A Method to Estimate Probability Level for Loss Reserves

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Abstract

This paper explores the collective risk model as a vehicle for estimating the probability distribution for reserves. Though this basic model has been suggested in the past and it provides a direct means to estimate process uncertainty, it does not directly address the potentially more significant problem of parameter uncertainty. This paper presents some techniques to estimate parameter uncertainty and, to some extent, also uncertainty regarding projection model selection inherent in reserve estimates.

A METHOD TO ESTIMATE PROBABILITY LEVELS FOR LOSS RESERVES

1. Introduction

The collective risk model, see for example Beard, Pentikäinen and Pesonen [1], provides a conceptually simple framework to model total claims in the insurance process. In its simplest form this model calculates the total loss from an insurance portfolio as the sum of N random claims chosen from a single claim size distribution where the number N is itself a random variable. With some fairly broad assumptions regarding the number and size of claims we can draw conclusions regarding the various moments of distribution of total claims. Thus this model seems to be a reasonable choice as a starting point in estimating the distribution of reserves for an insurer.

The distribution resulting from this simple collective risk model provides an estimate of the potential variation in total payments assuming all distributions are correct. We often refer to this variation as process variation, that inherent due to the random nature of the process itself. Not directly addressed in this simple collective risk model is the possibility that the estimates of the parameters for the underlying distributions, are incorrect. Variation due to this latter uncertainty is often called parameter variation.

Parameter variation is itself an important aspect in assessing the variability inherent in insurance related estimates. Meyers and Schenker [2] discuss this aspect of collective risk applications. They conclude, not surprisingly, that for a "large" volume of claims, that expected to be experienced by most insurers, parameter uncertainty is a much more significant contributor to overall variability than the random, or process, portion.

As indicated above, the collective risk model does not directly address parameter uncertainty nor does it address the methodology used in obtaining reserve estimates themselves. In practice actuaries often apply several methods, based on different underlying assumptions, to derive different projections of required reserves. The actuary then selects a "best estimate" of required reserves, based on the various projections used, keeping in mind the nature of the data and the assumptions inherent in each of the methods. Complicating matters further is the fact that most of the generally accepted actuarial projection methods currently in use are not stochastic in nature, that is, they do not have specific assumptions regarding underlying probability distributions. Thus, in many cases, they only provide "point estimates" without any indication as to the statistical nature of those estimates.

Even if the actuary uses stochastic methods, methods that make assumptions regarding the underlying distributions, the result will usually be a single distribution of total losses or reserves. It is possible that different methods may lead to different estimates of the distribution of reserves. This raises another area of uncertainty that should be considered in estimating probability levels for loss reserves; that of uncertainty that the model applied is indeed the correct one. This is sometimes termed specification uncertainty.

Though many of the stochastic methods we have seen attempt to provide estimates of process variation and sometimes even parameter variation within the framework of the particular model those methods do not provide a convenient means of measuring the possibility that the model itself may be incorrect. Even regression related approaches with regimens in selecting which independent variables to include can only claim to provide the "best" estimate within a particular family of models and do not generally address whether another family is indeed better for a particular situation.

For these reasons this paper will deal with an application of collective risk theory to estimate probability levels in loss reserves. Though the method that we present follows the general approach described in

Hayne [3] we cover ground not covered there, especially in the area of estimating the impact of parameter uncertainty in probability levels.

2. The Collective Risk Model

The basic collective risk model, as described above, can probably be seen best as the implementation of the following algorithm:

Algorithm 2.1

1. Randomly select N, the number of claims.

2. Randomly select N claims, $X_1, X_2, \ldots, X_N$ from the claim size distribution.

3. Calculate aggregate loss as $T = X_1 + X_2 + \ldots + X_N$.

4. Repeat steps 1 through 3 "many" times.

The distribution of T then represents the distribution of total losses given the distributions of the individual claims X_i and the distribution of N, the number of claims. Assuming these distributions are correct the result of this algorithm provides an estimate of the inherent process variation. It does not, however, provide a means of incorporating parameter uncertainty.

