

The Valuation of a Pure Risk Element  
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## The Valuation of a Pure Risk Element

**Abstract:** It is a generally accepted principle of financial theory that an assumption of risk entitles the assuming party to a higher expected return on investment. This is paralleled in property/casualty insurance by the concept of a risk/contingency loading, or underwriting profit margin, which varies directly with the riskiness of the business written. A risky liability can be separated into two distinct components: a fixed liability, and a pure-risk element which is neither an asset nor a liability, but which negatively impacts net worth. It is demonstrated that, under certain assumptions: 1) the dollar value of a given risky liability is inversely related to the net capitalization of the entity assuming or retaining it, and 2) the transfer of risk from a lower-capitalized entity to a higher-capitalized entity for an appropriate premium results in gain for both parties, allowing them to achieve higher rates of return than would otherwise be available. This implies that insurance offered at an appropriate premium creates net economic value for both parties, aside from the value created by the "pooling" of risks. A fair premium is defined to be the premium which equalizes the gains to both parties.

### Introduction

Insurance is a transaction which is designed to generate economic value for both parties, the customer and the insurer. Implicit to an insurance transaction's creation of value is that the economic cost of the risk being transferred is greater to the insured than to the insurer. This is in contrast to a fixed, riskless liability which has the same discounted cost to all parties. In analyzing the cost of a risky liability, it is useful to separately identify the portion of the liability which is pure risk, apart from the portion which is a definite, fixed liability. We will present an analysis of risk value and a method which calculates the difference in risk values between the customer and the insurer, apart from diversification or risk-pooling considerations.

### Measuring the Economic Value Created by an Insurance Transaction

Suppose that, at 1/1/95, Party C (Customer) has cash assets of \$600, and Party N (Insurer) has cash assets of \$1,000. In addition, C has a risk which will come to fruition at 1/1/96 (no timing risk involved). The risk consists of either a gain of \$100 or a loss of \$100, each with probability 50%. For this example, N has assets only, with no risks. Assume the risk-free rate is constant at 8% per year. Then, if no transfer of risk takes place, N will have assets of \$1,080 on 1/1/96, and C will have assets of either \$748 or \$548, each with probability 50%. The rates of return are 8.00%, 24.67%, and -8.67%, respectively.

If an investor invests \$100,000 for two years, at 5% for one year and 9% for the other year, he will end up with  $(\$100,000)(1.05)(1.09) = \$114,450$ . If instead the investment had been at 7% for both years, the result would be  $(\$100,000)(1.07)(1.07) = \$114,490$ . Thus, there is a loss of

income associated with deviation from the arithmetic mean. If the investor makes a long-term investment in which the annual return is either 5% or 9% each year, always with 50% probability, and dividends are reinvested for compounding, then the effective annual rate will approach:

$$\sqrt{(1.0500)(1.0900)} - 1 = 6.98\%$$

with near certainty. This is just the principle that the geometric mean is less than the arithmetic mean, and recognition of the fact that compound interest is geometric. (A fuller technical discussion of this aspect of the model appears in Appendix 1.)

If we apply the above principle to Party C's position, we get a long-term effective annual rate of:

$$\sqrt{(1.2467)(0.9133)} - 1 = 6.71\%$$

a 6.71% return-including-risk. Receiving an effective 6.71% return when the available risk-free rate is 8.00% is equivalent to the Customer's having risk-discounted capital of:

$$(600.00)(1.0671)/(1.0800) = \$592.81$$

Thus, the value of ceding the risk from C's standpoint is  $\$600.00 - \$592.81 = \$7.19$ . By ceding the risk for a premium that is less than \$7.19, C will be able to invest the remaining assets (net of premium), and obtain a better return than by retaining the risk. If C cedes the risk for a premium of \$6.10, the long-term effective rate of return will be:

$$(600.00 - 6.10)(1.0800)/(592.81) = 8.20\%$$

Suppose N assumes the risk on 1/1/95 for a premium of \$6.10. Then, N invests \$1,006.10 at 8% and has assets of either \$986.59 or \$1186.59 at 1/1/96, for a long-term effective rate of return of:

$$\sqrt{(0.98659)(1.18659)} - 1 = 8.20\%$$

This is an improvement over the 8.00% rate available without assuming any risk. To the two decimal places calculated, the returns to both parties are equal - therefore, the premium is "fair". The transaction has made it possible for both parties to realize a higher effective rate of return than would be available without the risk transfer.

