Review of "Risk Loads for Insurers"
PCAS LXXVII, (1990 by Sholom Feldblum)
by Glenn G. Meyers, FCAS

"Risk Loads for Insurers" by Sholom Feldblum

Discussion by Glenn Meyers

1. Introduction

For many years now, a theoretical war has been raging on the subject of risk loads. Some favor the classical premium calculation principles, such as the standard deviation principle, the variance principle or the expected utility principle. Others favor the modern portfolio theories, represented most often by the Capital Asset Pricing Model, also known as the CAPM. Mr. Feldblum presents arguments against the classical premium calculation principles, calling them theoretically unsound, and presents arguments for the Capital Asset Pricing Model.

In this discussion, I will address the same issues as Mr. Feldblum from a different viewpoint. Historically, actuaries have not always derived premium calculation principles from economic and/or statistical assumptions. More often their approach would be to simply state a principle, then check to see if it has desirable properties[1]. While a mathematical derivation from explicitly stated economic principles is certainly desirable, I see no reason why it should be required. I find it difficult to attach much meaning to Mr. Feldblum's use of the term "theoretically unsound" in this context.

However, the list of "desirable properties" can be, and often is, at issue. My personal view is that the list of "desirable properties" should be consistent with competitive market economic principles. Moreover, we should be able to observe behavior in the insurance marketplace which is consistent with these desirable properties.

From this viewpoint I will make the following arguments.

1. The standard deviation principle is not acceptable. It predicts behavior that is opposite of what is observed in the insurance marketplace.
2. The variance principle and the expected utility principle predict some behavior which can be observed in the insurance marketplace, but much is left unexplained.

[1] A recent analysis of this type can be found in "Why Standard Deviation should be replaced by Absolute Deviation" by D. Dennenberg, *Astin Bulletin*, November, 1990, p. 181.

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3. *The CAPM was designed as a tool for pricing securities (including those of insurance companies).* While some may argue that it is oversimplified, it provides a tremendous amount of economic insight and predicts behavior which is consistent with activity observed in the securities market. Many, however, try to make the CAPM into a premium calculation principle by treating a line of insurance or even an individual insurance policy as if it were a security in which one could invest. I will argue that such an treatment is inappropriate. I will further argue that many statements made by those who attempt this treatment are inconsistent with behavior observed in the insurance marketplace.

4. Instead of trying to mold premium calculation principles into the framework of the CAPM result, one should apply the principles underlying CAPM to the problem that exists -- calculating premiums. This work has been done. The result is a premium calculation principle called the Competitive Market Equilibrium risk load formula[2].

2. The Classical Premium Calculation Principles

Let X be a random loss faced by a prospective insured. Let μ_X, σ_X and σ_X^2 denote the mean, standard deviation and variance of X respectively. The classical premium calculation principles provide different formulas for calculating the premium, P, to be charged for insurance against this loss[3]. The standard deviation principle can.be stated as:

$$P = \mu_X + \lambda \cdot \sigma_X$$

The risk load, R, for the standard deviation principle is given by $\lambda \cdot \sigma_X$.

The variance principle can be stated as:

$$P = \mu_X + \lambda \cdot \sigma_X^2$$

The risk load, R, for the variance principle is given by the expression $\lambda \cdot \sigma_X^2$.

[2]This formula is described in detail in "The Competitive Market Equilibrium Risk Load for Increased Limits Ratemaking", by Glenn Meyers *PCAS LXXVII*, 1992.

[3]*Here, and elsewhere, parameters which are extraneous to the argument, such as insurer* expenses or initial wealth of the insured are suppressed.

It turns out that the standard deviation principle and the variance principle imply contradictory behavior with respect to excess of loss reinsurance. For a random loss, X, let:

$$X_1 = \begin{cases} X \text{ if } X \le L \\ L \text{ if } X > L \end{cases} \qquad \text{and} \qquad X_2 = \begin{cases} 0 \quad \text{if } X \le L \\ X - L \text{ if } X > L \end{cases}$$

We have:

$$X = X_1 + X_2$$

Let ρ be the coefficient of correlation between X_1 and X_2. If $\mu_{X_2} \neq 0$, we have that $0 < \rho < 1$[4].

