Workers Compensation Reserve Uncertainty

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Introduction

Actuaries have developed a host of techniques for producing point estimates of indicated reserves. Current regulatory concerns, as reflected in the NAIC's risk-based capital requirements, and developing actuarial practice, as reflected in the American Academy's vision of the future role of the Appointed Actuary, now stress the uncertainty in the reserve estimates in addition to their expected values. This paper demonstrates how the uncertainty in property-casualty loss reserves may be analyzed, and it draws forth the implications for capital requirements and actuarial opinions.

Genesis of this Paper

The analysis in this paper has been stimulated by the NAIC's risk-based capital efforts and by the American Academy of Actuaries vision of the valuation actuary:

❶ The reserving risk charge, which measures the potential for unanticipated adverse loss development by line of business, is the centerpiece of the NAIC property-casualty risk-based capital formula, accounting for about 40% of total capital requirements before the covariance adjustment and about 50% after the covariance adjustment.[1] Because good actuarial analyses of loss reserve uncertainty are still lacking, the present reserving risk charges are based on simple extrapolations from past experience, with a large dose of subjective judgment to keep the results reasonable.

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[1] Readers might wonder: "Why should the percent of capital requirements attributable to the reserving risk charge differ between the before-covariance and the after-covariance figures?" The covariance adjustment sets the total capital requirements as

$$\text{TCR} = \text{total capital requirements} = (\Sigma\ C_i^2)^{0.5},$$

where the "C_i" are the capital requirements for each individual risk. The *marginal* capital requirements for any risk "j" equals

$$\partial\text{TCR}/\partial C_j = 0.5(\Sigma\ C_i^2)^{-0.5} * 2C_j.$$

In other words, the marginal (post-covariance) charge for an additional dollar of any pre-covariance risk charge is proportional to the total dollars in that risk category. Risk categories with large pre-covariance charges, such as reserving risk, provide a high post-covariance contribution *for each dollar of risk charge.* Risk categories with low pre-covariance charges, such as default risk, provide a low post-covariance contribution *for each dollar of risk charge.*

❷ The Appointed Actuary presently opines on the reasonableness of the Annual Statement's point estimate of loss and loss adjustment expense reserves. The American Academy of Actuaries envisions an expanded role, in which the actuary opines on the financial strength of company under a variety of future conditions. The greater the uncertainty in the reserves, the greater the range of reasonable financial conditions that the actuary must consider.

Issues Addressed

This paper focuses on the uncertainty in workers' compensation loss reserves. Specifically, it addresses the following issues:

❶ How should the uncertainty in loss reserves be measured? Many actuaries respond: "Uncertainty should be measured by the variability of the loss reserve estimates." This only begs the question: How might the variability in the loss reserve estimates best be quantified?

❷ What insurance characteristics, such as payment patterns and contract obligations, affect reserve uncertainty? For instance, average payment lags are about the same for general liability and workers compensation loss reserves. So why is the variability of general liability loss reserves so much greater than the variability of workers compensation loss reserves?

❸ How does the measure of variability that underlies risk-based capital requirements differ from the measure of variability that underlies the actuarial opinion? More specifically, how does the variability of the discounted, "net" reserves [i.e., loss obligations after consideration of return premiums and additional premiums on retrospectively rated policies, valued on an economic basis] differ from the variability of the undiscounted, "gross" reserves?

The Mixing of Lines

Why concentrate on workers compensation? Why not discuss property-casualty loss reserves in general, of which workers compensation is but one instance?

This is the primary error that has hampered past analyses of loss reserve variability. Many observers have contrasted short-tailed lines like Homeowners and Commercial Property with long-tailed lines like General Liability and Automobile Liability, and they have noted the greater reserve uncertainty associated with the latter lines of business. Consequently, they have reasoned that reserve uncertainty is associated with reserve "duration": that is, reserves with longer average payment lags have greater uncertainty.[2]

To see the error in this reasoning, let us extend the comparison to life insurance reserves. Single premium traditional life annuities have the longest reserve duration of all insurance products. Yet these products have low reserving risk, since the benefits are fixed at policy inception and mortality fluctuations are low.[3]

The bulk of workers' compensation loss reserves that persist more than two or three years after the accident date are lifetime pension cases. These are disabled life annuities, with long duration and low fluctuation reserves. The longest workers' compensation reserves are often low risk reserves.[4]

The Peculiarities of Compensation Reserves

The quantification of reserve uncertainty must begin with the characteristics of the line of business. Four aspects of workers compensation reserves which affect the level of uncertainty are dealt with in this paper:

- *Duration and Discount:* The previous section noted that most compensation

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[2] We use the term "duration" here in its widespread sense, as a substitute for "average payment lag." Thus, we speak of "long duration" reserves to mean reserves with long average payment lags. In its more precise meaning, "duration" refers to the effect of interest rate changes on the market value of the asset or liability. Duration is not a measure of time (i.e., the loss payment lag) but of the magnitude of the correlation between interest rate changes and market value changes.

For fixed income securities with no call options, the magnitude of the correlation is a direct function of the average time to payment (calculated on a discounted basis). A bond with twice as long an average time to payment (where the payment dates are weighted by the present value of each payment), has twice as great a duration (in the correlation sense). This explains the association of "duration" with "payment lags." However, the duration (in the correlation sense) of assets (such as common stocks) or liabilities (such as casualty loss reserves) that vary with inflation is exceedingly difficult to determine.

[3] These products do have significant interest rate risk, which is indeed affected by the "duration" of the liabilities. For the quantification of interest rate risk for property-casualty insurance companies and the implications for risk-based capital requirements, see Douglas M. Hodes and Sholom Feldblum, "Interest Rate Risk and Capital Requirements for Property-Casualty Insurance Companies" (CAS Part 10 examination study note).

[4] See Sholom Feldblum, "Author's Reply to Discussion by Stephen Philbrick of 'Risk Loads for Insurers,'" *Proceedings of the CAS*, Volume 80 (1993), pages 371-373, which compares reserves uncertainty among four property casualty lines of business: workers' compensation, automobile liability, products liability, and property.

reserves that persist more than two or three years after the accident date are lifetime pension cases. We compared these to life annuities, which are low risk reserves. But the analogy is incomplete, since the statutory accounting treatment differs for these two types of business. Life annuities are discounted at rates close to current corporate bond rates.[5] Most companies discount the indemnity portion of workers' compensation lifetime pension cases at 3.5% or 4% per annum, which is well below their actual investment earnings. The low fluctuations in these reserves, combined with the large "implicit interest margin," create enormous hidden "equity" in statutory balance sheets.

❷ *Statutory Benefits:* What about non-pension cases? Do non-pension compensation reserves have the same uncertainty as General Liability and automobile liability reserves? After all, industry studies have found similarly strong underwriting cycles and "reserve adequacy" cycles in all these lines of business.[6]

Yes, underwriting results are driven by industry cycles, and so underwriting results vary greatly from year to year, whether in workers compensation, general liability, or automobile liability. But underwriting cycles reflect primarily the movement of premium levels, not fluctuations in loss experience. Reserve adequacy cycles are a secondary effect, which are driven by management desires to smooth calendar year operating results. They reflect the accounting treatment of company results, not the uncertainty inherent in the reserves themselves.

When a general liability or medical malpractice accident occurs, the claim may not be reported for some time. Even after the claim is reported, the case may not be settled until years later, and the amount of the loss liability depends on the vagaries of court decisions, societal opinion, and jury awards. This is the source of reserve uncertainty in the liability lines of business.

In workers compensation, almost all claims are reported immediately to the insurer. [It is hard for the employer to be unaware that a worker has been injured on the job and is on disability leave.] Benefits are mandated by statute, and disputes are

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5 The exact discount rate varies by type of product, as prescribed by the 1990 Standard Valuation Law. The discount rate rose as high as 13.25% in the early 1980's, when corporate bond yields were high. The statutory rate for single premium immediate annuities – the life insurance product most comparable to workers' compensation pension cases – issued in the first half of the 1990s is about 7% per annum.

6 See especially Robert P. Butsic, "The Underwriting Cycle: A Necessary Evil?" *The Actuarial Digest,* Vol. 8, No. 2 (April/May 1989).

generally resolved quickly by administrative judges.[7] The paid loss link ratios, or "age-to-age" factors, are extremely stable in workers' compensation, both for pension and for non-pension cases, unlike the comparable factors for the liability lines of business.

❸ *Tail Development:* But don't workers compensation reserve estimates need large "tail factors," just as liability reserve estimates need? And aren't these tail factors highly uncertain, even as the liability tail factors are?

The highly volatile General Liability tail factors reflect the emergence or the settlement of claims – often toxic tort and environmental liability claims – decades after the occurrence of the accident. This is true reserve uncertainty.

The volatility of workers compensation tail factors stems from two causes.

- First, changes in company philosophy regarding reserve margins and implicit discounts affects the selected tail factors. A company seeking stronger reserve margins may choose larger loss development tail factors. A company seeking to implicitly discount its reserves may choose smaller tail factors. Our primary interest here is the inherent uncertainty in the reserves. We are less interested in the accounting illusions caused by changing company philosophies.

- Second, workers compensation tail factors are affected by monetary inflation, both for cost of living adjustments to indemnity benefits and for all aspects of medical benefits. Inflation levels, especially for 30 or 40 years into the future, are extremely uncertain.

  Indeed, this creates great uncertainty in the undiscounted reserve, and the actuary opining on reserve adequacy for statutory statements should consider a wide range of "reasonable" estimates. But the economic value of the reserve is less affected by long-term inflation rates. In the short-term – that is, for periods less than a year or two – inflation rates and interest rates may differ from each other. Over the long-term, the "Fisher effect" holds: the inflation and interest rates are strongly correlated. If the loss reserve discount rate varies with the long-term inflation rate, then inflation-induced changes in the tail factor are offset by changes in the discount rate.[8]

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[7] The resolution may not be as quick as some claimants and companies would like, but they are much shorter than the delays in the liability lines of business.

[8] The appropriate discount rate is not the same as the statutory yield earned by the insurer on its investment portfolio. [The statutory investment yield, of course, does not necessarily move in tandem with inflation rates.] As Butsic argues cogently in "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," *Evaluating Insurance*

● *Loss Sensitive Insurance Contracts:* A high percentage of the workers compensation contracts covering large employers are retrospectively rated. That is, the premium paid by the employer (the insured) is a function of the losses actually incurred. On a retrospectively rated contract, loss liabilities and premiums receivable move in tandem. If loss reserves develop adversely, the insurer will collect retrospective premium adjustments from the employer.

For loss sensitive contracts, estimates of reserve uncertainty must be distinguished from their implications for capital requirements and actuarial opinions. Risk-based capital requirements reflect the equity needs of the insurer. Similarly, the envisioned future role of the appointed actuary is to opine on the financial strength of the insurer under various future conditions. If adverse loss development on a book of business is offset by favorable premium development, the financial condition of the insurer is unaffected, and there is no need for additional equity.

Summary: We may summarize the previous four points as follows: The novice actuary sees an insurer's large book of compensation reserves, notes the long payment lags and the strong underwriting cycles, and concludes: "There must be great uncertainty here. Moreover, unexpected development may severely affect the insurer's financial condition, so much additional capital is needed to guard against this risk." To which the experienced actuary replies: "No, because of the steady compensation payment patterns, the long duration of these claims, and the correspondence of adverse loss development with offsetting premium development, the reserving risk is so low that it is outweighed by the implicit interest margin in the reserves."

Reserve Uncertainty: Regulatory vs. Actuarial Measures

We have differentiated above between the inherent uncertainty in reserve estimates and the accounting illusions caused by discretionary adjustments of reported reserves. Similarly, we may differentiate between "actuarial" measures of reserve uncertainty and "regulatory" measures of reserve uncertainty.

The Solvency Regulator and the Actuary

Suppose that the solvency regulator sees wide fluctuation in reported reserve levels and concludes that there is great uncertainty in the reserve estimates. The company

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Company Liabilities (Casualty Actuarial Society 1988 Discussion Paper Program), pages 147-188, the economic value of loss reserves depends on the characteristics of the reserves, such as average payment lag, and characteristics of the financial markets, such as risk-free interest rates, not on the particular assets held by the insurer.

actuary responds that the actual reserve indications have been stable. The shift in reported reverse levels from year to year stems simply from a desire to smooth calendar year earnings.[9]

> *"What difference does that make?"* replies the solvency regulator. *"We are concerned that the reported reserves may not be sufficient to cover the loss obligations of the company. What difference does it make whether the insufficiency stems from an inherent uncertainty in the reserve indications or from discretionary adjustment of the reported reserves?"*

The regulator is correct. We must differentiate between two types of reserve fluctuations:

- The valuation actuary tells the company's management how much capital it should hold to guard against unexpected adverse events. Suppose the actuary's reserve analysis yields a point estimate of $800 million with a range of $650 million to $950 million, and the company is reporting $700 million on its statutory statements. The actuary's recommendation might be that the company needs $250 million of capital: $100 million for reserve "deficiencies" (the difference between the point estimate and the held reserves) and $150 million for reserve uncertainties.[10]

- The solvency regulator can not easily distinguish between adverse loss development stemming from unanticipated random occurrences and adverse loss development stemming from reserve inadequacies. The regulator estimates the variability of reported reserves and applies this figure to some base number.[11]

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9 For an analysis of workers compensation reserve strengthening and weakening in accordance with industry underwriting cycles, see Kevin M Ryan and Richard I. Fein, "A Forecast for Workers Compensation," *NCCI Digest*, Volume III, Issue IV (December 1988), pages 43-50.

10 Because of the statutory requirement to report even long-term claim reserves at undiscounted values, this is a common situation, particularly for toxic tort and environmental liability exposures. In practice, the implicit interest margin in statutory reserves must be included in the valuation actuary's recommendation. To complete the illustration in the text, the actuary might add that there is $200 million of implicit interest margin in the statutory reserves, so only $50 million of additional capital is needed.

11 The "base number" might be the company's reported reserves (if believed by the regulator) or an independent estimate of the company's reserve needs (if the regulator lacks confidence in the company's financial statements).

Regulators concerned with reserve uncertainty take the latter viewpoint. Our primary interest in this paper is with the uncertainty inherent in the reserve indications themselves, the former viewpoint.

The difference is not in the *magnitude* of the uncertainty, but in the *method* of quantifying the uncertainty. The solvency regulator begins with the reserves reported by companies. How the companies determined these reserves is irrelevant. The actuary examines the factors used to quantify reserve needs, such as age-to-age "link ratios," to determine the uncertainty in the reserve indications. How various companies deviate from the reserve indications in their financial statements is irrelevant.

Measures of Uncertainty

Finding a measure to quantify reserve uncertainty is not easy. The appropriate measure is a probability distribution – but probability distributions are opaque to most reviewers of a company's reserves. One might convert the results to a simple percentile distribution – showing perhaps the 95th percentile, the mean, and the 5th percentile. But this produces only a few figures, and it discards the information conveyed by the shape of the probability distribution. Moreover, it is often hard to find meaning in these numbers. We need a yardstick to measure reserve uncertainty.

We use two measures of reserve uncertainty, one in the text of this paper and one in the appendix. For the analysis in the text of the paper, we use the "expected policyholder deficit" (EPD) concept developed by Robert Butsic as the yardstick for the uncertainty in the reserve estimates.[12] The EPD ratio allows us to translate "reserve uncertainty" into a "capital charge," thereby transforming an abstruse actuarial concept into concrete business terms. In the appendix to this paper, we discuss the "worst case year" concept used to measure reserve uncertainty and thereby to determine the reserving risk charge in the NAIC risk-based capital formula.[13]

Some readers will rightfully ask: "The NAIC worst case year concept is a simple but

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[12] See Robert P. Butsic, "Solvency Measurement for Property-Liability Risk-Based Capital Applications," *Journal of Risk and Insurance*, Volume 61, Number 4 (December 1994), pages 656-690.

[13] For the NAIC worst case year concept, see Allan M. Kaufman and Elise C. Liebers, "NAIC Risk Based Capital Efforts in 1990-91," *Insurer Financial Solvency* (Casualty Actuarial Society 1992 Discussion Paper Program), Volume I, pages 123-178, or Sholom Feldblum, "Risk-Based Capital Requirements" (CAS Part 10 Examination Study Note, Second Edition, 1995).

arbitrary yardstick that is not supported by financial or actuarial theory. Why include it even in the appendix of an actuarial paper?"

The answer is important. This paper demonstrates that the implicit interest margin in full-value workers compensation reserves exceeds the capital needed to guard against unexpected reserve volatility. Some readers, aware of the 11% workers compensation reserving risk charge in the NAIC's risk-based capital formula, may mistakenly conclude that the "regulatory" and "actuarial" approaches to this problem yield different answers.

This is not so. The NAIC "regulatory" approach yields a similar result to that arrived at here. However, the workers compensation charges were subjectively modified to produce capital requirements that seemed more reasonable to some regulators. In fact, the "unreasonableness" of the NAIC formula indications to these regulators stemmed from a misunderstanding of statutory accounting and of the risks of workers compensation business, not from any artifacts in the risk-based capital formula. A full discussion of the NAIC approach to reserve uncertainty embodied in the risk-based capital formula is presented in Appendix A.

The Quantification of Uncertainty

Attempts to measure reserve "uncertainty" often dissolve for failure to make clear (i) what exactly we seek to measure and (ii) how we ought to measure it. This task is particularly difficult because of the variety of loss reserve estimation procedures and the slipperiness of "uncertainty."

This paper combines three elements to analyze the uncertainty of loss reserve estimates:

❶ A statistical procedure to quantify the uncertainty, relying on a stochastic simulation of the loss reserve estimation process.

❷ A yardstick to measure the uncertainty, relying on the expected policyholder deficit ratio.

❸ A set of factors that explain the amount of uncertainty, focusing on payment patterns, inflation and interest rate effects, and loss sensitive contracts.

Actuarial Procedures

Loss reserve estimates stem from empirical data, such as reported loss amounts or paid loss amounts, combined with actuarial procedures, such as chain ladder development methods. Loss reserve uncertainty stems from both of these components.

- Random loss fluctuations may cause past experience to give misleading estimates of future loss obligations.
- Imperfect actuarial analysis of the data may lead to invalid reserve estimates.

The two causes are intertwined. The ideal reserving actuary is ever watchful of data anomalies and will adjust the reserving procedures to avoid the most likely distortions.[14]

In this paper we do not measure the uncertainty stemming from imperfect actuarial practice. Rather, we assume a standard reserving technique which often used for workers' compensation: a paid loss chain ladder development method.[15]

In practice, reserving actuaries use a variety of techniques. Even when employing a paid loss chain ladder development method, rarely does the reserving actuary follow the method by rote, with no analysis of unusual patterns. To the extent that actuarial judgment improves the reserve estimate, this paper "overestimates" the reserve uncertainty. To the extent that actuarial "judgment" masks the true reserve indications, one might say that this paper "underestimates" the reserve uncertainty.

Let us clarify: this paper measures the uncertainty inherent in the empirical data used to produce actuarial reserve estimates. It does not attempt to measure the "uncertainty" added or subtracted by the quality of actuarial analysis.

Empirical Data

How should we measure the uncertainty inherent in the empirical data? Two methods have been used:

❶ We may simulate experience data, develop reserve indications, then continue the

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[14] See, for instance, James R. Berquist and Richard E. Sherman, "Loss Reserve Adequacy Testing: A Comprehensive Approach," *Proceedings of the Casualty Actuarial Society*, Volume 64 (1977), pages 123-185, and the discussion by J. O. Thorne, Volume 65 (1978), pp. 10-34.

[15] We choose this technique, rather than a reported loss chain ladder development technique or Bornhuetter-Ferguson (expected loss) techniques, because it is dependent on claim payment patterns, not on individual company case reserving practices. Thus, we are measuring the uncertainty caused by fluctuations in actual claim patterns, not by changes in company case reserving practices.

simulation to see how accurately the indications forecast the final outcomes.[16] This method is entirely theoretical. The amount of "uncertainty" depends on the simulation procedure, which is not always grounded in actual experience.

❷ We may look at actual experience, develop reserve indications at intermediate points in time, and then compare the indications with the actual outcomes.[17] This method is "practical" – so practical, in fact, that the uncertainty measurements are generally overwhelmed by historical happenstance.

Our procedure charts a middle course. We use stochastic simulation of the experience data to ensure statistically valid results. But the simulation parameters are firmly grounded in 25 years of actual paid loss histories from the country's largest workers compensation carrier.

We describe the three elements of the analysis: (i) the stochastic simulation, (ii) the expected policyholder deficit ratio "yardstick," and (iii) the explanatory factors.

The Stochastic Simulation

We begin with 25 years of countrywide paid loss workers compensation experience, separately for indemnity and medical benefits, for accident years 1970 through 1994. From these data we develop 22 columns of paid loss "age-to-age" link ratios, as shown in Exhibits C-1 and C-2.[18]

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[16] See, for instance, James N. Stanard, "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques," *Proceedings of the Casualty Actuarial Society*, Volume 72 (1985), pages 124-148, and the discussion by John P. Robertson, pages 149-153.

[17] This is the procedure used by the NAIC risk-based capital formula to estimate reserve uncertainty by line of business.

[18] Analysis of the uncertainty inherent in workers compensation loss reserve estimates *must be grounded in actual workers compensation experience. The empirical data is the* experience of the country's largest workers compensation carrier, with about 10% of the nation's experience during the historical period. To ensure confidentiality of the data, the dollar figures are normalized to a $100 million indicated undiscounted reserve.

Upon reviewing an earlier version of this paper, Stephen Lowe points out that "Because of its large market share, [your company's] experience probably does not respond to changes in mix of business by hazard group or state. . . . For smaller companies, changes in mix of business may add uncertainty beyond what is captured in your model."

