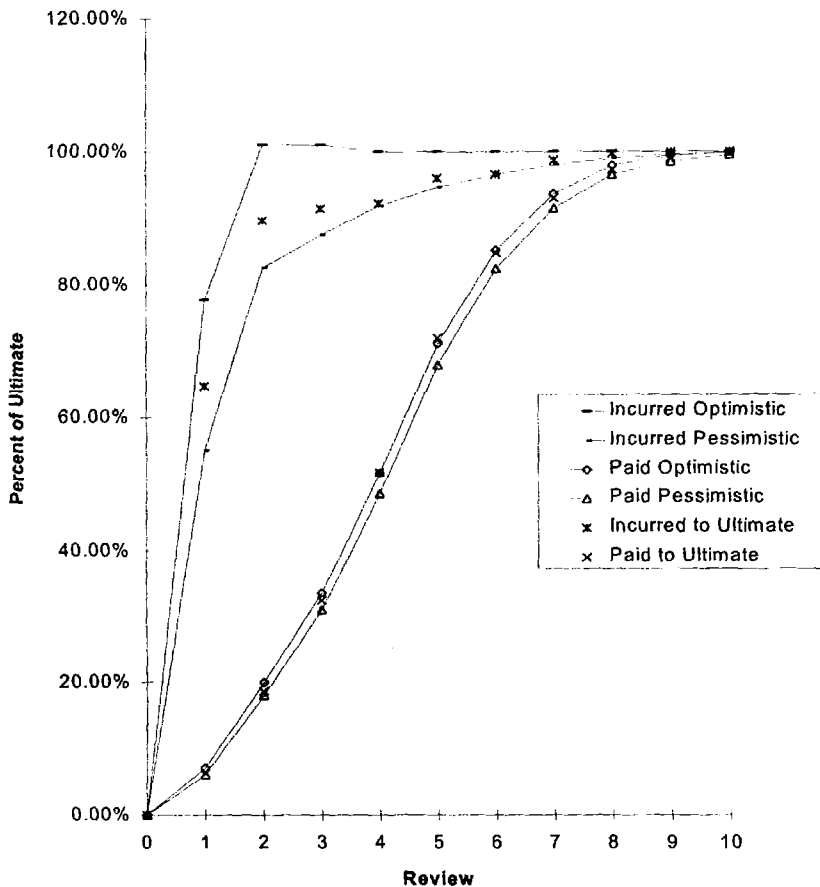
Paid Loss Development											Exhibit 51
Accident Year	Review										
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10		
1987	3.099	1.636	1.501	1.390	1.224	1.110	1.046	1.017	1.006		
1988	3.078	1.643	1.532	1.400	1.225	1.110	1.049	1.019			
1989	2.980	1.660	1.541	1.387	1.211	1.100	1.043				
1990	2.920	1.689	1.568	1.398	1.211	1.098					
1991	2.819	1.673	1.559	1.384	1.199						
1992	2.869	1.692	1.561	1.371							
1993	2.886	1.720	1.573								
1994	2.858	1.721									
1995	2.914										
All-years Averages											
Arithmetic	2.936	1.679	1.548	1.389	1.214	1.105	1.046	1.018	1.006		
Volume	2.932	1.679	1.549	1.389	1.213	1.104	1.046	1.018	1.006		
Three-year Averages											
Arithmetic	2.886	1.711	1.565	1.385	1.207	1.103	1.046	1.018	1.006		
Volume	2.887	1.711	1.564	1.385	1.207	1.103	1.046	1.018	1.006		
Selected											
Optimistic	2.850	1.680	1.540	1.375	1.200	1.100	1.045	1.015	1.005	1.001	
Pessimistic	3.000	1.720	1.565	1.400	1.215	1.110	1.055	1.020	1.010	1.005	
Cumulative											
Optimistic	14.280	5.011	2.982	1.937	1.408	1.174	1.067	1.021	1.006	1.001	
Pessimistic	16.654	5.551	3.228	2.062	1.473	1.212	1.092	1.035	1.015	1.005	
Percent of Ultimate											
Optimistic	7.00%	19.96%	33.53%	51.63%	71.00%	85.20%	93.72%	97.93%	99.40%	99.90%	
Pessimistic	6.00%	18.01%	30.98%	48.49%	67.88%	82.48%	91.55%	96.59%	98.52%	99.50%	
Ratio of Paid Loss to Ultimate Loss											
Accident Year	Review										
	1	2	3	4	5	6	7	8	9	10	
1987	0.065	0.201	0.329	0.493	0.686	0.840	0.932	0.975	0.992	0.998	
1988	0.063	0.193	0.318	0.487	0.681	0.835	0.927	0.972	0.991		
1989	0.066	0.197	0.328	0.505	0.700	0.847	0.932	0.973			
1990	0.065	0.189	0.319	0.501	0.700	0.848	0.931				
1991	0.070	0.196	0.328	0.511	0.707	0.848					
1992	0.069	0.198	0.336	0.524	0.718						
1993	0.066	0.191	0.329	0.517							
1994	0.066	0.188	0.324								
1995	0.064	0.186									
1996	0.062										

The first "tests" derived from Exhibits 50 and 51 appear graphically in Exhibit 52. The range of paid development curves is plotted as is the range of incurred curves. The ratios of the current paid loss to ultimate (the last diagonal from the bottom of Exhibit 51) and the ratios of the current reported incurred loss to ultimate (the last diagonal from the bottom of Exhibit 50) are also graphed.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Exhibit 52

Paid and Incurred Development Tests



The plotted ratios fall within the range of development curves, (other than the paid to ultimate ratios for accident years 1992 and 1993 which are slightly above the range) suggesting that the estimates "pass" this test. This should be no surprise since the estimates are based on the historical development curves. It is also interesting to note that the relative stability of the historical paid factors leads to a narrower range of paid development curves.

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The second set of "tests" derived from Exhibits 50 and 51 involves further inspection of the ratios in the lower portion of each exhibit. The ratios within each column should be consistent, or any variation within each column should be consistent with the information noted in the analysis of the underlying data.

Beginning with accident year 1987, the oldest year, we have assumed that the case reserves on the 57 open claims for that year are exactly adequate. We have not extrapolated a tail factor. Thus the incurred to ultimate ratio for accident year 1987 is 1.000 and the paid to ultimate ratio is 0.998.

The ratios of 0.999 and 0.991 for accident year 1988 are consistent with the analogous hindsight ratios for 1987 of 0.998 and 0.992, respectively.

Similarly, the ratios of 0.997 and 0.973 for accident year 1989 are reasonable.

The ratios of 0.987 and 0.931 for accident year 1990 are close to the hindsight ratios of prior years. Recalling, however, evidence of a speed up in closure, we might expect the paid to ultimate ratio to be slightly higher than prior years ratios. The incurred to ultimate ratio is below the average of the prior accident years, which is inconsistent with the observed case reserve strengthening. Thus, it might be reasonable to lower the ultimate loss estimate slightly to increase the ratios slightly.

Accident year 1991 shows ratios of 0.966 and 0.848. This would be consistent with a very slight speed up in closure, but not with the observed case reserve strengthening. Thus, it might be reasonable to lower the ultimate loss estimate slightly to increase the ratios slightly.

Accident year 1992 shows ratios of 0.960 and 0.718. Keeping in mind that if we revise the ultimate estimates for 1990 and 1991, the hindsight ratios for those years will change as well, we note that the paid to ultimate ratio may be consistent with a speed up in closure, but may not if we revise the 1990 and 1991 estimates. We also note that the incurred to ultimate ratio is consistent with recent case strengthening, but the ratio is still below the hindsight ratios of 1987 to 1989. The ultimate estimate for this year may be reasonable but slightly conservative.

Accident year 1993 shows ratios of 0.923 and 0.517. The latter ratio is not consistent with a speed up in closure. The former ratio is consistent with modest case strengthening. Thus, it might be reasonable to lower the ultimate loss estimate to increase the ratios.

Accident year 1994 shows ratios of 0.915 and 0.324, consistent with some case strengthening (which should change if we revise the ultimate estimates for prior years) and inconsistent with a speed up in closure. Thus, it might be reasonable to lower the ultimate loss estimate to increase the ratios.

Loss Reserve Testing: Beyond Popular Methods
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Accident year 1995 shows ratios of 0.897 and 0.186, consistent with some case strengthening (which should change if we revise the ultimate estimates for prior years) and inconsistent with a speed up in closure. Thus, it might be reasonable to lower the ultimate loss estimate to increase the ratios.

