PRICING OF EXCESS LIABILITY FOR MUNICIPALITIES: A CASE STUDY

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Pricing a new product line with limited data poses a major challenge to the actuary. Standard actuarial methods require a quantity and consistency of data that may not be available. Therefore, unique solutions may be required. This does not mean that the actuary must develop an entire new methodology. Instead it is often possible to use a combination of techniques found in actuarial literature in reaching a solution. The application of these techniques may require the use of equal portions of actuarial art and science.

This paper relates the method used to develop a pricing framework for a new excess liability insurer. A standard pure loss rate technique was used along with a "curve-fitting" approach. The paper highlights how the limitations imposed by the availability of data were addressed.

LEON R. GOTTLIEB, FCAS, MAAA

Senior Vice President and Senior Actuary of Advanced Risk Management Services of Willis Corroon Corporation. Mr.Gottlieb graduated magna cum laude with a Bachelor of Science degree in mathematics from the City College of New York, and received a Master of Science degree in Operations Research and Statistics from Rensselaer Polytechnic Institute in Troy, New York. Leon attained the fellowship designation from the Casualty Actuarial Society in 1976.

Since joining Willis Corroon in 1990, Leon has performed actuarial studies and consulting for clients ranging from insurance pools to large corporations. He has also authored an actuarial study for a due diligence review and is providing the actuarial support for a newly-forming mutual insurer. Leon has been designated the representative of Willis Corroon's Construction Industry Division.

Leon shares the senior management responsibilities for the casualty actuarial unit, providing actuarial consulting services to clients and prospective clients of Willis Corroon.

Prior to joining Willis Corroon, Leon had almost 18 years of previous experience with the Royal Insurance Group. For the last ten years of that time he was Vice President and Actuary accountable for the corporate actuarial and statistical departments. The mission of these units included recommending loss reserve levels to management, filing of various regulatory and statistical reports, preparation of corporate management reports, and performing various types of financial analysis, including the impact of loss reserves on tax liabilities.

Additional areas in which Leon has experience include planning, budget, and pricing of both individual risks and manual ratemaking. As regards the last aspect, he served on several actuarial committees of the Insurance Service Office and the National Council on Compensation Insurance. Leon's service at Royal included a two year assignment in the Liverpool, England office, where he introduced American actuarial methods.

Leon has served on various committees and task forces of the Casualty Actuarial Society and the American Academy of Actuaries. He is a founder and past president of Casualty Actuaries of the Southeast, a regional affiliate of the CAS. Leon's paper, "Projecting Professional Liability Losses for Design Professionals", was published in the January/February, 1993 issue of *Construction Business Review.*

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I. Introduction

The actuary is often confronted with situations where fully suitable data is not available. An important example of this situation is when a new product line is being developed or a new specialty insurance company is being formed.

The author was requested to provide actuarial pricing indications for a newly forming mutual insurance company which will provide excess liability insurance for municipalities in California. Currently, most of these municipalities obtain this coverage through one of several pools. In 1989, the Government Accounting Standard Board (GASB) issued Standard Number Ten which requires a governmental entity to recognize as a liability its share of likely assessments from pools of which it is a member, and for any anticipated unrecoverable claims. The City of Gardena saw this new accounting standard as creating a need for an alternative to pools, and proposed to provide the capital to form a mutual insurance company which would provide this coverage. This would then free the insured municipalities from the requirement of GASB 10 to establish reserves for potential assessments.

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II. Loss Data

Because most of the existing excess insurance that is provided for California municipalities is through pools, there is no meaningful published data. The City of Gardena was able to obtain loss experience directly from nine other municipalities. They ranged in size in terms of population from about 25,000 to over 250,000. The cities were located in metropolitan areas of both Northern and Southern California, as well as non-metropolitan areas. It is not anticipated that the new company will insure the largest

California cities. These are the cities with most of the historic claims in the upper layers of coverage. Use of data from these largest cities, even if it had been available, would not be representative of the cities to be insured by the mutual insurer.

For five of the cities, complete loss listings were provided. For the other five cities, listings of losses with value of $10,000 or greater were obtained. There was little consistency in the format of the data provided by the ten cities. A database was created that captured the essential elements on as a consistent basis as was feasible. At times, judgments were made when some data element was not explicit in the source data.

The loss data that was used in the pricing consisted only of large losses. This was for a number of reasons.

- For half of the cities, only losses valued at $10,000 or more were available.

