Interpreting Model Output—
The California Earthquake Authority and the Cost of Capital of the Reinsurance Layer

by Giuseppe Russo, Ph.D.
Oakley E. Van Slyke, FCAS, ASA

Interpreting Model Output

The California Earthquake Authority and the Cost of Capital of the Reinsurance Layer

Introduction

Actuaries and other financial analysts had had difficulty interpreting the voluminous data that is typically output by a dynamic financial model. This paper illustrates the use of the decision-theoretic approach of Borch (1962) and Van Slyke (1995) to produce a simple illustration of the meaning of the results of 10,000 simulations of the financial results of a reinsurance program. The illustration in Figure 1 relates the model's results to the cost of capital in international financial markets.

The California Earthquake Authority

Earthquakes in California have accounted for large losses to homeowner insurers in the past ten years. The Loma Prieta earthquake in San Francisco in 1989 accounted for $1 billion of insured commercial and residential losses and the Northridge earthquake in Los Angeles in 1993 accounted for an additional $8.2 billion of insured residential losses. There are predictions from many experts that these regions are due for another large earthquake in the near future. These large losses to insurers have created a crisis in California. These losses suggest limiting underwriting. However, state statute requires that all companies selling homeowners insurance in California offer earthquake insurance. In the present market, earthquakes in California have become an uninsurable risk.

In response to this problem, California legislators have proposed two significant changes to earthquake insurance issued in California. First, AB 1366 proposes changes in the minimum coverage to earthquake policies issued in California, known as the "mini-earthquake policy." Coverage A (Structure) has an increased deductible of 15%; it includes the basic structure, foundations and walkways but excludes any external masonry veneer. The coverage is on a replacement cost basis and requires insurance to be purchased to value. Coverage B (Contents) limits loss to $5,000 at replacement cost excluding obvious items such as computers, glassware, securities and money, etc. Coverage C (Loss of Use) limits losses to $1,500 with no deductible. There are similar changes to the condominium and renters form of the insurance. Mini-earthquake policies will provide significant reductions in loss exposure; estimates show Northridge earthquake losses would have total $4.3 billion under mini-earthquake policies.

The second, and more significant change, is the introduction of AB 13. AB 13 proposes to establish an agency of the State of California known as the California Earthquake Authority (CEA). This agency will be the insuring mechanism for all mini-earthquake policies sold in California by *participating* members. This authority is unique in that it relies on various sources for insurance coverage: the insurance market, state government, and the financial markets. If an insurance company joins the CEA, the CEA will sell all mini-earthquake policies for this insurer. At 100% participation, the CEA will have sufficient capacity to pay $10.5 billion in earthquake losses in the first year. All capacities will be prorated downward based on the percentage of participation in the CEA. If an insurer does not join the CEA, the insurer must offer, at a minimum, the mini-earthquake policy along with all home insurance policies sold in California.

The CEA capacity consists of seven different layers of coverage. The first layer of coverage comprises of initial industry capital contributions. At 100% participation, the total coverage amount for this layer is $1 billion with each participant contributing an amount proportion to their total market share. This contribution layer is not reinstated after a loss. The second layer is comprised of retained earnings. This layer will increase the paying capacity of the agency in the first two years. If earthquake losses are low enough to lead to retained earnings in these years, the CEA will use this layer to reduce the coverage exposure in the first and second assessment layer. The next $3 billion of capacity is the first industry assessment layer. Participants in the CEA must pay an amount proportional to their market share if losses penetrate this layer. Any portion of this layer used to pay for losses or reduced by the favorable increases in the retained earnings layer cannot be reinstated. The next $2 billion dollars of losses is covered by a reinsurance layer. For the first two years of the CEA, this reinsurance is basically $2 billion coverage excess of $4 billion plus retained earnings. This study note discusses this layer in great detail, including comparing the risk/reward payoff of this layer to other risky investments. The remaining three layers to do not affect the reinsurance layer in any way but we will discuss them for completeness. The fifth layer is comprised of up to $1 billion of general revenue bonds issued by the State of California. If penetrated, the state legislator can reinstate this layer through the passage of subsequent legislation. It will be repaid by a surcharge to mini-earthquake policyholders of up to 20%. The sixth layer is also a very interesting and unique layer. It comprises of $1.5 billion of coverage through the sale of Cat Bonds to the financial markets. Only the interest paid on these bonds is contingent; any losses penetrating this layer will reduce proportionally the payment of interest to these bonds. If the entire layer is used to pay for losses, no future interest payments will be made on these bonds. The bonds are to be paid back 10 years after the inception of the CEA, whether or not a loss penetrates this layer. Finally, the remaining $2 billion of coverage is the second industry assessment layer. This layer is identical to the first industry assessment layer except that it is reduced only if the assets of the CEA exceed $6 billion for at least 6 months.

CEA Reinsurance Layer

One of the arguments against the formation of the California Earthquake Authority is the extraordinary cost of the reinsurance layer. The goal of this study is to quantify the cost of this risk and compare it to comparable investments in the financial markets. In particular, we will compare risks and rewards of underwriting this reinsurance layer to the risks and rewards of investments in Baa subordinate bonds and investments in the S&P 500 index.

At 100% participation level, the reinsurance layer is basically $2 billion dollars of coverage excess of $4 billion less retained earnings. This coverage will cost the CEA a total of $575 million in premium over two years. Since the CEA will underwrite new mini-earthquake policies as homeowner policies are renewed throughout the year, we assumed the exposure of loss to the CEA would increase proportionally in the first year of exposure, starting from no exposure at inception to full exposure of loss at the end of the first year. At this premium level, our calculations show a loss ratio of 33% to 42%, depending on the uncertainty one has with EQECAT loss estimates.

EQECAT Catastrophe Management is the firm responsible for estimating the catastrophic earthquake losses for the CEA (EQECAT Report No. 710003.001, 12/95.) They modeled twelve scenario earthquake events; these simulations examined earthquake losses in highly populated urban areas in California. They calibrated their model using the San Francisco earthquake 1906 and the Northridge earthquake as benchmarks. Their estimate of total expected annual loss for California was $742 million. We simulated the earthquake losses for this study using the annual loss distribution estimated by EQECAT.