Accident year 1996 shows ratios of 0.647 and 0.062, consistent with some case strengthening (which should change if we revise the ultimate estimates for prior years) and inconsistent with a speed up in closure. Thus, it might be reasonable to lower the ultimate loss estimate to increase the ratios.

Although the ultimate loss estimates produce paid to ultimate and incurred to ultimate ratios which seem to fall within reasonable bounds, further scrutiny of the ratios to ultimate, including hindsight ratios to ultimate for older accident years, uncovers a slight internal inconsistency within the selected estimates, suggesting that the estimates are above the high end of the reasonable range of reserve estimates.

Exhibits 53 through 56 contain tests of the hindsight reserves calculated by subtracting from the selected ultimate loss estimates the paid to date at each valuation for each accident year.

Exhibit 53

Accident Year	Hindsight Reserves (000 omitted) Review									
	1	2	3	4	5	6	7	8	9	10
1987	92,841	79,329	66,645	50,304	31,182	15,913	6,755	2,464		213
1988	120,620	103,826	87,823	66,083	41,020	21,276	9,408	3,608	1,200	
1989	133,833	115,041	96,376	70,994	43,006	21,880	9,679	3,900		
1990	135,148	117,175	98,346	72,119	43,282	21,970	10,000			
1991	115,715	99,996	83,592	60,784	36,380	18,900				
1992	109,117	93,979	77,890	55,810	33,000					
1993	105,926	91,748	76,127	54,750						
1994	110,603	96,096	80,000							
1995	132,286	115,000								
1996	160,000									

Hindsight reserves decrease for accident years 1991 to 1993 and increase each year after that.

Loss Reserve Testing: Beyond Popular Methods

IV. Tests of Reserve Estimates

To determine if the changes are consistent with a change in the underlying exposure or if they must be explained by something else, we consider the ratios of the hindsight reserve estimates to the earned premium.

Exhibit 54

Accident Year	Ratio of Hindsight Reserves to Earned Premium									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.840	0.718	0.603	0.455	0.282	0.144	0.061	0.022	0.007	0.002
1988	0.864	0.744	0.629	0.473	0.294	0.152	0.067	0.026	0.009	
1989	0.831	0.714	0.599	0.441	0.267	0.136	0.060	0.024		
1990	0.847	0.735	0.617	0.452	0.271	0.138	0.063			
1991	0.755	0.652	0.545	0.397	0.237	0.123				
1992	0.763	0.657	0.545	0.390	0.231					
1993	0.749	0.648	0.538	0.387						
1994	0.729	0.634	0.528							
1995	0.740	0.644								
1996	0.759									

These loss ratios should be considered in light of the paid and incurred loss ratios in Exhibits 27 and 28, respectively. The ratios in Exhibit 54 show loss ratio improvement in 1991. This is consistent with Exhibits 27 and 28.

The hindsight reserves should also reflect the volume of claims remaining to be settled. The hindsight open plus IBNR claim counts, which appear in Exhibit 48, were calculated based on the ultimate claim count estimates in Exhibit 14.

Exhibit 55

Accident Year	Hindsight Reserve per Open + IBNR Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	14,244	23,617	21,478	16,327	11,368	7,820	5,700	4,869	4,447	3,736
1988	15,955	26,002	23,526	17,850	12,461	8,932	6,740	5,877	5,106	
1989	16,909	28,775	25,007	18,624	13,012	9,501	7,737	7,400		
1990	19,181	32,740	28,011	20,820	14,747	10,952	9,217			
1991	19,286	33,657	28,716	21,418	15,273	11,879				
1992	21,320	36,898	30,921	22,863	16,418					
1993	22,809	40,012	33,374	24,943						
1994	24,432	43,581	36,347							
1995	26,790	48,281								
1996	29,874									

This test assumes that the estimates of the ultimate claim counts are accurate. The average unpaid amount per unsettled claim appears consistent with continual inflation increases.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

The implied annual inflation rates are shown in Exhibit 56.

Exhibit 56

Accident Year	Growth in Hindsight Reserve per Open + IBNR Claim									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.120	1.101	1.095	1.093	1.096	1.142	1.182	1.207	1.148	
1989	1.060	1.107	1.063	1.043	1.044	1.064	1.148	1.259		
1990	1.134	1.138	1.120	1.118	1.133	1.153	1.191			
1991	1.005	1.028	1.025	1.029	1.036	1.085				
1992	1.105	1.096	1.077	1.067	1.075					
1993	1.070	1.084	1.079	1.091						
1994	1.071	1.089	1.089							
1995	1.097	1.108								
1996	1.115									

These growth rates should be consistent with the trends in Exhibits 34, 35 and 36. Because of the evidence of a slight speed up in closure, these growth rates might tend to be slightly higher than the paid growth rates. With the exception of accident year 1992, where the paid growth rates were unusually high, the hindsight reserve growth rates are well above the paid growth rates. Thus, the ultimate loss estimates would need to be revised downward to bring these growth rates more in line with (but still slightly above) the emerged paid rates.

Exhibit 57 shows our final test of the hindsight reserves.

Exhibit 57

Accident Year	Hindsight Reserve per Ultimate Claim									
	1	2	3	4	5	6	7	8	9	10
1987	10,890	9,305	7,818	5,901	3,658	1,867	792	289	93	25
1988	12,024	10,349	8,754	6,587	4,089	2,121	938	360	120	
1989	12,614	10,843	9,084	6,691	4,053	2,062	912	368		
1990	13,896	12,048	10,112	7,415	4,450	2,259	1,028			
1991	13,797	11,923	9,967	7,247	4,338	2,253				
1992	15,111	13,015	10,787	7,729	4,570					
1993	15,962	13,826	11,472	8,250						
1994	16,840	14,631	12,180							
1995	18,065	15,704								
1996	19,534									

This test is consistent with Exhibit 55.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Similar tests of the hindsight IBNR reserves calculated by subtracting from the selected ultimate loss estimates the reported incurred to date at each valuation for each accident year appear in Exhibits 58 through 62.

Exhibit 58

Accident Year	Hindsight IBNR Reserve (000 omitted) Review									
	1	2	3	4	5	6	7	8	9	10
1987	5,244	-19,371	-12,423	-5,777	-2,104	-350	318	313	155	-0
1988	15,111	-16,751	-9,581	-3,998	-655	1,092	1,253	777	126	
1989	26,233	-9,599	-5,252	-736	2,365	2,945	2,105	434		
1990	33,315	-2,040	838	5,178	6,612	5,406	1,879			
1991	32,639	5,213	7,988	10,502	9,674	4,211				
1992	36,126	10,737	13,038	13,180	4,659					
1993	45,129	20,934	20,462	8,781						
1994	50,954	27,679	10,059							
1995	68,184	14,537								
1996	60,146									

Hindsight IBNR reserves are consistent with a case reserve weakening through calendar year 1995 and a case reserve strengthening in calendar year 1996.

Exhibit 59

Accident Year	Ratio of Hindsight IBNR to Earned Premium Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.047	0.175	0.112	0.052	0.019	0.003	0.003	0.003	0.001	0.000
1988	0.108	0.120	0.069	0.029	0.005	0.008	0.009	0.006	0.001	
1989	0.163	0.060	0.033	0.005	0.015	0.018	0.013	0.003		
1990	0.209	0.013	0.005	0.032	0.041	0.034	0.012			
1991	0.213	0.034	0.052	0.069	0.063	0.027				
1992	0.253	0.075	0.091	0.092	0.033					
1993	0.319	0.148	0.145	0.062						
1994	0.336	0.183	0.066							
1995	0.382	0.081								
1996	0.285									

The ratios of the hindsight IBNR reserves to earned premium also reflect a case reserve weakening through calendar year 1995 and strengthening in 1996.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Exhibit 60

Accident Year	Hindsight IBNR per IBNR Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	5,340	-1,937,072	0	0	0	0	0	0	0	0
1988	12,166	-728,316	-9,581,345	0	0	0	0	0	0	0
1989	21,989	-342,839	-5,252,497	0	0	0	0	0		
1990	30,592	-97,124	419,058	0	0	0	0			
1991	34,393	347,546	0	0	0	0				
1992	44,326	671,070	13,038,087	0	0					
1993	60,821	996,868	10,231,064	0						
1994	69,992	1,258,127	0							
1995	77,933	663,701								
1996	64,543									

The low IBNR counts caused by the fast reporting of claims on this line of business, and the changing case reserve adequacy, make tests of the hindsight IBNR reserves to IBNR counts less meaningful. Since the IBNR reserve in this case is mostly case reserve development, perhaps a more meaningful severity test would be the hindsight IBNR per open plus IBNR claim.

