Reserving For Excess Layers: A Guide to Practical Reserving Applications

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Actuaries analyzing reserves for excess insurance layers are confronted with circumstances distinctly different from those faced in primary layer reserving. Although many excess reserving techniques are similar to primary techniques, the low-frequency/high-severity nature of excess exposures creates some difficulties not typically experienced in primary analyses. However, this same low frequency nature of excess layers allows the application of some reserving methods that would be impractical for many primary coverages.

This paper presents a compilation of excess loss reserving methods, describes their application using a common data base, and tests their performance in projecting ultimate loss amounts. Throughout this paper there are several underlying ideas that we believe are important to keep in mind:

- Excess reserving involves some considerations different from those for primary layer reserving. However, with adjustments, many standard reserving techniques can be applied to reserving for excess layers.
- The availability of data often dictates the type of analysis performed. In some cases the application of the method can be modified based on the data available. In other cases, it will be impossible to use a method due to a lack of data.
- The strength of several of the techniques described is that they help establish an initial estimate of expected losses. These initial estimates can be used as a *priori* assumptions in a Bornhuetter-Ferguson[1] calculation.
- The most important overall consideration is that the nature of the business for which you are estimating a reserve must be understood.

Section I describes several commonly used excess loss reserving techniques and provides practical, spreadsheet examples. The intent of *Section* I is to provide a reference guide of reserving methods applicable to excess layers. Some of the methods discussed are similar to commonly used methods for reserving primary layers and will already be familiar to many readers. Section *II* presents an alternative excess reserving technique that uses modern day computing power as a tool to develop a range of potential costs for the layer of coverage. *Section III* applies the methods from the first two sections to a common set of data and retrospectively tests the performance of each method.

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[1]Bornhuetter, Ronald L. and Ferguson, Ronald E., "The Actuary and IBNR," PCAS LIX, 1982

Section I - Commonly Used Reserving Techniques for Excess Layers

An excess layer provides coverage for that portion of losses greater than a specified attachment point and is typically limited to a maximum per occurrence and/or aggregate amount. Direct excess policies involve the original insured and their high layer insurer. This differs from excess of loss reinsurance coverage, which would involve the original insured purchasing a policy from a primary insurer and then the primary insurer purchasing excess of loss coverage from a reinsurer. There are many differences between these two types of excess coverage, including regulations governing the contracts, the loss reporting pipeline and actual contract wording. Many reinsurance contracts "follow the fortunes" of the primary carrier, while direct excess policies may cover different perils or have different definitions for an occurrence as compared to the provisions of lower layer policies. Despite these differences, the methods described can be used in either a direct excess or excess-of-loss reinsurance reserving context, provided the necessary data is available.

In any case, understanding the book of business for which you are reserving is a key element in properly applying a methodology. At a minimum, the following information should be known in any reserving assignment:

- What type of losses are covered (e.g., 3rd party bodily injury liability, property damage catastrophe, workers compensation, professional liability, etc)?
- What event triggers coverage by the policy (e.g., reporting of a claim to the insurer, reporting of the claim to the policyholder, occurrence of an injury, "injury-in-fact", etc)?
- How do the attachment point and limit respond to a claim (or combination of claims)? Are there any reinstatement provisions for the limits?
- How does the policy respond to costs spent defending the original insured from lawsuits (i.e., is defense covered within the limits, outside the limits, or not covered at all)?
- Are declaratory judgment costs an issue (i.e., costs spent defending the insurer in coverage disputes with the original insured)?
- What is the mechanism or series of steps that results in a claim presented to the insurer?

This list is not intended to be exhaustive, but should provide a basic understanding of the coverage provided and the losses that can occur. Each reserving assignment will dictate areas where additional detail is required.

Differences from reserving for primary contracts

Reserving for excess layer contracts is, in many ways, similar to reserving for primary insurance, and many of the same methods are commonly used. However, there are also differences between the primary insurance process and excess layers that make reserving for these contracts unique and, in some ways, more difficult. The following is a partial list of these differences:

- The claim reporting lag for excess layers is generally longer than for primary contracts. This is especially true of casualty excess-of-loss contracts.
- Excess layer policy forms and language vary to a much greater extent than primary contracts. In addition, data grouped by "line of business" for an excess carrier may contain contracts with a wide array of underlying exposures as well as different attachment points and limits.
- The heterogeneity of excess contracts makes it difficult to obtain useful industry statistics. Furthermore, the potential use of additional case reserves necessitates further considerations when applying industry statistics compiled from reinsurers.
- Low claim frequency and high claim severity make the available data more volatile and less useful for predicting future loss emergence.
- Rating information may not be available.
- For excess-of-loss reinsurance, differing reserving philosophies of cedants can create inconsistencies in historical data.
- Inflationary effects on attachment points and policy limits force adjustments to the historical data.

Typical actuarial loss development methods are predicated on the existence of a relatively stable, homogeneous group of underlying exposures. To achieve homogeneity, data are typically segregated into groupings such as annual statement lines of business. For an actuary charged with the responsibility of estimating loss reserves, the level of detail from the annual statement line of business may not be sufficient to achieve homogeneity. Therefore, the data may be further segregated by state or coverage. Given the relatively low frequency of claims for excess of loss contracts, segregation beyond even the highest level of detail may result in a data base of insufficient size to derive credible reporting statistics.

Primary companies faced with the problem of insufficient data to derive credible reporting and payment patterns will typically turn to "industry statistics". For workers compensation, the National Council on Compensation Insurance (NCCI) annually publishes the Statistical Bulletin and prepares rate filings for a number of states. These data sources contain useful information for preparing workers compensation industry reporting and payment patterns by state. In addition, A.M. Best and the Insurance Services Office (ISO) publish industry loss reporting and payment patterns for certain lines of business. The A.M. Best data provide summaries of the statistics for the largest carriers in the market and aggregate statistics for all carriers completing an annual statement. The Reinsurance Association of America (RAA) publishes a summary of casualty excess reinsurance loss development statistics every two years from approximately thirty member companies. The reviewed lines of business include automobile liability, general liability, medical malpractice and workers compensation.

Like any industry data, the application of these statistics to develop losses to ultimate values must be performed with care. However, industry data for reinsurance development statistics are particularly heterogeneous in nature. Before using these statistics in any application, the following factors (among others) should be considered:

- the mix of claims-made versus occurrence policies in the book of business being reviewed (RAA data is primarily reported on an occurrence basis, medical malpractice data is the exception),
- whether the business is broker market or direct,
- the extent of excess over primary versus retrocessions (i.e. excess over excess),
- the presence of unusual loss events (pollution, asbestos, breast implants), and
- the prevalence of additional case reserves contained in the data (RAA contains additional case reserves).

Aside from pure working layer excess of loss contracts, such as $75,000 excess of $25,000 on commercial automobile, most excess policies do not anticipate incurring a significant number of claims. The claims that do exceed the attachment point are large (high severity) losses. Therefore, except in extremely rare cases, large claims are not excluded from the data base as is sometimes the practice when reserving for primary contracts in an effort to provide an additional measure of stability. For medium and higher layer excess contracts, some years may have no reported claims at several evaluations, and may ultimately incur no losses. What is a reasonable reserve provision for an accident year that is several years old, but has no associated incurred claims? How many claims should be expected to emerge in the accident year? What is the average severity of claims ultimately reported and settled? There is no one right method for determining the range of reserves for excess policies, consequently, we will present several methods that can be compared and evaluated for their relative strengths and weaknesses and ultimately used to determine a reasonable range of reserves.

In the following sections, we assume the reader is already familiar with basic actuarial techniques such as the loss development (chain ladder) method. Additionally, the methods presented below utilize incurred loss data only. In general, paid losses lag incurred losses, but for excess layers the lag can be very substantial. The lag in claim payments accentuates the leverage problem associated with development techniques. For these reasons, additional care should be used in substituting paid losses in excess reserving methods.

Throughout *Section 1*, we will use the same underlying data to demonstrate the application of the methods being described. For example let's assume that we are estimating reserves for workers compensation losses in the layer $250,000 excess of $250,000 per occurrence with no aggregate limit.

Several techniques described below provide a method of estimating ultimate losses without directly considering reported losses in the layer. As a result, a mechanical application of these methods will never produce an estimate of ultimate losses that results in zero reserves, even if no claims are ever reported to the excess layer. Therefore, the best use of these methods is often as an *a priori* estimate of expected losses in a Bornhuetter-Ferguson analysis.

Development of losses in the layer

<u>SUMMARY OF DEVELOPMENT OF LOSSES IN LAYER METHOD:</u>

INCURRED LOSSES IN EXCESS LAYER ÷ EXPECTED % REPORTED (IN THE LAYER) = ESTIMATED ULTIMATE LOSSES IN LAYER

TWO PROCEDURES ARE DESCRIBED FOR ESTIMATING THE EXPECTED PERCENT REPORTED,

1. SELECTED PATTERNS BASED ON HISTORICAL DATA, BENCHMARKS AND JUDGMENT
2. PATTERNS DERIVED BASED ON THE PINTO AND GOGOL PROCEDURE

Loss development (i.e., chain ladder) is the most commonly applied actuarial method used to estimate ultimate losses and thus, reserves. The basic technique of loss development can be used for reserving excess layers. In Exhibit I, Page 1 we demonstrate the familiar technique. Column (2) displays the reported losses in the layer $250,000 excess of $250,000. Column (3) displays the expected percentage reported for losses in excess of the attachment point. These patterns can be determined based on a mix of historical data triangles, representative industry statistics, and judgment.

Excess losses (W) can be described as the truncated and shifted distribution of ground-up losses (X). The following definition is provided by Hogg and Klugman[2]:

$W = X - d$,	$X > d$, where d is the truncation (attachment) point,
$W = 0$,	otherwise.

Therefore, W represents the dollars of loss in excess of the attachment point. When constructing the development triangle of excess losses, we must consider the impact on the excess loss distribution caused by changes to the underlying size of loss distribution. Over time, the impact of inflation causes the ground-up size of loss distribution to increase. This increase in ground-up losses affects the distribution of excess losses. For example, assume the following three claims occur in 1992: $95,000, $125,000, $150,000. If we are analyzing claims in excess of $100,000, we would have two claims: $25,000, and $50,000 (total excess losses of $75,000). However, given a

[2]Hogg, Robert V. and Klugman, Stuart A., *Loss Distributions*, J Wiley and Sons, Inc., 1984

15% annual inflation, the same three claims in 1993 would result in three losses excess of $100,000: $9,250, $43,750, and $72,500 (total excess losses of $125,500).

When constructing a development triangle of excess losses, we must be careful to account for the impact of trend on the underlying size of loss distribution. To do this, the most common procedure is to use a detrended attachment point. For instance, if we are compiling data on losses in excess of $100,000, we would select $100,000 as the attachment point for the latest year, say 1994. We would then compare the ground-up value of claims in the 1994 year (as at 12/31/94) to the attachment point of $100,000 to determine which claims contribute to the excess distribution. Assuming a 10% trend factor, the attachment point used to compile data for the 1993 year would be $90,900 ($100,000/1.1). Therefore, for 1993, we would compare the value of the ground-up losses (as at 12/31/94 and 12/31/93) to an attachment point of $90,900 to determine which claims contribute to the excess distribution. Using this procedure, we determine the claims in each year that will be used to construct the loss triangle.

Once the historical data have been compiled, the development statistics for the company specific data can be analyzed and compared to industry development statistics for a corresponding attachment point (if available). The selected reporting pattern is based on this analysis of alternative factors. It is worth noting that many industry benchmarks are based on development of losses above a particular attachment point with an unknown upper limit (i.e., carriers submit data censored at various limits). Therefore, it is possible that unadjusted factors could overstate the potential losses in a layer of coverage if losses are implicitly developed above the policy limits.

A second procedure for estimating the expected percent of losses reported is to derive a pattern to estimate the development of losses in the specific layer (versus development of losses in excess of an attachment point with some unknown limit, as is the case with many industry statistics). One such method for adjusting the reporting and payment patterns is presented by Pinto and Gogol[3]. In their paper, "An Analysis of Excess Loss Development", the authors introduce the formula for the excess development factor as follows:

$$LDF_{c,d} \quad \frac{f(c) \quad f(d)}{e_{c,n} \quad e_{d,n}}$$

where, $LDF_{c,d}$ = loss development factor to ultimate for losses in the layer c to d
c = attachment point
d = Attachment point plus the limit
f(c) = ratio of losses in excess of c to ground-up ultimate losses
f(d) = ratio of losses in excess of d to ground-up ultimate losses
$e_{c,n}$ = f(c) divided by the loss development factor to ultimate, for the retention c and month *n*
$e_{d,n}$ = f(d) divided by the loss development factor to ultimate, for the retention *d* and month *n*

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[3]Pinto, E. and Gogol, D.F., "An Analysis of Excess Loss Development," PCAS LXXIV

A brief inspection of this equation reveals that the numerator is an estimate of the percentage of ground-up ultimate losses in the layer, while the denominator is an estimate of the expected reported losses in the layer (as a percentage of ground-up ultimate losses) as of month *n*. For workers compensation, the NCCI excess loss factors provide a suitable estimate for f(c) and f(d) (except for *considerations discussed in the section on* ELFs, *below*).