This view is consistent with Allan Kaufman's recommendation that a "small company charge" be added to the risk based capital formula because small companies experience greater fluctuation

We fit each column of "age-to-age" link ratios to lognormal curves, determining "mu" (μ) and "sigma" (σ) parameters for each.[19] We perform 10,000 sets of stochastic simulations. Each simulation produces 22 "age-to-age" link ratios (one for each column).[20] These are the age-to-age factors that drive the actual loss payments.[21]

The 10,000 simulations produce 10,000 reserve amounts. We ask: "How tight is this distribution of reserve amounts?" We answer in two ways.

☛ We show the standard deviation, the mean, and two other percentiles of the

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in underwriting results and in adverse reserve development. For political reasons, the small company charge was never added to the risk-based capital formula; see Feldblum, "Risk-Based Capital Requirements" (op. cit.). In a review of the 1994 risk-based capital results, Michael Barth, a senior research associate in the NAIC's research department, similarly concludes that "the R4 RBC [i.e., reserving risk] for companies with large reserves may be higher than necessary, relative to smaller companies" (see Michael Barth, "Risk-Based Capital Results for the Property-Casualty Industry," *NAIC Research Quarterly*, Volume II, Issue I (January 1996), pages 17-31).

Lowe, Kaufman, and Barth are correct. Small companies, or companies entering new markets or developing new products, may experience greater reserve uncertainty than implied here.

19 We use the method of moments to fit lognormal curves to the development part of each set of observed paid loss "age-to-age" link ratios: i.e., to the link ratios minus one. We performed the same analysis using other curve families, particularly the Gamma. We chose the lognormal for our final analysis to add conservatism to our results. The lognormal family gave the greatest amount of "uncertainty." Our analysis shows that the uncertainty in workers' compensation reserve estimates is low. In particular, the capital that would be needed to ensure a 1% EPD ratio is substantially less than the "implicit interest margin" in undiscounted (or even partially discounted) workers' compensation reserves. This result would be all the more true if we had used Gamma curves to fit the paid loss "age-to-age" link ratios.

20 Twenty-five accident years yields 24 columns of "age-to-age" factors. The last two columns contain only 2 and 1 historical factors, so instead of fitting these columns to lognormal curves we include these development periods in the "inverse power curve" tail. See Appendix C for a full description of the reserve estimation and simulation procedures.

21 To simplify the mathematics, we assume that the same "actual" age-to-age factor will occur in each subsequent accident year. For instance, Exhibit C-5 shows a simulated indemnity plus ALAE age-to-age factor of 1.113 in the "48 to 60 months" development period for one of the 10,000 simulations. We assume that the 1.113 factor occurs in each subsequent accident year: that is, for accident years 1991, 1992, 1993, and 1994. A more complex procedure would be to perform separate simulations for each subsequent accident year. Our procedure is "conservative." Using separate simulations for each future accident year would dampen the effects of outlying factors, making the distribution of required reserves more compact.

distribution (5% and 95%). For instance, the table below shows that for discounted reserves with no adjustments for inflation, the mean reserve amount is \$52.7 million, the standard deviation is \$3.4 million, the 95th percentile is \$58.7 million, and the 5th percentile is \$47.9 million.

- To facilitate the comparison of reserve uncertainty with other types of risk, we use the "expected policyholder deficit (EPD) ratio" as a yardstick. We ask: "How much additional capital must the insurer hold to have a 1% EPD ratio?" The table below shows that for discounted reserves with no adjustments for inflation, the required capital for a 1% EPD ratio is \$2.4 million.[22]

	Average Reserve Amount	Standard Deviation of Reserve	95th Percentile of Reserve	5th Percentile of Reserve	Capital Needed for 1% EPD Ratio
Undiscounted	100.0	19.5	135.3	74.0	31.0
Discounted: 6.75%	52.7	3.4	58.7	47.9	2.4

Trends and Correlations

The simulation procedure skips quietly over two issues of importance to reserving actuaries: correlations among link ratios and trends in link ratios.

1. *Correlations*: The simulation procedure assumes that the link ratio in any column is independent of the preceding link ratio in the same row. If the link ratios are not independent, the results may be overstated or understated.

   For instance, suppose that accident year 1988 shows a high paid loss link ratio *from 24 to 36 months. Should one expect a higher than average or a lower than* average link ratio from 36 to 48 months?

   The answer depends on the cause of the high 24 to 36 month link ratio. If it is caused by a speeding up of the payment pattern, but the ultimate loss amount has not changed, then one should expect a lower than average link ratio from 36 to 48 months. If it is caused by higher ultimate loss amounts (e.g., because of lengthening durations of disability for indemnity benefits, or because of greater utilization of medical services), then one should expect a higher than average link

[22] For a complete explanation of the expected policyholder deficit calculations and analysis, see Appendix B.

ratio from 36 to 48 months.[23]

- *Trends*: Our procedure uses unweighted averages of all observed link ratios in each column. During the 1980s, industry-wide paid loss link ratios showed strong upward trends, though this trend has ceased in the early 1990s.[24] How would the

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[23] For further explanation, see the discussion by H. G. White to Ronald L. Bornhuetter and Ronald E. Ferguson's, "The Actuary and IBNR," in the *Proceedings of the Casualty Actuarial Society*, Volume 60 (1973), pages 165-168, as well as J. Eric Brosius, "Loss Development Using Credibility" (CAS Part 7 examination study note, December 1992).

Compare also Randall D. Holmberg, "Correlation and the Measurement of Loss Reserve Variability," *Casualty Actuarial Society Forum* (Spring 1994), Volume I, pages 247-278:

> *There are different reasons we might expect development at different stages to be correlated. For instance, if unusually high loss development in one period were the result of accelerated reporting, subsequent development would be lower than average as the losses that would ordinarily be reported in those later periods would have already been reported. In this instance, correlation between one stage and subsequent stages would be negative. Positive correlation would occur if there were a tendency for weaker-than-average initial reserving to be corrected over a period of several years. In that case, an unusually high degree of development in one period would be a warning of more to come.* (page 254)

Holmberg looks at incurred loss development. To circumvent the effects of company case reserving practices on the variability of reserve estimates, we use paid loss development chain ladder estimates in this analysis.

Holmberg also discusses possible correlation among accident years. Our procedure uses the same projected link ratios for all future accident years, thereby overstating the potential variability in the reserve estimates. Choosing different simulated link ratios in each column for each future accident year, and incorporating any correlations among the accident years, would show slightly less variability in the total reserve estimate. Since the computational effort involved is enormous, and this would only marginally strengthen the conclusions in this paper, we have not undertaken the analysis.

Roger Hayne also discusses the possible correlations in the reserve estimation procedure, though he deals with them in a different fashion; see his "A Method to Estimate Probability Level for Loss Reserves," *Casualty Actuarial Society Forum* (Spring 1994), Volume I, pages 297-356.

Roger Bovard, commenting upon an earlier draft of this paper, says: *"According to my eye . . . the columns of development factors are not independent . . . I would not characterize it as a lockstep, but the development factors appear to move up and down together."* Bovard's observation is correct; see below in the text of the paper for the cause of this phenomenon.

[24] See Sholom Feldblum, "Workers' Compensation Ratemaking" (Casualty Actuarial Society Part 6 examination study note), section 7, and the references cited therein.

recognition of such trends affect the variability of the reserves estimates as discussed here?[25]

These two issues are related. First, the observed correlations among the columns of link ratios in the historical data result from the trends in these link ratios. When the trends are removed, the correlations largely disappear. Second, the trends affect the proper reserve estimate. The reserving actuary must investigate these trends and their causes, and then project their likely effect on future loss payments. That is not our interest in this paper. Rather, we ask: "What is the inherent variability in the reserve estimation process itself?"

Let us take each of these issues in turn.

➊ *Correlations*: Suppose one has two columns of observed link ratios, each from accident years 1971 through 1993, from 12 to 24 months and from 24 to 36 months, and that they are *not* correlated. We then apply a strong upward trend to both columns. That is, we increase the accident year 1972 link ratios by 1.02, the accident year 1973 link ratios by $(1.02)^2$, the accident year 1974 link ratios by $(1.02)^3$, and so forth.

The resulting link ratio show a strong positive correlation. Indeed, we observe such a correlation in the historical link ratios used in our simulation. But if we remove the trend, the correlation disappears.

This trend was caused primarily by the increasing liberalization of workers compensation benefit systems between the mid-1970s and the late 1980s. This liberalization, along with its associated effects (increasing paid loss link ratios, statewide rate inadequacies, growth of involuntary markets) ceased by the early 1990s, and has even reversed in many jurisdictions.

➋ *Trends*: Yes, there were trends, at least in the 1980s. Moreover, there are

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25 Roger Bovard notes these trends as well: *"My eye tells me that the actual data contains trends and turning points . . . A trend means that a development factor occurring far into the future could be materially above its estimate calculated in the present from a historical average. A turning point means that history could be pointing in one direction while the actual result is in the other direction."*

Similarly, Stephen Lowe, in a review of an earlier draft of this paper, says: "The model looks at one reserving methodology, paid loss development. Suppose that I were able to take the *same data* and use a different method on it, and that an unbiased application of that method produced a *mean undiscounted reserve indication* of 140.0 rather than the 100.0 produced by the paid loss method." Lowe's implication is that the use of a solitary reserving method underestimates the reserve volatility. See the comments in the text for further discussion of this subject.

multiple reserving methods. The mark of the skilled actuary is to take the various reserve indications and the manifold causes for discrepancies between them and to project an estimate as close as possible to the true, unfolding loss payments.

In our analysis, we have used the full column of observed link ratios to fit the lognormal curve, and then we have compared the simulated loss payments with their averages. Had we incorporated the "trends," and had we ignored old link ratios (because they are not relevant for today's environment), we might have produced tighter reserve distributions.

If one places faith in the skills of reserving actuaries, then the use of a solitary reserving method overstates the uncertainty of the reserving process. Suppose the simulation produces actual loss payments considerably higher than the reserve estimate. Oftentimes, the experienced actuary would have noted signs that the paid loss estimate was underestimating the actual reserve need, and that other methods were giving higher indications. By combining the indications from several methods, the actuary would come closer to the actual reserve need, thereby reducing the uncertainty in the estimates.

Our analysis, however, is based on science, not on faith. Perhaps uncertainty can be reduced by actuarial judgments of trends and by actuarial weighing of various indications. Our question is simpler: even in rote applications of basic reserving techniques, how much uncertainty is produced by the fluctuations in loss data?

Reserve Discounting

We are primarily concerned with the economic values, or discounted values, of the reserves, not with undiscounted amounts. Much of the variation in statutory reserve requirements stems from fluctuations in "tail factors." This fluctuation depends in part on inflation rates. For discounted reserves, the effects of changes in the long-term inflation rate are offset by corresponding changes in the discount rate. Moreover, tail factor uncertainty has a relatively minor effect on the present value of loss reserves, even if the discount rate is held fixed. Thus, the distribution of discounted loss reserve amounts is more compact than the distribution of undiscounted loss reserve amounts.

Exhibits 1 and 2 show the shapes of the probability distributions for the discounted and the undiscounted reserves. Exhibit 1 has no adjustment for expected inflation. Rather, the inflation implicit in the historical link ratios is presumed to continue into the future. Exhibit 2 uses explicit assumptions about future inflation rates, as discussed below in the text (and in Appendix D).

A common view is that discounted reserves are simply smaller than undiscounted reserves, but they exhibit the same degree of variability. This is not correct. As Exhibits 1 and 2 show, the probability distributions for undiscounted reserves are wide, whereas the corresponding probability distributions for discounted reserves are far more compact. The rationale for this is that much of the reserve variability comes from uncertainty in distant "tail" factors, which strongly wag estimates of undiscounted reserves but have relatively little effect on discounted reserve estimates.

Because statutory accounting mandates that insurers hold undiscounted reserves, we show results both for discounted and for undiscounted reserves in the exhibits. Moreover, the difference between the discounted and undiscounted reserve amounts is the "implicit interest margin" in the reserves, which is important for assessing the implications of the reserve uncertainty on the financial position of the insurance company.

Length of the Development

The paid loss development for 25 years is based on observed data. Workers' compensation paid loss patterns extend well beyond 25 years. For each simulation, we complete the development pattern as follows:

- ❶ Given the 22 paid loss "age-to-age" link ratios from the set of stochastic simulations on the fitted lognormal curves, we fit an inverse power curve to provide the remaining "age-to-age" factors.[26] This fit is deterministic.
- ❷ The length of the development period is chosen (stochastically) from a linear *distribution of 30 to 70 years.*

The EPD Yardstick

Probability distributions developed from simulation runs are hard to grasp. The user is left to wonder: "What do the results mean?"

As a yardstick to measure reserve uncertainty, we use the "expected policyholder deficit" (EPD) ratio developed by Robert Butsic for solvency applications. The EPD ratio allows us to

- ☛ Compare reserve uncertainty across different lines of business,
- ☛ Compare reserve uncertainty with either explicit margins in held reserves or with the "implicit interest margins" in undiscounted reserves,

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[26] On the use of the inverse power curve, see Richard Sherman, "Extrapolating, Smoothing, and Interpolating Development Factors," *Proceedings of the Casualty Actuarial Society*, Vol 71 (1984), pages 122-192, as well as the discussion by Stephen Lowe and David F. Mohrman, Vol 72 (1985), page 182, and Sherman's reply to the discussion, page 190.

- ☛ Quantify the effects of various factors (such as loss sensitive contracts) on reserve uncertainty, and
- ☛ Translate actuarial concepts of reserve uncertainty into more established measures of financial solidity.[27]

The Expected Policyholder Deficit

Were there no uncertainty in the future loss payments, then the insurer need hold funds just equal to the reserve amount to meet its loss obligations. Since future loss payments are not certain, funds equal to the expected loss amount will sometimes suffice to meet future obligations and will sometimes fall short.

When the future loss obligations are less than the funds held by the insurance company to meet these obligations, the "deficit" is zero. When the future loss obligations are greater than the funds held, the "deficit" is the difference between the two. The "expected policyholder deficit" is the average deficit over all scenarios, weighted by the probability of each scenario. In the analysis here, the expected deficit is the average deficit over all simulations, each of which is equally weighted.

Let us illustrate with the workers compensation reserve simulations in this paper. Suppose first that the company holds no capital besides the funds supporting the reserves. For the discounted analysis, the average reserve amount is $52.7 million. About half the simulations give reserve amounts less than $52.7 million. In these cases, the deficit is zero. The remaining simulations give reserve amounts greater than $52.7 million; these give positive deficits. The average deficit over all 10,000 simulations is the expected policyholder deficit, the EPD. The "EPD ratio" is the ratio of the EPD to the expected losses, which are $52.7 million in this case.

Clearly, if the probability distribution of the needed reserve amounts is "compact," or "tight," then the EPD ratio will be relatively low. Conversely, if the probability distribution of the needed reserve amounts is "dispersed" – that is, if there is much uncertainty in the loss reserves – then the EPD ratio will be relatively great.

We have two ways of proceeding:

- ☛ We could assume that the company holds no assets besides those needed to support the expected loss obligations and compare EPD ratios for different lines of business or operating environments.

- ☛ We may "fix" the EPD ratio at a desired level of financial solidity and determine how

27 Fur further comments on the benefits of the EPD yardstick for measuring uncertainty, see Appendix B.

much additional capital is needed to achieve this EPD ratio.

The second approach translates EPD ratios into capital amounts, which are more readily understood by business managers, so we follow this approach. We use a 1% EPD ratio as our benchmark, since this is the ratio which Butsic uses for risk-based capital applications. Since our interest here is in reserve uncertainty, not in capital requirements, any EPD ratio will suffice, as long as we hold it constant throughout the analysis.

Suppose the desired EPD ratio is 1%. If the reserve distribution is extremely compact, then even if the insurer holds no capital beyond that required to fund the expected loss payments, the EPD ratio may be 1% or less. If the reserve distribution is more dispersed, then the insurer must hold additional capital to achieve an EPD ratio of 1%. The greater the reserve uncertainty, the greater the required capital.

Results

The results for the base case, with discounted reserves and no adjustments for inflation, are shown in Exhibit 2. The average discounted reserves are $52.7 million, and additional capital of $2.4 million is needed to achieve a 1% EPD ratio.

The corresponding full value reserves are $100.0 million. Statutory accounting permits tabular discounts on life-time pension cases at discount rates between 4% and 5%. Most insurers discount at least the indemnity portion of "identified" pension cases at a rate between 3.5% and 5%. Some insurers also use tabular discounts on "unidentified" pension cases or "implicitly discount" long-term medical benefits, by not fully accounting for future inflation. Industry practices vary. In general, most insurers would report reserves between $80 million and $90 million in this situation.

A common view is that workers compensation reserve estimates are highly uncertain, because of the long duration of the claim payments and because of the unlimited nature of the insurance contract form. This uncertainty creates a great need for capital to hedge against unexpected reserve development. In fact, the opposite is true. There is indeed great underwriting uncertainty in workers compensation, and regulatory constraints on the pricing and marketing of this line of business have disrupted markets and contributed to the financial distress of several carriers. But once the policy term has expired and the accidents have occurred, little uncertainty remains. The difference between the economic value of the reserves and the reported (statutory) reserves, or the "implicit interest margin," is many times greater than capital that would be needed to hedge against reserve uncertainty.[28]

Statutory Benefits

On average, workers compensation reserves have about the same payment lags as General Liability reserves. There is great uncertainty in GL reserves, as an equivalent analysis to that shown in this paper would show.[29] The causes of the GL reserve uncertainty illuminate the reasons for the compactness of the workers compensation reserve distribution.

➊ *IBNR Emergence*: Many GL claims are not reported to the insurer until years after the accident. For toxic tort and environmental impairment exposures, claims are still being reported decades after the exposure period.[30] In contrast, most workers compensation claims are known to the employer within days of the accident, and insurance companies are notified soon thereafter.

➋ *Claim Payment Patterns*: General Liability loss costs depend upon judicial decisions and jury awards. Ultimate costs may not be known until years after the claim has been reported to the insurer. Even cases settled out-of-court are often settled

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28 The implications for capital allocation to lines of business are important. For companies which carry adequate statutory reserves, the capital needed to support compensation reserves is negative, though positive capital is needed to support workers compensation underwriting operations. This is indeed the capital allocation procedure used in our own company, which carries strong statutory reserves but which seeks markets in all jurisdictions, even when regulatory restraints and legislative changes hamper underwriting operations. It is in contrast to the statutory accounting procedures used in NCCI's surplus allocation method in its internal rate of return pricing model; see Sholom Feldblum, "Pricing Insurance Policies: The Internal Rate of Return Model" (Casualty Actuarial Society Part 10A Examination Study Note, May 1992), as well as the Cummins/NCCI dispute there on the proper funding of the underwriting loss in this model.

29 A full actuarial study of reserve uncertainty would apply the techniques used in this paper to all lines of business and compare the reserve distributions, EPD ratios, or capital requirements among them. A serious analysis must take into account the factors specific to each line that affect reserve fluctuations. For instance, just as we examine loss sensitive contracts for workers compensation, we must examine latent injury claims, such as those stemming from asbestos and pollution exposures, for general liability. The extent of such analysis, of course, puts it beyond the scope of this paper.

30 For current reviews of pollution reserves and loss costs, see the Insurance Services Office December 1995 report, "Superfund and the Insurance Issues Surrounding Abandoned Hazardous Waste Sites" (and the references cited therein) and the January 1996 *BestWeek* report by Eric M. Simpson, W. Dolson Smith, and Cynthia S. Babbitt, "P/C Industry Begins to Face Environmental and Asbestos Liabilities."

"on the courthouse steps," after pre-trial discovery and litigation efforts have provided good indications of the probable judicial outcome.

Workers compensation benefits, in contrast, are fixed by statute, both in magnitude and in timing. The benefits may be determined either by agreement between the insurer and the injured worker or by a workers compensation hearing officer. The major uncertainty in indemnity benefits is the duration of disability on non-permanent cases and the mortality rates on permanent cases. For sufficiently large blocks of business, both of these show relatively compact distributions. The major uncertainty for medical benefits is the rate of inflation and the extent of utilization of medical services. Over a large enough block of business, these risks also show relatively compact distributions, particularly when reserves are valued on a discounted basis.

Inflation

The preceding discussion, which shows highly compact distributions for workers compensation reserves, still *overstates* the uncertainty in these reserves. Proper actuarial analysis of two further items, the effects of inflation on the reserve estimates and the effects of loss-sensitive contracts on the net financial results of the company, further reduce the uncertainty in the loss reserves.

Inflation affects workers compensation medical benefits through the payment date. In about half of the U.S. jurisdictions, indemnity payments that extend beyond two years have cost of living adjustments that depend on inflation.[31]

Unadjusted paid loss development patterns combine true development with the effects of inflation. If future expected inflation rates are not equal to past inflation

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31 On the effects of inflation through the "payment date" versus through the "accident date," see Robert P. Butsic, "The Effect of Inflation on Losses and Premiums for Property-Liability Insurers," in *Inflation Implications for Property-Casualty Insurance* (Casualty Actuarial Society 1981 Discussion Paper Program), pages 51-102, and the discussion by Rafal J. Balcarek, pages 103-109.

The statutory rules for cost of living adjustments for indemnity benefits vary greatly by state. Some states have no COLA adjustments. Among the states which do have COLA's, most apply them only to disabilities extending beyond a certain time period, such as two years. In addition, many of these states cap the COLA's at specific levels, such as 5% per annum.

Properly quantifying the effect of the COLA adjustments on workers compensation indemnity reserve indications requires extensive work. For this paper we applied the inflation adjustment to medical benefits only, where a single index can be used countrywide. Performing a similar analysis for indemnity benefits would further reduce the uncertainty in the loss reserve indications, though the effect is not great.

rates, a rote application of a paid loss chain ladder development technique produces misleading reserve indications. A more sophisticated reserving technique would "strip" out past inflation from the historical triangles, determine the paid loss "age-to-age" link ratios, then restore expected future inflation to the indicated (future) link ratios.

