Downward Bias of Using High-Low Averages for Loss Development Factors

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Abstract

This paper studies the downward bias associated with high-low averages. The bias occurs when the high-low averages are applied to data that exhibits a long-tailed property. This research included a comprehensive review of insurance industry data when 3-of-5 averages are used to determine the age-to-age development factors in setting reserves; 140 paid and incurred loss triangles from 70 insurers were compiled from the AM Best Database (1996) to analyze the downward bias by:

- line of business
- data size
- development age
- paid and incurred loss development methods.

The study assumes the age-to-age development factors are lognormally distributed. The 3-of-5 average was selected as the representative high-low average because it is commonly used by property and casualty actuaries. Results for the 3-of-5 average can be generalized to other types of high-low averages. The study also used large-scale simulations to review the effect of limited volume of data on the downward bias.

CONTENTS

1. HIGH-LOW AVERAGES

Downward Bias of Using High-Low Averages for Age-to-Age Factors

Property and casualty actuaries often use an averaging technique that excludes the same amount of data at both ends to calculate the age-to-age development factors. These will be called high-low averages in this paper.

Many types of high-low averages exist - for example, the middle 3 of the latest 5 years (3-of-5 averages) and the middle 6 of the latest 8 quarters (6-of-8 averages).

The purpose of using high-low averages is to exclude outliers and their disproportional influence on results. This requires a great deal of caution, however. According to Neter, Wasserman, and Kutner (1989):

> *"an outlying influential case should not be automatically discarded, because it may be entirely correct and simply represents an unlikely event. Discarding of such an outlying case could lead to the undesirable consequences of increased variances of some of the estimated regression coefficients."*

Applying high-low averages to calculate loss development factors will result in a systematic downward bias if the loss development factors exhibit a long-tailed property (Wu (1996)). This can be illustrated by the following example based on a lognormal assumption.

First, assume:

- At development age i, the aggregate reported loss or paid loss is equal to L_i.
- From age i to i+1, a total loss of l_{i+1} is reported or paid.
- Since insurance losses have a long-tailed property, both L_i and l_{i+1} can be approximated by lognormal distributions. That is, both $\ln(L_i)$ and $\ln(l_{i+1})$ are normally distributed. For the use of lognormal distributions to approximate insurance losses, please see Bowers, et al (1986), Finger (1976), and Hogg and Klugman (1984).

Based on the above assumptions, the age-to-age development factor from age i to i+1 can be expressed as follows:

$$D_{i,i+1} = (L_i + l_{i+1}) / L_i$$

$$\ln(D_{i,i+1}) = \ln((L_i + l_{i+1}) / L_i) = \ln(1 + l_{i+1} / L_i) = c + \ln(l_{i+1}) - \ln(L_i)$$

where c is a constant.

Since both $\ln(L_i)$ and $\ln(l_{i+1})$ are normally distributed, $\ln(D_{i,i+1})$ is normally distributed also. That is, $D_{i,i+1}$ is lognormally distributed and should have a long-tailed property:

$$\ln(D_{i,i+1}) \sim N(\mu_i, \sigma_i^2)$$

where μ_i is the mean and σ_i^2 is the variance of the normal distribution for $\ln(D_{i,i+1})$.

One advantage of assuming lognormal distributions for age-to-age development factors is that age-to-ultimate factors and, consequently, the ultimate loss estimates are also lognormally distributed:

$$UD_i = D_{i,i+1} \times D_{i+1,i+2} \times D_{i+2,i+3} \ldots$$

where

$$\ln(UD_i) = \ln(D_{i,i+1}) + \ln(D_{i+1,i+2}) + \ln(D_{i,i+1}) \ldots$$

and

$$\ln(UD_i) \sim N(\mu_i + \mu_{i+1} + \mu_{i+2} + \ldots, \sigma_i^2 + \sigma_{i+1}^2 + \sigma_{i+2}^2 + \ldots)$$

The fact age-to-age development factors may have a long tail does not go unnoticed. Hayne's study (1985) in quantifying the variability of loss reserves assumes age-to-age development factors are lognormally distributed. Kelly (1992) and McNichols (1992) also conclude a lognormal assumption is better in describing age-to-age development factors than a normal assumption, because lognormal distributions can take only positive values and their long-tailed property reflects no upper boundary for the development factors.

However, if $D_{i,i+1}$ is lognormally distributed, using high-low averages to estimate it will result in a downward bias - represented by the percentage difference between the mean and the conditional mean (given that the data lie between a specified lower and upper pair of percentile points). The bias is expressed in the following formula (detailed derivations are in Appendix A.1):

$$\text{Bias} = \frac{E(D_{i,i+1})'}{E(D_{i,i+1})} - 1 = \frac{1}{(1-2p)}[\Phi(\Phi^{-1}(1-p)-\sigma_i)) - \Phi(\Phi^{-1}(p)-\sigma_i)] - 1 \qquad (1)$$

where,

$E(D_{i,i+1})$: Expected value of $D_{i,i+1}$

$E(D_{i,i+1})'$: Expected value of $D_{i,i+1}$ given that $D_{i,i+1}$ lies between its upper and lower p percentile points: $\frac{1}{1-2p}\int_{d_1(p)}^{d_2(p)} d \times f(d)$

f(d): Probability density function for $D_{i,i+1}$

F(d): Cumulative density function for $D_{i,i+1}$

p: Percentile

$d_1(p)$: Value of $D_{i,i+1}$ when F(d) = p

$d_2(p)$: Value of $D_{i,i+1}$ when F(d) = 1-p

Φ: Standard normal distribution function, $\int_{-\infty}^{x} \frac{\exp(\frac{1}{2}x^2)}{\sqrt{2\pi}} d(x)$

Equation (1) indicates the degree of bias depends only on p and σ_i, the percentage of data being excluded and the shape factor, but not on μ_i, the location parameter. This suggests the more data excluded or the more skewed and volatile the distribution, the higher the downward bias. Exhibit 1 illustrates the downward bias graphically.

Please note that we are not limited to only the lognormal assumption. For example, another commonly used long-tailed distribution is the Pareto distribution. The bias formula similar to Equation (1) for the Pareto distribution is derived in the Appendix A.2. Further analysis indicates, for the age-to-age development factors reviewed in this study, no significant difference in bias results between the lognormal distribution and the Pareto distribution.

Modified High-Low Averages for the Correction of Downward Bias

Results from Equation (1) can be extended to the high-low averages used by property and casualty actuaries. For example, a 3-of-5 average also excludes the upper and lower 20% of the data. The only difference between Equation (1) and a high-low average is that the high-low average is based on limited volume of data and a sample distribution function, while Equation (1) is based on very large volume data and a cumulative distribution function.

Equation (1) provides a basis to correct the downward bias for the sample high-low average:

Modified High-Low Average = Sample High-Low Average / (1+Bias) (2)

where the bias is given in Equation (1).

Exhibits 2-5 displays, through an example, how to correct the downward bias for the 3-of-5 averages based on Equations (1) and (2). This example uses the total industry paid loss data for medical malpractice claims-made insurance from the AM Best Database (1996).