We used the formula described above to determine a reporting pattern for losses in the layer $250,000 excess of $250,000. The variables f(250) and f(500) were calculated using methods similar to those described by Gillam[4]. The following table provides the excess loss factors (f(250) and f(500)) and reporting patterns for accident years 1990 to 1994 used to calculate the excess loss development patterns.

Table 1 - Pinto and Gogol Reporting Pattern

Accident Year	*Percent Reported Excess $250,000*	*Percent Reported Excess $500,000*	*Percent of Losses Excess $250,000*	*Percent of Losses Excess $500,000*	*Pinto & Gogol Percent Reported*
(1)	(2)	(3)	(4) = f(c)	(5) = f(d)	(6)
1990	60.27%	53.45%	21.65%	12.87%	70.28%
1991	53.42%	46.12%	22.73%	13.56%	64.23%
1992	43.84%	36.53%	23.85%	14.28%	54.74%
1993	34.25%	30.54%	25.01%	15.04%	39.84%
1994	17.12%	15.12%	26.22%	15.83%	20.18%

(2) & (3) Selected based on industry statistics.
(4) & (5) are derived based on adjusted NCCI ELF table.
(6) is based on the application of Pinto & Gogol's formula. $\frac{[(2) \times (4)] - [(3) \times (5)]}{(4) - (5)}$

We applied the loss development factors calculated in the table above to the losses in the layer $250,000 excess of $250,000 and derived a second set of estimates of the ultimate losses in the layer. The detailed calculations are displayed in *Exhibit I*, Page 2.

The results of the two procedures using the loss development method are displayed in Table 2 below.

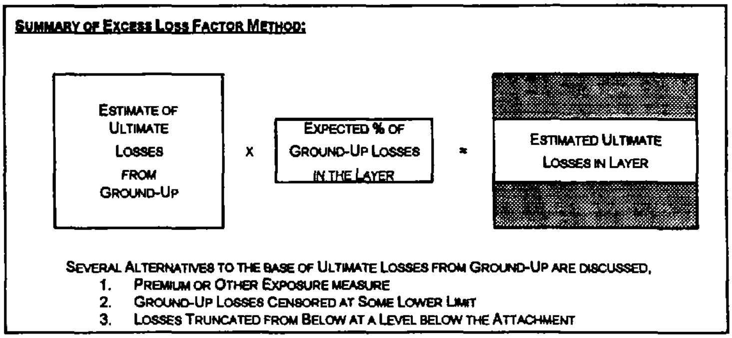
Table 2 - Results of Loss Development Method

Accident Year	*Traditional Loss Development*	*Pinto & Gogol Loss Development*
1990	$15,298,414	$13,120,925
1991	$20,155,392	$16,765,635
1992	$19,475,016	$15,596,445
1993	$11,106,116	$ 9,547,318
1994	$ 3,927,236	$ 3,333,119

[4]Gillam, W.R., "Retrospective Rating: Excess Loss Factors," PCAS LXXVIII, 1992

Steps for Development of Losses in Layer: 1. Obtain data for losses above the attachment point and censored at the upper limit 2. Determine a development factor to ultimate (or percent unreported) A. Selected based on inflation adjusted historical data, benchmarks, and judgment B. Derived using the Pinto & Gogol procedure 3. Divide the incurred losses in the layer by the percent reported to estimate ultimate losses in the layer
Advantages 1. Simplicity 2. understanding 3. acceptance
Disadvantages 1. Result is leveraged due to the small percent reported in excess layers, particularly for immature periods 2. Result is zero if no claims are reported 3. Loss data may be unstable or historical data insufficient to construct a loss triangle

Excess Loss Factor Methods



A second method of estimating ultimate losses, and thus reserves, for an excess layer is the excess loss factor method. Excess loss factors (ELFs) are usually expressed as a percentage of a specific premium (or unlimited benefits for loss cost states) and represent the portion of total unlimited benefits expected to exceed a specific attachment point. For workers compensation, the methodology used to estimate ELFs is described by Gillam[5]. On reviewing this methodology, we note the following:

- The curves used to model size of loss distributions are not developed beyond fifth report. A significant amount of development takes place beyond fifth report, especially for excess claims.
- Permanent total and fatal injuries are discounted at 3.5%. These injury types contribute a significant portion of benefits in the excess layers as compared to other injury types.
- The data used in the development of the curves are organized on a per claimant basis, not a per occurrence basis. A significant number of occurrences that reach excess layers have multiple claimants.
- Allocated loss adjustment expenses are excluded from the NCCI ELF analysis.

These characteristics of the workers compensation ELFs may all serve to understate the estimated actual excess losses when these factors are used for deriving reserving assumptions (which is not the purpose for which the ELFs were intended). Therefore, when applying the excess loss factor method, special attention should be given to the selected ELFs, and appropriate adjustments should be made to account for the potential understatement. A detailed description of these adjustments is outside the scope of this paper. Additionally, if the data base contains a sufficient volume of data, consideration should be given to deriving ELFs from the actual data. A frame-work for the necessary calculations is provided by Gillam[4].

The ELF method can be applied to either an exposure measure (e.g., premium) or an estimate of ultimate losses. The ELFs as published by NCCI represent losses in excess of a retention expressed as a percent of premium. Dividing these factors by the expected loss ratio yields ELFs applicable to ultimate losses. Depending on the data available, it may be desirable to use the ELF method applied to premium, ultimate ground-up losses, censored losses, or truncated losses. The following equations show examples of excess layer losses estimated from different bases of data, where ELF_{250} is the excess loss factor for a \$250,000 retention:

$$(Premium) \times (ELF_{500} - ELF_{250})$$

— Using Premium Based ELFs

$$(Ultimate\ Loss) \times (ELF_{500} - ELF_{250})$$

$$(Ultimate\ Loss\ Limited\ to\ \$200{,}000\ per\ Occurrence) \times \left(\frac{ELF_{500} - ELF_{250}}{ELF_{200}} \right)$$

$$(Ultimate\ Loss\ Excess\ of\ \$100{,}000\ per\ Occurrence) \times \left(\frac{ELF_{500} - ELF_{250}}{1 - ELF_{100}} \right)$$

— Using Loss Based ELFs*

* Loss based ELFs are equal to premium based ELFs divided by the expected loss ratio

Table 3 displays excess loss factors applicable to the 1994 accident year calculated using the NCCI methodology adjusted to account for the characteristics mentioned above. For our sample data, we modified the premium based ELFs to be percentages of unlimited losses (i.e., divide ELFs by expected loss ratio).

Table 3 - Excess Loss Factors

Retention	Label	Excess Loss Factor
\$50,000	ELF_{50}	58.4%
\$100,000	ELF_{100}	45.4%
\$250,000	ELF_{250}	26.2%
\$500,000	ELF_{500}	15.8%

Based on the factors in Table 3, one would expect 10.4% (ELF_{250} - ELF_{500}) of ground-up ultimate losses to be in the layer \$250,000 excess of \$250,000 for accident year 1994.

For coverages other than workers compensation, a suitable estimate of the ELFs may be determined using increased limits tables published by ISO. For a discussion of the use of increased limits factors in ratemaking and insight into how they may be used for reserving, see Miccolis[5].

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[5]Miccolis, Robert S., "On the Theory of Increased Limits and Excess of Loss Pricing," PCAS, LXIV, 1977

Once suitable excess loss factors have been obtained, the next step is to determine the base estimate to which the ELFs will be applied, an exposure measure or some portion of ultimate losses. For our sample data, we chose to estimate the ultimate losses for an attachment point (the data limit) which is below the attachment point of the layer in question. For example, a data limit of $50,000, $25,000, or even $0 (i.e., from ground-up) could be used as a starting point for estimating losses in the layer $250,000 excess $250,000. An estimate of losses from ground-up is preferred because ground-up losses typically provide the highest level of stability and because benchmark statistics (i.e., loss development patterns, loss ratios, pure premiums, etc.) are more readily available if company specific data prove less than 100% credible. However, when ground-up loss information is not available, the ELFs can be adjusted to accommodate other data limits.

Ultimate losses for ground-up data (or other relatively low data limits) can be esimtated using well documented methods such as loss development, Bornhuetter-Ferguson[1], expected loss ratio, frequency/severity and others. For our example, we elected to use the incurred loss development method. However, since the excess loss factor method is dependent on the accuracy of the base data to which the ELFs are applied, in "real life" applications, we recommend using several of the techniques to derive the estimates of ground-up losses. The final selection of an ELF procedure will depend on the data available, the confidence in the base data estimate (e.g., premiums, ground-up ultimate losses, etc.) and the reasonableness of the ELFs.

Exhibit II, Page 1 displays the details of our analysis for accident years 1990 to 1994 using a data limit of $100,000.

The excess loss factors displayed in Table 3 above represent the percentage of ground-up ultimate losses in excess of the specified attachment points. Since we elected to use a data limit of $100,000, we needed to adjust the estimate of the expected percentage of losses in the layer to be stated as a percentage of ultimate losses in excess of $100,000. Based on the factors in Table 3, for accident year 1994, we would expect 22.9% ($[ELF_{250}$ - $ELF_{500}]$ ÷ ELF_{100}) of ultimate losses in excess of $100,000 to be in the layer $250,000 excess of $250,000. Therefore, the ultimate losses in the layer for accident year 1994 can be calculated as 22.9% times the ultimate losses in excess of $100,000. Table 4, below, displays the results of the excess loss factor method applied to the sample data base.

A primary strength of the ELF method is its use as an *a priori* estimate in a Bornhuetter-Ferguson analysis. *Exhibit II*, Page 2 displays the application of the Bornhuetter-Ferguson method. The reporting pattern utilized to calculate the "IBNR factors" (i.e., the percentage of losses unreported) is the pattern used in the traditional loss development method in the prior section (*Exhibit I*, page 1).

Table 4 - Results of Excess Loss Factor Method

Accident Year	*Excess Loss Factor Method*	*Bornhuetter-Ferguson Method*
1990	$21,669,165	$17,829,260
1991	$23,264,882	$21,612,962
1992	$23,779,770	$21,892,754
1993	$13,533,504	$12,702,207
1994	$ 4,786,222	$ 4,639,135

Steps for Excess Loss Factor Method:

1. Estimate ultimate ground-up losses (or losses in excess of some data limit)
2. Calculate the percent of ultimate ground-up losses in the excess layer of coverage using increased limits or excess loss factors
3. Multiply the ground-up losses by the percent of losses in the layer. (1) x (2)

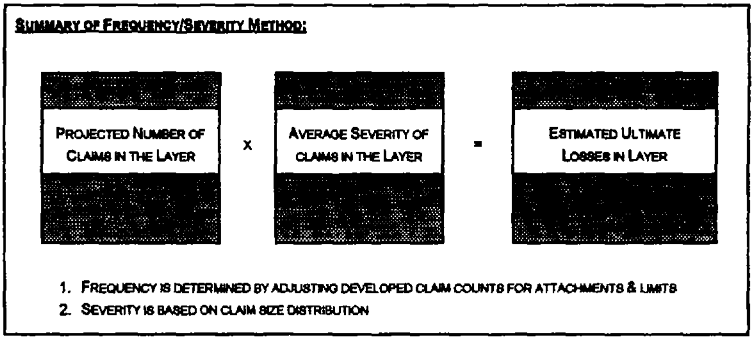
Advantages

1. The starting point is data at lower attachment points (or Ground-up) which may be more stable
2. Industry statistics used to estimate excess loss factors are readily available

Disadvantages

1. Necessary data for estimating ultimates at a lower attachment points may not be available
2. Ignores actual loss emergence in the layer (Unless used as a *priori* estimate for Bornhuetter-Ferguson approach)
3. Dependent on accuracy of the base data estimate and ELFs

Frequency/Severity Based Method[6]



As with projecting ultimate losses for a primary layer, a frequency/severity method can be used to project ultimate losses in an excess layer. The basic methodology does not change: project an ultimate number of claims and multiply by an average severity to produce an estimate of ultimate losses. However, estimating the number of claims and the average size of a claim in an excess layer is typically more complicated than deriving these estimates for primary layers.

As with the expected loss approach, a primary strength of the frequency/severity method estimates is their use as *a priori* assumptions for a Bornhuetter-Ferguson analysis.