Inflation Rates and Discount Rates

To account for the effects of inflation, we make the following adjustments to the stochastic simulation:

❶ We convert the paid losses to "real dollar" amounts by means of an appropriate inflation index. For workers compensation medical benefits, we use the medical component of the CPI.[32] We then determine paid loss "age-to-age" link ratios from the deflated figures, fit lognormal curves to each column, and run the simulation 10,000 times to determine the future link ratios.

❷ For each simulation, we select three (3) future inflation rates: 4%, 6%, and 8% per annum. We combine the simulated link ratios and the future (expected) inflation rate to determine the required reserves. For instance, one set of simulated link ratio may show required reserves of $75 million at a 4% future inflation rate and of $100 million at an 8% future inflation rate.

❸ For each set of simulated link ratios and expected future inflation rate, we determine three (3) required reserve amounts.

- The undiscounted (full value) reserve,
- The reserve discounted at the company's current investment yield, which was 6.75% per annum at the time of our analysis, and
- The reserve discounted at a rate midway between the company's current investment yield and the expected (future) inflation rate.

The third method in the list above is the most meaningful. It mimics the response of investment yields to the assumed future inflation rate. The investment yield is not fully responsive to inflation because (i) yields on long-term investments do not rise immediately with inflation and (ii) the assumed inflation rate applies only to medical inflation, not to overall inflation, which would be more closely correlated with interest rates.

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32 Actuaries differ on the appropriate deflator for each line of business. Our concern in this paper is the method of measuring uncertainty in loss reserves, not with specifying the proper deflator. Consistency and reasonableness in the choice of deflator are important. Slight differences in the index numbers have no significant effect on the measure of uncertainty.

❹ For each future inflation rate and discounting procedure, we obtain a distribution of required reserve amounts. As before, we determine the mean, the standard deviation, the two percentiles (5% and 95%), and the required (additional) capital to achieve a 1% EPD ratio. These figures are shown in the table below.

Medic Infla-tion		Average Reserve Amount	Standard Deviation of Reserve	95th Percentile of Reserve	5th Percentile of Reserve	Capital Needed for 1% EPD Ratio
4%	Undisc	77.2	11.1	96.4	61.6	14.0
4%	Disc: 6.75%	45.0	2.6	49.2	41.3	1.5
4%	Disc: 5.375%	48.6	3.0	53.6	44.2	1.9
6%	Undisc	86.8	16.7	117.1	64.4	25.0
6%	Disc: 6.75%	46.7	2.7	51.2	42.7	1.7
6%	Disc: 6.375%	47.7	2.8	52.4	43.4	1.8
8%	Undisc	105.9	32.3	166.5	68.2	63.0
8%	Disc: 6.75%	49.2	3.2	54.5	44.3	2.2
8%	Disc: 7.375%	47.1	2.8	51.8	42.9	1.8

To see the effects on reserve uncertainty, let us consider the middle case: historical inflation is removed from the paid loss "age-to-age" link ratios and then expected future inflation of 6% per annum is restored to the indicated link ratios. The required reserves are discounted at a 6.375% rate, which is midway between the current investment yield of 6.75% and the expected inflation rate of 6%.[33]

33 Some readers, citing Butsic's work, may wonder: "Should we not use a 'risk-adjusted discount rate,' which would be less than the risk-free rate and presumably less than the midpoint of the current investment yield and the expected inflation rate?" Butsic assumes that the reserves are uncertain, and he uses the risk adjustment to compensate the company or its investors for the uncertainty in these reserves. We want to first measure the uncertainty; for this purpose, we must use a discount rate that is not adjusted for risk. Once we have quantified the uncertainty, we proceed in one of two paths:

A. We may require additional capital, held by the company "below the line" (i.e., in the surplus account), or

B. We may discount the reserves at a lower rate, thereby holding additional funds "above the line" (i.e., in the reserve account).

The discounted reserves are $47.7 million. The full value (undiscounted) reserves are $86.8 million. Most companies use tabular discounts for lifetime pension indemnity benefits, and some companies do not fully account for inflation of medical benefits. For most companies, the held statutory reserves would be between $70 million and $80 million, for an "implicit interest margin" of about $30 million.

The additional capital required to achieve a 1% EPD ratio because of the reserve uncertainty is $1.8 million, which is only a small fraction of the "implicit interest margin" in the reserves themselves. Removing historical inflation from the observed link ratios, and then restoring expected future inflation to the indicated link ratios, makes the loss reserve distribution more compact and reduces the "uncertainty."

Loss-Sensitive Contracts

In the preceding sections, we have examined the uncertainty in the loss reserves. For business written on loss-sensitive contracts, such as retrospectively rated plans for large workers compensation risks or reinsurance treaties with sliding scale reinsurance commissions, companies are concerned with the uncertainty in the net reserves, or the future loss payments after adjustment for retrospective premiums and variable commissions.[34]

It would be "double-counting" to first discount the reserves at a risk-adjusted rate and then determine the additional capital needed to achieve a 1% EPD ratio.

Our approach, in concept, is similar to that use by Stephen Philbrick in his "Accounting for Risk Margins" (*Casualty Actuarial Society Forum* (Spring 1994), Volume I, pages 1-90). Philbrick first discounts reserves at the risk-free rate, and he uses Butsic's "expected policyholder deficit" procedure to determine the needed risk margins. He then adds part of this risk margin to the reserves (i.e., "above the line"), and he places the remaining portion in an allocated surplus account (i.e., "below the line"). As he notes, the portion of the risk margin that he places "above the line" is equivalent to discounting the loss reserves at a rate lower than the risk-free rate. For the equivalence of Butsic's and Philbrick's results, compare Butsic's derivation of the risk-adjustment in his "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach" (*Evaluating Insurance Company Liabilities* (Casualty Actuarial Society 1988 Discussion Paper Program), pages 147-188) with Philbrick's derivation of the "narrow risk margin" in his "Accounting for Risk Margins." Both Butsic and Philbrick produce figures that allow investors to achieve their desired returns even when the insurance company's investments are yielding only a risk-free rate.

[34] The effects on loss sensitive contracts on reserve uncertainty has become a significant regulatory and actuarial issue in recent years. The NAIC risk-based capital formula contains an offset of 15% to 30% to the reserving risk charge for business written on loss-sensitive contracts, based upon analysis begun by the authors of this paper and continued by the American

When the retrospective rating plan contains loss limits or premium maxima/minima, reserving risk remains, though it is dampened. These plans are more risky in some ways and less risky in other ways than traditional first dollar coverages are. The "pure insurance portion" of the plan is more risky, since

- The consideration paid by the insured is the "insurance charge" and

- The benefits paid by the insurer are the difference between (a) the value of the uncapped and unbounded premium and (b) the value of the capped and bounded premiums.[35]

The "pure insurance portion" is like excess-of-loss reinsurance, where the loss limit provides coverage like that of per-accident excess-of-loss and the premium bounds

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Academy of Actuaries task force on risk-based capital; see Sholom Feldblum, "Risk-Based Capital Requirements," *op cit.* For the 1996 Annual Statement, a new Part 7 has been added to Schedule P to measure the premium sensitivity to losses on loss-sensitive contracts; see Sholom Feldblum, "Completing and Using Schedule P," third edition (CAS Part 7 examination study note, 1996).

The text of this paper assumes familiarity with retrospective rating plans and with their parameters, such as loss limits and premium maximums and minimums, as well as with standard reserving techniques for retrospective premiums. More detailed information on the retrospective rating plan pricing parameters may be found in LeRoy J. Simon, "The 1965 Table M," *Proceedings of the Casualty Actuarial Society*, Volume 52 (1965), pages 1-45; David Skurnick, "The California Table L," *Proceedings of the Casualty Actuarial Society*, Volume 61 (1974), pages 117-140; Yoong-Sin Lee, "The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach," *Proceedings of the Casualty Actuarial Society*, Volume 75 (1988), pages 49-78; William R. Gillam and Richard H. Snader, "Fundamentals of Individual Risk Rating" (1992); and Robert K. Bender, "Aggregate Retrospective Premium Ratio as a Function of the Aggregate Incurred Loss Ratio," *Proceedings of the Casualty Actuarial Society*, Volume 81, Numbers 154 and 155 (1994), pages 36-74, along with the discussion by Howard C. Mahler, pages 75-90. The retrospective premium reserving techniques that underlie the analysis in this paper are discussed in Walter J. Fitzgibbon, Jr., "Reserving for Retrospective Returns," *Proceedings of the Casualty Actuarial Society*, Volume 52 (1965), pages 203-214; Charles H. Berry, "A Method for Setting Retro Reserves," *Proceedings of the Casualty Actuarial Society*, Volume 67 (1980), pages 226-238; and Michael T. S. Teng and Miriam Perkins, "Estimating the Premium Asset on Retrospectively Rated Policies" (*Proceedings of the Casualty Actuarial Society*, forthcoming).

[35] "Caps" refer to the loss limits; "bounds" refer to the premiums maximum and minimum. "Ratable losses" are paid by the insurer but reimbursed by the employer, so there is no insurance risk. Acquisition expenses, underwriting expenses, and adjustment expenses are paid by the insurer but reimbursed in the basic premium and in the loss conversion factor, again eliminating most of the risk to the insurer.

provide coverage like that of aggregate excess-of-loss. The variability of reserves for excess layers of coverage, per dollar of reserve, is generally greater than the corresponding variability of reserves for first dollar coverage.

If the retrospectively rated policy is considered as a whole – both the insurance portion and the "pass-through" portion – the retrospectively rated plan is less risky, per dollar of loss, than traditional first dollar coverage. In fact, if there are no loss limits and no maximum or minimum bounds on the premium, then the insurance contract becomes simply a financing vehicle and the insurance company serves as a claims administrator, not as a risk-taker. There is no underwriting or reserving uncertainty at all.[36]

Premium Sensitivity

How potent are loss sensitive contracts in reducing "net" loss reserve uncertainty? [By "net" loss reserve uncertainty, we mean the variability in the insurer's total reserves, or loss reserves minus retrospective premium reserves. "Accrued retrospective premium reserves" are carried as an asset on statutory financial statements, whereas loss reserves are carried as a liability.] The answer depends on the "premium sensitivity" of the plan: that is, the amount of additional premium generated by each additional dollar of loss.

We quantify the net loss reserve uncertainty in the same fashion as we did earlier, by asking: "How does reserve uncertainty affect the financial condition of the insurer?" For instance, if the required reserves turn out to be 15% higher than our current estimates, how much additional funds will the company need to meet its loss obligations?

For business which is not written on loss sensitive contracts, the answer is simple. The additional funds needed equal the additional dollars of loss minus the amount of any implicit interest cushion in the reserves.[37]

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36 There is still "credit risk." In the event of a large loss, the insured may be unable or unwilling pay the additional premiums, though the insurer is still liable to the injured employee for benefit payments. The credit risk may be mitigated by obtaining letters of credit to secure the insurance commitments. For further comments on the credit risk, see Howard W. Greene, "Retrospectively-Rated Workers Compensation Policies and Bankrupt Insureds," *Journal of Risk and Insurance*, Volume 7, No. 1 (September 1988), pages 52-58.

37 As noted above, a more sophisticated analysis is needed if the amount of the implicit interest cushion varies with the magnitude of the adverse development, as is true for workers compensation, where both the size of the tail factor and the amount of implicit interest cushion vary with inflation.

For business written on loss sensitive contracts, the answer is more complex, as the following illustration shows. Suppose that the indicated workers compensation reserves are $800 million. As a conservative range to guard against reserve uncertainty, the valuation actuary chooses an upper bound of $1,050 million as the worst case reserve estimate. The actuary estimates that there would be about $200 million of implicit interest margin in this scenario, so the capital needed to guard against reserve uncertainty is $50 million.[38]

Suppose now that half of the company's workers compensation business is written on retrospectively rated policies, of two types.

- Large accounts have plans with wide swings: loss limits and premium maximums are high, so each additional dollar of loss generates about a dollar of premium.

- Small and medium-size accounts have plans with narrower swings. Loss limits and premium maximums are lower and constrain the retro premiums. On average, each additional dollar of loss generates about 65¢ of additional premium.

For the entire book of retrospectively rated contracts, the premium sensitivity is 80%: that is, each additional dollar of loss generates about 80¢ of additional premium.

How much capital should this insurer hold to guard against reserve uncertainty? Well, suppose the needed reserves increase to the "worst case" scenario of $1,050 million. Half of this business is written on retrospectively rated plans, and the average premium sensitivity is 80%. In other words, of the adverse loss development of $250 million, $125 million occurs on retrospectively rated business. With a premium sensitivity of 80%, adverse loss development of $125 million generates $100 million of additional premium.

We add the $100 million of additional premium to the $200 of implicit interest margin to arrive at a solvency cushion of $300 million. Since the worst case adverse loss development is $250 million, the company already has a $50 million surplus solvency

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38 For ease of illustration, we assume here that the company wishes to hold a margin for reserve uncertainty even greater than the implicit interest margin. The preceding sections of this paper show that for workers compensation this implies a very low EPD ratio. Alternatively, using a "probability of ruin" perspective, the valuation actuary may desire to hold a margin such that there is an extremely low probability that the reserves plus the margin will be insufficient to pay the claims.

cushion in the carried reserves, so no additional capital is needed.[39]

In sum, loss sensitive contracts have potent implications for the quantification of reserve uncertainty. We examine this subject from two perspectives:

- A theoretical perspective, showing the factors affecting the risks in loss sensitive contracts, and

- A simulation perspective, showing the effects of loss sensitive contracts on our measures of reserve uncertainty.

Underwriting Risk and Reserving Risk

Our primary concern in this report is with reserve uncertainty. We can not answer this question empirically; rather, we must use simulation techniques, as we do in the previous sections of this paper.

The practical actuary is skeptical of simulation. So let us broaden our inquiry and ask: "To what extent do retrospectively rated policies mitigate underwriting uncertainty?" We can answer this question empirically, by comparing the variability of standard loss ratios and net loss ratios on a large and mature book of retrospectively rated workers' compensation policies.

- *Standard loss ratios* are incurred losses divided by standard earned premium. These loss ratios are influenced by random loss occurrences and premium rate fluctuations, and they vary considerably over time.

- *Net loss ratios* are incurred losses divided by the final earned premiums, as modified by retrospective adjustments. These adjustments counteract both the random loss occurrences and the fluctuations in manual rate levels, so the net loss ratios should be more stable over time.

Exhibit 3 shows these loss ratios for retrospectively rated policies issued by a large workers' compensation insurer. Only mature policies are used in this comparison, to

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39 Depending on the type of retrospective rating plan, an adjustment may be needed to bring the accrued retrospective premiums to present value. For "incurred loss" retro plans, the additional premium is billed and collected when the case reserves develop adversely, so no adjustment is needed. For "paid loss" retro plans, the additional premium is collected only when the losses are paid, so the present value of the retro premium is less than $100 million. In this illustration, the implicit interest margin in the loss reserves is $200 million ÷ $1,050 million, or 19%. If all the retro plans in this illustration were paid loss retros, and the additional premium is collected when the losses are paid, the present value of the additional premiums is $81 million.

ensure that the net loss ratios are not subject to significant additional retrospective adjustments.[40]

As expected, the mean loss ratios are similar for standard and net: 77.0% for standard and 78.8% for net. [The net loss ratios are slightly higher, since more retrospective premiums are returned than are collected.] The variances and standards deviations, however, differ greatly. The standard loss ratios show a variance of 46.9% and a standard deviation of 68.5%. Retrospective rating dampens the fluctuations in the loss ratios, leading to a variance of 11.2% and a standard deviation of 33.4%.

Reserve Uncertainty

Exhibit 3 deals with (prospective) underwriting risk, or the risk that future underwriting returns will be lower than anticipated.[41] Let us return now to reserving risk. We ask "To what extent is adverse development on existing losses mitigated by loss sensitive contracts?"

To resolve this issue, we must know the premium sensitivity of the retrospective rating plans, or the amount of additional premium received for each dollar of additional loss. Let us examine the variables that affect the premium sensitivity: the plan parameters, the current loss ratio, and the maturity of the reserves.[42]

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40 An earlier exhibit from this same book of business, produced by Dr. J. Eric Brosius, was provided by the authors to the American Academy of Actuaries task force on risk-based capital. It was used by the Tillinghast consulting firm to support the recommendations of the task force regarding a loss-sensitive contract offset to the reserving and underwriting risk charges in the NAIC risk-based capital formula. The exhibit in this paper, along with the variances and standard deviations, was produced by Miriam Perkins.

41 The NAIC risk-based capital formula called this "written premium" risk, since the capital requirements are dependent on the most recent year's premium volume. Part of the risk is that premium collections will be lower than expected, because of, say, underwriting cycle downturns or severe marketplace competition, and part of the risk is that losses will be higher than expected, because of, say, misestimation of claim frequency or severity or simply random loss occurrences.

42 Compare Robert K. Bender, "Aggregate Retrospective Premium Ratio as a Function of the Aggregate Incurred Loss Ratio," *Proceedings of the Casualty Actuarial Society*, Volume 81, Numbers 154 and 155 (1994), pages 36-74: "The aggregate premium returned to a group of individual risks that are subject to retrospective rating depends upon the retrospective rating formula, the aggregate loss ratio of the risks, and the distribution of the individual risks' loss ratios around the aggregate" (page 36).

● Plan Parameters

If the retrospective rating plan had no loss limits and no constraints on the final premium, then the premium sensitivity would equal the loss conversion factor, which is equal to or greater than unity. In most cases – and particularly for smaller risks – the loss limits and the premium maximums constrain the swing of the plan, and the premium sensitivity is lower than unity.

Generally, larger insureds choose retrospective rating plans with wide swings, while smaller insureds choose more constrained plans. To quantify premium sensitivity, therefore, the book of business should be divided into relatively homogeneous groups by size of risk, such as between medium sized risks and "national accounts."[43] [Small risks rarely use retrospective rating plans.]

The differences are dramatic. National accounts in our own book of business, with annual premium of $2 million or more per risk, almost always have wide swing plans, and the average premium sensitivity is close to unity. Medium sized risks, with more constrained plans, have an average premium sensitivity of about 65%.[44]

There are several additional items which should also be examined for a complete analysis of the effects of loss-sensitive contracts on reserve uncertainty. As noted earlier, we should look at the effects of "incurred loss" retros versus "paid loss" retros on the implicit interest margin in the accrued retrospective premiums. For ease of analysis, we assume here that all plans are paid loss retros: since the additional loss payments and the additional premium collections occur at the same time, the age-to-age link ratios can simply be adjusted for the premium sensitivity. This analysis is conservative. Incurred loss retros would show even greater dampening of the loss reserve uncertainty: since the premiums have less implicit interest margin, the effective premium sensitivity is greater than a nominal dollar analysis indicates.

In addition, a complete analysis should look at the effects of the plan parameters on the credit risk of the company and on the size of the implicit interest margin. The accrued retrospective premiums are a receivable, not an investable asset. As is true for losses, they are held on statutory financial statements at ultimate value, not at present value. If loss reserves are backed by accrued retrospective premiums, then either these premium reserves should be reduced to present value or the implicit interest margin in the loss reserves should be reduced.

[43] This subdivision of the data by size of insured or by "underwriting market" is generally available in company files. Of course, if the company keeps data by type of plan (wide swing plans vs. narrow swing plans and so forth), this more accurate subdivision is preferable.

[44] These are empirical figures, using actual ratios of retrospective premium collected to historical loss development. Robert Bender, "Aggregate Retrospective Premium Ratio as a Function of the Aggregate Incurred Loss Ratio," *Proceedings of the Casualty Actuarial Society*, Volume 81, Numbers 154 and 155 (1994), pages 36-74, using theoretical relationships

❷ Loss Ratio

The premium maximum and the loss limits constrain the swing of the plan. Ideally, we wish to know whether adverse loss development causes the retrospectively rated premium on each policy to hit the premium maximum or the loss to hit the loss limit. However, we do not have information on each individual change in reported losses. Actuaries estimate from aggregates, not from details. We must determine what aggregate statistics predict the average amount of retrospective premium that will be collected.

Given the parameters of any retrospectively rated plan, the loss ratio determines whether retrospective premium will be capped at the maximum. Given a distribution of loss ratios in a book business, all of which are written on similar retrospectively rated plans, we can determine the percent of plans which will hit the maximum premium. If the shape of this distribution does not depend significantly upon the average loss ratio of the book of business, and if we know the average loss ratio, then we can determine the percent of plans which will hit the maximum premium.

Premium sensitivity declines as the aggregate loss ratio increases. During poor underwriting years, when loss ratios are higher, adverse loss development leads to less additional premium than in good underwriting years, when loss ratios are lower.

❸ Reserve Duration

In workers' compensation, adverse loss development at early maturities stems from delayed reporting of some cases and primarily from the reclassification of non-serious cases to serious cases. For instance, almost all lower back sprains are initially classified as short-term temporary total cases. Significant case reserve development is expected in the first two or three years, as some of these claims develop into

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based on the NCCI's "Table M," estimates premium sensitivity for various risk sizes. Bender's analysis is a useful check on our procedure, but it is not a substitute. His analysis posits that the Table M relationships are correct, and that compensation carriers actually use the NCCI Table M insurance charges to price their retrospectively rated policies. In practice, insurers use a variety of plans for their large insureds, and they often negotiate the loss limits, premium maximum, and plan parameters in each case for their national accounts. We rely, therefore, on the actual results of the plans sold "on the street," not on *a priori* actuarial expectations.

As emphasized in Howard Mahler's discussion of Bender's paper (pages 75-90 of *PCAS*, Volume 81), the premium sensitivity is strongly dependent on the size of the risk. Bender analyzes primarily small risks, where the premium sensitivity is weak. The sensitivity rises rapidly with the size of the risk; see especially Bender's Table 5 on page 50, which shows the "slope" of the plan as a function of the "loss group," and Mahler's comments on pages 76-78.

permanent partial or permanent total cases. Much of this development is within the "ratable" area of the retrospective rating plan: for instance, a $10,000 claim is reclassified as a $100,000 claim, so premium sensitivity is high.