The Cost of Capital for CEA Reinsurance

Appendix 1 lists the assumptions used in the model of the CEA. We performed simulation runs using different levels of market participation and found the results to be insensitive to this assumption. For a realistic model, we settled on a market participation level of 80%. This reduces the total premium to $460 million and the maximum loss to $1,600 million.

Reinsurance underwriters do not know whether the EQECAT earthquake loss estimates reflect the true probabilities of losses to the CEA. By assuming that the yearly earthquake loss incurred by the CEA is the product of the earthquake loss estimated by EQECAT and a random variable representing uncertainty about the EQECAT estimates. (We assume this random variable is lognormal with a mode, or most likely value of 1.0, and uncertainty measured by the parameter σ.) When the uncertainty parameter equals 0.0, the CEA earthquake losses are equivalent to losses generated by the EQECAT model. When the uncertainty parameter equals 1.0, the mean earthquake loss of the simulation model is 65% greater than the mean earthquake loss of the EQECAT estimate.

We choose values of σ from 0.0 to 1.0 because this range reflects a wide variation in uncertainty. As we noted, $\sigma = 0.0$ denotes no uncertainty in EQECAT estimates. An uncertainty of 1.0 is about the same uncertainty as in auto collision claims about the average for a given make and model of car. That is, $\sigma = 0.0$ is like being told "pay the average collision loss of $2,500"; $\sigma = 1.0$ is like being told "pay the actual collision loss, the average is $2,500."

Let R denote a random variable representing the total ceded loss over the two years of exposure. Similar to the CEA earthquake losses, ceded losses also increase with increases in σ but in a more complicated manner.

Table 1 displays the results of the simulations for various values of σ. As a percentage of premium, the expected loss to the reinsurer ranges from 33% with no parameter uncertainty to 42% when $\sigma = 1$. At first glance, it may seem that this expected loss ratio is too low but note the other statistics in Table 1. In particular, if the reinsurer does incur a loss, the expected loss is more than two and one half times the $460 million dollars of premium. Furthermore, there is approximately a 50% probability that if a loss occurs, the reinsurer will have to pay the probable maximum loss, $1,600 million.

σ	0.0	1.0
Probability of no loss	85.3%	83.7%
Probability of total loss	6.6%	8.9%
Probability of total loss, given a loss occurs	44.8%	54.4%
Expected loss to reinsurer, given a loss occurs (millions)	1,043	1,172
Expected loss to reinsurer, given a loss occurs but it is not a full loss (millions)	591	660
Expected loss to reinsurer (millions)	153	192
Expected loss ratio	33%	42%

Table 1: Earthquake Loss Distribution of CEA Reinsurance Layer

Clearly, the distribution of R is not the typical normal random variable that is familiar to everyone (including the critics). In particular, there are two point masses associated with the distribution of R; a large point mass at \$0 and a second probability of loss at the probable maximum loss of \$1,600 million. For R between \$0 and \$1,600 million, we found the truncated exponential distribution to fit well. For further details, we refer the reader to Appendix 3.

To quantify the cost of risk in this coverage, we used a risk-adjusted value (RAV_c) characterized by an exponential utility function. Some literature refers to the "economic value added" of a transaction as the change in risk-adjusted value. Appendix 2 contains a thorough discussion of these functions. There are many appealing properties of this approach including the specification of risk capacity c and a measure of cost associated with this value. Some other important properties of RAV are:

- First, if we know the ceded loss with certainty then the risk-adjusted value of this loss is simply the loss itself, regardless of the insurer's commitment of risk capacity.

- Second, if two risks are independent then we can separately determine their risk-adjusted values.

- Third, reinsurance losses can be pooled. If one underwrites $1/k$ of the total, one would expect to get $1/k$ of the premium. Furthermore, the reinsurer's risk-adjusted value of a loss is proportional to the capacity it puts up to bear the risk. In other words, if the total ceded exposure is too large for an individual reinsurer with a specified risk capacity c, we can divide the risk amongst k reinsurers, each with capacity c; each reinsurer is responsible to an amount R/k. This property is precisely what allows the California Earthquake Authority to obtain a large reinsurance layer of coverage. There is no one reinsurer that is willing to allocate all the capacity needed to underwrite the entire ceded loss. Rather, the ceded loss exposure is divided amongst a large number of reinsurers with varying commitments of capacity; each reinsurer is willing to underwrite a small portion of the total loss relative to their committed risk capacity.

- Fourth, the risk-adjusted value of an uncertain loss is always greater than the pure premium. Furthermore, the risk capacity is critical to the value of the risk-adjusted value. In particular, the more capacity the insurer chooses to apply to underwriting a random loss exposure R, the less is the risk-adjusted value charged for this exposure. That is,

  Risk Premium at Capacity c

  = Risk-Adjusted Value at Capacity c – Expected Losses

  or

  Estimate Risk Premium = Observed Market Price – Estimated Pure Premium

We refer to the difference between the risk-adjusted value and the pure premium as the risk premium. In this model, the measure of risk to reward is not the absolute size of the risk premium, or the absolute size of the risk capacity, but rather the amount of risk premium one receives relative to one's risk capacity.

- Fifth, in a competitive market in rough equilibrium, the amount of risk capacity will rise as the risk premium increases.

The concept of risk-adjusted value and risk premium is not limited to insurance problems. We may apply this model to any form of risk including investments in stocks and bonds. One can compare directly the percentage of risk premium to risk capacity for various investments to determine if the risk/reward payoffs are sufficient.

Figure 1 displays risk premium curves for the CEA reinsurance layer as a function of risk capacity. This figure displays two risk premium curves; one curve associated with $\sigma = 0.0$ and the second curve associated with $\sigma = 1.0$. Each of these curves shows the intersection of the observed risk premium with the risk premium curve. The solid line between these two points displays the location of similar points for values of σ between 0.0 and 1.0. Notice that as uncertainty increases, the risk premium curves shift upward and to the right. The farther these curves are from the origin, the higher the risk. Recall from the discussion above that although this graph is for the whole reinsurance layer, one can obtain the corresponding graph for an individual reinsurer by scaling the figure downward according to the level of the reinsurer's participation in the CEA.

