

*A Stochastic Planning Model for the
Insurance Corporation of
British Columbia*

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Summary

A stochastic planning model is a representation to an appropriate level of detail of all of the cash flows of an insurance company, where the variables are stochastic (randomly generated). The variables are connected by simple econometric equations whose form and parameters are generated by the relevant underlying data. The main virtue of stochastic planning models is that all the probability levels, and not just the mean, are available for any financial variable. Such a model has been built for a large Canadian automobile carrier, with the primary application to surplus requirements under different management decisions.

Although it is considerably more complex than the spreadsheet approach to risk-based capital being proposed by the NAIC¹ (for reasons of simplicity), a stochastic model gives surplus requirements as a function of both risk appetite and management scenarios. The data and analysis requirements for a detailed model are substantial.

One of the by-products is a model of stochastic loss development involving accident period, development period, and payment period changes. Taken with the stochastic investment treatment and a projected zero future premium income, the run-off position variability can be quantified, i.e. the distribution of the adequacy of loss reserves can be ascertained.

Introduction

Both for proprietary reasons and because the data is specific to a very specialized situation, this paper for the most part discusses methodology rather than numerical results. Our intent is to encourage other actuaries in the creation and use of models such as this one.

What is a Stochastic Planning Model?

A stochastic planning model is a simulation model that represents to an appropriate level of detail all the cash flows of an insurance company. The variables are all stochastic (randomly generated). They are connected by

econometric equations generated by the relevant underlying data. The loss model here is Compound Poisson, with the severities being linked to the inflation rate. Claims are incurred on an underwriting period basis. The actual cash flow during each calendar period is a combination of stochastic runoff from all open underwriting periods, with calendar inflation of the severity distributions. Market share is represented by the total number of ultimate claims for each underwriting period. Investment yields and asset growth are correlated both to themselves at prior periods and to inflation. Expenses are also linked appropriately.

There are many places where management policies must be made explicit, e.g., in investment scenarios, in expense projections, and in the size of loads to be added to the pure premium to get the rates. The loads may depend upon management goals for cross-subsidization of lines, for market share, for profitability, for solvency, or for any other goal which is explicitly codifiable.

Any one realization is an explicit random choice of all the stochastic variables, moving forward through time to the desired horizon. The output from one realization is one complete delineation of all of the cash flows of the company under the given management policies. The output from the simulation of many realizations is a probability distribution for any financial quantity at any point in time. **The advantage of stochastic simulation is that the mean values are available, as in most planning models, and the probabilities of being far from the mean are also available.** The primary application is to surplus, for solvency testing, and the resulting distributions are a sophisticated way of doing risk-based capital. In particular, by setting future income and exposures to zero a distribution of reserve adequacy can be obtained which incorporates loss, investment, and expense variability. However, one should remember that not just surplus, but all income statement and balance sheet items are available.

What is the Insurance Corporation of British Columbia?

The Insurance Corporation of British Columbia (ICBC) is a virtually monopolistic carrier of automobile insurance in the Canadian province of British Columbia. Current premium volume is around two billion Canadian dollars annually. It is a Crown corporation that pays no taxes (other than premium taxes) or dividends. It is mandated to provide coverages at cost. It is not subject to regulatory scrutiny in the same manner as private insurance corporations.

At the same time, it has a need for surplus. Although in principle a deficit could be made up by increased taxation, this is politically unpalatable. On the other hand, a large surplus would not only be a tempting target for other uses, but could lead to charges that the rates had been too high, which is also undesirable. Thus, ICBC needed a way of quantifying a defensible surplus.

ICBC represents a particularly simple case for modeling. The market share is essentially constant, so no supply-demand curves or market elasticities need be created. There are no income taxes, so the US. complications of tax status, alternative minimum tax, and balancing between taxable and non-taxable investments need not be considered. There are no dividends or stockholders, so the usual management decisions with respect to investment analysts are irrelevant. The investments are conservative, mostly in British Columbia and Canadian government bonds. Since the writings are confined to one province, the British Columbia consumer price index provides the relevant inflation index.

However, even with all these simplifications, this turned out to be a major project requiring considerable data preparation by and consultation with the actuarial department of ICBC over a period of about a year and a half. Their chief actuary, Dave Lalonde, was instrumental in the creation of the model and in setting many of the key actuarial assumptions.

What are ICBC's uses for the model?

The original and major application for ICBC is to surplus under various management policies. For any given policy, the model gives the distribution of surplus at future times. Management's appetite for risk was stated as "What are acceptable probabilities that the surplus will be negative at the end of one year/five years?" Given those numbers, the first question was what initial surplus was necessary to obtain them with rate increases following projected claim costs. The second question was how the numbers changed if a previously announced set of future rate increases were followed independent of claim costs.

The third question was what impact various forms of reinsurance would have: *a priori*, reinsurance should decrease the negative variability of the results, thus increasing the probability of positive surplus, while at the same time having an average cost, which would reduce the mean surplus growth.

Other possible questions that may be investigated include the effects of expense savings and of re-aligning the investment strategy to accept more risk and more profit.

Annual Model

The actual model used for ICBC was a quarterly model. In the interests of clarity this discussion of an annual model is presented first as it contains all the key concepts. The modifications required for quarterly work along with some of the data are presented later. For any of these models, a salient requirement is

parsimony: the individual equations should be simple and intuitive so that the overall model with its complex behavior is believable. There should also be as few equations as possible. A corollary of this is that there is no point in trying to model a detail whose behavior is masked by the random noise created by other terms.