At later maturities, adverse loss development stems primarily from re-estimation of the costs of permanent cases. For a plan with low or even moderate loss limits, almost all the adverse loss reserve development after five or six years occurs in the "non-rateable" portion of the retrospective rating plan. For instance, a $300,000 claim is re-estimated at $400,000, where average premium sensitivity is low.

Furthermore, many companies "close" their retrospective rating plans after, say, five or seven years, with a final accounting between the company and the insured. Adverse development occurring after this date would not affect the retrospective premiums.

Effects on the Simulation

For the simulation, we use premium sensitivity factors based on observed long-term patterns by market and by reserve duration in our countrywide book of business.[45]

❶ The market dimension is binary: large (national) accounts vs medium-sized accounts. As noted above, our national accounts have wide swing plans with high premium sensitivities. Our medium sized accounts have more constrained plans with more moderate premium sensitivities.

❷ For reserve duration, we use annual periods demarcated by retrospective premium adjustment points. In other words, we look at the change in incurred losses from first to second retrospective premium adjustment, from second to third adjustment, and so forth, and we match these with the change in retrospective premium from first to second adjustment, from second to third adjustment, and so forth. There is a delay of about 4 to 5 months between the loss change and the corresponding premium change. That is, the standard workers compensation first retrospective adjustment may be done at 21 months subsequent to policy inception, using losses evaluated at 18 months, with the additional or return

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[45] To avoid undue complexity, we do not use aggregate loss ratio in the simulation analysis. To incorporate the aggregate loss ratio dimension, we would have to evaluate the effect of each simulated link ratio on the new accident year loss ratio, and determine a new premium sensitivity factor for every cell in every simulation. Moreover, since we are using paid loss age-to-age factors, we would have to convert paid loss ratios to incurred loss ratios. The benefits from these refinements is far less than the additional effort.

premium booked in the 23nd or 22rst month.[46]

From the empirical data we produce two curves, each showing premium sensitivity by reserve duration, one for national accounts and one for medium-sized risks. We weight these two curves by the volume of business in these two markets.

In the simulation analysis, we first repeat the steps outlined earlier. Based upon historical experience, we estimate (deterministically) the amount of case reserves associated with each cumulative paid loss amount at each duration. From the change in reported losses, we determine the change in retrospective premiums, and thereby the change in "net reserves."

For the exhibits in this paper, we vary this procedure. The effects of loss sensitive contracts vary greatly by type of plan and by company practice. As some reviewers of an earlier draft of this paper have pointed out to us: "Your company writes primarily large accounts and uses highly sensitive, wide swing plans. For this type of business, the "net" reserve uncertainty is clearly mitigated. What about other companies, which use less sensitive plans, recognize the adverse development later, and close their plans after several years? Would they also show a significant reduction in net reserve uncertainty?"

This criticism is legitimate. For the exhibits in this paper, we make three adjustments, to model the loss sensitive contracts often used for smaller risks:

- *The retrospective plans are relatively insensitive. In fact, for the most recent* accident year, the assumed premium sensitivity is 49%.
- We assume that most adverse development is recognized late.
- We assume that the plans are closed, on average, about five to ten years after policy inception. With the late recognition of the adverse development and the relative early closure of the plans, even the limited premium sensitivity is markedly reduced for earlier accident years.

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46 See the paper by Teng and Perkins, "Estimating the Premium Asset on Retrospectively Rated Policies" (*op. cit.*) for a complete discussion of this. In truth, the premium sensitivity is the change in booked premium between the 23rd and the 35th month compared to the change in incurred losses between the 18th and the 30th month. Because we have data only for quarter-end points, and because premiums bookings on some risks are late, we use either a two quarter or a three quarter lag, not a five month lag.

As noted above, premium sensitivity varies by plan and by company. The new Part 7 of Schedule P, which is designed to measure premium sensitivity on loss sensitive contracts, has no lag; see Sholom Feldblum, "Completing and Using Schedule P, Third Edition (1996), for a more complete discussion of this. Thus, the aggregate industry data provided by Schedule P, Part 7, will not be of much aid in estimating premium sensitivities.

Even with these adjustments, the projected reserve distribution is more compact, and there is less "reserve uncertainty." The table below compares the results for the loss-sensitive contracts versus non-loss-sensitive contracts, for discounted reserves.

As before, the figures are normalized to an undiscounted "gross" loss reserve of $100 million. The discounted "gross" (i.e., loss only) reserve is $52.7 million, whereas the discounted "net" (i.e., loss minus premium) reserve is $46.5 million, for a reduction of 12%. The capital needed to achieve a 1% EPD ratio decreases from $2.4 million to $1.9 million, for a reduction of 21%. In other words, the required capital declines from about 5% of required reserves to about 4% of required reserves.

Discounted reserves at 6.75% annual rate:	Average Reserve Amount	Standard Deviation of Reserve	95th Percentile of Reserve	5th Percentile of Reserve	Capital Needed for 1% EPD Ratio
Loss only	52.7	3.4	58.7	47.9	2.4
Loss – premium	46.5	2.8	51.3	42.6	1.9

We now combine the effects of loss sensitive contracts with the inflation adjustments discussed earlier. The table below shows the results for discounted and undiscounted reserves, with inflation stripped out and then built back in at 4%, 6%, and 8% per annum. These figures should be compared with the figures shown in the previous section for the "gross" (loss only) reserve uncertainty.

Medic Inflation		Average Reserve Amount	Standard Deviation of Reserve	95th Percentile of Reserve	5th Percentile of Reserve	Capital Needed for 1% EPD Ratio
4%	Undisc	69.4	9.9	86.8	55.4	12.4
4%	Disc: 6.75%	36.1	2.2	39.6	32.5	1.5
4%	Disc: 5.375%	39.4	2.5	43.4	35.7	1.6
6%	Undisc	77.3	14.7	104.4	57.8	21.7
6%	Disc: 6.75%	37.4	2.2	41.1	33.9	1.3
6%	Disc: 6.375%	38.4	2.2	42.1	34.9	1.3
8%	Undisc	93.6	27.5	145.8	60.8	52.3
8%	Disc: 6.75%	40.1	2.1	43.7	37.1	1.1
8%	Disc: 7.375%	38.4	2.0	41.8	35.4	1.1

Exhibit 4 graphs these probability distributions, for

- Loss sensitive versus non-loss sensitive contracts, and
- Both discounted and undiscounted reserves, with
- Future inflation projected at 6% per annum, and
- A 6.75% discount rate (for the discounted reserves).

The probability distributions for the loss sensitive contracts are not just shifted to the left, since the "net" (loss minus premium) reserve is less than the "gross" (loss only) reserve. Rather, the probability distributions for the loss sensitive contracts are also more compact; there is less uncertainty.

Conclusions

Casualty actuaries have developed numerous methods of estimating required loss reserves. But reserves are uncertain, and actuaries are now being asked to quantify the uncertainty inherent in the reserve estimates.

Many past attempts to address this subject have foundered on one of two shoals. Some attempts are silver vessels of pure theory: loss frequencies are simulated by Poisson functions, loss severity is simulated by lognormal distributions, inflation is simulated by Brownian movements, and the results are much prized by hypothetical companies. Other attempts are steel vessels of actual experience: actual reserve changes, taken from financial statements, reveal how companies have acted in the past, though they offer imperfect clues about the uncertainties inherent in the reserve estimation process itself.

This paper glides between the shoals. Loss reserve uncertainty must be tied to the line of business. The uncertainty in workers' compensation reserves is different from the uncertainty in general liability reserves even as it is different from the uncertainty in life insurance or annuity reserves. We begin with extensive data: twenty five years of experience from the nation's premier workers compensation carrier.

These data allow the actuary to develop reserve indications. Our concerns in this paper are different. We fit these data to families of curves to develop probability distributions of required reserves. The power of stochastic simulation techniques enables us to develop thousands of potential outcomes that are solidly rooted in the empirical data.

The analysis shows that workers compensation reserves, when valued on a discounted basis, have a highly compact distribution. To measure uncertainty, we use an "expected policyholder deficit (EPD) ratio" yardstick. For workers' compensation, the amount of capital needed to achieve a 1% EPD ratio is only a small fraction of the "implicit interest margin" in the reserves themselves.

The vicissitudes of inflation are a major cause of workers compensation reserve fluctuations, particularly for full value (undiscounted) estimates. The paper shows the effects of different future expected inflation rates on the uncertainty inherent in the reserve estimates.

Insurers are risk averse, even as other economic actors are. They use policy exclusions and contract provisions to mitigate the risks that they undertake. The paper shows how the use of retrospectively rated workers' compensation policies dampen the uncertainty in the reserve estimates.

The combination of rigorous actuarial theory with an extensive empirical database enables us to examine the uncertainty in the reserves themselves. Similar analyses should be performed for other lines of business, such as automobile insurance or general liability. Comparisons among the lines, as well as comparisons of reserve uncertainty with underwriting risks and with asset risks, would allow us to exchange preconceived notions with well-supported facts.

EXHIBIT 1

Distributions of Reserves, Without Inflation Adjustments

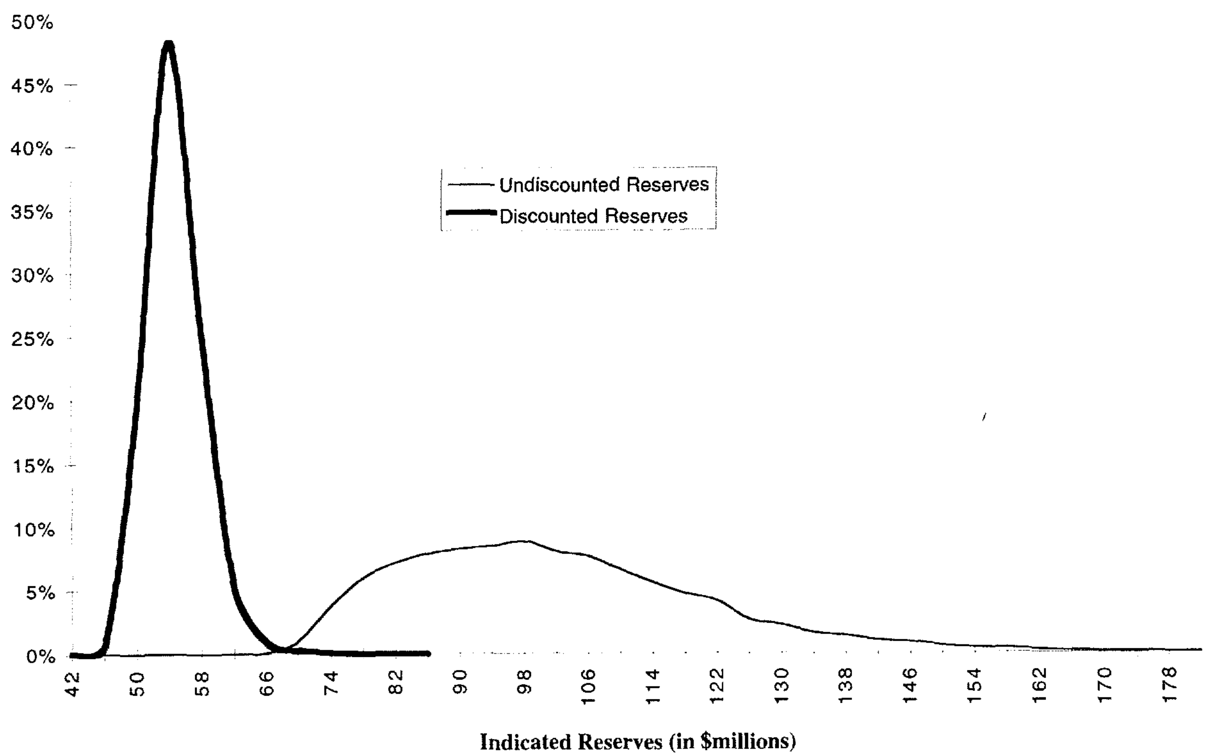
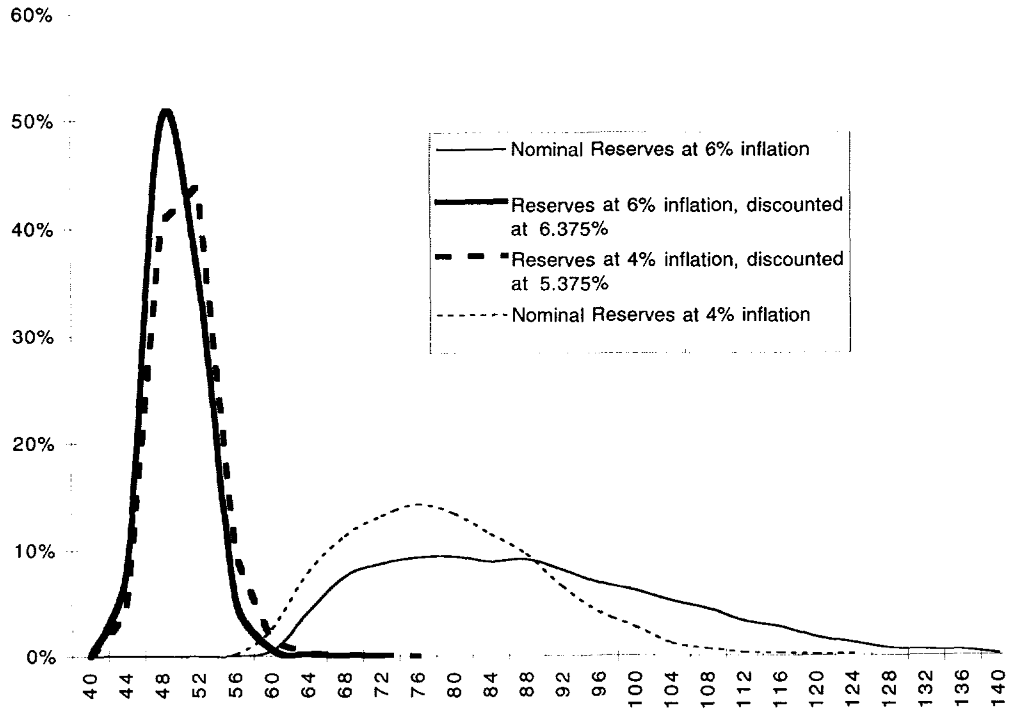


EXHIBIT 2

Distributions of Reserves, with Inflation Stripped Out and Built Back at the Specified Rates



Indicated Reserves (in $millions)

EXHIBIT 3

WORKERS COMPENSATION
NATIONAL ACCOUNTS RETRO POLICIES

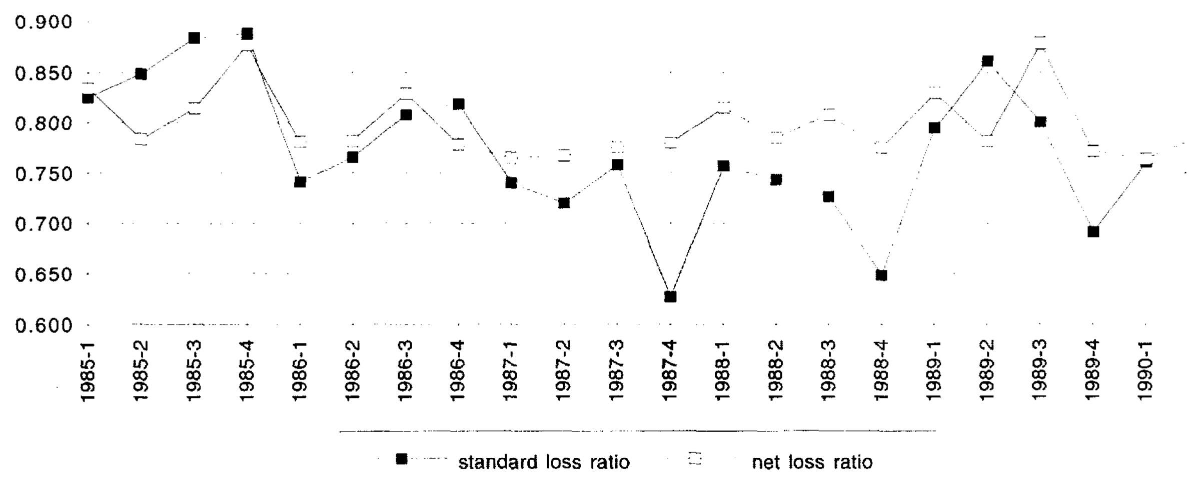
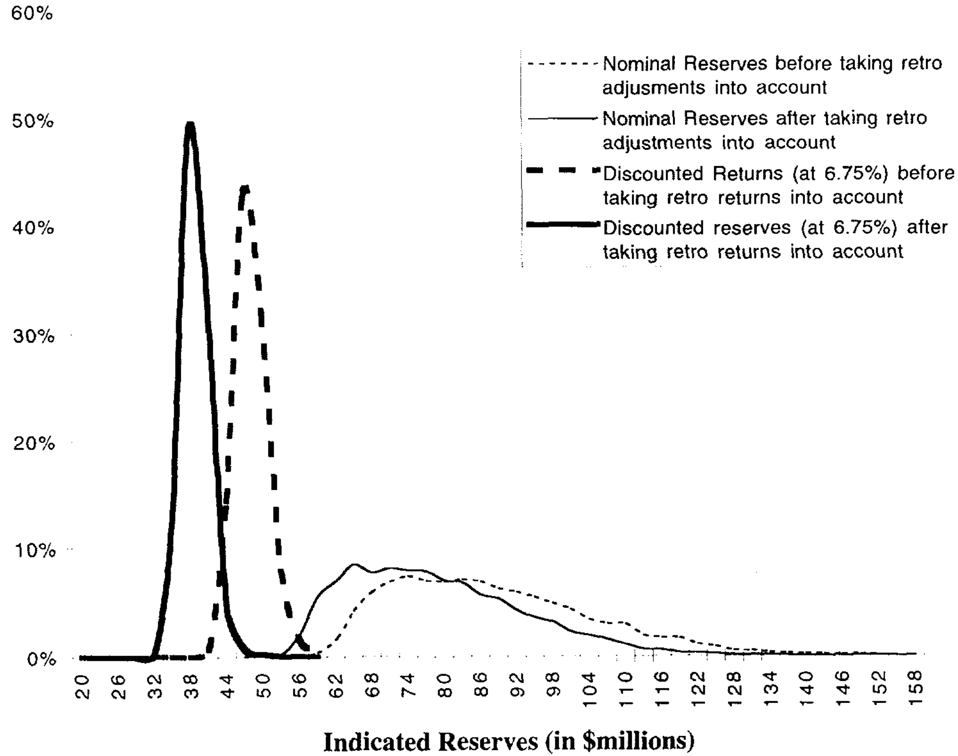


EXHIBIT 4

Distributions of Reserves, With and Without Retro Adjustments (Assuming Inflation at 6%)



Appendix A: Workers Compensation Reserves and Risk-Based Capital Requirements

The text of this paper distinguishes between "regulatory measures" of reserving risk, as used in the NAIC's risk-based capital formula, and "actuarial measures" of reserving risk, as quantified here. The analysis in this paper shows that the volatility inherent in workers compensation reserve estimates is well below the implicit interest margin in statutory (undiscounted) reserves. The NAIC risk-based capital formula, however, has a reserving risk charge of 11% for workers compensation, even after incorporation of the expected investment income on the assets supporting the reserves.

An actuary unfamiliar with the development of the workers' compensation reserving risk charge in the risk-based capital formula might conclude that "regulatory measures" of workers compensation reserving risk give high capital charges whereas "actuarial measures" give low charges. This is not correct. The risk-based capital formula gives a low charge for workers compensation reserving risk, even as the actuarial analysis in this paper provides. The final 11% charge in the risk-based capital formula is an *ad hoc* revision intended to provide more "reasonable" capital requirements.

The workers compensation reserving risk charge was one of the most contested aspects of the risk-based capital formula, and the derivation of the final 11% charge was never publicly revealed. This appendix explains the issues relating to the workers' compensation reserving risk charge, and it shows the charge resulting from the NAIC "worst-case year" method.

Adverse Development and Loss Reserve Discounting

The reserving risk charge in the risk-based capital formula bases the capital requirements on the historical adverse loss development in each line of business. The "worst-case" industry-wide adverse loss development as a percentage of initial reserves is determined from Schedule P data, and this figure is then reduced by a conservative estimate of expected investment income.

For workers compensation, the original risk-based capital formula produced a charge of 0.4%.[47] Current Best's *Aggregates and Averages* data show a gross "worst-case year" adverse development of 24.2%, as derived in Exhibit A-1.

Two considerations related to loss reserve discounting complicate the estimation of the reserving risk charge for workers compensation.

- Statutory accounting conventions for property-casualty insurers are conservative, particularly with regard to the reporting of loss reserves. Life insurers show discounted loss reserves, with sufficient margins in the valuation rate to ensure that benefit obligations are met. Property-casualty insurers show undiscounted reserves, leaving a large margin in the reserves themselves, particularly for long-tailed lines of business.

  In other words, property-casualty insurers have two potential margins to ensure adequacy of loss reserves: an implicit interest margin in the reserves themselves, and an explicit capital requirement provided by the reserving risk charge. To avoid "double counting," the risk-based capital formula offsets the implicit interest margin against the explicit reserving risk charge.

- The "double margin" occurs when reserves are reported on an undiscounted basis. But some property-casualty reserves are reported on at least a partially discounted basis. For instance, many carriers use tabular discounts for workers compensation lifetime pension claims. The special statutory treatment of workers compensation lifetime pension cases necessitates adjustments to the reserving risk charge.

Both the NAIC Risk-Based Capital Working Group and the American Academy of Actuaries task force on risk-based capital spent months working on these two topics. The issues are complex, and no clear explanation is available for either regulators or for industry personnel. Some actuaries, in fact, presume that the 11% charge for workers compensation results from the effects of tabular reserve discounts. To clarify the issues, this appendix discusses the treatment of the implicit interest margin in statutory reserves and the adjustments needed for tabular loss reserve discounts in workers' compensation.

