

*A Model for Reserving Workers  
Compensation High Deductibles*  
Jerome J. Siewert, FCAS

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## Abstract

Several approaches for estimating liabilities under a high deductible program are described. Included is a proposal for a more sophisticated approach relying upon a loss distribution model. Additionally, the discussion addresses several related issues dealing with deductible size and mix, absence of long-term histories, as well as the determination of consistent loss development factors among deductible limits. Lastly, approaches are proposed for estimating aggregate loss limit charges, if any, and the asset value for associated servicing revenue.

## Biography

Jerry Siewert is an Assistant Vice President and Actuary with Wausau Insurance Companies. He currently manages the reserving unit of the actuarial department. His previous experience includes managing several pricing units and serving on various industry ratemaking committees. Prior to joining Wausau, he taught mathematics for four years at the secondary level.

Jerry is a Member of the American Academy of Actuaries and became a Fellow of the Casualty Actuarial Society in 1985. He graduated from the University of Wisconsin - Stevens Point (1972) earning bachelor degrees in both mathematics and psychology.

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### 1. Abstract

Several approaches for estimating liabilities under a high deductible program are described. Included is a proposal for a more sophisticated approach relying upon a loss distribution model. Additionally, the discussion addresses several related issues dealing with deductible size and mix, absence of long-term histories, as well as the determination of consistent loss development factors among deductible limits. Lastly, approaches are proposed for estimating aggregate loss limit charges, if any, and the asset value for associated servicing revenue.

### 2. Introduction

With the advent of the high deductible program in the early '90s, actuarial efforts focused principally on pricing issues. Insurers initially developed this program to provide both themselves and insureds many advantages, including:

1. achieving price flexibility while passing additional risk to larger insureds in what was considered at that time an unprofitable line of business,
2. ameliorating onerous residual market charges and premium taxes in some states,
3. realizing cash flow advantages similar to those of the closely related product - the paid loss retro,
4. providing insureds with another vehicle to control losses while protecting them against random, large losses, and
5. allowing "self-insurance" without submitting insureds to sometimes demanding state requirements.

Now as the program matures, the focus shifts to the liability side. Questions are being asked as to what losses are actually emerging and, more importantly, what will they ultimately cost insurers. For the actuary, the question is how best to estimate these liabilities when losses are not expected to emerge above deductible limits for many years. Many issues need to be addressed:

1. In the absence of long-term development histories under a deductible program, how can the actuary construct reasonable development factors?
2. How can the actuary determine development patterns that reflect the diversity of both deductible size and mix?
3. How should the actuary determine consistent development factors between limited and excess values?
4. What is a reasonable approach for the indexing of deductible limits over time?
5. How can the actuary estimate the liability associated with aggregate loss limits, if any?

6. Is there a sound way to determine the proper asset value for associated service revenue?<sup>1</sup>

In the remainder of this paper I describe possible approaches dealing with those issues.

### 3. Development Approaches

#### *Overview*

The development approach presented relies heavily upon my company's extensive history of full coverage workers compensation claim experience. In effect, I create deductible/excess development patterns as needed. Of course, this approach poses problems if credible histories of full coverage losses are not readily available.

Once I establish the appropriate development factors, I apply them at the account level and determine the overall aggregate reserve by summarizing estimated ultimates for each account. I argue this is a reasonable approach, if you view each account as belonging to a cohort of policies with similar limit characteristics. Determining the overall reserve in such a fashion allows me to address the issue of deductible mix by reflecting each account's unique limits.

Later I describe the possible use of a loss distribution model to enforce consistent results between deductible/excess development factors. Once the parameters of the distribution are set, it is possible to determine development factors, as needed, for any deductible size. Perhaps, the use of such a model may even provide an alternative approach for determining tail factors through the projection of the distribution parameters.

#### *Loss Ratio*

In the absence of credible development histories, a common approach for determining liabilities is to apply loss ratios to premiums arising from the exposures. Historically, as that element was required to first price the product, loss ratios for the various accounts written should be readily available. For immature years, where data is sparse, applying loss ratios is probably the most practical approach to take. Given the long-tailed nature of this business, actual experience over deductible limits emerges slowly over time. Also the expected experience is readily converted to an accident year basis based upon a pro rata earnings of the policy year exposures.

The loss ratio approach requires a database of individual accounts and pricing elements. The database should include an estimate of the full coverage loss ratio. From a pricing standpoint, that number can come from a variety of sources. One approach would be to use company experience by state, reflecting the individual account's premium distribution. Possibly, that experience to the extent credible could be blended with industry experience. As with other

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<sup>1</sup> Similar in usage to a loss conversion factor in retro rating, loss multipliers are applied to deductible losses to capture expenses that vary with loss.

pricing efforts, that experience ought to be developed to ultimate, brought on level, and trended to the appropriate exposure period.

Besides an estimate of the full coverage loss ratio, the database should include estimates of excess losses for both occurrence and aggregate limits. For the occurrence limit, several approaches are possible including estimating excess ratios based upon company experience. A potentially more credible approach uses excess loss pure premium ratios underlying industry-based excess loss factors used in retro rating. Besides their availability by multiple limits, excess loss factors can easily be adjusted to a loss basis and reflect hazard groups with differing severity potential. Utilizing account-based excess ratios reflecting unique state and hazard group characteristics should lead to reasonable estimates of per occurrence excess losses:

$$(3.1) \quad P \cdot E \cdot \chi$$

where P = premium, E = expected loss ratio, and  $\chi$  = per occurrence charge

Regarding the aggregate loss charge, if any, an approach I prefer uses a process similar to that for determining insurance charges in a retro rating program. Those charges would, in turn, rely on the National Council on Compensation Insurance's (NCCI) Table M. I refer the interested reader to the Retrospective Rating Plan [1] for further details. The process reflects the size of the account, deductible, state severity relativities, prospective rating period, and appropriate rating plan parameters:

$$(3.2) \quad P \cdot E \cdot (1 - \chi) \cdot \phi$$

where P = premium, E = expected loss ratio,  $\chi$  = per occurrence charge, and  $\phi$  = per aggregate charge

Applying this procedure to each account and aggregating leads to an estimate of ultimate accident year losses. I show in Table 1 a hypothetical case of how to apply those factors to determine the ultimate liabilities. Incurred But Not Reported (IBNR) amounts are easily determined by subtracting known losses from the ultimate estimate.

Again, this approach is particularly useful when no data is available or the data is so immature as to be virtually useless. Obviously, loss ratio estimates can be consistently tied to pricing programs, at least at the outset. This procedure also benefits from its reliance on a more credible pool of company and/or industry experience. On the negative side, a loss ratio approach ignores actual emerging experience, which in some circumstances may differ significantly from estimated ultimate losses. For this reason alone, the loss ratio approach is not particularly useful after several years of development. Another shortcoming of this method is that it may not properly reflect account characteristics, as development may emerge differently due to the exposures written.