Overview

This model is based¹ primarily on the 1989 work of Pentikainen *et al.*² "Insurance Solvency and Financial Strength" and on the 1990 Daykin and Hey³ paper "Managing Uncertainty in a General Insurance Company". Losses, premiums, expenses, and investments are explicitly modeled. Both of these papers assume that the high end of the loss distribution is well-behaved, if necessary through appropriate reinsurance. One of the major tasks here was to obviate that assumption by treating the reinsurance and large claims explicitly, using parametrized distributions derived from the data.

For each time period (year, in an annual model), the evolution is modeled at three points. The first point is just after the start of the period when all of the last period data is known. At this point, projections of loss characteristics and market size for current underwriting are created. When combined with management-determined loads, the rates are generated. Premium dollars and market share of claims are returned by the market. Reinsurance premium is paid. The premiums and any investment yields are then invested in various assets, again determined by management goals. If asset allocations are to be re-balanced, this is when it happens.

The second point is at the middle of the period, when all losses and expenses are assumed to happen. Explicit realizations of all the stochastic variables are taken: first inflation, then loss payments, expenses, and investment yields and asset growth. If it is appropriate to re-evaluate any fundamental econometric parameters (such as the long-term inflation rate), this is the point. If assets need to be sold to raise additional cash, there is a liquidity penalty. Reinsurance recoveries are taken to be immediate, although a delay could be introduced.

Lastly, just before the end of the period the discounted and undiscounted reserves are calculated. Results of investment yields and asset growth are evaluated at this time. In the more general case, taxes and dividends would be paid. All income statement and balance sheet items are created.

¹ Subsequent to the writing of this paper, the book "Practical Risk Theory for Actuaries" by Daykin, Pentikainen, and Pesonen was published by Chapman and Hall (1994). This book contains an updating and summarization of all the early papers, and is strongly recommended.

The above is repeated for each period out to the chosen time horizon. It is typically suggested that five years is the most that can reasonably be chosen because of possible divergence of the extrapolation from current data. The whole process comprises one simulation. In order to have confidence in information above the 1% level, it is not unreasonable to use 20,000 to 100,000 simulations.

Inflation

The key external economic variable that connects to all the other variables through econometric equations is the inflation. In the ICBC case, the British Columbia consumer price index was used. The form suggested for the econometric equation for the inflation at time "t" is

$$\text{infl}[t] = \text{avg_infl} + \text{regress} * (\text{infl}[t-1] - \text{avg_infl}) + \text{uncertainty}$$

Long-term average inflation is **avg_infl**. This can be allowed to be a function of time in order to allow for updating of this parameter. However, it will be slowly changing since by definition it requires a number of years of data for evaluation. The autoregressive coefficient for inflation is **regress**, and **uncertainty** is a random term reflecting of the variability of the inflation.

If the term **regress** were zero, then inflation would be essentially a random walk about its long-term average. However, it is known that inflation is "sticky": when it is high at one time it tends to remain high and conversely for low values. The autoregressive term reflects this. The value of the coefficient should be below one for stability. The value of **regress** found on annual Canadian data is 0.69. The expected value for the inflation going forward from some fixed time approaches the average geometrically, although the actual values will only exhibit this behavior when the uncertainty is small compared to the other terms. In general, for the inflation and all other variables, the behavior going forward from $t=0$ will depend upon the parameters, on the immediately preceding (historic) values, and on the realizations of the uncertainties.

The values of the coefficients emerge from regression analysis on the data. In fact, the model structure - the number of lags involved and indeed the form of the equation itself - is given by appropriate analyses. However, it is to be emphasized again that simplicity and ease of interpretation are virtues not lightly to be dismissed in the choice of a model. It happens in the present case that the above form is not only simple, but also works well on the actual data.

Claims

The first problem is to define the claim lines. In the model these are the fundamental pricing units as well as the exposure and reserving bases. For

ICBC eight claim lines were used: Bodily Injury, Property Damage, Comprehensive, Death Benefits, Loss of Use and Collision, Medical Rehabilitation, Special Coverages, and Weekly Benefits. The information needed by the model is the frequencies, the severity distributions, and the runoff patterns for each line. These are needed both to run the histories forward from $t=0$ and to get the parameters for the distributions and econometric equations.

The underwriting for one period consists of receiving, by line, premium and an exposure (market share) in the form of an ultimate number of claims. This version of the model takes the claims to have one payment and to be closed when paid. One could possibly treat partial payments as separate claims. The number of claims closed from each underwriting period during each calendar period is given by a Poisson draw on the number expected; the latter is determined as the exposure times the appropriate element of the runoff pattern times a stochastic structure functionⁱⁱ.

For each claim closed in each line, a random draw is made from a severity distribution that is inflating with calendar time. The claim inflation by line is the overall inflation plus a line dependent claim excess and a stochastic term. The claim excess is the average amount by which the claim inflation exceeds the overall inflation. As usual, the parameter values come from the actual data. Thus, during each calendar period, for each line, each open underwriting period contributes a stochastic number of claims which have severities drawn from a distribution whose mean inflates in a stochastic manner with payment time.

It is perhaps worthy of mention why payment date inflation is relevant. When, for example, auto parts are bought, it is the current price *at the time of purchase* which governs, independent of how much earlier the actual accident occurred. This economically very reasonable statement is equivalent to payment quarter inflation. If inflation is constant- which the data on the Exhibit 1 indicates is certainly not true - then payment quarter inflation is equivalent to accident quarter inflation.