Exhibit 61

Accident Year	Hindsight IBNR per Open + IBNR Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	804	-5,767	-4,003	-1,875	-767	-172	268	619	868	-0
1988	1,999	-4,195	-2,567	-1,080	-199	458	897	1,265	534	
1989	3,314	-2,401	-1,363	-193	716	1,279	1,682	824		
1990	4,728	-570	239	1,495	2,253	2,695	1,732			
1991	5,440	1,755	2,744	3,700	4,061	2,647				
1992	7,059	4,216	5,176	5,399	2,318					
1993	9,718	9,130	8,971	4,000						
1994	11,256	12,553	4,570							
1995	13,808	6,103								
1996	11,230									

The hindsight IBNR per open plus IBNR claim is consistent with a case reserve weakening through calendar year 1995 and strengthening in 1996.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

The final test of hindsight IBNR is in Exhibit 62.

Exhibit 62

Accident Year	Hindsight IBNR per Ultimate Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	615	-2,272	-1,457	-678	-247	-41	37	37	18	-0
1988	1,506	-1,670	-955	-399	-65	109	125	77	13	
1989	2,472	-905	-495	-69	223	278	198	41		
1990	3,425	-210	86	532	680	556	193			
1991	3,892	622	952	1,252	1,153	502				
1992	5,003	1,487	1,806	1,825	645					
1993	6,801	3,155	3,084	1,323						
1994	7,758	4,214	1,532							
1995	9,311	1,985								
1996	7,343									

This exhibit is consistent with case reserve deterioration through 1995 followed by strengthening in 1996.

Loss Reserve Testing: Beyond Popular Methods
IV. Tests of Reserve Estimates

Exhibit 63

Ratio of Case Reserve to Indicated Reserve

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.944	1.244	1.186	1.115	1.067	1.022	0.953	0.873	0.805	1.000
1988	0.875	1.161	1.109	1.061	1.016	0.949	0.867	0.785	0.895	
1989	0.804	1.083	1.054	1.010	0.945	0.865	0.783	0.889		
1990	0.753	1.017	0.991	0.928	0.847	0.754	0.812			
1991	0.718	0.948	0.904	0.827	0.734	0.777				
1992	0.669	0.886	0.833	0.764	0.859					
1993	0.574	0.772	0.731	0.840						
1994	0.539	0.712	0.874							
1995	0.485	0.874								
1996	0.624									

Exhibit 63

Ratio of Case Reserve to Indicated Reserve

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.944	1.244	1.186	1.115	1.067	1.022	0.953	0.873	0.805	1.000
1988	0.875	1.161	1.109	1.061	1.016	0.949	0.867	0.785	0.895	
1989	0.804	1.083	1.054	1.010	0.945	0.865	0.783	0.889		
1990	0.753	1.017	0.991	0.928	0.847	0.754	0.812			
1991	0.718	0.948	0.904	0.827	0.734	0.777				
1992	0.669	0.886	0.833	0.764	0.859					
1993	0.574	0.772	0.731	0.840						
1994	0.539	0.712	0.874							
1995	0.485	0.874								
1996	0.624									

The final test presented in Exhibit 63 shows that the ratios of the case reserves to hindsight reserves are consistent with our observations regarding changing case reserve adequacy.

The single ratio of 1.000 in the 10th review column is the result of our assumption that the case reserves are adequate by the 10th review. The much lower values in the preceding columns would appear to refute that assumption. The reserve tests are dependent on the ability to accurately estimate the ultimate loss for the most mature accident year. It would seem that an IBNR reserve equal to some fraction of the case reserve would be appropriate. That amount would not likely be material to the overall analysis.

To summarize, some tests of the reserve estimates derived from using the popular projection methods indicate that the estimates might be near or above the conservative end of the range of reasonable estimates while the estimates appear to "pass" other tests. It appears that a better point estimate or "best" estimate in this case might be lower than the average of the rote estimates produced by the "popular" methods.

Loss Reserve Testing: Beyond Popular Methods

IV. Tests of Reserve Estimates

We now have the following findings and conclusions:

- Claim reporting pattern: Claims have historically been reported in a consistent manner, with nearly 90% of all claims being reported within two years, over 99% being reported within three years, and no claims historically reported after four years.
- Claim closing pattern: At the first review, the claim closing pattern sped up until 1991 and slowed after 1991. In general, as the years passed, temporary disability claims and other small claims were closed more quickly after the first review, and more serious claims increasingly tended to close within a few years of the fourth review.
- Claim payment pattern: Paid losses are driven more by the number of reported claims than by the number of closed claims. After the second review, most payments are on open claims. Paid loss ratios are fairly consistent, except for an improvement in 1991.
- Case reserve adequacy: It appears that case reserves deteriorated gradually through 1995, then were dramatically strengthened during calendar year 1996.
- Rate level: Rates were made more adequate in 1991. In other years, premium is a good measure of exposure to loss.
- Claim frequency: Frequencies have been quite consistent, with a slight reduction noted in 1991.
- Claim severity: The average cost per claim declined in 1991. Perhaps there is a slight increase from year to year, but observed annual inflation rates are distorted by the shifting settlement rate and case reserve strengthening.
- The reserve estimate of \$477 million is high.
- The reserve estimate for 1996 of \$160 million is particularly high.

These findings and conclusions are consistent with one another. They are in an important sense a model—a simplified description—of the loss experience.

We have presented a number of reserve tests. This is by no means an exhaustive list of possible tests. The actuary should devise whatever tests are warranted by the situation at hand. The appropriate tests will be determined by the data that is available and the observations that seem important.

The loss reserve tests are especially useful when reserve estimates must be brought up to date. Our experience has taught us that when previous estimates have been developed that

Loss Reserve Testing: Beyond Popular Methods

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in some sense pass a good set of loss reserve tests, it is a straightforward process to revise the estimates in light of new data or new insights. The revised estimates can be shown to be consistent with the tests in light of the new data or insights. This is a simple exercise, comparable to applying the popular methods, but it yields a much more meaningful actuarial report.

The reserve tests could be repeated using revised estimates, with the revised estimates being more consistent with the observations about the experience in light of the various tests. This is beyond the scope of this paper. When estimates are selected that are consistent with the observations about the loss experience, the estimates will appear to be high in some tests and low in others (because loss data is, after all, subject to random influences not reflected in the "model"), and thus will not appear to be biased.

In general, the actuary can develop estimates that are unbiased in light of the observations from the experience. That is, such estimates in some sense "pass" the loss reserve tests. Such estimates are more reliable than estimates based on the popular methods because they are consistent with observations of the processes of loss development and trend for the problem at hand. Moreover, estimates that pass a number of loss reserve tests are less sensitive to random data fluctuations than are simple averages of the indications of various popular methods.

In addition, the reserve tests could be repeated using loss reserve estimates designed to be conservative. The selected estimates would be the highest reserves that still provide for a consistent set of observations and conclusions about the loss process, and thus could serve as an upper boundary of the reasonable range of estimates. This, too, is outside the scope of this paper.

The assumptions inherent in the popular actuarial methods demonstrated in Section II are discussed in Appendix A. Appendix A also identifies the assumptions which caused the methods to overestimate the reserve need in this case.

Loss Reserve Testing: Beyond Popular Methods

V. Conclusion

The approach of any observational science is to first make observations, including both quantification of the subject and determining the causal relationships underlying the numbers, and to then draw inferences from those observations. The loss reserve testing approach uses various statistics to make observations of the processes of loss development and trend, attempts to explain those observations, and finally draws conclusions regarding the reasonableness of the loss reserve estimates being tested.

The tables below summarize the results of the "typical" actuarial reserve review and the conclusions regarding those results derived from this example of the concept of loss reserve testing. The tables also compare those conclusions with the costs of losses that underlie the simulated loss development data.