- The coverage to be priced is excess insurance over a self-insured retention (SIR). It is expected that almost all cities insured will retain at least $100,000 per occurrence.

- The single parameter Pareto was used to model the size of loss distribution. This distribution is defined only above a specified lower limit. A lower limit of $100,000 was selected.

Since the excess coverage will attach over an SIR inclusive of allocated loss adjustment expense, the losses in the database were tabulated to include those expenses. Throughout this paper when losses are referred to, it should be understood that it also includes allocated loss adjustment expense.

III. Exposure Data

California employers submit reports to the state of California of what is defined as "DE3" payrolls. DE3 payroll was collected directly from the ten cities in the database. For nine of the cities, payrolls were available for from four to eight years. For the tenth city, only estimated payroll for fiscal year ending 1992 was provided. In each case care was taken to remove the payroll for operations that will be excluded from the excess insurance program, such as airports and transportation systems.

Loss information was available for some cities in years where the DE3 payroll had not been compiled. In order to make use of all the loss information, DE3 payrolls were extrapolated backward in time for the missing years. This was accomplished using the assumption that payroll for a city will grow in proportion to its population and with an adjustment for wage inflation.

IV. Other Considerations

It was essential that the new mutual insurance company be supported by reinsurance. Therefore, it was considered important that the pricing structure be acceptable to prospective reinsurers and that the pricing for reinsurance coverage be consistent with the pricing for the gross charged rates. Since the coverage will always attach over an SIR, it was also important that the pricing be equitable over the various SIR levels to prevent adverse selection.

In consideration of the GASB 10 requirements, the new mutual company would be required to operate on a non-assessable basis. This created a need for assurance that the rates would be adequate and be able to absorb some degree of adverse experience. Therefore, a "loss fluctuation" provision was explicitly added. This provision also needed to be consistently applied over all SIR's.

A final consideration was the various audiences that would be evaluating the ratemaking methodology. These included the backers of the mutual insurer, potential reinsurers and the State of California Department of Insurance. The limited database needed to be exploited to the maximum extent. Sophisticated models seldom alleviate the imperfections of a slim database. Simplicity has it merits. Therefore, when choices needed to be made, the author generally opted for the most straight-forward assumptions and model design.

The available data is limited. In fitting a loss distribution to this data and extrapolating the results to high layers of loss, there was significant potential for variation. Nevertheless, pricing decisions needed to be made. The degree of accuracy achieved will only be known over time. The addition of a provision for adverse experience or mis-estimation was therefore crucial. It was expected that this procedure would also provide a benchmark against which the past and future loss experience of the municipalities that are insured by this mutual company can be measured.

V. Calculation of Pure Premium Rate

The first step was to calculate a pure premium rate at a "base layer" of coverage which was selected to be the $900,000 excess of $100,000 layer. The mutual insurer will provide $5 million of coverage over the insured's self-insured retention. The SIR will range from as low as $50,000 up to $1 million per occurrence. It is expected that the most commonly selected SIR levels will be between $100,000 and $250,000. A second consideration in choosing the lower bound for the base layer was to ensure that no losses would be "missing" from the layer, after trend is applied. Some of the cities in the sample only supplied losses in excess of $10,000. A $10,000 loss for the earliest years would trend to over $50,000. Thus, a lower bound of $100,000 was considered appropriate. It was also desired that no single loss have an excessive influence on the pure premium loss rate. Therefore some limitation was needed. An upper bound of $1,000,000 was selected for the base layer of coverage.

The single parameter Pareto loss distribution, which is described in Section VI, was used to adjust the pure premium rate for the $900,000 xs $100,000 layer to $5 million excess of the various SIR's. This is shown in Step 8 of Section VI. The elements of ratemaking, such as loss trending and loss development were considered separately for each of the two stages (pure premium rate, and size of loss distribution).

Loss Trend

Using a trend factor of 10% per annum, each of the individual claims was brought to the expected cost level during the period that the rates would be effective. The trend factor was selected after examining broad indices of liability claim costs.

After applying the trend factor to each claim, those with a value of $100,000 or more were selected for the pricing. These losses were used in two ways:

1) To develop a pure loss cost for losses for the $900,000 excess of $100,000 layer; and

2) To produce an empirical size of loss distribution, which was then fitted to a statistical distribution.

Loss Development

A majority of the losses included in the final database were still open at the valuation date. Loss development was addressed in distinct ways for each of the two loss projection phases:

1) In projecting pure loss costs, excess layer loss development factors were applied to total losses in excess of $100,000.

