A Survey of Methods Used to Reflect Development in Excess Ratemaking

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Abstract

This paper discusses the strengths, weaknesses, and application of several methods used to obtain an estimated ultimate loss distribution from data whose valuation is less than final. The central issues are introduced by examining several basic methods via a simple example. This foundation is followed by a description of three additional methods which rely on industry loss distributions as a basis for obtaining the ultimate loss distribution using limited data. Finally, a more robust method is introduced which accommodates slightly more refined, but not atypical, data.

Introduction

There is a substantial amount of published material on fitting statistical distributions to sample data[1]. In actual practice the sample data usually consist of individual claims and the distributions fit to this claim data are referred to as loss distributions. Several authors[2] have illustrated the use of loss distributions in estimating various insurance pricing factors, such as deductible credits, increased limits factors, and excess loss factors. However, most of these materials tiptoe around the issue of loss development. For instance, the current staple reference of the profession regarding loss distributions, "Loss Distributions" by Messrs. Hogg and Klugman[3], directly fits loss distributions to property and liability claims and then immediately uses these curves in further computations. This process essentially makes the development assumption that the individual case reserves are correct and that unreported claims will basically be no different in nature than the claims which have been reported. While this assumption may be appropriate for direct use in estimating deductible factors, increased limits factors, etc., for property claims, the errors arising are often too large to be ignored without adjusting the distribution for the effects of loss development for non-property claims. Appendix A discusses some of the literature references to this problem.

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[1] Hogg, R.V., and Klugman, S.A., Loss Distributions, 1984
Hogg, R.V., and Craig, A.T., Introduction to Mathematical Statistics (Fourth Edition)
Hossack, I.B., Pollard, J.H., and Zehnwirth, B., Introductory Statistics with Applications in General Insurance, 1983, Chapters 4 (all sections), 5 (all sections), 6 (6.1-6.4), and 8 (excluding 8.7).
[2] Gary S. Patrik, "Reinsurance," Foundations of Casualty Actuarial Science (Second Edition), Casualty Actuarial Society, 1992
Keith D. Holler, Review of "The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach," Forum, Spring 1992
Stephen W. Philbrick, A Practical Guide to the Single Parameter Pareto Distribution, PCAS LXXII, 1985
[3] Hogg, R.V., and Klugman, S.A., Loss Distributions, 1984

This paper discusses some of the issues regarding the recognition of loss development when estimating liability loss distributions. It is separated into three sections. These sections are organized as follows:

Section I — This section will illustrate the nature and potential magnitude of the problem of using an artificially simplified data set containing a handful of claims. This section also demonstrates that several standard adjustments for development do not sufficiently address the problem.

Section II — This section illustrates three intermediate techniques which provide a more complete solution to the problem:

- Use shape of industry curves without adjustment
- Use shape of industry curves with adjustment for mean values
- Use industry curves incorporating the latest evaluation date of individual claim data.

Section III — This section will discuss a more rigorous approach to account for loss development in the case that more complete historical data on individual claims is available.

Section I - Examining The Basic Problem

In order to examine the basic dynamics surrounding the development issue, this section will discuss a simple empirical sample which at the time of evaluation, consists of 10 claims. Continuous loss distributions, other than the empirical distribution, will not be discussed in this section as the basic concepts are unaffected by the transition from the actual data to a continuous loss distribution that seeks to model the process underlying the actual data. While we realize that this simple example may not totally reflect reality, it is provided to familiarize the reader with some of the fundamental issues surrounding the problem and thereby more fully prepare the reader for the latter sections of the paper.

Given the following ten claims from a specific accident period, the goal will be to estimate a $1,250 deductible credit, a $12,500 increased limits factor (ILF), and a $7,500 excess loss factor (ELF). Assume that the base limit is $5,000.

Claim Number	Status	Paid Value	Incurred Value
1	Closed	$600	$600
2	Closed	800	800
3	Closed	1,100	1,100
4	Closed	1,300	1,300
5	Closed	1,600	1,600
6	Closed	1,800	1,800
7	Closed	2,500	2,500
8	Open	0	3,000
9	Closed	11,000	11,000
10	Open	0	12,000
Total	10	$20,700	$35,700

The goal is to estimate the following quantities for all claims occurring during the specific accident period after they have each been reported, settled, and closed.

Deductible Credit[4] = $\frac{\text{Sum losses limited to the deductible}}{\text{Sum of unlimited losses}}$

Increased Limits Factor[5] = $\frac{\text{Sum of losses limited to \$12,500}}{\text{Sum of losses limited to the basic limit (\$5,000)}}$

Excess Loss Factor = $\frac{\text{(Sum of unlimited losses - Sum of losses limited to \$7,500)}}{\text{Sum of unlimited losses}}$

The process of loss development consists of the reporting of claims to the insurer and the adjustment of those claims until each claim is closed. Typically, as a body of claims develops, the total value and the average value of the claims increase on both a paid and an incurred basis[6]. Assume, for the present, that no IBNR claims are reported. We will revisit this assumption later. Assume that the final settlement values for each claim are as follows:

Claim Number	Status	Final Value
1	Closed	$600
2	Closed	800
3	Closed	1,100
4	Closed	1,300
5	Closed	1,600
6	Closed	1,800
7	Closed	2,500
8	Closed	15,000
9	Closed	11,000
10	Closed	20,000
Total	10	$55,700

Based on the ultimate distribution of the 10 claims from this accident period, the actual factors should be:

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[4] Calculation of deductible credits in the context of workers compensation coverage normally uses the sum of unlimited losses as the denominator. In liability coverages, it is more usual to use losses at some limit as a denominator. The reader is invited to restate the data if a different convention is preferred.

[5] The concept of increased limits factors rarely occurs directly in workers compensation. However, the pricing of excess layers often uses techniques that are mathematically equivalent to an ILF approach, so we believe the issue associated with appropriate adjustments to ILF's to account for development also apply to workers compensation.

[6] This is apparent if one divides the paid or incurred losses by the reported counts for the industry in total using data from Best's Aggregates and Averages. It is also apparent upon examining the average loss by settlement lag implied by ISO's selected loss distributions for general liability.

Item	Calculation	Value
Deductible Credit (DED)	11,250/55,700	0.202
Increased Limits Factor (ILF)	45,700/24,700	1.850
Excess Loss Factor (ELF)	(55,700 - 32,200)/55,700	0.422

These are the actual factors for this accident period that we are trying to estimate. The key is that these factors are usually estimated using a body of claims that are not fully developed. In this example, we need to estimate the actual factors using the original ten claims.

The following are several basic approaches that might be used:

- Estimation using the eight closed claims;
- Estimation using the incurred value of the ten claims;
- Estimation using the incurred values after adjusting by a single loss development factor; and
- Estimation using the incurred values of the ten claims after adjusting the open claims by a single total case reserve development factor, which includes a provision for the unreported claims.

Each of these approaches will mis-estimate the actual factors.

The purported justification for using closed claims is that they are settled and their values will not be subject to change. The problem with using closed claims is that the eight closed claims do not represent the same loss distribution as the ultimate body of ten claims. The estimated factors using the closed claims only are:

Item	Calculation	Value
DED	8,750/20,700	0.423
ILF	20,700/14,700	1.408
ELF	(20,700-17,200)/20,700	0.169

The deductible credit is overstated because the larger claims, which develop and are closed later, add more to the denominator of the calculation of the deductible credit than they do to the numerator. The ILF is understated because the future development of the larger claims tends to push a greater percentage of the losses into the higher layers. The excess loss factor is understated for the same reasons.

One could use the incurred values of the ten claims. The use of incurred values attempts to include more information about the claims than is contained in the actual paid values alone. In essence, the incurred values recognize more of the development in the claims than the paid values. The use of the incurred claim values results in the following estimates:

Item	Calculation	Value
DED	11,250/35,700	0.315
ILF	35,700/22,700	1.573
ELF	(35,700-27,700)/35,700	0.224

The use of incurred claim values produces errors in the same direction as the use of closed claims. However, the magnitude is smaller.

Once again, this is due to the fact that the average claim value and the spread of the claims tend to increase with time. The increase in the average claim value over time is supported by statistics from Best's aggregates and averages[7], while the increase in the spread of claims is supported by actual data and common sense. There are two common sense arguments supporting the latter phenomenon. The first is that if the average value increases, then if all claims have been reported, the total increase must originate from the open claims. If one

[7] Average incurred claim size can be calculated from incurred dollars and reported counts shown in the industry aggregate Schedule P exhibits. A comparison of subsequent years' values will demonstrate the increase over time.

assumes that the average open claim is larger than the average closed claim, then increasing the values of open claims should increase the variance of the total body of claims. The second common sense argument is the "big bang" theory. The big bang theory suggests that adjusters do a pretty good job on most of the claims, but usually get surprised by one or two claims. Thus, given a body of claims with ten open claims, eight might settle within a relatively small percentage of the case reserve, but the remaining two claims "explode" and settle at much more than the case reserve. The net effect is that the two problem claims spread the distribution of all the claims and account for a substantial portion of the total dollar development.