We have that:

$$\sigma_X^2 = \sigma_{X_1}^2 + 2 \cdot \rho \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2 > \sigma_{X_1}^2 + \sigma_{X_2}^2$$

This implies that total risk load is reduced by excess of loss reinsurance for the variance principle.

We also have that:

$$\sigma_X = \sqrt{\sigma_{X_1}^2 + 2 \cdot \rho \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2} < \sqrt{\sigma_{X_1}^2 + 2 \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2} = \sigma_{X_1} + \sigma_{X_2}$$

This implies that the risk load is *increased* by excess of loss reinsurance for the standard deviation principle.

The fact that excess of loss reinsurance arrangements are common in the insurance business provides evidence that the variance principle predicts results which are consistent with observed marketplace behavior, while the standard deviation principle predicts results which are contradictory to observed marketplace behavior. The insurance industry does not take on the extra expense of reinsurance for the purpose of increasing its total risk.

[4]Shown as part of a demonstration of risk reduction by layering in "Increased Limits and Excess of Loss Pricing" by Robert S. Miccolis *PCAS LXIV*, 1977.

The expected utility principle is generally regarded as the most complete of the classical premium calculation principles. It usually addresses the problem from the point of view of the insured. If the insured, with utility function, u, is faced with a random loss, X, it calculates the risk load, R, as the solution of the equation:

$$E[u(X)] = u(\mu_X + R) \equiv u(P)$$

i.e. the insured is indifferent between the variable loss, X, and the certain premium, $P = \mu_X + R$.

It should be noted that this premium represents the maximum premium the insured will pay for insurance against the random loss. If an insurer offers a lower price, the insured will surely accept it.

The variance principle and the expected utility principle are closely related. Consider the approximation[5]:

$$P \approx \mu_X + \tilde{\lambda} \cdot \sigma_X^2$$

where[6]:

$$\tilde{\lambda} = -\frac{1}{2} \cdot \frac{u''(-\mu_X)}{u'(-\mu_X)}$$

This approximation is based on a Taylor series expansion in which the approximation becomes increasingly accurate as σ_X^2 gets smaller.

The formula is exact in some cases. One example is when the utility function is exponential and the losses have a normal distribution[7].

[5]This expression is derived on page 21 of *Actuarial Mathematics*, by Bowers, Gerber, Hickman, Jones and Nesbitt, Society of Actuaries, 1986.

[6]$\tilde{\lambda}$ will be positive under the usual assumptions that $u' > 0$ (more is better) and $u'' < 0$ (risk averse).

[7]Bowers, *et al*, *op. cit.*, p. 11.

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Thus far, we have addressed utility from the point of view of the insured. We now consider utility theory from the point of view of the insurer. It should be noted that many proponents of CAPM say that it is improper to use utility theory in this context. This will be addressed below.

The minimum premium, G, necessary for an insurance company to voluntarily write an insured is given by[8]:

$$u(0) = E[u(G - X)]$$

i.e. the insurer is indifferent between doing nothing and accepting the uncertain liability, X, in exchange for the premium, G.

If the maximum premium, P, an insured is willing to pay is greater than the minimum premium, G, an insurer must receive, a deal can be made to benefit both parties. Utility theory says nothing about where the final price of the insurance policy will lie between P and G. This is determined by the economic laws of supply and demand. For this reason, it could be said that utility theory provides an incomplete description of insurance pricing.

Insurance Services Office (ISO) originally used a risk load based on the variance principle, but in the mid 1980's it was changed to the standard deviation principle. It is true, as Mr. Feldblum states, that "ISO simply chooses an overall risk load by line of business, and then spreads this risk load by size of policy limit using the standard deviation or variance method." This is done by adjusting the λ parameter so that the average risk load, in ISO's judgment, is reasonable. In describing this practice he uses terms such as "theoretically unsound". It is certainly true that the λ parameter is not derived with the consideration of any kind of utility function, or risk aversion. I tend to think this reflects a narrow view of what is "theoretically sound". ISO has always viewed this as a good practical solution which has been deemed acceptable by many actuaries. It should be mentioned that the Competitive Market Equilibrium risk load formula, referenced below, does provide an explicit justification for its version of this practice.