2) In generating the size of loss distribution, no development was applied to individual losses.

On an aggregate basis, it is expected that there will be upward loss development. There are three reasons for this:

1) Some claims that have not yet been reported will exceed $100,000.

2) Some claims currently valued at less than $100,000 will settle for amounts in excess of $100,000.

3) On claims currently valued at $100,000 or more, there will likely be more upward development than downward development.

Therefore, loss development must be reflected in the pure loss cost projection.

No historical loss information was available for the cities in the database to directly measure loss development on either a first dollar or excess basis. Pinto and Gogol(1), in their 1987 paper, used insurance industry loss development data to model excess loss development factors. The paper displays separate development factors for OL&T, M&C, and Products Liability. All three types of claims, or claims of comparable severities, are experienced by municipalities. For example, cities have extensive OL&T exposure from the properties they own and occupy. The streets and roads hazard is analogous to M&C. While police operations are not products liability per se, the severity of the potential causes of action such as false arrest or civil rights violation are comparable. Therefore, the three tables published in the Pinto & Gogol paper were each given equal weighting and a set of composite loss development factors for losses excess of $100,000 was calculated. While these factors were based on policy year data, they were applied in this study to fiscal accident year losses in order to provide a measure of conservatism.

Ideally a size of loss distribution would be based only on closed claims. Due to the sparsity of data, it was important to use all the available information. Some will settle for more, others for less than the current reserve. It was felt that in determining the shape of the size of loss distribution the latest valuation would provide the best indication, since the individual loss assessments represent the most recent best estimate of each claim's ultimate value.

Pure Premium

Exhibit 1 shows the calculation of pure loss rates using standard methodology. The payrolls were adjusted for subsequent wage trend, based on assumed changes in California public sector wage rates. In each year, the DE3 payroll includes only those cities for which loss data was available. The loss development and trend factors were described previously. The losses were censored at $1,000,000 (after trending each individual claim).

Projected losses per $100 of DE3 payroll were then calculated and various averages were taken. The 1983-88 weighted average is $0.78 and a straight average over those six years is $0.82. The earlier years (1981-82) are based on only two cities, while the most recent years (1989-90) were judged to be too immature for consideration. A pure loss rate of $0.79 was selected. Also shown on Exhibit 1 is reported excess claim frequency per $100 million of adjusted payroll. These claim counts have not been developed to an ultimate basis.

VI. Modeling the Loss Severity Distribution

The single parameter Pareto distribution was selected to model the loss severity distribution. As Philbrick[2] described in his 1985 paper, this family of statistical distributions often provides a good representation of loss severity. Its characteristics make it especially useful if estimates of losses within various size ranges will be required. Since it is a relatively simple distribution, various statistical properties can be computed directly, rather than through the use of simulations.

One additional advantage of this distribution is that there is a simple formula for estimating the single parameter:

$$Q = \frac{n}{\sum_i \ln x_i}$$

EXHIBIT 1

LOSSES TRENDED AT 10% AND CENSORED AT $1,000,000

YEAR*	DE3 PAYROLL	INFLATION ADJUSTED DE3 PAYROLL	TRENDED LOSSES $100,000+	TRENDED LOSSES XS$100,000	XS LDF	ULTIMATE LOSSES XS$100,000	LOSS PER $100 PAYROLL	# XS CLAIMS	XS CLAIM FREQ PER $100M DE3
1981	$322,975	$632,191	$121,750	$21,750	1.123	$24,425	$0.04	1	1.582
1982	351,164	654,636	302,198	202,198	1.152	232,932	0.36	1	1.528
1983	477,880	848,437	1,582,665	882,665	1.190	1,050,371	1.24	7	8.250
1984	1,507,133	2,548,373	1,612,116	812,116	1.241	1,007,836	0.40	8	3.139
1985	1,934,406	3,115,084	2,675,459	1,375,459	1.313	1,805,978	0.58	13	4.173
1986	2,366,620	3,629,622	4,373,284	3,073,284	1.422	4,370,210	1.20	13	3.582
1987	3,029,764	4,371,278	3,082,140	1,782,140	1.598	2,847,860	0.65	13	2.974
1988	3,222,931	4,374,388	3,390,949	1,890,949	1.910	3,611,713	0.83	15	3.429
1989	3,452,142	4,407,798	893,333	393,333	2.556	1,005,359	0.23	5	1.134
1990	2,194,715	2,636,197	326,195	126,195	4.389	553,870	0.21	2	0.759
TOTAL	18,859,730	27,218,004	18,360,089	10,560,089		16,510,554	0.61	78	2.866
1983–88	12,538,734	18,887,182	16,716,613	9,816,613		14,693,968	0.78	69	3.653
SELECTED							0.79		