From the individual reinsurer's perspective of underwriting a risk, one must decide whether the risk premium one receives for bearing the risk is sufficient in light of the amount of committed capacity. If we know the premium the reinsurer is requesting for the risk, we can determine an *implied* capacity that corresponds to the intersection of the observed risk premium with the risk premium curve. If the implied capacity is less than the total capacity the reinsurer is willing to commit, given the premium, then the risk is acceptable.

Because the premium for the reinsurance layer is fixed at $460 million, the observed risk premium decreases with increases in uncertainty associated with EQECAT loss estimates. Furthermore, because the risk premium curves are also shifting upward and to the right with increases in uncertainty, the implied risk capacity of the reinsurer increases with increases in uncertainty. Intuitively, this graph says that with the premium fixed at $460 million, the more one is uncertain of EQECAT loss estimates, the less return one will receive from this investment. Table 2 displays the results of the risk-adjusted value calculations for the CEA. The percentage of risk premium to risk capacity ranges between 53% with no parameter uncertainty and 38% with large parameter uncertainty.

σ	0.0	1.0
Observed Risk Premium	307	268
Implied Risk Capacity	575	713
Ratio of Risk Premium to Risk Capacity	53%	38%

Table 2: Risk-adjusted Value Analysis of CEA Reinsurance Layer

Is a ratio of risk premium to risk capacity of between 38% and 53% too high as opponents to the CEA would argue? To answer this question, let us compare these values to other investment alternatives in the market.

The Cost of Capital for Stocks and Bonds

Figure 2 displays the risk premium curve versus risk capacity for an investment of \$5 million in the S&P 500 composite index. We assume the S&P 500 follows a log-normal random process with drift parameter μ_S and volatility σ_S. Changes in the drift parameter do not result in any significant shifts in the risk premium curve but do affect the observed risk premium. Changes in the volatility do not affect the observed risk premium but they do affect the risk premium curve. The region displayed in the graph is the intersection of the observed risk premium with the risk premium curve for various combinations of μ_S and σ_S; the actual combination for a particular investment in the S&P 500 index depends on market timing. This region also does not display all possible values of μ_S and σ_S; it only represents values that we believe are realistic in today's investment environment. Specifically, we let μ_S vary between 9.5% and 10.5% with the associated σ_S varying between 0.5% and 1.0% above μ_S.

Figure 3 shows the risk premium versus risk capacity for a \$2 million investment in Baa subordinate debt. The displayed curve assumes a coupon rate of 7.5% and a time to maturity of 20 years. We used default rates and severity estimates from Moody's. By varying the assumption about the coupon rate from 6.5% to 8.5% and the time to maturity from 15 to 20 years, one obtains a risk premium -- risk capacity combination somewhere in the circled region.

Allocation of Capital and Return on Investment

The level of risk depends on the amount one is willing to invest in each of these instruments. We measure this risk level by the distance from the origin to the point on the risk premium curve that corresponds the intersection of the observed risk premium and risk capacity. For example, a $1 million reinsurance limit in the CEA reinsurance layer has an associated risk capacity of approximately $400,000 and provides a risk premium of approximately $160,000. This is the same level of risk and reward as an investment of approximately $1.7 million in an issue of a Baa subordinate bond or an investment of approximately $3.6 million in the S&P 500 index.

The $160,000 of risk premium is not the total return on the investment. It is only the premium for the *risk* associated with the investment. The total return on each investment is the expected return plus the risk premium. For example, for the investment in an issue of Baa debt, the total return is the risk-free return of about 6.2% plus an additional return of about 1.1% for the cost of capital. (The cost of defaults accounts for the balance of the quoted yield on the Baa bond.) In contrast, allocating $750,000 of capital together with a premium of $287,500 to secure a commitment to a $1 million limit of CEA reinsurance, the risk premium of approximately $160,000 over a two-year period would be 10.2% per year in addition to the risk-free return.

These numbers are approximate since that actual level depends on the uncertainty about EQECAT loss estimates. If one believes there is no uncertainty then the required investment amount is lower whereas if one believes a high level of uncertainty then the required investment amount is higher.

Is CEA Reinsurance Overpriced?

Comparing the CEA reinsurance risk to the S&P 500 and Baa subordinate bonds, one sees that the percentage of risk premium to risk capacity remains between 35% and 55%. This suggests that the level of risk/reward associated with the CEA reinsurance layer is no smaller and no greater than investments in the stock market or the bond market.

It is also interesting that as little as a $75 million increase or decrease in premium would have a significant effect on these conclusions. Figures 4 and 5 show that decreasing the premium by $75 million would undercharge this risk exposure whereas increasing the premium by $75 million would overcharge this risk exposure.

Conclusion

This study measures the risks and rewards associated with the CEA reinsurance layer for the first two years of the agency. It quantifies the uncertainty one has with the EQECAT loss data and associates a measure of risk and reward with this uncertainty. We show that there is no measurable difference between the premium paid for this reinsurance layer and premiums paid for other risky investments including Baa subordinate bonds and S&P 500 index.

One can use this model to evaluate the risk/reward relationship for other loss exposures including future extensions of the CEA reinsurance coverage and investments in the Capital Market Layer.

Bibliography

Borch, K., "Equilibrium in a Reinsurance Market, " *Econometrica*, 30 (1962): 424:444.

Van Slyke, O. E., "A General Theory of Finance", *Proceedings of the 5th AFIR International Colloquium*, (1995), Vol. II, 585:635.