**Table 1**  
**Countrywide Insurance Enterprise**  
*Account: Widget, Inc.*  
Expected Deductible/Aggregate Loss Charges

| (1)                 | (2)           | (3)         | (4)              | (5)             | (6)                          | (7)                     | (8)                         |
|---------------------|---------------|-------------|------------------|-----------------|------------------------------|-------------------------|-----------------------------|
|                     |               |             | (2) x (3)        |                 | (4) x (5)                    |                         | [(4) - (6)] x (7)           |
| State               | Premium       | ELR         | Expected<br>Loss | Excess<br>Ratio | Deductible<br>Loss<br>Charge | Aggre-<br>gate<br>Ratio | Aggregate<br>Loss<br>Charge |
| Arkansas            | 9,084         | .567        | 5,151            | .062            | 319                          | .02                     | 97                          |
| Illinois            | 573,066       | .532        | 304,871          | .105            | 32,011                       | .02                     | 5,457                       |
| Iowa                | 373,072       | .588        | 219,366          | .096            | 21,059                       | .02                     | 3,966                       |
| Kansas              | 70,549        | .644        | 45,434           | .071            | 3,226                        | .02                     | 844                         |
| Minnesota           | 1,012,622     | .457        | 462,768          | .143            | 66,176                       | .02                     | 7,932                       |
| South Carolina      | 22,980        | .522        | 11,996           | .048            | 576                          | .02                     | 228                         |
| <u>South Dakota</u> | <u>94,401</u> | <u>.697</u> | <u>65,797</u>    | <u>.211</u>     | <u>13,883</u>                | <u>.02</u>              | <u>1,038</u>                |
| Total               | 2,155,774     | .517        | 1,115,383        | .123            | 137,250                      | .02                     | 19,562                      |

*Implied Development*

There are many ways to incorporate actual emergence in high deductible reserve estimates. Determining excess development implicitly is one possibility. By implied development, I mean an approach that works as follows:

1. Develop full coverage losses to ultimate.
2. Next, develop deductible losses to ultimate by applying development factors reflecting various inflation indexed limits.
3. Finally, determine ultimate excess losses by differencing the full coverage ultimate losses and the limited ultimate losses.

A variety of the usual development techniques could be applied to determine full coverage losses. Those methods include paid and incurred techniques designed consistently with the company's reserving procedures for full coverage workers compensation. However, care should be exercised in determining a full coverage tail factor consistent with the limited loss tail factors. In particular, the actuary should avoid developing limited losses beyond unlimited losses, or even losses for lower limits beyond those of higher limits.

When calculating development factors for the various deductibles, it is appropriate to index the limits for inflationary effects. Adjusting the deductible by indexing keeps the proportion of deductible/excess losses constant about the limit from year to year, at least, in theory. For example, if inflationary forces drive claim costs ten percent higher each year, the percentage of losses over a \$100,000 deductible for one year equate to those of a \$110,000 deductible in the next. Indexing of deductible limits allows for the possibility of combining differing experience years in the determination of development factors.

There is really no set method for determining the indexing value. One approach would be to determine that index by fitting a line to average severities over a long-term history. Another simpler approach might be to use an index that reflects the movement in annual severity changes. In any event, the actuary needs to be cognizant that a constant deductible over time usually implies increasing excess losses.

An advantage of the implied development approach is that it provides an estimate of excess losses at early maturities even when excess losses have not emerged. Also, the development factors for limited losses are more stable than those determined for losses above the deductible. This procedure also provides an important byproduct in the estimation of assets under the high deductible program. Specifically, estimating deductible losses helps determine the asset represented by revenue collected from the application of a loss multiplier to future losses. Despite these advantages, this approach does appear to have its focus misplaced, as one would like to explicitly recognize excess loss development.

#### *Direct Development*

This approach explicitly focuses on excess development, though it relies upon development factors implicit from the previous technique. That is, given development factors for limited as well as full coverage losses, excess loss development factors are fixed. It is important to recognize here that excess development is part of overall development, and the actuary should strive to determine excess factors in conjunction with limited development factors that balance back to full coverage development. That is not to say that reserve indications from implicit and explicit methods necessarily will be the same, but only that the underlying loss development factors should be.

Again, a variety of paid and incurred techniques are applicable here. I see several disadvantages to directly determining excess development factors and applying them to excess losses. Those factors tend to be quite leveraged and extremely volatile, making them difficult to select. Additionally, if excess losses have not actually emerged at any particular stage of development, it is not possible to get an estimate of the required liability.

#### *Credibility Weighting Techniques/Bornhuetter-Ferguson*

Given the significant drawbacks mentioned for the previous approaches to determining excess liabilities for the deductible product, the next approach described offers greater promise. It relies on credibility weighting indications based upon actual experience with expected values, preferably based on pricing estimates. This method requires that the actuary determine a suitable set of weights or credibilities. The Bornhuetter-Ferguson [2] technique offers one possible approach for determining credibilities that are specified as reciprocals of loss development factors.

$$(3.3) \quad L = O_t \cdot LDF_t \cdot Z + E \cdot (1 - Z) \quad (\text{Credibility viewpoint})$$

where  $L$  = ultimate loss estimate,  $O_t$  = observed loss at time  $t$ ,  $LDF_t$  = age to ultimate development factor,  $Z$  = credibility, and  $E$  = expected ultimate loss

Letting  $Z = \frac{1}{LDF_t}$  leads to:

$$(3.4) \quad L = O_t + E \cdot \left( \frac{LDF_t - 1}{LDF_t} \right) \quad (\text{Bornhuetter-Ferguson viewpoint})$$

Using the Bornhuetter-Ferguson approach allows the actuary to determine liabilities either directly or indirectly. This procedure affords the ability to tie into pricing estimates for recent years where excess losses have yet to emerge. Also, it provides more stable estimates over time, rather than the volatility arising from erratic emergence or leveraged development factors. Hopefully, a credibility weighting approach like this provides better estimators of ultimate liabilities as well. Of course, a disadvantage of this technique is that it ignores actual experience to the extent of the complement of credibility. That drawback suggests finding alternative weights or credibilities that may be more responsive to the actual experience as desired.

#### 4. Development Model

This section deals more specifically with a number of the issues I described at the outset. How best can the actuary determine development factors in the absence of a long-term history under the deductible program? How can the actuary determine development patterns that reflect the diversity of both deductible size and mix? What is a reasonable approach for indexing deductible limits over time? How best should the process relate development for various limits consistently? Determining development factors for a high deductible program is really an exercise in partitioning development about the deductible limit. Is it possible to develop consistent tail factors among the limits the company is exposed to?

##### *Some Possible Approaches*

As I stated earlier, in the absence of long-term experience under the deductible program, I suggest making extensive use of a company's history of full coverage workers compensation claims, if available. It is also appropriate to apply an indexed limit to the claims in order to determine a series of accident year loss development histories by limit. In some of the analyses I performed, I looked at selected limits ranging from \$50,000 to \$1,000,000. I focused, however, on the more common deductible sizes in the neighborhood of \$250,000. I used case losses that included indemnity, medical, and any subject allocated claim expense. The histories I reviewed ran out for 25 years but were not further separated by account, injury, or state. That suggests eventually creating alternative development patterns that do reflect those types of break-out. I show in Table 2, age-to-age development factors by indexed limit resulting from my preliminary studies.