* Fiscal Calendar/Accident Years Ending June 30.

where x_j's are the observed values divided by the lower bound, in this case \$100,000. In selecting the Q value to be used, trended losses excess of \$100,000 were examined, both on an unlimited basis, and limited to \$1 million and \$2 million per occurrence. In addition, Q parameters were calculated separately for each accident year. The "Q's" for the older years, for which most claims are closed, were fairly stable. It should be noted that because of the limited sample size (82 occurrences), there is considerable potential for parameter variance. Please note that in addition to the 78 large losses for the period 1981-1990 as shown in Exhibit 1, there were an additional four large losses in earlier or later years.

Expected Loss and Fluctuation Provision

The following section shows how the properties of the Pareto distribution were used to derive the loss costs and fluctuation provisions for each SIR. The calculations will be shown for the \$500,000 SIR, and results are tabulated after each step also at SIR's of \$100,000, \$250,000, and \$1,000,000. Recall that in each case, \$5 million of limits of coverage will be provided. Therefore, the upper loss limit for the \$500,000 SIR is \$5,500,000.

Some of the important the properties of the single parameter Pareto are:

A. The cdf of the single parameter Pareto is:

$$F(x) = 1 - \left(\frac{x}{C}\right)^{-Q} \tag{6.1}$$

Where C is the lower limit (the SIR) and Q is the parameter.

B. The unlimited mean claim size is:

$$C\left[\frac{Q}{Q-1}\right]$$

C. The limited mean claim size is:

$$C\left[\frac{Q-B^{1-Q}}{Q-1}\right] \tag{6.2}$$

Where U is the upper limit, and:

$$B=\frac{U}{C}$$

D. If the claim frequency is assumed to be Poisson distributed with parameter M, the total loss variance for the covered layer (between C and U) is:

$$\sigma^2 = C^2M\left\{\frac{Q-2B^{2-Q}}{Q-2}-2\left(\frac{Q-B^{1-Q}}{Q-1}\right)+1\right\}Q\neq 2,\ Q\neq 1 \tag{6.3}$$

These formulas are from Philbrick's paper (with notation modified).

1. Calculation of the frequency of excess losses relative to $100,000 SIR using the cumulative distribution function formula (6.1), and Q = 1.4 yields.

$$\left(\frac{SIR}{100{,}000}\right)^{-Q}=\left(\frac{500{,}000}{100{,}000}\right)^{-1.4}=0.1051$$

SIR	Relative Frequency
$ 100,000	1.0000
250,000	0.2773
500,000	0.1051
1,000,000	0.0398

2. Average gross loss for losses in layer $5 million excess of SIR using formula (6.2).

$$= SIR\left[\frac{Q-B^{1-Q}}{Q-1}\right] \text{ where } B=\frac{\textit{Upper Limit}}{\textit{Lower Limit}(=SIR)}$$

$$\textit{For the } \$500{,}000\ SIR,\ B=\frac{5{,}500{,}000}{500{,}000}=11$$

The average loss limited to $5,500,000 is calculated as:

$$Average\ Loss = (\$500,000)\left[\frac{1.4 - 11^{(1-1.4)}}{1.4 - 1}\right] = \$1,270,981$$

SIR	B	Average Gross Loss
$ 100,000	51	$298,131
250,000	21	690,076
500,000	11	1,270,981
1,000,000	6	2,279,102

3. Average net loss in the layer $5M excess of SIR = **average gross loss - SIR**.
   For $500,000 SIR = $1,270,981 - $500,000 = $770,981.

SIR	Average Net Loss
$ 100,000	$198,131
250,000	440,076
500,000	770,981
1,000,000	1,279,102

4. *The relative expected loss cost for the various SIR's is the frequency adjusted* average net loss = **Average net loss x Frequency relative to $100,000 SIR**

   For $500,000 SIR = $770,981 x .1051 = $81,030

SIR	Frequency Adjusted Average Net Loss
$ 100,000	$198,131
250,000	122,033
500,000	81,030
1,000,000	50,908

The next steps will relate the frequency adjusted net loss to the base layer of coverage. First the frequency adjusted loss cost for the base layer is found.