Estimating Frequency: Estimating the number of claims in the excess layer typically begins with an estimate of the number of claims in excess of some attachment point (*the data limit*) below the attachment point for the layer in question. The number of claims above this data limit will serve as a starting point to produce the expected number of claims above the attachment point for the layer in question. After estimating ultimate claims above this data limit, a size of loss distribution can be used to project the estimated number of claims in the layer.

An alternative to this approach is to directly estimate ultimate claim counts in the layer using a development technique (similar to the development of losses in the layer) Estimating the frequency directly from the data may provide insight into the claim reporting process. However, for excess layers, the volume of the claim count data is usually insufficient for a development type approach. Therefore, the use of a size-of-loss distribution may be necessary.

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[6]Michael Angelina, "Using Pareto Distribution to Estimate Excess Losses, A Practical Guide," Presentation at the Casualty Loss Reserve Seminar, 1996

To apply this method to our sample information, we selected a data limit of $200,000 for the 1994 accident year. The selected data limit must be high enough to assure that the detrended data limit for any accident year does not fall below the lowest available data attachment. Assuming a data limit of $200,000 and an annual inflation trend of 12%, the detrended data limit in 1988 (the earliest year in our data base) was $101,326 (just above our lowest attachment point of $100,000).

Using these detrended data limits, we constructed the triangle of claim development displayed in *Exhibit III*, Page 3. It is important to remember that this triangle contains only claims known to be greater than the detrended data limits. Therefore, it is not unreasonable to expect a greater magnitude of report-to-report factors and a longer tail than might be expected when analyzing claim development for primary coverages. Using the chain ladder method, we calculated report-to-report factors, and, based on various averages of the factors, selected age-to-ultimate claim count development factors. The selected development factors were then used to develop known claims greater than the data limit. *Exhibit III*, Page 1, shows the development of reported claim counts greater than the detrended data limit.

The next step involves using a size-of-loss distribution to estimate the percentage of claims greater than the attachment point. For our sample data, we found that the single-parameter pareto distribution, as described by Philbrick[7], was well suited for this application. Solving the selected size of loss distribution for F(x) yields the probability that a claim will be *less* than or equal to *x*. The complement of this figure is the probability that a claim will be *greater* than *x*. Using our example, in *Exhibit III*, page 1, the estimated ultimate claims greater than $200,000 for 1990 is 215.

To estimate a value for *q*, we applied the formulas described by Philbrick[7] to the claims in the data base. Based on the results of the calculations, we judgmentally selected a value of 1.800 for the q parameter. Using the single parameter pareto distribution and with a parameter q = 1.800, we estimated the number of claims greater than the $250,000 attachment point as follows;

$$\begin{aligned}
\textit{Claims Over } \$250{,}000 &= (\textit{Ult Claim Estimate}) \; x \; (\textit{\% of those Claims Over } \$250{,}000) \\
&= 215 \; x \; [1.0 - F(x)] \\
&= 215 \; x \left(1.0 - \left[1.0 - \left(\frac{\$250{,}000}{\$200{,}000} \right)^{-1.8} \right] \right) \\
&= 215 \; x \left(\frac{\$250{,}000}{\$200{,}000} \right)^{-1.8} \\
&= 144
\end{aligned}$$

[7]Philbrick, Stephen W., "A Practical Guide to the Single Parameter Pareto Distribution," *PCAS*, LXXII

This calculation is performed for each accident year in column (6), of *Exhibit III*, Page 1. In column (5) we show a similar calculation using the two-parameter pareto model. In practice, we recommend comparing the results of several size-of-loss distributions.

As an aside, one additional consideration in the process of estimating the frequency for the excess layer is the impact of a potential mixture of primary policy limits included in the data base. For instance, assume the analysis is for a $300,000 excess of $200,000 reinsurance coverage on automobile policies. Furthermore, assume that the data provided include claims over $100,000. Some of these claims over $100,000 may arise from primary policies with limits of $150,000 (or some other figure between $100,000 and $200,000). Claims from these policies will never reach the reinsurance layer, so an adjustment based on the distribution of policy limits should be made to eliminate these claims from the analysis.

Estimating Severity: To estimate the average claim size (severity) in the layer, we utilize the pareto distribution with a q parameter of 1.800. In column (9) of *Exhibit III*, Page 1, we apply the single-parameter pareto formula for estimating the average size of a claim in the layer $250,000 to $500,000. Again, for comparison purposes, column (8) shows the average claim size using a two-parameter pareto distribution. A brief inspection of the estimated average severities show that the single-parameter pareto distribution projects the same average claim size for each accident year (i.e., the distribution does not adjust the average claim in the layer for the effects of trend). This is because, unlike most distributions, the q value of the single-parameter pareto distribution is unaffected by trend. This may seem counter-intuitive given the well-documented leveraged effect of trend on excess losses discussed by Miccolis[5]. However, as Philbrick[7] notes, the leveraged effect is on the *total excess dollars*, not necessarily on the *average* excess claim size. In addition, the leveraged effect is somewhat reduced due by the application of policy limits. This implies that the major impact of trend is to increase the *frequency* of excess claims rather than the severity.

According to Hogg and Klugman[2], a two-parameter pareto distribution with parameters b and q before the effects of trend will have parameters $(1+i)b$ and q after trend of i%. In Table 5 we compare the average severities produced by the two-parameter pareto distribution assuming a 12% annual trend and the single-parameter pareto distribution. As shown in the table, the expected impact of trend appears to be minimal.

Table 5 - Comparison of Average Severities in the Layer $250,000 excess of $250,000

Accident Year	Single-parameter	Two-Parameter
1990	133,016	133,182
1991	133,016	133,202
1992	133,016	133,224
1993	133,016	133,249
1994	133,016	133,277

The estimate of the ultimate losses in the layer is the product of the number of claims greater than the attachment point (frequency) and the average claim size (severity) in the layer. Table 6 displays the results of the frequency/severity method for the layer $250,000 excess of $250,000.

As discussed above, the estimate produced by the frequency/severity method can be used as the *a priori* estimate in a Bornhuetter-Ferguson method. The Bornhuetter-Ferguson method recognizes actual losses as they emerge or, if no losses emerge, will eventually reduce to an estimate of zero.

Table 6 - Results of Frequency Severity Method

Accident Year	*Frequency/Severity Method*	*Bornhuetter-Ferguson Method*
1990	$19,110,465	$16,812,790
1991	$26,937,902	$23,314,369
1992	$43,293,056	$32,852,271
1993	$31,189,065	$24,236,205
1994	$21,325,839	$18,346,626

Comparing the results of the frequency/severity method for our sample data to the results of the other two methods, the frequency/severity method produces much higher (and perhaps more reasonable) results for the two latest years, 1993 and 1994. This is because the frequency/severity method is not dependent on the application of a highly leveraged loss development factor to actual reported losses for an immature accident year. In this case, the emerged losses in 1993 and 1994 were less than might be expected based on our selected reporting pattern. Therefore, the indicated ultimate losses from the development factor approaches (and excess loss factor method, which, in our example, used loss development for base ultimate losses) are producing low results relative to the frequency/severity method which is not dependent on actual emerged losses in the layer.

STEPS FOR FREQUENCY/SEVERITY METHOD:

1. ESTIMATE ULTIMATE CLAIM COUNTS ABOVE SOME DATA LIMIT (WHICH IS LOWER THAN THE LAYER ATTACHMENT POINT)
2. ADJUST THE ULTIMATE CLAIM COUNTS FOR THE PERCENT ABOVE THE ATTACHMENT POINT
3. ESTIMATE THE AVERAGE SEVERITY DISTRIBUTION FOR CLAIMS IN THE LAYER
4. MULTIPLY THE ESTIMATED FREQUENCY TIMES THE ESTIMATED SEVERITY TO PRODUCE AN ESTIMATE OF ULTIMATE LOSSES.

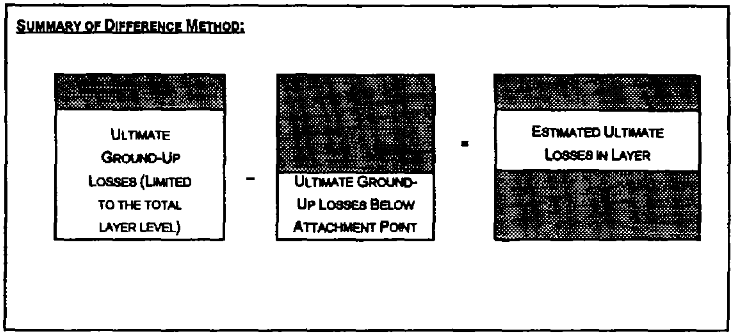
ADVANTAGES

1. LESS DEPENDENT ON A HIGHLY LEVERAGED LOSS DEVELOPMENT FACTOR
2. PROVIDES ADDITIONAL INSIGHT AS TO THE POSSIBLE DRIVING FORCES BEHIND THE TOTAL EXCESS DOLLARS (I.E., FREQUENCY SEPARATE FROM SEVERITY)

DISADVANTAGES

1. CLAIM COUNT DATA MAY BE UNSTABLE
2. TESTING OF THE SIZE OF LOSS ASSUMPTIONS AND THE FIT OF THE CURVE CAN BE CUMBERSOME
3. UNDERLYING DATA MAY NOT BE AVAILABLE
4. IGNORES ACTUAL LOSS EMERGENCE IN THE LAYER (UNLESS THE ESTIMATES PRODUCED BY THE METHOD ARE USED AS THE *A PRIORI* ESTIMATE FOR A BORNHUETTER-FERGUSON METHOD)

Difference Method



One method for estimating ultimate losses in an excess layer bears mention because of its frequent use. For lack of a better name, we call it the difference method. The three methods described above derive estimates of ultimate losses in the layer directly. The difference method produces an estimate of the ultimate losses in the layer as a by product of estimating two other quantities: ultimate losses limited to the attachment point and ultimate losses limited to the attachment point plus the limit.

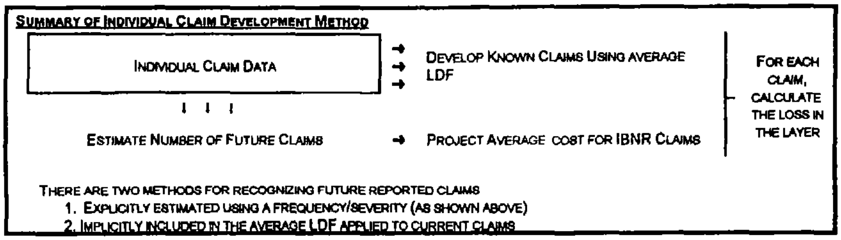
In *Exhibit IV*, Page 1 we use the chain ladder method to develop losses limited to $250,000 and losses limited to $500,000 to ultimate values. The loss development factors were selected based on an analysis of historical losses censored at the appropriate limit. Our data base contains workers compensation claims greater than $100,000 on a combined medical and indemnity basis. Therefore, the two quantities that we estimated were actually claims greater than $100,000 limited to the attachment point (i.e., $250,000) and claims greater than $100,000 limited to the attachment point plus the limit (i.e., $500,000). Given that the claims below $100,000 were not included in the data base, it is not surprising that the loss development factors selected for the two different limits are very similar.

Table 7 - Difference Method

Accident Year	*Difference Method*
1990	$23,066,616
1991	$32,396,723
1992	$40,353,269
1993	$32,089,593
1994	$21,777,263

<u>Steps for Difference Method:</u> 1. Project Ground-Up ultimate losses capped at the attachment point. 2. Project Ground-Up ultimate losses capped at the attachment point plus layer limit. 3. Subtract (2) from (1) to produce an estimate of ultimate losses in the layer.
<u>Advantages</u> 1. Simplicity 2. Acceptability and common use 3. Availability of benchmarks for lower limit data
<u>Disadvantages</u> 1. Simplicity (Since not directly using layer data, may not provide much insight) 2. Ignores the characteristics of the losses in the layer 3. Data at lower limits may not be available to excess carriers

Individual Claim Development



The last technique that we will describe in Section I is the development of individual claims. In this method, the ground-up losses for individual claims greater than a selected data limit are developed to ultimate, and the resulting values are compared to the attachment point and limit to determine the ultimate value of losses in the layer. This method relies on the weak assumption that all losses develop equally.

The first step is to select a data limit below the attachment point for the layer in question. This permits accounting for claims currently below the attachment point that will ultimately exceed that attachment point. For example, if the current incurred value of a claim is $125,000 and the loss development factor is 3.0, then the projected ultimate value is $375,000. If the attachment point of the layer in question is $250,000, then this claim will contribute $125,000 of excess losses to the layer. Given that the losses in our data base are all greater than $100,000, we will again use $100,000 as the data limit.