This procedure implies that the distribution for claims from a given underwriting period that pay late has the same shape (but different mean) as that for claims that pay early. This is probably not true, but perhaps more true in automobile than in, say, general liability. Given sufficient data and motivation, one could construct distributions with a severity shape dependent upon lag and incorporate them into this type of model.

ⁱⁱ The latter term is used to account for some correlation and/or parameter variation, which was otherwise not treated in this early model.

The principal computational difficulty in the model is the creation of an algorithm to evaluate all the claim payments in less than real timeⁱⁱⁱ. A workable solution is to separate the Compound Poisson (for a given severity distribution and number of claims) into two groups based on a cutoff value. The severity distribution then becomes two distributions: one limited by the cutoff and one above the cutoff. The compound Poisson process becomes a sum of two compound Poisson processes, with the expected number of claims in each part proportional to the probabilities above and below the cutoff value in the original severity distribution.⁴

The claims above the cutoff are simulated individually. This makes it possible to model excess reinsurance coverage explicitly, with or without aggregate deductibles and/or aggregate limits. In principle, any conceivable reinsurance arrangement can be modeled. The claims below the cutoff are evaluated by calculating the first three moments of the aggregate distribution and constructing the corresponding three-parameter gamma distribution. The stochastic value of the aggregate is then chosen as a random gamma deviate.

The lower the cutoff value is taken the more accurate this procedure becomes but also the more claims have to be simulated individually. In order to check out different possibilities, a test bed was constructed using some five billion simulations of individual claims. This gave us an "exact" cumulative distribution function against which individual approximations were then tested. We found that for our distributions, reducing the cutoff to a point where the skewness of the aggregate was less than 0.3 produced an acceptable compromise between accuracy and speed^{iv}.

Expenses

Expenses are taken to be those proportional to premium, those proportional to the number of claims, those proportional to size of claims, and those independent of premium and claims. In the ICBC case, ALE was included with loss, and the premium-based expenses were the taxes and agents' commissions. All other expenses were combined and projected from past history including a stochastic term. Management objectives for future expense reduction were included in the projection. Fixed expenses were allocated to claim line by a fixed ratio.

Investments

ⁱⁱⁱ That is, in order to be useful a model with a projection horizon of five years should not take five years to run. In fact, if the runs are longer than a few days the model becomes very difficult to use.

^{iv} In particular, the error in the approximation was less than 0.5% everywhere in the distribution.

ICBC invests principally in provincial and Canadian government bonds, with some commercial investment-grade bonds and a tiny stock portfolio. The assets were taken as three kinds: cash, bonds, and stocks. Except for the stocks, the investment yield and asset growth indices used track well with the overall inflation. Investment transaction costs were negligible compared to the variability of returns and were ignored. It was also assumed that all instruments were available at any time and that there was no liquidity problem, as outlined below.

Investment performance is split up into two components - cash return (dividends) and asset growth. For both of these, parsimonious equations are defined that link the current value to past investment returns and to the current and past realizations of inflation. The Wilkie⁵ approach was used, although further work could involve the use of dynamic investment models.

The investment strategy was taken as keeping a fixed ratio⁶ of different classes of assets. No attention was paid to asset-liability matching. If this were desired, then the investments would need to be broken out more finely, including the durations.

If, during a particular realization, the dollar outflow from losses and expenses exceeds the cash available, then other assets are converted to cash to cover the shortfall. No liquidity penalty except for the loss of interest is incurred - that is, there is no loss of value assumed for a forced sale of assets.

Reserves

There are many methods for calculating reserves. This model uses the expected value of the discounted and undiscounted cash flows from the exposures, which is in line with using the cash flows as our primary variables. As mentioned earlier, over time the expected inflation geometrically approaches the long-term average. It is this series of values that is used in estimating the values of the future payments that make up the reserves.

Clearly, the model can be run with no future premium income or exposures, and no assets other than those which make up the reserves. In that case, the distribution of the surplus at the end of the runoff is the distribution of the reserve adequacy. This distribution includes the variability due to losses, investments, and expenses. The model can be run to ascertain how much additional surplus is necessary to achieve any desired probability of non-negative final surplus. Additionally, other reserving methodologies could be explored⁷

The discounted reserves are calculated based on the expected present value with no risk loading. The discount rate is tied to the expected future yield for cash, although any type of method could be used. Perhaps a better alternative would be to tie the yield into the current investment portfolio.

Other Income/Payments

The model also includes income and payments difficult to associate with losses, investments, or expenses. In particular, there are revenues from late fees and penalties. There are also bulk payments relating to the use of ambulance services and hospitals. Both of these cash flows were modeled as linear projections with uncertainty. Both of these were relatively small compared to the large components, but ICBC felt they warranted specific attention.

Quarterly Model

The previous discussion was based on an annual model, as were the European models referenced earlier. ICBC required a more detailed model, one which would identify key seasonal differences throughout the year. It was therefore decided to construct a quarterly model.

In analyzing the data there were clear seasonalities, and therefore even a parsimonious model had to be more complex than described previously. A most interesting result arose in that the severity data exhibited not only accident quarter seasonality, as would be expected, but also strong payment quarter seasonality. That is, regardless of when the claim occurred, there was seasonality according to when it was paid. This apparent mystery was resolved by the ICBC actuary who remarked on the effect of summer vacations taken by people in the legal system.

The description of the quarterly model will parallel the annual. The general commentary is the same and will not be repeated, but only the specifics to the actual equations used.