Exhibit 2 shows 2 types of averages: 5-years straight and 3-of-5. These are factor averages, not volume-weighted averages. Because the data covers 10 years of experience, the 3-of-5 averages can be applied only to the first 5 development ages. After the fifth development age, all years averages are used. Note the tail factor selected in Exhibit 2; 1.0515 is the ratio of incurred loss to paid loss for the earliest year in the triangle.

Results from Exhibit 2 clearly indicate the 5-year averages result in higher estimates than the 3-of-5 averages. This is consistent with assuming age-to-age loss development factors have a long-tailed property.

Fitting lognormal distributions to the age-to-age development factors in Exhibit 2 produces the parameter estimates in Exhibit 3. First, μ_i and σ_i^2 are calculated for each development age. All data in each development age is used to calculate the sample parameters, although only the latest 5 data points are used to select age-to-age development factors; this increases the credibility of the sample parameters. Parameters for the age-to-ultimate factors for a development age are the sum of all parameters of the age-to-age factors from that age to ultimate.

Given these lognormal parameter estimates, the 3-of-5 averages in Exhibit 2 can be modified to correct the downward bias for the averages. The modified 3-of-5 factors are in Exhibit 4. For example, the lognormal parameters for the 12-to-24 development factors are: μ_1 =0.8918, and σ_1^2 =0.0174. With p=20%, a bias of -0.68% is indicated for the 3-of-5 average based on Equation (1).

Exhibit 4 shows the indicated downward bias for each development age and the modified 3-of-5 averages. Exhibit 5 compares the estimated ultimate losses and reserves across 5-year averages, 3-of-5 averages, and modified 3-of-5 averages. For example, the total reserve for the 3-of-5 averages is approximately 4.4% lower than for the 5 year averages, while the total reserve for modified 3-of-5 averages is approximately 0.5% higher than for 3-of-5 averages.

The downward bias illustrated in this example is relatively minor because data is for the total industry and therefore is very large. As will be shown in Section 3, the bias will become higher for individual company data and more volatile lines of business.

Limited volume of data

As mentioned, the bias formula in Equation (1) is based on very large volume data and a cumulative distribution function, while the real-world data is limited. Note these issues regarding limited volume of data:

- Additional parameter variation is introduced because sample parameters are assumed for true parameters. There is a booth-trap procedure because excluded data are used to calculate the sample parameters, which in turn are used to calculate the degree of bias to modify the high-low averages.
- Even though the true parameters are known, the indicated bias when sample size is small will not be the same as the indicated bias when sample size is large.

Analyzing these issues through statistical theories is very difficult, if not impossible, and is beyond the scope of this study. Instead, large scale simulations have been conducted; results are in Section 3.

2. STUDY PURPOSE, DATA, AND APPROACH

Purpose

Many questions remain considering the results given in the previous section:

- Do the real-world loss development factors really exhibit a long-tailed property?
- What is the level of the downward bias when high-low averages are used in setting reserves?
- Does the downward bias vary by line of business, data size, development age, and paid and incurred loss development methods?
- What is the effect of limited volume of data on the bias?

This study attempts to answer these questions through a comprehensive review of industry data and large scale simulations.

Data

Data from the AM Best Database (1996) was gathered for the following major liability lines:

- Workers compensation
- Private passenger automobile liability
- Commercial automobile liability
- Medical malpractice - occurrence

- Medical malpractice - claims-made
- Product liability
- Other liability

For each line of business, paid loss and incurred loss triangles on an annual basis were compiled from 10 randomly selected insurers - a total of 140 triangles. The loss triangles have 10 years of experience and cover 1986 to 1995.

The data was further broken down into 2 groups, based on size:

- Group A contains large multiline and multistate insurers.
- Group B contains small local and regional companies.

Exhibit 6 shows the range of annual earned premium for the companies within each group.

Approach

Loss development procedures used to review the AM Best data were the same as those in Exhibits 2 to 5. The following list summarizes important assumptions in the approach:

- The 3-of-5 average was selected as the representative high-low average. Results for the 3-of-5 average can be extended to other types of high-low averages.

- Because the loss triangle data has only 10 years of history, the 3-of-5 averages can be applied only to the first 5 development ages. For development ages after 72 months, all years' averages were used.

- No tail development is assumed for the incurred loss method. For the paid tail, the ratio of incurred to paid loss for the oldest accident year in the triangle was used.

- All data points in each development age were used to calculate the lognormal parameters to increase credibility. However, only the latest 5 points were used to select the age-to-age development factors.

- Large scale simulations were conducted to study the effect of limited volume of data on the bias when sample parameters are assumed as the true parameters. The simulations also measure the differences between the simulated bias and the bias based on Equation (1).

3. RESULTS AND DISCUSSION

Long-Tailed Property for Age-to-Age Development Factors

The reserve indications between the 5-year averages and 3-of-5 averages for the AM Best data were compared. The comparison results by line of business, company size, and paid and incurred loss methods are in Exhibit 6.

Exhibit 6 indicates approximately 70% of the data reviewed has lower reserve indications for 3-of-5 averages. This is consistent with assuming age-to-age development factors may have a long tail and the use of high-low averages will result in a downward bias.

Exhibit 6 also suggests the long tail assumption is more prevailing for more volatile lines such as medical malpractice and product liability. On the other hand, the assumption is equally prevailing for both large and small groups and for incurred as well as paid methods.

Results by Line of Business

Exhibits 7 to 13 give 2 types of downward bias by line of business: the bias for age-to-age development factors and for reserve indications. The reserve indications include both the total reserve and the incurred but not reported reserve (IBNR). In each exhibit, the downward bias is indicated by company size and paid and incurred methods.

The indicated bias in these exhibits is based on Equation (1). For example, Exhibit 11 shows that for the malpractice claims-made data of Group A, the indicated minimum, maximum, and average downward biases associated with 3-of-5 averages for the 12-24 paid factors are -0.86%, -2.88%, and -2.06%, respectively.

The indicated bias for reserve indications is the difference between the 3-of-5 averages and modified 3-of-5 averages. For example, Exhibit 11 shows that for the malpractice claims-made data of Group A, the indicated minimum, maximum, and average downward biases for total reserves of the paid method are -0.61%, -2.86%, and -1.87%, respectively.

Exhibits 7-13 lead to the following observations:

- The indicated bias for age-to-age factors decreases as the loss data matures. For workers compensation, private passenger automobile liability, and commercial automobile liability, the bias appears insignificant after 72 months of development. On the other hand, the bias is still noticeable after 72 months for medical malpractice, product liability, and other liability.

- The bias for reserve indications can be substantial, especially for the highly volatile medical malpractice, product liability, and other liability. The use of high-low averages can easily lead to a double digit downward bias for these lines of business.

- In general, the data of small companies shows higher downward bias than the data of large companies. This is because the age-to-age factors become more volatile as data size decreases.