To analyze historical development applicable to the current claims it is necessary to create multiple *data triangles. For instance, the 1992 year as of year-end 1994 shows a certain number of claims* over the selected data limit after 24 months. To develop that specific group of claims, a triangle must be constructed which examines the group of claims for each accident year that exceed the detrended attachment after 24 months. To project claims from the prior accident year, 1991 as of year-end 1994, a triangle must be constructed such that, for each accident year, there are only those amounts related to claims over the detrended attachment after 36 months.

With these development factors selected, the method proceeds in two steps. First, we apply the selected loss development factors to each of the known claims and determine the ultimate value of known claims in the layer. Second, we use a frequency/severity technique to estimate the number of claims in the layer still to be reported and the average cost associated with these claims. The final estimate of losses in the layer will be the sum of the known and unknown claims in the layer. From an IBNR perspective, this approach segregates the IBNR into two components: (1) case development on known claims (incurred but not enough reported, IBNER), and (2) claims that have not yet been reported that will penetrate the layer.

An alternative procedure is to implicitly include the emergence of unreported claims in the development factor applied to known claims. This approach requires only a single triangle of amounts over the detrended attachment points at all evaluations. For this alternative, there is no need to construct multiple triangles or to perform a frequency/severity estimate of unreported claim amounts and, therefore, eliminates a substantial amount of the analysis discussed above.

Exhibit V, Page 1 displays a sample of the claims in our data base and the application of the loss development technique to individual claims. In this exhibit, we apply a loss development factor for ground-up losses above the particular data limit to incurred losses on a claim-by-claim basis to determine the ultimate value of individual claims. The estimates of ultimate losses for the individual claims are then compared to the attachment point and limit of the layer in question and the ultimate value is truncated and shifted as discussed earlier. The result of this calculation is the expected amount of losses in the layer for known claims and the expected number of known claims that will ultimately exceed the attachment point. This second piece of information will be useful in step two of this method.

Depending on the actual provisions for the particular coverage being analyzed, it may not be necessary to estimate unreported claims. For instance, if the contract is reinsurance and there is a specified sunset clause that limits the reporting period for claims or if the policy is a claims-made policy, it may not be necessary to incorporate a specific provision for unreported claims. It is important in these instances that the policy language, judicial precedence and precautionary notice provisions are carefully understood, since not all "claims reported" policies eliminate the potential for future claim emergence. Depending on the results of this investigation, step two of this procedure may not be necessary.

The second step applies a frequency/severity technique to estimate the number of unreported claims and the average amount of a claim in the layer for each unreported claim. In review, the following steps are used to determine the expected number of claims in the layer:

- Determine the number of reported claims that exceed the selected data limit.
- Using a triangle of reported claims (based on a detrended data limit), calculate an expected reporting pattern for claims in excess of the data limit (see *Exhibit III*, Page 3).
- Multiply the reported number of claims in excess of the data limit by the expected loss development factor to determine the expected ultimate number of claims in excess of the data limit.
- Using a size of loss distribution, estimate the percentage of losses in excess of the data limit that will also exceed the attachment point.

- Apply the percentage from (4) to the estimated ultimate number of claims in excess of the data limit from (3) to determine the ultimate number of claims in excess of the attachment point.

Once we have an estimate of the ultimate number of claims in excess of the attachment point, we can determine the expected number of unreported claims by simple subtraction (IBNR claims equal ultimate claims (step (5)) minus the number of reported claims over the attachment point after applying the development factors).

Similarly, the estimate of severity for unreported claims will once again call upon the procedures described in the section on the frequency/severity method. In short, a size of loss distribution is applied to determine the estimate of the average severity in the layer. For a more detailed discussion, see the frequency/severity method discussion above.

In *Exhibit V*, Page 2 we apply the methods to determine the number of unreported claims in the layer and the average severity. The final estimate of ultimate losses in the layer is the sum of the losses *in the layer as the result of unreported claims (step two) and the estimate of the ultimate losses in* the layer as the result of known claims (step one).

Table 8 summarizes the results of the individual claim development method.

Table 8 - Individual Claim Development Method

Accident Year	*Individual Claim Dev.*
1990	$16,256,487
1991	$25,989,930
1992	$41,265,154
1993	$30,691,852
1994	$22,037,874

If we elected to implicity include the emergence of unreported claims in the selected development factors, the use of the frequency/severity method would not be necessary.

STEPS FOR INDIVIDUAL CLAIM DEVELOPMENT:
1. DETERMINE AVERAGE DEVELOPMENT FACTORS AND APPLY TO INDIVIDUAL CLAIMS A. EXCLUDING EMERGENCE OF NEW CLAIMS INTO THE DATA LIMIT (SEPARATELY FOR EACH AGE) B. INCLUDING THE EMERGENCE OF NEW CLAIMS INTO THE LAYER 2. IF (A), USE FREQUENCY/SEVERITY APPROACH TO ESTIMATE COSTS RELATING TO UNREPORTED CLAIMS 3. DETERMINE LOSSE IN LAYER FOR EACH CLAIM USING ATTACHMENT AND LIMIT INFORMATION
ADVANTAGES 1. CAN PRODUCE SEPARATE ESTIMATE OF DEVELOPMENT ON KNOWN CLAIMS AND UNREPORTED CLAIMS 2. APPLIES THE EXCESS LAYER COVERAGE PROVISIONS TO INDIVIDUAL CLAIM EXPERIENCE
DISADVANTAGES 1. ASSUMES THERE IS NO VARIATION IN INDIVIDUAL CLAIM DEVELOPMENT 2. REQUIRES DATA WHICH IS BELOW THE ATTACHMENT POINT (MAY BE UNAVAILABLE) 3. METHOD (A) REQUIRES ANALYSIS OF MULTIPLE DEVELOPMENT TRIANGLES 4. METHOD (B) INCLUDES AN ARTIFICIAL IBNR PROVISION FOR EACH CLAIM

One of the implicit assumptions (and limitations) of the loss development technique as applied to individual claims is that each claim will develop in a fashion similar to the reporting pattern for all claims in total (i.e., there is no variation in individual loss development factors between claims). Of course, in reality, individual claim development is much more volatile than the aggregate development of all claims. In fact, development factors for aggregate losses can be thought of in terms of a mean development factor and a distribution of loss development factors that apply to individual claims. For instance, suppose our claims data base consists of the following 10 claims with reported losses at 12 months and 24 months as displayed in Table 9.

Table 9 - Example of Individual Claim Loss Development Factors

Claim Number	*Incurred @ 12 months*	*Incurred @ 24 months*	*Age-to-Age Factor*
1	$1,000	$1,000	1.000
2	$1,000	$1,000	1.000
3	$1,000	$1,200	1.200
4	$2,500	$2,500	1.000
5	$2,500	$2,500	1.000
6	$2,500	$2,200	0.880
7	$5,000	$5,000	1.000
8	$5,000	$7,500	1.500
9	$10,000	$12,725	1.273
10	$10,000	$15,000	1.500
Total	$40,500	$50,625	1.250

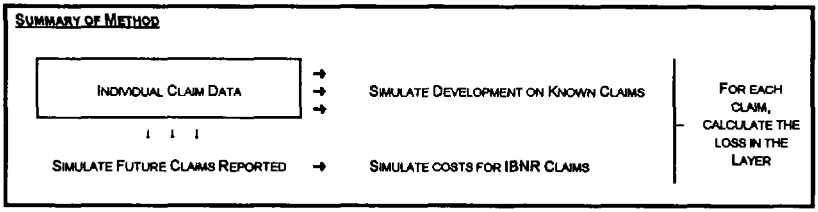
The aggregate age-to-age factor for these claims would be 1.250; the sum of the incurred losses at 24 months divided by the sum of the reported losses at 12 months. This quotient is simply the weighted average of the age-to-age factors for each of the ten claims with the weights equal to the incurred losses at 12 months. Thus, the aggregate factor of 1.250 could be viewed as the mean of the distribution of individual claim development factors shown in the Table 10.

Table 10 - Hypothetical Distribution of Individual Claim Development

Age-to-Ultimate Factor	*Number of Claims*	*Percentile*
0.880	1	10%
1.000	5	50%
1.200	1	10%
1.273	1	10%
1.500	2	20%

In *Section II*, we expand these concepts to construct a simulation model of individual claim loss development that can be used to estimate a distribution of ultimate losses in the layer.

SECTION II - A Simulation Procedure for Individual Claim Development



This section describes a method for excess layer reserving that combines aspects of the other projection techniques and uses the computing power of current desktop computers. The method is a simulation routine that projects multiple alternative scenarios of potential ultimate costs. These multiple alternatives form a distribution of possible outcomes and, hence, a distribution of potential reserves.

Note that we are describing this method in a separate section because it is unique, not because it is expected to consistently outperform other methods.

The concept for this procedure extends from methods described in Section 1, most notably the individual claim development method. The individual claim development method assumes that the selected aggregate development factor applies to each individual claim in the accident period. As described above, the individual claim development method is performed in three steps:

- Develop known claims greater than the data limit to an ultimate value,
- Project IBNR claims and average severities in the layer, and
- Apply the coverage provisions to determine the losses in the layer.

In some ways, this method is an expansion of the ideas presented by Ferguson[8]. In his paper, Ferguson notes that an expected gross loss of $129,280 does not imply an expected loss of $50,000 for a $50,000 retention nor does it imply an excess reserve of $79,280 for the layer above $50,000. This is because there is a non-zero probability that the claim will settle for less than $50,000 or for more than $129,280.

Similarly, because the average development factor for a group of claims is 1.75, we cannot assume that each claim will develop by the same average amount. For any claim there is a non-zero probability that it will develop by more or less than the 1.75 average. However, there is a problem in identifying which claims will develop more than the average amount and which will develop less.

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[8] Ferguson, Ronald, E., "Actuarial Note on Workmen's Compensation Loss Reserves," PCAS LVIII

The simulation program helps to answer that question using a high-tech trial-and-error routine. Instead of assuming that all claims develop equally, this method randomly selects individual development factors from an assumed group of possible values and applies them to each claim. The layer losses are then calculated and saved as the result of one trial. Having saved the result, the routine starts over again and randomly selects another set of individual development factors to apply to the incurred claim values, and so on. This process continues until a sufficient sample of trials is generated to represent a distribution of possible ultimate loss outcomes.

Simulation programs are now relatively easy to create in current spreadsheet environments or can be performed using pre-packaged computer software. Essentially, a simulation routine consists of two steps:

- select a range of possible outcomes for one or more of the input assumptions
- generate multiple potential outcomes by randomly choosing input assumptions from the selected ranges.

A simple example may help clarify the process. Assume you want to estimate the number of home runs Ken Griffy Jr. will hit in a season. Let's say that you've chosen the following equation to estimate the number of home runs:

$$\textit{Number of Home Runs} = (\textit{Number of At-Bats}) \times (\textit{Probability of Home Run})$$

For Ken, assume the average number of at-bats per season is 420 and the percent of home runs during any season averages 8.1% (about one every 12.3 at bats). A simple point estimate may be 34 home runs per season (34 = 420 x 0.081). However, assume that you have enough information to determine the distribution of at-bats, taking into account possibility of injury, players' strike, lock-outs, hold-outs, etc. Based on this analysis assume that the distribution of the number of at-bats is given by the following equation:

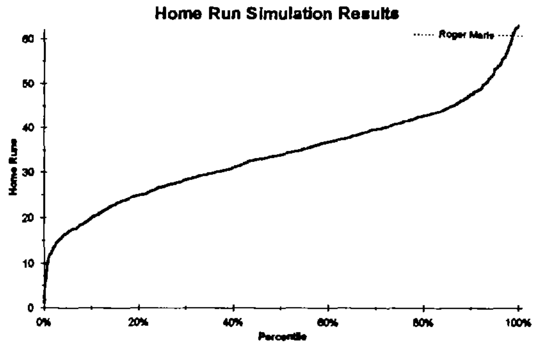
$$\textit{Number of At-Bats} = 250\,(9x)\,(0.30x^2)\,(9.5x^3) \qquad \textit{For } 0 < x < 1$$

Furthermore, assume that for any given year, the percentage of home runs to total at-bats is approximately normally distributed with a mean of 8.5% and standard deviation of 2.5%. Also assume that the percentage of home runs never goes below 0% and never above 10%. (The mean of this censored distribution is 8.1%.) Using these two input assumptions, a simulation routine can be created to determine a distribution of potential home run totals for a season.

The simulation program selects a random number between 0 and 1 and substitutes this number into the at-bats equation to estimate a possible result for this parameter. Then, assuming that the percent of home runs is independent from the number of at-bats, the simulation selects another random number and determines an estimate from the normal distribution according to the assumed mean, standard deviation and limitations. The program multiplies the random at-bats by the random percent of home runs, records the results, and begins the procedure from the beginning. Table 11 displays the results of 1,000 trials of the simulation procedure.