Inflation

Exhibit 1 shows the British Columbia quarterly CPI change. The average of this data over the period 1968-1992 was used for long-term quarterly mean inflation. For the equation, intuition suggests that there should be a similar form to the annual model: average inflation plus some autoregression. In particular, correlation with the preceding quarter and the preceding year are appealing.

Piecewise linear regression was used to examine correlations between the lags; auspiciously enough, the statistically favored equation was the one suggested by intuition, with lag one and lag four. Exhibit 2 shows the standardized residuals, along with the actual values of the coefficients, and Exhibit 3 shows the check of residuals for Normality. Similar work was prepared for all the appropriate econometric equations.

Claims

The claims data has three pieces: the claim frequencies, the shape of the claim severity distributions, and the runoff and payment quarter severities. The frequencies were obtained by projecting the total number of claim counts to ultimate after extracting the accident quarter seasonality in the data. Fortunately, for this book of business the IBNR counts die out rapidly and we had mostly reliable data for ten years.

The severity distributions by line were fit by using a parametrized distribution—often lognormal—above some demarcation point in the data, and using the empirical moments below. The data we had were binned (numbers of losses in size classes rather than individual losses), so that in order to get the first three moments from the low-end empirical distribution we had to approximate the claims as all at the class midpoint. The error introduced by this becomes more severe as the demarcation point gets larger, so there is a compromise between the goodness of the fit (which typically gets worse as the demarcation point lowers) and the error from the empirical data. It is also necessary to be aware of the necessity of keeping the high-end tail correct, as this is where a substantial fraction of the dollars are. This latter consideration is perhaps the most significant, especially when reinsurance is involved.

As a technical note, although the usual tendency is to use a Chi-squared test on binned data, such tests are notoriously sensitive to the tails and should not be used when the expected frequency is small.⁸ In order to create bins with enough points in them to be usable, much of the top end would be collapsed into a single bin, losing a great deal of information. A way around this difficulty is to use a maximum likelihood solution, evaluating the differences of the parametrized CDF at the end points of the bins. This also allows for an infinite end point, such as values listed only as "greater than \$5,000,000." The price paid is the technical difficulty of actually doing the calculation and minimizing the negative log-likelihood.

The third piece, for the runoff and payment quarter severities, is one of the more interesting calculations. Each line followed the same analysis: First an accident quarter triangle of incremental quarterly payments was converted into a partial severity triangle by dividing by the appropriate ultimate claim count. Each

payment cell represents the product of (1) an accident quarter seasonality, (2) a lag quarter runoff, and (3) a payment quarter mean severity. A least-squares fit to the data was done numerically, which involved many variables. For example, in Property Damage the fit to the data had three accident quarter variables (the four seasonalities are constrained), 28 development quarter variables, and 39 payment quarter variables (there was a data glitch), for a 70-variable minimization! Fortunately, the usual accident quarter runoff gives a good starting point. It was found that whereas the simplex minimization was unusably slow, a less robust but more sophisticated technique based on Powell's conjugate direction set method worked quite well.⁹

The result of the minimization is the accident quarter seasonalities, the runoff factors, and the payment quarter severities. The surprising result alluded to earlier was the presence of seasonality in the payment quarters. When this was removed, the comparison to the historical inflation was made to get the claim excess. Typically, the size of the uncertainty was large compared to the average value of the excess.

Reversing the process for the simulation, the econometric equations and their realizations are on the de-seasonalized data. The consequent realization of the mean severity results from the blending of inflation, accident quarter seasonality, and payment quarter seasonality. Mean frequency is similarly a combination of trend, runoff pattern and accident quarter seasonality.

Premiums

Since the policies were assumed to be annual, as most actually are, there is a significant unearned premium reserve as well as the spread of counts to accident quarter. Although there is a notable fraction of the policies that incept on March 1, the model took uniform premium writings.

Expenses and Investments

The expenses were available only on an annual basis, so they were assumed uniform over the quarters. On the investment side, the quarterly econometric equations were not as clean and satisfying as for inflation. They did give statistically significant coefficients, which tended to be lags 1 and 3 or 4 on the variable, and the current and lag 1 values of the inflation.

Results

A sanitized version of the principal result can be seen on Exhibit 4, which shows the mean level and the 1% and 5% probability levels for surplus at different time horizons beginning at a given fixed initial value. For heuristic reasons, this same

exhibit shows in light tones a few of the simulations that underlie the statistical values. The whole calculation was repeated for several different starting points, and comparison of the results provided an answer to the question "What initial value of surplus is necessary in order that the probability of negative surplus at the end of one year is less than 1%?"

Implicit in the result of Exhibit 4 was that the rates were set to exactly cover costs. Thus, as claim costs rose in some realizations, so did the rates. Another question of interest was what would happen if a particular pre-determined set of rates were used. Exhibit 5 shows the values corresponding to Exhibit 4, but generated by this imposed rating structure. Even though the mean values are not too dramatically affected, the risk-significant levels are shifted considerably. For state regulators who want to fix rates or limit rate increases independent of costs, this kind of exhibit should provide food for thought on the increased risk of insolvency.

The third question of interest was the effect of reinsurance on the probability spread. Exhibit 6 shows the effect of reinsurance over a fixed \$100,000 excess. The mean line is dropped due to the (on average) cost of reinsurance, but the low percentile levels are significantly raised because the down-side of the loss is considerably restricted. This is equivalent to requiring less initial surplus for a given risk appetite.