Each party's net economic gain can be calculated directly. Customer C has ceded a risk worth \$7.19 for a premium of \$6.10, realizing a gain of \$1.09, or +0.18% on initial risk-discounted capital. Insurer N began with assets of \$1,000.00 and no liabilities. After the transaction, N has assets of \$1,006.10 and the assumed risky liability, for \$1,001.83 in risk-discounted capital, a gain of \$1.83, or +0.18%. Note that the parties' gains are equal as ratios to their respective pre-transaction risk-discounted capital. This transaction gain combined with the annual yield of

8.00% gives the post-transaction rate-of-return of 8.20% calculated earlier.

One point of this analysis is that risk should be evaluated in terms of rate-of-return on capital against which the risk is assumed, rather than independently of supporting capital. The point carries with it the concept that the value of risk is *relative* to the net capitalization of the party assuming or retaining the risk, and does not have a fixed value independent of supporting capital. This model does not use a discounting of future liabilities with an adjusted discount rate in order to value risk. The "risk element" liability in the example above has expected value of zero, and therefore cannot be assigned a nonzero value via the application of a discount factor, yet it has a negative value since a rational investor would not accept it without an accompanying premium.

The economic cost of the risk element in the above example can be calculated for both the customer and the insurer. It has already been shown that, for the customer, the risk element has a cost, or negative value, of \$7.19. For the insurer, the negative value of the risk element following the transaction is:

$$\$1,006.10 - \$1,001.83 = \$4.27$$

The difference between the customer's and insurer's risk values is  $\$7.19 - \$4.27 = \$2.92$ . It is this **risk-value gap** that allows the insurance transaction to generate net value by the transfer of risk. If the "fair premium" condition is met, the risk-value gap is split between the customer and the insurer, in proportion to the pre-transaction risk-discounted net capital of each; thus the economic gains as percentages of risk-discounted capital are equalized.

### Theoretical Considerations

Any risky loss liability can be split into two parts: 1) a definite **fixed liability**, equal to the expected loss amount (discounted at the risk-free rate for time value), and 2) a **pure-risk element** with expected value of zero, equal to the loss distribution (also discounted for time value) minus the expected loss amount. For example: a loss liability with possible values \$100 and \$300, each with probability 50%, is equivalent to a fixed \$200 liability (the expected value of the loss) combined with the zero-expected-value pure risk element from the above example. The same principles apply - the "fair" premium is calculated so as to equalize transaction gains for customer and insurer. The results are shown on Exhibit 1. After the premium calculation, the premium amount can be allocated into two parts: the discounted \$200 fixed liability, and the premium for the risk element. The value of the discounted fixed liability is  $200.00/1.0800 = \$185.19$ . If C begins with assets of \$785.19 (\$600.00 as before, plus \$185.19 to offset the fixed liability), the risk element's values and the fair risk premium are the same as calculated in the earlier example. This equivalent example is shown in Exhibit 2. The case is identical to the prior pure-risk case, with the addition of a fixed liability and offsetting assets to be transferred along with the pure-risk element.

**Mathematical Development**

**Notation:** We begin by defining the risk-discounted assets of an entity, X, as:

$$\Pi_x = \pi(x, \langle \$, p \rangle) = \Pi(x - \$)_j^p$$

where:                    x = assets of entity X (assume no asset risk for simplicity)  
                               j = index for loss distribution  
                               \$ = discounted loss amount  
                               p = probability of loss amount  
                               <\$, p> = loss distribution

This is the general formula associated with the above analysis. Next, define liability value by:

$$L_x = x - \Pi_x$$

Liability value is the difference between unadjusted capital and risk-discounted capital. It is also the sum of the **fixed component** of liability, F=expected loss amount, and the **risk value**, V:

$$L_x = F_x + V_x, \text{ where } F_x = \sum p_j \$_j$$

If P represents the premium paid for the transfer of the liability, then the **net transaction gain** as a percentage of risk-discounted assets can be calculated for both C (customer) and N (insurer):

$$k_c = \frac{c-P}{\Pi_c} - 1, k_n = \frac{\Pi_n}{n} - 1, \text{ where } \Pi_n = \pi(n+P, \langle \$, p \rangle)$$

Note that  $\Pi_n$  is a post-transaction value.