- While the paid development factors are larger than incurred development factors, there is no systematic difference in the bias level between paid and incurred factors. As indicated in Equation (1), the bias depends on data skewing and volatility (represented by σ_i), but not on data level or magnitude (represented by μ_i). This suggests the paid and incurred development factors are skewed to a similar degree.

Large Scale Simulations for Limited volume of data

Since real-world data is limited, it will deviate somewhat from the assumptions for Equation (1). For the limited volume of data, the true means and variances are usually unknown and the sample means and variances must used. Also, Equation (1) assumes the data size is very large, while the 3-of-5 average, for example, uses only 5 data points.

This study used large scale simulations to analyze these issues. The simulation procedures follow:

- Select a set of μ_i and σ_i. The range for μ_i is 0.1 to 2.0 and the range for σ_i is 0.002 to 1.2. These ranges are based on the AM Best data reviewed in the study.
- Generate 4,000 lognormal replicates based on the selected μ_i and σ_i. Each replicate contains 5 random data.
- Calculate sample parameters for each of the 4,000 replicates. Calculate the bias using Equation (1) and the sample parameters. Compare the result to the bias based on the true parameters of μ_i and σ_i. This is the effect of using the sample parameters.
- Finally, calculate 3-of-5 averages for the 4,000 replicates. Compare these to the bias based on Equation (1). This is the effect of limited volume of data.

Exhibit 14 suggests the bias will be understated if sample parameters are used for true parameters. For example, when $\sigma_i = 1.2$ and $\mu_i = 1.0$, the bias on average will be understated by 8.5%.

Exhibit 15 indicates that the bias is tempered somewhat for limited volume of data. For example, when $\sigma_i = 1.2$ and $\mu_i = 1.0$, the simulated bias for the 3-of-5 average is approximately 67.5% of the bias calculated by Equation (1) for large volume data.

Exhibits 14 and 15 also show that the effects of limited volume of data on bias depend primarily on σ_i, not μ_i. The effects diminish quickly as σ_i decreases.

4. SUMMARY AND CONCLUSIONS

The current study presents strong evidence, through a comprehensive review of industry data, that a downward bias will occur when high-low averages are used to determine age-to-age development factors. The review results display the level of the bias by line of business, development age, data size, and paid and incurred methods. These results indicate the downward bias can be substantial, especially for small companies and highly volatile lines.

Equations (1) and (2) provide a basis to quantify and correct the bias. Equation (1) is based on large volume data, while only limited volume of data is available for most real-world applications. The simulation results suggest the bias for limited volume of data is somewhat lower than indicated by Equation (1).

Exhibit 16 further provides a quick bias estimate instead of Equation (1) and simulations. For example, if the average of loss development factors at a development age is approximately 1.5 and the maximum factor is approximately 3.0, the potential downward bias of using the 3-of-5 average for the development factors will be -7% to -12%.

The real-world data that actuaries deal with daily may have an even higher bias than indicated in this study. For example, the bias will increase if less mature data or quarterly data is used, and the bias for the tail may be significant.

As in most research, many assumptions used in this study are ideal. Attempts to study the bias under more complicated assumptions are beyond this scope because they require advanced statistical knowledge. They can be the topics for future research, however. For example, explaining results for limited volume of data call for the knowledge of Order Statistics. Another interesting study could cover the bias when loss development factors between development ages are highly correlated.

5. REFERENCES

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APPENDIX

DOWNWARD BIAS FOR 2 LONG-TAILED DISTRIBUTIONS

This Appendix shows the derivations of downward bias based on cumulative distribution functions for 2 long-tailed distributions- lognormal and Pareto. Many details of these distributions can be found in Hogg and Klugman (1984) or other statistical texts.

The following lists global notations for the 2 distributions:

E(X): Expected value for random variable X

E(X)': Expected value of X when excluding the upper p% and lower p% of data

F(x): Cumulative probability function

f(x): Probability density function

p: Percentile

x_1: Value of X when F(x) = p

x_2: Value of X when F(x) = 1-p

F: Standard normal distribution function = $\int_{-\infty}^{x} \frac{\exp(\frac{1}{2}x^2)}{\sqrt{2\pi}} dx$

f: Standard normal density function = $\frac{\exp(-\frac{1}{2}x^2)}{\sqrt{2\pi}}$

A.1 Lognormal Distribution

- Probability Density Function:

$$f(x) = \frac{\exp(\frac{1}{2}(\frac{\ln x - \mu}{\sigma})^2)}{x\sigma\sqrt{2\pi}}$$

- Cumulative Probability Function:

$$F(x) = \int_0^{\infty} \frac{\exp(\frac{1}{2}(\frac{\ln x - \mu}{\sigma})^2)}{x\sigma\sqrt{2\pi}} dx$$

Let $x = e^{\sigma y + \mu}$, then $y = \frac{\ln x - \mu}{\sigma}$, and $dx = e^{\sigma y + \mu}\sigma dy$.

$$F(x) = \int_{-\infty}^{\frac{\ln x - \mu}{\sigma}} \frac{e^{-y^2/2} e^{\sigma y + \mu}\sigma}{e^{\sigma y + \mu}\sigma\sqrt{2\pi}} dy = \Phi(\frac{\ln x - \mu}{\sigma})$$

$$F(x_1) = \Phi(\frac{\ln x_1 - \mu}{\sigma}) = p, \ x_1 = e^{(\Phi^{-1}(p)\sigma + \mu)}$$

$$F(x_2) = \Phi(\frac{\ln x_2 - \mu}{\sigma}) = 1-p, \ x_2 = e^{(\Phi^{-1}(1-p)\sigma + \mu)}$$

- Expected Value of X:

$$E(X) = \int_0^{\infty} x \frac{e^{-\frac{1}{2}(\frac{\ln x - \mu}{\sigma})^2}}{x\sigma\sqrt{2\pi}} dx = \int_0^{\infty} \frac{e^{-\frac{1}{2}(\frac{\ln x - \mu}{\sigma})^2}}{\sigma\sqrt{2\pi}} dx$$

Let $y = \frac{\ln x - \mu - \sigma^2}{\sigma}$, then $x = e^{\sigma y + \mu + \sigma^2}$, and $dx = e^{\sigma y + \mu + \sigma^2}\sigma dy$

$$E(X) = \int_0^{\infty} \frac{e^{-\frac{1}{2}(y+\sigma)^2} e^{\sigma y + \mu + \sigma^2}\sigma}{\sigma\sqrt{2\pi}} dx = e^{(\mu + \frac{1}{2}\sigma^2)} \int_0^{\infty} \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}} dx = e^{(\mu + \frac{1}{2}\sigma^2)}$$

- Expected Value of X when Excluding Upper p% and Lower p% of Data:

$$E(X)' = \int_{x_1}^{x_2} x \frac{e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2}}{(1-2p)x\sigma\sqrt{2\pi}}dx = \int_{x_1}^{x_2} \frac{e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2}}{(1-2p)\sigma\sqrt{2\pi}}dx$$