In order to more properly recognize claim development, one might suggest that we simply multiply each of the incurred claims by a development factor and then compute the statistics. This approach has the continuous loss distribution analog of multiplying the individual losses by a development factor before fitting the loss distribution. In the example, assume that the incurred development factor is known with certainty to be 1.560 (55,700/35,700). The resulting factors are:

Item	Calculation	Value
DED	12,184/55,700	0.219
ILF	44,815/29,815	1.503
ELF	(55,700-34,815)/55,700	0.375

If all the claims are reported, the deductible factor is still overstated because the use of a uniform development factor increases the value of the small closed claims too much. The ILF and the ELF for the larger limits are understated because the majority of the true upper layer development is distributed by the use of uniform factor to the more frequent smaller and moderate sized claims. Once again, the actual change in the spread of claims is not completely captured. For more moderate limits, the ILF and the ELF would be overstated because the values of many moderate-valued claims are increased beyond what they will actually settle for. This is due to the dollars that must be accounted for in the loss development factors arising from increases in large value claims.

Finally, one might decide to apply a development factor to the open reserves[8]. This factor can be calculated directly from historical data, or solved for using paid and incurred development factors. In this case, assume that the development factor applicable to open reserves is 2.333 ((20,000 + 15,000)/(12,000 + 3,000)). Applying this factor to the open claim reserves, and calculating the resulting factors yields:

Item	Calculation	Value
DED	11,250/55,700	0.202
ILF	40,200/24,700	1.628
ELF	(55,700-31,700)/55,700	0.431

The deductible factor is right on (although if we construct an example with open reserves well below the deductible amount, we would still get an error). The ELF is reasonably close while the ILF is still not close.

The calculations to this point have been based upon the assumption that no new claims are reported and the only development arises from known, open claims. In the usual situation in which IBNR claims do emerge, the picture becomes more complicated.

[8] The technique of using a factor applicable to open reserves only is not as widespread as other traditional methods. Part of the reason is the correct perception that the factor can be leveraged - at close to maturity, only a small portion of total incurred is still outstanding and the factors may swing widely based upon the actual prior settlements of just a few claims. However, at less mature ages, the perception of instability may be false. While the factors appear larger and more volatile, it should be noted that an incurred development factor can be derived from an outstanding development factor by adding a constant (paid dollars) to both numerator and denominator. While it should be clear that adding such a constant does force the resulting value closer to one, it can be argued that it is an artificial dampening of results. In any event, we think that this method should not be rejected simply because the typical factors are larger and appear to be more volatile.

Let us assume that, at ultimate, our total incurred is supplemented by two IBNR claims, one at $5,000 and one at $25,000. Then our ultimate distribution is as follows:

Claim Number	Status	Paid Value	Incurred Value
1	Closed	$ 600	$ 600
2	Closed	800	800
3	Closed	1,100	1,100
4	Closed	1,300	1,300
5	Closed	1,600	1,600
6	Closed	1,800	1,800
7	Closed	2,500	2,500
8	Closed	15,000	15,000
9	Closed	11,000	11,000
10	Closed	20,000	20,000
11	Closed	5,000	5,000
12	Closed	25,500	25,000
Total		$85,700	$85,700

With these ultimate claims, the correct factors are as follows:

Item	Calculation	Value
DED	13,750/85,700	0.160
ILF	63,200/34,700	1.821
ELF	(85,700-44,700)/85,700	0.478

The calculation of estimated factors using closed only claims does not change. For convenience, the results are repeated here:

Item	Calculation	Value
DED	8,750/20,700	0.423
ILF	20,700/14,700	1.408
ELF	(20,700-17,200)/20,700	0.169

Note that the deductible and ELF factors are further away from the correct values. The ILF is only marginally closer.

Similarly, the calculation using incurred amounts is unchanged and reproduced here:

Item	Calculation	Value
DED	11,250/35,700	0.315
ILF	35,700/22,700	1.573
ELF	(35,700-27,700)/35,700	0.224

Assuming that we have accurate incurred loss development factors reflecting IBNR emergence, we can update the incurred development method with the revised factor of 2.401 as follows:

Item	Calculation	Value
DED	12,500/85,700	0.146
ILF	55,487/37,284	1.488
ELF	(85,700-45,487)/85,700	0.469

Note that the derived deductible factor is now too low (as contrasted to the situation where we assumed no IBNR). This understatement results because the IBNR claims would include a full deductible, but the development factor applied to known claims with incurred values above the deductible produces no new deductible losses. The ELF factor is reasonably close, but the ILF factor is still substantially off.

When we update our open reserve development method, we can consider the possibility that the entire development factor should be applied to the open reserves, but a few moments reflection should indicate that this does not make much sense. To increase open reserves for anticipated development of open claims is plausible, but to increase individual claim amounts to account for newly reported counts seems unreasonable.

Given that our goal is to estimate the ultimate distribution as opposed to the total incurred, it would be entirely appropriate to ignore the IBNR claims if these claims had the same expected size distribution as known claims. However, experience tells us that IBNR claims tend to have higher loss amounts than previously reported claims. Loading pure IBNR dollars into known claims is clearly wrong, but ignoring these counts also introduces errors.

We have reproduced below the open case development approach using only expected development on known claims:

Item	Calculation	Value
DED	11,250/55,700	0.202
ILF	40,200/24,700	1.628
ELF	(55,700-31,700)/55,700	0.431

While this method produces better results than incorporating the entire development into the open reserves, our correct factors have now changed and we see that the resulting factors no longer match the correct factors.

The relative error of these procedures depends upon the context in which the results will be used. In the rating factors being estimated in the example, one must keep in mind how the factors will be applied and the nature of the overall objective. For example, the ILF will be applied to an adequate base rate to estimate losses/premiums above the base limit but below the increased limit. Similarly, the deductible factor and ELF might be used to layer the unlimited losses for workers compensation, which are assumed to be reflected in the rate. We will use the following error functions for each:

Deductible Credit Error:

(Estimated Dollar Deductible Credit -
Actual Dollar Deductible Credit)/
Actual Dollar Deductible Credit

This Equals:

$$\frac{\text{Estimated Deductible Factor}}{\text{Actual Deductible Factor}} - 1$$

Increased Limits Factor Error:

(Estimated dollar increased limits cost -
Actual dollar increased limits cost)/
Actual dollar increase limits cost

This Equals:

$$\frac{(\text{Estimated ILF - Actual ILF})}{\text{Actual ILF - 1}}$$

Excess Loss Factor Error:

$$\frac{\text{Estimated Dollar Excess Losses - Actual Dollar Excess Losses}}{\text{Actual Dollar Excess Losses}}$$

This Equals:

$$\frac{\text{Estimated ELF}}{\text{Actual ELF}} - 1$$

The table below displays the errors associated with the methods discussed under the assumption that no IBNR claims would be reported:

ESTIMATION ERROR - without true IBNR			
Method	**Deductible**	**ILF**	**ELF**
Closed Only	109%	-52%	-60%
Incurred Only	56%	-33%	-47%
Incurred Developed	8%	-41%	-11%
Open Developed	0%	-26%	2%

The next table displays the errors associated with the methods under the assumption of two IBNR claims:

Method	Deductible	ILF	ELF
Closed Only	163%	-50%	-65%
Incurred Only	96%	-30%	-53%
Incurred Developed	-9%	-41%	-2%
Open Developed	26%	-24%	-10%

Although this is a simple example, the magnitude of the errors should be unappealing for most. In subsequent sections of this paper we will discuss several techniques being used to reflect loss development in distributions.

The final introductory topic regards trend. The deductible credits, ILF's and ELF's are probably being estimated for a prospective period. Even if the ultimate loss distribution is estimable based on prior claims, it still must be adjusted to reflect the economic cost levels of the prospective period being considered.

For the sake of simplicity, we will assume in this section that trend is uniform. Loss distributions are very malleable under this assumption[9]. Unfortunately, little research that we are aware of has been performed in the area of non-uniform trend, although a recent article by Philbrick[10] did discuss the issues surrounding the problem.

If the accident period being projected is four years later than the accident period of the sample claims, then the estimated claim values at ultimate for the projected period, assuming a

[9] Hogg and Klugman, p. 179
[10] Stephen W. Philbrick, "Brainstorms," Actuarial Review, August 1994

uniform annual trend of 5%, would be (working only with the original reported claims and ignoring the IBNR claims):

Claim Number	Original Ultimate Claims	Trended Ultimate Claims
1	600	729
2	800	972
3	1,100	1,337
4	1,300	1,580
5	1,600	1,945
6	1,800	2,188
7	2,500	3,039
8	15,000	18,233
9	11,000	13,371
10	20,000	24,310
Total	55,700	67,704

The trended claim values equal the ultimate value multiplied by 1.05^4. The resulting deductible credit, ILF, and ELF are .173, 1.84, and .49. As one might expect, the deductible credit decreases with trend and the ILF and ELF increase. Fortunately, assuming uniform trend, the estimation error can be eliminated by multiplying the original claim values by the trend index of 1.05^4 before fitting the loss distribution. Alternatively, the original claims may be indexed to an overall severity of 1 before fitting. The selected ultimate indexed distribution may then be scaled to the ultimate severity. This approach is discussed in further detail in Section II of this paper.

Section II - Some Practical Methods to Reflect Development

Quite often the data available for use in a loss distribution fitting process consists of individual claim values for a particular valuation and other aggregate projections from more traditional triangular methods. In less optimal cases, only the aggregate data may be available. This section discusses three methods for developing loss distributions, which reflect loss development, using data provided in one of these formats. These methods will rely substantially on "industry" analysis performed by the Insurance Services Office (ISO) and the National Council on Compensation Insurance (NCCI).