We will follow Heckman and Meyers [4] and consider a revised collective risk algorithm that incorporates parameter uncertainty in both the claim count and claim size distributions. We assume that the number of claims N has a Poisson distribution with mean λ, and hence variance $\mathrm{Var}(N) = \lambda$. We also assume that χ is a random variable with $E(\chi) = 1$, and $\mathrm{Var}(\chi) = c$. The variable χ then will be used to reflect the uncertainty with the selection of the expected claim count parameter λ. If χ is assumed to have a

Gamma distribution then Heckman and Meyers show that the resulting N will have a negative binomial distribution with

$$E(N) = \lambda, \text{ and}$$
$$\mathrm{Var}(N) = \lambda + c\lambda^2$$

In this case $\mathrm{Var}(N) \geq E(N)$, with equality only if $c = 0$.

As Heckman and Meyers point out, the Poisson distribution assumes that claims during two disjoint time periods are independent, that the expected claims in a time interval is dependent only on the length of the interval and not on the starting point of that interval and that no more than one claim can occur at a time. They introduce the contagion parameter c to allow for dependence of the number claims in one time interval on claims in prior interval(s). The above modification with $c > 0$ assumes that the number of claims in one interval is positively correlated with the number in past intervals. For example, a successful liability claim may lead to an increased number of future claims.

Similarly it is possible that the existence of past claims may decrease the possibility of future claims. An example that Heckman and Meyers point out in this situation is with a group of life insurance policies where claims in an earlier period reduces the number of claims in a later period. They model this by assuming that the final claim count distribution will be Binomial. In this case $\mathrm{Var}(N) < E(N)$, which can be accomplished with an appropriate negative value for c, even though a negative value does not make sense in the original derivation of the distribution for N. We will thus assume that N has either a Binomial distribution ($c < 0$), a Poisson distribution ($c = 0$), or a Negative Binomial distribution ($c > 0$).

The modification of Algorithm 2.1 also reflects uncertainty in the overall mean of the claim size distribution. For this we assume that β is a random variable with $E(1/\beta) = 1$ and $\mathrm{Var}(1/\beta) = b$. With these added distributions Heckman and Meyers present the following modified collective risk algorithm:

Algorithm 2.2

1. Randomly select a number N from the assumed claim count distribution.
2. Select N claims $X_1, X_2, \ldots, X_N$ from the assumed claim size distribution.
3. Randomly select a number β from the assumed distribution.
4. Calculate the aggregate loss as $T = \frac{1}{\beta}(X_1 + X_2 + \ldots + X_N)$.
5. Repeat steps 1 through 4 "many" times.

We note that in the case that $b = c = 0$, that is, no parameter uncertainty, Algorithm 2.2 simply reduces to Algorithm 2.1 with an assumed Poisson claim count distribution.

Following Heckman and Meyers we will assume that β has a Gamma distribution. We follow their caution that this is selected for its mathematical convenience rather than for a specific property of parameter uncertainty. We refer readers to page 31 of [4] for a further discussion of this assumption.

The collective risk model has some useful properties, for example, if we know the moments of the claim count and claim size distributions, assuming independence of the various distributions, we can determine the corresponding moments of the final aggregate distribution. These properties hold for both the formulation in Algorithm 2.1 and the formulation in Algorithm 2.2. In particular under the above conventions we have:

$$E(T) = \lambda E(X) \tag{2.1}$$

$$\mathrm{Var}(T) = \lambda E(X^2)(1+b) + \lambda^2 E^2(X)(b + c + bc) \tag{2.2}$$