Payment Patterns and Discount Rates

The amount of the implicit interest margin, or the difference between undiscounted (full-value) reserves and discounted (economic) reserves, depends on two items: the

[47] For a full description of the risk-based capital reserving risk charges, see Sholom Feldblum, "Risk-Based Capital Requirements" (Casualty Actuarial Society Part 10 examination study note).

payout pattern of the loss reserves and the interest rate used to discount them.

For most lines of business, the NAIC risk-based capital formula uses the IRS loss reserve payment pattern along with a flat 5% discount rate. These choices were made for simplicity. Using the IRS discounting pattern avoids the need to examine loss reserve payout patterns, and using a flat 5% discount rate avoids the need to examine investment yields. For some lines of business, these choices are acceptable proxies for good solvency regulation. For workers' compensation, greater complexities arise.

- *Payment Pattern*: The IRS procedure assumes that all losses are paid out within 15 years. Moreover, the pattern is based on the industry data for the first 10 years as reported in Schedule P.

  For short-tailed lines of business, this is not unreasonable, since most losses are indeed paid out before the Schedule P triangles end. Workers compensation reserves, however, have a payout schedule of about 50 years, since permanent total disability cases – which are a small percentage of the claim count but a large percentage of the dollar amount – extend for the lifetime of the injured worker.

- *Discount Rate*: For its discount rate, the IRS uses a 60 month rolling average of the federal midterm rate, which is defined as the average yield on outstanding Treasury securities with maturities between 3 and 9 years. Since 1986, the IRS discount rate has ranged between 7 and 8%.

  Actual portfolio yields have been about 100 to 200 basis points higher, since insurance companies invest not only in Treasury securities but also in corporate bonds, common stocks, real estate, and mortgages. However, these latter investment vehicles have additional risks, such as default risks, market risks, and liquidity risks. As a loss reserve discounting rate, many casualty actuaries would prefer the 7 to 8% "risk-free" Treasury rate to the 8 to 10% portfolio rate, particularly for statutory financial statements, which emphasis solvency.

  The NAIC risk-based capital formula uses a flat 5% discount rate. A variety of justifications have been given, such as

  - The 5% rate is simple, obviating any need to examine actual investment yields and cutting off any arguments about the "appropriate" rate.

  - The 5% rate adds an additional margin of conservatism, since it is 2 to 3 points lower than the corresponding IRS rate.

  For lines of business where the implicit interest margin in the reserves is small, the difference between the 5% NAIC rate and the 7 to 8% IRS rate is not that important in setting capital requirements. For a line of business like workers compensation, however, where the discount factor ranges from 60% to 83%, depending upon the assumptions, the choice of discount rate has a great effect.

We begin the analysis below with the current NAIC risk-based capital assumptions to see the unadjusted charge produced by the formula. We then turn to actual payment patterns and investment yields to address the fundamental questions: "What is the risk associated with workers compensation loss reserves? And how much capital ought insurance companies to hold to guard against this risk?"

The IRS Discount Factor

The IRS determines the loss reserves payout pattern by examining the ratio of paid losses to incurred losses by line of business for each accident year from Part 1 of Schedule P. The data are drawn from Best's *Aggregates and Averages*, and the payout pattern is redetermined every five years.

Schedule P shows only 10 years of data, though several lines of business, such as workers compensation, have payout schedules extending up to 50 years. The IRS allows an extension of the payout pattern beyond the 10 years shown in Schedule P for up to an additional 6 years. The extension of the payout pattern does not rely on either empirical data or financial expectations. Rather, the payout percentage in the tenth year is repeated for each succeeding year until all reserves are paid out.

Accident Years vs. Aggregate Reserves

The IRS determines a discount factor for each accident year. Companies determine discounted reserves by multiplying the statutory reserves by the discount factor for the appropriate line of business and accident year. The risk-based capital formula uses a single discount factor for all accident years combined. Thus, one must use a weighted average of the discount factors, based on the expected reserves by accident year.[48]

Exhibit A-2 shows the workers compensation payment pattern using the IRS procedures and the most recent Best's *Aggregates and Averages* Schedule P data.

☛ The left-most column shows the payment year. Because workers' compensation

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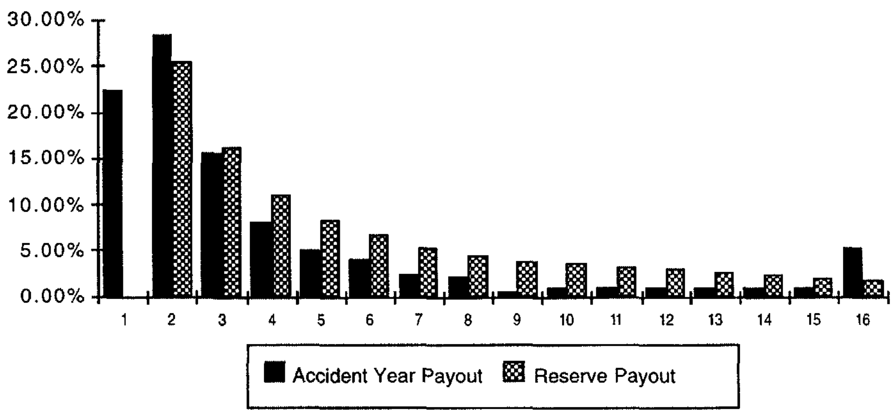
[48] For simplicity, the calculations in this paper assume that the volume of workers compensation business is remaining steady from year to year. A theoretical refinement would be to use the actual volume of industry wide workers compensation reserves in each of the past ten years, though there is no significant difference in the result.

reserves are paid out so slowly, the IRS extends the payment schedule for the full 16 years. It is still far too short, particularly for lifetime pension cases.

- The middle column shows the payment schedule for an individual accident year. This payment schedule says that 22.34% of an accident year's incurred losses are paid in the first calendar year, 28.36% in the next calendar year, and so forth.

- The right-most column shows the payment schedule for the aggregate reserves, assuming no change in business volume over the 16 year period. This payment schedule says that 25.42% of the reserves will be paid in the immediately following calendar year, 16.14% in the next calendar year, and so forth.

The graph below shows the payout patterns for an individual accident year and for the aggregate reserves. The horizontal axis represents time since the inception of the most recent accident year. The accident year payout pattern begins with the first losses paid on the policy, soon after the inception of the accident year. The valuation date of the reserves in the graph is the conclusion of the most recent accident year, so the payout pattern begins in the second year since inception.



The payout pattern is combined with an annual interest rate to give the discount factor, or the ratio of discounted reserves to undiscounted reserves. With an interest rate of 5% per annum, the discount factor for the reserves is 82.98%. The risk-based capital formula would therefore indicate a reserving risk charge of

$$[1.242 * 82.98\%] - 1 = 3.06\%.$$

The 3% reserving risk charge depends upon the conservative 5% annual interest rate the short IRS payment pattern. More realistic interest rates and payment patterns, even when still containing margins for conservatism, lead to a negative charge. We discuss these in conjunction with tabular loss reserve discounts below.

Discounted Reserves

What if an insurer holds discounted reserves, or partially discounted reserves? How should the reserving risk procedure described above be modified to account for the reserve discount?

This question is most relevant for workers compensation. Statutory accounting normally requires that insurers report undiscounted, or full-value, reserves. An exception is made for workers compensation lifetime pension cases, where insurers are allowed to value indemnity (lost income) reserves on a discounted basis. State statutes often mandate conservative discount rates, usually between 3.5% and 5% per annum, with the most common being 4%. These reserve discounts are termed "tabular" discounts, since they are determined from mortality tables, not from aggregate cash flow analyses.

Adverse Development and Interest Unwinding

The combination of three factors – (a) adverse development, (b) the unwinding of interest discounts, and (c) weekly claim payments – produces intricate results that are difficult even for the most technically oriented readers to follow. So let us begin with a simple example, which illustrates the concepts discussed above.

Suppose we have one claim, which will be used for determining both the "worst case" adverse loss development and the interest discount factor. The claim occurred in 1987, and it will be paid in 1997 for $10,000.

Suppose first that the company accurately estimates the ultimate settlement amount and sets up this value at its initial reserve. Adverse loss development in this "worst case year" is 0%. Since there is a substantial implicit interest offset – the claim is paid 10 years after it occurs – the final reserving risk charge will be negative.[49]

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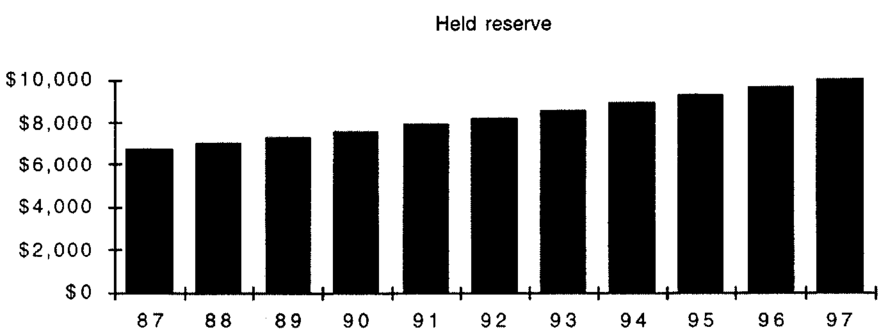
[49] In practice, there are no negative charges in the NAIC risk-based capital formula, since all charges are bounded below by 0%. However, the example in the text is a heuristic illustration. It is meant to demonstrate the principles, not to reflect actual industry experience or all aspects of the NAIC formula. For instance, the industry "worst case year" would not be determined from one claim, and the "implicit interest margin" is based on the IRS loss reserve discounting procedure, not the cash flow pattern of the losses. As the illustrations proceed, however, we remove the unrealistic assumptions, so that readers can see the actual workers' compensation industry-wide loss reserve payment patterns.

How large is the offset for the implicit interest discount? For a claim that is paid ten years after it occurs, with a 5% per annum discount factor, the offset is $1 \div (1.05)^{10}$ = 61.39%. The final reserving risk charge in this simplified illustration is –38.60%.

What if the company holds the reserve on a discounted basis, using a 4% per annum discount rate? In 1987, the company sets up a reserve of [$10,000 ÷ $(1.04)^{10}$], or $6,756. In 1988, the discounted reserve increases to [$10,000 ÷ $(1.04)^{9}$], or $7,026. In 1989, the discounted reserve increases to [$10,000 ÷ $(1.04)^{8}$], or $7,307.

The increases in the held reserve, from $6,756 to $7,026 in 1988, and from $7,026 to $7,307 in 1989, stem from the "unwinding" of the interest discount. However, they show up in Schedule P of the Annual Statement just like any other adverse development.[50]

The chart below shows the unwinding of the 4% interest discount over the course of the ten years that the reserve is on the company's books. Between 1987 and 1992, the held reserve increases from $6,756 to $8,219, for observed adverse loss development during this period of 21.67% [= (8,219 – 6,756) ÷ 6,756].



The unwinding of the interest discount during 1987 through 1992 is reflected in the observed adverse development, so it is picked up by the NAIC calculation of the reserving risk charge. That is,

☛ A valuation basis that uses undiscounted reserves shows no adverse loss

50 This was true for the *pre-1995* Schedule P, when Part 2 was net of tabular discounts, though it was gross of non-tabular discounts. In 1995 and subsequent Annual Statements, Part 2 of Schedule P is gross of all discounts, so the unwinding of the interest discount no longer shows up as adverse development. The NAIC risk-based capital reserving risk charges were derived from the 1992 Schedule P.

development on this claim.

- A valuation basis that uses reserves discounted at a 4% annual rate shows 21.67% of observed loss development.

The higher risk-based capital reserving risk charge generated by the discounted reserves is offset by the lower reserves held by the company.

Future Interest Unwinding

The unwinding of the interest discount continues from 1992 through 1997. Since this future unwinding is not yet reflected in the Schedule P exhibits of historical adverse loss development, a modification of the standard reserving risk charge calculation is needed.

What adjustment is needed? Consider the assumptions underlying the reserving risk charge. The derivation of the reserving risk charge said:

> *Let us select the "worst case" adverse loss development that happened between 1983 and 1992, and let us assume that it might happen again.*

This procedure assumes that the 1992 reserves are adequate. That is to say, we should not *expect* either adverse or favorable development of the 1992 reserves.[51]

This is the proper assumption for the risk-based capital formula. The observed adverse loss development is meant to capture unanticipated external factors that cause higher or lower settlement values for insurance claims. A line of business may show adverse loss development even if the initial reserves were properly set on a "best estimate" basis. If a company is indeed holding inadequate reserves, it is the task of the financial examiners of the domiciliary state's insurance department to correct the situation. This is not the role of the generic risk-based capital formula.

If the reserves are valued on a discounted basis, however, they will continue to show (apparent) adverse development until all the claims are settled. In the example above,

- The unwinding of the interest discount between 1987 and 1992 is reflected in the observed adverse loss development, and no further adjustments are needed.
- The unwinding of the interest discount between 1992 and 1997 is not reflected

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51 We do not *expect* either adverse or favorable development of the 1992 reserves. The risk-based capital requirement guards against *unexpected* adverse development of the reserves.

anywhere, so an adjustment to the calculation procedure must be made.

Alternative Adjustments

There are two ways to make this adjustment: either in the "worst case year" industry adverse loss development or in the offset for the implicit interest discount.

➊ *Adverse loss development*: One might add the expected future unwinding of the interest discount that will occur after the final valuation date to the "worst case year" observed adverse loss development. In the example above, the observed adverse loss development from 1987 to 1992 is \$1,464, giving a factor of +21.7% as a percentage of beginning reserves. We expect further adverse loss development of \$1,781 from 1992 to 1997 because of continued unwinding of the interest discount. The total adverse loss development is therefore \$3,245, or +48.0% as a percentage of beginning reserves.

➋ *Implicit interest discount*: The further unwinding of the actual interest discount in the reserves may be used to reduce the offset for the implicit interest discount. In the example above, the observed adverse loss development is offset by ten years of implicit interest discount at a 5% annual rate. However, there are five years of unwinding of the actual 4% interest discount that are still to come (1992 through 1997), and that are not reflected in the observed adverse development.

In our illustration, ten years of implicit interest discount at a 5% annual rate gives a discount factor of 61.4%. Five future years of actual interest unwinding at a 4% annual rate gives a discount factor of 82.2%. The interest margin that should offset the "worst case year" adverse loss development is the *excess* of the implicit interest cushion over the actual interest discount, or 74.7% [= 61.2% ÷ 82.2%].

Diversity and other Obstacles

In practice, the needed adjustments for tabular discounts are difficult to determine, for a variety of reasons.

➊ *Industry Practice*: There is great disparity among insurance companies in the use of tabular reserve discounts. The prevalent practice is to use tabular discounts on indemnity benefits for lifetime pension cases. But there are companies that do not use tabular reserve discounts at all, and that report aggregate loss reserves on a full-value basis.[52]

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52 More precisely, the case reserves generally show the tabular discounts. However, these discounts are "grossed up," or eliminated, by the actuarial "bulk" reserves.

❷ *Pension Identification*: Some companies show tabular discounts only for claims that have been identified as lifetime pension cases. Other companies show tabular discounts for the expected amount of claims that will ultimately be coded as lifetime pension cases.

The distinction between "identified" and "unidentified" lifetime pension cases is analogous to the distinction between "reported" and "IBNR" claims. A workers' compensation claim may be reported to the company soon after it occurs, but it may remain "unidentified" as a lifetime pension case for several years.[53]

❸ *Indemnity vs. Medical Benefits*: Workers compensation benefits comprise two

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53 An example should clarify this. Suppose an insurer has 100 claims at the conclusion of a given accident year:

- Ten of the claims are lifetime pension cases, with undiscounted reserves of $300,000 apiece. These is a $100,000 tabular discount on each claim, so the discounted reserves are $200,000 apiece.
- The other 90 non-pension cases are given case reserves of $5,000 apiece.
- Based upon historical experience, the company's actuaries expect that five of these "non-pension" claims will eventually become lifetime pension cases, since the injured workers will prove unable to return to work. These five cases will require $300,000 of undiscounted reserves apiece, or $200,000 of discounted reserves.

Number	Case Reserves		Revised Reserves	
of Claims	Undiscounted	Discounted	Undiscounted	Discounted
10	300,000	200,000	300,000	200,000
85	5,000	—	5,000	—
5	5,000	—	300,000	200,000

The total case reserve initially held by the company is

$$(10 * \$200,000) + (90 * \$5,000) = \$2,045,000.$$

An actuarial bulk reserve is needed for the adverse development on known cases, or the required reserve minus the case reserve. But how large a bulk reserve is needed here? Is it

$$[5 * (\$300,000 - \$5,000)] = \$1,475,000,$$

or

$$[5 * (\$200,000 - \$5,000)] = \$975,000?$$

In other words, should one consider the tabular reserve discount only on claims that have *already* been identified as pension cases or also on the expected number of claims that will *ultimately* be identified as lifetime pension cases?

parts: indemnity benefits, which cover the loss of income, and medical benefits, which cover such expenses as hospital stays and physicians' fees.

Lifetime pension cases may show continuing payments of both types. For instance, an injured worker who becomes a quadriplegic may receive a weekly indemnity check for loss of income as well as compensation for the medical costs of around-the-clock nursing care.

Some insurers will discount only the indemnity benefits, since the weekly benefits are fixed by statute.[54] Other insurers will discount the medical benefits as well, since the payments are regular and do not vary significantly, even if they are not fixed by statute.

❹ *Interest Rates*: The interest rate use for the tabular reserve discounts varies by company and by state of domicile. Some companies use a 3.5% annual rate, since this is the interest rate used in the NCCI statistical plan. Several New York and Pennsylvania domiciled companies use a 5% annual rate, since this is the rate permitted by statute in these states. Other companies may use a 4% annual rate, since this is the most common rate in other state statutes.

Pension Discounts

The 3.06% reserving risk charge calculated above uses the conservative 5% interest rate in the risk-based capital formula and the short IRS payment pattern. But our interest extends beyond simply replicating the formula results. We want to analyze the actual reserving risk, and to determine how much capital an insurance company must hold to guard against this risk.

As we have discussed above, the NAIC reserving risk charge presumes that loss reserves are reported at undiscounted values. If reserves are valued on a discounted basis – as is true for certain workers compensation cases – then one expects future "adverse development," so the NAIC procedure is incomplete.

What is the expected effect of tabular discounts on the reserving risk charge for workers compensation? Analysts unfamiliar with workers' compensation are tempted to say: *It should increase the charge.*

This would indeed be true if lifetime pension cases had the same payment pattern as other workers compensation claims and the only difference between pension cases and other compensation claims were that the pension cases are reported on a

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[54] In some states, the indemnity benefit may depend on cost of living adjustments, so the amounts are not entirely "fixed."

discounted basis whereas the other compensation claims are reported on an undiscounted basis. But this is not so. In fact, the very reason that tabular reserve discounts are permitted for lifetime pension cases is that they are paid slowly but steadily over the course of decades.

In other words, to properly incorporate tabular discounts into the workers' compensation reserving risk charge, two changes are needed:

- One must increase the "worst case year" adverse development to include the future unwinding of the interest discount on the pension cases. Alternatively, one may adjust the "implicit interest discount" offset to account for the discount already included in the reported reserves.

- One must adjust the payout pattern from the IRS sixteen year pattern to the longer pattern appropriate for lifetime pension cases.

The net effect is to reduce the reserving risk charge. In fact, the indicated charge becomes negative, so it would be capped at 0% by the NAIC formula rules.

This is expected. The NAIC risk-based capital formula imposes a reserving risk charge when the "worst case" adverse development exceeds the implicit interest margin in the reserves. For lines of business like products liability and non-proportional reinsurance, the potential adverse development may far exceed the implicit interest margin, so companies must hold substantial amounts of capital to guard against reserving risk. For workers compensation "non-pension" cases, the mandated statutory benefits reduces the risk of adverse development while the slow payment pattern increases the implicit interest discount, so that the latter almost entirely offsets the former, resulting in the 3% charge calculated above with the RBC formula's exceedingly conservative assumptions. For workers compensation lifetime pension cases, true adverse development practically disappears, since mortality rates do not fluctuate randomly, and only the unwinding of the tabular discount remains. Because of the extremely long payout pattern for lifetime pension cases and the low interest rate allowed for tabular discounts, the implicit interest margin in lifetime pension reserves is well in excess of the "worst case" adverse development.

What is the appropriate reserving risk charge for workers compensation, after taking into consideration the tabular discounts on lifetime pension cases? To calculate this charge, we make the two adjustments discussed above.

❶ We replace the IRS payment pattern with a 50 year payment pattern derived from the historical experience of the nation's largest compensation carrier. At a 5% per annum interest rate, the present value of the reserves is 65.6% of the ultimate value, as shown in Exhibit A-3.[55]

❷ We increase the "worst case year" adverse development to incorporate the future interest unwinding on lifetime pension cases. The observed "worst case year" adverse development is 24.2% of initial reserves, from the 1985 statement date to the 1992 statement date. This includes the unwinding of tabular interest discount between 1985 and 1992. The post-1992 unwinding of interest discount on these pension cases adds between 6 and 8% to this figure. To be conservative, we use the 8% endpoint, giving a total adverse development of 34.1%.[56]

❸ The resulting reserving risk charge is (1.341 *x* 0.656) – 1, or –14.1%. In other words, industry-wide workers compensation reserves have always been adequate on a discounted basis, even during the worst of years.

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55 Are statistics from a single carrier, no matter how large, a valid proxy for industry-wide figures? For loss ratios, expense ratios, and profit margins they are not appropriate, since each carrier has its own operating strategy. But workers compensation payment patterns are determined by statute; they do not differ significantly among companies.