Conclusion

The results and further use of the model allow ICBC management to know the risk being assumed under different scenarios and their trade-offs. Prepared with this information and knowledge of external factors, ICBC can take an optimum stance for its risk appetite.

Their optimum surplus value is a dynamic number, depending on risk appetite, rate constraints, investment profiles, developing trends in losses, expenses, and investments, and reinsurance arrangements. In order that it best reflect current reality, the model should be retested in its assumptions and re-parametrized on an annual basis, as well as having the input data updated quarterly. Each quarter, out of all the possible simulation paths implicit in the model, one will be selected by actuality. The management reaction to the situation may require a change in the assumptions of the model, as some of them directly reflect management decisions.

Since the model is a complete but simple version of all the cash flows of ICBC, it can be used to analyze many other questions besides the appropriate value of surplus. For example, it can be used as an "early warning system", in that it highlights potential areas that are likely to have large swings and suggests

careful monitoring of these areas. Control engineers often define a band of variability inside of which a process is considered in control, and outside of which action needs to be taken to restore balance. Whereas the claims department is (properly) focused on its payouts, and the investment department on its returns, senior management needs to know each of these and the overall result. It needs to have control bands at all levels.

The model can also be used as a planning tool to explore the consequences of other decisions, such as changing the target investment mix or introducing a new product line. In the latter case, however, one must be extremely careful with the actuarial assumptions involved for the line if they are not supported by comparable data from elsewhere.

For the actuary, the important conclusion is that a model of this type can be built, and it provides a useful tool for management planning and control. For any individual company, the specifics of the claim lines, investments, etc. that are considered will depend upon the company's current environment and plans for the future. One of the areas left untouched here is the whole question of what happens in a competitive environment as rates move. Classically, this means the creation of the supply and demand curves for the company's products. The forthcoming book by Pentikainen¹⁰ has a treatment of this and other approaches to competition.

¹As of this writing (4/93), the final version has not emerged. For a preliminary version, see "Property-Casualty Risk-Based Capital Requirement - A Conceptual Framework" in the Spring 1992 edition of the Casualty Actuarial Society Forum.

²T. Pentikainen, H. Bonsdorff, M. Pesonen, J. Rantala, M. Ruchonon, Insurance Solvency and Financial Strength, Finnish Insurance Training and Publishing Company Ltd., Helsinki, 1989

³C.D. Daykin and G.B. Hey, "Managing Uncertainty in a General Insurance Company", in Journal of the Institute of Actuaries, Vol. 117, Part II, pp. 173ff.

⁴Newton L. Bowers, Hans U. Gerber, James C. Hickman, Donald A. Jones and Cecil J. Nesbitt, Risk Theory, Section 11.4.

⁵A.D. Wilkie, "A stochastic investment model for actuarial use", in Transactions of the Faculty of Actuaries, #39, 1986, pp.341-373.

⁶ This is the approach that has been taken by ICBC. A discussion of rebalancing is given in A.J. Wise, "The Investment Return from a Constantly Rebalanced Asset Mix" in the 3rd AFIR Colloquium, 1993 pp. 349-358.

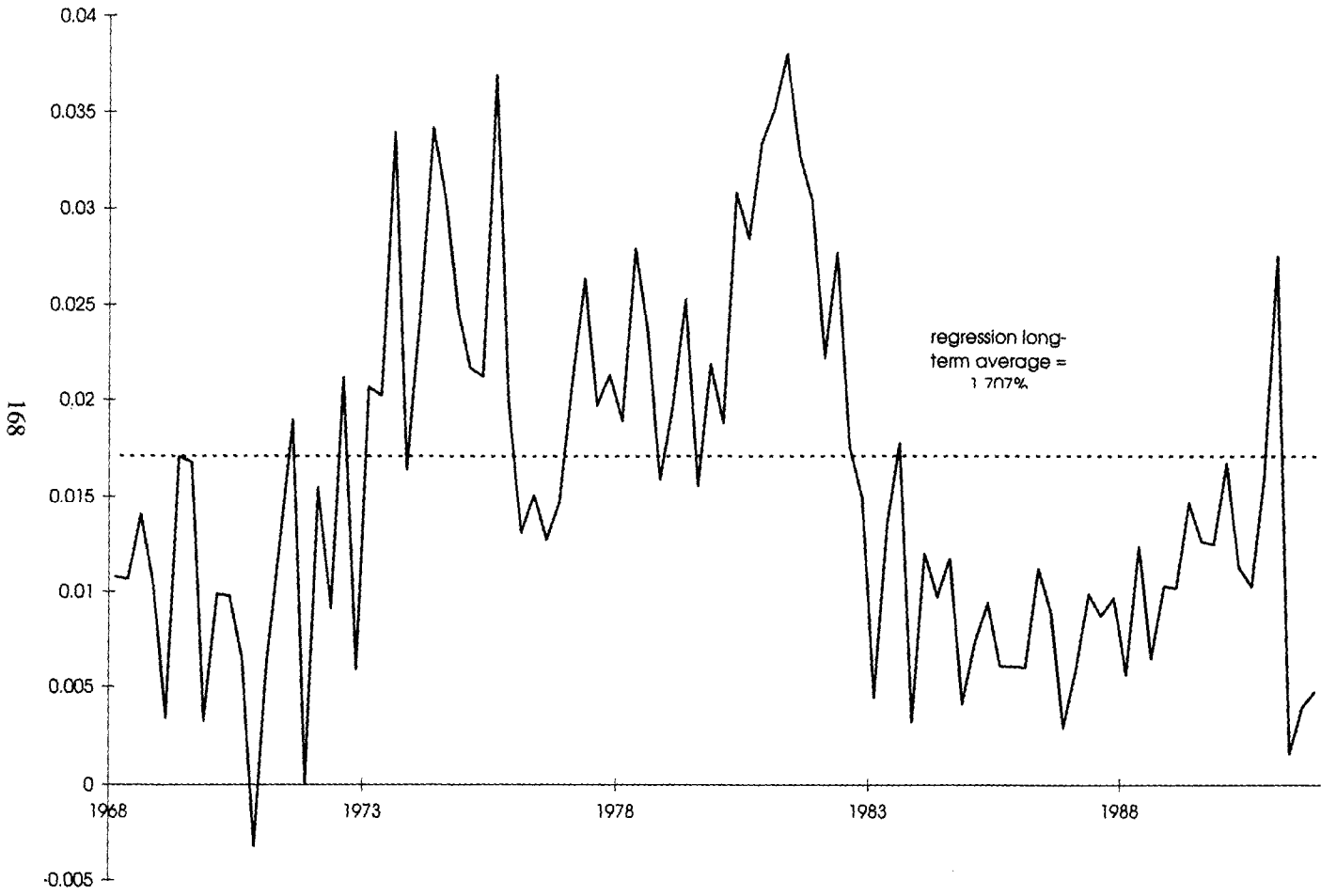
⁷ See for example T. Pentikainen and J. Rantala, "A simulation Procedure for Comparing Different Claims Reserving Methods" in the Astin Bulletin, Volume 22, No. 2 (November 1992) p. 191.