The "fair premium" as defined above is the value of P for which  $k_c = k_n$ . When this is the case, solving the  $k_c$  equation for P yields:

$$P = c - (1+k)\Pi_c$$

It can be shown that:

$$k = \frac{\Delta V}{n + \prod_C}, \text{ where } \Delta V = V_C - V_N$$

$\Delta V$  is the risk-value gap - the difference in risk values between the insurer and the customer. The risk-value gap actually quantifies the total net value generated by the risk-transfer transaction. It can be shown (Appendix 2) that  $\Delta V$  is closely approximated by:

$$\Delta V \approx \pi(n + L_C, \langle \$, p_f \rangle) - n = \pi(n + V_C, \langle r_f, p_f \rangle) - n, \text{ where } r_f = \$_j - F_C. \text{ (Note: } \sum p_f = 0.)$$

The distribution  $R = \langle r_f, p_f \rangle$  represent the pure risk element associated with the discounted loss distribution  $\langle \$, p_f \rangle$ . As noted above,  $E[R] = \sum r_f p_f = 0$ .

Both P and k can be calculated by iteration, but close approximations can be obtained without iteration by using the above three formulae. Substituting from the earlier example,

$$\Delta V \approx (1,000.00 + 7.19 + \frac{100.00}{1.0800})^{1/2} \cdot (1,000.00 + 7.19 + \frac{100.00}{1.0800})^{1/2} - 1,000.00 = \$2.92$$

Then,

$$k \approx \frac{2.92}{1,000.00 + 592.81} = 0.18\%, \quad P \approx 600.00 - 592.81(1.0018) = \$6.12$$

closely approximating the exact answers obtained by iteration.

### Generalization of Model

The geometric function used to evaluate risk comes from a model which assumes: 1) at any given time, a party will assume or acquire risk which is proportional in magnitude to the party's assets, and 2) the shape of the pure risk density function is always the same. The model does not consider any externally generated infusions of capital. This is the simplest model for performing calculations and illustrating the concepts being presented. Other functions based on alternative modeling assumptions can be used to value risk as a function of capitalization.

To generalize the mathematics above, let  $V^R(x)$  be a risk-value function which maps capitalization net of discounted expected losses to risk value, for a given pure risk element (R). Thus V is parameterized by R, and:

$$V^R(x) = x - x',$$

where  $x'$  = risk-discounted value of net capital  $x$  combined with pure risk R.

Two basic tenets of this paper are: 1)  $V^R(x) > 0$  for all  $x$  and all R, and 2)  $V^R(x)$  is a decreasing

function of  $x$  for any fixed  $R$ . The formulas given earlier for the geometric model generalize to:

$$k_c = \frac{V^R(c) - P}{c - V^R(c)}$$

$$k_n = \frac{P - V^R(n + P)}{n}$$

$$P = V^R(c) - kc'$$

$$k = \frac{\Delta V}{n + c'}$$

$$\Delta V = V^R(c) - V^R(n + P) \text{ (by definition)}$$

$$\Delta V \approx V^R(c) - V^R[n + V^R(c)]$$

The last formula is significant, because it gives the approximate value of an insurance transaction using only the capitalization of both parties, the risk-value function  $V$ , and the risk element  $R$ .