The first method uses the projected ultimate unlimited severity (the average claim) for an individual risk or line of business and an appropriate coefficient of variation (CV) based on industry data to obtain parameters for three two-parameter loss distributions. The second method adjusts actual industry distributions to produce the projected average claim of the individual risk being considered. This process is referred to as "scaling" the industry curves and is described by Venter[11]. The third method, unlike the first two methods, requires individual ground up claim information for a single valuation. This claim detail is used to estimate an immature CV. Industry loss distribution development patterns are employed to develop the immature CV to ultimate. The ultimate CV is combined with the projected average claim, as in the first method, to obtain the parameters of several two-parameter loss distributions.

[11] Gary G. Venter, "Scale Adjustments to Excess Expected Losses," Proceedings, May 23, 24, 25, 26, 1982, Vol. LXIX, Part 1, No. 131.

Using Industry Coefficients of Variation

For each of the three methods discussed, we will assume that an estimated ultimate average severity for the accident year in question is available. This estimate could be developed by projecting ultimate losses and non-zero claim counts to ultimate using traditional actuarial techniques. An exponential regression could then be fit to the indicated historical ultimate average values and used to estimate the ultimate severity for the prospective period[12]..

Some of the more familiar loss distributions such as the Pareto, lognormal, and the gamma distribution have two parameters which define them. Estimates of these parameters can be obtained if two quantities about the distribution are known. These quantities might be the mean, mode, median, second moment, variance, CV, 99th percentile, etc.[13] As discussed, it is assumed that a projected average cost per claim or mean is available. The second quantity used to parameterize the loss distributions in this first method will be CV's based on ISO and NCCI (industry) published information.

In order to obtain an industry CV for workers compensation, the loss distributions used by the NCCI in developing excess loss premium factors for use in retrospective rating may be used. However, we are interested in a total CV, whereas the NCCI distributions are by injury type. Further, the NCCI distributions have been "indexed" so that the expected value for each distribution is one. For a more complete description of this process see Gillam[14].

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[12] It should not be inferred that this is the only or even the best method to determine these values. This approach is suggested as one specific method. Other methods do exist and may be appropriate.
[13] It should be understood that the calculation of parameters from the various quantities cannot be performed arbitrarily. Issues of bias and efficiency are important. Certain pairs of quantities could be very poor choices for the determination of parameters. The selection of parameters is discussed in many good statistics references and is beyond the scope of this paper.
[14] William R. Gillam, Parameterizing the Workers Compensation Experience Rating Plan, PCAS LXXIX, 1992

In order to develop a CV for the general distribution underlying any workers compensation claim, we need to describe the mixture process somewhat further. Appendix B contains a discussion regarding the mixture of models. This process is often confused with the addition or convolution of two random variables. The NCCI developed three loss distributions:

1. Fatal claims (D)
2. Permanent total (PT) and major permanent partial (major)
3. Minor permanent partial (minor) and temporary total (TT)

The cumulative distribution function (cdf) for the general workers compensation claim is:

$$F_T(x) = P_D F_D(x) + P_{PT/major} F_{PT/major}(x) + P_{TT/minor} F_{TT/minor}(x) + P_{mo} F_{mo}(x)$$

Where P_i is the probability that a claim is from injury type *i*, F_i is the cdf for injury type *i*, and MO claims are claims with medical losses only.

The expected value for the general workers compensation claim is:

$$E[X] = P_D E[D] + P_{PT/major} E[PT/major] + P_{TT/minor} E[TT/minor] + P_{MO} E[MO]$$

Where E[*i*] is the expected value of injury type *i*.

The second moment for the general workers compensation claim is:

$$E\left[X^2\right] = P_D E\left[D^2\right] + P_{PT/major} E\left[PT/major^2\right] + P_{TT/minor} E\left[TT/minor^2\right] + P_{mo} E\left[MO^2\right]$$

Based on NCCI published data and other internal data, we developed the following table.

	INJURY TYPE			
	Fatal	**PT/Major**	**Minor/TT**	**Med Only**
Average Claim	$210,000	$180,000	$8,500	$400
Probability	.0006	.0270	.2264	.7460
Second Moment	1.34E11	2.08E11	2.80E8	213,333

The second moment by injury type is the major item which was estimated using the NCCI loss distributions. It should be noted that the second moment for PT/Major claims is undefined in the NCCI information. The value shown here is calculated by capping the PT/Major loss distribution at $50 million. This capping allows us to calculate a second moment. We believe this is an acceptable adjustment, however, this means that the value shown should not be characterized as an NCCI value. Additionally, we assumed that the distribution for medical only claims is uniform from zero to twice the mean. Finally, the NCCI produces two different loss distributions for fatal claims, based on state benefit types. We averaged the CVs for the two distributions to produce our single second moment. Similarly, the NCCI has two separate distributions for PT/Major claims based on state benefits, which we averaged.

The CV of the total distribution is the standard deviation of the total distribution divided by the mean. The standard deviation can be calculated using total mean and second moment[15]. The resulting total CV is 10.474. If the total claim process being considered excludes medical only claims the CV is 5.441.

Some extensive client data was available for testing the three methods presented in this section, as well as the more robust method presented in Section III. The data consisted of the incurred value, paid value, accident date, report date, and closure date (if closed) for all claims occurring subsequent to 1/1/75 and prior to 3/31/95. The claims were due to products liability self-insured exposures for a diverse manufacturer. We transformed the actual claim amounts to preserve the confidentiality of the client's data. Therefore, all of the loss distributions fit to this sample data are for illustrative purposes only and are not suitable for use in any other circumstances.

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[15] $\text{Variance}[T] = \text{Second Moment}[T] - (\text{Mean}[T])^2$

Given the estimated industry CV of 10.474 and an estimated ultimate average claim size of $133,892, a lognormal and Pareto distribution were parameterized[16]. The table below displays the limited expected values (LEVs) and the cdf for the two loss distributions and the undeveloped trended empirical data.

Loss	Lognormal		Pareto		Empirical	
Limit	F(x)	LEV	F(x)	LEV	F(x)	LEV
1,000	0.121	935	0.015	993	0.215	872
5,000	0.333	3,957	0.070	4,821	0.404	3,519
10,000	0.456	6,953	0.134	9,308	0.481	6,296
25,000	0.622	13,671	0.289	21,081	0.602	13,090
100,000	0.829	32,011	0.671	57,336	0.808	33,321
500,000	0.955	65,037	0.955	105,807	0.965	64,342
1,000,000	0.978	80,676	0.986	118,261	0.984	76,434

This method assumes that no individual claim data is available. This makes it difficult to select the most appropriate loss distribution. Because of development, we would expect that the empirical cdf and LEV would be less than that of the ultimate loss distributions. It appears that both of the distributions had trouble handling the combination of the large CV and the large unlimited severity. Therefore, it would probably be better to use a CV from ISO's products liability distributions, discussed next, as the data consists of products liability losses and the average ISO claim sizes are much more consistent with the client data.

ISO has estimated distributions for the premises and operations (PremOPs) and products and completed operations (products) lines of business[17]. ISO has generated three compound distributions for each, tables 1-3 for PremOps and tables A-C for products. The tables

[16] Given the unlimited mean, M, and the CV, the parameters of the three distributions are: lognormal sigma = sqrt(ln(1+CV^2)), mu = ln(M) - .5 sigma^2; gamma alpha = 1/CV^2, beta = alpha/M; and Pareto alpha = 1/CV^2+2, lamda=Mx(alpha-1).
[17] Insurance Services Office, Inc., Revision of Premises/Operations and Products/Completed operations Increased Limits and Deductible Discount Factors. Filing GL95-ICDD1-Louisiana March 17, 1995.

represent increasing degrees of hazard within the line. The parameters for these tables are included as Section II, Exhibit 1a-1b.

The total loss distribution for a given line for a given hazard group is a compound process. All of the accidents for a given accident year are separated by settlement lag. Settlement lag is the number of years after the accident year in which an individual claim settles. Two Pareto distributions represent the group of claims in each settlement lag. The total cdf for the process is:

$$F(x) = \sum_{i=1}^{7} q_i \left[P_i F_{i2}(x) + (1 - P_i) F_{i2}(x) \right]$$

Where i is the settlement lag, F_{ij} is the jth Pareto distribution for the ith settlement lag, P_i is the weight for the first Pareto distribution in settlement lag i, and q_i is the relative percentage of claims which settle in settlement lag i.

The mean and variance of the compound process are calculated as noted previously for the total workers' compensation process. The resulting CV's for each distribution are displayed in the table below.

Coefficients of Variation					
PremOps			Products		
Table	Mean	CV	Table	Mean	CV
1	10,920	10.298	A	11,372	19.393
2	24,996	13.142	B	66,356	9.392
3	95,772	7.389	C	276,832	4.423

As with the NCCI distributions, we calculated a limited mean and variance assuming the losses were capped at $50 million, where the unlimited variance did not exist. Of interest is the observation that the CVs for the Products tables decrease as the hazard increases.

The estimated ultimate average claim size of the sample data combined with the ISO products table B CV produce the following parameters:

Distribution	Distribution Parameters First Parameter	Second Parameter
Lognormal	9.559	2.119
Pareto	2.011	135,410
Gamma	0.011	8.467E-08

The table below displays the cdf and ILF's, assuming a $25,000 base limit, for two of the loss distributions and the ISO products table B distribution.

Loss Limit	Lognormal F(x)	Lognormal ILF	Pareto F(x)	Pareto ILF	ISO Table B	
10,000	0.435	0.505	0.134	0.441	0.767	0.602
25,000	0.606	1.000	0.289	1.000	0.861	1.000
100,000	0.822	2.353	0.671	2.720	0.947	1.921
250,000	0.912	3.653	0.878	4.146	0.976	2.701
1,000,000	0.978	5.889	0.986	5.611	0.994	3.932

It appears that the two parameter distributions do not capture the skew or diversity present in the ISO multimodal process.