⁸ Robert V. Hogg, Allen T. Craig, Introduction to Mathematical Statistics, Macmillan Publishing Co., Inc., 1990, pages 269ff.

⁹ See for example Press, Flannery, Teukolsky and Vetterling Numerical recipes in C - The Art of Scientific Computing, Cambridge University Press, 1988 p. 309ff.

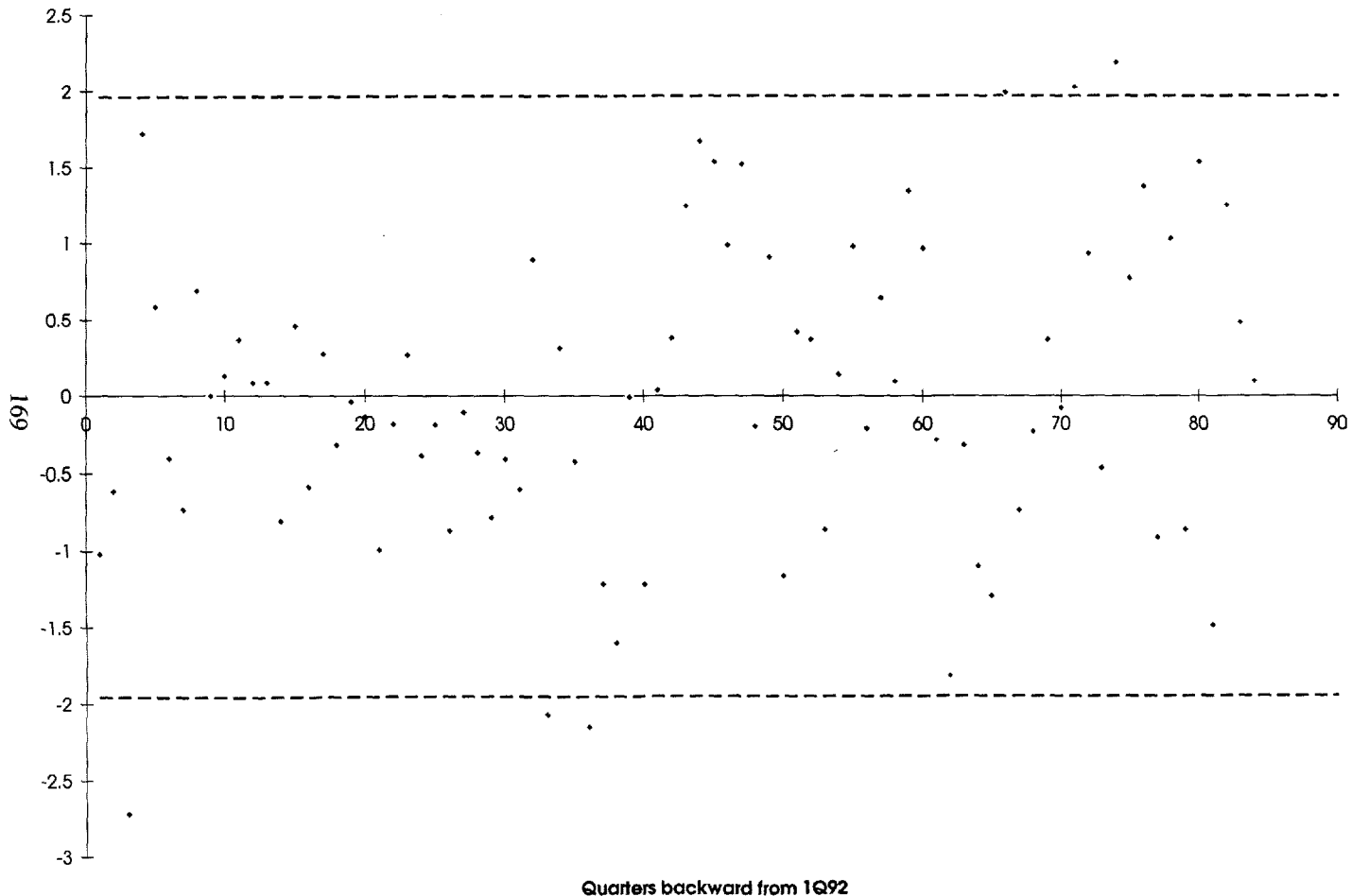
¹⁰ T. Pentikainen, private communication at the 3rd AFIR International Colloquium.