Let  $V = V^R(c)$  and  $W = V^R(n + P)$ . Then, from the above definition,  $\Delta V = V - W$ . Also,  $c' = c - V$ . Combining the above formulas for  $k$  and  $P$  and reintroducing discounted expected losses ( $F$ ), algebra yields:

$$P = \frac{Vn + Wc'}{n + c'} + F$$

This is the general premium formula for pricing a risky liability, based on estimates of the pure risk element's respective values to the customer and insurer ( $V$  and  $W$ ), capitalizations net of discounted expected losses ( $c$  and  $n$ ), and the amount of discounted expected losses being transferred ( $F$ ). The fraction term is the risk load. It is a cross-weighted average of the customer's and insurer's risk values, each weighted on the other party's net capitalization. This creates the even split of the insurance transaction gain.

Example: Consider the following situation:

Customer Assets	=	2,500
Discounted Exp.'d Losses (F)	=	500
Customer Net Assets (c)	=	2,500 - 500 = 2,000
Insurer Assets (n)	=	10,000
Risk Value to Customer (V)	=	400
Risk Value to Insurer (W)	=	250

Then  $c' = c - V = 1,600$ , and:

$$P = \frac{(400)(10,000) + (250)(1,600)}{10,000 + 1,600} + 500$$

$$P = \$879.31$$

$$\text{Risk Load} = \$879.31 - \$500.00 = \$379.31$$

$$\text{Customer Transaction Gain} = \$400.00 - \$379.31 = \$20.69 = 1.29\% \text{ of } \$1,600$$

$$\text{Insurer Transaction Gain} = \$379.31 - \$250.00 = \$129.31 = 1.29\% \text{ of } \$10,000$$

### Premium vs. Transaction Gain

For the customer, the transaction gain is inversely related to the premium paid. For the insurer, the transaction gain is directly related to the premium received. This relationship is depicted in Exhibit 3, which shows gain vs. premium for both the customer (the downsloping curve) and the insurer (upsloping curve). The point of intersection indicates the "fair" premium  $P$  and gain  $k$  described earlier. A mutually profitable transaction can occur at any premium in the range between the two points where the curves intersect the x-axis. This **competitive premium range** varies with the two parties' capitalizations. The width of the competitive premium range equals the risk-value gap. The plot of the customer and insurer gain-vs.-premium curves provides a picture of the risk-value gap.

### Capitalization and Price Competition

As insurer capitalization increases, the cost to the insurer of assuming a given risk decreases, creating a larger risk value gap. At first, it appears that higher insurer capitalization would imply greater potential for total net (customer plus insurer) transaction gain, and therefore a lower fair premium. In fact, this is true only up to a point. As Exhibit 4 shows, there is an amount of insurer capitalization which minimizes the fair premium. This is because the insurer's transaction gain is equal to premium minus assumed risk value, divided by initial assets. Therefore, a larger insurer must receive more premium net of assumed risk value to achieve the same return as a smaller insurer. Up to a certain capitalization, the risk value decreases at a rate which is fast enough to provide additional return to both the customer and the insurer. Beyond that point, the decrease in risk value is not sufficient to support the increased insurer capitalization, and additional premium must be collected.



Exhibit 5 shows the gain vs. premium curves for two insurers, L (low capitalized) and H (high capitalized). For higher premium values, L has an advantage since the same amount of premium received produces a larger percentage return on capital. For lower premium values, H shows an advantage over L due to the lower risk value. There is a range of premiums for which H shows a gain whereas L shows a loss. By offering a premium in this range, H provides the customer with a lower premium than L can afford to offer. This advantage is present whenever H's capital exceeds L's capital. To exercise this competitive pricing advantage, H might have to accept a smaller transaction gain than the fair premium would give. The conclusion is that the dynamic of capacity and demand in the competitive marketplace may preclude equal transaction gains. The transaction can still be financially beneficial to both parties, and can be considered "fair" from a supply-and-demand perspective.

### Reducing Risk by Combining Independent Risks

The discussion so far has focused exclusively on the value generated by a single insurance transaction between two parties, and the value generated by transferring risk from a party with less capital to a party with more capital, for an appropriate premium. It is also possible for value to be generated by an insurance transaction through the combining of two or more independent risk elements. When the resulting combined risk element is supported by capitalization equal to the total of the original capitalizations that supported the individual risk elements, the risk value will be less than the sum of the individual elements' risk values, creating a risk-value gap.