Figure 1: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $460 Million

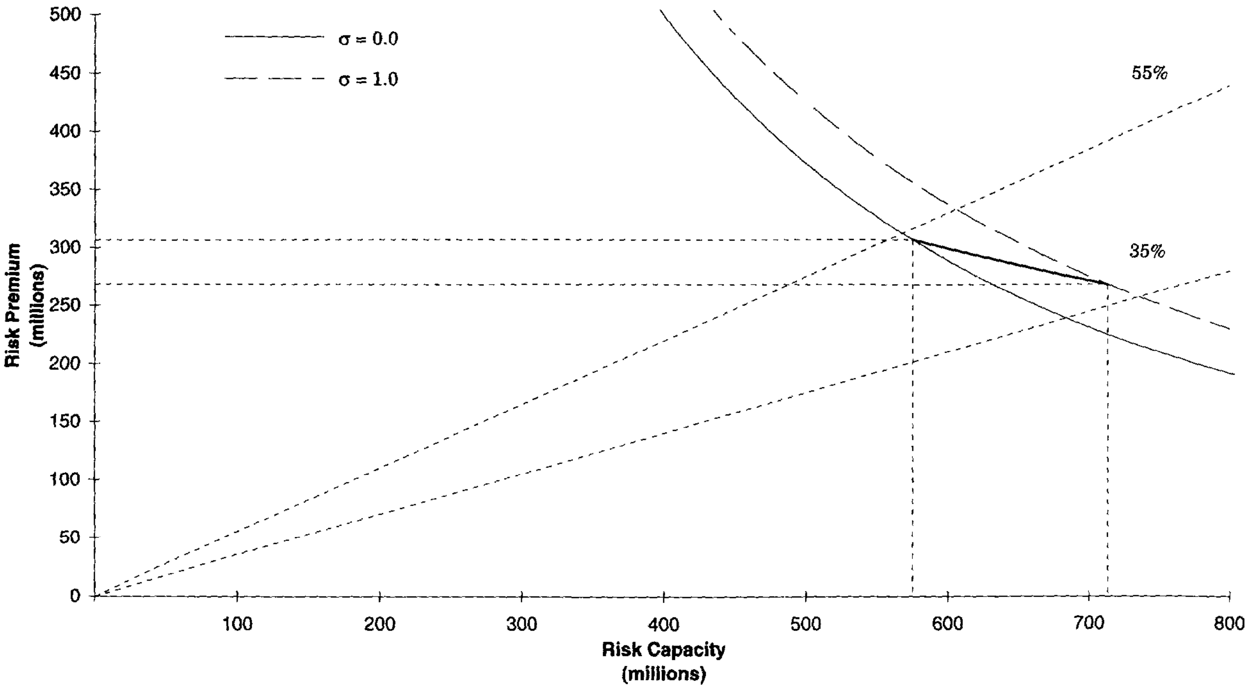

Figure 1 displays risk premium curves for the CEA reinsurance layer as a function of risk capacity. Risk premium is the amount by which premium exceeds expected losses and expenses. This figure displays two risk premium curves; one curve associated with $\sigma = 0.0$ and the second curve associated with $\sigma = 1.0$. Each of these curves shows the intersection of the observed risk premium with the risk premium curve. The percentage of risk premium to risk capacity ranges between 53% for $\sigma = 0.0$ and 38% for $\sigma = 1.0$. This graph assumes an 80% participation in the CEA.

Figure 2: Risk Premium vs Risk Capacity
$5 Million Investment in S&P 500 Index

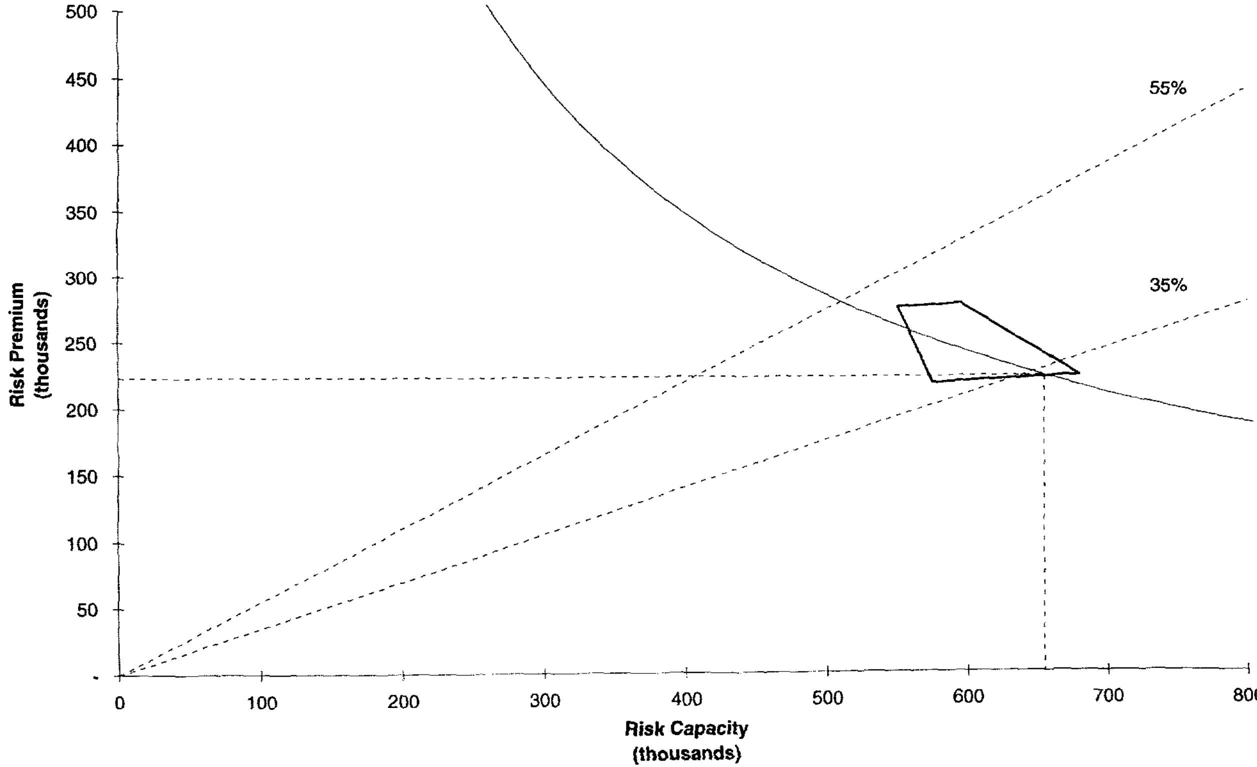

Figure 2 displays the risk premium curve versus risk capacity for an investment of $5 million in the S&P 500 composite index. We assume the S&P 500 follows a log-normal random process with drift parameter μ_S and volatility σ_S. Changes in the drift parameter do not result in any significant shifts in the risk premium curve but do affect the observed risk premium. Changes in the volatility do not affect the observed risk premium but they do affect the risk premium curve. The region displayed in the graph is the intersection of the observed risk premium with the risk premium curve for various combinations of μ_S and σ_S; the actual combination for a particular investment in the S&P 500 index depends on market timing.