56 For the unwinding of the tabular interest discount, it is no longer appropriate to use a single company's experience as a proxy for the industry. Insurers vary in whether they use tabular discounts at all, what types of benefits they apply the discounts to, and what interest rate they use to discount the reserves. The "6 to 8%" range in the text results from extended observation of reserving practices in workers compensation, along with detailed analysis of one company's own experience. With the reporting of tabular discounts in the 1994 Schedule P, more refined estimates of industry-wide practice may soon be available.

EXHIBIT A-1

NAIC METHOD

Consolidated Industry 1992 Schedule P, Part 2D (Workers Compensation)
Incurred Losses and ALAE

	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
All Prior	18,141,872	18,124,544	18,133,835	18,522,890	18,876,893	19,168,300	19,695,156	20,083,948	20,568,671	21,085,073
1983	10,285,007	10,518,014	10,615,001	10,800,631	10,904,709	11,053,667	11,087,456	11,163,710	11,309,445	11,364,446
1984		11,935,500	12,483,704	12,996,457	13,398,843	13,641,258	13,807,452	13,890,249	14,025,270	14,170,486
1985			13,506,212	14,148,315	14,560,000	15,036,193	15,289,931	15,451,016	15,648,178	15,824,280
1986				15,657,270	16,137,074	16,618,301	16,738,489	16,875,565	17,142,404	17,341,361
1987					18,543,543	18,630,232	18,849,648	18,945,479	19,228,271	19,492,604
1988						21,144,056	21,525,659	21,824,122	22,103,365	22,403,642
1989							23,337,805	23,983,219	24,549,997	24,863,843
1990								25,687,116	26,642,155	26,948,591
1991									27,107,842	27,477,716
1992										25,391,687
Total Incurred	28,426,879	40,578,058	54,738,752	72,125,563	92,421,062	115,292,007	140,331,596	167,904,424	198,325,598	226,363,729
Latest View of Incurreds	32,449,519	46,620,005	62,444,285	79,785,646	99,278,250	121,681,892	146,545,735	173,494,326	200,972,042	226,363,729
Adverse Development	4,022,640	6,041,947	7,705,533	7,660,083	6,857,188	6,389,885	6,214,139	5,589,902	2,646,444	0

Consolidated Industry 1992 Schedule P, Part 3D (Workers Compensation)
Paid Losses and ALAE

	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
All Prior	0	3,644,371	6,120,130	7,945,566	9,435,974	10,681,184	11,778,823	12,725,216	13,559,492	14,283,870
1983	2,595,880	5,414,887	6,989,233	8,016,341	8,714,150	9,206,673	9,565,906	9,842,305	10,035,523	10,182,271
1984		3,098,456	6,475,507	8,592,479	9,951,477	10,858,138	11,474,586	11,929,005	12,268,081	12,519,733
1985			3,307,517	7,223,536	9,609,598	11,175,251	12,188,709	12,884,942	13,409,641	13,785,641
1986				3,399,423	7,693,744	10,430,068	12,205,296	13,319,794	14,100,702	14,642,580
1987					3,823,180	8,916,751	12,037,953	13,992,209	15,236,501	16,062,480
1988						4,517,537	10,522,224	14,272,224	16,542,809	17,976,821
1989							4,923,056	11,851,679	16,021,809	18,519,232
1990								5,283,149	12,856,717	17,435,376
1991									5,481,562	12,644,529
1992										4,795,009

Consolidated Industry 1992 Schedule P, [(Part 2D) - (Part 3D)] (Workers Compensation)
Loss and ALAE Reserves

	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
All Prior	18,141,872	14,480,173	12,013,705	10,577,324	9,440,919	8,487,116	7,916,333	7,358,732	7,009,179	6,801,203
1983	7,689,127	5,103,127	3,625,768	2,784,290	2,190,559	1,846,994	1,521,550	1,321,405	1,273,922	1,182,175
1984		8,837,044	6,008,197	4,403,978	3,447,366	2,783,120	2,332,866	1,961,244	1,757,189	1,650,753
1985			10,198,695	6,924,779	4,950,402	3,860,942	3,101,222	2,566,074	2,238,537	2,038,639
1986				12,257,847	8,443,330	6,188,233	4,533,193	3,555,771	3,041,702	2,698,781
1987					14,720,363	9,713,481	6,811,695	4,953,270	3,991,770	3,430,124
1988						16,626,519	11,003,435	7,551,898	5,560,556	4,426,821
1989							18,414,749	12,131,540	8,528,188	6,344,611
1990								20,403,967	13,785,438	9,513,215
1991									21,626,280	14,833,187
1992										20,596,678
Reserves Held (1)	25,830,999	28,420,344	31,846,365	36,948,218	43,192,939	49,506,405	55,635,043	61,803,901	68,812,761	[illegible]
Adverse Development (2)	4,022,640	6,041,947	7,705,533	7,660,083	6,857,188	6,389,885	6,214,139	5,589,902	2,646,444	
(2) / (1)	15.6%	21.3%	24.2%	20.7%	15.9%	12.9%	11.2%	9.0%	3.8%	[illegible]

Worst Year Development: 24.2%

EXHIBIT A-2

IRS Payment Pattern

Year	Payment Pattern (Single Accident Year)	Payment Pattern (Stationary Book)
	Accident Year Payout	Reserve Payout
1	22.34%	0.00%
2	28.36%	25.42%
3	15.49%	16.14%
4	8.23%	11.07%
5	5.14%	8.37%
6	4.16%	6.69%
7	2.41%	5.33%
8	2.31%	4.54%
9	0.52%	3.78%
10	0.96%	3.61%
11	0.96%	3.30%
12	0.96%	2.98%
13	0.96%	2.67%
14	0.96%	2.35%
15	0.96%	2.03%
16	5.25%	1.72%

EXHIBIT A-3

Workers Compensation Payment Pattern

Year	Payment Pattern (Single Accident Year)	Payment Pattern (Stationary Book)
1	0.190	
2	0.213	0.127
3	0.127	0.094
4	0.083	0.074
5	0.057	0.061
6	0.041	0.052
7	0.032	0.045
8	0.025	0.041
9	0.021	0.037
10	0.016	0.033
11	0.014	0.031
12	0.013	0.028
13	0.011	0.026
14	0.010	0.025
15	0.009	0.023
16	0.009	0.022
17	0.009	0.020
18	0.007	0.019
19	0.006	0.018
20	0.006	0.017
21	0.006	0.016
22	0.005	0.015
23	0.006	0.014
24	0.005	0.013
25	0.005	0.013
26	0.004	0.012
27	0.004	0.011
28	0.004	0.010
29	0.004	0.010
30	0.004	0.009
31	0.004	0.009
32	0.003	0.008
33	0.003	0.008
34	0.003	0.007
35	0.003	0.006
36	0.003	0.006
37	0.003	0.006
38	0.003	0.005
39	0.003	0.005
40	0.003	0.004
41	0.003	0.004
42	0.003	0.003
43	0.003	0.003
44	0.002	0.003
45	0.002	0.002
46	0.002	0.002
47	0.002	0.001
48	0.002	0.001
49	0.002	0.001
50	0.002	0.000
Total (Excluding first 12 months)	0.810	1.000
PV @ 5%	**0.767**	**0.656**

Appendix B: The "Expected Policyholder Deficit" Yardstick

Quantifying Reserve Uncertainty

Reserve uncertainty is a slippery concept, difficult to grasp and even more difficult to quantify. Actuaries are accustomed to estimating dollar figures, such as premium rates, reserve requirements, or company values. In truth, for each of these there is a range of reasonable values. But the actuary's skill is in forming a "best estimate" that accords with the data and that is appropriate for the particular business environment, such as the insurance marketplace for the premium rates, a statutory financial statement for the reserve requirements, or a merger transaction for the company valuation.

Quantifying reserve uncertainty is a more complex task. A statistician might discuss reserve uncertainty as a probability distribution. One might show the mean of the distribution, its variance, and its higher moments; one might show various percentiles; one might even try to fit the empirical distribution to a mathematical curve. Accordingly, the exhibits in this paper show the mean, the standard deviation, the 95^{th} *percentile, and the* 5^{th} *percentile of each of the distributions.*

Probability Distributions

Yet probability distributions are an awkward way of speaking about reserve uncertainty, for several reasons.

- Few non-actuaries are comfortable with standard deviations or higher moments of probability distributions. If actuarial analyses are to have much influence with other company personnel, they must be couched in language that others understand.

- Property-casualty reserves are uncertain by definition, since they are only estimates of future loss payments. Similarly, future underwriting results are uncertain, future stock returns are uncertain, and so forth. The question is not: "Are *reserves uncertain?"* Rather, *the fundamental question is:* "How does the uncertainty of loss reserves compare with the uncertainty in other parts of the *company's operations?"* To *answer this question, we need a consistent measure* of uncertainty that can be used for various types of risk.

- We want to measure the effect of various factors on reserve uncertainty. For instance, we want to quantify the effect of loss sensitive business on reserve uncertainty. It is hard to do this if we must speak about probability distributions. To facilitate the analysis, we seek a measuring rod for uncertainty.

Ideally, we seek a yardstick that expresses uncertainty (i) as a dollar figure and (ii) that has an intuitive meaning to other financial analysts. Moreover, the yardstick should apply to all sources of uncertainty, whether of assets or of liabilities.

Capital Requirements

In recent years, state and federal regulators have been setting capital requirements for financial institutions, such as for banks and insurance companies. In theory, "risk-based capital requirement" relate the capital requirements to the uncertainty in various balance sheet items. In practice, most of the risk-based capital formulas that have been implemented in recent years use crude, generic charges that are based more on *ad hoc* considerations of what constitutes a "reasonable" charge than on rigorous actuarial or financial analyses.

Risk-based capital theory, however, is a siren for some actuaries and academicians, who have examined the relationship between uncertainty and capital requirements. In an ideal risk-based capital system, capital requirements should be calibrated among the balance sheet items in proportion to the risk that each poses to the company's solvency. Suppose a company has $100 million of bonds and $100 of loss reserves, and the theoretically correct risk-based capital system says that the company needs $5 million of capital to guard against the uncertainty in the bond returns and $15 million of capital to guard against the uncertainty in the loss reserve payments. Then we can say that the uncertainty in the loss reserve portfolio is "three times as great" as the uncertainty in the bond portfolio.

Of course, we don't really mean that "uncertainty" is a absolute quantity that can be three times as great as some other figure. Rather, our measuring rod gives us a figure that we use as a proxy for the amount of uncertainty.

Moreover, our interest is not in absolute capital requirements but in the *relative* uncertainty among the company's various components. The regulator must indeed calibrate the absolute capital requirements, deciding between (i) $5 million of capital for bond risk and $15 million of capital for reserve risk versus (ii) $10 million of capital for bond risk and $30 million of capital for reserve risk. For the measurement of uncertainty, however, we are most interested in relative figures, such as the relative amount of capital needed to guard against reserve risk versus the amount needed to guard against bond risk, or the percentage reduction in capital for business written on loss sensitive contracts.

Calibrating Capital Requirements

There are two "actuarial" methods of calibrating capital requirements.

- The "probability of ruin" method says: How much capital is needed such that the chance of the company's insolvency during the coming time period is equal to or less than a given percentage?

- The "expected policyholder deficit" method says: How much capital is needed such that the expected loss to policyholders and claimants during the coming time period – as a percentage of the company's obligations to them – is equal to or less than a given *amount?*[57]

In this paper, we use the "expected policyholder deficit" (EPD) approach. The results would be no different if we used a "probability of ruin" approach.[58]

Computing the Expected Policyholder Deficit

The "expected policyholder deficit" is a relatively new concept, having first been introduced in 1992. Many casualty actuaries, even if they are conversant in reserve estimation techniques and familiar with curve fitting and Monte Carlo simulation, have little experience with EPD analysis. This appendix provides a brief outline of the EPD analysis used in the paper.

Let us repeat the underlying question, which (at first) sounds complex. The EPD

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57 The "probability of ruin" method is used in Chris D. Daykin, Teivo Pentikäinen, and M. Pesonen, *Practical Risk Theory for Actuaries*, First Edition (Chapman and Hall, 1994). Probability of ruin analysis has long been used by European actuaries; see especially R. E. Beard, T. Pentikainen, and E. Pesonen, *Risk Theory: The Stochastic Basis of Insurance*, Third Edition (London: Chapman and Hall, 1984), and Newton L. Bowers, Jr., Hans U. Gerber, James C. Hickman, Donald A. Jones, and Cecil J. Nesbitt, *Actuarial Mathematics* (Itasca, IL.: Society of Actuaries, 1986). The "expected policyholder deficit" method is discussed by Robert P. Butsic, "Solvency Measurement for Property-Liability Risk-Based Capital Applications," *Journal of Risk and Insurance*, Volume 61, Number 4 (December 1994), pages 656-690.

58 We use the expected policyholder deficit approach partly because it has already been calibrated to the NAIC's risk-based capital formula; see below in the text. In theory, the probability of ruin approach takes the company's perspective, whereas the expected policyholder deficit approach takes the policyholders' perspective. The company's management is concerned primarily with the company's survival. Since the company's officers are not liable for the company's debt upon its demise, they are not concerned with the potential magnitude of that debt. The policyholders, however, are concerned with how much they stand to lose if the company becomes insolvent, not simply with the probability of insolvency.

analysis says: "Given a probability distribution for an uncertain balance sheet item, how much capital must the company hold such that the ratio of the expected loss to policyholders to the obligations to policyholders is less than or equal to a desired amount?" The format of the analysis depends on the type of probability distribution.

- For a simple discrete distribution, we can work out by hand the exact capital requirement. The type of simple discrete distribution that we illustrate below never occurs in real life. We use it only as a heuristic example, since the same procedure is used in our simulation analysis.

- The full simulation analysis is a complex time-consuming procedure; see the next bullet item. If the empirical probability distribution can be modeled by a mathematically tractable curve, a closed-form analytic expression for the EPD can sometimes be found. In his previously cited paper, Robert Butsic does this for the normal and lognormal distributions, which can serve as reasonable proxies for many balance sheet items.

  For the analysis in this paper, we use as few preconceived assumptions about the probability distributions of loss reserves as possible. Thus, we avoid such statements as "Assume that loss frequency follows a Poisson distribution." Instead, we use the following method to determine the "expected policyholder deficit" ratio.

- The distributions in this paper are derived by means of stochastic simulation. Each distribution results from 10,000 Monte Carlo simulations, with each simulation using stochastic values for each of the 24 "age-to-age" link ratio and for the length of the tail. For each distribution we determine the amount of capital needed to achieve a desired EPD ratio, as explained below.

Let us begin with the first case, the simple discrete distribution, to illustrate how the analysis proceeds. The extension to the full stochastic simulation merely requires greater computer power; there is no difference in the structure of the analysis.

Scenarios and Deficits

The distributions used in this paper are based on 10,000 simulations each. Think of this as 10,000 different scenarios. In fact, however, these simulations are *stochastic*. We do not know what these simulations are until after they have been realized. In other words, there are an infinite number of *possible* scenarios, 10,000 of which will be realized in the simulation.

It is cumbersome to follow a "pencil and paper" analysis of an infinite set of possible scenarios. So to clarify the meaning of the "expected policyholder deficit," let us assume that only two future scenarios are possible:

- In the *favorable* scenario, the company's assets are $350 million, and the company must pay losses of $200 million.

- In the *adverse* scenario, the company's assets are $250 million, and the company must pay losses of $300 million.

Suppose also that there is a 60% chance of the favorable scenario being realized and a 40% chance of the adverse scenario being realized.[59]

What is the expected policyholder deficit? In the favorable scenario, the company has a positive net worth at the end. Since we are concerned only with deficits, a positive outcome of any size is considered a $0 deficit.

In the adverse scenario, the final deficit is a $50 million deficit, or –$50 million. Since there is a 60% chance of a favorable outcome and a 40% chance of an adverse outcome, the <u>expected</u> policyholder deficit is

$0 million * 60% + (–$50 million * 40%) = –$20 million.

The EPD <u>Ratio</u>

The definition of the EPD ratio is as

The average insolvency cost per dollar of obligation to policyholders, or "the ratio of the expected policyholder deficit to expected loss."

The numerator of this ratio is the expected policyholder deficit. The denominator is the obligations to policyholders, or the "expected loss."

In the numerator, the expected policyholder deficit, the loss payments to policyholders and claimants enter as a probability distribution (either discrete or continuous). In the denominator, the "obligations to policyholders," or the "expected loss," is an

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59 In the simulation analysis in this paper, only reserves are uncertain; assets are not uncertain. However, the same type of analysis applies to both assets and liabilities. Indeed, a more complete model would examine the external (economic and financial) factors that lead to variability in ultimate loss reserves and it would analyze their effects on asset values as well.

In the simulation analyses used here, each simulation has a 0.01% chance of realization, since there are 10,000 equally likely runs. There is no need, however, for a uniform distribution along the range of possible outcomes, as shown by the example in the text.

absolute dollar amount. [In the simulation analysis in this paper, the "expected loss" is the mean of the loss reserve probability distribution. When performing an EPD analysis for other risks (such as asset risk), the numerator and denominator of the EPD ratio may be unrelated.]

In the example above, there is a 60% chance of a $200 million payment to claimants and a 40% chance of a $300 million payment to claimants. Thus, the "obligations to policyholders" is

($200 million * 60%) + ($300 million * 40%) = $240 million.

These figures may be either "economic values" (i.e., discounted reserve values) or undiscounted ("ultimate") values. Moreover, the discounted values may use various discount rates, such as a "risk-free" rate, a "risk-adjusted" rate, or a new-money investment yield. We show the analysis for both undiscounted and discounted values in our exhibits. For the discounted values, we generally use the company's new-money investment yield as the discount rate.

Consistency

We use a 1% expected policyholder deficit ratio to determine the capital requirements. We use 1% to be consistent with the charges in the NAIC risk-based capital formula. In memoranda submitted to the American Academy of Actuaries task force on risk-based capital, Butsic estimates that the overall industry-wide reserving risk charge in the NAIC risk-based capital formula amounts to approximately a 1% EPD ratio.

This allows us to compare the workers' compensation loss reserve uncertainty to other sources of insurance company risk. If one believes that the overall capital requirements in the NAIC risk-based capital formula are reasonable, so a 1% EPD ratio is appropriate, then the degree of workers' compensation loss reserve uncertainty measured in this paper can be viewed in light of the other NAIC capital requirements. As Butsic says

> *The amount of risk-based capital for each source of risk (e.g., underwriting, investment, or credit) must be such that the risk of insolvency (or other applicable impairment) is directly proportional to the amount of risk-based capital for each source of risk.*

Discrete Distributions

Our purpose in this section is to understand how capital requirements are determined by means of an expected policyholder deficit analysis. So let us suppose that only liabilities (not assets) are uncertain, where

- X and Y are the two possible loss outcomes,
- p is the probability of the true loss being equal to X, and
- Z is the expected policyholder deficit ratio.

	Asset Amount	Loss Amount	Probability	Loss Payment	Deficit
Scenario #1	A	X	p	E	G
Scenario #2	B	Y	(1−p)	F	H
Expected Value	C	D			
Capital		I			
EPD Ratio		Z			

We must calculate the risk-based capital amount, or C − D, which equals the assets minus the expected loss amount. Note that C = B = A in this example; i.e., the liability is uncertain, but the asset value is certain. This is the format of our simulation analysis as well, except that (i) there are 10,000 scenarios, (ii) the probability of each scenario is 0.01%, and (iii) the scenarios are stochastic, not deterministic.

An Illustration

To calculate the value of C − D, we must determine the other values in this chart: A through H. To make the explanation clear, let us fill in sample values for X, Y, p, and Z. Suppose we are told that

> An insurance company faces a single uncertain loss. There is a 25% chance that the loss will be paid for $1,000 and a 75% chance that the loss will be paid for $5,000. The risk-based capital requirements use a 1% expected policyholder deficit (EPD) ratio. Using an EPD analysis, calculate the risk-based capital requirement for this risk.

The table below shows these input figures as well as the value for cell "D," the expected loss amount. We must "solve" this exhibit for the risk-based capital requirement.

	Asset	Loss	Proba-	Loss	
	Amount	Amount	bility	Payment	Deficit
Scenario #1	A	$1,000	25%	E	G
Scenario #2	B	$5,000	75%	F	H
Expected Value	C	$4,000			
Capital		I			
EPD Ratio		1%			

The expected loss is

"D" = 25% * $1,000 + 75% * $5,000 = $4,000.

If the expected loss is $4,000, then the company must hold *at least* $4,000 in assets. If the actual loss amount is $1,000, the company will be able to pay the entire claim and the "deficit" will be zero. Thus, cell "E" is $1,000, and cell "G" is $0.

The EPD ratio is the expected policyholder deficit divided by the obligations to policyholders. The denominator is the expected loss amount, or $4,000. The numerator is the EPD. The EPD is the deficit in cell "H" times the probability of 75%. In other words

EPD ratio = 75% * H ÷ $4,000 = 1%.

This gives

H = $4,000 * 1% ÷ 75% = $53.33.

The deficit is the loss amount minus the claim payment. Thus

$5,000 – claim payment = $53.33, or

Cell "F" = claim payment = $4,946.67.

The company makes a claim payment less than the claim amount only if it exhausts all its assets in doing so. Thus, cell "B" equals $4,946.67. Since the initial asset amount does not depend on the eventual claim payment, cells "A" and "C" also equal $4,946.67.

The company's capital is the asset value minus the expected loss payment, or

Cell "I" = capital = \$4,946.67 − \$4,000 = \$946.67.

These figures are shown in the table below.

	Asset Amount	Loss Amount	Probability	Loss Payment	Deficit
Scenario #1	\$4,946.67	\$1,000	25%	\$1,000	\$0
Scenario #2	\$4,946.67	\$5,000	75%	\$4,946.67	\$53.33
Expected Value	\$4,946.67	\$4,000			
Capital		\$946.67			
EPD Ratio		1%			

Full Simulation

The full analysis in this paper proceeds in the same fashion. The 10,000 simulations are run, each of which produces a "realization" for the loss amount. The average of these 10,000 realizations is the expected loss. The probability of each realization is 0.01%.