British Columbia Composite Quarterly CPI Change



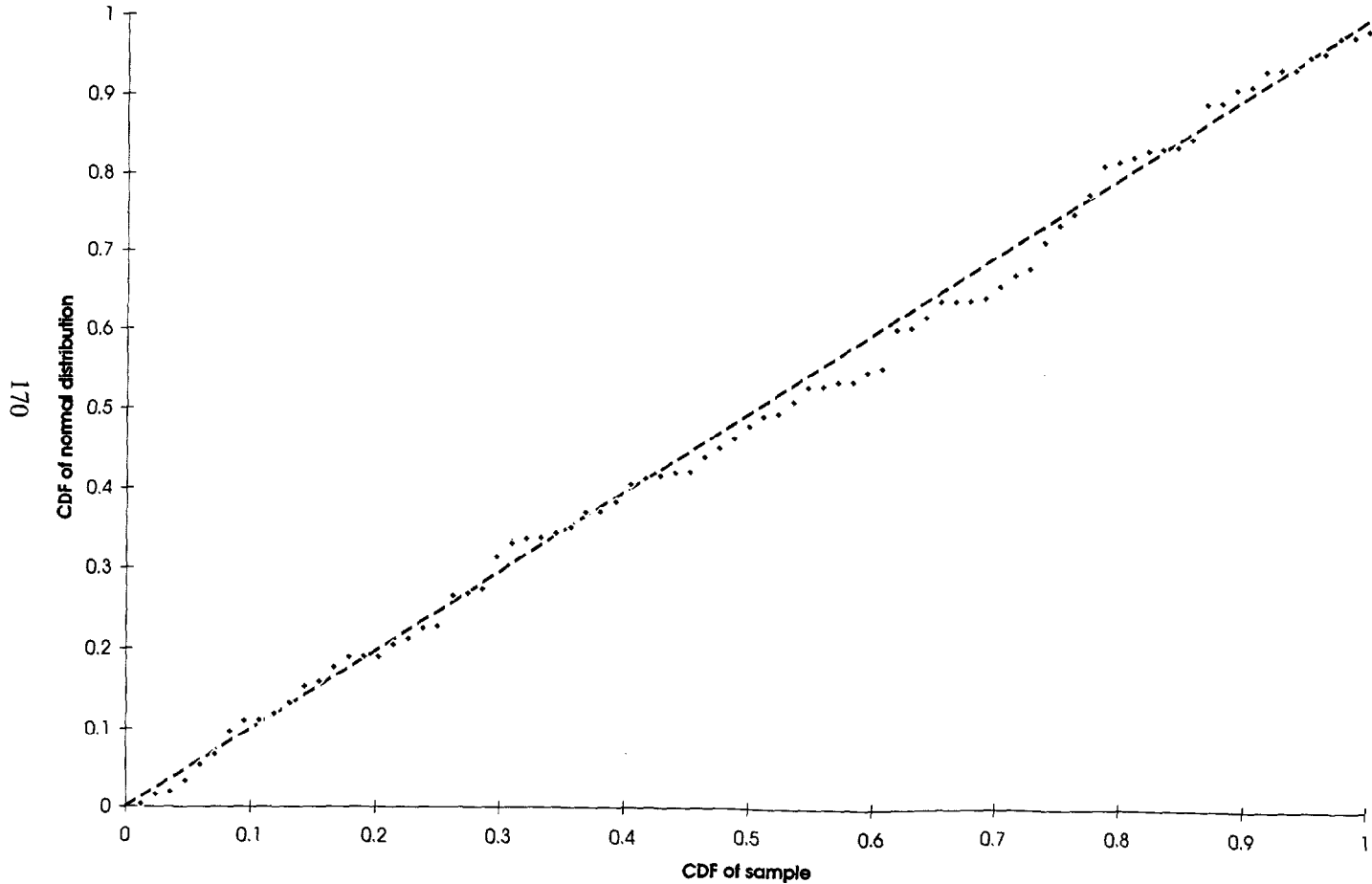
B.C. CPI Change Standardized Residuals from regression

long-term mean = 1.707%; autoregressive Lag 1 = 41.0% and Lag 4 = 40.9%; deviation = 0.637%