Since Algorithm 2.1 is a special case of Algorithm 2.2 with $b = c = 0$, equations (2.1) and (2.2) will still hold. In this case, however, the last term in the formula for Var(T) disappears and equation (2.2) becomes:

$$\mathrm{Var}(T) = \lambda \mathrm{E}\left(X^2\right) \qquad (2.3)$$

The difference between these two variance equations is notable. In the case of equation (2.3), the variance of the average claim, i.e. $\mathrm{Var}(T/\lambda)$, will approach 0 as λ gets large. However, in the case of equation (2.3), if either b or c is non-zero, $\mathrm{Var}(T/\lambda)$ approaches $\mathrm{E}^2(X)(b+c+bc)$. Thus introduction of parameter uncertainty introduces uncertainty in the average that cannot be overcome by increasing the number of claims, or by diversifying the risk. In financial terms, parameter uncertainty in this manner introduces undiversifyable risk.

Heckman and Meyers present an algorithm for approximating the distribution of T in the case that the cumulative density function for the claim size distribution is a step function. Since any smooth function can be approximated within any required tolerance by a step function, this is not a restrictive assumption. We will use that algorithm in the method presented here.

3. Point Estimates of Reserves

Exhibit 1 presents summaries of various medical malpractice loss statistics that were derived from the data used by Berquist and Sherman [5]. To keep the numbers to a manageable size, all losses and claim counts in that paper were divided by 10 and the dates were changed to make the exhibits here appear more current. In addition, page 2 of Exhibit 1 shows projected ultimate reported claims. This projection is based on a development factor method applied to reported counts using volume weighted averages as selected factors. Though the data are hypothetical, they do reflect characteristics of actual loss data.

In addition, we included another example of our calculations and estimates of probability levels in the appendix to this paper. That example is based on the data set used in the Advanced Case Study session of the 1992 Casualty Loss Reserve Seminar.

As pointed out by Berquist and Sherman a comparison of the trends in average case reserves and average loss payments, as shown in Exhibit 2, indicates a potential change in relative reserve adequacy. This change, if it is occurring, could affect the incurred loss projections.

In addition, reference to ratios of closed to projected ultimate claims, as shown in Exhibit 3, seems to indicate a change in the rate at which claims are being closed. This could affect projections based on paid losses.

Since there appear to be occurrences that can influence forecasts based on either paid or incurred data we considered two sets of forecasts; one based on the data shown in Exhibit 1 without any adjustment and the second based on data adjusted in an attempt to remove the influences of these apparent changes. The resulting adjusted paid and incurred loss data appear in Exhibit 4.

We used methods similar to those presented in [5] to adjust the paid losses for apparent changes in the rate of claims closing. We calculated the adjusted incurred as the sum of the adjusted paid losses plus the product of adjusted average reserves times adjusted claims open. We calculated the adjusted reserves as suggested in [5].

Exhibit 3 also shows the triangle of adjusted closed claims. We obtained this triangle as the product of the forecast ultimate reported claims for an accident times the most recent percentage of ultimate claims closed at that particular valuation point. For example, the estimate of 210 claims closed for 1989 at 36 months is the product of 42.3%, the percent of ultimate closed at 36 months for the most recent accident year (1990) times 497, the projected ultimate claims for 1989.

We used four different projection methods on each set of data; paid loss development, incurred loss development, a severity projection method and a hindsight average outstanding loss method. In both of the development factor methods we used an exponential curve fit to the difference of selected development factors minus 1 to estimate development after 96 months. In the severity projection method we reviewed the average costs per ultimate claim and inherent trends in those averages at the various stages of development to "square the triangle" of average payments, see, for example [5] for examples of this technique.

For the hindsight average outstanding loss method we calculated the average unpaid loss per open and incurred but not reported (IBNR) claim at various stages of development. We calculated these averages as the ratios of the difference of initial forecast ultimate losses minus paid losses to date divided by the difference of forecast ultimate claims minus claims closed to date. We used the unweighted average of the other three projections as the initial selection in this case. We then reviewed these averages and inherent trends at each stage of development and selected a representative average for the accident year currently at that age. We then used the product of that average and the number of open and IBNR claims as an estimate of the future payments for that year. Our ultimate loss projection for this method was then the sum of this outstanding loss estimated and the amount paid to date.