Table 11 - Results of Home Run Simulation

Percentile	*Number of Home Runs*
10%	19.8
25%	26.7
50%	33.8
75%	40.9
85%	44.3
90%	47.5
95%	53.2
98%	58.8
99%	61.7
Average	33.9



Care must be taken when using a simulation model. The model's design should reflect your ability to estimate the parameters and the relationships between parameters. A problem with some simulation models is that they are overparameterized, that is, the model has too many input assumptions and the relationships between variables are not easily determined or clearly defined. Without key parameter relationships, the results of the simulation will not fully reflect true potential events, but rather a conglomeration of arbitrary outcomes.

For instance, assume in our model of home runs that the probability of a home run is negatively correlated with the number of at-bats in a season (i.e., too many games without rest decreases the ability to hit one over the fence). Without this correlation, the model will simulate too many high home run seasons and the tail of the distribution will be too fat.

Benefits of this method include the distribution of results produced, the ability to investigate individual iteration results, and, hopefully, a better understanding of the underlying loss process obtained in determining the input assumptions. The individual iteration results of the simulation procedure represent random points in the distribution of actual potential ultimate loss outcomes. If a sufficient number of iterations are performed, the results can be sorted to determine the percentile rank of any given outcome. Thus, the results can be reported as "*Given the input assumptions*, there is a 65% probability that ultimate losses will be less than or equal to $5 million."

In addition, detailed simulation output allows you to investigate the various combinations of parameters that result in specific loss outcomes and promotes the understanding of the conditions under which certain results are produced (e.g., whether a high-dollar indication is the result of a large number of IBNR claims reported or significant development on several key claims).

Finally, the process of determining the input assumption parameters can be insightful. There are a number of aspects of the loss process that may be overlooked when performing a chain-ladder, frequency/severity or Bornhuetter-Ferguson analysis. For instance, when selecting a report-to-report factor, it is usually not important whether a consistent 20% upward development is the result of all claims increasing by 20% or whether the 20% increase is caused by a small handful of significant deteriorations while the majority of claims remain constant. Another issue may be the possible correlation between the size of a reported claim the future development on the claim. The importance of these questions begins to become apparent when performing the individual claim development method. The investigations needed to properly parameterize a simulation model should provide a better insight into the loss process and, ideally, enable better application of other loss projection methods.

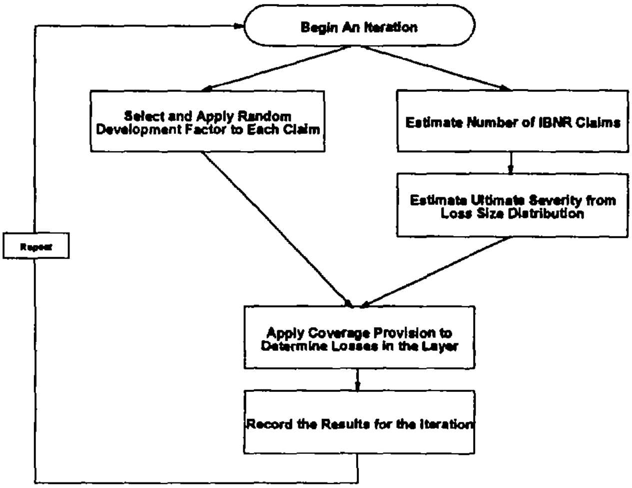
With that introduction, let's look at the application of the simulation method to reserving for excess loss layers. The description and example below assume that the method is applied to a single accident year (or report year, underwriting year, etc.). There are four major steps in the simulation approach,

- Estimate the *number of future claims to be reported.*

- Estimate the potential ultimate costs for each claim. For current claims, this is accomplished by simulating individual development factors based on an assumed distribution. For IBNR claims, costs are projected directly from a size of loss distribution.

- Apply the coverage details to each loss generated in (2) to determine an estimate of losses in the layer and record the results.

- Repeat steps (1) to (3) for a selected number of trials to determine a range of results.



Determining the range of potential future claims reported for step (1) is similar to the techniques described for the frequency/severity method. When reserving for the layer $250,000 excess of $250,000 using a data limit of $100,000, the goal of step (1) is to estimate the number of claims currently not reported or with reported values below $100,000 that will eventually become $100,000 or greater. This procedure is slightly different than the determination of claims in the layer used in the frequency/severity method. In addition to estimating the expected number of additional claims reported, the analysis should include an estimate of the variation of potential future claims reported. If the policy form precludes the possibility of additional claims (i.e., sunset clause or claims-made policy), then this step would not be necessary.

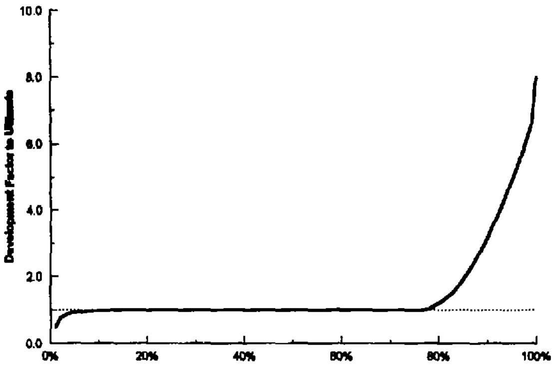
Since the main input to the simulation method is a listing of claim values, this method will be most useful for years that have a fair number of claims already reported. For years that have very few claims reported (i.e., immature years), the simulation method is essentially a complex frequency/severity procedure. As the year matures and more claims are reported, more weight will be given to the development of known claims and less on the IBNR claims.

The goal for step (2) is to determine the distribution of potential claim costs for claims greater than $100,000 as well as the distribution of development factors for the claims already reported. The distribution of potential claim costs for claims greater than $100,000 is used to estimate costs for the unreported claims generated from step (1), while the distribution of development factors will be used to project potential costs for known claims.

In estimating the distribution of development on current claims, it is important to consider the possible correlation of development factors with factors such as age/maturity, line of business, size of loss, and type of loss (injury type). Furthermore, it is important to know how the shape of the distribution changes due to higher/lower average development factors.

For instance, assume the analysis of development factors for a group of claims reveals that the average incurred development factor to ultimate is 1.50. The analysis of the distribution for individual claims shows that the majority of claims settle for costs very close to their incurred value, while a few claims deteriorate significantly, as shown in the graph below.

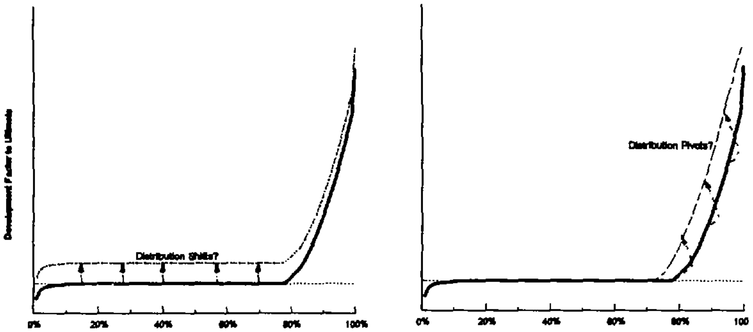
Distribution of Development Factors



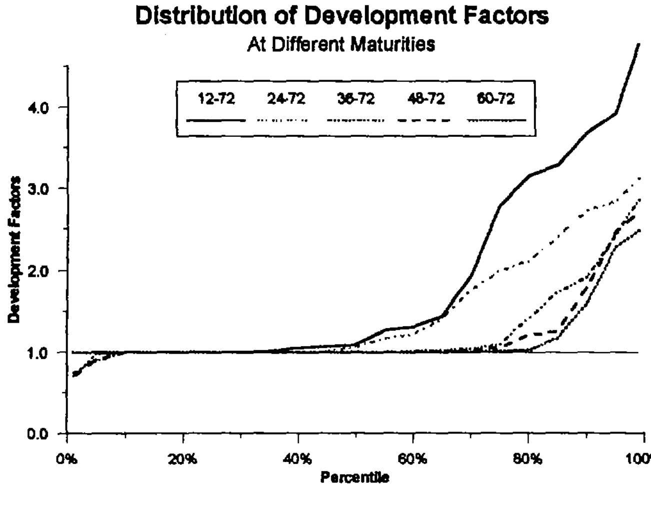
This graph shows that a small percentage (approximately 10%) of claims settle for less than their incurred value, while the majority of claims settle for the incurred amounts (development factor of 1.0). Slightly more than 20% of claims settle for amounts greater than their incurred value.

It may not be practical to replicate this analysis each time reserves are projected, therefore, it is important to know how this distribution would be affected by a change in the aggregate development factor to, say 2.00 (instead of 1.50). Would the expected development for each and every claim rise by 33.3% (2.0/1.5) so that the vast majority of claims would increase by 33.3% over their incurred value (i.e., the distribution shifts upward)? Or, perhaps, the majority of claims still settle for the incurred estimates and a larger percentage are settling for amounts greater than their incurred values (i.e., the distribution pivots)? These two alternatives are illustrated below.

Changes in Development Distributions



To answer this question, we analyzed a data base of claims at several different maturity levels (i.e., evaluations). As the year matures, we expect that the aggregate development factor will decrease and the shape of the distribution will change. The following graph depicts the results of our analysis.



This figure seems to show that the distribution tends to pivot rather than shift. There appears to be a portion of claims at all maturities that settle for amounts very close to their incurred value (i.e., incurred development factor of 1.00). The tail of the distribution seems to expand for less mature periods (or for higher average development factors for groups of claims at similar maturities).

Based on the data available, other analyses of development factor correlations should be investigated. We examined the possible correlation of the individual claim development with the size of the incurred claim amount. Our analysis shows a slight negative correlation between incurred claim size and the magnitude of development factors (large claims tend to develop less). The negative correlation might reflect a greater attention paid to high-dollar claims and, hence, a greater accuracy (or conservatism) in their case reserves. Or perhaps, more simply, it reflects the fact that an equal dollar amount of deterioration will result in a smaller percentage change for a large loss.

The third step of this analysis is to randomly generate potential losses based on input parameters determined in steps (1) and (2). This can be accomplished by using pre-packaged software or by writing a program that selects random figures (independent or possibly correlated) from all input distributions. Each set of random results represents one possible ultimate outcome of claim counts and amounts.

The final step is to apply the coverage provisions to the claims within each iteration that yields an estimate of the potential losses in the layer. It may also be desirable to apply any outwards reinsurance coverage so that each iteration results in an estimated gross and net liability amount for the layer in question.

The following paragraphs describe the application of this method using the example from the individual claim development in Section 1. Specifically, we examine a group of claims over $100,000 for reserves in the layer $250,000 excess of $250,000. *Exhibit VI*, Page 2, displays the calculations for a single iteration of this method for a sample of claims. In the top half of the exhibit, we develop an estimate of ultimate losses in the layer for each reported claim. In the bottom half of the exhibit, we address the issue of losses from unreported claims.

For reported claims, column (4) shows the incurred value as of December 31, 1994. Column (5) displays the percentile ranking for the incurred values in column (4). This percentile ranking is used to adjust the randomly selected development factor for correlation with size of loss. Column (6) displays a random value selected from a uniform distribution between zero and one. Column (7) shows the selected correlation factor between size of loss and magnitude of development factor. Column (8) is a calculation using the incurred loss percentile in column (5), the random factor in column (6), and the loss size correlation factor in column (7) to determine the random value adjusted for correlation. The footnotes to the exhibit describe the actual calculation. Based on the adjusted random lookup value from column (8), a development factor for the claim is selected from the distribution and recorded in column (9). Column (10) is the calculation to determine the developed ultimate loss for the particular claim (i.e., column (4) times column (9)) and column (11) calculates the indicated ultimate losses in the layer.

For IBNR claims, the process is somewhat simpler. First we sample from a distribution of IBNR claim counts to determine the number of claims not reported or reported but currently with values below $100,000. Next, for each IBNR claim, we sample from a size of loss distribution to determine the ground-up loss severity for a claim that is known to be greater than $100,000. This value is recorded in column (10) and the losses in the layer for IBNR claims are determined in column (11) by applying the attachment point and limit to the value in column (10). The sum of the values in column (11) is the estimated ultimate losses in the layer. This process is repeated for the selected number of trials.

Once the simulation routine has been completed, the results are compiled and analyzed. In *Exhibit VI*, Page 1, we display the percentile distributions for the results of a simulation for a single accident year. This information is useful in comparing results with other methodologies. These results can form the basis for questioning assumed parameters for the simulation model or those assumptions used in the other methods.