Exhibit 64

Comparison of Indicated Reserves			
	(1)	(2)	(3)
Accident Year	Popular Methods (\$,000)	Reserve Testing Indications	Model Values (\$,000)
1987	\$213	ok	\$223
1988	1,200	ok	1,126
1989	3,900	ok	3,623
1990	10,000	slightly high	8,482
1991	18,900	slightly high	15,300
1992	33,000	slightly high	29,466
1993	54,750	high	47,766
1994	80,000	high	72,775
1995	115,000	high	104,805
1996	<u>160,000</u>	high	<u>143,554</u>
Total	\$476,963	HIGH	\$427,118

Exhibit 65

Comparison of Indicated IBNR Reserves			
	(1)	(2)	(3)
Accident Year	Popular Methods (\$,000)	Reserve Testing Indications	Model Values (\$,000)
1987	\$0	ok	\$10
1988	126	ok	52
1989	434	ok	157
1990	1,879	slightly high	361
1991	4,211	slightly high	611
1992	4,659	slightly high	1,124
1993	8,781	high	1,797
1994	10,059	high	2,834
1995	14,537	high	4,342
1996	<u>60,146</u>	high	<u>43,700</u>
Total	\$104,832	HIGH	\$54,988

Loss Reserve Testing: Beyond Popular Methods

V. Conclusion

Exhibits 64 and 65 demonstrate that the loss reserve tests work. They successfully assessed the reasonableness of the loss reserve estimates. They led us to the conclusion that a loss reserve estimate of \$477 million is outside the range of reasonable estimates because it is inconsistent with the other observations about changes in the costs of claims. The loss reserve tests correctly detected biases in the estimates determined using the popular methods. In particular, the decline in claim frequency in 1991, the speed-up of claim closure after 1991, and the strengthening of case reserves in calendar year 1996 lead the popular methods to overestimate the appropriate reserve. The overestimation is greatest in the years with the most uncertainty regarding the future emergence of claims costs.

The conclusion that the \$477 million total reserve estimate is above the high end of the reasonable range of estimates was reached without a full actuarial study to determine the "best" estimate or the "range" of estimates. In general, for audit purposes and when one is testing the work of another actuary, it is sufficient to perform tests such as these, and it is not necessary to develop new estimates. In fact, doing so can lead to an unnecessary debate. Many times, arguments between differing estimates of loss reserves center around the selections of parameters such as age-to-age link ratios or tail factors. Seemingly minor differences in the selected parameters might lead to significant differences in the final loss reserve estimates. In reality, there is no convincing argument that any selected age-to-age link ratio or tail factor will repeat indefinitely into the future independent of accident year. The process of testing estimates of loss reserves shifts the focus away from "who is right," and on to whether a given set of loss reserve estimates is reasonable.

Will we continue into the next millennium relying on methods first developed over half a century ago on columnar pads? Modern computing technology should have rendered the popular actuarial methodologies obsolete. At best, the popular methods might serve as a starting point for a complete analysis of claims costs. We hope this article will serve as a springboard towards the development of sound methods for assessing the reasonableness of loss reserve estimates which take full advantage of the ongoing technological explosion.

Loss Reserve Testing: Beyond Popular Methods

Appendix A Popular Methods and Their Assumptions

Paid and Incurred Loss Development (Chain-Ladder) Methods

The underlying assumptions of these methods are: i) the age-to-age link ratios in each development interval are independent and identically distributed random variables, ii) the link ratios among the various development intervals are independent of each other, iii) the claims settlement process unfolds at the same rate over time, and iv) an estimate of the mean of the independent, identically distributed development ratios will continue into the future indefinitely, independent of accident year. An additional assumption of the incurred method is: v) the case reserves are estimated consistently.

If there are changes in the loss development process, these assumptions are met only when those changes are random, are independent of accident year, and operate multiplicatively. If there are changes in the loss process that do not operate in this limited way, then the assumptions of the loss development methods are not met.

Unfortunately, when there is a change in the loss process over time the models' assumptions are most inappropriate for the most recent period—just the one with the greatest uncertainty about the appropriate estimate of ultimate loss. There are two reasons for this. First, the change in the loss process operates longest on that period. The loss process has had and will have more opportunity to depart from the historical average.

Second, the method works by multiplying together a number of age-to-age link ratios, and as that number of ratios increases, the uncertainty of the result increases. As Brown has noted, "Stability is not a characteristic of such a model."³ This problem can be mitigated by computing the ratio of historical total paid (or reported) to estimated ultimate, but this step is usually not taken in practice.

In the example presented above, as typically happens, the link ratios within a given development interval are neither independent nor identically distributed. The many factors which have affected the loss development process in the past shape the mind set of both the personnel adjusting the claim and the claimant as well, violating the independence assumption. Also, changes in the underlying process will cause the mean to change within the same development interval from one accident year to the next. In the example presented in this paper, we have seen shifts in closure and payment patterns, and in case reserve adequacy, and so the link ratios within each development interval are not identically distributed.

³ Robert L. Brown, "Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance", Actex, 1993, p. 125.

Loss Reserve Testing: Beyond Popular Methods

Appendix A: Popular Methods and Their Assumptions

The assumption of independence from one development interval to another is also often violated. For example, when unusually high losses are reported at one review, all else equal, the link ratio **to** that review will be high, and the link ratio **from** that review will be low. The ratios are, in fact, perfectly correlated in their movements in response to a loss payment. The incurred method, relying as it does on case reserves, creates link ratios that are perfectly correlated with respect to a change in the adequacy of case reserves. As with the link ratios within each development interval, the changes in settlement and case reserve adequacy are inconsistent with the independence assumption.

The third assumption that the claims settlement process unfolds at the same rate over time is clearly violated in this case, and typically is not an appropriate assumption. The fourth assumption is also inappropriate because of the changing patterns and the observations regarding the different accident years unfolding in different ways. Finally, we have noted shifting case reserve adequacy, violating the fifth assumption.

All of the assumptions of the paid and incurred methods are violated in this example, which more often than not is the case in practice. The effect of the violated assumptions is not counteracted by averaging the estimates resulting from the two methods. Averaging estimates based on two or more methods reduces the uncertainty of the resulting estimate when the various methods all provide unbiased estimates of the ultimate loss (or when two methods are equally biased in opposite directions). Presumably, this is what actuaries have in mind when they average estimates. Unfortunately, the paid and incurred loss development methods are unbiased only when all of the assumptions of the methods are appropriate. Moreover, when biased, they are usually biased in the same direction. Averaging several estimates made from inappropriate model assumptions does nothing to improve the uncertainty of the resulting estimate, except by chance, and here the benefits of chance are mitigated by imposing on the two different methods many of the same incorrect assumptions about the loss development process.

This can be seen clearly by a thought experiment involving a single type of claim in which the loss process is slowing down from year to year. In this hypothetical situation, the slowing in any year may be small or large, but each year's losses are paid and reported a little more slowly than the year before. The true age-to-age link ratios are increasing with each passing year. In this situation, the paid method will understate the ultimate loss cost because it relies on averages of past link ratios, and all future link ratios will be larger than any historical average. The incurred method will also underestimate the ultimate loss for the same reason. Because the paid link ratios are larger (farther from 1.000) than the incurred link ratios, the paid method will understate the ultimate cost of losses more than will the incurred method. That is, the two methods will identify a range of possible loss costs that begins and ends with numbers **less** than the ultimate cost of losses. The converse is true in our example, where a steady pattern of speeding up the settlement process is combined with a recent increase in case reserve adequacy, causing the paid and incurred methods to give a range that is above the true value. The more different the two estimates are, the more clear it is that the assumptions are not appropriate, and the more inappropriate it is to average them.

Loss Reserve Testing: Beyond Popular Methods

Appendix A: Popular Methods and Their Assumptions

Frequency and Severity Methods

The link-ratio or chain-ladder method can be applied to the historical claim count data to provide an estimate of the ultimate number of claims. This is the first component for the frequency-severity method in the typical actuarial report. Clearly this approach to estimating the ultimate number of claims relies on the same assumptions as the paid and incurred development methods.

There are additional assumptions, of course, in the frequency-severity method: i) frequency and severity are independent, and ii) severity is changing from year to year in a way that can be estimated independently of the loss development data.

Estimates of severity—the average cost per claim—in principle can be made in such a way that they are independent of the estimate of the number of claims. In practice, however, this is often not done. In practice, the severity in the earliest years is often estimated by dividing estimates of ultimate by estimates of the ultimate number of claims. In Exhibit 14, illustrating typical frequency and severity projections, the severity parameters are based on the developed ultimate loss and claim count estimates for the first five accident years. Therefore, in this example, the estimates of severity are not independent of the estimates of claim counts.

When there are few claims the loss development data can be distorted by the reserves or payments on a few claims in the loss development history. Typically, ultimate severities are estimated for each experience period for all claims, both open and closed, rather than estimating the average unpaid cost for open and unreported claims. The former procedure is inappropriate when the emerged experience is distorted by the presence or absence of any atypically large or unusual claims.

The assumption that frequency and severity are independent can also be violated by a change in the reporting of small claims. Small claims contribute to the claim count but contribute little to the loss costs. If in one period (say, the first three years) the smallest claims are recorded as "suspense" until a payment is made, and then in a more recent period (say, the last seven years) the small claims are included in the reported claim count, then the severity of claims in the second period will be less than in the first period (all else equal). Changes in data processing procedures often change the counting of small claims. Thus, frequency and severity are not independent.