Let y $= \frac{\ln x-\mu-\sigma^2}{\sigma}$, then $x = e^{\sigma y+\mu+\sigma^2}$, and $dx = e^{\sigma y+\mu+\sigma^2}\sigma dy$.

$$E(X)' = \frac{e^{(\mu+\frac{1}{2}\sigma^2)}}{(1-2p)} \int_{\frac{\ln x_1-\mu-\sigma^2}{\sigma}}^{\frac{\ln x_2\mu-\sigma^2}{\sigma}} \frac{e^{-\frac{1}{2}y^2}}{\sqrt{2\pi}}dx$$

$$= \frac{e^{(\mu+\frac{1}{2}\sigma^2)}}{(1-2p)}(\Phi(\frac{\ln x_2-\mu-\sigma^2}{\sigma})-\Phi(\frac{\ln x_1-\mu-\sigma^2}{\sigma}))$$

$x_1 = e^{(\Phi^{-1}(p)\sigma+\mu)}$ and $x_2 = e^{(\Phi^{-1}(1-p)\sigma+\mu)}$, then

$$E(X)' = \frac{e^{(\mu+\frac{1}{2}\sigma^2)}}{(1-2p)}[\Phi(\Phi^{-1}(1-p)-\sigma))-\Phi(\Phi^{-1}(p)-\sigma)]$$

- Downward Bias for Excluding Upper p% and Lower p% of Data:

$$\text{Bias} = \frac{E(x)'}{E(x)} - 1 = \frac{1}{(1-2p)}[\Phi(\Phi^{-1}(1-p)-\sigma))-\Phi(\Phi^{-1}(p)-\sigma)]-1$$

The above result indicates the degree of bias depends only on p, the percentage of data being excluded, and σ, the shape factor. The bias does not depend on μ, the location parameter.

A.2 Pareto Distribution

- Probability Density Function:

$$f(x) = \alpha\lambda^{a}(\lambda+x)^{-a-1},\ x>0$$

- Cumulative Probability Function:

$$F(x) = \int_0^x \alpha\lambda^{\alpha}(\lambda+x)^{-\alpha-1}dx = -\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}\Big|_0^x = 1-\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}$$

$F(x_1) = p$, then $x_1 = \lambda\times\left(\frac{1}{(1-p)^{1/\alpha}}-1\right)$

$F(x_2) = 1-p$, then $x_2 = \lambda\times\left(\frac{1}{p^{1/\alpha}}-1\right)$

- Expected Value of X:

$$E(X) = \int_0^\infty x\alpha\lambda^{\alpha}(\lambda+x)^{-\alpha-1}dx = -\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}x\Big|_0^\infty + \int_0^\infty \lambda^{\alpha}(\lambda+x)^{-\alpha}dx$$

$$= \int_0^\infty \lambda^{\alpha}(\lambda+x)^{-\alpha}dx = -\frac{\lambda}{\alpha-1}\left(\frac{\lambda}{\lambda+x}\right)^{-(\alpha-1)}\Big|_0^\infty = \frac{\lambda}{\alpha-1}$$

- Expected Value of X when Excluding Upper p% and Lower p% of Data:

$$E(X)' = \int_{x_1}^{x_2} x\frac{\alpha\lambda^{\alpha}(\lambda+x)^{-\alpha-1}}{1-2p}dx = -x\frac{\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}}{1-2p}\Big|_{x_1}^{x_2} + \int_{x_1}^{x_2}\frac{\lambda^{\alpha}(\lambda+x)^{-\alpha}}{1-2p}dx$$

$$= = -x\frac{\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}}{1-2p}\Big|_{x_1}^{x_2} - \frac{\lambda\left(\frac{\lambda}{\lambda+x}\right)^{(\alpha-1)}}{(\alpha-1)(1-2p)}\Big|_{x_1}^{x_2}$$

Since $\frac{\lambda}{\lambda+x_1} = \frac{\lambda}{\lambda+\lambda\left(\frac{1}{(1-p)^{1/\alpha}}-1\right)} = (1-p)^{1/\alpha}$, and $\frac{\lambda}{\lambda + x_2} = \frac{\lambda}{\lambda + \lambda(\frac{1}{p^{1/\alpha}} - 1)} = p^{1/\alpha}$,

then,

$$E(X)' = \frac{\lambda}{1-2p}[-p^{\frac{\alpha-1}{\alpha}}(1-p^{\frac{1}{\alpha}}) + (1-p)^{\frac{\alpha-1}{\alpha}}(1-(1-p)^{\frac{1}{\alpha}}) - \frac{p^{\frac{\alpha-1}{\alpha}}}{\alpha-1} + \frac{(1-p)^{\frac{\alpha-1}{\alpha}}}{\alpha-1}]$$

$$= \frac{\lambda}{(\alpha-1)(1-2p)}[\alpha(-p^{\frac{\alpha-1}{\alpha}} + (1-p)^{\frac{\alpha-1}{\alpha}}) - (\alpha-1)(1-2p)]$$

- Downward Bias for Excluding Upper p% and Lower p% of Data:

$$\text{Bias} = \frac{E(X)'}{E(X)} - 1 = \frac{\alpha}{(1-2p)}[-p^{\frac{\alpha-1}{\alpha}} + (1-p)^{\frac{\alpha-1}{\alpha}} - (1-2p)]$$

Again, the degree of bias for Pareto distribution depends only on p and α, the percentage of excluded data and the shape factor, but not on λ, the location parameter.

EXHIBITS

Exhibit 1. Downward Bias of High-Low Average for A Lognormal Distribution

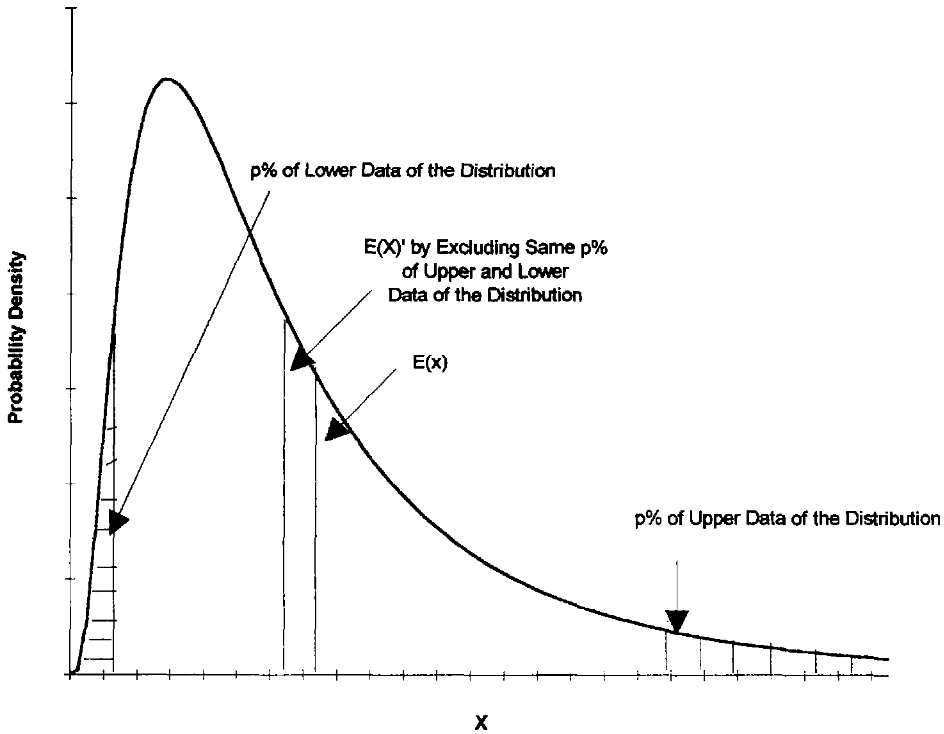

Exhibit 2. Paid Loss and Loss Development Factor Triangles for Industry Medical Malpractice Claims-Made Insurance*