We first assume that the asset amount equals the expected loss, and we determine the loss payment and the deficit in each realization.

- If the loss amount is less than the asset amount, then the loss payment equals the loss amount, and the deficit is zero.

- If the loss amount exceeds the asset amount, then the loss payment equals the asset amount and the deficit is the difference between the loss amount and the asset amount.

We sum the deficits in the 10,000 realizations, and we divide by 10,000. This gives the expected policyholder deficit. We then divide by the expected loss amount to give the EPD ratio.

If the probability distribution for the loss reserves is extremely compact, then the EPD ratio may be less than 1% even if no additional capital is held. For instance, suppose that the probability distribution is uniform over the range \$100 million $\pm$ \$4 million. Then the expected policyholder deficit is 1% if no additional capital is held.[60] This makes sense: if the loss payments are practically certain, there would be no need for additional capital.

In practice, of course, the loss payments are not certain, and the EPD ratio would be greater than 1% if no additional capital is held. We proceed iteratively. We add capital and redetermine the loss payment and deficit in each scenario. This gives a new expected policyholder deficit and a new EPD ratio. If the EPD ratio still exceeds 1%, we must add more capital. If the EPD ratio is now less than 1%, we can subtract capital. With sufficient computer power, we quickly converge to a 1% EPD ratio.

60 If the actual loss is less than \$100 million, then the deficit is zero. If the actual loss exceeds \$100 million, then the deficit is uniform over [\$0, \$4 million], for an average of \$2 million. The expected deficit over all cases is therefore \$1 million, for an EPD ratio of 1%.

Appendix C: The Simulation Procedure

Casualty *actuaries are accustomed to providing point estimates of indicated* reserves. The traditional procedures – such as a chain ladder loss development using 25 *accident years of experience, supplemented by an "inverse power curve"* tail factor – provide a sound basis for estimating workers' compensation reserve needs. The actuary's task is to examine the historical experience for trends, evaluate the effects of internal (operational) changes on case reserving practices and settlement patterns, and forecast the likely influence of future economic and legal developments on the company's loss obligations.

Our perspective in this paper is different. We are not determining a point estimate of the reserve need; rather, we are determining a probability distribution for the reserve need. We use the same procedure and the same data as we would use for the point estimate: a chain ladder loss development based on 25 accident years of experience, along with a tail factor based on an inverse power curve fit. But now each step turns stochastic, and the probability distribution is determined by a Monte Carlo simulation.

The traditional procedures for determining point estimates are documented in various *textbooks. This appendix shows the corresponding procedures for determining the* probability distribution.

Data

We use a chain ladder *paid* loss development, since payment patterns for workers' compensation are relatively stable whereas case reserving practices often differ from company to company and from year to year. This enables readers to replicate our results using their own companies' data.

We begin with 25 accident year triangles of cumulative paid losses, separately for indemnity (wage loss) and medical benefits. Indemnity and medical benefits have different loss payment patterns, and they are affected by different factors. For *instance, medical benefits are strongly affected by medical inflation and by changes in* medical utilization rates.[61]

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[61] Numerous other segmentations of the data can be used. For instance, many companies divide their experience by type of injury, separating lifetime pension cases (i.e., fatalities and permanent total disabilities cases) from other claims. Similarly, other reserving techniques can be used, such as incurred loss development chain ladder procedures and various types of "expected loss" (Bornhuetter-Ferguson) procedures. The analysis in this paper can be extended to other data sets and other reserving procedures, as required by individual company

From the historical data we determine paid loss "age-to-age" factors (or "link ratios). Exhibit C-1 shows 22 columns of paid loss age-to-age factors for countrywide indemnity plus ALAE benefits. For instance, the column labeled "12-24" shows the ratio of cumulative paid indemnity losses at 24 months to the corresponding cumulative paid indemnity losses at 12 months for each accident year. Similarly, Exhibit C-2 shows the paid loss age-to-age factors for countrywide medical benefits.

Point Estimates versus Realizations

The reserving actuary, when determining a point estimate, would examine these factors for trends. For instance, the average of the most recent five factors in the indemnity plus ALAE "12 to 24 months" column in Exhibit C-1 is 2.514, whereas *all* the previous factors are less than 2.500. For a point estimate, the reserving actuary might use an average of the most recent five factors, instead of an average of all the factors in the column.

In this paper, our goal is to estimate the uncertainty in the reserve indications. Just as there was an upward trend in the age-to-age factors during the 1980s, there may be subsequent upward or downward trends in the 1990s. We therefore use the entire column of factors in our analysis. An "outlying" factor that is not a good estimator of the expected future value is an important element in measuring the potential variability of the future value.

We want to use the historical factors to simulate future "realizations." We do this by fitting the observed factors to a mathematical curve, thereby obtaining a probability distribution for the "12 to 24" age-to-age factors. Note carefully: This is *not* the probability distribution of the loss reserves, which will be the *output* of the simulation and which is *not* modeled by any mathematical function. This is the probability distribution of the age-to-age factors, which is the *input* to the simulation and is modeled by a mathematical curve.

Lognormal Curve Fitting

In this analysis, we used lognormal curves, which gave good fits to the data. Exhibit C-3 shows the curve fitting procedure for the first column of "indemnity plus ALAE" age-to-age factors.

For the lognormal curve, the probability distribution function is

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needs.

$$f(x) = \frac{e^{-.5[\frac{\ln(x)-\mu}{\sigma}]^2}}{x\sigma\sqrt{2\pi}}$$

and the cumulative distribution function is

$$F(x) = \Phi(\frac{\ln(x)-\mu}{\sigma})$$

We fit the function with the "development" part of the link ratios, or the "age-to-age factor minus one," as shown in column 2 of Exhibit C-3. Column 3 shows the natural logarithms of the factors in column 2. Using the method of moments to find the parameters of the fitted curve, the "mu" (μ) parameter is the mean of the figures in column 3 and the "sigma" (σ) parameter is the standard deviation of the figures in column 3.[62]

We do the same for each "age-to-age" development column. The fitted parameters shown in the box in Exhibit C-3 are carried back to the final two rows in Exhibit C-1. Thus, each column has its own lognormal probability distribution function. We do this for development through 276 months. There is still paid loss development after 276 months, but there is insufficient historical experience to generate the factors, so we use an inverse power curve to estimate the loss development "tail" (see below).

For each run, we use a random number generator [Excel's "RAND" built-in function] to obtain simulated "age-to-age" factors in each column. Column 3 of Exhibit C-4 shows the results of one simulation for indemnity plus ALAE payments.[63] For instance, the simulated age-to-age factor for 12 to 24 months of development is 2.409. The simulations for each of the 22 columns are independent of each other. For instance, the simulated 1.413 factor for "24 to 36" months in column 3 of Exhibit C-4 is independent of the simulated 2.409 factor for "12 to 24" months.[64]

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62 For a more complete discussion of curve fitting procedures, see R. V. Hogg and Stuart A. Klugman, *Loss Distributions* (Somerset, NJ: John Wiley and Sons, 1984).

63 For a more complete explanation of simulation techniques, see the Society of Actuaries study note 130-33-86, "An Introduction to Stochastic Simulation."

64 Our analysis assumes independence between columns and complete dependence among future accident years. Dependence among columns may raise or lower the reserve variability, depending on whether the columns are positively or negatively correlated with each other. See the text of this paper for further discussion of trends in "age-to-age" factors on any observed

Tail Development

Exhibit C-4 shows the fitting of the inverse power curve for one simulation. To clarify the procedure, let us *contrast* this with fitting an inverse power curve for a "best-estimate" reserve indication. For the "best-estimate" indication, we would use "selected" age-to-age factors in column 3, such as averages of the factors in each column, or averages of the most recent years, or perhaps averages that exclude high and low factors. For the indemnity plus ALAE "12 to 24" months factor, the overall average is 2.352 and the average of the most recent five factors is 2.514. For a "best estimate," we would probably choose a factor such as 2.500.

In our analysis, the 22 factors in column 3 are the results of *simulations* from the 22 fitted lognormal curves. For instance, the 2.409 factor is a simulation from the lognormal curve representing the probability distribution for the 12 to 24 month column.

From these *simulated* age-to-age factors, we fit an inverse power curve to estimate the "tail" development.[65] The inverse power curve will vary from simulation to simulation, since we have different "age-to-age" factors in each run.

The inverse power curve models the age-to-age ("ATA") factors as

$$\text{ATA} = 1 + at^{-b}$$

where "t" represents the "development year," and "a" and "b" are the parameters that we must fit. In workers compensation, the shape of the loss payment pattern differs greatly between the first several years and subsequent years. In early years, there are many temporary total claims, with rapid payment patterns. By the tenth year, most of the remaining reserves are for lifetime pension cases (fatalities and permanent total disability cases), with slow payment patterns. Therefore, we fit the inverse power curve using the simulated factors from the tenth through the 22nd

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correlations between columns, as well as the paper by Randall Holmberg, "Correlation and the Measurement of Loss Reserve Variability" [*Casualty Actuarial Society Forum* (Spring 1994), Volume I, pages 247-278], for methods of quantifying these correlations.

65 For the rationale of using an inverse power curve for the tail development, see Richard Sherman, "Extrapolating, Smoothing, and Interpolating Development Factors," *Proceedings of the Casualty Actuarial Society*, Volume 71 (1984), pages 122-192.

columns only.[66]

Column 4 and 5 of Exhibit C-4 show the fitting procedure. Column (4) is the natural logarithm of the development year in column (2), and the column (5) is the natural logarithm of the "simulated age-to-age [ATA] factor minus one" in column (3). The inverse power curve can be written as

$$\ln(ATA - 1) = \ln(a) - b * \ln(t).$$

We use a least squares procedure to determine the parameters "a" and "b" from the figures in columns (4) and (5), giving ln (a) = 0.194, or a = 1.214, and b = 1.822, as shown in the box at the bottom of Exhibit C-4.

The fitted inverse power curve provides age-to-age factors for development years 23 through 70. We don't really know how long paid loss development continues for workers compensation. Moreover, the factors are small. For development years 30 through 39 in this simulation, the age-to-age factors are about 1.002, and for development years 40 through 70, the factors are about 1.001.[67] We therefore choose the length of the tail development stochastically; that is, the length of the total development is chosen randomly from a uniform distribution between 30 and 70 years.

Selected Factors

In the simulation shown in Exhibit C-5, the stochastic selection produced a development period of 54 years. We therefore have three sets of age-to-age factors:

- For development years 1 through 22, we use the simulated age-to-age factors generated by the lognormal curves for each column. For these development years, the "selected ATA" in column (4) equals the "simulated ATA" in column (2), not the "fitted ATA" in column (3).
- For development years 23 through 53, we use the age-to-age factors from the fitted inverse power curve. For these development years, the "selected ATA" in column (4) equals the "fitted ATA" in column (3).
- For development years 54 through 70, we use age-to-age factors of unity.

We now have all the age-to-age factors for this simulation. We "square the triangle"

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66 For actual reserve indications, one would probably segment the data between non-pension cases (temporary total and permanent partial cases) and lifetime pension cases (fatalities and permanent total cases).

67 The actual factors, of course, differ in the subsequent decimal places.

in the standard reserving fashion to determine ultimate incurred losses, and we subtract cumulative paid losses to date to obtain the required reserves.[68] Exhibit C-6 shows the determination of the required medical reserves for one simulation. The "ultimate paids" in Exhibit C-6 are the "paid-to-date" times the "age-to-ultimate" factors, and the "indicated reserves" are the "ultimate paids" minus the "paid-to-date." The right-most two columns of Exhibit C-6 show the determination of the present value of the reserves. The "present value factors" are discussed in Appendix D, which has a full explanation of inflation effects.

We perform this simulation 10,000 times, giving a complete probability distribution of the required reserves, and we determine the mean, standard deviation, 95th percentile, and 5th percentile of this distribution. For the manner of determining the "capital required to achieve a 1% expected policyholder deficit ratio" (the right-most column of the exhibits in the text of this paper), see Appendix B.

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[68] Note that we are using the same simulated age-to-age factors for each subsequent accident year. In theory, we could use separate simulations for each cell in the lower triangle of the square (i.e., for each age-to-age factor that we are forecasting). This would enormously complicate the work, particularly since we would have to fit separate inverse power curves for each accident year, without much gain in the quality of our results. Moreover, our procedure is "conservative." By assuming perfect dependence among the accident years that we are forecasting, we *increase* the variability in the loss reserve probability distribution.

EXHIBIT C-1

PAGE 1

Age-to-Age Factors for Indemnity and ALAE

Period	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144
1970 Dec					1.055	1.040	1.028	1.021	1.018	1.016	1.012
1971 Dec				1.094	1.055	1.041	1.026	1.024	1.016	1.011	1.010
1972 Dec			1.168	1.093	1.065	1.043	1.032	1.025	1.016	1.018	1.018
1973 Dec		1.386	1.169	1.096	1.062	1.049	1.033	1.025	1.020	1.017	1.012
1974 Dec	2.334	1.385	1.164	1.093	1.068	1.044	1.034	1.022	1.019	1.016	1.013
1975 Dec	2.310	1.398	1.190	1.116	1.076	1.051	1.037	1.026	1.021	1.016	1.013
1976 Dec	2.262	1.388	1.195	1.117	1.069	1.048	1.031	1.027	1.020	1.017	1.013
1977 Dec	2.192	1.397	1.191	1.111	1.070	1.048	1.031	1.023	1.019	1.016	1.015
1978 Dec	2.246	1.407	1.193	1.113	1.068	1.048	1.031	1.027	1.022	1.019	1.016
1979 Dec	2.199	1.409	1.192	1.109	1.068	1.045	1.036	1.027	1.023	1.020	1.019
1980 Dec	2.169	1.400	1.209	1.107	1.074	1.050	1.038	1.030	1.023	1.020	1.017
1981 Dec	2.191	1.400	1.185	1.115	1.075	1.055	1.041	1.032	1.025	1.019	1.017
1982 Dec	2.179	1.395	1.207	1.131	1.098	1.059	1.046	1.043	1.026	1.024	1.020
1983 Dec	2.283	1.437	1.227	1.140	1.088	1.064	1.048	1.037	1.025	1.022	1.017
1984 Dec	2.345	1.473	1.228	1.134	1.089	1.064	1.044	1.033	1.027	1.018	
1985 Dec	2.422	1.473	1.245	1.140	1.087	1.057	1.041	1.030	1.020		
1986 Dec	2.377	1.500	1.237	1.133	1.085	1.055	1.038	1.026			
1987 Dec	2.452	1.496	1.234	1.127	1.080	1.053	1.034				
1988 Dec	2.496	1.498	1.228	1.126	1.074	1.047					
1989 Dec	2.502	1.512	1.231	1.121	1.068						
1990 Dec	2.666	1.520	1.232	1.109							
1991 Dec	2.529	1.507	1.217								
1992 Dec	2.454	1.470									
1993 Dec	2.426										
1994 Dec											
Lognormal Parameters:											
mu [=avg. of ln(ATA-1)]	0.30	-0.82	-1.58	-2.16	-2.62	-3.00	-3.33	-3.59	-3.86	-4.03	-4.21
sigma = [std. dev. of ln(ATA-1)]	0.102	0.114	0.124	0.133	0.154	0.139	0.167	0.182	0.157	0.174	0.204

EXHIBIT C-1
PAGE 2

144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276
1.013	1.009	1.008	1.007	1.021	*1.001*	1.004	1.006	1.005	1.004	1.004
1.011	1.007	1.010	1.012	1.009	1.005	1.006	1.005	1.005	1.004	1.006
1.010	1.012	1.011	1.008	1.008	1.008	1.007	1.007	1.006	1.006	1.008
1.013	1.012	1.011	1.008	1.008	1.007	1.007	1.007	1.005	1.007	
1.010	1.013	1.009	1.008	1.009	1.007	1.006	1.008	1.007		
1.014	1.012	1.011	1.010	1.009	1.010	1.010	1.010			
1.013	1.012	1.008	1.011	1.009	1.009	1.009				
1.013	1.011	1.010	1.009	1.009	1.009					
1.014	1.012	1.012	1.011	1.009						
1.013	1.011	1.012	1.012							
1.017	1.011	1.011								
1.016	1.011									
1.015										
-4.34	-4.52	-4.61	-4.65	-4.64	-5.21	-5.00	-4.95	-5.19	-5.23	-5.12
0.163	0.177	0.161	0.170	0.291	0.986	0.280	0.229	0.190	0.226	0.280

EXHIBIT C - 2
PAGE 1

Development Factors (Medical)

Year	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120
1					1.013	1.011	1.010	1.008	1.005
2				1.019	1.015	1.014	1.008	1.010	1.010
3			1.045	1.026	1.018	1.014	1.013	1.012	1.009
4		1.105	1.044	1.030	1.017	1.017	1.012	1.012	1.011
5	1.895	1.108	1.050	1.028	1.021	1.013	1.013	1.010	1.014
6	1.898	1.122	1.055	1.034	1.023	1.019	1.017	1.014	1.011
7	1.893	1.113	1.056	1.035	1.026	1.019	1.016	1.016	1.014
8	1.865	1.119	1.055	1.035	1.020	1.021	1.014	1.013	1.011
9	1.912	1.122	1.057	1.036	1.025	1.018	1.019	1.016	1.014
10	1.869	1.120	1.056	1.036	1.025	1.020	1.016	1.015	1.013
11	1.849	1.126	1.063	1.031	1.030	1.023	1.017	1.016	1.014
12	1.836	1.127	1.054	1.037	1.028	1.021	1.018	1.018	1.015
13	1.808	1.126	1.063	1.040	1.025	1.022	1.020	1.016	1.013
14	1.898	1.135	1.071	1.041	1.030	1.028	1.022	1.020	1.018
15	1.948	1.158	1.072	1.047	1.036	1.027	1.024	1.020	1.013
16	1.949	1.156	1.081	1.048	1.035	1.028	1.022	1.016	1.015
17	1.808	1.162	1.082	1.051	1.032	1.026	1.019	1.015	
18	1.906	1.172	1.084	1.049	1.037	1.025	1.017		
19	1.871	1.178	1.083	1.051	1.030	1.021			
20	1.934	1.172	1.079	1.043	1.024				
21	1.898	1.171	1.071	1.036					
22	1.869	1.153	1.058						
23	1.773	1.126							
24	1.772								
Lognormal Parameters:									
mu [=avg. of ln(ATA-1)]	-0.14	-1.99	-2.77	-3.31	-3.71	-3.93	-4.14	-4.25	-4.41
sigma = [std. dev. of ln(ATA-1)]	0.060541	0.170376	0.204201	0.247302	0.286232	0.266582	0.292769	0.246563	0.276408
Simulated ATA	1.874	1.101	1.057	1.042	1.022	1.025	1.014	1.008	1.011

EXHIBIT C-2
PAGE 2

120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276
1.007	1.006	1.008	1.007	1.007	1.005	1.006	*1.001*	1.007	1.005	1.007	1.008	1.009
1.008	1.009	1.011	1.012	1.008	1.008	1.007	1.007	1.008	1.011	1.012	1.012	1.008
1.013	1.010	1.011	1.018	1.011	1.008	1.010	1.007	1.009	1.010	1.011	1.008	1.010
1.010	1.010	1.009	1.010	1.009	1.009	1.008	1.012	1.007	1.007	1.009	1.008	
1.009	1.009	1.011	1.010	1.007	1.014	1.014	1.009	1.009	1.007	1.012		
1.009	1.012	1.012	1.011	1.011	1.014	1.015	1.012	1.010	1.009			
1.014	1.012	1.009	1.010	1.010	1.010	1.011	1.010	1.009				
1.011	1.008	1.011	1.009	1.009	1.010	1.010	1.008					
1.013	1.017	1.015	1.014	1.013	1.012	1.012						
1.011	1.014	1.012	1.012	1.011	1.009							
1.014	1.013	1.010	1.017	1.011								
1.015	1.012	1.010	1.007									
1.013	1.010	1.009										
1.014	1.012											
1.010												
-4.49	-4.53	-4.56	-4.52	-4.66	-4.64	-4.63	-5.06	-4.79	-4.87	-4.61	-4.72	-4.68
0.218529	0.238770	0.158673	0.292236	0.213223	0.281293	0.308683	1.047670	0.151740	0.313104	0.233627	0.209965	0.119079
1.010	1.010	1.013	1.018	1.011	1.007	1.012	1.002	1.008	1.007	1.008	1.008	1.007

EXHIBIT C-3

Illustration of Fitting Lognormal Distributions to Age-to-Age Factors

	(1) 12-24 Factors for Indemnity & ALAE	(2) Age-to-Age Factor minus 1 (1) - 1	(3) Natural Logs of (Age-to-Age Factors minus 1) ln (2)
1974 Dec	2.334	1.334	0.288
1975 Dec	2.310	1.310	0.270
1976 Dec	2.262	1.262	0.232
1977 Dec	2.192	1.192	0.175
1978 Dec	2.246	1.246	0.220
1979 Dec	2.199	1.199	0.181
1980 Dec	2.169	1.169	0.156
1981 Dec	2.191	1.191	0.175
1982 Dec	2.179	1.179	0.165
1983 Dec	2.283	1.283	0.249
1984 Dec	2.345	1.345	0.297
1985 Dec	2.422	1.422	0.352
1986 Dec	2.377	1.377	0.320
1987 Dec	2.452	1.452	0.373
1988 Dec	2.496	1.496	0.403
1989 Dec	2.502	1.502	0.407
1990 Dec	2.666	1.666	0.510
1991 Dec	2.529	1.529	0.425
1992 Dec	2.454	1.454	0.375
1993 Dec	2.426	1.426	0.355
Average	2.352	1.352	0.296
Standard Deviation	0.139	0.139	0.102

Lognormal Parameters:	
mu [= mean of the logs of (ATA-1)]	0.296
sigma [= standard deviation of logs of (ATA-1)]	0.102

Simulated ATA*	2.464

* The simulated age-to-age factor is a single pick from a lognormal distribution with the fitted parameters. The factor is picked by inverting the cumulative density of a lognormal. The Excel formula for the simulated age-to-age factor is: 1+LOGINV(RAND(),mu,sigma). [Note that the "1+" at the start of the expression is needed because we fit the curve to (ATA - 1).]