In preparing this paper, we noted several random observations about CV's. First, it is possible for a line which would normally be considered highly skewed to have a CV which appears small. This is partly due to the fact that these lines generally have larger average claim sizes. Because the CV is a ratio to the mean, a smaller CV with a larger mean can still produce a skewed distribution.

Second, some people fall into a normal distribution thought process when considering CV's. A distribution consistent with the normal distribution would have a 97.5 percentile at 1.96 standard deviations. Unfortunately, insurance distributions, such as the Pareto distribution, tend to be skewed. A Pareto distribution with parameters 3 and 10,000 has a 97.5 percentile which is 2.8 standard deviations from the mean.

Third, we have found in other studies of empirical data, support for the statement that inflation does not affect claims uniformly. In these studies, lognormal distributions were fit to claims for individual accident years of common maturities. The distributions were not rejected by various goodness of fit statistics and produced reasonable and increasing overall severities. The CV's for these distributions were clearly decreasing. If inflation impacted claims uniformly, the CV's would have been constant, assuming no substantial changes in loss adjustment procedures or the mix of business. Finally, care must be taken when trying to examine CV's behavior through simulation. CV's are ratios and many simulation techniques produce biased results for ratios.

Scaling the Industry Curves

Given a projected unlimited severity for a risk or book of business, the industry loss distributions can be modified to produce a mean equal to the risk's average claim. This process essentially accepts the shape of the industry loss distribution and shrinks or expands the industry distribution to match the risks average claim. This adjustment is made to the scale parameter of the distribution and basically assumes a uniform inflationary effect to "scale" the original distribution. Hence, the name of the method. This procedure can also be applied with some modification to a limited average claim size, if a credible unlimited severity is unavailable.

The following table displays the approximate parameters for the NCCI distributions, and our estimates of representative probabilities for each injury type.

Workers' Compensation Distributions by Injury Type						
Benefit Type	Fatal NonEscalating or Limited	Fatal Escalating	PT/Major NonEscalating	PT/Major Escalating or Limited	Minor/TT All	Med Only All
Distribution Type	Gamma	Gamma	Shifted T-Beta[18]	Shifted T-Beta	Trans-Beta	Uniform
Alpha	.5500	.4450	1.4900	1.4000	63.4960	N.A.
Lamda	381,818	471,910	66,616	64,772	4,024,363	N.A.
Gamma	N.A.	N.A.	1.2200	1.2500	.6410	N.A.
Tau	N.A.	N.A.	1.5000	1.4000	.9670	N.A.
Mean	210,000	210,000	180,000	180,000	8,500	400
Probability	.06%	.06%	2.70%	2.70%	22.64%	74.60%

The medical only distribution is not based on the NCCI distributions. The Lamda parameter is the scale parameter. The actual Lamda parameters used by the NCCI produce a mean of one for each distribution. However, in order for scaling to be feasible, a representative industry severity for each injury type is required. We have adjusted the Lamda parameters to produce these representative severities.

The overall mean for the industry distribution is $7,209, the weighted averages of the industry severities. In the following calculations, we will assume limited fatal benefits and non-escalating PT/Major benefits. The projected severity for the sample data is $133,892. Given the large difference between the average claim sizes, it is probably inappropriate to scale the NCCI distributions in this example. However, we will continue with the process for purposes of illustration. Each of the Lamda parameters and distribution means need to be increased by 18.573 ($133,892/$7,209). The new and original PT/Major shift points are 25% of the new and original means, respectively. The medical only distribution is adjusted by multiplying the upper bound by 18.573. The adjusted average severities by injury type are displayed in the table below:

[18] The PT/Major distributions are truncated and shifted by 25% of the average claim. Therefore, the actual mean of the distribution is $135,000 or 75% of $180,000.

Adjusted Severities					
Injury Type	Fatal	PT/Major	TT/Minor	Med Only	Total
Adjusted Severity	$3,900,423	$3,343,220	$157,874	$7,429	$133,892

The resulting LEV's, ILF's, and ELF's are displayed in the table below.

NCCI Scaled Distributions			
Limit	LEV[19]	ILF	ELF
$ 10,000	7,793	0.729	0.942
25,000	10,685	1.000	0.920
50,000	14,780	1.383	0.890
100,000	21,340	1.997	0.841
250,000	34,656	3.243	0.741
1,000,000	66,595	6.232	0.503

Establishing a worksheet in Excel or Lotus which readily handles the distributions is not an insurmountable task. Both software packages contain the functions necessary to construct cdf, LEV, and moment functions for each of the distributions used. Even the truncated shifted PT/Major distribution can be handled.

In order to scale the ISO compound Pareto distribution, the Pareto scale parameters must be multiplied by the overall severity adjustment factor. The overall industry mean for the products table B is $66,356. Given the projected severity for the sample data of $133,892, the B

[19] When estimating the PT/Major component of the LEV, if the limit is less than 25% of the average claim (the shift point) then the LEV is the limit. If the limit is greater than the shift point then the LEV is computed using the transformed Beta distribution with the scaled lamda parameter. The actual limit used in computing the LEV with the transformed Beta distribution is the original limit less the shift point. After the LEV is calculated via the transformed Beta distribution, the shift point must be added back in to obtain the final LEV.

parameters need to be multiplied by 2.018 ($133,892/$66,356). The adjusted B parameters are displayed in the following table.

Adjusted B Parameters		
Lag	B1	B2
1	13,504	7,367
2	20,104	10,968
3	58,482	31,905
4	104,184	56,838
5	130,974	71,453
6	153,230	83,595
7	150,894	82,321

The table below displays the adjusted cdf, LEV's, and ILF's which result from scaling the ISO medium hazard products loss distribution.

Scaled Products Table B			
Loss Limit	F(x)	LEV	ILF
$ 10,000	0.665	5,268	0.577
25,000	0.792	9,136	1.000
50,000	0.860	13,347	1.461
100,000	0.911	18,861	2.065
250,000	0.956	28,087	3.074
1,000,000	0.988	44,448	4.865

The major assumption made when one scales industry loss distributions is that the shape of the industry distribution is appropriate for the individual risk or book of business. If the data has an inordinate number of small losses, perhaps due to an incident reporting procedure, the overall projected severity will be reduced. This will distort the accuracy of the adjusted distribution. For example, suppose we are given the following risk and industry losses.

Industry Losses		Risk Losses	
Loss Amount	Number	Loss Amount	Number
$ 100	500	$ 110	500
1,000	50	1,100	50
10,000	10	11,000	10
50,000	5	55,000	5
250,000	1	275,000	1

Obviously, the risk's average severity of $1,360 is 10% higher than the industry average severity of $1,237. Scaling the industry distribution up 10% would be appropriate. However, assume the risk had an additional 500 claims valued at $25 each. The risk's total severity is now $734. The industry distribution scaled down by 41% (1-734/1,237) would be as follows:

Scaled Industry Losses

Loss Amount	Number
$59	500
594	50
5,935	10
29,677	5
148,383	1

Now the ELF for a $25,000 limit based on the scaled industry distribution is .35, whereas the actual ELF is .51. Problems pertaining to a larger than normal or less than expected number of smaller claims can often be discovered by examining the empirical and theoretical cdf and LEV's at smaller loss limits.

Developing Empirical CV's

The general approach for developing an ultimate CV based on an entity's individual ground up claim detail or a book of business and industry data consists of the following steps:

1. Using the individual claim detail available, estimate the risk's CV. This is done by first trending the individual claims to the prospective period. The sample CV can then be computed by dividing the sample standard deviation by the sample mean.

2. Estimate an industry CV which is appropriate for the overall maturity of the sample data and the inflationary level of the prospective period.

3. Estimate an ultimate industry CV for the prospective period based on the industry distributions.

4. Develop the ultimate sample CV by multiplying the sample CV from step 1 by the ratio of the ultimate industry CV and the undeveloped industry CV.

5. Use the projected unlimited severity and the estimated ultimate CV to parameterize a two parameter loss distribution as in method 1.

Unfortunately, attempting this process with NCCI data is problematic. The NCCI curves were developed by fitting a distribution for each injury type for a single policy year at 3rd, 4th, and 5th report. After examining the progression of these parameters, the NCCI selected ultimate parameters. In order to estimate an industry CV for a given maturity mix of data, estimates of the NCCI distributions would be required at additional maturities. While these distributions might be estimable, it is not currently possible to obtain all of the immature total workers compensation distributions. In order to obtain the total claim distribution, weights by injury type for each valuation are needed. This would require a claim count distribution for each maturity by type of injury, which is unavailable other than on a state by state basis for the first few reports. ISO's current methodology lends itself to this procedure much more readily than the NCCI data.

Section II, Exhibit 2a displays the estimated developed CV based on a portion of the settled claims from the sample data. Section II, Exhibit 2b displays the notes to the calculations. Estimated "premiums" for each year are combined with a rate change and trend index to develop a relative volume index by year in column (5). This volume index is combined with the estimated percentage of claims settled to obtain the cumulative lag weights by lag in column (8). The relative weights are used as the probabilities in a compound process to obtain the overall immature industry standard deviation and mean, columns (9)-(10). The ultimate industry CV is divided by the immature industry CV yielding the CV development factor. This is applied to the sample data CV to obtain the estimated ultimate CV for the sample data, row (14).