Paid Losses: (in Millions)

Accident Year	Earned Premium	Development Age, Month 12	24	36	48	60	72	84	96	108	120
1986	$ 14,322	$ 559	$ 1,532	$ 2,807	$ 4,082	$ 5,299	$ 6,130	$ 6,674	$ 7,104	$ 7,362	$ 7,505
1987	$ 17,371	$ 556	$ 1,737	$ 3,075	$ 4,395	$ 5,680	$ 6,497	$ 7,105	$ 7,504	$ 7,695	
1988	$ 17,340	$ 1,006	$ 2,185	$ 3,676	$ 5,445	$ 6,624	$ 7,456	$ 8,063	$ 8,410		
1989	$ 16,493	$ 1,105	$ 2,441	$ 4,470	$ 6,053	$ 7,257	$ 8,214	$ 8,791			
1990	$ 16,582	$ 1,061	$ 2,885	$ 4,643	$ 6,318	$ 7,628	$ 8,507				
1991	$ 16,272	$ 1,351	$ 2,896	$ 4,751	$ 6,411	$ 7,632					
1992	$ 15,785	$ 1,326	$ 2,904	$ 4,830	$ 6,567						
1993	$ 15,902	$ 1,304	$ 3,085	$ 4,898							
1994	$ 16,853	$ 1,348	$ 3,320								
1995	$ 17,102	$ 1,402									

Age-to-Age Factors:

Accident Year	Earned Premium	Development Age, Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail**
1986	$ 14,322	2.7436	1.8318	1.4541	1.2982	1.1568	1.0888	1.0644	1.0363	1.0195	
1987	$ 17,371	3.1250	1.7700	1.4294	1.2925	1.1437	1.0936	1.0562	1.0255		
1988	$ 17,340	2.1724	1.6825	1.4811	1.2166	1.1257	1.0814	1.0430			
1989	$ 16,493	2.2090	1.8311	1.3542	1.1989	1.1318	1.0703				
1990	$ 16,582	2.7188	1.6092	1.3607	1.2073	1.1152					
1991	$ 16,272	2.1446	1.6404	1.3493	1.1904						
1992	$ 15,785	2.1905	1.6630	1.3595							
1993	$ 15,902	2.3659	1.5876								
1994	$ 16,853	2.4625									
1995	$ 17,102										
Age-to-Age Development Factors:											
5 Years Average***		2.3764	1.6663	1.3810	1.2211	1.1346	1.0835	1.0545	1.0309	1.0195	1.0515
3-of-5 Average***		2.3396	1.6376	1.3581	1.2076	1.1337	1.0835	1.0545	1.0309	1.0195	1.0515
Age-to-Ultimate Development Factors:											
5 Years Average***		9.5669	4.0257	2.4160	1.7495	1.4327	1.2627	1.1654	1.1051	1.0720	1.0515
3-of-5 Average***		8.9953	3.8448	2.3479	1.7287	1.4315	1.2627	1.1654	1.1051	1.0720	1.0515

* Industry total data AM Best Database (1996).

** The tail factor of 1.0515 is the ratio of incurred to paid loss for 1986.

*** For the last 4 development ages, straight averages are used for 3-of-5 averages.

Exhibit 3. Lognormal Parameters for Loss Development Factors

Natural Logarithm Transformation of the Age-to-Age Factors in Exhibit 2:

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
	Development Age, Months								
1986	1.0093	0.6053	0.3744	0.2610	0.1456	0.0851	0.0624	0.0356	0.0193
1987	1.1394	0.5710	0.3572	0.2566	0.1343	0.0895	0.0547	0.0251	
1988	0.7758	0.5203	0.3928	0.1960	0.1184	0.0783	0.0421		
1989	0.7925	0.6049	0.3032	0.1814	0.1238	0.0679			
1990	1.0002	0.4757	0.3080	0.1884	0.1090				
1991	0.7629	0.4950	0.2996	0.1743					
1992	0.7841	0.5086	0.3071						
1993	0.8611	0.4622							
1994	0.9012								
1995									
Age-to-Age Development Factors:									
Lognormal Mean - All-Year Average	0.8918	0.5304	0.3346	0.2096	0.1262	0.0802	0.0531	0.0304	0.0193
Logonormal Variance - All-Year Average	0.0174	0.0032	0.0015	0.0015	0.0002	0.0001	0.0001	0.0001	0.0000
Age-to-Ultimate Development Factors:									
Lognormal Mean - All-Year Average	2.2756	1.3838	0.8534	0.5188	0.3091	0.1829	0.1027	0.0497	0.0193
Logonormal Variance - All-Year Average	0.0241	0.0067	0.0035	0.0020	0.0004	0.0002	0.0002	0.0001	0.0000

Exhibit 4. Modified High-Low Averages for Loss Development Factors

Age-to-Age Factors in Exhibit 2:

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
1986	2.7436	1.8318	1.4541	1.2982	1.1568	1.0888	1.0644	1.0363	1.0195	
1987	3.1250	1.7700	1.4294	1.2925	1.1437	1.0936	1.0562	1.0255		
1988	2.1724	1.6825	1.4811	1.2166	1.1257	1.0814	1.0430			
1989	2.2090	1.8311	1.3542	1.1989	1.1318	1.0703				
1990	2.7188	1.6092	1.3607	1.2073	1.1152					
1991	2.1446	1.6404	1.3493	1.1904						
1992	2.1905	1.6630	1.3595							
1993	2.3659	1.5876								
1994	2.4625									
1995										
Age-to-Age Development Factors:										
5-Year Average	2.3764	1.6663	1.3810	1.2211	1.1346	1.0835	1.0545	1.0309	1.0195	1.0515
Lognormal Parameters from Exhibit 3:										
Lognormal Mean - All-Year Average	0.8918	0.5304	0.3346	0.2096	0.1262	0.0802	0.0531	0.0304	0.0193	
Logonormal Variance - All-Year Average	0.0174	0.0032	0.0015	0.0015	0.0002	0.0001	0.0001	0.0001	0.0000	
3-of-5 Average	2.3396	1.6376	1.3581	1.2076	1.1337	1.0835	1.0545	1.0309	1.0195	1.0515
% of High and Low Data Excluded	20.0%	20.0%	20.0%	20.0%	20.0%					
Indicated Downward Bias	-0.68%	-0.12%	-0.06%	-0.06%	-0.01%					
Modified 3-of-5 Average	2.3557	1.6396	1.3590	1.2083	1.1338	1.0835	1.0545	1.0309	1.0195	1.0515
Age-to-Ultimate Development Factors:										
5-Year Average	9.5669	4.0257	2.4160	1.7495	1.4327	1.2627	1.1654	1.1051	1.0720	1.0515
3-of-5 Average	8.9953	3.8448	2.3479	1.7287	1.4315	1.2627	1.1654	1.1051	1.0720	1.0515
Modified 3-of-5 Average	9.0799	3.8545	2.3509	1.7299	1.4317	1.2627	1.1654	1.1051	1.0720	1.0515