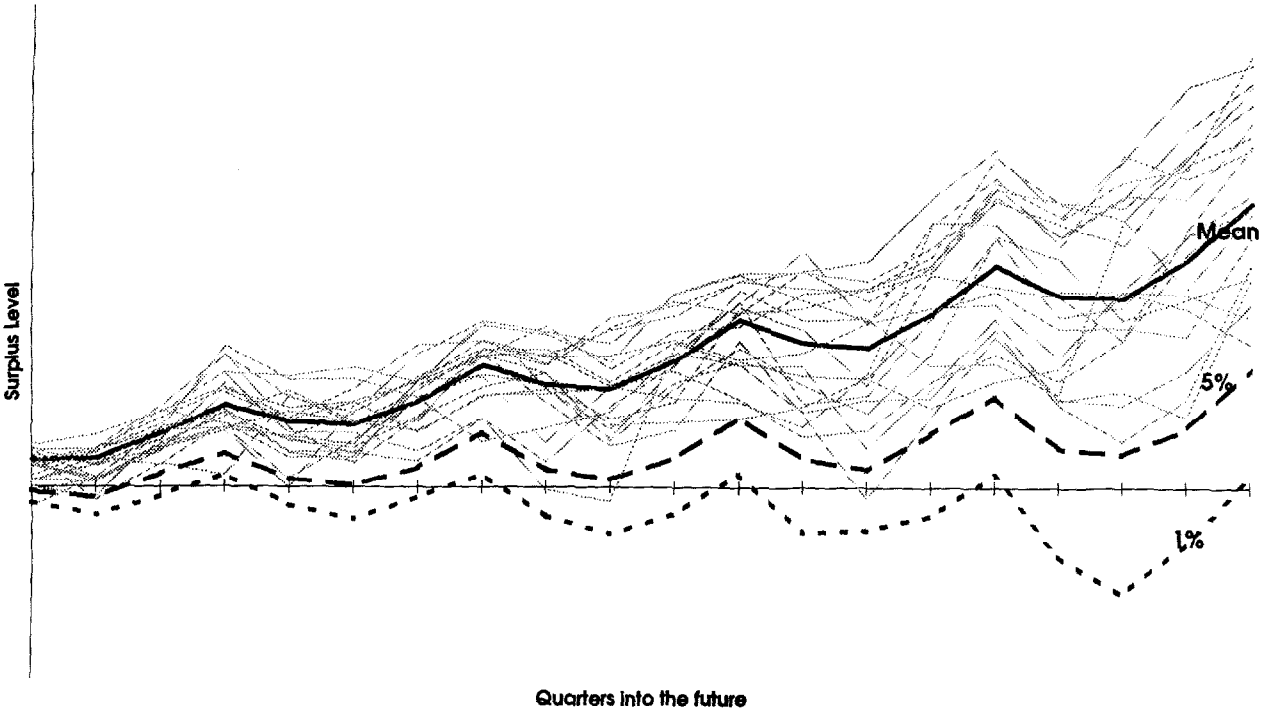
Regression on B.C. CPI Change
Check of residuals for normality

Exhibit 3

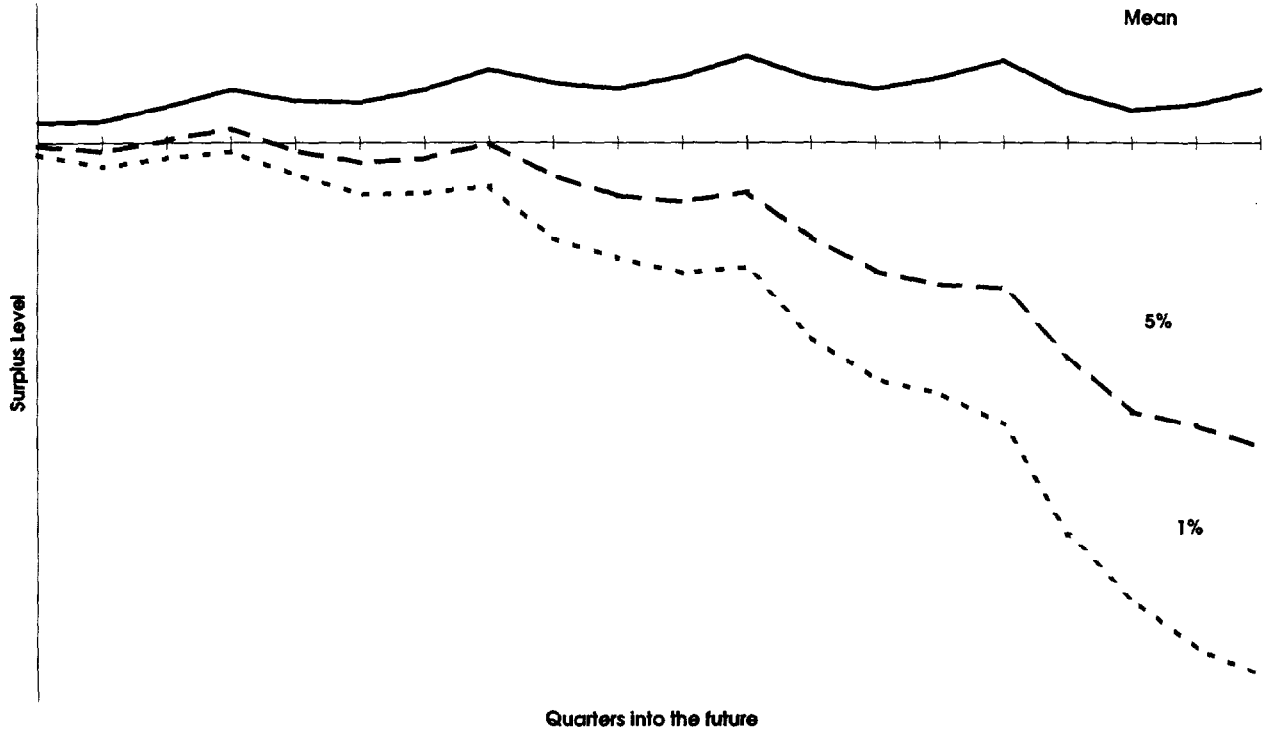


Surplus by Quarter for rates following costs

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Surplus by Quarter
for fixed rates



Surplus by Quarter for fixed rates with and without reinsurance