5. Average gross loss in layer $900,000 x $100,000 using formula (6.2).

$$= \$100,000\left[\frac{1.4 - 10^{(1-1.4)}}{1.4 - 1}\right] = \$250,473$$

6. Average net loss in layer $900,000 x $100,000

= **Gross Loss - SIR** = $250,473 - $100,000

= $150,473

= Frequency adjusted average net loss, since relative frequency = 1.00

7. Expected loss costs relative to $900,000 excess of $100,000

= **Frequency Adjusted Average Net Loss for SIR ÷ $150,473** from above.

$$\textit{For the } \$500{,}000 \textit{ SIR} = \frac{\$81{,}030}{\$150{,}473} = 0.539$$

SIR	Relative Loss Cost
$ 100,000	1.3167
250,000	0.8110
500,000	0.5385
1,000,000	0.3383

This provides the relative cost between $5 million of coverage excess of each SIR and the base layer. This is then converted to an expected loss cost by multiplying each of the relativities by the rate for the base layer.

8. Expected loss cost per $100 DE3 payroll = **$0.79 x relative loss cost**. For $500,000 SIR = $0.79 X .5385 = $0.425.

SIR	Expected Loss Cost
$ 100,000	$1.040
250,000	0.641
500,000	0.425
1,000,000	0.267

9. Using Steps 3 and 8, an implied claim frequency may be calculated.

Using the SIR of $100,000 as the basis:

Average net loss $5 million xs $100,000 = $198,131 (Step 3)

Expected losses = $1.04 per $100 DE3 payroll (Step 8)

or

$1,040,000 per $100 Million DE3 payroll

$$\frac{\$1{,}040{,}000}{\$198{,}131} = 5.25 \text{ Claims xs } \$100{,}000 \text{ per } \$100 \text{ million of payroll.}$$

Note that this projected frequency is somewhat above the undeveloped empirical excess loss frequencies shown on the last column of Exhibit 1.

Frequencies for other SIR's can be calculated using relative frequencies from Step 1: For the \$500,000 SIR = 5.25 X .1051 = 0.552.

SIR	Claims Per \$100 Million DE3
\$ 100,000	5.250
250,000	1.456
500,000	0.552
1,000,000	0.209

10. As noted previously, Philbrick has shown that if the claim frequency is assumed to be Poisson distributed, the total process variance for a loss layer may be calculated from the formula below. This does not quantify the parameter variance.

$$\sigma^2 = \left(SIR^2\right) M\left[\frac{Q-2B^{2-Q}}{Q-2} - 2\left(\frac{Q-B^{1-Q}}{Q-1}\right) + 1\right] \qquad (6.3)$$

M is the expected number of claims excess of the SIR.

If the first year insured DE3 payroll is \$500 million, then M, the expected number of claims would be 2.76 (= 0.552 x 5) where 0.552 is from Step 9 for the \$500,000 SIR.

The table below shows the quantity within the brackets, the "partial variance", for each SIR. Recall that in Step 2, B was calculated as $\left(\frac{Upper\ Limit}{SIR}\right)$:

$$For\ SIR\ of\ \$500{,}000\ B = \frac{\$5{,}500{,}000}{\$500{,}000} = 11$$

SIR	M	B	Partial Variance
\$ 100,000	26.25	51	27.9753
250,000	7.28	26	13.8575
500,000	2.76	11	7.6340
1,000,000	1.05	6	3.8757

11. One standard deviation of the total losses for each layer can then be found as the SIR multiplied by the square root of the product of the expected number of claims and the variance from Step 10.

The standard deviation of the expected losses for the $500,000 *SIR*

$= \$500,000\sqrt{2.76 \; X \; 7.6340} = \$2,295,095$

Where $500,000 is the SIR, and 2.76 (the number of expected claims excess of $500,000), and 7.6340 (the partial variance) are from Step 10.

SIR	Expected Claims (M)	Standard Deviation
$ 100,000	26.25	$2,709,892
250,000	7.28	2,511,008
500,000	2.76	2,295,095
1,000,000	1.05	2,017,296

The standard deviation of the total losses is seen to be dependent on the expected number of claims or insured payroll. As the number of municipalities insured by the mutual insurance company increases, the standard deviation will decrease relative to the expected losses.

12. The loss fluctuation provision will be expressed as a percentage of expected losses for each SIR. To do this, expected losses corresponding to $500 million of DE3 payroll is calculated as:

Expected Claims x Average Net Loss

For the $500,000 SIR, this is 2.76 X $770,981 = $2,127,908. The 2.76 is from Step 10 and the $770,981 is the average net loss from Step 3.