Development Age, Months

Exhibit 5. *Comparison of Ultimate Losses and Reserves Across Different Averaging Techniques*

(in Millions)

		Age-to-Ultimate Loss Development Factors			Ultimate Losses			Total Reserves		
Accident Year	Undeveloped Paid Losses	5-Year Average	3-of-5 Average	Modified 3-of-5 Average	5-Year Average	3-of-5 Average	Modified 3-of-5 Average	5-Year Average	3-of-5 Average	Modified 3-of-5 Average
1986	$ 7,505	1.0515	1.0515	1.0515	$ 7,891	$ 7,891	$ 7,891	$ 387	$ 387	$ 387
1987	$ 7,695	1.0720	1.0720	1.0720	$ 8,249	$ 8,249	$ 8,249	$ 554	$ 554	$ 554
1988	$ 8,410	1.1051	1.1051	1.1051	$ 9,294	$ 9,294	$ 9,294	$ 884	$ 884	$ 884
1989	$ 8,791	1.1654	1.1654	1.1654	$ 10,244	$ 10,244	$ 10,244	$ 1,454	$ 1,454	$ 1,454
1990	$ 8,507	1.2627	1.2627	1.2627	$ 10,741	$ 10,741	$ 10,741	$ 2,234	$ 2,234	$ 2,234
1991	$ 7,632	1.4327	1.4315	1.4317	$ 10,934	$ 10,925	$ 10,926	$ 3,302	$ 3,293	$ 3,294
1992	$ 6,567	1.7495	1.7287	1.7299	$ 11,488	$ 11,352	$ 11,359	$ 4,922	$ 4,785	$ 4,793
1993	$ 4,898	2.4160	2.3479	2.3509	$ 11,833	$ 11,499	$ 11,514	$ 6,935	$ 6,602	$ 6,616
1994	$ 3,320	4.0257	3.8448	3.8545	$ 13,366	$ 12,765	$ 12,797	$ 10,046	$ 9,445	$ 9,477
1995	$ 1,402	9.5669	8.9953	9.0799	$ 13,416	$ 12,615	$ 12,733	$ 12,014	$ 11,212	$ 11,331
Total:	$ 64,726				$ 107,457	$ 105,576	$ 105,749	$ 42,731	$ 40,850	$ 41,024

Exhibit 6. AM Best Data

Group A: Multistate, Multiline Insurance Companies/Groups:

	Number of Companies Sampled	Annual Earned Premium From 1986 to 1995 (in Millions) Minimum	Maximum	Average	Data With Lower Reserve Indications for 3-of-5 Averages* Paid Loss Method	Incurred Loss Method
Workers Compensation	5	$ 426	$ 1,823	$ 1,029	3	3
Private Passenger Automobile Liability	5	$ 543	$ 14,126	$ 3,651	2	3
Commercial Automobile Liability	5	$ 151	$ 682	$ 354	2	2
Medical Malpractice - Occurrence	5	$ 14	$ 270	$ 71	5	5
Medical Malpractice - Claims-Made	5	$ 44	$ 700	$ 186	4	3
Product Liability	5	$ 43	$ 218	$ 115	5	5
Other Liability	5	$ 199	$ 1,221	$ 611	3	3
Total	35				24	24

Group B: Regional or Single State Insurance Companies:

	Number of Companies Sampled	Annual Earned Premium From 1986 to 1995 (in Millions) Minimum	Maximum	Average	Data With Lower Reserve Indications for 3-of-5 Averages* Paid Loss Method	Incurred Loss Method
Workers Compensation	5	$ 14	$ 137	$ 60	2	3
Private Passenger Automobile Liability	5	$ 26	$ 122	$ 62	3	3
Commercial Automobile Liability	5	$ 19	$ 99	$ 47	3	2
Medical Malpractice - Occurrence	5	$ 2	$ 53	$ 17	5	5
Medical Malpractice - Claims-Made	5	$ 20	$ 64	$ 39	5	3
Product Liability	5	$ 5	$ 50	$ 29	5	5
Other Liability	5	$ 12	$ 98	$ 54	5	3
Total	35				28	24

Group A and Group B Combined:

	Number of Companies Sampled	Annual Earned Premium From 1986 to 1995 (in Millions) Minimum	Maximum	Average	Data With Lower Reserve Indications for 3-of-5 Averages* Paid Loss Method	Incurred Loss Method
Workers Compensation	10	$ 14	$ 1,823	-	5	6
Private Passenger Automobile Liability	10	$ 26	$ 14,126	-	5	6
Commercial Automobile Liability	10	$ 19	$ 682	-	5	4
Medical Malpractice - Occurrence	10	$ 2	$ 270	-	10	10
Medical Malpractice - Claims-Made	10	$ 20	$ 700	-	9	6
Product Liability	10	$ 5	$ 218	-	10	10
Other Liability	10	$ 12	$ 1,221	-	8	6
Total	70				52	48

* Reserve indications were compared between 5-year averages and 3-of-5 averages. This is the data where 3-of-5 averages have a lower reserve indication.

Exhibit 7. Review Results of AM Best Workers Compensation Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-0.25%	0.00%	0.00%	0.00%	0.00%
	Maximum	-2.68%	-0.10%	-0.02%	-0.02%	0.00%
	Average	-0.80%	-0.05%	-0.01%	-0.01%	0.00%
Group B - Small to Medium Companies	Miminum	-0.03%	-0.02%	-0.01%	0.00%	0.00%
	Maximum	-0.72%	-0.22%	-0.20%	-0.09%	-0.09%
	Average	-0.25%	-0.07%	-0.05%	-0.02%	-0.02%

Incurred 3-of-5 Averages:		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-0.10%	-0.01%	-0.01%	-0.01%	-0.01%
	Maximum	-0.78%	-0.06%	-0.05%	-0.03%	-0.02%
	Average	-0.37%	-0.03%	-0.02%	-0.02%	-0.01%
Group B - Small to Medium Companies	Miminum	-0.07%	-0.02%	-0.01%	0.00%	0.00%
	Maximum	-1.07%	-0.16%	-0.13%	-0.05%	-0.02%
	Average	-0.57%	-0.10%	-0.05%	-0.02%	-0.01%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.05%	-0.11%
	Maximum	-1.37%	-2.92%
	Average	-0.37%	-0.85%
Group B - Small to Medium Companies	Miminum	-0.06%	-0.15%
	Maximum	-1.37%	-3.63%
	Average	-0.38%	-0.96%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.11%	-0.32%
	Maximum	-0.30%	-0.77%
	Average	-0.22%	-0.54%
Group B - Small to Medium Companies	Miminum	-0.09%	-0.32%
	Maximum	-0.73%	-1.73%
	Average	-0.45%	-1.04%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.