Table 12 - Results of Simulation Model ($000,000)

	Accident Year				
Percentile	*1990*	*1991*	*1992*	*1993*	*1994*
10%	19.7	12.5	23.3	16.2	15.3
25%	20.7	14.4	26.6	22.2	21.2
50%	21.6	16.2	31.4	28.3	25.8
75%	22.2	18.6	37.3	32.2	28.9
85%	22.9	19.5	39.7	34.0	32.1
95%	24.1	21.5	45.4	40.2	37.4
99%	25.9	22.7	47.3	47.0	42.4
Mean	21.6	16.4	32.1	27.3	25.4
Selected	21.8	17.1	32.5	29.3	26.7

Steps:

1. Analyze the potential for future claims
2. Project ultimate cost using (a) size of loss distribution for IBNR claims and (b) distribution of development factors for reported claims
3. Simulate multiple potential scenarios based on (1) and (2)
4. Apply policy provisions to determine the costs in the layer for each iteration in (3)

Advantages:

1. Distribution of results
2. Understanding of input assumptions and loss process gained through research of parameters
3. Insights into what situations cause certain outcomes

Disadvantages:

1. Parameterization may be difficult and lead to over-complexity or over-simplifications
2. Too much variation for immature years
3. Complexity of method requires much more time to perform than other methods
4. Involves a number of miscellaneous considerations including selecting a sampling procedure, maintaining confidentiality of claimants, and preserving proper relationships between variables

Two final considerations mentioned under the disadvantages (item (4) above) deserve further explanation: the sampling procedure and confidentiality. The Monte Carlo method is the traditional sampling procedure used to generate random (or quasi-random) numbers. Monte Carlo techniques are meant to be entirely random, i.e., any given outcome will fall anywhere within the range. The advantage of a Monte Carlo method is its simplicity and availability, but its disadvantage is that for small samples the results can cluster and the full range of outcomes are not sufficiently represented. An alternative structured simulation procedure is the Latin Hypercube. This method segments the range of inputs and samples evenly from each segment so that small samples are more likely to recreate the assumed probability distribution. The choice of sampling procedure should be considered when deciding on how many iterations to run.

Another important consideration regarding the simulation procedure (or any method that lists individual claims) is that of confidentiality. It is important to note that the simulation result for any given claim is based on assumptions derived from a review of data for the group of claims in aggregate and may differ from results derived from an analysis of a single claim in isolation. In addition, a coding system should be used to protect the identity of individual claimants.

Section III - Testing and Conclusions

Armed with the various methods described in Sections I and II, the question becomes which method produces the "best" estimate. You might also ask whether there are certain conditions under which a particular method should be expected to perform well or poorly.

In this section, we apply each of the methods described above separately to two different data bases. The two data bases have different features that illustrate some (but not all) of the performance characteristics of the reserving methods. Due to the availability and volume of data, we selected two data bases of workers compensation information. Table 13 highlights important aspects of each data base used for testing the reserving methods.

Table 13 - Data Bases Used for Testing Reserving Methods

	Data Base 1	Data Base 2
Source	Industry Workers Comp Data	Actual Workers Comp Reserve Assignment
Years	Accident years 1987 to 1990	Underwriting Years 1984 to 1991
Evaluations	Year-end 1991 to 1994	Year-end 1991 to 1994
Data Available	Loss data only on individual claim basis for all occurrences over $100,000	Premium by year and all ground-up loss data on individual claim basis
Layer Evaluated	$250,000 excess $250,000 per occurrence with no aggregate limit	Varies by year from $270,000 excess $30,000 per occurrence to $425,000 excess $75,000 per occurrence
Other Features	Case reserve adjustments between year-end 1992 and 1993	Changes in excess layer by underwriting year and additional exposures after 1988
"Best" Estimate	Mechanical calculation of high/low average across results for all method at year-end 1994	Based on actual selected estimates for year-end 1996 analysis

For the purposes of testing the reserving methods we relied on a relatively mechanical application of each technique to the subject data in order to highlight the performance of the method rather than the reasonableness of any judgments. This is particularly true for the first data base where even our selected best estimate used for our performance test is based on a simple average (excluding the highest and lowest projection) at the latest evaluation. The methods applied to data base two include much more judgment since they reflect an actual analysis that formed the basis of reserve opinions at successive year-ends.

Table 13 displays the selected ultimate losses for data base one for each accident year, along with the mean test result and the standard deviation of the results for all methods.

Table 13 - Selected Ultimate Losses in the Layer for Data Base 1

Accident Year	*Mean Test Result*	*Standard Deviation*	*Selected Ultimate Loss*
1987	$15,920,117	$1,986,017	$16,064,887
1988	$17,178,577	$1,706,517	$17,240,675
1989	$17,053,008	$2,369,002	$16,762,793
1990	$18,827,028	$2,591,081	$18,738,157
Total			$68,806,512

A "perfect" method would produce the best estimate under any conditions (i.e., at an early evaluation, no losses actually reported in the layer, case reserves are strengthened by the claims department, higher than normal emergence of claims, an unusual loss occurs, etc.). However, to our knowledge, no method for estimating ultimate losses, excess layers or otherwise, can produce the best estimate under any conditions. Therefore, it is instructive to understand the set of circumstances under which each method produces a good estimate (close to our best estimate) or a bad one (inconsistent with our best estimate). In general, each method is founded on some underlying assumptions about the loss process. It is typically the violation of one or more of the assumptions that results in a bad estimate. When we compare the results of each method produced at several successive evaluations to the overall best estimate, we can observe which methods perform well and which perform poorly and examine the reasons for the performance.

Our test statistic will be the percentage deviation of the indicated ultimate loss to the best estimate.

$$\%\ Deviation = \frac{Indicated\ Ultimate\ -\ Best\ Estimate}{Best\ Estimate}$$

In each instance we examined the results of the methods to evaluate how they performed relative to the overall best estimate and how they performed relative to the other reserving techniques.

Loss Development Method: The loss development method is highly leveraged upon the selected loss development factor, especially for immature accident years. By this we mean that, at early evaluations of an accident year, large or small changes in the reported data can result in significant variation in the indicated ultimate losses. In addition, if there are no losses reported in the layer (which is not uncommon for early evaluations of medium- and high-layer excess covers), the loss development method will produce an indication of zero.

Reviewing the results of the retrospective testing, as expected, we found the incurred development method performed best for the more mature accident periods and that the total for all accident periods improved as the data matured. While you might generally assume that all methods improve with maturity, that was not always the case. In additional, we found that the total for all accident periods for the loss development method improved relative to the other methods as the data matured. In other words, when selecting an ultimate loss, more weight should be placed on the loss development method for more mature accident years and less weight on immature accident years.

Excess Loss Factor Method: The accuracy of the excess loss factor method is heavily dependent on the estimate of base data to which the ELFs are applied. In the first data base, we applied loss-based ELFs to ultimate losses greater than our data limit. In the second data base, we applied premium-based ELFs to written premium. The ELF method applied to the first data base was heavily dependent on our estimate of ultimate losses above the data limit. While we calculated a relatively mechanical estimate of losses over the data limit, in practice, several methods (i.e., loss development, frequency/severity, Bornhuetter-Ferguson, etc.) could be used to estimate losses in excess of the data limit.

An additional consideration is the selection of accurate excess loss factors. It is important that the ELFs applied are consistent with the underlying business and potential loss severities.

In our analysis on the first data base, the ELFs were applied to losses with a data limit of $100,000 and the selected estimate of ultimate losses greater than the data limit was based on the incurred loss development method. Because the accuracy of the incurred loss development method generally improves relative to other methods and to the best estimate as the data matures, the ELF method, in this case, improved slightly as the data matured.

For the second data base, the ELFs were applied to premium which did not vary from evaluation to evaluation. Thus, the results for the second data base did not improve with maturity.

Because actual losses in the layer are not used for the ELF procedure, the results can be inconsistent with emerged losses to date (e.g., estimated ultimate losses less than actual losses emerged to date). Therefore, it is often preferable to perform a Bornhuetter-Ferguson analysis using the results of the excess loss method as an *a priori* estimate (excess loss-BF method). The Bornhuetter-Ferguson method places an increasing weight on the actual losses emerged as the data matures.

Reviewing the results of the retrospective testing for both data bases, we found that the all-accident year total for the excess loss-BF methods improved relative to the best estimate as the data matured, while the results of the ELF method alone did not. In fact, for the second data base, the all-accident year results of the excess loss-BF method were within 1.0% of the all-accident year best estimate as of year-end 1994, while the results of the excess loss method were 13.3% above the best estimate. In addition, we found that the all-accident year results of the excess loss-BF method improved relative to the other estimates as the data matured. It appears that the combination of the excess loss method and the Bornhuetter-Ferguson method is preferable to the excess loss method alone, and that more weight can be applied to the excess loss-BF method as the data mature. Note that the excess loss-BF method is slightly more responsive if ELFs are applied to losses since the base data will also reflect actual emerged experience.

Frequency/Severity Method: The frequency/severity method is heavily dependent on the parameterization of the loss distribution used to estimate the number of claims and severity of a claim in the layer. For our first data base we selected the single-parameter pareto distribution to model claim frequency and severity in the layer. Our selection was based on several factors:

- We believe that the single-parameter pareto distribution provides a reasonably good fit to the actual data,
- Parameterization of the distribution is relatively easy and, in general, can be accomplished based on empirical data, and
- The distributional form of the single-parameter pareto is easy to manipulate to obtain estimates for the frequency and severity in the layer (complex, multi-parameter distributions often produce results very similar to less complex distributions).

We also tested the two-parameter pareto distribution on the first data base and found that the results were very similar to the single-parameter pareto. While we believe that the pareto distribution is a reasonable distribution for modeling excess frequencies and severities, we encourage you to review Hogg and Klugman[1] for further information regarding other distributions.

For the second data base we used a pre-packaged software program to fit alternative curves to the actual ground-up size of loss data. We examined the results of several test statistics (Chi-square and Kolmogorov-Smirnov) and found the lognormal generally produced superior results compared to other distributions. In applying the lognormal distribution, we varied our assumed distribution means to reflect trend in the average loss size.

Like the excess loss method, the frequency/severity method does not directly utilize actual emerged losses in the layer. Therefore, the potential for results that are inconsistent with actual emerged data exists. In addition, this property makes the frequency/severity method less suited to mature accident years. However, by using the results of the frequency/severity method as the *a priori* estimate for a Bornhuetter-Ferguson method (frequency/severity-BF), actual loss emergence can be recognized.

Based on a review of the retrospective analysis, the all-accident year totals for the frequency/severity method generally produced the poorest results at the more mature years. However, the results of the frequency/severity-BF method improved relative to the other methods as the data matured, although, for the first data base the results were still generally not consistent with the best estimate.

We believe the relatively poor performance of this method applied to data base one can be attributed to the following factors:

- The volume of claims was not high enough to produce a stable triangle of claims used to select claim development factors, and,
- Not using actual losses in the layer resulted in indicated ultimate losses that were not consistent with actual data.

As with the excess loss factor method, it appears that the combination of the frequency/severity method with the Bornhuetter-Ferguson method is preferable to the frequency/severity method alone. In either case, a decreasing weight should be applied to the indicated ultimate results of the this method as the data mature.

Difference Method: The difference method is an indirect application of the loss development method. Ultimate losses below the attachment point are estimated and subtracted from an estimate of ultimate losses limited to the attachment point plus the limit. In our description of the method, both quantities are estimated using a development factor approach. Because the results of the loss development method are highly leveraged in immature accident years, we had concluded above that less weight should be applied to the loss development method at early evaluations.

This conclusion may *appear* to apply to the difference method, and certainly would if we were concerned with the individual estimates of losses below the attachment point and losses below the attachment point plus the layer. However, for the purpose of estimating losses in the layer, we focus on the *difference* between the two estimates, not on the estimates themselves. In addition, since the data used to estimate losses below the attachment point are a subset of the data used to estimate losses up to the limit of the layer, it would seem to follow that aberrations in the data affecting one set of data would affect both sets of data, and in the same direction. For instance, if losses below the attachment point emerge at a rate that is less than expected, then the losses below the attachment point plus the layer will also emerge at a rate that is less than expected. Therefore, any understatement of the ultimate costs will have an impact on both estimates.

As with several of the ELF and frequency/severity methods, we considered the results of a Bornhuetter-Ferguson approach using the estimates of the difference method as the *a priori* assumption (difference-BF). For the first data base, which included case reserve adjustments between the 1992 and 1993 evaluation, we found that the difference method and the difference-BF method seemed to outperform the other methods when compared to our selected best estimate. In fact, for data base one at year-ends 1991 to 1994, the difference-BF method ranked in the top three results (i.e., closest to the best estimate) of all methods and combinations of methods considered and the difference method ranked first at year-end 1992 and in top four at the other evaluations. In addition, the indicated all-accident year results of the difference-BF method were within 7.0% of the best estimate at each of the year-end evaluations. In this case, it appears that the difference method somewhat minimized the distortions created by the case reserve changes.