SIR	Expected Losses
$ 100,000	$5,200,939
250,000	3,203,753
500,000	2,127,908
1,000,000	1,343,057

13. It was considered appropriate for this new insurer that actual losses should not exceed the provision for losses and fluctuation more often than one year out of six. Using the Normal approximation, this is approximately equivalent to one standard deviation.

The provision for loss fluctuation can then be expressed as a percentage of the expected losses, and then as a rate per $100 of DE3 payroll.

For the $500,000 SIR using Steps 11 and 12, the percentage load for loss fluctuation is:

$$\frac{\$2,295,095}{\$2,127,908} = 107.9\%$$

107.9% of the $0.425 expected losses per $100 DE3 payroll is $0.459 for a total loss and fluctuation provision of $0.884. Therefore, a $0.884 loss provision will be adequate about 84% of the time. This assumes that the only source of variation is process variance.

SIR	One Standard Deviation as Percent of Expected Losses	Fluctuation Provision	Indicated Loss Rate
$ 100,000	52.1%	$0.542	$1.582
250,000	78.4%	0.503	1.144
500,000	107.9%	0.459	0.884
1,000,000	150.2%	0.401	0.668

The indicated loss rates can then be increased for an expense and profit provision to produce manual rates.

VII. Excess Reinsurance

The model described in the previous section can be used to price excess reinsurance. Both the expected losses and the loss fluctuation provision can be calculated for any desired layer.

Since parallel layers of coverage (e.g. first $x million excess of each SIR) will have different relative and absolute loss costs due to different attachment points, separate calculations must be made for each SIR.

Sample calculations are shown in the Appendix.

While the expected losses for consecutive non-overlapping layers will sum to the expected losses over the entire coverage, the loss fluctuation provisions are not additive. This is because of the simplifying assumption that was made to define the loss fluctuation provision in terms of the standard deviation of the expected losses.

The non-additivity can be handled in several ways. One way is to prorate the fluctuation provisions of the individual layers to equal the provision for the entire ($5 million) coverage. Another way is to give explicit consideration to the greater spread of risk available to the reinsurers. This is not the only risk covered by the reinsurer. In fact, in adding this treaty to its portfolio, the overall variance of its portfolio may even be reduced. This suggests that an acceptable loss fluctuation provision for the reinsurer may be less than one standard deviation. Alternately, in computing the percentage loading for loss fluctuation, a higher expected number of expected losses can be used for the reinsurer.

This will reduce the ratio of the standard deviation to expected losses within the reinsured layer.

VIII. Application of the Model for Monitoring Purposes

There are several reasons why the loss costs estimated by the analysis described in this paper may produce deficient or redundant rates. These include:

1. The possibility that the Pareto is not a suitable distribution;
2. The Pareto distribution may be suitable, but the selected parameter (Q = 1.4) may not be;
3. One or more of the elements used in the selection of the base pure loss rate (e.g. trend, loss development) may not be suitable;
4. The cities for which data was available may not be representative of the cities that are ultimately insured by the mutual.

The method developed in this paper can be applied to the book of business that is eventually insured by the mutual. The underwriters will have available, at a minimum, the type of data that was used in calculating the rates. Thus, the base pure loss rate and value of Q can be re-calculated from that data (either for individual municipalities or the entire book). This will test whether the cities used for the pricing were representative of the insured population.

A second test will be on the experience of the mutual as it develops. While it will be many years until the size of loss distribution can be tested (because of both claim volume and maturity of claims), it should be possible to test the overall "base rate" calculation at an earlier stage. Because of claim notification requirements, data will be available for amounts well below the SIR, if not on a first dollar basis.

IX. Conclusion

Pricing a new product line with limited data poses a major challenge to the actuary. Standard actuarial methods require a quantity and consistency of data that may not be available. Therefore, unique solutions may be required. This does not mean that the actuary must develop an entire new methodology. Instead it is often possible to use a combination of techniques found in actuarial literature in reaching a solution. The application of these techniques may require the use of equal portions of actuarial art and science.

References

(1) Pinto, E and Gogol, D.F. "An Analysis of Excess Loss Development", *PCAS*, LXXIV, pp. 227-271.

(2) Philbrick, S. W. "A Practical Guide to the Single Parameter Pareto Distribution", *PCAS*, LXXII, 1985, pp. 44-84.