Exhibit 5 then shows a summary of the various projections and our weighted average selection, based on the weights shown in the bottom portion of that exhibit. We judgmentally selected the weights shown but they reflect our view of the extent that the hypotheses of the indicated projection method fit with what has been occurring in the data.

We recognize that these methods and selections are based on judgment and that different actuaries may have different opinions than we do. However, we believe that the method to estimate variation that we will present is sufficiently adaptable to accommodate different selections or even different underlying forecasting methods.

If we had estimates of the variances of the different projection methods another weighting presents itself. If we assume the various projections are independent then the weighted average with the least variance is that which assigns a weight to a random variable proportional to the inverse of its variance. This is intuitively appealing since, in this case, uncertain projections, identified by high variances, are given relatively less weight than more precise ones.

4. Estimate of Process Variation

We will estimate the process variation, that which is due only to random fluctuation, using the unadjusted collective risk model as described in Algorithm 2.1. Later we will examine an approach to include parameter uncertainty in the estimates and to use Algorithm 2.2.

Since we will be using the collective risk model we will need estimates of the distributions of the number of claims and of the size of individual claims. We will use the results of our reserve forecasts as a starting point.

Columns (1) through (7) of Exhibit 6 shows the calculation of indicated reserves and resulting indicated average loss per outstanding and IBNR claim by accident year. We will assume that the total outstanding claims have a lognormal distribution and that the loss data, and corresponding reserves, represent losses at $500,000 policy limits. We make these assumptions to maintain simplicity in the presentation. In practice the actuary will need to make appropriate estimates for these distributions.

We have also selected the coefficient of variation (ratio of the standard deviation to the mean) for the lognormal distribution, as shown in column (8). Though the selections here are judgmental they are based on two assumptions:

1. In ratemaking for this line of business we have selected a lognormal distribution with a coefficient of variation of 5.0 in calculating our increased limits distributions.

2. As time progresses the book of open and IBNR claims become more homogeneous and thus we would expect the coefficient of variation to decrease.

In practice we would have to derive estimates for these parameters too. One approach would be to consider the distribution of open and IBNR claims at various stages of development for older accident years that are completely, or at least nearly completely, closed out. Such a review would provide better insight in the selection of the coefficient of variation.

We have selected a lognormal distribution here primarily for its computational convenience. All of the concepts we will present will apply for most commonly used claim size distributions, though some of the specific formulae we will use may need to be modified.

Also, for convenience, we will assume that open claims and IBNR claims have the same claim size distribution and that they are independent. A potential refinement would be to separately estimate the distributions for open and IBNR claims. Again, this could be accomplished by reviewing distributions for older accident years, but we will not explore this further here.

There may be some argument with the assumption of independence. It is possible that settlement of open claims, and resulting precedent, may influence the distribution of IBNR claims, or even that of other open claims. The inclusion of the mixing parameter by Heckman and Meyers will essentially affect all claims in the same way, adjusting the aggregate losses either up or down uniformly, thereby building in some dependence. We recognize that notwithstanding the use of a mixing parameter our assumptions may slightly understate the spread of reserves if the distributions for open and IBNR claims are not independent.