EXHIBIT C-4

Illustration of Fitting an Inverse Power Curve to the Simulated Age-to-Age Factors

(1) Development Period	(2) Year	(3) Simulated ATA	(4) ln(year) ln(2)	(5) ln(ATA - 1) ln[(3)-1]	(6) Fitted ATA 1 + a x (2)^[-b]
12 - 24	1	2.409			2.214
24 - 36	2	1.413			1.343
36 - 48	3	1.272			1.164
48 - 60	4	1.113			1.097
60 - 72	5	1.068			1.065
72 - 84	6	1.042			1.046
84 - 96	7	1.029			1.035
96 - 108	8	1.022			1.027
108 - 120	9	1.031			1.022
120 - 132	10	1.021	2.303	-3.869	1.018
132 - 144	11	1.013	2.398	-4.374	1.015
144 - 156	12	1.017	2.485	-4.055	1.013
156 - 168	13	1.014	2.565	-4.258	1.011
168 - 180	14	1.009	2.639	-4.706	1.010
180 - 192	15	1.010	2.708	-4.634	1.009
192 - 204	16	1.007	2.773	-4.987	1.008
204 - 216	17	1.003	2.833	-5.971	1.007
216 - 228	18	1.008	2.890	-4.819	1.006
228 - 240	19	1.007	2.944	-4.978	1.006
240 - 252	20	1.004	2.996	-5.640	1.005
252 - 264	21	1.005	3.045	-5.246	1.005
264 - 276	22	1.007	3.091	-4.933	1.004

Fitting a least squares line to columns (4) and (5), with (5) as the dependent variable gives the following fitted paramenters:

slope = -1.822
Intercept = 0.194

Since the inverse power curve can be written in the form: ln(ATA-1) = ln(a) - b ln(t), we have the following parameters for the inverse power curve:

a = exp(intercept) = 1.214
b = -slope = 1.822

EXHIBIT C-5

Illustration of Selecting Age-to-Age Factors

(1) Year	(2) Simulated ATA	(3) Fitted ATA a= 1.214 b= 1.822	(4) Selected ATA Cut-off for tail* 54
1	2.409	2.214	2.409
2	1.413	1.343	1.413
3	1.272	1.164	1.272
4	1.113	1.097	1.113
5	1.068	1.065	1.068
6	1.042	1.046	1.042
7	1.029	1.035	1.029
8	1.022	1.027	1.022
9	1.031	1.022	1.031
10	1.021	1.018	1.021
11	1.013	1.015	1.013
12	1.017	1.013	1.017
13	1.014	1.011	1.014
14	1.009	1.010	1.009
15	1.010	1.009	1.010
16	1.007	1.008	1.007
17	1.003	1.007	1.003
18	1.008	1.006	1.008
19	1.007	1.006	1.007
20	1.004	1.005	1.004
21	1.005	1.005	1.005
22	1.007	1.004	1.007
23		1.004	1.004
24		1.004	1.004
25		1.003	1.003
26		1.003	1.003
27		1.003	1.003
28		1.003	1.003
29		1.003	1.003
30		1.002	1.002
31		1.002	1.002
32		1.002	1.002
33		1.002	1.002
34		1.002	1.002
35		1.002	1.002

(1) Year	(3) Fitted ATA a= 1.214 b= 1.822	(4) Selected ATA Cut-off for tail* 54
36	1.002	1.002
37	1.002	1.002
38	1.002	1.002
39	1.002	1.002
40	1.001	1.001
41	1.001	1.001
42	1.001	1.001
43	1.001	1.001
44	1.001	1.001
45	1.001	1.001
46	1.001	1.001
47	1.001	1.001
48	1.001	1.001
49	1.001	1.001
50	1.001	1.001
51	1.001	1.001
52	1.001	1.001
53	1.001	1.001
54	1.001	1.000
55	1.001	1.000
56	1.001	1.000
57	1.001	1.000
58	1.001	1.000
59	1.001	1.000
60	1.001	1.000
61	1.001	1.000
62	1.001	1.000
63	1.001	1.000
64	1.001	1.000
65	1.001	1.000
66	1.001	1.000
67	1.001	1.000
68	1.001	1.000
69	1.001	1.000
70	1.001	1.000

* The cut off for the tail models the actuarial uncertainty in when to cut off the development from the invers power curve. The cut-off is based on a uniform distribution from 30 to 70.

EXHIBIT C-6

Calculation of Required Reserves for a Single Simulation (Medical Payments Only)

Year	Paid to date	Age-to-Ultimate	Ultimate Paids	Indicated Reserves	Present Value Factor	Present Value of Reserves
1994	1,787,601	3.203	5,725,294	3,937,693	0.675	2,659,636
1993	3,324,538	1.709	5,680,459	2,355,921	0.516	1,214,503
1992	4,208,871	1.551	6,528,911	2,320,040	0.470	1,089,486
1991	7,017,997	1.468	10,299,947	3,281,950	0.440	1,444,103
1990	7,547,277	1.408	10,627,859	3,080,582	0.414	1,274,652
1989	7,905,743	1.378	10,895,652	2,989,908	0.408	1,221,093
1988	8,507,321	1.345	11,442,028	2,934,707	0.394	1,156,938
1987	7,629,124	1.326	10,117,196	2,488,072	0.395	981,650
1986	6,621,638	1.315	8,709,026	2,087,388	0.405	845,872
1985	5,398,367	1.300	7,019,675	1,621,309	0.410	664,608
1984	3,997,086	1.288	5,147,204	1,150,117	0.418	480,222
1983	3,198,587	1.275	4,077,616	879,029	0.424	372,952
1982	2,895,279	1.258	3,642,920	747,641	0.424	316,754
1981	2,929,995	1.236	3,621,690	691,695	0.409	282,863
1980	2,704,128	1.222	3,304,949	600,822	0.406	244,159
1979	2,552,368	1.213	3,096,693	544,325	0.413	224,961
1978	2,375,139	1.199	2,848,514	473,374	0.407	192,665
1977	1,986,508	1.197	2,377,166	390,658	0.428	167,106
1976	1,680,001	1.187	1,994,188	314,187	0.432	135,664
1975	1,321,413	1.178	1,557,089	235,677	0.438	103,113
1974	1,154,614	1.169	1,349,561	194,947	0.440	85,775
1973	1,004,449	1.160	1,164,746	160,297	0.442	70,808
1972	908,372	1.152	1,046,035	137,663	0.446	61,378
1971	782,100	1.144	895,081	112,981	0.452	51,090
1970	776,907	1.138	883,852	106,945	0.459	49,083
Total	90,215,423		124,053,352	**33,837,929**		**15,391,133**

Appendix D: Inflation Adjustments

For certain long-tailed casualty lines of business, much reserve uncertainty stems from changes in the rate of inflation. For workers compensation medical benefits, as an example, the employer is responsible for all physician fees, which are affected by the rate of inflation up through the date that the service is rendered.

Paid loss development analyses often overstate the uncertainty in reserve indications, particularly if one is concerned with the economic value of the reserves, not their nominal value. For instance, suppose that the cumulative paid losses *in real dollar terms* will increase by 30% over the coming year, for a "real dollar" age-to-age factor of 1.300. If inflation is high, the nominal age-to-age factor may be 1.350. If inflation is low, the nominal age-to-age factor may be 1.320.

To some extent, this is "apparent" reserve uncertainty, not real reserve uncertainty. We can get a better estimate of reserve uncertainty by

- Stripping inflation out of the historical paid losses,
- Determining "age-to-age" factors in real dollar terms,
- Using the "real dollar" factors to produce all the simulations, and
- Restoring nominal inflation, based upon current inflation expectations, to determine ultimate losses.

These adjustments are even more important for standard "point estimates" of indicated reserves than for quantification of the uncertainty in the reserves. Nominal dollar paid loss "age-to-age" factors have the historical inflation rate built into them.[69] *If future inflation is expected to be different from past inflation, a rote application of* the paid loss chain ladder technique gives misleading reserve indications. By properly adjusting the reserve analysis for the actuary's estimate of future inflation, the reserve uncertainty is slightly reduced.

Exhibit D-1 shows the procedure used to put the paid loss experience into real dollar terms (at a 1994 price level). We demonstrate the procedure for medical benefits, which we assume to be fully inflation sensitive. Indemnity benefits, in contrast, are only partially inflation sensitive. About half the states have "cost of living" adjustments for wage loss benefits, but generally these adjustments apply only to certain cases (such as cases that extend for two years or more) and they are often capped (say, at 5% per annum).

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69 See Charles F. Cook, "Trend and Loss Development Factors," *Proceedings of the Casualty Actuarial Society*, Volume 57 (1970) pages 1-26

We begin with the medical component of the Consumer Price Index, shown on the second row of Exhibit D-1. During the 1980s, the rate of increase in workers compensation medical benefits exceeded the medical CPI. This additional WC medical inflation is related to increases in utilization rates, or perhaps to the incurral of medical services to justify claims for increased indemnity benefits.

For ratemaking, we would need a "loss cost trend factor" for workers compensation medical benefits, of which the medical CPI is but one component. For our purposes, we are concerned only with medical inflation. Changes in utilization rates remain embedded in the paid loss development factors. If the reserving actuary believes that future changes in utilization rates will differ from past changes in utilization rates, this expected difference must be separately quantified.

We must convert the *incremental* paid losses during each calendar year to their "real dollar" (calendar year 1994) values. For ease of application, the one dimensional index in the second row of Exhibit D-1 is converted to a two-dimensional triangle. For instance, the "1.32" in column (2) for accident year 1989 means that accident year losses paid between 12 and 24 months (i.e., from January 1, 1990, through December 31, 1990) must be increased by 32% to bring them to 1994 levels. The 1.032 factor is derived from the inflation index: 1.032 = 1.0885 * 1.0805 * 1.0667 * 1.0536.

We now redo the entire simulation procedure, as documented in Appendix C, using the paid losses that have all been adjusted to a 1994 cost level. Exhibit D-2 shows the results of one such simulation. Lognormal curves were fitted to each column of "real dollar" age-to-age factors, 22 age-to-age factors (through 276 months of development) were generated stochastically from these lognormal curves, an inverse power curve was fitted to these simulated factors, and then a 36 year length for the full development was generated stochastically from a uniform distribution of 30 to 70 years. [These steps are not shown in Exhibit D-2, since the procedure is identical to that discussed in Appendix C.]

The "age-to-ultimate" factors in column (3) are the backward product of the "simulated age-to-age" factors in column (2). For instance, the 2.446 factor in year 1 is the product of 1.378 (column 3, year 2) and 1.776 (column 2, year 1); the 1.378 factor in year 2 is the product of 1.247 (column 3, year 3) and 1.105 (column 2, year 2); and so forth.

The "payment pattern" in the fourth column is derived directly from the "age-to-ultimate" factors in the third column. The 2.446 "age-to-ultimate" factor for year 1 means that for every dollar paid so far, we expect another $1.446 to be paid in the future. Thus, the percentage paid so far is $1 ÷ ($1 + $1.446) = 0.409, or 40.9%.

The 1.378 "age-to-ultimate" factor for year 2 means that for every dollar paid through the end of year 2, we expect another $0.378 to be paid in the future. Thus, the percentage paid so far is $1 ÷ ($1 + $0.378) = 0.726, or 72.6%. Since 40.9% is paid in the first year, the difference is assumed to be paid in the second year: 72.6% – 40.9% = 31.7%, or 0.317.

The same procedure is used for all the entries in the column. The entries sum to unity, except for slight rounding discrepancies.

Restoring Inflation

This "payment pattern" is derived from paid loss histories where are dollar amounts are put on 1994 levels. It is as though there were no inflation in the past, and no inflation will occur in the future.

To properly estimate reserves, we must "restore" future inflation, at the rate assumed by the reserving actuary. This exhibit illustrates an expected future medical inflation rate of 6% per annum. [The tables in the text of this paper show results for expected future inflation rates of 4%, 6%, and 8%.]

Consider the entries for year 1. If there is no inflation, then 40.9% of losses have been paid by the end of year 1, and the age-to-ultimate factor is 2.446. What is the proper age-to-ultimate factor if future losses will inflate at a 6% annual rate?

For simplicity, we assume that losses are paid in mid-year. To clarify the procedure, let us suppose that the ultimate losses in real dollar terms are $1,000,000. By the end of the first year, 40.9% of losses have been paid, or $409,000. In the next calendar year, an additional 31.7% of the losses will be paid, or $317,000 in real dollar terms. Since these losses will be paid in mid-year, inflation will affect them for half a year, so the nominal payment will be $317,000 * $(1.06)^{0.5}$, or $326,371. In the next calendar year, an additional 7.6% of losses will be paid. This is $76,000 in real dollar terms, or $76,000 * $(1.06)^{1.5}$ = $82,942 in nominal terms.

We continue in this fashion to determine all the future expected payments in nominal terms. Summing these payments gives $1,206,500. Since $409,000 is paid by the end of the first year, the percentage paid is $409,000 ÷ $1,206,500 = 33.9%, or 0.339, which is the entry for year 1 in column (5), captioned "cum % paid."

The "inflated ATU" in column (6) is the reciprocal of the "cum % paid" figure. It is the "age-to-ultimate" factor appropriate for a paid loss pattern where there is *no* past inflation but there is 6% per annum future inflation. This factor is applicable to the analysis in this paper: it assumes that past payments have been brought to current monetary levels but future payments will inflate at the assumed rate. [It is *not* a loss development factor that is proper for loss payment patterns that are either on an entirely nominal basis or on an entirely real dollar basis.]

An explanation of the factors for subsequent years should clarify this. Consider the entries for year 3. Cumulative payments by the end of year 3, when all payments are at 1994 monetary levels, are 40.9% + 31.7% + 7.6% = 80.2%. Supposing, as before, that total payments are $1 million, $802,000 has been paid so far.

The future payments, assuming 6% annual inflation and payments made in mid-year, are

$$
\begin{aligned}
& \$4{,}600 * (1.06)^{0.5} \\
+\ & \$3{,}900 * (1.06)^{1.5} \\
+\ & \$2{,}000 * (1.06)^{2.5}, \text{ and so forth.}
\end{aligned}
$$

The total payments are $1,138,000, giving a "cumulative percentage paid" of 70.5%. The "inflated ATU" is the reciprocal of this figure, or 1.419. This factor assumes *no* inflation for the three years of history and 6% annual inflation for future payments.

Payment Patterns

The present value factors in the right-most column of Exhibit D-2 are derived from the appropriate payment pattern for each accident year. Column (5), which is labeled "Cum % paid," looks like a payment pattern, but it is *not* a payment pattern.

Think of a matrix, where each column is a payment pattern. The first column is the appropriate payment pattern when there is no inflation in year 1 but 6% inflation in subsequent years; the second column is the appropriate payment pattern when there is no inflation in years 1 and 2 but 6% inflation in subsequent years; and so forth. The first column is the appropriate payment pattern for the most recent accident year in our analysis; the second column is the appropriate payment pattern for the previous accident year; and so forth. The column labeled "Cum % paid" in Exhibit D-2 is the diagonal of this matrix, as shown below.

Accident Year:	X	X–1	X–2	X–3	X–4	X–5
Development Yr						
1	0.339	.....	.....	.....	.....	.....
2	.....	0.625	.....	.....	.....	.....
3	.....	.....	0.705	.....	.....	.....
4	.....	.....	.....	0.758	.....	.....
5	.....	.....	.....	.....	0.804	.....
6	.....	.....	.....	.....	.....	.....

The "present value factors" in the right-most column of Exhibit D-2 *are not* determined from the *diagonal* of factors that is shown in the "Cum % paid" column of Exhibit D-2. Rather, the present value factor for the current accident year's reserves (the 0l754 in column 7 of Exhibit D-2) is determined from the payment pattern for accident year "X" in the box directly above this paragraph. Similarly, the next present value factor, *or 0.635, is derived from the payment pattern for accident year* "X–1."

The full matrix is not shown in this appendix. In fact, the matrix changes for each simulation, since it is determined from the "simulated age-to-age" factors.[70]

Present Value Factors

The present value factors in Exhibit D-2 *use a 6.375% discount rate.* The discount rate was chosen as the average of the assumed future inflation rate and the company's "new-money" investment yield, which was 6.75% when this analysis was performed. This assumes that the appropriate discount rate moves with the inflation rate but is not perfectly correlated with it.

The present value factors are calculated as follows. Suppose there are 5 years in the payment stream, with the payment pattern being 30% – 25% – 20% – 15% – 10%. [The present value factors in Exhibit D-2 use 70 year payment patterns, but the *procedure is the same.]* Again, *we assume that all payments are made in mid-year.*

At the end of the first year, 30% of payments have been made and 70% of losses are still unpaid. The discounted amount of these unpaid losses is

$$25\%*(1.06375)^{-0.5} + 20\%*(1.06375)^{-1.5} +15\%*(1.06375)^{-2.5} +10\%*(1.06375)^{-3.5}$$
$$= 24.2\% + 18.2\% + 12.9\% + 8.1\% = 63.4\%.$$

The discount factor for these reserves is therefore 63.4% ÷ 70.0% = 90.5%.

70 In the spreadsheet used for this analysis, the matrix is a matrix of formulas. In each run, the simulated "age-to-age" factors are determined, the inverse power curve is fit, the formulas in the matrix are replaced by figures, and the "present value factors" are derived.

EXHIBIT D-1

Stripping Medical Inflation from the Losses

Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
Medical Inflation	6.65%	6.40%	4.75%	3.65%	6.65%	10.65%	10.75%	9.55%	9.00%	8.80%	10.10%	10.85%	11.15%	10.20%	7.50%	6.25%	6.90%	7.05%	6.55%	7.10%	8.35%	8.85%	8.05%	6.67%	5.36%

Index for use in Calendar Year

(Multiplying the corresponding element in the triangle by this factor puts the loss at the 1994 medical price level)

Accident Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1970	6.26	5.89	5.62	5.42	5.08	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00
1971	5.89	5.62	5.42	5.08	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00	
1972	5.62	5.42	5.08	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00		
1973	5.42	5.08	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00			
1974	5.08	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00				
1975	4.59	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00					
1976	4.15	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00						
1977	3.79	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00							
1978	3.47	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00								
1979	3.19	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00									
1980	2.90	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00										
1981	2.62	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00											
1982	2.35	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00												
1983	2.14	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00													
1984	1.99	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00														
1985	1.87	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00															
1986	1.75	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00																
1987	1.63	1.53	1.43	1.32	1.21	1.12	1.05	1.00																	
1988	1.53	1.43	1.32	1.21	1.12	1.05	1.00																		
1989	1.43	1.32	1.21	1.12	1.05	1.00																			
1990	1.32	1.21	1.12	1.05	1.00																				
1991	1.21	1.12	1.05	1.00																					
1992	1.12	1.05	1.00																						
1993	1.05	1.00																							
1994	1.00																								

EXHIBIT D-2

Building Medical Inflation at 6% into the Projected Losses

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cut-off for tail					**6.0%**	
36				Cum % paid, assuming future		PV Factor for
		0%		inflation, but no past inflation [NOT		Reserves from
Year	Simulated ATA	Age-to-Ultimate	Payment Pattern	a payment pattern, since it applies	Inflated ATU	AY
				separately to each accident yr.]		**6.375%**
1	1.776	2.446	0.409	0.339	2.954	0.754
2	1.105	1.378	0.317	0.625	1.601	0.635
3	1.057	1.247	0.076	0.705	1.419	0.595
4	1.046	1.180	0.046	0.758	1.320	0.565
5	1.023	1.128	0.039	0.804	1.244	0.524
6	1.013	1.103	0.020	0.831	1.203	0.505
7	1.010	1.088	0.012	0.850	1.176	0.501
8	1.008	1.077	0.010	0.866	1.154	0.499
9	1.004	1.068	0.008	0.880	1.136	0.501
10	1.007	1.064	0.004	0.890	1.124	0.518
11	1.005	1.057	0.006	0.901	1.110	0.521
12	1.005	1.052	0.005	0.911	1.098	0.529
13	1.005	1.047	0.004	0.920	1.087	0.538
14	1.003	1.042	0.005	0.929	1.077	0.544
15	1.003	1.038	0.003	0.936	1.069	0.556
16	1.003	1.035	0.003	0.942	1.062	0.570
17	1.002	1.032	0.003	0.948	1.055	0.582
18	1.002	1.030	0.001	0.952	1.050	0.606
19	1.003	1.028	0.002	0.957	1.045	0.624
20	1.002	1.024	0.003	0.963	1.039	0.633
21	1.002	1.022	0.002	0.967	1.034	0.649
22	1.002	1.020	0.002	0.971	1.030	0.668
23	1.002	1.018	0.002	0.975	1.026	0.681
24	1.002	1.016	0.002	0.978	1.023	0.700
25	1.002	1.014	0.002	0.981	1.020	0.720
26	1.002	1.012	0.002	0.983	1.017	0.741
27	1.001	1.011	0.002	0.986	1.014	0.763
28	1.001	1.009	0.001	0.988	1.012	0.785
29	1.001	1.008	0.001	0.990	1.010	0.809
30	1.001	1.007	0.001	0.992	1.008	0.833
31	1.001	1.005	0.001	0.994	1.007	0.858
32	1.001	1.004	0.001	0.995	1.005	0.885
33	1.001	1.003	0.001	0.996	1.004	0.912
34	1.001	1.002	0.001	0.998	1.002	0.940
35	1.001	1.001	0.001	0.999	1.001	0.970
36	1.000	1.000	0.001	1.000	1.000	#DIV/0!