The industry distributions should be trended to the prospective period level before calculating the standard deviation and mean for each year. This was accomplished by trending the ISO B-parameters. The relative volume weights could be estimated via an ultimate claim count projection for each accident year. The settlement pattern might be adjusted based on the individual risks data. However, the settlement pattern represents the percentage of claims closed for the industry by lag and should be treated accordingly.

Prior to the most recent ISO ILF filings, ISO used the incurred claim data in its ILF estimation process. As a result, loss distributions for each valuation of a current diagonal of claims were developed. One might think that these could be used in a fashion similar to the process described for Exhibit 2 and the settled data. However, the size of loss distributions by valuation age are not independent. Therefore, the CV cannot be estimated without an estimate of the covariances for each distribution pair. As incurred data often contains more information regarding individual large claims than settlements only, it would be helpful if someone developed a process to account for the covariance in the NCCI and ISO incurred distributions by valuation age.

It has been suggested that one might construct a triangle of CV's from incurred claims and develop these CV's to ultimate. This procedure would be similar, in its basic nature, to the prior approach used by ISO. Before such a procedure could be relied upon, a more complete understanding of the underlying statistical assumptions, particularly regarding independence, would need to be obtained. Any additional research in this area would certainly be welcome.

The following table displays the cdf and LEV using the developed CV and the projected average claim to parameterize these distributions. Due to the large CV, the gamma distribution was not tractable.

Loss Distributions Via Developed CV's				
	Lognormal		Pareto	
Limit	F(x)	LEV	F(x)	LEV
10,000	0.338	7,997	0.133	9,314
25,000	0.526	16,326	0.287	21,109
100,000	0.787	39,413	0.670	57,519
250,000	0.899	60,998	0.878	87,710
1,000,000	0.978	95,141	0.986	118,624

This section presented three simplified methods of developing loss distributions using minimal sample data. Method 1 and method 3 result in a two-parameter loss distribution. Such a simple distribution will probably not capture all of the variation in the underlying loss process. This is one reason why ISO and NCCI have developed such robust compound processes.

Scaling the industry distributions as in method 2 retains the same diversity as the industry distributions but may not correctly address the shape issue. However, all of these methods attempt to address development and require only basic summary data for the most part. Section III introduces a more refined method which requires a minimal amount of extra data and a few additional loss distribution parameters.

Section III - A More Refined, Practical Approach

When considering the problem of development and loss distributions, one is tempted to jump to the utopic extreme and begin thinking about what the best procedure would be if there were no limitations on the data available. After this perfect method was created, the minimal amount of alterations could be made to the assumptions of the method to account for the actual data available in a given situation. We decided to approach the problem from another direction. The first question we asked was what is the format of the data most likely to be available for this type of project. For many practical business applications, this data consists of listings of individual claims in excess of a fixed retention and summary loss and claim count information. The procedure described in this section is designed to use data in this format.

A two part loss distribution is developed in order to estimate ILF's and ELF's. Because the data format does not include individual claim information regarding the smaller claims, the distribution developed will not be applicable to smaller deductibles. The first part pertains to the smaller claims for which no individual claim detail is available. This part of the distribution is estimated using aggregate loss data and more traditional triangular approaches. The second part involves fitting a loss distribution based on the individual large claim data. The technique employed is somewhat different from more traditional approaches.

The complete ultimate loss distribution is similar to the old five parameter ISO distributions. The distribution consists of two parts: one for the smaller claims below the loss limit and another for the larger claims about which we have more detail. The distributional formula is:

$$F(x) = \begin{cases} \dfrac{px}{L} & x < L \\ p + (1-p)G(x) & x \geq L \end{cases}$$

Where L is the loss limit, p is the ultimate proportion of claims below the loss limit, and G(x) is the cdf of a truncated lognormal distribution.

The function form below the loss limit is essentially immaterial because (1) we are estimating ILF's and ELF's for limits that are greater than the loss limit and (2) there is no individual loss data below the loss limit. The product of the proportion of all claims less than the loss limit (p) and the average severity of these small claims (S) is important. An ILF for a given limit K is estimated by the following formula:

$$\frac{pS+(1-p)E[X;K]}{pS+(1-p)E[X;B]}$$

Where E[X;Y] is the limited expected value of X limited to Y, B is the basic limit, and S is the average severity of all claims less than the loss limit. One property of this estimator is that it is not distorted by the addition of a large number of very small claims.

Handling the Small Claims

The quantities that must be estimated for the smaller claims are p and S. By subtracting the incurred losses and claim counts for the large claims from the total aggregate information, we constructed a small claim loss and count triangle. These are then developed to ultimate to produce the estimated historical severity by accident year. A regression was fit to these to both smooth the indications and project the severity S for the prospective period.

A total claim count triangle was developed to ultimate. The projected ultimate small claim counts divided by the total count projection yields historic p ratios by accident year. A regression was fit to these to smooth and project the p value for the prospective period.

There are a few points to note here. First, the inclusion or exclusion of a large amount of small claims will effect both p and S. However, the product of pS will not be materially effected. Therefore, if the historic projections for either p or S are not very smooth, one may wish to regress on pS.

Second, the possibility of error from misestimation is greater if the value of p is very large or very small. The final unlimited severity should be multiplied by the projection of ultimate claim counts and checked for reasonableness against an independent projection of the total ultimate losses.

Addressing the Large Claims

The process used to estimate the parameters of the lognormal distribution consisted of the following steps:

1. For each accident year and valuation, count the cumulative number of claims in each of 8 fixed layers;

2. Convert the count distributions to percentage distributions;

3. Develop a function for each layer which most accurately reflects the changes in the percentage distributions as each accident year matures;

4. For each accident year, estimate an ultimate percentage distribution using the development function;

5. Select an ultimate percentage distribution for the prospective period; and

6. Fit a lognormal distribution to the selected ultimate percentage distribution for the prospective period.

In order to determine the shape of the ultimate size of loss distribution, we computed "development factors" for selected size of loss ranges. We were provided with individual claim amounts in the excess of $25,000 evaluated at 1990, in addition to values evaluated at 1995. This information gives us two "snapshots" of the development process separated by five years.

Each of the individual claims greater than or equal to $25,000 were grouped into categories. The following categories were established:

$50,000—claim amount greater than $25,000 but less than $50,000

$100,000—claim amount greater than $50,000 but less than $100,000

$250,000—claim amount greater than $100,000 but less than $250,000

$500,000—claim amount greater than $250,000 but less than $500,000

$750,000—claim amount greater than $500,000 but less than $750,000

$1,000,000—claim amount greater than $750,000 but less than $1,000,000

$2,500,000—claim amount greater than $1,000,000 but less than $2,500,000

$6,000,000—claim amount greater than $2,500,000 but less than $6,000,000

The counts in each category were compared at the 1990 and 1995 evaluations by accident year. For example, the number of claims in the $50,000-$100,000 category for the 1985 accident year as of 1990 was 41, and five years later, the number in that category as of 1995 was 30. From this information, we wish to determine a set of development factors which can be used to estimate the movement of claims between categories.

In Section III, Exhibit 1, we have calculated the relative proportions of the large claims at the two evaluation dates. For example, the 1985 Accident Year had 31.8% of the large claims with an incurred value between $50,000 and $100,000 as of 1990. Five years later, the proportion of large claims in this size category had dropped to 25.2%. Conversely, the proportion of large claims in excess of $1,000,000 had increased from 3.1% to 4.2% over the same time period.

When there is sufficient claim count experience by layer, the actual claim count development factors by layer may be used in the fitting process discussed later to obtain the selected layer claim count development factors. However, the sample data included in this analysis was sparse in some of the upper layers. In particular, there were problems associated with individual cells which had no claims. There were several multimillion dollar claims which needed to be reflected in the procedure. Therefore, each of the accident year claim count distributions were smoothed by fitting a lognormal loss distribution for each of the two valuations. These smoothed distributions are displayed in Section III, Exhibit 2.

The results of the smoothed distributions were employed to obtain the fitted distribution layer development factors in Section III, Exhibit 3. Continuing to focus on Accident Year 1985, the proportion of $100,000 claims as of 1995 is divided by the proportion as of 1990. The resulting ratio is 0.904. Similarly , the factor for the $2,500,000 range is 1.934, reflecting the fact that a higher proportion of the claims are in this size category at the later evaluation. (It is important to keep in mind that the data reflects relative proportions of claims, not the absolute number of claims. The absolute number of claims will be discussed later).

A review of these five-year development factors shows a clear trend. The proportion of claims under $250,000 drops steadily over time, faster at early evaluations and slower at later evaluations. The proportion of claims in the largest categories grows steadily over time, fast at first and then slower. The larger the size category, the larger the growth. In other words, over time we have a migration of claims. At early intervals we have a certain proportion of claims in each size category. Over time, there is a tendency to depopulate the smaller size categories. Some of these claims become larger, and some become smaller. The larger size categories tend to show an overall net increase in the proportion of claims.

At the same time, a small proportion of claims "drop out," that is, are settled with a zero indemnity amount. However, the remaining claims show a pronounced trend toward higher size categories.

The table in Section III, Exhibit 3 essentially has overlapping development factors evaluated at a five year interval. We need to convert these to one-year development factors. One approach is to approximate the annual development factor by the fifth root of the five year age-to-age factor, then calculate the average of the factors with the same "maturities." However, most development factors have the property that the age-to-age factor is not a constant factor over a period of time, but rather a decreasing factor. In order to apportion the five year factors into annual amounts, we fit the development factors for a given layer to a curve of the form EXP[exp(a(x+5)+b)-exp (ax+b)], where x is the development year of each individual accident year. This curve provided a good fit to the factors.