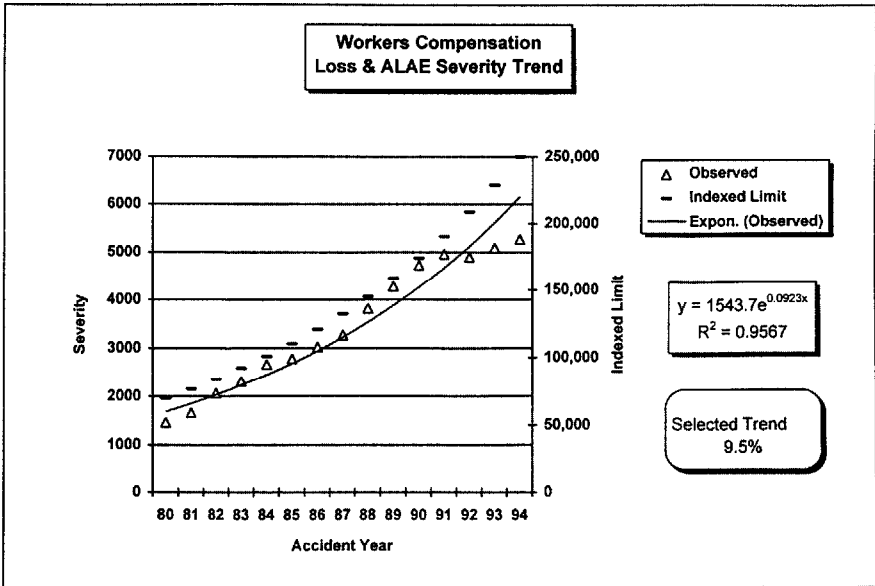


**Table 2**  
**Workers Compensation - High Deductibles**  
Limited Loss & ALAE Age-to-Age Development Factors  
*by Indexed Limit (Middle 6 of Last 8)*

| Limit       | 12:24 Months | 24:36 Months | 36:48 months | 48:60 Months | 60:72 Months |
|-------------|--------------|--------------|--------------|--------------|--------------|
| \$50,000    | 1.5031       | 1.0418       | 1.0038       | 1.0025       | 1.0020       |
| \$100,000   | 1.6225       | 1.0727       | 1.0151       | 1.0063       | 1.0080       |
| \$250,000   | 1.6791       | 1.1300       | 1.0451       | 1.0207       | 1.0060       |
| \$500,000   | 1.6827       | 1.1393       | 1.0684       | 1.0322       | 1.0170       |
| \$750,000   | 1.6816       | 1.1408       | 1.0720       | 1.0359       | 1.0214       |
| \$1,000,000 | 1.6811       | 1.1411       | 1.0728       | 1.0371       | 1.0229       |
| Unlimited   | 1.6876       | 1.1430       | 1.0749       | 1.0391       | 1.0196       |

In order to determine those development factors, I combined several years of experience based upon indexed limits. For example, for the most recent year, limits were used as stated. But for the first prior year, I adjusted limits downward by an indexing factor of 1.095. For the current year, I assumed a limit of \$250,000 was the equivalent of a limit of \$228,311 for the first prior year. Each limit was adjusted by the same index, back for as many years as needed, to generate the desired development factors.

**Chart 1**



I simply based the selected indexing factor of 1.095 upon a long-term severity history. As I alluded to earlier, other approaches may be better. Possibly varying the indexing factor by year or adjusting for the distorting effects of larger claims are but a couple of examples of improvements that could be explored. I show in Chart 1 the exponential trend line fit through known data points determining the long-term indexing factor of 1.095. Also depicted is the indexed \$250,000 loss limit.

The approach I recommend requires separating claim count development from severity development. In my work to date I focused on the counts for full coverage losses rather than worrying about emergence of claims over a specific deductible limit. I feel it is much easier to recognize development in this fashion, as there is generally very little true claim count IBNR after about three years. This is true even for the larger claims, as they will be reported early on just like the other claims, but their true severity will not be known for some time.

**Table 3**  
**Workers Compensation**  
Age-to-Age Development Factors  
*Full Coverage Claim Count*

| <u>Accident Year</u> | <u>12:24 Months</u> | <u>24:36 Months</u> | <u>36:48 months</u> | <u>48:60 Months</u> |
|----------------------|---------------------|---------------------|---------------------|---------------------|
| 1988                 | -                   | -                   | -                   | 0.9999              |
| 1989                 | -                   | -                   | 0.9999              | 0.9994              |
| 1990                 | -                   | 1.0026              | 0.9999              | 1.0001              |
| 1991                 | 1.1111              | 1.0022              | 1.0002              | -                   |
| 1992                 | 1.1305              | 1.0017              | -                   | -                   |
| 1993                 | 1.1283              | -                   | -                   | -                   |
| Last 3               | 1.1233              | 1.0022              | 1.0000              | 0.9998              |
| Selected             | 1.1250              | 1.0025              | 1.0000              | 1.0000              |
| Age to Ultimate      | 1.1278              | 1.0025              | 1.0000              | 1.0000              |

In order to handle the issue of how to develop limited losses to ultimate, I relied upon an inverse power curve recommended by Richard Sherman [3] to model the development arising in the tail. Specifically, I used a three parameter version of the curve depicted as follows:

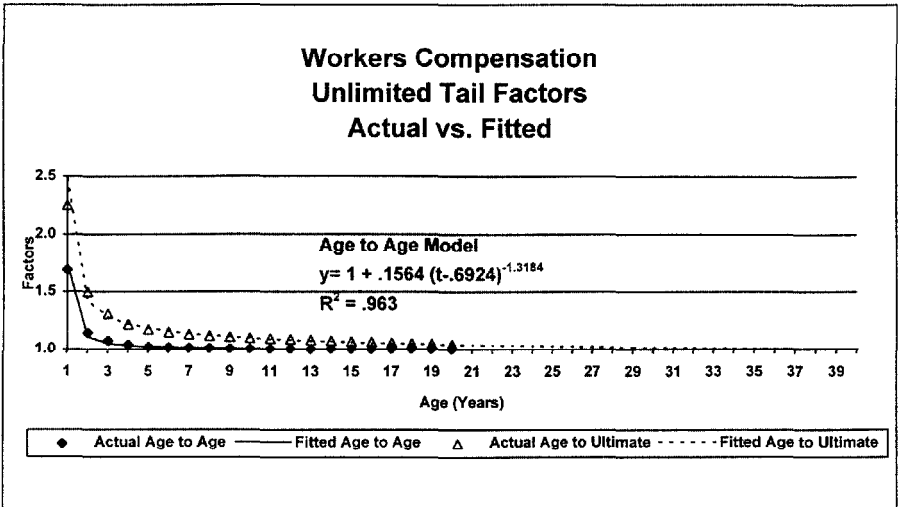
$$(4.1) \quad y = 1 + a \cdot (t + c)^{-b}$$

Again, my concern was to determine consistent tail factors by limit. Starting with the unlimited loss development and fitting an inverse power curve to known age-to-age factors allowed me to project ultimate unlimited losses. As the inverse power curve continues indefinitely, there is a need to select a time at which the projection should end. At this point I tied this approach to a similar method used for determining our full coverage tail factor that relies upon extended

development triangles. That procedure suggested that I could get an equivalent result from the inverse power curve model by stopping its projected age-to-age development factors at 40 years. Compounding the age-to-age factors from the fitted curve leads to the desired completion or tail factors.