Columns (9) and (10) of Exhibit 6 show the μ and σ parameters for the selected lognormal distribution. In this case we selected the following parameterization for the lognormal probability density function:

$$f(x) = \frac{e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}}{x\sigma\sqrt{2\pi}}$$

With this parameterization, if X is the lognormal variable, μ and σ represent the mean and standard deviation respectively of the normal distribution of ln(X). In addition, the coefficient of variation (c.v.) for the unlimited distribution and expected loss limited to L respectively are given by:

$$\text{c.v.} = \sqrt{e^{\sigma^2} - 1}$$

$$E(X|L) = e^{\mu+\frac{1}{2}\sigma^2}\Phi\left(\frac{\ln L - \mu}{\sigma} - \sigma\right) + L\left[1 - \Phi\left(\frac{\ln L - \mu}{\sigma}\right)\right]$$

Here $\Phi(X)$ denotes the probability that a standard normal variable will not exceed X. This and other formulae regarding the lognormal distribution can be found in [6] among other sources. We solved the first of these equations directly for σ. Given σ, then, we used numerical methods to estimate the value of μ that would yield a mean limited to $500,000 equal to the selected average reserve shown in column (7). Many commercially available software and spreadsheet packages contain such algorithms, one could also write a simple algorithm using interval halving since the function $E(X|L)$ is an increasing function of μ for a fixed L.

Exhibit 7 shows the selected step function approximations for the claim size distributions. Since these distributions will be used as input for the Heckman and Meyers algorithm, the probability for an indicated amount does not correspond to the probability that the limited mean will not exceed that amount. Rather these represent step function approximations for the lognormal distribution which have means equal to the expected limited losses.

We will assume that the number of open claims is certain, that is, it has 0 variance. This is equivalent to a contagion parameter $c = -1/\lambda$. We will assume that the IBNR claims have a Poisson distribution. Claims that close without payment may add some technical complexity to the selection of these distributions. We can include this in a number of ways. Probably the most straight-forward would be to include a positive probability of \$0 losses in the claim size distribution. We note that the positive probability of a \$0 loss may present problems with the algorithm presented in [4]. This practical problem can be overcome by using a small loss amount such as \$0.01 instead of \$0 for the claim size distribution input. Again, in order to keep these discussions relatively simple we will not make this refinement here, although the example we present in the appendix to this paper does deal with such a situation.

Another potentially complicating factor with these assumptions is the presence of reopened claims. We have assumed that the claim count data includes a reopened claim as a separate count and we have thus included provision for reopened counts in our estimates for IBNR claims. Again, we could adjust the claim count distribution for open claims to accommodate reopens. Another option would be to model reopened claims separately, similar to the way we treat IBNR claims.

We note another option in representing the combined distribution of open and IBNR claims. Let λ_O denote the number of open claims and λ_I the number of expected number of IBNR claims. We have assumed that the number of open claims is certain and that the number of IBNR claims has a Poisson distribution. Then the number of combined claims has mean $\lambda_O + \lambda_I$ and variance λ_I. We see that a claim distribution with mean $\lambda_O + \lambda_I$ using contagion parameter

$$c = -\frac{\lambda_O}{(\lambda_O + \lambda_I)^2}$$

will also have variance equal to λ_I. This is one potential short-cut in the calculations. If one assumes that open and IBNR claims have the same distributions then this assumed claim count distribution could replace the two separate distributions in the calculations.

We note, however, that this value of c is negative, resulting in the use of a binomial distribution which has a maximum number of possible claims. This may be undesirable in applications. However, we calculated aggregate loss distributions using both this single distribution and using separate distributions for open and IBNR claims and we found no discernible difference in the results.

Making use of the algorithm in [4] we calculated the resulting distribution of aggregate reserves for each accident year separately. We then used the same algorithm to calculate the aggregate distribution for all years combined, using the output of the algorithm to estimate the aggregate reserves for individual accident years. In this case we assumed 1 "claim" and used contagion factors of -1 for each year (implying a zero claim count variance) to estimate the distribution for aggregate reserves.

The user of this algorithm should be aware that the output provides estimates of the value of the cumulative density function at selected values of the aggregate reserves. These correspond to the valuation of that function at those points. Though this is valuable information, it does not directly provide a step function approximation to the aggregate reserve function that maintains expected values. We thus modified the output, similar to the modification for the individual claim size distributions, to obtain better step function approximations to the indicated cumulative density function before using them as input for the final calculations.