[8]Bowers, *et al*, *op. cit.*, p. 10.

3. The Capital Asset Pricing Model

The recognition of the ability of individual investors to diversify their investment risk has been the main contribution of the modern portfolio theories. The CAPM models the effect of the ability to diversify on the price of securities. The significance of these models has been recently recognized by the awarding of the Nobel Prize in Economics to three of the originators of the theory.

We begin with an examination of a derivation of the Capital Asset Pricing Model. I have found the following derivation based on a constrained optimization to be particularly illuminating. What follows is a direct quote of the statement of the problem by Thomas E. Copeland and J. Fred Weston[9].

"We assume that portfolio cash flows for the *i*th individual are generated at the end of the period and that they are normally distributed with mean, e_i, and variance, σ_i^2. The *i*th individual's utility is a function of the mean and variance of his end-of-year cash flows. His utility function is written

$$U_i(e_i, \sigma_i^2)$$

"We further assume that the marginal utility of expected cash flows is positive, and the marginal utility of the variance of cash flows is negative.

$$\delta U_i/\delta e_i > 0, \qquad \delta U_i/\delta \sigma_i^2 < 0$$

"Finally, all assets are marketable and infinitely divisible, transactions costs and taxes are zero, and there are no constraints on short sales. The expected end-of-period cash flows to an individual are the payments from risky assets less any interest on debt:

$$e_i = \sum_j X_{ij} \cdot E[\widetilde{D}_j] - r \cdot d_i$$

where

X_{ij} = fraction of *j*th firm held by the *i*th individual.

r = $(1 + R_f)$, where R_f is the one period risk-free borrowing/lending rate.

d_i = the net personal debt issued by the *i*th individual.

$\widetilde{D}_j$ = net end of period cash flow paid by the *j*th firm.

[9] Copeland, T. E., and Weston, J. F., *Financial Theory and Corporate Policy*, Addison-Wesley, 1979, Appendix to Chapter 7: An Alternative Derivation to the CAPM.

"The variance of the end of period cash flows for the ith individual is

$$\sigma_i^2 = \sum_j \sum_k X_{ij} \cdot X_{ik} \cdot \mathrm{Cov}[\widetilde{D}_j, \widetilde{D}_k]$$

"The individual investor's problem is to find the set of weights, X_{ij}, and borrowing, d_i, which maximize his expected end-of-period utility subject to his budget constraint.

$$\underset{X_{ij},\, d_i}{\mathrm{MAX}}\ EU_i(e_i, \sigma_i^2)$$

subject to $$\sum_j X_{ij} \cdot V_j - d_i = W_i$$

where

V_j = the total market value of the jth firm at the beginning of the period.

W_i = the total wealth of the individual at the beginning of the period."

The derivation of the CAPM assumes that all investors behave in the manner described above, and that the market is in equilibrium. The equilibrium value of the jth asset is then demonstrated to be:

$$V_j = \frac{1}{r} \cdot \left(E[\widetilde{D}_j] - \theta \cdot \mathrm{Cov}[\widetilde{D}_j, \widetilde{D}_m] \right) \tag{1}$$

where

θ = $(E[\widetilde{D}_m] - r \cdot V_m)/\mathrm{Var}[\widetilde{D}_m]$

$\widetilde{D}_m$ = the cash payouts for all firms in the market.

V_m = the value of the market portfolio at the beginning of the period.

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The above equation can be converted into rates of return if we define the rate of return on the jth asset as

$$\widetilde{R}_j = (\widetilde{D}_j - V_j)/V_j \tag{2}$$

Using Equation 2 in Equation 1, we obtain

$$E[\widetilde{R}_j] = R_f + \lambda \cdot \mathrm{Cov}[\widetilde{R}_j, \widetilde{R}_m] \tag{3}$$

where

$$\widetilde{R}_m = \sum_j V_j \cdot \widetilde{R}_j / V_m.$$

$$\lambda \;= (E[\widetilde{R}_m] - R_f)/\mathrm{Var}[\widetilde{R}_m].$$

Equation 3 is the familiar CAPM.