The curve is used to apportion each five-year development factor into an annual amount. The resulting annual factors are accumulated in the normal manner to produce age-to-ultimate factors. The resulting factors are then applied to the current proportion of claims in each size category, which yields an estimate of the ultimate proportion of claims by size for each accident year. Based on the projected distributions for the most recent years, an ultimate distribution is selected for the prospective period in Section III, Exhibit 4. It is to this ultimate distribution that the lognormal loss distribution is fit.

The table below displays the resulting ILF's and ELF's from this method, as well as some of the methods from Section II.

	Section III		ISO CV Pareto		Scaled ISO		Developed CV Lognormal	
Limit	ILF	ELF	ILF	ELF	ILF	ELF	ILF	ELF
$ 25,000	1.000	0.922	1.000	0.843	1.000	0.932	1.000	0.878
50,000	1.759	0.863	1.729	0.728	1.461	0.900	1.605	0.804
100,000	2.973	0.768	2.720	0.572	2.065	0.859	2.414	0.706
250,000	5.315	0.586	4.146	0.347	3.074	0.790	3.736	0.544
1,000,000	9.418	0.266	5.611	0.116	4.865	0.668	5.828	0.289

There are three aspects of this method which are appealing. First, the data required is frequently available. Second, the final loss distribution is fit to data at ultimate. It is possible that a particular family of loss distributions may be rejected if fit to immature data, where the distribution would have been appropriate for the ultimate distribution. This procedure avoids this possibility when the empirical data is not smoothed.

Finally, one may have noticed that after giving the sermon in Section I on the evils of not trending the data, there is no explicit trend adjustment in the method provided in Section III. The method in Section III recognizes trend implicitly in the actual percentage claim count distribution by layer and its migration. The fact that trend is addressed without making the usual uniform assumption is appealing. The method could probably be improved in this respect if the layer boundaries were actually indexed to a smoothed average severity by year for each age. However, because the individual loss data is provided above a fixed retention, indexing would be problematic.

There are some problems with this method, which arise primarily from variability and small sample sizes. For example, the layer boundaries must be sufficiently refined and still contain a sufficient number of claims by grouping for each accident year. In order to accomplish this one might consider grouping accident years together. In addition, the availability of intermediate valuations may not provide sufficiently stable information for use.

Loss distributions are an invaluable tool. However, the actuary should be aware of the possible effects of development on loss distributions used for many casualty exposures. We attempted to illustrate the potential problems which may result if development is not considered via a simple example in Section I. In Section II, some practical methods for reflecting development were discussed. In particular, two methods were provided which did not require any individual claim information. Finally, Section III presented a practical method for use with aggregate data and individual loss data for losses in excess of a fixed amount. There still remains several unanswered questions, such as "what is the utopic procedure for recognizing loss development?" and "what is the actual impact of trend on claims?" However, the concepts presented in the paper do not hinge upon the answers to these questions, so we will leave them for another day.

Appendix A

The observation that loss development has a material impact on the size of loss distributions is made in Harwayne's article "Accident Limitations for Retrospective Rating." [20] He notes the significance of the impact of loss development, yet suggests only that the NCCI use fourth reports instead of third reports. While this increases the development age of claims from 42 months to 54 months, workers compensation claims show a stubborn tendency to continue development beyond 20 years.

A paper presented by Dr. Shaw Mong at the 1980 Discussion Paper Program, "Estimating Aggregate Loss Probability and Increased Limit Factor"[21] recognized the importance of loss development on the size-of-loss distribution but did not feel the need to provide techniques. He states, "In our model, we assume that all the losses have already been adjusted to the present or ultimate level. That is: losses have been developed to the ultimate; IBNR has been adjusted and inflation has been trended to the forecasting year, etc. The reason that we did not discuss those in here is because they are rather standard actuarial techniques practiced in most areas of rate-making and have been covered extensively elsewhere in the literature." He supplies two references in the literature. However, the Hewitt and Lefkowitz paper referenced only deals with inflation adjustments. The Miccolis paper[22] referenced discusses the need to adjust for development. The author notes, "Loss development also poses certain problems in working with severity distributions...It is very likely that this distribution of immature claim values

[20] Frank Harwayne, "Accident Limitations for Retrospective Rating," Proceedings, May, 1976, Vol. LXIII, Part 1, No. 119.
[21] Dr. Shaw Mong, "Estimating Aggregate Loss Probability and Increased Limit Factor," Pricing Property and Casualty Insurance Products, May 11-14, 1980.
[22] Robert S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," 1977 Proceedings, Vol. LXIV, p. 49.

will change considerably as these claims develop..." but when it comes to specific techniques, the author notes "Hachemeister describes a technique of estimating such loss development distributions conditioned on the age of the claim and its estimated value...The actual procedure for adjusting a severity distribution for loss development using the Hachemeister technique will be left to the interested reader..." (The methodology used in Section III of this paper is intended to follow the spirit of the Hachemeister proposed technique.)

Gary Patrik notes in his excellent paper on fitting loss distributions, "I decided to use undeveloped and incomplete data for this example so as not to get involved in the question of how to develop and complete it..."[23]

As recently as 1987, Pinto and Gogel noted that, "There is very little information available regarding excess loss development, despite its importance in excess of loss pricing and reserving..." and "There is a paucity of published information regarding both reported and paid excess loss development..."[24] They went on to explore the impact of loss development on various sizes of claims in one of the few papers to address the subject.

One other paper directly discusses specific techniques. Venter's paper, "Scale Adjustments to Excess Expected Losses"[25] illustrates one of the techniques discussed in Section II of this paper.

[23] Gary Patrik, "Estimating Casualty Insurance Loss Amount Distributions," PCAS, Vol. LXVII, 1980.
[24] Emanuel Pinto and Daniel F. Gogol, "An Analysis of Excess Loss Development," Proceedings, November 4, 5, 6, 1987, Vol. LXXIV, Part 2, No. 142.
[25] Gary G. Venter, "Scale Adjustments to Excess Expected Losses," Proceedings, May 23, 24, 25, 26, 1982, Vol. LXIX, Part 1, No. 131.

Appendix B

Combining Distributions

In the normal course of actuarial work, it is often necessary to combine two or more distributions, and calculate relevant statistics (such as the mean and the variance) of the composite distribution. There is more than one way to create such a composite distribution, and the formulas differ. While the correct calculation of the mean is usually straightforward, the calculation of higher moments is trickier. This appendix will clarify the distinction between a convolution of two variables, and a mixture of two variables, as well as the appropriate formulas for mean and variance of each. The NCCI and ISO distributions discussed in Section II of this paper are mixtures of models.

Severity distributions for workers compensation provide a good working example, because an overall workers compensation severity distribution can be viewed as both a mixture and a convolution. First consider one injury type, such as PT (Permanent Total). A typical PT claim has an indemnity component, and a medical component. Assume that we analyze the severity distribution of indemnity amounts and call this random variable X and its associated distribution F_x. Similarly, analyze the medical distribution, call the random variable Y and the distribution F_y.

Now assume that we are interested in the distribution of a PT claim, including both the indemnity and medical amounts. We can define a new random variable,

$$Z = X + Y$$

which has distribution F_z. F_z is the **convolution** of X and Y.

(One important caution. It is likely that there is some correlation between medical and indemnity. Unfortunately, the calculation of the convolution requires independent distributions. For purposes of this discussion, we will make the simplifying assumption that X and Y are independent.)

Under convolution, the mean of the resulting distribution is the sum of the means of the two distributions being combined. The variance of the resulting distribution is the sum of the variances of the two distributions.

After we calculate a severity distribution for PT, we might also calculate severity distributions for other injury types. Now, we may be interested in an overall workers compensation severity distribution. The process of combining the severity distributions of the various injury types into an overall distribution is a **mixture**. The resulting distribution is not formed by adding a death amount to a PT amount, but by combining the distributions such that the resulting distribution has the appropriate proportion of each injury type.

For simplicity, assume we have only two injury types, death and PT. Assume the proportion of death claims is p and the severity distribution is F_x. Assume that the proportion of PT claims is q, that is, 1-p , *and its distribution is* F_y. *We form the composite distribution by mixing the two* distributions. Using Z to represent the resulting random variable, we can describe Z as:

$$
\begin{aligned}
Z &= X \text{ with probability } p \\
& Y \text{ with probability } q
\end{aligned}
$$

The formula for the resulting mean is straightforward, but the formula for the variance is slightly less intuitive because the exponents on p and q do not allow the usual simplification. The formulas for convolution and mixtures are summarized below:

	Convolution (assuming X, Y independent)	Mixture (assuming X, Y independent)
	$Z = X + Y$	$Z = X$ *with probability* p Y *with probability* q
Mean	$E(Z) = E(X) + E(Y)$	$E(Z) = pE(X) + qE(Y)$
Variance	$Var(Z) = Var(X) + Var(Y)$	$Var(Z) = pE\left[X^2\right] + qE\left[Y^2\right] - \left(pE[X] + qE[Y]\right)^2$

It may also be helpful to think of these concepts in terms of an urn model. Assume we can represent severity distributions by values in urns X and Y. We form Z by selecting one value from urn X, one value from urn Y, and adding the values together. Each draw selects two values which are added together. The resulting random variable Z, has distribution F_z which is the convolution of F_x and F_y.