Exhibit 8 shows the estimated distribution of aggregate reserves for each accident year and for all accident years combined. To facilitate comparison between the years we show the estimated probability levels for various multiples of the expected values (shown in the first line). Heckman and Meyers refer to these ratios as "entry ratios."

As can be seen from this exhibit, the distributions of reserves for earlier accident years appear to be more disperse than those for later years. In addition, the distribution of aggregate reserves for all accident years is quite tight. This is a result of the law of large numbers. Even with this substantial narrowing of the ranges, in this case random fluctuation alone could result in reserves of more than 110% of the expected value approximately 5% of the time, with an approximate 0.1% chance of exceeding 120%. In this case roughly 90% of the aggregate reserve distribution falls between $\pm$10% of the expected value. We stress that only accounts for random fluctuations assuming all our hypotheses are correct. We have not yet addressed uncertainty in these assumptions.

5. Estimate of the Contagion Parameter

We first address uncertainty in the expected claim count parameter, λ. For this we consider projected ultimate frequencies by accident year as shown in Exhibit 9. A review such as this may be conducted in conjunction with a periodic rate review and all factors considered in such a review should be included in these projections. Here we selected an average annual frequency trend of 2.3% as indicated by an exponential fit through the frequencies for all years.

Assuming that 1993 will have an estimated 8,700 earned exposures column (6) shows the indicated 1993 claims assuming the respective historical frequencies, adjusted to 1993 level using the 2.3% assumed trend. We see that this results in an average of 516 claims per year with an unbiased estimate of the variance of 3,158 as compared with the expected variance of 516 if the distribution were Poisson. We thus assume a contagion parameter of 0.0099 by solving the equation $3{,}158 = 516 + c \times 516^2$ for c. We will then assume that the distributions of IBNR claims for all accident years have this same factor to reflect parameter uncertainty.

6. Estimates of Mixing Parameters

Returning to our ultimate loss, and hence reserve, selections described in section 3 (*Point Estimates of Reserves*) we note that our selected weights can be thought of as providing our subjective judgment regarding the likelihood that the underlying assumptions for the various methods are met in this particular data set. This may be thought of as a form of Bayesian á-priori probability estimate.

Following this thought, we can calculate the variance of the projection methods about the weighted average, using the same weights as used in the selections. In particular, if, for a fixed accident year, Z_i denotes the projection for method i and w_i denotes the relative weight given to method i then our selection and corresponding variance can be calculated as:

$$E(Z) = \sum_{i=1}^{n} w_i Z_i$$

$$\text{Var}(Z) = \sum_{i=1}^{n} w_i \left(Z_i - E(Z)\right)^2$$

These estimates are shown in column (8) of Exhibit 10. If we then assume that the methods that we applied consider all different sets of alternative hypotheses then the variance in the methods is an indication of the overall variance of the estimates, and hence reserves, for a particular year.

As indicated above, we can explain a portion of the variance experienced by process variation and in uncertainty in the claim counts. In particular, using formula (2.2) separately for open and IBNR claims we derive:

$$\begin{aligned} \mathrm{Var}(Z_O) &= \lambda_O\left(\mathrm{E}(X_O^2|L) - \mathrm{E}^2(X_O|L)\right) \\ \mathrm{Var}(Z_I) &= \lambda_I \mathrm{E}(X_I^2|L) + c\lambda_I^2 \mathrm{E}^2(X_I|L) \end{aligned} \qquad (6.1)$$

The first of these equations assumes a contagion parameter $c = -1/\lambda_O$, and both follow directly from equation (2.2) with b = 0. With our assumption that the reserves for open and IBNR claims are independent then the total variance is the sum of the variances.