Figure 3: Risk Premium vs Risk Capacity
$2 Million Investment In Baa Subordinate Bonds

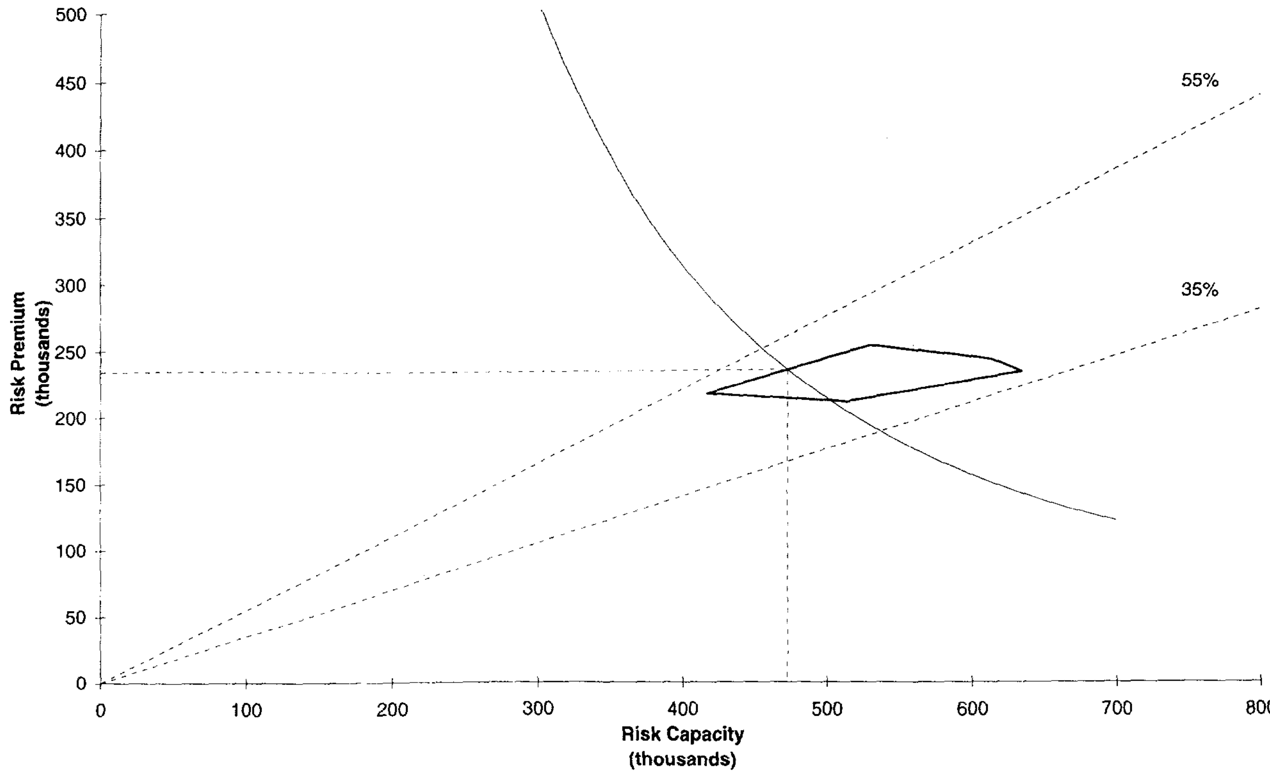

Figure 3 shows the risk premium versus risk capacity for a $2 million investment in Baa subordinate debt. The displayed curve assumes a coupon rate of 7.5% and a time to maturity of 20 years. We used default rates and severity estimates from Moody's. By varying the assumption about the coupon rate from 6.5% to 8.5% and the time to maturity from 15 to 20 years, one obtains a risk premium -- risk capacity combination somewhere in the circled region.