Knowledge of the claim size process and how it is changing can be brought to bear to improve the estimate of severity. Unfortunately, this is not always done in practice. Often only some average inflation rate is brought over from some other source and applied to a starting point average severity from a relatively mature exposure period to get estimates of severity for all subsequent exposure periods. This is not very robust, considering the variety of factors which affect claim sizes.

Loss Reserve Testing: Beyond Popular Methods

Appendix A: Popular Methods and Their Assumptions

The Bornhuetter-Ferguson Method

The Bornhuetter-Ferguson method permits the introduction of information about loss costs from ancillary sources in a direct way. The assumptions are: i) The fraction of total expected loss cost that is unpaid (or unreported) at any stage of development can be estimated, and ii) there exists ancillary data about exposure to loss that provides a dollar scale for the expected total losses in each year.

Estimates of the fraction of total expected loss costs unpaid (or unreported) are typically an algebraic manipulation of the cumulative development factors derived in conjunction with the paid and incurred development approaches. The assumptions inherent in those methods, discussed above, apply here as well.

In principle, the Bornhuetter-Ferguson method can be applied using a robust estimate of the cost per unit of exposure and reliable exposure data. Estimates of the pure premium can often be made from the experience of similar risks. If a reasonable measure of exposure exists, then a good estimate of expected loss costs can be made with this approach.

Unfortunately, this kind of effort is seldom done in actuarial reports. Rather, the typical actuarial report applies the loss ratio estimated for earlier years to earned premium as the basis for estimating expected losses. This introduces the additional assumption that premium is an accurate measure of exposure. If other data is used, it is usually to make some adjustment to the expected loss ratio—as the 80% in Exhibits 15 and 16 illustrate. Such an adjustment must be regarded as yet another assumption unless it can be supported with observations from other relevant, credible cases.

Indeed, in practice the estimate of the loss ratio is sometimes adjusted so as to get from the Bornhuetter-Ferguson method an estimate of ultimate loss that is consistent with the paid and incurred development methods. When this is done, it contributes no information at all to the estimation process.

Loss Reserve Testing: Beyond Popular Methods

Appendix B Creation of Data

The data generated for this study was simulated to mimic the functions of a workers' compensation company. The data used to calibrate the model is fictitious; any resemblance to any existing insurance company, whether it is a workers' compensation company or not, is entirely coincidental. As with any set of simulated data, extensive analysis of patterns in the data will result in observations that typically would not occur with a real set of data. We tried to minimize this with this model but as Exhibit 23 illustrates, this effort was not entirely successful.

The model simulated each claim in the insurance company from policies issued from 1986 through 1996. It was important to include 1986 policies since some covered accidents from these policies appear in the accident year 1987 row of the triangles. Exhibit B-1 displays the written premium, earned premium, exposure, ultimate claim counts, and ultimate loss from the model.

We derived exposure figures from earned premiums by reducing the earned premiums by inflation factors. This process is similar to the actuarial technique of computing premiums at current rate levels. Ultimate counts are proportional to exposure levels. Hence, any change in the level of claim counts is due to changes in the underlying exposure levels, not changes to earned premiums due to inflationary pressures.

The distribution of policies did not change between years. Exhibit B-2 displays the assumptions underlying the distribution of policies within each calendar year; there are a larger than average number of January, April, July, and October renewals with the majority of policies issued on the first day of each month. All other policies are assumed issued uniformly within each month.

In order for the triangles to have the appearance of different types of claims, the model simulated six different claim types: Medical Only, Temporary Disability, Minor-Minor Permanent Disability, Minor-Major Permanent Disability, Major-Minor Permanent Disability, and Total Permanent Disability. Exhibit B-3 to B-8 display the assumptions by type of claim for these policies.

The indemnity part of each claim was paid uniformly from time of reporting to time of closing. The medical part of each claim was paid according to payment schedules that varied by type of claim. Payment schedules did not vary by policy year. Medical payments were also paid from time of reporting to time of closing. Exhibit B-9 displays the assumed rate of medical payment for three claim types. These medical payment schedules did not vary between years.

Loss Reserve Testing: Beyond Popular Methods
Appendix B: Creation of Data

The model was also designed with a set of case reserving assumptions that varied by calendar year. Exhibit B-7 displays these assumptions. As stated in the paper, case reserves developed favorably for the first few accident years and steadily deteriorated through the 1990's. In calendar year 1996, there was a reversal and case reserves were strengthened to a "more appropriate" level.

Exhibit B-1

Premium, Exposure, Ultimates by Accident Year

Year	Written Premium (000 omitted)	Earned Premium (000 omitted)	Exposure (000 omitted)	Ultimate Count	Ultimate Loss (000 omitted)
1986	93,916	---	---	---	---
1987	121,404	110,538	105,100	8,525	99,288
1988	151,558	139,638	123,009	10,032	128,628
1989	167,200	161,017	131,625	10,610	143,046
1990	154,414	159,468	121,100	9,726	142,993
1991	152,572	153,300	107,657	8,387	120,758
1992	136,687	142,967	93,134	7,221	113,683
1993	144,645	141,499	85,086	6,636	106,459
1994	156,204	151,635	84,395	6,570	111,185
1995	193,346	178,664	91,847	7,325	131,124
1996	222,031	210,692	100,423	8,217	154,096

Exhibit B-2

Distribution of Policies with Each Calendar Year

Month	Exposure Distribution	First Day Proportion
January	27.12%	50.0%
February	5.08%	5.0%
March	5.08%	5.0%
April	8.47%	50.0%
May	5.08%	5.0%
June	5.08%	5.0%
July	13.56%	50.0%
August	5.08%	5.0%
September	5.08%	5.0%
October	10.17%	50.0%
November	5.08%	5.0%
December	5.08%	5.0%

Loss Reserve Testing: Beyond Popular Methods
Appendix B: Creation of Data

Exhibit B-3

Distribution of Claims

Year	Medical Only	Temporary Disability	Minor-Minor Permanent Disability	Minor-Major Permanent Disability	Major-Minor Permanent Disability	Total Permanent Disability
1986	40%	24%	20%	13%	3%	3
1987	39%	24%	20%	13%	4%	3
1988	39%	23%	20%	13%	5%	4
1989	42%	23%	17%	13%	5%	5
1990	41%	23%	17%	13%	6%	2
1991	40%	25%	17%	12%	5%	2
1992	42%	24%	16%	12%	6%	4
1993	41%	26%	15%	11%	7%	2
1994	41%	26%	14%	12%	7%	2
1995	43%	25%	14%	11%	8%	4
1996	45%	23%	13%	11%	9%	3

Note: Since Total Permanent Disability are infrequent and very costly, these claim count are generated separately from the other counts of claim types. Total PD column shows the actual number of claims used in the model, not a percentage of the total claim counts as the other columns show.

Exhibit B-4

Mean Severity Per Claim

Year	Medical Only	Temporary Disability	Minor-Minor Permanent Disability	Minor-Major Permanent Disability	Major-Minor Permanent Disability	Total Permanent Disability
1986	200	3,000	8,000	40,000	100,000	500,000
1987	240	3,000	8,320	41,200	102,000	550,000
1988	288	3,000	8,653	42,436	104,040	605,000
1989	346	3,000	8,999	43,709	106,121	665,500
1990	380	3,000	9,359	45,020	108,243	732,050
1991	418	3,000	9,733	46,371	110,408	805,255
1992	439	3,000	10,123	47,762	112,616	885,781
1993	461	3,000	10,527	49,195	114,869	974,359
1994	484	3,000	10,949	50,671	117,166	1,071,794
1995	508	3,000	11,386	52,191	119,509	1,178,974
1996	534	3,000	11,842	53,757	121,899	1,296,871

Loss Reserve Testing: Beyond Popular Methods
Appendix B: Creation of Data

Exhibit B-5

Mean Reporting Delay (days)

Year	Medical Only	Temporary Disability	Minor-Minor Permanent Disability	Minor-Major Permanent Disability	Major-Minor Permanent Disability	Total Permanent Disability
1986	25	30	40	90	150	150
1987	25	30	41	90	150	150
1988	24	30	42	90	150	150
1989	24	28	43	90	150	150
1990	23	28	43	90	150	150
1991	23	28	42	90	150	150
1992	23	28	41	90	150	150
1993	22	26	40	90	150	150
1994	22	26	39	90	150	150
1995	22	26	38	90	150	150
1996	22	26	38	90	150	150

Exhibit B-6

Mean Time to Closing (days)