Alternatively, we could form Z by selecting a value from urn X with probability p, and selecting a value from urn Y with probability q. Each draw selects precisely one value. The resulting random variable Z has distribution F_z, which represents the mixture of F_x and F_y.

INCREASED LIMITS FACTORS
GENERAL LIABILITY
SUPPORTING MATERIAL - INDEMNITY

Section II
Exhibit 1a

Table of Mixed Pareto Parameters
Average Accident Date of July 1, 1996
Products/Completed Operations Liability

Table A

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	2,515.12	1.20	0.915991663	2,200.53	3.20	0.531773315	1,653.70
2	3,744.31	1.20	0.746928275	3,275.96	3.20	0.286921419	4,301.54
3	10,892.26	1.20	0.624052128	9,529.83	3.20	0.079310877	14,904.08
4	19,404.31	1.20	0.534745107	16,977.18	3.20	0.044616820	28,826.35
5	24,393.84	1.20	0.469836301	21,342.62	3.20	0.025099466	38,599.45
6	28,538.96	1.20	0.422660243	24,969.26	3.20	0.014119858	46,955.28
7	28,103.97	1.20	0.388372440	24,588.68	3.20	0.018158245	48,447.35
Total/Average						1.000000000	7,093.34

Table B

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,692.52	1.15	0.921341244	3,651.13	3.15	0.418536937	3,419.68
2	9,963.26	1.15	0.752277856	5,435.49	3.15	0.277258043	10,126.17
3	28,983.32	1.15	0.629401709	15,811.95	3.15	0.094095287	34,316.06
4	51,633.12	1.15	0.540094688	28,168.64	3.15	0.064990169	64,650.57
5	64,909.82	1.15	0.475185881	35,411.79	3.15	0.044887711	85,654.41
6	75,939.63	1.15	0.428009824	41,429.14	3.15	0.031003252	103,251.17
7	74,782.16	1.15	0.393722021	40,797.68	3.15	0.069228601	107,070.92
Total/Average						1.000000000	26,127.78

Table C

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	9,359.41	1.10	0.816306461	3,701.05	3.10	0.304440622	7,865.08
2	13,933.50	1.10	0.647243074	5,509.81	3.10	0.233816271	18,835.47
3	40,532.82	1.10	0.524366926	16,028.14	3.10	0.091998387	57,430.38
4	72,208.31	1.10	0.435059905	28,553.77	3.10	0.073668491	102,373.05
5	90,775.62	1.10	0.370151099	35,895.95	3.10	0.058990671	132,179.64
6	106,200.66	1.10	0.322975042	41,995.57	3.10	0.047237281	156,654.04
7	104,581.96	1.10	0.288687239	41,355.47	3.10	0.189848278	161,900.30
Total/Average						1.000000001	65,557.43

* Limited Average Severity at the one million dollar policy limit.

SECTION2.XLS ISO
10/2/95

INCREASED LIMITS FACTORS
GENERAL LIABILITY
SUPPORTING MATERIAL - INDEMNITY

Section II
Exhibit 1b

Table of Mixed Pareto Parameters
Average Accident Date of July 1, 1996
Premises/Operations Liability

Table 1

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,723.15	1.70	0.829092621	1,990.24	3.70	0.555830405	2,203.36
2	8,047.56	1.70	0.436495006	2,382.30	3.70	0.294627117	6,643.19
3	29,432.05	1.70	0.273002096	8,712.70	3.70	0.070332575	28,909.45
4	45,198.29	1.70	0.204917292	13,379.94	3.70	0.037253873	46,656.98
5	59,490.73	1.70	0.176564133	17,610.89	3.70	0.019732692	61,810.38
6	72,580.29	1.70	0.164756775	21,485.76	3.70	0.010452045	74,767.81
7	80,979.51	1.70	0.159839732	23,972.17	3.70	0.011771294	82,770.33
Total/Average						1.000000001	9,928.87

Table 2

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	5,033.26	1.30	0.818071267	2,416.55	3.30	0.502090657	3,288.78
2	6,024.78	1.30	0.425473652	2,892.59	3.30	0.299264855	9,588.06
3	22,034.19	1.30	0.261980742	10,578.97	3.30	0.080330909	38,265.66
4	33,837.54	1.30	0.193895938	16,245.95	3.30	0.047845462	59,696.28
5	44,537.52	1.30	0.165542779	21,383.18	3.30	0.028496979	77,342.69
6	54,336.96	1.30	0.153735421	26,088.05	3.30	0.016972933	92,054.25
7	60,625.01	1.30	0.148818378	29,107.04	3.30	0.024998204	101,000.47
Total/Average						0.999999999	16,742.04

Table 3

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,928.10	1.10	0.914673388	3,319.51	3.10	0.481709595	3,764.32
2	8,292.88	1.10	0.522075773	3,973.43	3.10	0.298779652	16,098.35
3	30,329.25	1.10	0.358582863	14,531.87	3.10	0.083458413	60,279.55
4	46,576.11	1.10	0.290498059	22,316.34	3.10	0.051727363	91,466.07
5	61,304.23	1.10	0.262144900	29,373.13	3.10	0.032060519	115,885.35
6	74,792.81	1.10	0.250337542	35,836.01	3.10	0.019871048	135,446.14
7	83,448.07	1.10	0.245420499	39,983.07	3.10	0.032393410	147,065.78
Total/Average						1.000000000	27,556.07

* Limited Average Severity at the one million dollar policy limit.

SECTION2.XLS ISO
10/2/95

Developing Empirical CV's

Section II
Exhibit 2a

Accident Year	Premium	On-Level Rate Factor	Exposure Trend	On-Level Premium	Relative Volume Weights
	(1)	(2)	(3)	(4)	(5)
88	27,600	1.210	1.160	38,729	0.1530
89	29,400	1.210	1.131	40,249	0.1590
90	28,200	1.100	1.104	34,240	0.1352
91	29,900	1.100	1.077	35,419	0.1399
92	31,700	1.100	1.051	36,635	0.1447
93	32,100	1.000	1.025	32,903	0.1300
94	35,000	1.000	1.000	35,000	0.1382
Total	213,900			253,175	1.0000

Accident Year	Settlement Lag	ISO Lag Weight	Cumulative Lag Weight	Prospective Level Second Moment	Prospective Level Mean
	(6)	(7)	(8)	(9)	(10)
88	7	0.069229	0.010590	2.105E+12	327,291
89	6	0.031003	0.009671	2.021E+12	314,712
90	5	0.044888	0.020073	1.551E+12	248,251
91	4	0.064990	0.038155	1.048E+12	174,762
92	3	0.094095	0.068858	4.366E+11	80,559
93	2	0.277258	0.238929	8.589E+10	19,397
94	1	0.418537	0.418537	1.728E+10	5,362
Total		1.000000	0.804814	2.122E+11	38,005

(11) Adjusted Industry Immature CV	12.0796
(12) Adjusted Industry Ultimate CV	9.1692
(13) Sample Data Immature CV	7.8672
(14) Sample Data Ultimate CV	5.9717

SECTION2.XLS DevCV's
10/18/95

Notes to Section II, Exhibit 2a

(1)-(3) These would be based on the individual client or book of business. The purpose is to essentially develop an estimate of the ultimate number of claims by year, which is column (5).

(4) (1) x (2) x (3)

(5) (4) / (4) Total

(7) From Section II, Exhibit 1a, Lag Weight Table B

(8) [Sum (5)] x (7). For example, for lag 2 column (5) is summed for accident years 88-93. This sum is then multiplied by the lag 2 weight in column (7).

(9) The second moment from ISO Table B for the specific lag. The B parameters have been trended forward from 7/1/96, the midpoint of the filing, to the midpoint of the prospective period. The total second moment is sum [(8) x (9)] divided by (8) Total.

(10) The mean from ISO Table B for the specific lag. The B parameters have been trended forward from 7/1/96, the midpoint of the filing, to the midpoint of the prospective period. (10) Total is [sum ((8) x (10))]/(8) Total.

(11) (9) Total/(10) Total

(12) Based on ISO Products Table B using trended B parameters.

(13) Based on client data for settled claims.