Figure 4: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $385 Million

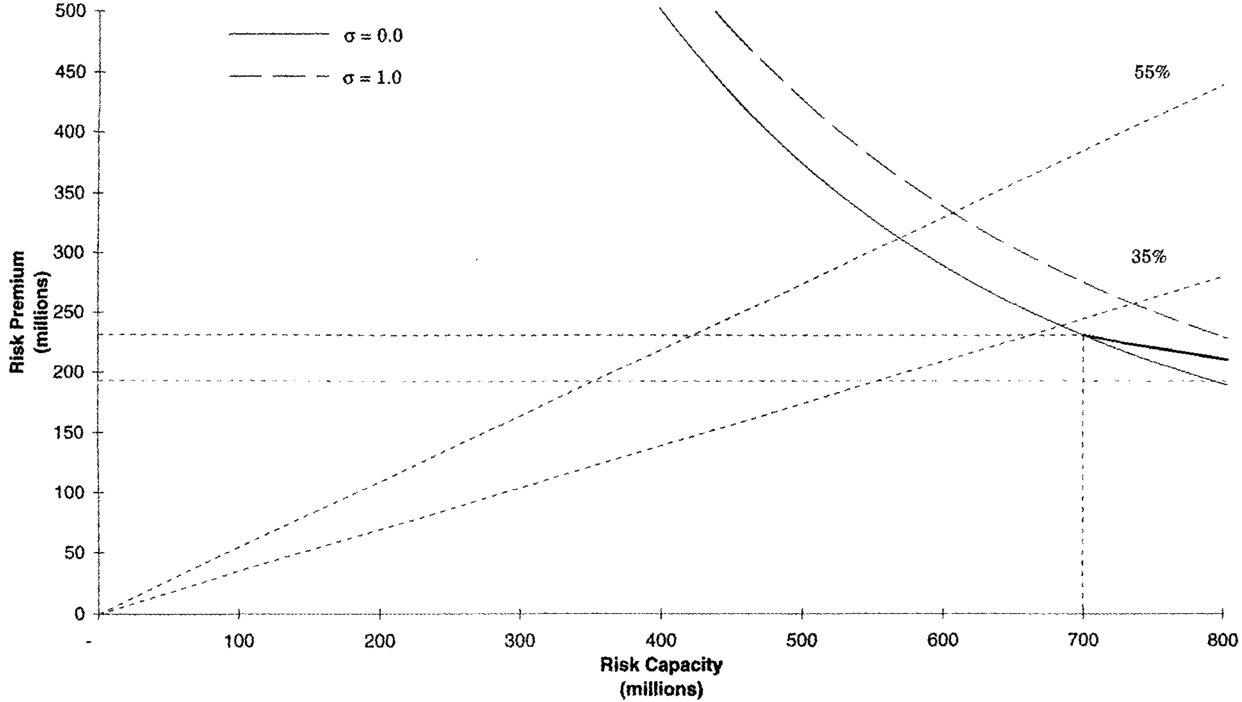

Figure 4 displays the effect a decrease in premium of about 15%, or $75 million, has on the risk premium and the implied capacity displayed in Figure 1. Since the ratio of observed risk premium to implied risk capacity is below the 35% to 55% interval, the risk/reward associated with this premium is lower than other comparable investments in the financial markets. This suggests that this premium is too low compared to the associated risk. This graph assumes an 80% participation in the CEA.

Figure 5: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $535 Million

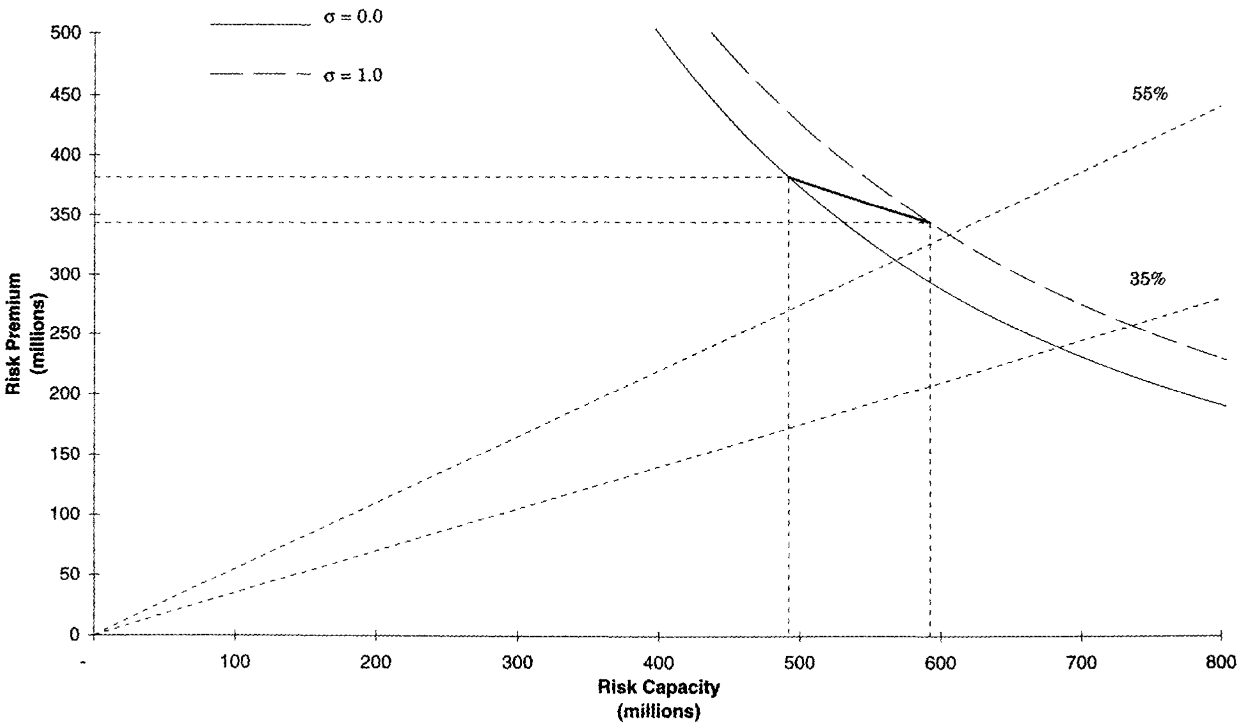

Figure 5 displays the effect a increase in premium of about 15%, or $75 million, has on the risk premium and the implied capacity displayed in Figure 1. Since the ratio of observed risk premium to implied risk capacity is above the 35% to 55% interval, the risk/reward associated with this premium is higher than other comparable investments in the financial markets. This suggests that this premium is too high compared to the associated risk. Th an 80% participation in the CEA.

Appendix 1: Simulation Assumptions

Number of simulations	10,000
Percentage Market Participation	80.0%
Layer Assumptions at 100% Participation (millions)	
Initial Capital Layer	1,000
1st IAL Layer	3,000
Reinsurance Layer	2,000
Bond Layer	1,000
Capital Market Layer	1,500
2nd IAL Layer	2,000
Premium Assumptions	
Total Annual Premium at 100% Participation	1,000
Annual Growth in Premium	0.0%
Expense Description	
Commissions Per Premium	10.0%
General Expenses Per Premium	5.0%
Interest Assumptions	
Yield on Cash and Investments	6%
Capital Markets Cat Notes Rate	8%
Capital Markets Term Notes Rate	11%
California State Bond Rate	5%

Exposure to losses rose proportionally in the first year from 0% at inception to 100% at the end of the first year.

Appendix 2: Risk-adjusted Value

We use a risk-adjusted value formula derived from the following set of axioms:

1. There is a frontier of opportunities that are optimal for the firm. The role of the decision-maker is to quantify and identify this frontier.

2. The decision-maker should be risk averse.

3. There are no riskless arbitrage opportunities. That is, the decision-maker would never pay more to avoid a loss than the amount of the loss. In turn, no individual would be able to sell insurance for a premium greater than the amount of the exposure.

4. The evaluation of an alternative is robust with respect to the input data. That is, a small change in an input parameter should not lead to a large change in the evaluation of an alternative.