Year	Medical Only	Temporary Disability	Minor-Minor Permanent Disability	Minor-Major Permanent Disability	Major-Minor Permanent Disability	Total Permanent Disability
1986	120	365	2,008	2,190	2,555	1,460
1987	120	347	1,962	2,154	2,500	1,460
1988	120	329	1,916	2,117	2,446	1,460
1989	120	310	1,871	2,081	2,391	1,460
1990	120	292	1,825	2,044	2,336	1,460
1991	120	274	1,779	2,008	2,281	1,460
1992	120	256	1,734	1,971	2,227	1,460
1993	120	237	1,688	1,935	2,172	1,460
1994	120	219	1,643	1,898	2,117	1,460
1995	120	201	1,597	1,862	2,062	1,460
1996	120	183	1,551	1,825	2,008	1,460

Loss Reserve Testing: Beyond Popular Methods
Appendix B: Creation of Data

Exhibit B-7

Mean Adjustments to Case Reserves (Percentage of Ultimate) by Calendar Year

Year	Medical Only	All Other Claim Types
1986	120%	150%
1987	115%	140%
1988	110%	130%
1989	105%	120%
1990	100%	110%
1991	100%	100%
1992	100%	90%
1993	95%	80%
1994	90%	70%
1995	85%	65%
1996	100%	100%

Exhibit B-8

Percentage Medical Costs of Total Costs

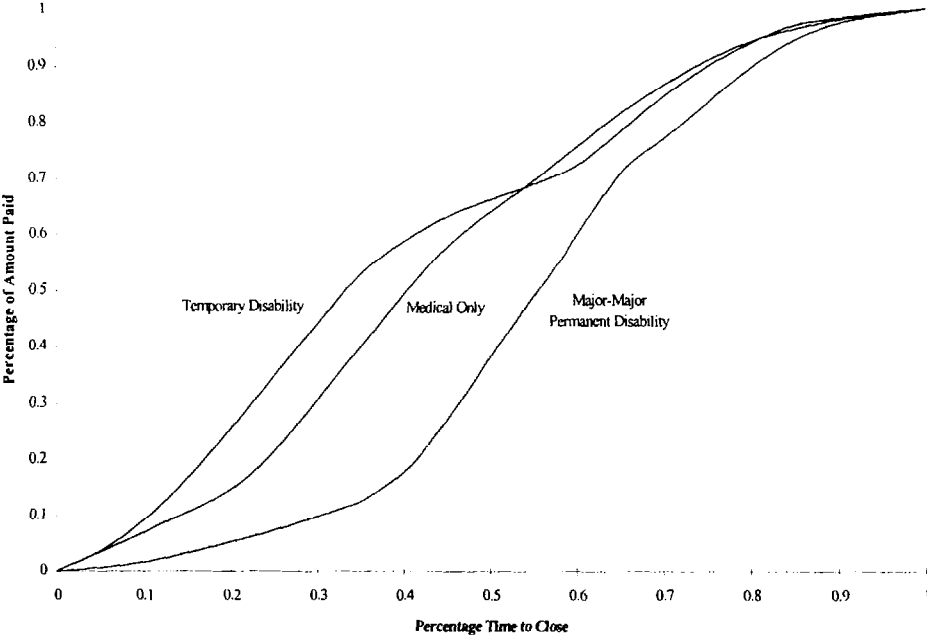
Type of Claim	
Medical Only	100%
Temporary Disability	50%
Minor-Minor Permanent Disability	50%
Minor-Major Permanent Disability	50%
Major-Major Permanent Disability	50%
Total Permanent Disability	50%

Loss Reserve Testing: Beyond Popular Methods

Appendix B: Creation of Data

Rate of Medical Payments on Types of Claims

Exhibit B-9



Loss Reserve Testing: Beyond Popular Methods

Appendix C Reserve Tests applied to Model "Actual" Ultimates

In this section, we show the reserve tests applied to the "actual" ultimates generated by the model. Exhibit C-18 is comparable to Exhibit 18 in the text. Similarly, Exhibits 45 through 63 are updated based on the "actual" ultimate losses from the model and presented as Exhibits C-45 through C-63. Note that Exhibits 1 through 17 and 19 through 44 are not affected by a change in the ultimate loss estimates.

Exhibit C-18

Summary							
Accident Year	(1) Earned Premium	(2) Paid Loss	(3) Case Reserve	(4) Incurred Loss	(5) Model Reserve	(6) Model Ultimate	(7) Ultimate Loss Ratio
1987	\$110,537,942	\$99,064,933	\$212,975	\$99,277,908	\$223,067	\$99,288,000	89.82%
1988	139,638,242	127,501,531	1,074,481	128,576,012	1,126,469	128,628,000	92.12%
1989	161,016,927	139,423,412	3,465,635	142,889,047	3,622,588	143,046,000	88.84%
1990	159,468,213	134,511,457	8,120,505	142,631,963	8,481,543	142,993,000	89.67%
1991	153,300,139	105,458,369	14,688,993	120,147,362	15,299,631	120,758,000	78.77%
1992	142,966,702	84,217,437	28,341,302	112,558,738	29,465,563	113,683,000	79.52%
1993	141,499,363	58,693,102	45,969,037	104,662,140	47,765,898	106,459,000	75.24%
1994	151,634,759	38,410,177	69,940,501	108,350,678	72,774,823	111,185,000	73.32%
1995	178,663,700	26,319,255	100,462,965	126,782,219	104,804,745	131,124,000	73.39%
1996	210,691,772	10,541,955	99,854,302	110,396,257	143,554,045	154,096,000	73.14%
Total	\$1,549,417,760	\$824,141,628	\$372,130,696	\$1,196,272,324	\$427,118,372	\$1,251,260,000	80.76%

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-45

Ratio of Reported Count to Ultimate Count

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.885	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1988	0.876	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1989	0.888	0.997	1.000	1.000	1.000	1.000	1.000	1.000		
1990	0.888	0.998	1.000	1.000	1.000	1.000	1.000			
1991	0.887	0.998	1.000	1.000	1.000	1.000				
1992	0.887	0.998	1.000	1.000	1.000					
1993	0.888	0.997	1.000	1.000						
1994	0.889	0.997	1.000							
1995	0.881	0.997								
1996	0.886									

Exhibit C-46

Ratio of Closed Count to Ultimate Count

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.235	0.606	0.636	0.639	0.678	0.761	0.861	0.941	0.979	0.993
1988	0.246	0.602	0.628	0.631	0.672	0.763	0.861	0.939	0.977	
1989	0.254	0.623	0.637	0.641	0.689	0.783	0.882	0.950		
1990	0.276	0.632	0.639	0.644	0.698	0.794	0.888			
1991	0.285	0.646	0.653	0.662	0.716	0.810				
1992	0.291	0.647	0.651	0.662	0.722					
1993	0.300	0.654	0.656	0.669						
1994	0.311	0.664	0.665							
1995	0.326	0.675								
1996	0.346									

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-47

Accident Year	Indicated Number of IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	982	10	0	0	0	0	0	0	0	0
1988	1,242	23	1	0	0	0	0	0	0	0
1989	1,193	28	1	0	0	0	0	0		
1990	1,089	21	2	0	0	0	0			
1991	949	15	0	0	0	0				
1992	815	16	1	0	0					
1993	742	21	2	0						
1994	728	22	0							
1995	875	22								
1996	932									

Exhibit C-48

Accident Year	Indicated Number of Open + IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	6,518	3,359	3,103	3,081	2,743	2,035	1,185	506	179	57
1988	7,560	3,993	3,733	3,702	3,292	2,382	1,396	614	235	
1989	7,915	3,998	3,854	3,812	3,305	2,303	1,251	527		
1990	7,046	3,579	3,511	3,464	2,935	2,006	1,085			
1991	6,000	2,971	2,911	2,838	2,382	1,591				
1992	5,118	2,547	2,519	2,441	2,010					
1993	4,644	2,293	2,281	2,195						
1994	4,527	2,205	2,201							
1995	4,938	2,382								
1996	5,356									

Exhibit C-49

Accident Year	Ratio of Open to Open + IBNR Claims									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.849	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1988	0.836	0.994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
1989	0.849	0.993	1.000	1.000	1.000	1.000	1.000	1.000		
1990	0.845	0.994	0.999	1.000	1.000	1.000	1.000			
1991	0.842	0.995	1.000	1.000	1.000	1.000				
1992	0.841	0.994	1.000	1.000	1.000					
1993	0.840	0.991	0.999	1.000						
1994	0.839	0.990	1.000							
1995	0.823	0.991								
1996	0.826									