As noted above, the CAPM was put forth as a model to explain the price of securities. Mr. Feldblum, along with many others, has tried to use the CAPM to calculate risk loads for insurers. I believe this attempt has failed. I offer two complaints.

My first complaint has to do with the treatment of risk as it applies to insurers. The most direct statement of the prevailing sentiment by proponents of CAPM is given by Cummins who states[10]: "Firms should not be risk averse." The reasoning behind such a statement is that individual investors can "eliminate this type of risk by holding diversified portfolios." The implication of such a statement is that an insurance firm should be indifferent between insuring low and high limit policies.

By my own observations, and by the observations of others, managers of insurance firms are risk averse. The existence of reinsurance provides an objective verification of these observations.

Another way to view this complaint is to note that much of the risk that insurers face is deemed "diversifiable" and CAPM proponents claim that the market should not reward such risks. Examples given of such diversifiable risks include the risk faced by insurers who accept high limit policies.

[10]Cummins, J.D., "Asset Pricing Models and Insurance Ratemaking", *ASTIN Bulletin*, November, 1990, p.125.

The flaw in these statements can be addressed by the CAPM itself. Nowhere in the above development of the CAPM is one required to label a particular risk as being diversifiable or non-diversifiable. Diversification of the investor's risk is a result of the investor finding the optimal X_{ij}'s. Consider the case when there are two otherwise identical insurance firms, Firm #1 which writes only low limit policies and Firm #2 which writes only high limit policies. Each firm uses the same percentage risk load. Since the variability of the results will be less for Firm #1, each risk averse investor will compete more for Firm #1's securities. Firm #1 will have a higher value. The only counter to this problem is for the managers of Firm #2 to charge a higher risk load.

My second complaint is about the allocation of surplus. In their attempt to use CAPM to calculate risk loads, proponents treat a line of insurance as if it were a free-standing insurance company. It is then maintained that multiline insurers can allocate their surplus by line of business, and all will be well. What is missing from the argument is an acceptable method of allocating surplus by line of insurance. Most methods used by insurance regulators for allocating surplus simply choose an arbitrary premium to surplus ratio. Others, including myself, believe that it is not appropriate to even attempt to allocate surplus. The case against allocating surplus was made most eloquently at the March 1990 CAS Ratemaking Seminar by Charles F. McClenahan who, after noting that the purpose of surplus was protection against insolvency, stated "The protection against insolvency afforded by a $100 million dollar surplus for a free-standing automobile insurance company is not comparable to the protection afforded by a multiline insurance company with $100 million dollars of surplus allocated to automobile insurance."

4. The Competitive Market Equilibrium Risk Load Formula

The case has been made that it is inappropriate to adapt the CAPM to calculate risk loads for insurers. In this section, I will discuss an alternative model, called the Competitive Market Equilibrium (CME) risk load formula. Curiously enough, its development parallels the above development of the CAPM. Since it is being described in a separate paper, I will only show the parallels between its development, and the CAPM.

Both the CAPM and the CME risk load formula assume that each market participant (investor or insurer) behaves in a "rational" manner. By matching supply with demand, an equilibrium price for each product (security or insurance policy) is calculated. What follows are the definitions of "rational".

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CAPM Statement of Individual Investor's Problem

The individual investor's problem is to select investments in such a way as to maximize his utility subject to a constraint on his total wealth.

CME Statement of Insurer Management's Problem

The insurer management's problem is to select amounts of exposure in lines of insurance and policy limits in such a way as to maximize the total risk load subject to a constraint on the variance of the insurer's book of business.

The CME risk load formula addresses most of the shortcomings of the premium calculation principles described above. It provides a more complete description of the premium than that provided by utility theory. Since each insurer has a constraint on the variance of its book of business, the insurer is assumed to be risk averse. Since each insurer chooses the amount of exposure for each line of insurance, it is not necessary to allocate surplus by line of insurance.

Many of these issues are discussed more fully in the CME paper.

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