In the second data base, the difference method estimates produced reasonably consistent estimates at each of the four evaluations, however, it did not out perform the other projections. We believe it is appropriate to apply a consistent weight to this method over time.

Individual Claim Development: The individual claim development method is based primarily on an assumption we know to be inaccurate, that is, all current claims develop by the same amount. While we accept that all methods will perform poorly in some situations, we should rarely choose to rely on a method that contains such an oversimplification. That issue aside, the individual claim development method is used by some actuaries to estimate potential losses in the excess layer. Overall, the method performed consistency over all four evaluations, improving slightly as the data matured.

For the first data base, the individual claim development method performed better relative to the other techniques at the earlier evaluations than at the later evaluations. This result may have occurred because a greater portion of the ultimate losses were generated by the frequency/severity portion of this method (i.e., the unreported claims). However, as the accident years mature, loss projections are primarily driven by the development of current claims. Since this development on current claims involves a significant over-simplification, the accuracy of the method does not improve.

There were not a significant number of unreported claims for the second data base (perhaps due to the lower attachment point), and thus, the frequency/severity portion of this method did not figure heavily in the projection of ultimate losses. For the second data base, the results were among those estimates closest to the best estimate at the earliest evaluation and improved slightly as the data matured. Because this method is based on erroneous assumptions, we do not recommend placing much weight on the results.

Simulation Method: The simulation method is based on a number of underlying assumptions that will vary based on how the model is constructed. In our analysis of both data bases these assumptions are:

- The development factor applied to an individual claim is dependent on the age of the claim (accident year), the size of the incurred losses, and the estimated average development factor for the group of claims. The distribution of development factors can be estimated based on historical distributions.

- The number of unreported claims emerging will be proportional to the number of claims already reported in the layer (claim development method). The distribution of potential unreported claims is approximately normal.
- The average severity of unreported claims is independent of the time the claim emerges. A size of loss distribution is used to project costs associated with unreported claims (pareto for the first data base and lognormal for the second data base).

A significant benefit of this method is the additional analysis that goes into producing the projections as well as the questions that are raised when reviewing the results in comparison to the other methods. For instance, the results of the simulation method can be analyzed, and values such as the ultimate frequency and severity can be compared to the corresponding values implied by other methods. In addition, ratios of losses in the layer to ultimate losses in excess of a lower limit can be used to test the reasonableness of the ELF assumptions. In fact, since claim detail figures are produced, virtually any comparison can be made from current reported information to projections of ultimate values.

For the first data base, the simulation method for immature accident years relied primarily on the projection of unreported claims from frequency/severity portion of the analysis. As the accident years matured, more weight is placed on the assumed distribution of development factors. The simulation method appears to have performed reasonably well relative to the overall best estimate and in comparison with the other methods for all accident years and evaluations. Our relatively mechanical application of this method to the first data base produced results the departed from our best estimates and, in an actual application, would indicate an area for further investigation.

For the second data base, the simulation method produced results that, in total, were among the best estimates at each of the four evaluations. We note that the results produced by the simulation method are sensitive to many of the input assumptions, therefore, a great deal of care must be exercised when relying on these estimates for a reserve projection.

Summary: Comparing the results of our analysis for both data bases, as expected, shows that no single technique consistently outperforms the other methods. Each method, with perhaps the exception of the individual claim development technique, provides some insights to potential ultimate losses for the excess layer. In general, the excess loss-BF, frequency/severity-BF, and difference methods seem to perform better at early evaluations, while the incurred loss development method seemed to perform better at later evaluations. The simulation method provides some valuable insights and can be expected to produce reasonable results, however, a greater degree of care (and time) is required to ensure the model is properly parameterized.

In conclusion, ***the process of selecting an ultimate loss value relating to a particular group of claims cannot be reduced to a purely mechanical process.*** The most valuable time spent by an actuary when performing a review of the liabilities for a particular book of business is understanding the coverage provided, the manner in which claims are reported and data are collected, and the process by which reported claims are reserved on a case-by-case basis. This information can generally be collected only by interviewing the responsible party(ies) in the claims, underwriting and data-reporting areas of the company. This information better enables the reasonable application of loss reserve methods and proper interpretation of the results. Without this type of information, the assumptions regarding the data will more likely lead to inaccurate estimates.

Appendix A - Summary of Pinto/Gogol Method of deriving loss development factors for excess layers.

The general method used to derive loss development factors applicable to losses in an excess layer as described by Pinto and Gogol[2] is as follows:

- *Derive reporting patterns applicable to a range of retention points.* Incurred loss triangles for losses in excess of selected detrended retentions were constructed, and loss development factors were selected by applying a chain ladder technique. The selected retention point for the most recent policy year was detrended for application in earlier policy years. Pinto and Gogol used an ISO data base of commercial general liability claims. Their selected factors by retention are displayed in *Exhibits 1 to 3*. The selected age-to-age development factors are referred to by Pinto and Gogol in *Exhibit 4* as "a_n values-actual."

- *Fit a curve to the selected excess development factors by age.* The selected age-to-age development factors are then used to estimate curves which, in addition to smoothing the underlying factors, allow for extrapolation to maturities that were not available in the ISO data base. The paper does not appear to specify the particular curve used to fit the development factors' maturity. For the purpose of the example in this paper, we used a curve of the type $y = ax^b$, where x is the maturity in months and $y = a_n - 1$. Values of a and b for each retention were determined by fitting the values of ln(x) and ln(y) to a least squares line which gives $\ln(y) = \ln(a) + b*\ln(x)$.

- *Fit a curve to the excess development factors by retention.* For each development interval, a curve is fit to smooth the fitted age-to-age factors by retention. The curve selected to fit the excess development factors by retention was $y = ax^b$, where x is the retention divided by $10,000 (the selected data limit), a is the development factor excess of $10,000, and y is the fitted factor. In practice, we found that we could reproduce the b-values by retention displayed in exhibits 5, 6 and 7, by using a curve of the type $y = x^b$, where x is the development factor for the retention divided by the development factor for the $10,000 retention (i.e., a relativity of the excess loss development factor for the retention relative to the $10,000 retention) . Values of ln(x) and ln(y) were fit to a least squares line passing through the origin.

Using this method, we can produce a table of excess loss development factors by retention limit. However, we are interested in excess loss development factors by layer. In section 5, Pinto and Gogol introduce the following formula to calculate excess loss development factors by layer using the excess loss development factors and excess loss factors (ELFs):

$$LDF_{c,d} = \frac{f(c) - f(d)}{e_{c,n} - e_{d,n}}$$

where, $LDF_{c,d}$ = loss development factor to ultimate for losses in the layer c to d
c = attachment point
d = Attachment point plus the limit
f(c) = ratio of losses in excess of c to ground-up ultimate losses
f(d) = ratio of losses in excess of d to ground-up ultimate losses
$e_{c,n}$ = f(c) divided by the loss development factor to ultimate, for the retention *c* and month *n*
$e_{d,n}$ = f(d) divided by the loss development factor to ultimate, for the retention *d* and month *n*

A brief inspection will reveal that the numerator is an estimate of the percentage of ground-up ultimate losses in the layer, while the denominator is an estimate of the expected reported losses (as a percentage of ground-up ultimate losses) as of month *n* in the layer. For workers compensation, the NCCI excess loss factors provide a suitable estimate for f(c) and f(d) (except for the issues mentioned in *Section I*).

Reserving for Excess Layers
Incurred Loss Development

Accident Year	Age in Months @ @12/31/94	Incurred Losses In Layer	Percent Reported	Projected Ultimate Losses
	(1)	(2)	(3)	(4)
1990	60	9,220,962	60.27%	15,298,414
1991	48	10,767,949	53.42%	20,155,392
1992	36	8,536,993	43.84%	19,475,016
1993	24	3,803,464	34.25%	11,106,116
1994	12	672,472	17.12%	3,927,236
		33,001,840		69,962,173

Notes:

(2) Reported losses in the layer $250,000 excess of $250,000
(3) Expected percentage of ultimate losses excess of $250,000 reported
(4) = (1)/(2)

Reserving for Excess Layers
Loss Development Method using Pinto & Gogol Loss Development Factors

Accident Year	Percent Reported for Losses Excess 500k	Percent Reported for Losses Excess 250k	Excess Loss Factor 250k	Excess Loss Factor 500k	Pinto & Gogol LDF	Incurred Losses In Layer	Percent Reported	Projected Ultimate Losses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1990	53.45%	60.27%	21.65%	12.87%	1.423	9,220,962	70.28%	13,120,925
1991	46.12%	53.42%	22.73%	13.56%	1.557	10,767,949	64.23%	16,765,635
1992	36.53%	43.84%	23.85%	14.28%	1.827	8,536,993	54.74%	15,596,445
1993	30.54%	34.25%	25.01%	15.04%	2.510	3,803,464	39.84%	9,547,318
1994	15.12%	17.12%	26.22%	15.83%	4.957	672,472	20.18%	3,333,119
						33,001,840		58,363,442

Notes:
(1) *Reportng pattern for losses excess of a $500,000 attachment point*
(2) Reporting pattern for losses excess of a $250,000 attachment point
(3) Percentage of ultimate losses in excess of a $250,000 attachment point
(4) Percentage of ultimate losses in excess of a $500,000 attachment point
(5) = [(3)-(4)]/{[(3)*(2)]-[(4)*(1)]}
(6) Reported losses in the layer $250,000 excess of$250,000
(7) = 1/(5)
(8) = (6)/(7)

Reserving For Excess Layers
Excess Loss Factor Method

Accident Year	Incurred Losses	Percent Reported	Ultimate Losses	Excess Loss Factor 100k	Excess Loss Factor 250k	Excess Loss Factor 500k	Ultimate Losses In Layer
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1990	80,290,684	82.55%	97,264,441	39.41%	21.65%	12.87%	21,669,165
1991	81,448,233	78.42%	103,855,144	40.90%	22.73%	13.56%	23,284,882
1992	74,960,557	71.17%	105,331,706	42.39%	23.85%	14.28%	23,779,770
1993	35,329,211	59.31%	59,563,708	43.88%	25.01%	15.04%	13,533,504
1994	7,331,857	35.09%	20,895,383	45.36%	26.22%	15.83%	4,786,222
Total	279,360,542		279,360,542				279,360,542

Notes:
(1) Ground-up reported losses on claims in excess of $100,000
(2) Expected percentage of ultimate losses on claims > 100k reported
(3) Estimated ultimate losses on claims > 100k
(4) Percentage of ultimate losses in excess of a $100,000 attachment point
(5) Percentage of ultimate losses in excess of a $250,000 attachment point
(6) Percentage of ultimate losses in excess of a $500,000 attachment point
(7) = [((5)-(6))/(4)] * (3)

Reserving For Excess Layers
Bornhuetter - Ferguson Using ELF Method

Exhibit II
Page 2

Accident Year	Expected Ultimate Losses In Layer	Percent Unreporte	Expected Unreported Losses In Layer	Actual Losses Reported In Layer	Projected Ultimate Losses
	(1)	(2)	(3)	(4)	(5)
1990	21,669,165	39.73%	8,608,298	9,220,962	17,829,260
1991	23,284,882	46.58%	10,845,014	10,767,949	21,612,962
1992	23,779,770	56.16%	13,355,761	8,536,993	21,892,754
1993	13,533,504	65.75%	8,898,743	3,803,464	12,702,207
1994	4,786,222	82.88%	3,966,663	672,472	4,639,135
Total	87,053,543		45,674,479	33,001,840	78,676,319

Notes:
(1) Based on Excess Loss Factor Method
(2) 1 - [Exhibit I, Page 1, column (2))
(3) = (1) * (2)
(4) Actual losses reported in the layer $250,000 excess of $250,000
(5) = (3) + (4)

Reserving for Excess Layers
Frequency/Severity Method

Exhibit III
Page 1

Accident Year	Data Limit	Number of Claims > Data Limit	Claim Count Dvlp'mt Factor	Ultimate Counts In Excess of Data Limit	Counts in Excess of Reten. Two Parameter	Counts in Excess of Reten. Single Parameter	Detrend Factor	Average Severity In Layer Two Parameter	Average Severity In Layer Single Parameter	Ultimate Losses In Layer Two Parameter	Ultimate Losses In Layer Single Parameter
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
				(2)*(3)						(5)*(8)	(6)*(9)
1990	200,000	146	1.470	215	144	144	1.574	133,182	133,016	19,171,067	19,110,465
1991	200,000	147	2.059	303	203	203	1.405	133,202	133,016	27,027,368	26,937,902
1992	200,000	135	3.603	486	326	325	1.254	133,224	133,016	43,444,115	43,293,056
1993	200,000	51	6.845	349	234	234	1.120	133,249	133,016	31,189,065	31,074,794
1994	200,000	10	23.957	240	161	160	1.000	133,277	133,016	21,408,751	21,325,839
										142,240,365	141,742,056