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-50

Accident Year	Incurred Loss Development									
	Review									
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	1.262	0.941	0.941	0.965	0.983	0.993	1.000	1.002	1.002	
1988	1.280	0.951	0.960	0.975	0.986	0.999	1.004	1.005		
1989	1.306	0.972	0.970	0.978	0.996	1.006	1.012			
1990	1.318	0.980	0.970	0.990	1.009	1.025				
1991	1.299	0.977	0.978	1.007	1.048					
1992	1.313	0.978	0.999	1.082						
1993	1.354	1.005	1.126							
1994	1.345	1.194								
1995	1.734									
All-years Averages										
Arithmetic	1.357	1.060	0.992	1.000	1.004	1.006	1.005	1.003	1.002	
Volume	1.344	0.991	0.986	0.997	1.004	1.007	1.006	1.004	1.002	
Three-year Averages										
Arithmetic	1.478	1.059	1.034	1.026	1.017	1.010	1.005	1.003	1.002	
Volume	1.484	1.055	1.029	1.022	1.015	1.010	1.006	1.004	1.002	
Selected										
Optimistic	1.300	1.000	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Pessimistic	1.500	1.060	1.050	1.030	1.020	1.015	1.010	1.005	1.003	1.002
Cumulative										
Optimistic	1.287	0.990	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Pessimistic	1.816	1.211	1.142	1.088	1.056	1.035	1.020	1.010	1.005	1.002
Percent of Ultimate										
Optimistic	77.70%	101.01%	101.01%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Pessimistic	55.06%	82.59%	87.55%	91.93%	94.68%	96.58%	98.03%	99.61%	99.50%	99.80%

Ratio of Reported Incurred Loss to Ultimate Loss

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
	1987	0.947	1.195	1.125	1.058	1.021	1.003	0.997	0.997	0.998
1988	0.883	1.131	1.075	1.032	1.006	0.992	0.991	0.995	1.000	
1989	0.819	1.069	1.039	1.007	0.985	0.981	0.987	0.999		
1990	0.778	1.025	1.005	0.974	0.964	0.973	0.997			
1991	0.760	0.987	0.964	0.943	0.950	0.995				
1992	0.713	0.937	0.916	0.915	0.990					
1993	0.642	0.869	0.873	0.983						
1994	0.607	0.816	0.975							
1995	0.558	0.967								
1996	0.716									

Loss Reserve Testing: Beyond Popular Methods

Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-51

		Paid Loss Development								
		Review								
Accident Year	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
1987	3.099	1.636	1.501	1.390	1.224	1.110	1.046	1.017	1.006	
1988	3.078	1.643	1.532	1.400	1.225	1.110	1.049	1.019		
1989	2.980	1.660	1.541	1.387	1.211	1.100	1.043			
1990	2.920	1.689	1.568	1.398	1.211	1.098				
1991	2.819	1.673	1.559	1.384	1.199					
1992	2.869	1.692	1.561	1.371						
1993	2.886	1.720	1.573							
1994	2.858	1.721								
1995	2.914									
All-years Averages										
Arithmetic	2.936	1.679	1.548	1.389	1.214	1.105	1.046	1.018	1.006	
Volume	2.932	1.679	1.549	1.389	1.213	1.104	1.046	1.018	1.006	
Three-year Averages										
Arithmetic	2.886	1.711	1.565	1.385	1.207	1.103	1.046	1.018	1.006	
Volume	2.887	1.711	1.564	1.385	1.207	1.103	1.046	1.018	1.006	
Selected										
Optimistic	2.850	1.680	1.540	1.375	1.200	1.100	1.045	1.015	1.005	1.001
Pessimistic	3.000	1.720	1.565	1.400	1.215	1.110	1.055	1.020	1.010	1.005
Cumulative										
Optimistic	14.280	5.011	2.982	1.937	1.408	1.174	1.067	1.021	1.006	1.001
Pessimistic	16.654	5.551	3.228	2.062	1.473	1.212	1.092	1.035	1.015	1.005
Percent of Ultimate										
Optimistic	7.00%	19.96%	33.53%	51.63%	71.00%	85.20%	93.72%	97.93%	99.40%	99.90%
Pessimistic	6.00%	18.01%	30.98%	48.49%	67.88%	82.48%	91.55%	96.59%	98.52%	99.50%

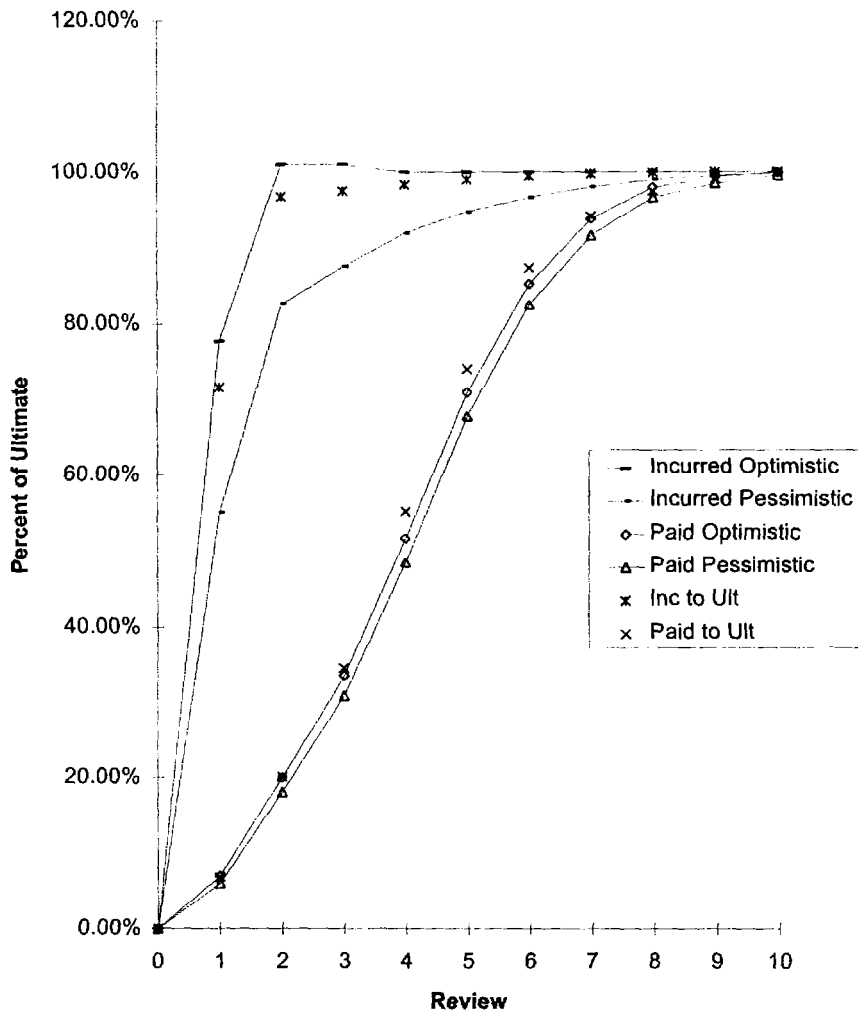
Ratio of Paid Loss to Ultimate Loss

		Review									
Accident Year	1	2	3	4	5	6	7	8	9	10	
1987	0.065	0.201	0.329	0.493	0.686	0.840	0.932	0.975	0.992	0.998	
1988	0.063	0.193	0.318	0.487	0.682	0.835	0.927	0.973	0.991		
1989	0.066	0.198	0.328	0.506	0.701	0.849	0.934	0.975			
1990	0.065	0.191	0.323	0.506	0.708	0.857	0.941				
1991	0.072	0.202	0.338	0.526	0.729	0.873					
1992	0.071	0.204	0.346	0.540	0.741						
1993	0.071	0.204	0.351	0.551							
1994	0.070	0.201	0.345								
1995	0.069	0.201									
1996	0.068										

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-52

Paid and Incurred Development Tests



Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-53

Accident Year	Hindsight Reserves (000 omitted)									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	92,851	79,339	66,655	50,314	31,192	15,924	6,765	2,474	806	223
1988	120,547	103,752	87,750	66,009	40,947	21,203	9,335	3,535	1,126	
1989	133,556	114,763	96,099	70,716	42,728	21,603	9,402	3,023		
1990	133,630	115,657	96,827	70,601	41,763	20,451	8,482			
1991	112,115	96,395	79,991	57,183	32,780	15,300				
1992	105,583	90,444	74,356	52,275	29,466					
1993	98,942	84,764	69,143	47,766						
1994	103,378	88,871	72,775							
1995	122,091	104,805								
1996	143,554									