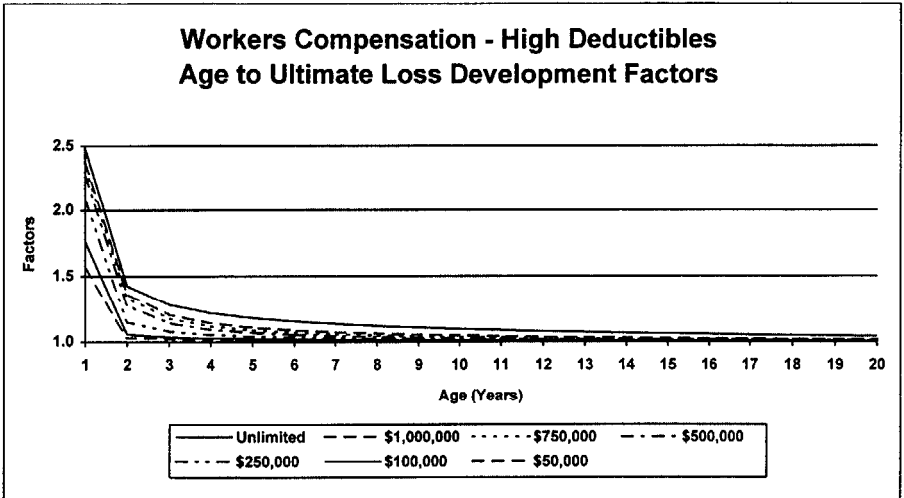
Once I set the ultimate age, I fit the inverse power curves to age-to-age factors for the various deductible limits under review and extended to that common maturity. Though this approach utilizes a consistent technique and generates uniformly decreasing tail factors, it is still an open issue whether the bias in extending all curves to a common maturity is significant or not. (At lower limits, development likely ceases well before forty years.) Chart 2 depicts the age-to-age model determined for the unlimited loss development.

Chart 2



In Chart 3 I show the pattern of age-to-ultimate limited loss development factors resulting from the inverse power curve model.

Chart 3



Another issue the actuary needs to be sensitive to is the relationship between loss development factors and limited severity relativities.<sup>2</sup> In some of my earlier efforts I attempted to uniquely develop losses by limit without regard to how they might relate to one another. This led to inconsistencies in development factors where completion factors for smaller deductibles, for example, sometimes exceeded factors for larger deductibles. Upon closer inspection, I found that any attempts to determine deductible development factors need to address the relationship between the full coverage loss development and severity relativities. The following formulas show the relationship between limited and excess development factors with the unlimited loss development and severity relativities.

$$(4.2) \text{LDF}^L = \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot \frac{R^L}{R_t^L}$$

where L = Deductible Limit, C = Counts, S = Severity, R = Severity Relativity, and t = age

$$(4.3) \text{XSLDF}^L = \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot \frac{(1-R^L)}{(1-R_t^L)}$$

where L = Deductible Limit, C = Counts, S = Severity, R = Severity Relativity, and t = age

<sup>2</sup> Limited severity relativities are defined simply as the ratio of the limited to unlimited severity.

## Motivation for Relationship

$$(4.4) \quad LDF_t = R_t^L \cdot LDF^L + (1 - R_t^L) \cdot XSLDF^L$$

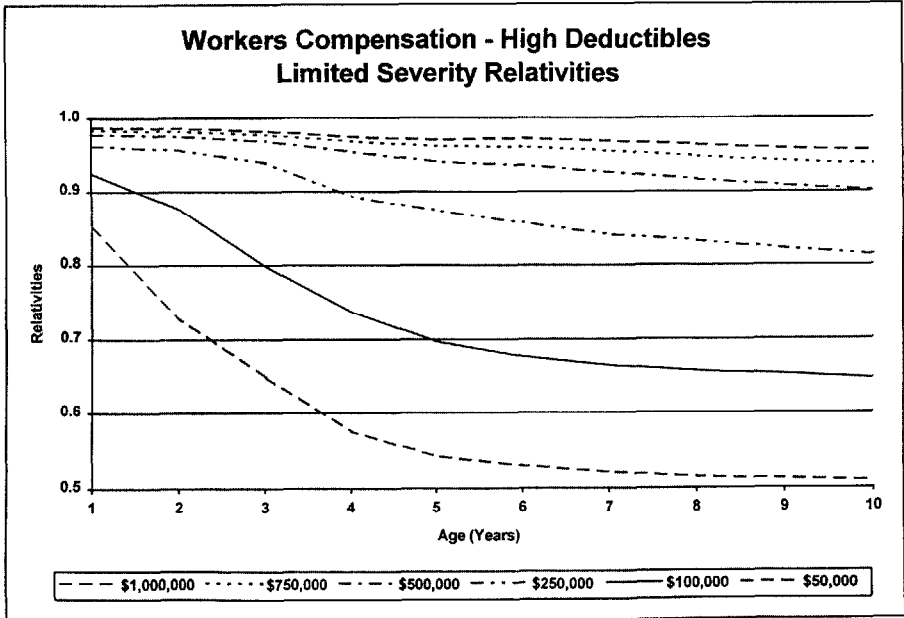
$$(4.5) \quad LDF_t = R_t^L \cdot \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot \frac{R^L}{R_t^L} + (1 - R_t^L) \cdot \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot \frac{(1 - R^L)}{(1 - R_t^L)}$$

$$(4.6) \quad LDF_t = \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot R^L + \frac{C}{C_t} \cdot \frac{S}{S_t} \cdot (1 - R^L)$$

$$(4.7) \quad LDF_t = \frac{C}{C_t} \cdot \frac{S}{S_t}$$

The motivation for these relationships results from the desire to partition total loss development in a consistent fashion between limited and excess development. I show in Chart 4 how the historical limited severity relativities ought to relate to one another and change over time.

**Chart 4**



In Table 4 I show age-to-age development about a \$250,000 deductible limit.

**Table 4**  
**Workers Compensation**  
**High Deductibles**

| <i>Age-to-Age Loss &amp; ALAE Development Factors</i> |              |              |              |              |              |
|---|--------------|--------------|--------------|--------------|--------------|
| <u>(Unlimited)</u>                                    |              |              |              |              |              |
| Accident<br>Year                                      | <u>12:24</u> | <u>24:36</u> | <u>36:48</u> | <u>48:60</u> | <u>60:72</u> |
| 1989  | 1.7063       | 1.1756       | 1.0929       | 1.0359       | 1.0273       |
| 1990  | 1.8219       | 1.1574       | 1.0744       | 1.0387       | -            |
| 1991  | 1.7724       | 1.1506       | 1.0737       | -            | -            |
| 1992  | 1.6912       | 1.1398       | -            | -            | -            |
| 1993  | 1.6044       | -            | -            | -            | -            |
| Average   | 1.7192       | 1.1559       | 1.0803       | 1.0373       | 1.0273       |
| <i>Age-to-Age Loss &amp; ALAE Development Factors</i> |              |              |              |              |              |
| <u>(\$250,000 Deductible)</u>                         |              |              |              |              |              |
| Accident<br>Year                                      | <u>12:24</u> | <u>24:36</u> | <u>36:48</u> | <u>48:60</u> | <u>60:72</u> |
| 1989  | 1.7077       | 1.1598       | 1.0657       | 1.0221       | 1.0120       |
| 1990  | 1.7755       | 1.1509       | 1.0550       | 1.0247       | -            |
| 1991  | 1.7734       | 1.1461       | 1.0643       | -            | -            |
| 1992  | 1.6750       | 1.1363       | -            | -            | -            |
| 1993  | 1.6229       | -            | -            | -            | -            |
| Average   | 1.7109       | 1.1483       | 1.0617       | 1.0234       | 1.0120       |
| <i>Age-to-Age Loss &amp; ALAE Development Factors</i> |              |              |              |              |              |
| <u>(Excess of \$250,000 Deductible)</u>               |              |              |              |              |              |
| Accident<br>Year                                      | <u>12:24</u> | <u>24:36</u> | <u>36:48</u> | <u>48:60</u> | <u>60:72</u> |
| 1989  | 1.6646       | 1.6582       | 1.6742       | 1.1927       | 1.2011       |
| 1990  | 4.4890       | 1.3049       | 1.3151       | 1.2411       | -            |
| 1991  | 1.7373       | 1.3115       | 1.3675       | -            | -            |
| 1992  | 2.2474       | 1.2291       | -            | -            | -            |
| 1993  | 1.1684       | -            | -            | -            | -            |
| Average   | 2.2613       | 1.3759       | 1.4523       | 1.2169       | 1.2011       |

In Table 5 I show relativities and their changes for the selected deductible limit.