Exhibit 6. Review Results of AM Best Private Passenger Automobile Liability Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Minimum	-0.04%	0.00%	0.00%	0.00%	0.00%
	Maximum	-0.22%	-0.01%	-0.02%	0.00%	0.00%
	Average	-0.08%	-0.01%	-0.01%	0.00%	0.00%
Group B - Small to Medium Companies	Minimum	-0.09%	-0.01%	0.00%	0.00%	0.00%
	Maximum	-0.39%	-0.14%	-0.04%	-0.03%	-0.02%
	Average	-0.16%	-0.04%	-0.02%	-0.01%	0.00%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Minimum	-0.01%	0.00%	0.00%	0.00%	0.00%
	Maximum	-0.14%	-0.02%	-0.01%	0.00%	0.00%
	Average	-0.06%	-0.01%	0.00%	0.00%	0.00%
Group B - Small to Medium Companies	Minimum	-0.03%	-0.02%	-0.01%	0.00%	0.00%
	Maximum	-0.20%	-0.07%	-0.04%	-0.03%	-0.01%
	Average	-0.13%	-0.04%	-0.02%	-0.01%	0.00%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Minimum	-0.04%	-0.08%
	Maximum	-0.17%	-0.38%
	Average	-0.08%	-0.21%
Group B - Small to Medium Companies	Minimum	-0.11%	-0.59%
	Maximum	-0.59%	-1.55%
	Average	-0.27%	-0.98%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Minimum	-0.03%	-0.08%
	Maximum	-0.13%	-1.93%
	Average	-0.08%	-0.56%
Group B - Small to Medium Companies	Minimum	-0.08%	-0.23%
	Maximum	-2.31%	-7.39%
	Average	-0.60%	-2.14%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages

Exhibit 9. Review Results of AM Best Commercial Automobile Liability Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-0.03%	-0.01%	0.00%	0.00%	0.00%
	Maximum	-1.77%	-0.16%	-0.03%	-0.04%	-0.03%
	Average	-0.56%	-0.07%	-0.02%	-0.02%	-0.01%
Group B - Small to Medium Companies	Miminum	-0.10%	-0.15%	-0.02%	-0.01%	0.00%
	Maximum	-1.21%	-0.43%	-0.13%	-0.07%	-0.07%
	Average	-0.46%	-0.22%	-0.07%	-0.03%	-0.02%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-0.04%	-0.02%	-0.01%	0.00%	0.00%
	Maximum	-0.91%	-0.18%	-0.11%	-0.06%	-0.02%
	Average	-0.35%	-0.07%	-0.04%	-0.02%	-0.01%
Group B - Small to Medium Companies	Miminum	-0.16%	-0.04%	-0.01%	-0.01%	0.00%
	Maximum	-0.48%	-0.21%	-0.06%	-0.06%	-0.01%
	Average	-0.31%	-0.09%	-0.02%	-0.03%	-0.01%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.06%	-0.09%
	Maximum	-1.31%	-2.17%
	Average	-0.50%	-0.98%
Group B - Small to Medium Companies	Miminum	-0.39%	-0.77%
	Maximum	-1.32%	-7.32%
	Average	-0.78%	-2.66%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.07%	-0.12%
	Maximum	-0.92%	-1.85%
	Average	-0.29%	-0.66%
Group B - Small to Medium Companies	Miminum	-0.22%	-0.59%
	Maximum	-3.63%	-10.37%
	Average	-1.11%	-4.57%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.

Exhibit 10. Review Results of AM Best Medical Malpractice - Occurrence Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-3.84%	-2.02%	-0.34%	-0.12%	-0.09%
	Maximum	-23.00%	-14.79%	-6.24%	-2.01%	-1.55%
	Average	-14.39%	-5.46%	-1.96%	-0.87%	-0.52%
Group B - Small to Medium Companies	Miminum	-10.30%	-1.22%	-0.51%	-0.10%	-0.14%
	Maximum	-22.99%	-9.79%	-2.25%	-2.73%	-0.99%
	Average	-15.75%	-5.75%	-1.31%	-0.92%	-0.37%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-0.68%	-0.57%	-0.28%	-0.12%	-0.07%
	Maximum	-30.02%	-21.60%	-2.70%	-1.33%	-0.69%
	Average	-12.84%	-7.67%	-0.96%	-0.52%	-0.30%
Group B - Small to Medium Companies	Miminum	-0.23%	-0.32%	-0.27%	-0.14%	-0.07%
	Maximum	-7.88%	-6.08%	-1.33%	-3.61%	-4.88%
	Average	-4.69%	-2.09%	-0.93%	-1.13%	-1.52%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-3.19%	-9.40%
	Maximum	-13.49%	-24.56%
	Average	-9.92%	-15.81%
Group B - Small to Medium Companies	Miminum	-4.19%	-8.20%
	Maximum	-18.89%	-39.64%
	Average	-13.58%	-23.67%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-1.14%	-5.43%
	Maximum	-60.72%	-68.37%
	Average	-17.50%	-22.11%
Group B - Small to Medium Companies	Miminum	-0.76%	-1.35%
	Maximum	-43.22%	-283.92%
	Average	-16.65%	-91.94%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.

Exhibit 11. Review Results of AM Best Medical Malpractice - Claims-Made Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-0.86%	-0.10%	-0.08%	-0.04%	0.00%
	Maximum	-2.88%	-0.44%	-0.63%	-0.60%	-0.22%
	Average	-2.06%	-0.28%	-0.26%	-0.21%	-0.12%
Group B - Small to Medium Companies	Miminum	-1.45%	-0.39%	-0.11%	-0.05%	-0.01%
	Maximum	-6.95%	-2.31%	-1.04%	-0.24%	-0.78%
	Average	-4.49%	-1.30%	-0.39%	-0.10%	-0.21%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-0.27%	-0.19%	-0.12%	-0.03%	0.00%
	Maximum	-2.33%	-0.94%	-0.44%	-0.24%	-0.06%
	Average	-0.95%	-0.44%	-0.27%	-0.11%	-0.03%
Group B - Small to Medium Companies	Miminum	-0.49%	-0.07%	-0.07%	-0.04%	-0.03%
	Maximum	-1.45%	-0.36%	-0.32%	-0.26%	-0.54%
	Average	-0.98%	-0.26%	-0.17%	-0.12%	-0.16%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.61%	-3.10%
	Maximum	-2.86%	-13.79%
	Average	-1.87%	-6.82%
Group B - Small to Medium Companies	Miminum	-3.05%	-3.90%
	Maximum	-4.28%	-68.72%
	Average	-3.89%	-20.40%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.52%	-1.27%
	Maximum	-3.64%	-8.05%
	Average	-1.41%	-4.99%
Group B - Small to Medium Companies	Miminum	-1.41%	-0.94%
	Maximum	-7.36%	-51.12%
	Average	-3.01%	-13.71%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** *The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.*