Example: Consider two identical customers A and B. Each customer has assets of \$2,000.00 and a risky liability which will cost \$1,000.00 (discounted to present value) with probability 50%, or \$0.00 with probability 50%. The liabilities' future outcomes are independent of each other. Then, each liability has a fixed component of \$500.00 in expected losses and a pure risk element of:

+\$500.00 @ 50%  
-\$500.00 @ 50%

Calculation of the pure risk element's value relative to either customer yields  $V = \$85.79$  per liability, or \$171.58 in total for both.

Suppose N is an insurer with assets of \$2,870.38 and no liabilities. A and B both agree to transfer their liabilities to N along with a premium of \$564.81 each - \$500.00 for the fixed component, and \$64.81 for the risk element. After the transaction, N has assets of  $\$2,870.38 + 2(\$564.81)$ , or exactly \$4,000.00. This is equal to the combined original assets of the two customers. Thus, N is in the same position that would have been created if A and B had merged their assets and risky liabilities into one entity. The new pure risk element is:

+\$1,000.00 @ 25%  
\$0.00 @ 50%  
-\$1,000.00 @ 25%

The \$1,000.00 amount equals the sum of the original risk elements' amounts, but at half of their original 50% probability. Qualitatively, half of the original risk has been replaced by a 50% probability of no change. This is one advantage of viewing the loss distribution as a fixed liability plus a pure risk element - different pure risk elements can be compared to each other directly for qualitative evaluation. The risk value for N is \$87.05, which is about half of the original total risk value. The transaction gain in this example is equal for all parties, at 1.48%.

Alternatively, A and B could have agreed to reinsure each other for 50% of their respective liabilities. No premium would be involved in this symmetric liability "swap". The result is approximately the same - the risk elements are reduced similarly and the risk value is reduced by about half.

If the insurer in this example had less capital, the gain from combining independent risks would have been partially or completely offset by the reduction in total capital supporting the total risk. At some point, the capital limitations of an insurer create a "diminishing returns" effect with regard to the assumption of additional pure risk, even when the additional risk is independent of all risk currently retained by the insurer.

### **Conclusion**

An uncertain future liability can be viewed as the sum of two components: a fixed component equal to discounted expected losses, and a pure risk component that is a distribution of outcomes with expected value of zero. Since the pure risk element has an expected value of zero, it is neither a true asset nor a true liability, but it has a discounting effect on net capital (assets minus liabilities). The cost associated with a risk element is a function of the amount of net capital against which the risk is retained or assumed.

The cost of a given risk element is inversely related to the net capital of the entity. Transferring risk from a lesser-capitalized entity to a greater-capitalized entity by an insurance transaction generates economic value equal to the difference in the values of the risk to the parties. This total gain, the "risk-value gap", is divided between the parties via the risk load in the premium. An equitable premium sets the risk load so that the parties' transaction gains (as percentages of initial net capitalization) are equal.

A risk value function is a function parameterized by a pure risk element, which maps net capital to risk value. A risk value function should be a decreasing, positive function of net capital for any given pure risk element parameter. There are an unlimited number of possible candidates for risk value functions, one of which is the geometric mean function presented in this paper.

## **Appendix 1 : Technical Discussion of Geometric-Mean Model**

Let  $i_k$  represent an outcome (expressed as a percentage of initial capital) obtained by a risk-retaining entity in year  $k$ . Assume the  $i_k$  are independent and identically distributed. Then, define the equivalent level return  $j$  by:

$$(1 + j)^n = \prod_{k=1}^n (1 + i_k) - 1$$

The variable  $j$  represents the effective annual return that is achieved by the risk-retaining entity over the time period  $k=1$  to  $n$ . It follows that:

$$\log(1 + j) = \frac{1}{n} \sum_{k=1}^n \log(1 + i_k) - \text{Normal}(\log(1 + g), \sigma^2)$$

where  $1+g$  represents the geometric mean of the  $\{1+i_k\}$ . Therefore, the distribution of  $1+j$  is approximately lognormal for large  $n$ , with mode equal to  $1+g$ .