**Table 5**  
**Workers Compensation**  
**High Deductibles**

*Limited Severity Relativities*  
*(\$250,000 Deductible)*

| Accident<br>Year | <u>12 Months</u> | <u>24 Months</u> | <u>36 Months</u> | <u>48 Months</u> | <u>60 Months</u> | <u>72 Months</u> |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 1989             | 0.9675           | 0.9683           | 0.9553           | 0.9315           | 0.9191           | 0.9053           |
| 1990             | 0.9829           | 0.9578           | 0.9524           | 0.9353           | 0.9227           | -                |
| 1991             | 0.9723           | 0.9728           | 0.9690           | 0.9605           | -                | -                |
| 1992             | 0.9717           | 0.9623           | 0.9594           | -                | -                | -                |
| 1993             | 0.9593           | 0.9704           | -                | -                | -                | -                |
| Average          | 0.9707           | 0.9663           | 0.9590           | 0.9424           | 0.9209           | 0.9053           |

*Changes in Limited Severity Relativities*  
*(\$250,000 Deductible)*

| Accident<br>Year | <u>12:24</u> | <u>24:36</u> | <u>36:48</u> | <u>48:60</u> | <u>60:72</u> |
|------------------|--------------|--------------|--------------|--------------|--------------|
| 1989             | 1.0008       | 0.9866       | 0.9751       | 0.9867       | 0.9850       |
| 1990             | 0.9745       | 0.9944       | 0.9820       | 0.9865       | -            |
| 1991             | 1.0005       | 0.9961       | 0.9912       | -            | -            |
| 1992             | 0.9903       | 0.9970       | -            | -            | -            |
| 1993             | 1.0116       | -            | -            | -            | -            |
| Average          | 0.9955       | 0.9935       | 0.9828       | 0.9866       | 0.9850       |

Note how the change in limited loss development relates to the unlimited loss development. Also note how actual case loss development does not always conform to expectations, as the limited loss development factor sometimes exceeds the unlimited.

$$(4.8) \text{ LDF}^L = \text{LDF} \cdot \Delta R^L$$

For example, for accident year 1993, moving from 12 to 24 months, a limited factor of 1.6229 is observed. That is equivalent to the unlimited loss development factor of 1.6044 compounded with the change in severity relativities for the same time period of 1.0116.

Note also the relationship of the excess development to the unlimited loss development for the same year.

$$(4.9) \quad \text{XSLDF}^L = \text{LDF} \cdot \Delta(1 - R^L)$$

There the accident year 1993 excess development factor of 1.1684 is equivalent to the unlimited development factor compounded with the ratio of the complements of the severity relativities moving from 12 to 24 months. ( $1.1684 = (1.6044)(1 - 0.9704) / (1 - 0.9593)$ )

And, as desired, the weighted average of the limited and excess development factors using the relativity as the weight leads to the unlimited loss development factor.

$$(4.10) \quad \text{LDF}_t = R_t^L \cdot \text{LDF}_t^L + (1 - R_t^L) \cdot \text{XSLDF}_t^L$$

(Accident Year 1993:  $1.6044 = (0.9704)(1.6229) + (1 - 0.9704)(1.1684)$ )

#### *Distributional Model - A More Promising Approach*

Because of the concepts just described, this whole approach seems ideally suited for the application of some form of loss distribution model. That model helps to tie the relativities to the severities and consequently provides consistent loss development factors. Not only that, a distributional model easily allows for interpolation among limits and years, as needed.

The approach I propose models the development process by determining parameters of a distribution that vary over time. Once the distribution and its parameters are specified, it is possible to calculate the desired limited/excess severities. Comparing those severities over time leads to the needed development factors. Of course, care has to be exercised to recognize claim count development at earlier maturities.

For my work, I relied upon a Weibull distribution to specify the workers compensation claim loss distribution. That distribution has been commonly used for workers compensation claims and is familiar to actuaries working with distributional models. It is ideally suited for this type of work, as it gives a reasonable depiction of the loss distributions and is easy to work with.

Of course, the most difficult aspect of working with distributional models is estimating the parameters involved. There are various approaches that can be used, including Method of Moments as well as Maximum Likelihood. I tried an alternative approach that optimizes the fit between actual and theoretical severity relativities around the \$250,000 deductible size. Specifically, I minimized the chi-square between actual and expected severity relativities to determine the needed parameters. I made use of a solver routine incorporated in Microsoft Excel's spreadsheet application, which allowed me to constrain the optimization routine in such a fashion that the parameters generated produced the actual unlimited severity at the specified maturity.



I show in Table 6 an example of results used to determine age-to-ultimate loss development factors by limit from 48 months to ultimate. I selected 48 months in order to focus attention on changes in severity rather than changes in total claim counts assuming no IBNR count development after 36 months. (Please see Appendix I for details.)

**Table 6**  
**Workers Compensation High Deductibles**  
Actual Versus Fitted Limited/Excess Development Factors (@ 48 Months)  
*(using a Weibull Loss Distribution)*

| <i>Ultimate</i>    |                  |                    |                  |                                  |                  |                  |                 |
|--------------------|------------------|--------------------|------------------|----------------------------------|------------------|------------------|-----------------|
| Limit              | <u>Unlimited</u> | <u>\$1,000,000</u> | <u>\$750,000</u> | <u>\$500,000</u>                 | <u>\$250,000</u> | <u>\$100,000</u> | <u>\$50,000</u> |
| <i>Observed</i>    |                  |                    |                  |                                  |                  |                  |                 |
| Limited Severity   | 6,846.4          | 6,159.2            | 5,980.4          | 5,714.4                          | 5,094.8          | 3,939.6          | 3,036.5         |
| Relativity         | 1.0000           | 0.8996             | 0.8735           | 0.8347                           | 0.7442           | 0.5754           | 0.4435          |
| Excess Severity    | 0.0              | 687.2              | 866.0            | 1,132.0                          | 1,751.6          | 2,906.8          | 3,809.9         |
| <i>Fitted</i>      |                  |                    |                  |                                  |                  |                  |                 |
| Limited Severity   | 6,846.4          | 6,295.2            | 6,106.5          | 5,778.7                          | 5,064.4          | 3,926.7          | 3,043.8         |
| Relativity         | 1.0000           | 0.9195             | 0.8919           | 0.8440                           | 0.7397           | 0.5735           | 0.4446          |
| Excess Severity    | 0.0              | 551.2              | 739.9            | 1,067.7                          | 1,782.0          | 2,919.7          | 3,802.6         |
| Weibull Parameters |                  | Scale = 180.0      |                  | Shape = .2326                    |                  |                  |                 |
|                    |                  | Mean = 6,846.4     |                  | Coefficient of Variation = 10.07 |                  |                  |                 |
| <i>48 Months</i>   |                  |                    |                  |                                  |                  |                  |                 |
| Limit              | <u>Unlimited</u> | <u>\$1,000,000</u> | <u>\$750,000</u> | <u>\$500,000</u>                 | <u>\$250,000</u> | <u>\$100,000</u> | <u>\$50,000</u> |
| <i>Observed</i>    |                  |                    |                  |                                  |                  |                  |                 |
| Limited Severity   | 5,530.2          | 5,346.6            | 5,288.5          | 5,182.3                          | 4,824.0          | 3,807.5          | 2,937.1         |
| Relativity         | 1.0000           | 0.9668             | 0.9563           | 0.9371                           | 0.8723           | 0.6885           | 0.5311          |
| Limited LDF        | 1.2380           | 1.1520             | 1.1308           | 1.1027                           | 1.0561           | 1.0347           | 1.0338          |
| Excess Severity    | 0.0              | 183.6              | 241.7            | 347.9                            | 706.2            | 1,722.7          | 2,593.1         |
| Excess LDF         | -                | 3.7429             | 3.5830           | 3.2538                           | 2.4803           | 1.6874           | 1.4692          |
| <i>Fitted</i>      |                  |                    |                  |                                  |                  |                  |                 |
| Limited Severity   | 5,530.2          | 5,380.5            | 5,301.4          | 5,142.5                          | 4,722.4          | 3,894.0          | 3,144.1         |
| Relativity         | 1.0000           | 0.9729             | 0.9586           | 0.9299                           | 0.8539           | 0.7041           | 0.5685          |
| Limited LDF        | 1.2380           | 1.1700             | 1.1519           | 1.1237                           | 1.0724           | 1.0084           | 0.9681          |
| Excess Severity    | 0.0              | 149.7              | 228.8            | 387.7                            | 807.8            | 1,636.2          | 2,386.1         |
| Excess LDF         | -                | 3.6820             | 3.2338           | 2.7539                           | 2.2060           | 1.7844           | 1.5936          |
| Weibull Parameters |                  | Scale = 305.7      |                  | Shape = .2625                    |                  |                  |                 |
|                    |                  | Mean = 5,530.2     |                  | Coefficient of Variation = 7.35  |                  |                  |                 |