Appendix - Pricing Excess Reinsurance

Most of the Figures in this Appendix parallel a step in Section VI (Modeling the Loss Severity Distribution). The step number will be indicated in parentheses.

The layer of $1 million excess of $1 million excess of an SIR of $500,000 will be used for illustration. This is equivalent to $1 million excess of $1,500,000.

In Figure 1, the single parameter Pareto's cumulative distribution function is used to compute the relative claim frequency for the sum of the SIR and primary retention corresponding to each entry in the matrix. (Step 1)

$$\left(\frac{1,500,000}{100,000}\right)^{-1.4} = 0.0226$$

Figure 2 shows the average gross loss for each layer and SIR combination. The values of B shown are for the layer $1 million excess of $1 million excess of the SIR. Other values of B were used to compute the other layers. (Step 2)

$$B = \frac{1,000,000 + 1,500,000}{1,500,000} = 1.667$$

$$Average\ Loss = \$1,500,000\left(\frac{(1.4-(1.667)^{1-1.4}}{1.4-1}\right) = \$2,193,026$$

In Figure 3, the retention is subtracted to give the net loss for each combination. (Step 3)

$2,193,026 - $1,500,000 = $693,026

Figure 4 multiplies Steps 1 and 3 to compute the frequency adjusted average net loss cost at each SIR and layer of coverage combination. (Step 4)

$693,026 x 0.0226 = $15,662

In Figure 5 the expected loss cost is multiplied by 26.25, the number of claims expected in excess of $100,000 for $500 million of payroll. (Step 10)

$$26.25 \text{ x } \$15,662 = \$411,128$$

Figure 6 shows the percentage split of the total losses by layer, separately for each SIR. In the example, 19.33% of a covered losses excess of $500,000 are in the layer $1M excess of $1.5 million.

$$\frac{\$411,128}{\$2,127,038} = 19.33\%$$

In Figures 7 and 8, one standard deviation of total net losses is calculated. Notice that in Figure 8 the sum of the standard deviations of the layers is greater than the fluctuation margin for the (combined) coverage of $5 million excess of the SIR . (Steps 10 and 11)

$$\left[\frac{1.4-2(1.667)^{2-1.4}}{1.4-2}-2\left(\frac{1.4-(1.667)^{1-1.4}}{1.4-1}\right)+1\right]=0.27148$$

$$\sigma = 1,500,000\sqrt{.27148x(26.25x0.0226)} = 601,976$$

Where $1,500,000 = lower limit of the layer

0.27148 is from Figure 7.

26.25 is expected number of claims excess of $100,000 and

0.0226 is from Figure 1, the relative number of claims excess of $1,500,000.

In Figure 9, the fluctuation provision for each layer/SIR combination is expressed as a percentage of the expected losses in the layer. (Step 13)

$$\frac{\$601,976}{\$411,128} = 146.42\%$$

Where $601,976 is from Figure 8 and $411,128 is from Figure 5.

As expected, the higher layers have greater relative fluctuation provisions.

APPENDIX
FIGURE 1

FREQUENCY RELATIVE TO $100,000

$= (\text{LOWER BOUND}/100{,}000)^{-Q}$

Q = 1.4

UPPER LIMIT ($M)	RETENTION ($M)	SIR = $100,000	$250,000	$500,000	$1,000,000
1.0	0.0	1.0000	0.2773	0.1051	0.0398
2.0	1.0	0.0348	0.0291	0.0226	0.0151
5.0	2.0	0.0141	0.0128	0.0110	0.0086
5.0	0.0	1.0000	0.2773	0.1051	0.0398

APPENDIX
FIGURE 2

AVERAGE GROSS LOSS

$$= \frac{(\text{SIR} + \text{RETENTION})(Q - B^{(1-Q)})}{Q-1}$$

Q = 1.4

UPPER LIMIT ($M)	RETENTION ($M)	SIR = $100,000 B* = 1.909	$250,000 1.800	$500,000 1.667	$1,000,000 1.500
1.0	0.0	$254,196	$546,684	$944,507	$1,605,354
2.0	1.0	1,726,746	1,904,749	2,193,026	2,748,585
5.0	2.0	3,668,550	3,866,962	4,190,566	4,816,063
5.0	0.0	298,131	690,076	1,270,981	2,279,102

* Values of B shown for layer $1M xs $1M xs SIR
B = (Upper Limit + SIR)/(Retention + SIR)