The mean of the lognormal is higher than the mode because the lognormal is skewed, and the mean includes extreme, low-probability results in the tail. As  $n$  gets large, the standard deviation  $\sigma$  decreases, and the probability of actual results falling within a fixed interval around the mode  $1+g$  increases toward unity. Therefore, the mode is most representative of the anticipated long-term return for a single entity. In particular, it is more representative than the mean which is distorted by extreme values that are unlikely to be realized. The mean would be representative of average aggregate results for many independent entities (e.g., an entire industry), because a small but positive percentage of the entities could reasonably be expected to obtain the extreme results in the distribution's tail. Also, a single entity which can obtain new capital after a negative or low result could be expected to achieve an overall result closer to the mean, since such capital infusions would effectively generate repeated trials.

### **An Alternative Risk-Value Function for Shorter Term Results**

The model presented in this paper assumes that the hypothetical insurer will always follow a consistent policy of assuming risk in constant proportion to current capitalization. So, if the insurer's capital increases, it will assume more risk, and if capital declines, less risk will be assumed. While somewhat simplified, this assumption probably corresponds in some degree to actual conditions, since writings are dependent on surplus.

In the model, if the insurer never places 100% of capital at risk, it can never go broke. However, if the insurer does risk 100% of capital, even with a very small probability of total loss, the

geometric model discounts insurer capital to zero, because long-term pursuit of such a policy would inevitably lead to ruin.

Suppose a customer has a 99% probability of no loss for a single time period, and a 1% chance of total loss. According to the geometric model, the customer has risk-discounted assets of zero. This is counterintuitive, since the customer's assets combined with the low-probability risk should have positive value. Moreover, they should have more value than if the probability of total loss were higher (say 80%). Therefore, for a shorter-term scenario such as this one, an alternative risk-value function would be appropriate. One possibility is:

$$V^R(c) = c - [tg + (1 - t)m]$$

where  $m$  is the arithmetic mean, and  $t$  is a weighting parameter between 0 and 1. The probability of severe or total loss is incorporated in this formula by including the arithmetic mean,  $m$ . One way to calculate a value for the  $t$  parameter is to exclude the extreme upper and lower tails (selecting a percental cutoff point) of a lognormal distribution, take the mean  $M$  of the resulting distribution, set  $M$  equal to the expression in brackets in the above equation, and solve for  $t$ .

To the extent that a given insurer risks insolvency, the insurance transaction includes an implicit assumption of credit risk by the customer. Additionally, insurers assume and retain risk on a regular and controlled basis over many years. This is in contrast to customers, who are exposed to risk in what may be a more volatile or temporary manner. It is therefore reasonable for a risk-value model to severely discount insurer capital when a significant risk of insolvency exists, in relation to the probability of insolvency risk.

## Appendix 2 : Derivation of Risk-Value Gap Approximation Formula

By definition,  $V_C = L_C - F_C$ ,  $V_N = n + P - \Pi_N - F_C$ ,  $\Pi_N = \pi(n + P, \langle \$, p \rangle)$ ,  $\Delta V = V_C - V_N$ .

Approximate P in the  $V_N$  and  $\Pi_N$  formulas by  $P \approx L_C = c - \Pi_C$ . Then,

$$V_N \approx n + L_C - \pi(n + L_C, \langle \$, p \rangle) - F_C$$

$$\Delta V \approx V_C - [n + L_C - \pi(n + L_C, \langle \$, p \rangle) - F_C]$$

and since  $V_C = L_C - F_C$  the result follows:

$$\Delta V = \pi(n + L_C, \langle \$, p \rangle) - n.$$

**Fair Premium and Risk Values Calculation**

<b>CUSTOMER POSITION</b>		
<b>Pre-Transaction</b>		
<u>Assets:</u>	\$600.00	
<u>Liability:</u>	<u>Amount</u>	<u>Probab'ly</u>
	\$100.00	50%
	\$300.00	50%
<u>Risk-Disc't'd Assets:</u>	\$404.35	