Lastly, the following formulation shows how expected development can be partitioned about the deductible limit.

$$(4.11) \text{ Expected Development} = 1 - \frac{1}{LDF_t}$$

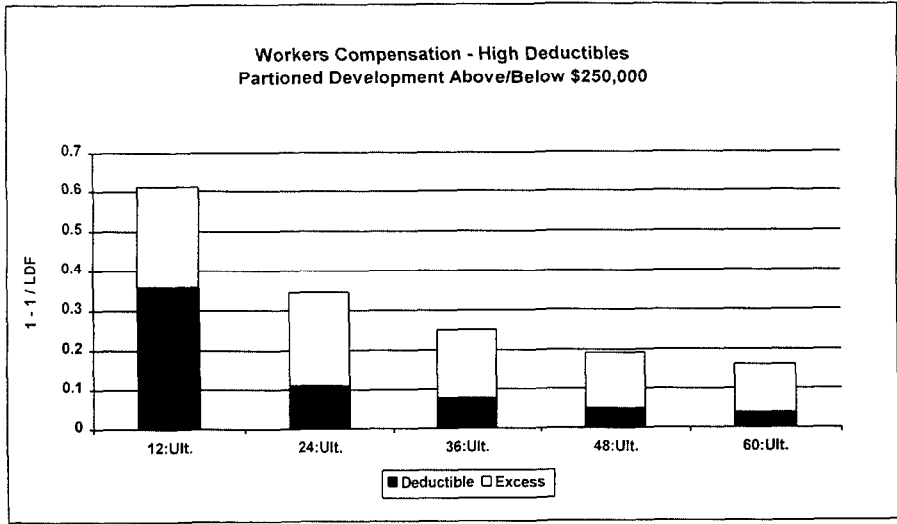
$$(4.12) \quad = \frac{LDF_t - 1}{LDF_t}$$

$$(4.13) \quad = \frac{R_t^L \cdot LDF_t^L + (1 - R_t^L) \cdot XSLDF_t^L - 1}{R_t^L \cdot LDF_t^L + (1 - R_t^L) \cdot XSLDF_t^L}$$

$$(4.14) \quad = \frac{R_t^L \cdot (LDF_t^L - 1) + (1 - R_t^L) \cdot (XSLDF_t^L - 1)}{R_t^L \cdot LDF_t^L + (1 - R_t^L) \cdot XSLDF_t^L}$$

I show graphically in Chart 5 partitioned development for a selected \$250,000 deductible limit based upon the previously described Weibull loss distribution model. Note the changing proportions of development over time. Not unexpectedly, excess development represents the vast majority of development with increasing age.

Chart 5



## 5. Other Elements

Several other elements associated with high deductible plans call for further discussion: aggregate limits, service revenue and allocated claim expense. Determining sound estimates for those items involves a fair amount of complexity. In the following discussion I recommend using advanced collective risk modeling techniques to estimate losses excess of aggregate limits. I also suggest an alternative procedure using the NCCI Table M, if collective risk modeling is not considered practical. The asset for service revenue, though not as difficult to determine, however, depends upon prior estimates of losses for deductible/aggregate limits. Treating allocated claim expense in a similar fashion to loss simplifies the estimation process for that liability, but separating the two pieces is problematic.

### *Aggregate Limits*

Some risks, besides choosing to limit their per occurrence losses, desire to limit all losses that they will pay under a high deductible program. Insurers satisfy that need by providing aggregate loss limits. Those limits are conceptually similar to maximum premium limitations used in retro rating plans.

Determining loss development factors for losses excess of aggregate limits is more complicated than for per occurrence limitations. However, the obligations arising from those aggregate limits are generally less significant than for per occurrence limits. Besides the additional complexity, the data needed to determine development factors for these limits is generally sparse and not likely to be very credible. Outside of actually attempting to gather data for development factors of this sort, I suggest making use of collective risk modeling techniques to determine the needed loss development factors. Such a model could utilize the loss distributions just described for the deductible limits in conjunction with selected claim frequency distributions.

I used a collective risk model described by Heckman and Meyers [4] to determine development factors for losses excess of aggregate limits. I show in Table 7 selected development factors using the same Weibull loss distribution I used previously to determine deductible development factors. I assumed a Poisson claim count distribution to model frequency. Though I did not incorporate any parameter risk in determining the development factors, the model does allow for that possibility. I refer the interested reader to a discussion by Meyers and Schenker [5] describing how to incorporate parameter risk into the collective risk model.

The sampling of development factors I calculated shows that development for losses excess of aggregate limits decreases more rapidly over time with smaller deductibles than larger ones. That is not unexpected as most of the later development occurs in the layers of loss above the deductible limits, which is not covered by the aggregate. Also, not unexpectedly, development is more leveraged for larger aggregate limits. There is one additional point the reader should note in reviewing Table 7. Though I show hypothetical results for risks of \$1 million and \$2.5 million in expected loss size, the limited expectations are considerably smaller.