(14) (13) x (12)/(11)

Empirical Claim Count Distribution
Valued as of 3/90 and 3/95

Section III
Exhibit 1

	AY_77		AY_78		AY_79		AY_80		AY_81		AY_82		AY_83		AY_84	
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	25	25	30	30	27	27	45	45	29	30	50	49	41	41	48	51
100,000	11	11	11	11	20	20	29	28	24	22	36	31	45	46	49	42
250,000	11	11	10	10	15	13	16	16	28	28	27	30	26	25	33	26
500,000	5	5	5	5	3	4	4	4	8	8	18	16	12	11	16	13
750,000	1	1	4	4	3	3	2	2	5	5	4	5	4	5	3	4
1,000,000	2	2	1	1	1	1	0	0	2	2	2	1	1	1	1	1
2,500,000	1	1	2	2	0	0	0	0	2	3	0	1	4	3	5	5
6,000,000	0	0	0	0	0	0	0	0	1	1	0	0	2	2	2	2
Total	56	56	63	63	69	68	96	95	99	99	137	133	135	134	157	144

	AY_77		AY_78		AY_79		AY_80		AY_81		AY_82		AY_83		AY_84	
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.446	0.446	0.476	0.476	0.391	0.397	0.469	0.474	0.293	0.303	0.365	0.368	0.304	0.306	0.306	0.354
100,000	0.196	0.196	0.175	0.175	0.290	0.294	0.302	0.295	0.242	0.222	0.263	0.233	0.333	0.343	0.312	0.292
250,000	0.196	0.196	0.159	0.159	0.217	0.191	0.167	0.168	0.283	0.283	0.197	0.226	0.193	0.187	0.210	0.181
500,000	0.089	0.089	0.079	0.079	0.043	0.059	0.042	0.042	0.081	0.081	0.131	0.120	0.089	0.082	0.102	0.090
750,000	0.018	0.018	0.063	0.063	0.043	0.044	0.021	0.021	0.051	0.051	0.029	0.038	0.030	0.037	0.019	0.028
1,000,000	0.036	0.036	0.016	0.016	0.014	0.015	0.000	0.000	0.020	0.020	0.015	0.008	0.007	0.007	0.006	0.007
2,500,000	0.018	0.018	0.032	0.032	0.000	0.000	0.000	0.000	0.020	0.030	0.000	0.008	0.030	0.022	0.032	0.035
6,000,000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.010	0.000	0.000	0.015	0.015	0.013	0.014

	AY_85		AY_86		AY_87		AY_88		AY_89		AY_90		AY_91	AY_92	AY_93	AY_94
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*
50,000	29	40	40	31	31	35	18	34	9	32	0	64	29	22	10	10
100,000	41	30	41	23	44	32	35	44	4	36	2	63	36	45	23	6
250,000	40	31	48	38	44	35	31	42	7	40	4	50	46	41	18	6
500,000	8	6	6	16	12	15	3	8	2	6	0	8	17	13	10	2
750,000	5	5	4	3	3	7	1	3	0	3	0	7	3	1	1	0
1,000,000	2	2	2	0	3	4	0	3	0	4	0	4	0	2	0	0
2,500,000	4	2	2	2	0	2	0	5	1	3	0	2	0	0	1	0
6,000,000	0	3	0	1	0	2	0	0	0	2	0	1	0	1	0	0
Total	129	119	143	114	137	132	88	139	23	126	6	199	131	125	63	24

	AY_85		AY_86		AY_87		AY_88		AY_89		AY_90		AY_91	AY_92	AY_93	AY_94
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*
50,000	0.225	0.336	0.280	0.272	0.226	0.265	0.205	0.245	0.391	0.254	0.000	0.322	0.221	0.176	0.159	0.417
100,000	0.318	0.252	0.287	0.202	0.321	0.242	0.398	0.317	0.174	0.286	0.333	0.317	0.275	0.360	0.365	0.250
250,000	0.310	0.261	0.336	0.333	0.321	0.265	0.352	0.302	0.304	0.317	0.667	0.251	0.351	0.328	0.286	0.250
500,000	0.062	0.050	0.042	0.140	0.088	0.114	0.034	0.058	0.087	0.048	0.000	0.040	0.130	0.104	0.159	0.083
750,000	0.039	0.042	0.028	0.026	0.022	0.053	0.011	0.022	0.000	0.024	0.000	0.035	0.023	0.008	0.016	0.000
1,000,000	0.016	0.017	0.014	0.000	0.022	0.030	0.000	0.022	0.000	0.032	0.000	0.020	0.000	0.016	0.000	0.000
2,500,000	0.031	0.017	0.014	0.018	0.000	0.015	0.000	0.036	0.043	0.024	0.000	0.010	0.000	0.000	0.016	0.000
6,000,000	0.000	0.025	0.000	0.009	0.000	0.015	0.000	0.000	0.000	0.016	0.000	0.005	0.000	0.008	0.000	0.000

REPORT.XLS
10/2/95

Fitted Claim Count Distribution
Valued as of 3/90 and 3/95

Section III
Exhbit 2

Layer	AY_77		AY_78		AY_79		AY_80		AY_81	
	90	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.398	0.398	0.392	0.392	0.381	0.389	0.467	0.470	0.285	0.289
100,000	0.251	0.251	0.250	0.250	0.279	0.274	0.289	0.287	0.260	0.255
250,000	0.188	0.188	0.190	0.190	0.214	0.207	0.176	0.175	0.250	0.244
500,000	0.077	0.077	0.078	0.078	0.075	0.075	0.047	0.047	0.110	0.110
750,000	0.029	0.029	0.030	0.030	0.023	0.024	0.011	0.012	0.039	0.040
1,000,000	0.015	0.015	0.016	0.016	0.010	0.011	0.004	0.005	0.019	0.020
2,500,000	0.029	0.029	0.031	0.031	0.015	0.016	0.005	0.005	0.029	0.033
6,000,000	0.012	0.012	0.013	0.013	0.003	0.004	0.001	0.001	0.007	0.009

Layer	AY_82		AY_83		AY_84		AY_85		AY_86	
	90	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.336	0.335	0.341	0.345	0.330	0.370	0.246	0.340	0.274	0.244
100,000	0.276	0.276	0.259	0.261	0.261	0.253	0.274	0.248	0.298	0.265
250,000	0.234	0.235	0.220	0.219	0.226	0.202	0.287	0.210	0.284	0.281
500,000	0.089	0.090	0.092	0.091	0.095	0.084	0.119	0.093	0.099	0.123
750,000	0.028	0.029	0.033	0.032	0.034	0.031	0.036	0.036	0.026	0.040
1,000,000	0.013	0.013	0.017	0.016	0.017	0.016	0.016	0.019	0.010	0.018
2,500,000	0.018	0.019	0.029	0.027	0.028	0.031	0.019	0.037	0.010	0.025
6,000,000	0.004	0.004	0.009	0.008	0.008	0.012	0.003	0.015	0.001	0.004

Layer	AY_87		AY_88		AY_89		AY_90	AY_91	AY_92	AY_93	AY_94
	90	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*	*95*
50,000	0.233	0.260	0.209	0.266	0.340	0.271	0.334	0.203	0.223	0.185	0.381
100,000	0.298	0.248	0.383	0.271	0.261	0.256	0.281	0.306	0.277	0.288	0.307
250,000	0.312	0.254	0.345	0.271	0.222	0.256	0.238	0.342	0.306	0.347	0.224
500,000	0.110	0.121	0.056	0.113	0.092	0.116	0.089	0.112	0.125	0.129	0.063
750,000	0.027	0.045	0.005	0.036	0.033	0.041	0.027	0.024	0.036	0.031	0.015
1,000,000	0.010	0.023	0.001	0.016	0.016	0.020	0.012	0.007	0.015	0.011	0.005
2,500,000	0.009	0.038	0.000	0.022	0.027	0.032	0.016	0.006	0.016	0.009	0.005
6,000,000	0.001	0.011	0.000	0.004	0.008	0.008	0.003	0.000	0.002	0.000	0.000

REPORT.XLS
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Fitted Distribution Layer Development Factors

Section III
Exhibit 3

Layer	AY_77	AY_78	AY_79	AY_80	AY_81	AY_82	AY_83
50,000	1.000	1.000	1.022	1.006	1.012	0.995	1.013
100,000	1.000	1.000	0.983	0.993	0.981	0.998	1.007
250,000	1.000	1.000	0.970	0.991	0.974	1.002	0.997
500,000	1.000	1.000	0.988	1.004	0.997	1.007	0.982
750,000	1.000	1.000	1.021	1.023	1.032	1.012	0.969
1,000,000	1.000	1.000	1.052	1.039	1.063	1.015	0.960
2,500,000	1.000	1.000	1.111	1.067	1.126	1.019	0.946
6,000,000	1.000	1.000	1.260	1.136	1.274	1.029	0.918

Layer	AY_84	AY_85	AY_86	AY_87	AY_88	AY_89
50,000	1.122	1.381	0.890	1.113	1.269	0.797
100,000	0.971	0.904	0.890	0.833	0.708	0.981
250,000	0.891	0.734	0.991	0.813	0.786	1.150
500,000	0.884	0.787	1.242	1.095	2.017	1.254
750,000	0.926	0.992	1.571	1.669	7.211	1.265
1,000,000	0.974	1.239	1.877	2.373	20.238	1.245
2,500,000	1.090	1.934	2.494	4.367	85.628	1.174
6,000,000	1.422	5.423	4.642	17.603	3808.624	0.986

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Layer	AY_87	AY_88	AY_89	AY_90	AY_91
50,000	27.31%	27.39%	27.18%	33.14%	20.36%
100,000	26.39%	26.07%	23.62%	25.03%	26.96%
250,000	26.34%	26.06%	23.59%	21.36%	30.50%
500,000	11.80%	11.88%	11.86%	9.03%	12.85%
750,000	3.77%	3.86%	4.66%	3.74%	4.47%
1,000,000	1.70%	1.80%	2.66%	2.38%	2.58%
2,500,000	2.27%	2.46%	4.70%	3.80%	2.03%
6,000,000	0.40%	0.48%	1.72%	1.53%	0.26%

				Selected	Fitted
Layer	AY_92	AY_93	AY_94	AY_96	AY_96
50,000	19.23%	16.83%	37.93%	15.16%	15.27%
100,000	20.20%	21.18%	22.94%	22.46%	20.70%
250,000	22.92%	26.75%	18.39%	27.24%	28.82%
500,000	13.62%	15.17%	8.28%	14.32%	16.98%
750,000	7.60%	8.00%	4.41%	9.71%	6.94%
1,000,000	6.92%	6.86%	4.13%	5.12%	3.62%
2,500,000	6.63%	4.21%	2.79%	4.92%	6.09%
6,000,000	2.88%	0.99%	1.13%	1.07%	1.58%

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