<b>INSURER POSITION</b>	
<b>Pre-Transaction</b>	
<u>Assets:</u>	\$1,000.00
<u>Liability:</u>	(None)
<u>Risk-Disc't'd Assets:</u>	\$1,000.00

<u>Fair Premium:</u>	\$193.86
<u>Fixed Liability Component (Disc't'd):</u>	\$185.19
<u>Fair Risk Premium Component:</u>	\$8.67

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<b>CUSTOMER POSITION</b>	
<b>Post-Transaction</b>	
<u>Assets @ 1/1/95:</u>	\$406.14
<u>Interest Rate:</u>	8.00%
<u>Assets @ 1/1/96:</u>	\$438.63
<u>Liability:</u>	(None)
<u>Risk-Disc't'd Assets:</u>	\$438.63
<u>Rate of Return:</u>	8.48%
<u>Transaction Gain:</u>	0.44%

<b>INSURER POSITION</b>		
<b>Post-Transaction</b>		
<u>Assets @ 1/1/95:</u>	\$1,193.86	
<u>Interest Rate:</u>	8.00%	
<u>Assets @ 1/1/96:</u>	\$1,289.37	
<u>Liability:</u>	<u>Amount</u>	<u>Probab'ly</u>
	\$100.00	50%
	\$300.00	50%
<u>Risk-Disc't'd Assets:</u>	\$1,084.77	
<u>Rate of Return:</u>	8.48%	
<u>Transaction Gain:</u>	0.44%	

Fair Premium and Risk Values Calculation

<u>CUSTOMER POSITION</u> Pre-Transaction		
<u>Assets:</u>	\$785.19	
<u>Liability:</u>	<u>Amount</u>	<u>Probab'ly</u>
	\$100.00	50%
	\$300.00	50%
<u>Risk-Disc'td Assets:</u>	\$592.81	

<u>INSURER POSITION</u> Pre-Transaction	
<u>Assets:</u>	\$1,000.00
<u>Liability:</u>	(None)
<u>Risk-Disc'td Assets:</u>	\$1,000.00

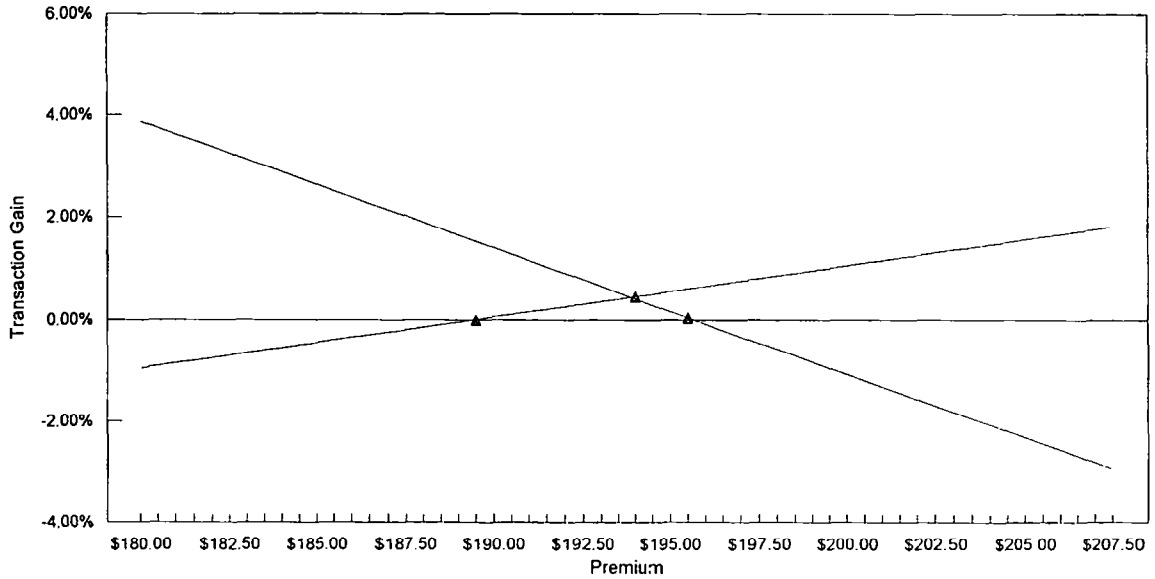
<u>Fair Premium:</u>	\$191.29
<u>Fixed Liability Component (Disc'td):</u>	\$185.19
<u>Fair Risk Premium Component :</u>	\$6.10