Two Parameter Pareto

B =	1,070
Q=	1.800
DL=	200,000
R=	250,000
L =	250,000

Single Parameter Pareto

Q = 1.800

Detrend = 1.12

Notes:

(1) Data limit
(2) Number of reported claims as at December 31, 1994 greater than the data limit
(3) Claim development factor; Derived from Exhibit III, Page 3
(4) = (2) * (3)
(5) = (4) * [(DL + B)/(R + B)]^Q
(6) = (4) * [(R/DL) ^ -Q]
(7) = Detrend Factor; 1.12^(1994-Year)
(8) = [(R + (B/(7)))/(Q - 1)]x{1 - [(R + (B/(7)))/(R + L + (B/(7)))]^(Q - 1)}
(9) = {R x [(Q - ((R + L)/R) ^ (1 - Q))]/(Q - 1)} - R
(10) = (5) * (8)
(11) = (6) * (9)

Reserving For Excess Layers
Bornhuetter - Ferguson Using Frequency/Severity Method

Exhibit III
Page 2

Accident Year	Expected Ultimate Losses In Layer	Percent Unreported	Expected Unreported Losses In Layer	Actual Losses Reported In Layer	Projected Ultimate Losses
	(1)	(2)	(3)	(4)	(5)
1990	19,110,465	39.73%	7,591,828	9,220,962	16,812,790
1991	26,937,902	46.58%	12,546,420	10,767,949	23,314,369
1992	43,293,056	56.16%	24,315,278	8,536,993	32,852,271
1993	31,074,794	65.75%	20,432,741	3,803,464	24,236,205
1994	21,325,839	82.88%	17,674,154	672,472	18,346,626
Total	141,742,056		82,560,422	33,001,840	115,562,262

Notes:
(1) Based on Frequency/Severity Method
(2) 1 - [Exhibit I, Page 1, Column(2)]
(3) = (1) * (2)
(4) Actual losses reported in the layer $250,000 excess of $250,000
(5) = (3) + (4)

Reserving For Excess Layers
Claim Count Development

Exhibit III
Page 3

Loss + Medical Claim Counts

Accident Year	Attachment Point (1.12)	12	24	36	48	60	72	84	96
1987	90,470	6	17	83	113	172	211	258	275
1988	101,326	2	45	64	135	207	278	327	
1989	113,485	18	26	54	126	227	293		
1990	127,104	11	20	43	202	295			
1991	142,356	5	20	147	241				
1992	159,439	6	74	173					
1993	178,571	8	61						
1994	200,000	10							

------ Report - to - Report Factors ------

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
1987	2.833	4.882	1.361	1.522	1.227	1.223	1.066
1988	22.500	1.422	2.109	1.533	1.343	1.176	
1989	1.444	2.077	2.333	1.802	1.291		
1990	1.818	2.150	4.698	1.460			
1991	4.000	7.350	1.639				
1992	12.333	2.338					
1993	7.625						

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	
Avg All	7.508	3.370	2.428	1.579	1.287	1.200	1.066	
Wtd All	2.720	2.291	1.767	1.498	1.263	1.196	1.465	
Avg 3yr	7.986	3.946	2.890	1.598	1.287			
Wtd 3yr	6.231	2.298	1.826	1.490	1.263			
Selected	5.000	2.250	1.900	1.500	1.250	1.175	1.065	1.100
Age-to-Ult	55.168	11.034	4.904	2.581	1.721	1.377	1.172	1.100

Reserving for Excess Layers
Difference Method

Estimate of Ultimate "Ceded" Losses

Accident Year	Reported Losses Limited to $500,000	LDF to Ultimate	Ultimate Losses $500,000 Retention	Reported Losses Limited to $250,000	% of Ultimate Reported	Ultimate Losses $250,000 Retention	Ultimate Excess Losses	Reported Excess Losses @12/31/94
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1990	76,299,050	62.52%	122,037,945	67,078,088	67.78%	98,971,329	23,066,616	9,220,962
1991	79,333,758	51.25%	154,808,077	68,565,809	56.01%	122,411,354	32,396,723	10,767,949
1992	74,855,226	38.53%	194,271,617	66,318,232	43.09%	153,918,348	40,353,269	8,536,993
1993	34,817,110	24.86%	140,059,202	31,013,646	28.72%	107,969,609	32,089,593	3,803,464
1994	7,331,857	8.57%	85,532,450	6,659,385	10.45%	63,755,187	21,777,263	672,472
Total	272,637,000		696,709,291	239,635,160		547,025,827	149,683,464	33,001,840

Notes:
(1) Reported Losses limited to $500,000
(2) Expected percentage of ultimate losses limited to $500,000 reported
(3) = (1)/(2)
(4) Reported Losses limited to $250,000
(5) Expected percentage of ultimate losses limited to $250,000 reported
(6) = (4)/(5)
(7) = (3) - (6)
(8) Reported losses in the layer $250,000 excess of $250,000

Reserving for Excess Layers
Individual Claim Development Method
Sample of Individual Claim Development Method

Claim Number	Accident Year	Incurred Loss @12/31/94	% of Ultimate Reported	Projected Ultimate Loss	Projected Layer Losses
(1)	(2)	(3)	(4)	(5)	(6)
42711	1987	233,451	89.79%	260,002	10,002
272955	1987	167,285	89.79%	186,311	-
464249	1988	278,950	87.91%	317,323	67,323
435112	1988	173,721	87.91%	197,618	-
169802	1989	398,637	85.36%	467,005	217,005
266644	1989	253,160	85.36%	296,578	46,578
34669	1990	279,488	82.55%	338,573	88,573
576948	1990	129,424	82.55%	156,785	-
649584	1992	197,409	71.17%	277,392	27,392
649821	1992	219,514	71.17%	308,453	58,453
676564	1993	174,526	59.31%	294,244	44,244
678312	1993	121,999	59.31%	205,686	-
712493	1994	142,809	35.09%	406,998	156,998
713480	1994	109,383	35.09%	311,735	61,735

Notes:
(1) Claim Number
(2) Accident Year
(3) Ground-up reported losses as of December 31, 1994
(4) Expected percentage of ultimate losses reported
(5) = (3)/(4)
(6) = max(500,000, (5)) - 250,000

Reserving For Excess Layers
Individual Claims Development Method

Exhibit V
Page 2

As at December 31, 1994

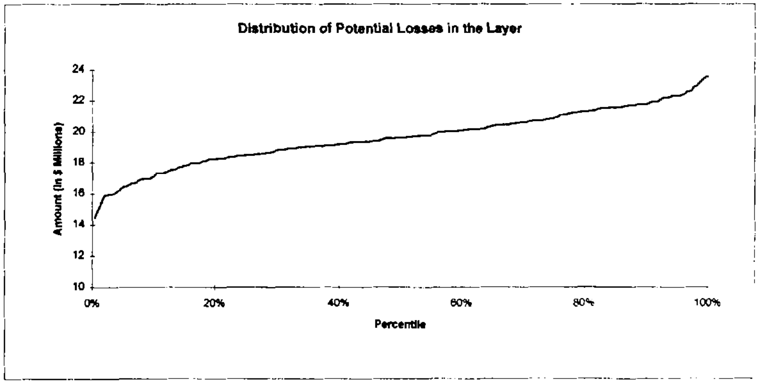
Accident Year	Projected Layer Losses	Projected Number of Claims In the Layer	Expected Ultimate Claims In the Layer	Expected IBNR Claims	Expected IBNR Severity	Expected IBNR Losses	Expected Ultimate Losses
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1987	14,968,210	110	87	0	133,016	-	14,968,210
1988	15,675,012	127	106	0	133,016	-	15,675,012
1989	14,458,486	121	109	0	133,016	-	14,458,486
1990	15,333,241	138	144	6	133,016	754,273	16,087,514
	60,434,949	496	445	6			61,189,223

Notes:
(1) Result of developing individual claims greater than $100,000 (data limit) to ultimate values and and applying coverage provisions to determine projected ultimate losses in the layer for known claims
(2) Number of claims with projected ultimate values greater than $250,000
(3) Expected ultimate number of claims greater than $250,000; based on single parameter pareto distribution from the Frequency/Severity Method [Exhibit III, Page 1]
(4) Max(0, (3)-(2))
(5) Expected ultimate severity of losses in the layer $250,000 excess of $250,000 [Exhibit III, Page 1]
(6) = (5) * (4)
(7) = (6) + (1)

Reserving for Excess Layers
Summary of Iteration Results for a Single Accident Period

	Claims in Excess of $100,000		Claims in the Layer		
Percentile	Projected Claim Counts	Projected Ultimate	Projected Claim Counts	Average Loss Size	Projected Ultimate
10%	324	85,764,005	127	130,101	17,290,639
25%	332	88,970,722	133	134,674	18,492,378
50%	343	92,100,169	142	138,754	19,646,394
60%	346	93,769,006	144	140,854	20,099,299
75%	353	95,817,186	149	143,123	20,899,313
90%	362	98,495,432	155	148,076	21,766,831
95%	368	100,188,848	158	150,348	22,306,988
97%	370	100,974,722	160	151,617	22,598,575
99%	373	102,030,969	162	153,061	23,353,312
Mean	343	92,313,573	141	138,917	19,621,573
Mode	341	91,000,192	145	135,979	19,000,012

Note that the percentile categories are independent of each other, i.e., the 10th Percentile projected claim counts does not necessarily correspond to the same iteration that produced the 10th percentile projected Ultimate



Reserving for Excess Layers
Sample Determination of Random Claim Development Factor

Record	Claim ID	Accident Year	Incurred Loss	Incurred Loss Percentile	Random Number	Loss Size Correlation	Random Look-up Value	Random Development Factor	Potential Projected Loss	Loss in Layer
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	28957	1987	132,977	40.7%	0.1099	-0.25	0.23070	1.000	132,979	-
2	42064	1987	222,967	75.0%	0.1732	-0.25	0.19239	1.000	222,967	-
3	16569	1987	651,395	97.3%	0.4063	-0.25	0.31145	1.000	651,448	250,000
4	72915	1987	129,091	37.5%	0.3612	-0.25	0.42714	1.000	129,123	-
5	83897	1987	592,863	96.0%	0.4634	-0.25	0.35756	1.000	592,942	250,000
6	66609	1987	113,748	19.0%	0.5032	-0.25	0.57992	1.007	114,598	-
7	43727	1987	157,602	57.2%	0.1222	-0.25	0.19868	1.000	157,602	-
8	26334	1987	147,200	51.9%	0.1106	-0.25	0.20318	1.000	147,200	-
9	37349	1987	120,199	24.3%	0.1044	-0.25	0.26752	1.000	120,203	-
10	25691	1987	683,031	98.0%	0.1266	-0.25	0.09992	0.979	668,613	250,000
11	13478	1987	161,069	60.5%	0.9125	-0.25	0.78314	1.282	206,432	-
12	04657	1987	184,798	66.4%	0.8415	-0.25	0.71515	1.052	194,349	-
13	73915	1987	111,768	18.4%	0.1550	-0.25	0.32028	1.000	111,779	-
14	84897	1987	178,244	65.1%	0.4372	-0.25	0.41517	1.000	178,281	-
15	67609	1987	158,515	58.5%	0.5386	-0.25	0.50767	1.001	158,644	-
:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:
151	50915	1987	101,557	2.6%	0.5747	-0.25	0.67449	1.030	104,605	-
152	62430	1987	601,205	96.7%	0.1221	-0.25	0.09984	0.979	588,514	250,000
153	69844	1987	122,545	27.6%	0.6221	-0.25	0.64760	1.020	125,054	-
154	Pure IBNR								135,420	-
155	Pure IBNR								238,306	-
156	Pure IBNR								336,804	86,804
157	Pure IBNR								409,235	159,235
:									:	:
:									:	:
340	Pure IBNR								114,507	-
341	Pure IBNR								247,985	-
342	Pure IBNR								600,000	250,000
343	Pure IBNR								600,000	250,000
344	Pure IBNR								433,746	183,746
345	Pure IBNR								168,868	

(1) *Data base Record Number*
(2), (3), & (4) Current claim data. Exists only for current reported claims.
(5) *Percentile ranking of incurred* losses by size.
(6) Random number
(7) Selected Correlation between size of loss and magnitude of development factor.
(8) Adjusted random number. (6)(8) + (7)[1.0 - abs(8)] - [(8) if negative, 0 if (8) is positive]
(9) Look up value of development factor associated with adjusted random figure in (9)
(10) (4) x (9)
(11) Losses over the attachment and limited to the upper cap.