5. The evaluation of an alternative is robust with respect to the analytical process one is using. For example, making small refinements to a particular scenario should not drastically change the evaluation of a particular alternative.

6. The evaluation of an alternative is robust to changes in the time scale. For example, changing the time intervals of the analysis from quarterly to monthly should not have a significant change in the evaluation of an alternative.

7. If there is no risk, one can determine the present value of a stream of future cash flows by discount factors derived from the term structure of interest rates.

These axioms imply that the firm, or decision-maker, must base his or her decisions using an exponential utility function. This conclusion does not say that either the individual making the decisions for the firm or the firm itself has an exponential utility function. It only says that if a firm would want to make decisions consistent with the above axioms then they must evaluate the alternatives assuming the firm has an exponential utility function.

This approach to evaluating risky investments satisfies the following properties:

1. $RAV_c(R) = R$, if R is known with certainty,

2. $RAV_c(R_1 + R_2) = RAV_c(R_1) + RAV_c(R_2)$, if R_1 and R_2 are independent,

3. $RAV_{kc}(kR) = k\, RAV_c(R)$,

4. $RAV_c(R)$ is equivalent to a variance load if R is distributed Normal,

5. $RAV_c(R) > E(R)$ for all c,

6. RAV_c is a decreasing function in c with $RAV_c(R) \to \infty$ as $c \to 0$ and $RAV_c(R) \to E(R)$ as $c \to \infty$.

One can use this theory to derive the risk-adjusted value corresponding to the CEA reinsurance layer. Let p_0 represent the probability of incurring no loss, p_1 represents the probability of incurring a total loss of \$1,600 million, and λ the parameter of the truncated exponential distribution. Then, the risk-adjusted value of the reinsurance layer is

$$RAV_c(R) = c\ln\left[E\left(e^{\frac{R}{c}}\right)\right], \text{ where } R \text{ is the ceded loss}$$

$$= c\ln\left[p_0 + p_1 e^{\frac{1600}{c}} + \left(1 - p_0 - p_1\right)\left(\frac{\lambda}{\lambda - c^{-1}}\right)\left(\frac{1 - e^{-1600\left(\lambda - c^{-1}\right)}}{1 - e^{-1600\lambda}}\right)\right]$$

Appendix 3: Modeling the Earthquake Loss Exposure

Simulated Earthquake Losses

The yearly earthquake loss, X, of the CEA simulation model is the product of two random variables:

$$X = YL \tag{1}$$

where Y is the random loss sampled from EQECAT earthquake loss estimates and L is a random variable representing uncertainty about the EQECAT estimates. We assume L to be log-normal with $\mu = 0$ and σ measuring the level of uncertainty. The mean increase in X over Y due to this uncertainty is

$$E(X) = E(Y)e^{\frac{1}{2}\sigma^2}$$

Reinsurer's Loss Distribution

There are three parts to the fitted model of earthquake loss to the reinsurer. First, we provided the distribution with point masses at the two end points of the loss exposure interval, namely R = \$0 and R = \$1,600 million. For losses between these two endpoints, we assumed a one parameter truncated exponential distribution. Table 3 shows the Kolmogorov-Smirnov goodness of fit statistics for four different fits of σ. Figures A-1 and A-2 plot the empirical and fitted distribution for $\sigma = 0.0$ and $\sigma = 1.0$ respectively. Although this study only discusses the results for two values of σ, we also performed these calculations on two other values of σ: $\sigma = 0.7$ and $\sigma = 0.9$. For $\sigma = 0.7$, $\sigma = 0.9$ and $\sigma = 1.0$, there is no evidence to suggest that there is any difference between the empirical distribution and the fitted distribution at the 5% significance level. For $\sigma = 0.0$, the fitted distribution is rejected at the 5% level. In Figure A-1, one can see that between losses of \$0 and \$700, the empirical distribution jumps up erratically four different times suggesting that there is no simple parametric distribution that fits this data. Since we wanted to keep the model simple, we used the truncated exponential distribution for all values of σ.

σ	Parameter Estimate	Kolmogorov-Smirnov statistic	5% critical value
0.0	1.63	0.0755	0.0477
0.7	1.34	0.0229	0.0483
0.9	1.15	0.0276	0.0490
1.0	1.07	0.0382	0.0498

Table 3: Fit of Truncated Exponential Distribution

Table 4 displays the fitted parameters of the ceded loss distribution using maximum likelihood approach. The fitted and simulated mean losses to the reinsurer, given that losses occurs, are very similar under all four scenarios.

σ	Probability of No Loss p_0	Probability of Complete Loss p_1	Estimate of Trunc. Exp. Parameter λ	Simulated Average Loss, Given Loss > \$0	Fitted Average Loss, Given Loss > \$0
0.0	85.3%	6.6%	1.632	1,043.60	1,043.29
0.7	85.1%	7.0%	1.340	1,084.24	1,084.20
0.9	84.2%	8.1%	1.148	1,137.15	1,137.14
1.0	83.7%	8.9%	1.073	1,171.52	1,171.52

Table 4: Estimated Parameters of Loss Distribution

Figure A-1: Fitted vs Simulated - Losses Between $0 and $1,600 Million
$\sigma = 0.0$