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<u>CUSTOMER POSITION</u> Post-Transaction	
<u>Assets @ 1/1/95:</u>	\$593.90
<u>Interest Rate:</u>	8.00%
<u>Assets @ 1/1/96:</u>	\$641.41
<u>Liability:</u>	(None)
<u>Risk-Disc'td Assets:</u>	\$641.41
<u>Rate of Return:</u>	8.20%
<u>Transaction Gain:</u>	0.18%

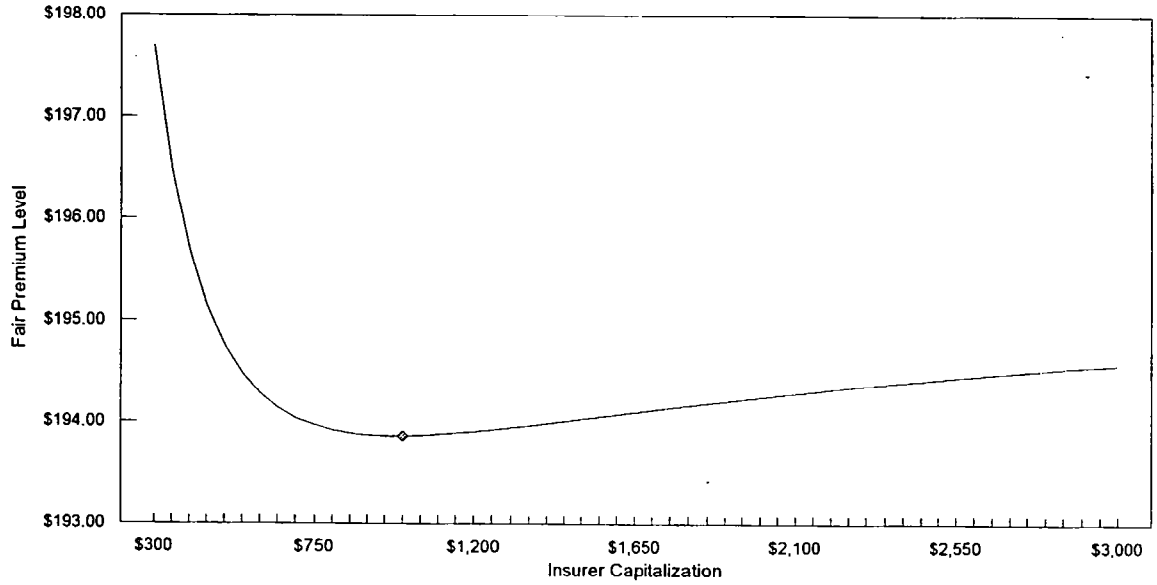
<u>INSURER POSITION</u> Post-Transaction		
<u>Assets @ 1/1/95:</u>	\$1,191.29	
<u>Interest Rate:</u>	8.00%	
<u>Assets @ 1/1/96:</u>	\$1,286.59	
<u>Liability:</u>	<u>Amount</u>	<u>Probab'ly</u>
	\$100.00	50%
	\$300.00	50%
<u>Risk-Disc'td Assets:</u>	\$1,081.98	
<u>Rate of Return:</u>	8.20%	
<u>Transaction Gain:</u>	0.18%	

**Exhibit 3**  
Gain vs. Premium for Customer and Insurer





**Exhibit 4**  
Fair Premium vs. Insurer Capitalization



**Exhibit 5**  
Gain vs. Premium, Customer and 2 Insurers