Exhibit 12. Review Results of AM Best Product Liability Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-2.44%	-1.45%	-1.02%	-0.30%	-0.16%
	Maximum	-42.19%	-35.08%	-10.36%	-2.04%	-7.65%
	Average	-17.40%	-9.39%	-2.93%	-1.00%	-1.73%
Group B - Small to Medium Companies	Miminum	-1.44%	-0.70%	-0.13%	-0.16%	-0.03%
	Maximum	-13.52%	-5.34%	-3.33%	-1.72%	-0.90%
	Average	-7.08%	-2.59%	-1.19%	-0.62%	-0.26%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-1.42%	-1.00%	-0.17%	-0.18%	-0.09%
	Maximum	-27.35%	-17.13%	-2.49%	-3.51%	-4.15%
	Average	-18.17%	-7.00%	-1.34%	-1.02%	-1.15%
Group B - Small to Medium Companies	Miminum	-4.23%	-0.85%	-0.50%	-0.34%	-0.06%
	Maximum	-21.73%	-6.71%	-4.27%	-3.70%	-1.83%
	Average	-9.84%	-3.34%	-2.64%	-1.70%	-0.73%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-3.04%	-6.11%
	Maximum	-68.50%	-77.61%
	Average	-22.20%	-27.14%
Group B - Small to Medium Companies	Miminum	-1.59%	10.47%
	Maximum	-5.82%	-15.54%
	Average	-3.26%	-1.52%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-1.94%	-4.63%
	Maximum	-39.88%	-45.01%
	Average	-22.00%	-28.70%
Group B - Small to Medium Companies	Miminum	-1.55%	-5.61%
	Maximum	-12.89%	-35.88%
	Average	-5.48%	-22.19%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.

Exhibit 13. Review Results of AM Best Other Liability Data

Indicated Downward Bias for 3-of-5 Age-to-Age Factors*:

Paid 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	60-72 Months
Group A - Large Companies	Miminum	-0.30%	-0.12%	-0.04%	-0.05%	-0.02%
	Maximum	-21.90%	-2.04%	-0.41%	-0.21%	-0.23%
	Average	-7.16%	-0.63%	-0.17%	-0.12%	-0.09%
Group B - Small to Medium Companies	Miminum	-1.03%	-0.40%	-0.17%	-0.03%	-0.02%
	Maximum	-8.18%	-3.97%	-4.41%	-0.67%	-0.24%
	Average	-2.98%	-2.28%	-1.29%	-0.33%	-0.10%

Incurred 3-of-5 Averages		12-24 Months	24-36 Months	36-48 Months	48-60 Months	48-60 Months
Group A - Large Companies	Miminum	-0.12%	-0.09%	-0.03%	-0.02%	-0.01%
	Maximum	-3.31%	-0.59%	-0.16%	-0.11%	-0.10%
	Average	-1.23%	-0.29%	-0.09%	-0.07%	-0.05%
Group B - Small to Medium Companies	Miminum	-0.42%	-0.38%	-0.07%	-0.05%	-0.02%
	Maximum	-21.96%	-2.24%	-1.53%	-0.50%	-0.32%
	Average	-8.06%	-0.87%	-0.47%	-0.20%	-0.12%

Indicated Downward Bias for 3-of-5 Reserve Indications:**

Paid Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.70%	-0.91%
	Maximum	-11.64%	-27.59%
	Average	-3.90%	-8.33%
Group B - Small to Medium Companies	Miminum	-1.47%	-3.91%
	Maximum	-14.28%	-21.24%
	Average	-5.27%	-8.29%

Incurred Loss Development Method		Total Reserves	IBNR Reserves
Group A - Large Companies	Miminum	-0.46%	-0.80%
	Maximum	-1.99%	-2.85%
	Average	-1.01%	-1.76%
Group B - Small to Medium Companies	Miminum	-1.14%	-2.08%
	Maximum	-9.29%	-18.19%
	Average	-4.45%	-9.50%

* The indicated downward bias for 3-of-5 factors is based on Equation (1).

** The indicated downward bias for reserves is the difference in reserve indications between 3-of-5 averages and modified 3-of-5 averages.

Exhibit 14. Effect of Sample Parameters

Ratio of Average Bias Based on Simulated Sample Parameters vs. True Parameters

		μ			
		2.000	1.000	0.500	0.100
	1.200	90.6%	91.5%	91.2%	91.8%
	0.900	93.2%	93.2%	94.9%	94.1%
σ	0.500	97.5%	97.7%	97.3%	97.9%
	0.100	99.5%	99.9%	99.5%	99.6%
	0.050	100.2%	98.8%	100.4%	100.9%
	0.002	99.4%	100.6%	100.9%	97.9%

Exhibit 15. Downward Bias for Limited Volume Data

Ratio of Simulated Bias to Bias Based on Equation (1) for 3-of-5 Averages

		μ			
		2.000	1.000	0.500	0.100
σ	1.200	68.3%	67.5%	67.4%	67.1%
	0.900	80.7%	80.2%	80.6%	80.6%
	0.500	93.1%	92.8%	93.6%	93.8%
	0.100	99.8%	99.8%	99.9%	99.7%
	0.050	99.9%	99.9%	99.9%	99.9%
	0.002	100.0%	100.0%	100.0%	100.0%

Exhibit 16. Downward Bias Level of 3-to-5 Averages *

Maximum of the Data	Average of the Data: 1.0	1.2	1.5	2.0	3.0	5.0	7.0	10.0
1.2	-0.3% ~ -0.5%							
1.5	-1.0% ~ -2.0%	-0.5% ~ -1.0%						
2.0	-3.0% ~ -7.0%	-2.0% ~ -5.0%	-1.0% ~ -2.0%					
3.0	-6.0% ~ -15.0%	-6.0% ~ -12.0%	-4.0% ~ -10.0%	-2.5% ~ -4.0%				
5.0			-10.0% ~ -20.0%	-7.0% ~ -15.0%	-3.0% ~ -6.0%			
7.0				-10.0% ~ -20.0%	-5.0% ~ -9.0%	-1.0% ~ -3.0%		
10.0					-12.0% ~ -25.0%	-3.0% ~ -8.0%	-1.0% ~ -3.0%	
15.0						-10.0% ~ -25.0%	-4.0% ~ -7.0%	-2.0% ~ -5.0%
30.0							-10.0% ~ -23.0%	-10.0% ~ -20.0%
50.0								-13.0% ~ -25.0%

* This exhibit provides approximations of the downward bias for 3-to-5 averages. For example, the bias is approximately -7% to -15% if 3-of-5 averages are applied to data where the average is 2.0 and the maximum is 5.0.