**Table 7**  
**Workers Compensation High Deductibles**  
*Development Factors for Losses Excess of Aggregate Limits*  
 (Collective Risk Model Utilizing Weibull Loss Distribution)

Expected Unlimited Losses of \$1,000,000

Aggregate Limit = 500,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | 9,253.6            | 13.024     | 114,646.0          | 1.051      | 120,523.3          |
| \$250,000         | 22,882.5           | 12.007     | 228,070.7          | 1.205      | 274,761.6          |
| \$500,000         | 28,653.6           | 13.255     | 289,389.2          | 1.312      | 379,794.3          |

Aggregate Limit = 750,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | 155.1              | 136.451    | 18,005.9           | 1.175      | 21,163.6           |
| \$250,000         | 1,844.9            | 63.845     | 84,475.1           | 1.394      | 117,788.5          |
| \$500,000         | 4,257.2            | 49.763     | 138,526.3          | 1.529      | 211,851.8          |

Aggregate Limit = 1,000,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | .8                 | 2,242.750  | 1,274.7            | 1.408      | 1,794.2            |
| \$250,000         | 94.5               | 418.531    | 23,343.1           | 1.694      | 39,551.2           |
| \$500,000         | 494.5              | 213.275    | 57,471.2           | 1.835      | 105,464.6          |

Expected Unlimited Losses of \$2,500,000

Aggregate Limit = 1,000,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | 39,703.2           | 11.761     | 456,498.9          | 1.023      | 466,934.1          |
| \$250,000         | 81,084.7           | 10.876     | 759,354.4          | 1.161      | 881,844.0          |
| \$500,000         | 95,069.6           | 12.021     | 912,976.1          | 1.252      | 1,142,866.6        |

Aggregate Limit = 1,250,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | 3,829.0            | 64.779     | 236,271.2          | 1.050      | 248,037.5          |
| \$250,000         | 17,740.7           | 36.191     | 522,364.3          | 1.229      | 642,046.5          |
| \$500,000         | 26,520.1           | 33.986     | 674,759.3          | 1.336      | 901,315.4          |

Aggregate Limit = 1,500,000

| <u>Deductible</u> | <u>12 Months</u>   |            | <u>48 Months</u>   |            | <u>Ultimate</u>    |
|-------------------|--------------------|------------|--------------------|------------|--------------------|
|                   | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> | <u>LDF</u> | <u>Excess Loss</u> |
| \$100,000         | 173.5              | 564.077    | 87,988.1           | 1.112      | 97,867.3           |
| \$250,000         | 2,693.1            | 158.522    | 318,464.5          | 1.341      | 426,916.3          |
| \$500,000         | 6,001.8            | 112.833    | 463,359.8          | 1.461      | 677,200.3          |

Given the volatility of losses excess of aggregate limits, I recommend using a Bornhuetter-Ferguson method to smooth out indications of ultimate liability. The example I show in Table 8 makes use of expected aggregate loss charges as well as expected development factors based upon the previously described collective risk modeling approach. The final indication adds together known losses excess of aggregate limits and IBNR based upon the modeled development patterns.

**Table 8**  
**Countrywide Insurance Enterprise**  
 Workers Compensation High Deductibles  
 Estimated Ultimate Aggregate Excess of Loss  
*(Utilizing Bornhuetter-Ferguson Methodology)*

| <u>Account</u>  | <u>Deductible</u> | <u>Known Loss (@ 48 Months)</u> |                            | <u>Aggregate Excess of Loss</u> |            |                  |
|---|-------------------|---------------------------------|----------------------------|---------------------------------|------------|------------------|
|   |                   | <u>Deductible</u>               | <u>Excess of Aggregate</u> | <u>Expected</u>                 | <u>LDF</u> | <u>Indicated</u> |
| <i>Expected Unlimited Loss - 1,000,000; Aggregate Limit - 750,000</i>   |                   |                                 |                            |                                 |            |                  |
| A   | 100,000           | 581,252                         | -                          | 21,164                          | 1.175      | 3,152            |
| B   | 250,000           | 703,027                         | -                          | 117,789                         | 1.394      | 33,292           |
| C   | 500,000           | 764,493                         | 14,493                     | 211,852                         | 1.529      | 87,789           |
| <i>Expected Unlimited Loss - 2,500,000; Aggregate Limit - 1,250,000</i> |                   |                                 |                            |                                 |            |                  |
| X   | 100,000           | 1,453,169                       | 203,169                    | 248,038                         | 1.050      | 214,980          |
| Y   | 250,000           | 1,757,616                       | 507,616                    | 642,047                         | 1.229      | 627,248          |
| Z   | 500,000           | 1,911,285                       | 661,285                    | 901,315                         | 1.336      | 887,963          |

An alternative approach for determining IBNR estimates for aggregate excess of loss coverage merits consideration. That procedure utilizes the NCCI methodology [1] for determining insurance charges in retrospective rating plans. I consider it a more practical approach than collective risk modeling, but its accuracy hinges upon determining the proper insurance charge table.

Essentially the IBNR is determined by subtracting insurance charges at different maturities. The process used to determine the ultimate insurance charge would be the same as that used for pricing purposes. The key to the NCCI procedure is the adjustment of expected losses reflecting loss limits. That adjustment increases expected losses used in determining the appropriate insurance charge table by use of the following formula:

$$(5.1) \text{ Adjustment Factor} = \frac{(1 + 0.8 \cdot \chi)}{(1 - \chi)}$$

where  $\chi$  = per occurrence charge

The intent of increasing expected losses for the use of a per occurrence limit is to utilize a less dispersed loss ratio distribution and, consequently, a smaller insurance charge. Though this adjustment for a loss limit moves the selection of an insurance charge table in the right direction, the question remains whether it does so in an appropriate manner. Additionally, the procedure generates smaller insurance charges by the use of limited losses in the entry ratio calculation.

In order to calculate the insurance charge at earlier maturities I suggest determining the per occurrence charge used in the NCCI procedure by relating undeveloped, limited losses to ultimate, unlimited losses. For example, using the fitted results depicted in Table 6 for a 250,000 deductible leads to a per occurrence charge of 31 percent ( $1 - 4722.4 / 6846.4$ ) at 48 months. Besides reflecting the impact of the limit, this approach also captures the effects of development. Again, the issue remains whether or not the adjustment for both the limit and development is appropriate.

I show in Table 9 a comparison of IBNR estimates determined using the NCCI Table M with estimates from the previously described collective risk modeling approach depicted in Table 8. I further detail IBNR estimates from the NCCI Table M in Appendix II.

**Table 9**  
**A Comparison of Aggregate Excess of Loss IBNR Estimates (@ 48 Months)**  
Collective Risk Model Versus NCCI Table M

| <u>Account</u>  | <u>Deductible</u> | <u>Collective Risk Model</u> | <u>NCCI Table M</u> |
|---|-------------------|------------------------------|---------------------|
| <i>Expected Unlimited Loss - 1,000,000; Aggregate Limit - 750,000</i>   |                   |                              |                     |
| A   | 100,000           | 3,152                        | 1,809               |
| B   | 250,000           | 33,292                       | 38,500              |
| C   | 500,000           | 73,296                       | 68,811              |
| <i>Expected Unlimited Loss - 2,500,000; Aggregate Limit - 1,250,000</i> |                   |                              |                     |
| X   | 100,000           | 11,811                       | 9,959               |
| Y   | 250,000           | 119,633                      | 103,000             |
| Z   | 500,000           | 226,678                      | 222,168             |

#### *Service Revenue*

A significant element that ought to be reflected on the asset side of the balance sheet is the revenue associated with servicing claims under a high deductible program. As I noted earlier, service revenue is generated in an analogous fashion to the use of a loss conversion factor in a retro rating plan. Generally, a factor is applied to deductible losses, limited by any applicable aggregate, to cover expenses that vary with those losses. In practice, however, other elements are captured by the loss multiplier, reflecting the desire of the individual accounts to fund the cost of the program as losses emerge. The service revenue is often collected as losses are paid, but it may also be gathered as a function of case incurred losses.