APPENDIX
FIGURE 3

AVERAGE CENSORED NET LOSS

= AVERAGE GROSS LOSS – RETENTION

UPPER LIMIT ($M)	RETENTION ($M)	SIR = $100,000	$250,000	$500,000	$1,000,000
1.0	0.0	$154,196	$296,684	$444,507	$605,354
2.0	1.0	626,746	654,749	693,026	748,585
5.0	2.0	1,568,550	1,616,962	1,690,566	1,816,063
5.0	0.0	198,131	440,076	770,981	1,279,102

APPENDIX
FIGURE 4

FREQUENCY ADJUSTED AVERAGE LOSS

= AVERAGE CENSORED NET LOSS x RELATIVE FREQUENCY

UPPER LIMIT ($M)	RETENTION ($M)	SIR = $100,000	$250,000	$500,000	$1,000,000
1.0	0.0	$154,196	$82,270	$46,718	$24,093
2.0	1.0	21,811	19,053	15,662	11,304
5.0	2.0	22,117	20,697	18,596	15,618
5.0	0.0	198,131	122,033	81,030	50,908

APPENDIX
FIGURE 5

EXPECTED NET LOSSES

= FREQUENCY ADJUSTED AVG LOSS x EXPECTED CLAIMS

EXPECTED CLAIMS XS OF $100,000 = 26.25

UPPER LIMIT ($M)	RETENTION ($M)	SIR =	$100,000	$250,000	$500,000	$1,000,000
1.0	0.0		$4,047,645	$2,159,588	$1,226,348	$632,441
2.0	1.0		572,539	500,141	411,128	296,730
5.0	2.0		580,571	543,296	488,145	409,973
5.0	0.0		5,200,939	3,203,366	2,127,038	1,336,335

APPENDIX
FIGURE 6

% OF EXPECTED NET LOSSES

= LOSSES IN LAYER / TOTAL LOSSES

UPPER LIMIT ($M)	RETENTION ($M)	SIR =	$100,000	$250,000	$500,000	$1,000,000
1.0	0.0		77.83%	67.42%	57.66%	47.33%
2.0	1.0		11.01%	15.61%	19.33%	22.20%
5.0	2.0		11.16%	16.96%	22.95%	30.68%
5.0	0.0		100.00%	100.00%	100.00%	100.00%

APPENDIX
FIGURE 7

"PARTIAL" VARIANCE OF TOTAL NET LOSSES*

Q = 1.4

UPPER LIMIT ($M)	RETENTION ($M)				
		SIR =	$100,000	$250,000	$500,000	$1,000,000
		B** =	1.909	1.800	1.667	1.500
1.0	0.0		7.63397	3.04829	1.33258	0.50835
2.0	1.0		0.44045	0.36195	0.27148	0.16950
5.0	2.0		0.84942	0.77135	0.66395	0.50835
5.0	0.0		27.97531	13.85750	7.63397	3.87565

* Uses Formula (3)
** Values of B shown for layer $1M xs $1M xs SIR
B = (Upper Limit + SIR)/(Retention + SIR)

APPENDIX
FIGURE 8

1 STANDARD DEVIATION OF NET TOTAL LOSSES

= (RETENTION + SIR) * SQRT(PARTIAL VARIANCE * N)

UPPER LIMIT ($M)	RETENTION ($M)					
		SIR =	$100,000	$250,000	$500,000	$1,000,000
		N* =	26.25	7.28	2.76	1.04
1.0	0.0		1,415,598	1,177,627	958,700	728,765
2.0	1.0		697,743	657,272	601,976	518,404
5.0	2.0		1,177,483	1,145,455	1,094,633	1,016,287
5.0	0.0		2,709,893	2,510,857	2,294,622	2,012,234
SUM OF LAYERS			3,290,824	2,980,354	2,655,309	2,263,456

* N = Expected number of claims excess of (SIR + Retention)
Values of N shown for layer $1M xs SIR
N = 26.25 x Relative Frequency from Figure 3
Variance is from Figure 7
SQRT signifies square root

APPENDIX
FIGURE 9

FLUCTUATION MARGIN AS % OF EXPECTED LOSSES

UPPER LIMIT ($M)	RETENTION ($M)	SIR = $100,000	$250,000	$500,000	$1,000,000
1.0	0.0	34.97%	54.53%	78.18%	115.23%
2.0	1.0	121.87%	131.42%	146.42%	174.71%
5.0	2.0	202.81%	210.83%	224.24%	247.89%
5.0	0.0	52.10%	78.38%	107.88%	150.58%