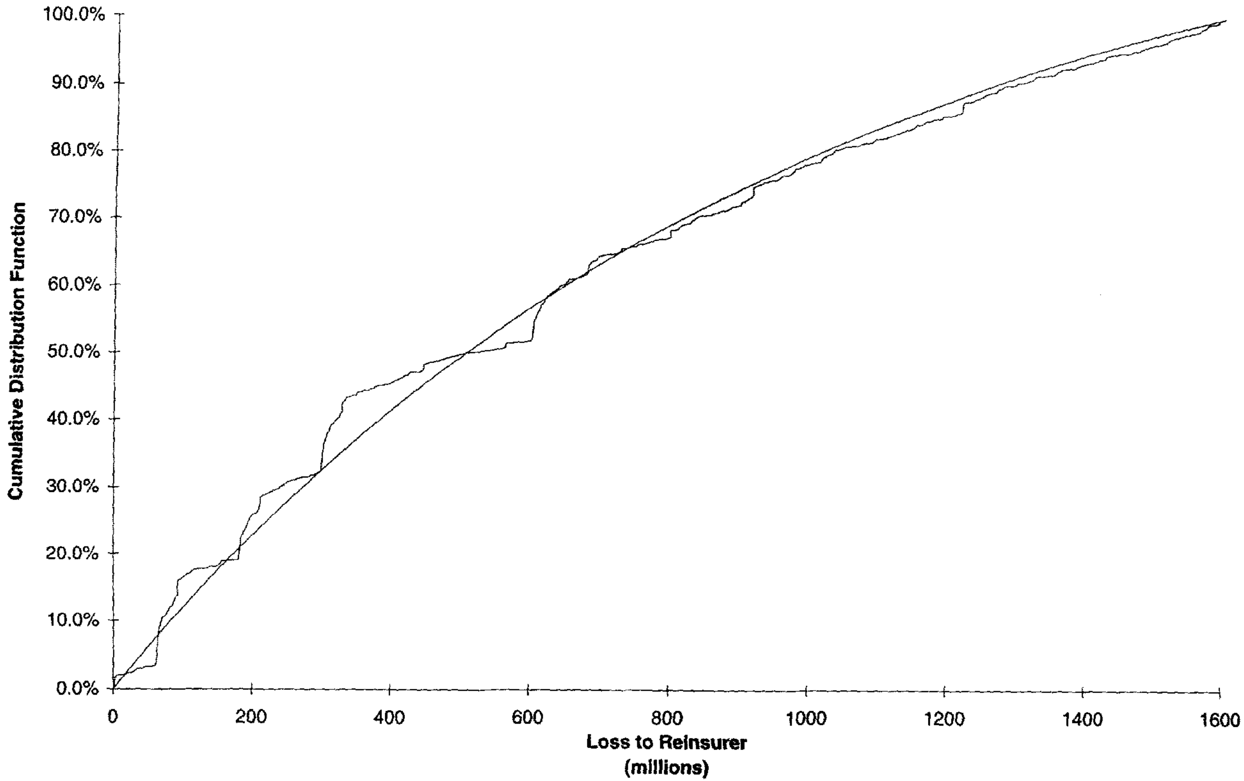

Figure A-1 plots the empirical and fitted ceded loss distribution for $\sigma = 0.0$. Using Kolmogorov-Smirnov goodness of fit test, this fitted distribution is rejected at the 5% significance level. One can see that between losses of $0 and $700, the empirical distribution jumps up erratically four different times suggesting that there is no simple parametric distribution that fits this data. This graph assumes an 80% participation in the CEA.

Figure A-2: Fitted vs Simulated - Losses Between \$0 and \$1,600 Million
σ = 1.0

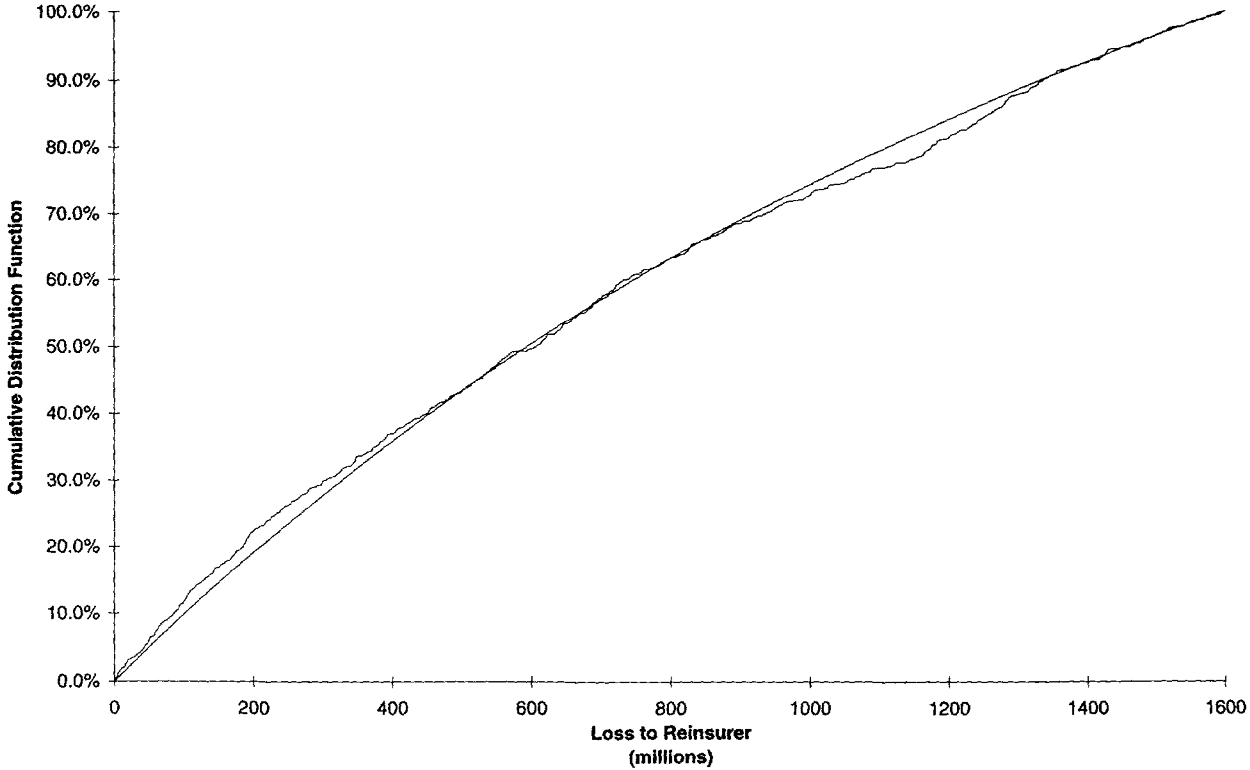

Figure A-2 plots the empirical and fitted ceded loss distribution for σ = 1.0. This fitted distribution cannot be rejected using Kolmogorov-Smirnov goodness of fit test at the 5% significance level. This graph assumes an 80% participation in the CEA.