I propose determining the asset in the following fashion:

1. Determine ultimate deductible losses at the account level.
2. Subtract ultimate losses excess of aggregate limits from ultimate deductible losses.
3. Apply the selected loss multiplier to the difference determined in step 2 to determine ultimate recoverables.
4. Determine the total asset by subtracting any known recoveries from the estimated ultimate recoverables and aggregate results for all accounts.

Table 10 shows an example of how in practice the asset for the service revenue might be determined.

**Table 10**  
**Countrywide Insurance Enterprise**  
**Workers Compensation - High Deductibles**  
**Estimated Ultimate Service Revenue**

*Expected Unlimited Loss - 2,500,000; Aggregate Limit - 1,250,000; Loss Multiplier - 10%*

| Account | Ultimate Loss    |                     | Multiplier       | Known Recoveries | Asset          |                  |
|---------|------------------|---------------------|------------------|------------------|----------------|------------------|
|         | Deductible       | Excess of Aggregate |                  |                  |                | Net of Aggregate |
| X       | 1,465,376        | 214,980             | 1,250,396        | 125,040          | 96,960         | 28,080           |
| Y       | 1,884,867        | 627,248             | 1,257,619        | 125,762          | 102,712        | 23,050           |
| Z       | <u>2,147,711</u> | <u>887,963</u>      | <u>1,259,748</u> | <u>125,975</u>   | <u>106,912</u> | <u>19,063</u>    |
| Total   | 5,497,954        | 1,730,191           | 3,767,763        | 376,777          | 306,584        | 70,193           |

*Allocated Claim Expense*

There are two principal means of handling allocated claim expense under a high deductible program. Either the account manages this expense itself or it is treated as loss and subjected to applicable limits. In the first instance development patterns reflecting loss only would be appropriate for determining liabilities, while a combination of loss and expense is appropriate for the second case. For this discussion I determined development factors combining loss and expense components assuming expenses were equivalent to additional loss dollars. Though different development patterns are likely for loss and expense versus loss only, the gain in precision is likely not worth the effort.

A remaining issue is how best to split loss and allocated claim expense for financial reporting purposes. Though splitting them proportionately based upon their full coverage counterparts is expeditious, other more actuarially sound approaches, even if available, may not be cost justifiable.

## 6. Conclusion

I intended with this discussion to suggest some possible approaches for estimating liabilities under a high deductible program. As with many actuarial procedures, much work and improvement are still needed. I hope my suggestions provoke further discussion as to how to better estimate these liabilities.

Although the reader probably has many ideas to improve upon the suggestions I have made, I feel several stand out including:

- Obtain longer histories of experience under the program better reflecting risk characteristics.
- Derive (Select) parameters (distributions) that provide better fits to the actual data.
- Determine better tail factors and/or parameters of the utilized loss distribution.
- Develop more advanced approaches to index loss limits.

None of these are really unknown issues for actuaries, who have long been confronted with developing either limited or excess losses. The availability of more comprehensive data in a workers compensation program allows for the application of more sophisticated loss distributional approaches that affords greater consistency to all of the pieces involved. To that end I hope this paper provides a few steps toward developing sounder actuarial techniques for analyzing workers compensation high deductible loss development.



## REFERENCES

- [1] National Council on Compensation Insurance, *Retrospective Rating Plan Manual for Workers' Compensation and Employers Liability Insurance - Appendix*
- [2] Bornhuetter, Ronald L., and Ferguson, Ronald E., "The Actuary and IBNR," *PCAS LIX*, 1972, p. 181
- [3] Sherman, Richard E., "Extrapolating, Smoothing, and Interpolating Development Factors," *PCAS LXXI*, 1984, p. 122
- [4] Heckman, Philip E., and Meyers, Glenn G., "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS LXX*, 1983, p. 22
- [5] Meyers, Glenn G., and Schenker, Nathaniel, "Parameter Uncertainty in the Collective Risk Model," *PCAS LXX*, 1983, p. 111

## Appendix I

### Weibull Distribution

1. Cumulative Distribution Function  $F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha}$ ; where  $x > 0, \beta > 0, \alpha > 0$

2. Probability Density Function  $f(x) = \frac{\alpha \cdot x^{\alpha-1}}{\beta^\alpha} \cdot e^{-\left(\frac{x}{\beta}\right)^\alpha}$

3.  $E(x) = \beta \cdot \Gamma\left(\frac{1}{\alpha} + 1\right)$ ; where  $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$

4.  $LEV(x) = \beta \cdot \Gamma\left(\frac{1}{\alpha} + 1\right) \cdot \Gamma\left(\frac{1}{\alpha} + 1; \left(\frac{x}{\beta}\right)^\alpha\right) + x \cdot e^{-\left(\frac{x}{\beta}\right)^\alpha}$

#### LDF calculations about \$250,000 deductible limit

*Severities at ultimate*

$$\beta = 180.0; \alpha = 2.326$$

$$E(x) = 180.0 \cdot \Gamma\left(\frac{1}{2.326} + 1\right) = 6846$$

$$LEV(x) = 6846 \cdot \Gamma\left[\frac{1}{2.326} + 1; \left(\frac{250000}{180}\right)^{2.326}\right] + 250000 \cdot \left(e^{-\left(\frac{250000}{180}\right)^{2.326}}\right) = 5064$$

$$E(x) - LEV(x) = 6846 - 5064 = 1782$$

*Severities at 48 Months*

$$\beta = 305.7; \alpha = 2.625$$

$$E(x) = 305.7 \cdot \Gamma\left(\frac{1}{2.625} + 1\right) = 5530$$

$$LEV(x) = 5530 \cdot \Gamma\left[\frac{1}{2.625} + 1; \left(\frac{250000}{305.7}\right)^{2.625}\right] + 250000 \cdot \left(e^{-\left(\frac{250000}{305.7}\right)^{2.625}}\right) = 4722$$

## Appendix I

$$E(x) - LEV(x) = 5530 - 4722 = 808$$

$$LDF_{48} = \frac{6846}{5530} = 1.238$$

$$LDF_{48}^{250000} = \frac{5064}{4722} = 1.072$$

$$XSLDF_{48}^{250000} = \frac{1782}{808} = 2.205$$

## Appendix II

### Determination of IBNR for an Aggregate Excess of 1,250,000

*Risk Characteristics: Expected Unlimited Loss - 2,500,000;*

*Severity - 6846.4; Frequency - 365.2*

|  | <u>48 Months</u>            | <u>Ultimate</u> |
|--|-----------------------------|-----------------|
| a. Severity: Deductible = 250,000                                  | 4722.4                      | 5064.4          |
| b. Frequency   | 365.2                       | 365.2           |
| c. Limited Loss: $a \cdot b$                                       | 1,724,620.5                 | 1,849,518.9     |
| d. Entry Ratio: $\text{Aggregate} / c$                             | 0.72                        | 0.68            |
| e. Loss Excess of Deductible: $1 - \text{LEV}(x) / E(x)$           | 0.310                       | 0.260           |
| f. Adjustment for Limit: $(1 + .8 \cdot e) / (1 - e)$              | 1.810                       | 1.633           |
| g. Adjusted Limited Loss: $\text{Expected Unlimited Loss} \cdot f$ | 4,525,000                   | 4,082,500       |
| h. 1994 Expected Loss Group  | 19                          | 20              |
| i. Insurance Charge Ratio  | .336                        | .369            |
| j. Insurance Charge Amount: $c \cdot i$                            | 579,472                     | 682,472         |
| k. IBNR  | 682,472 - 579,472 = 103,000 |                 |