Exhibit C-54

Accident Year	Ratio of Hindsight Reserves to Earned Premium									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.840	0.718	0.603	0.455	0.282	0.144	0.061	0.022	0.007	0.002
1988	0.863	0.743	0.628	0.473	0.293	0.152	0.067	0.025	0.008	
1989	0.829	0.713	0.597	0.439	0.265	0.134	0.058	0.022		
1990	0.838	0.725	0.607	0.443	0.262	0.128	0.053			
1991	0.731	0.629	0.522	0.373	0.214	0.100				
1992	0.739	0.633	0.520	0.366	0.206					
1993	0.699	0.599	0.489	0.338						
1994	0.682	0.586	0.480							
1995	0.683	0.587								
1996	0.681									

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-55

Hindsight Reserve per Open - IBNR Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	14,245	23,620	21,481	16,330	11,371	7,825	5,709	4,889	4,503	3,913
1988	15,945	25,983	23,506	17,831	12,438	8,901	6,687	5,757	4,793	
1989	16,874	28,705	24,935	18,551	12,928	9,380	7,515	6,874		
1990	18,965	32,315	27,578	20,381	14,229	10,195	7,817			
1991	18,686	32,445	27,479	20,149	13,761	9,616				
1992	20,630	35,510	29,518	21,415	14,659					
1993	21,305	36,967	30,312	21,761						
1994	22,836	40,304	33,064							
1995	24,725	44,000								
1996	26,803									

Exhibit C-56

Growth in Hindsight Reserve per Open + IBNR Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987										
1988	1.119	1.100	1.094	1.092	1.094	1.138	1.171	1.178	1.064	
1989	1.058	1.105	1.061	1.040	1.039	1.054	1.124	1.194		
1990	1.124	1.126	1.106	1.099	1.101	1.087	1.040			
1991	0.985	1.004	0.996	0.989	0.967	0.943				
1992	1.104	1.094	1.074	1.063	1.065					
1993	1.033	1.041	1.027	1.016						
1994	1.072	1.090	1.091							
1995	1.083	1.092								
1996	1.084									

Exhibit C-57

Hindsight Reserve per Ultimate Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	10,892	9,307	7,819	5,902	3,659	1,868	794	290	95	26
1988	12,016	10,342	8,747	6,580	4,082	2,114	931	352	112	
1989	12,588	10,817	9,057	6,665	4,027	2,036	886	341		
1990	13,739	11,892	9,955	7,259	4,294	2,103	872			
1991	13,368	11,493	9,538	6,818	3,908	1,824				
1992	14,622	12,525	10,297	7,239	4,081					
1993	14,910	12,773	10,419	7,198						
1994	15,740	13,531	11,080							
1995	16,673	14,312								
1996	17,526									

Loss Reserve Testing: Beyond Popular Methods

Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-58

Accident Year	Hindsight IBNR Reserve (000 omitted)									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	5,254	-19,361	-12,413	-5,767	-2,093	-340	328	323	165	10
1988	15,037	-16,825	-9,655	-4,072	-729	1,019	1,179	703	52	
1989	25,955	-9,877	-5,530	-1,013	2,088	2,668	1,827	157		
1990	31,796	-3,558	-680	3,660	5,094	3,887	361			
1991	29,039	1,613	4,387	6,901	6,074	611				
1992	32,591	7,203	9,504	9,645	1,124					
1993	38,145	13,950	13,478	1,797						
1994	43,729	20,454	2,834							
1995	57,988	4,342								
1996	43,700									

Exhibit C-59

Accident Year	Ratio of Hindsight IBNR to Earned Premium									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.048	0.175	0.112	0.052	0.019	0.003	0.003	0.003	0.001	0.000
1988	0.108	0.120	0.069	0.029	0.005	0.007	0.008	0.005	0.000	
1989	0.161	0.061	0.034	0.006	0.013	0.017	0.011	0.001		
1990	0.199	0.022	0.004	0.023	0.032	0.024	0.002			
1991	0.189	0.011	0.029	0.045	0.040	0.004				
1992	0.228	0.050	0.066	0.067	0.008					
1993	0.270	0.099	0.095	0.013						
1994	0.288	0.135	0.019							
1995	0.325	0.024								
1996	0.207									

Exhibit C-60

Accident Year	Hindsight IBNR per IBNR Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	5,350	-1,936,062	0	0	0	0	0	0	0	0
1988	12,107	-731,513	-9,654,876	0	0	0	0	0	0	
1989	21,756	-352,747	-5,529,909	0	0	0	0	0		
1990	29,198	-169,431	-340,171	0	0	0	0			
1991	30,599	107,522	0	0	0	0				
1992	39,989	450,168	9,503,651	0	0					
1993	51,409	664,291	6,739,013	0						
1994	60,067	929,710	0							
1995	66,280	198,228								
1996	46,894									

Loss Reserve Testing: Beyond Popular Methods
Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-61

Hindsight IBNR per Open + IBNR Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	806	-5,764	-4,000	-1,872	-763	-167	276	639	924	177
1988	1,989	-4,214	-2,586	-1,100	-221	428	845	1,145	221	
1989	3,279	-2,470	-1,435	-266	632	1,158	1,461	298		
1990	4,513	-994	-194	1,057	1,735	1,938	333			
1991	4,840	543	1,507	2,432	2,550	384				
1992	6,368	2,828	3,773	3,951	559					
1993	8,214	6,084	5,909	819						
1994	9,660	9,276	1,288							
1995	11,744	1,823								
1996	8,159									

Exhibit C-62

Hindsight IBNR per Ultimate Claim

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	616	-2,271	-1,456	-677	-246	-40	38	38	19	1
1988	1,499	-1,677	-962	-406	-73	102	118	70	5	
1989	2,446	-931	-521	-95	197	251	172	15		
1990	3,269	-366	-70	376	524	400	37			
1991	3,462	192	523	823	724	73				
1992	4,513	997	1,316	1,336	156					
1993	5,748	2,102	2,031	271						
1994	6,658	3,114	432							
1995	7,919	593								
1996	5,335									

Exhibit C-63

Ratio of Case Reserve to Indicated Reserve

Accident Year	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.943	1.244	1.186	1.115	1.067	1.021	0.952	0.869	0.795	0.955
1988	0.875	1.162	1.110	1.062	1.018	0.952	0.874	0.801	0.954	
1989	0.806	1.086	1.058	1.014	0.951	0.877	0.806	0.957		
1990	0.762	1.031	1.007	0.948	0.878	0.810	0.957			
1991	0.741	0.983	0.945	0.879	0.815	0.960				
1992	0.691	0.920	0.872	0.815	0.962					
1993	0.614	0.835	0.805	0.962						
1994	0.577	0.770	0.961							
1995	0.525	0.959								
1996	0.690									

Loss Reserve Testing: Beyond Popular Methods

Appendix C: Reserve Tests applied to Model "Actual" Ultimates

Exhibit C-61

Accident Year	Hindsight IBNR per Open + IBNR Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	806	-5,764	-4,000	-1,872	-763	-167	276	639	924	177
1988	1,989	-4,214	-2,586	-1,100	-221	428	845	1,145	221	
1989	3,279	-2,470	-1,435	-266	632	1,158	1,461	298		
1990	4,513	-994	-194	1,057	1,735	1,938	333			
1991	4,840	543	1,507	2,432	2,550	384				
1992	6,368	2,828	3,773	3,951	559					
1993	8,214	6,084	5,909	819						
1994	9,660	9,276	1,288							
1995	11,744	1,823								
1996	8,159									

Exhibit C-62

Accident Year	Hindsight IBNR per Ultimate Claim									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	616	-2,271	-1,456	-677	-246	-40	38	38	19	1
1988	1,499	-1,677	-962	-406	-73	102	118	70	5	
1989	2,446	-931	-521	-95	197	251	172	15		
1990	3,269	-366	-70	376	524	400	37			
1991	3,462	192	523	823	724	73				
1992	4,513	997	1,316	1,336	156					
1993	5,748	2,102	2,031	271						
1994	6,658	3,114	432							
1995	7,919	593								
1996	5,335									

Exhibit C-63

Accident Year	Ratio of Case Reserve to Indicated Reserve									
	Review									
	1	2	3	4	5	6	7	8	9	10
1987	0.943	1.244	1.186	1.115	1.067	1.021	0.952	0.869	0.795	0.955
1988	0.875	1.162	1.110	1.062	1.018	0.952	0.874	0.801	0.954	
1989	0.806	1.086	1.058	1.014	0.951	0.877	0.806	0.957		
1990	0.762	1.031	1.007	0.948	0.878	0.810	0.957			
1991	0.741	0.983	0.945	0.879	0.815	0.960				
1992	0.691	0.920	0.872	0.815	0.962					
1993	0.614	0.835	0.805	0.962						
1994	0.577	0.770	0.961							
1995	0.525	0.959								
1996	0.696									