Columns (1) through (5) of Exhibit 10 summarize estimates from Exhibits 1 and 6. Column (6) shows the value of $\mathrm{E}(X^2|L)$ using the following formula (see, for example, [6]):

$$\mathrm{E}(X^2|L) = e^{2\sigma^2+2\mu}\Phi\left(\frac{\ln L-\mu}{\sigma} - 2\sigma\right) + L^2\left[1-\Phi\left(\frac{\ln L-\mu}{\sigma}\right)\right]$$

Using these values and equations (6.1) we calculated the amount of variance that can be explained by process variation and the contagion parameter. This explained variance is shown in column (7).

As can be seen there, the explained variance exceeds the variance in the selection in accident years 1985 and 1986, but is less for the other years. Thus there is variance in the projections that is not explained by process variation or by uncertainty in the claim count projections. We will assume that this remaining uncertainty is explained by a non-zero mixing parameter, b. For this, we solve the following equation for b:

$$\mathrm{Var}(T) = Y + b\left[\lambda_o \mathrm{E}\left(X_o^2 \middle| L\right) + \lambda_o(\lambda_o - 1)\mathrm{E}^2\left(X_o \middle| L\right) + \lambda_i \mathrm{E}\left(X_i^2 \middle| L\right) + \lambda_i^2(c+1)\mathrm{E}^2\left(X_i \middle| L\right)\right] \quad (6.2)$$

Where Var(T) denotes the variance in selected in column (8) and Y denotes the explained variance in column (7). Column (9) shows the resulting b values. The b values we selected to estimate uncertainty in the expected value are shown in column (10).

We note that the indicated b parameter increases from 1985 through 1991 but decreases in 1992. This is primarily due to the decrease in the variance in the selected between 1991 and 1992 because of the wider range of forecasts for 1991 than 1992. Though it may seem counterintuitive for parameter uncertainty to decrease, it is possible that the wider range in 1991 may indicate that changes that appear to have influenced the 1991 forecasts more.

These b parameter estimates provide for parameter uncertainty regarding severity within each accident year. As yet unanswered is the question of uncertainty affecting all accident years. For this we chose an approach similar to that taken in estimating the c parameter.

As is often done in ratemaking applications, we used the trend inherent in the historical pure premiums to adjust historical pure premiums to present separate "observations" of 1993 pure premiums. We then used the variation inherent in these "observations" as an indication of the amount of overall uncertainty we have in the 1993 severity estimate. We then assumed, as in our estimates of the contagion parameter, that this uncertainty will apply to our total reserve estimates for historical years.

Calculations shown in Exhibit 11 derive estimates similar to those in Exhibits 9 and 10. Column (1) shows the limited severity implied by our projections while column (2) simply repeats our assumption that the losses will have a coefficient of variation of 5.0. Of course, if there were reason to believe that this coefficient will change over time we could modify the values in column (2). Column (3) then shows the unlimited severity for a lognormal distribution with the coefficient of variation shown in column (2) that would yield the severities limited to $500,000 shown in column (1).

Column (4) shows our selected frequency as shown in Exhibit 9 and column (5) shows the indicated unlimited pure premium. We then calculated an annual pure premium trend of 18.6% based on all observations of unlimited pure premiums in column (5). Similar to the analysis in Exhibit 9 we adjusted these observed pure premiums to our expected 1993 level using this indicated 18.6% trend. We elected to base our projections on the unlimited pure premium due to the damping effects of a fixed limit on limited severities.

We note that the usual arguments of additional variability in the unlimited averages that are cited as a reason for basing ratemaking analysis on limited data do not necessarily apply here. Since the unlimited loss estimates are based on the limited losses and a smooth distribution that does not change drastically from year to year, there is little additional fluctuation introduced in considering unlimited losses in this case.

Column (7) then shows the various indications of 1993 total losses, using the assumed 8,700 exposures as used in Exhibit 9. Using the estimated 516 claims for accident year 1993 from Exhibit 9, we derive the indicated unlimited severities shown in column (8). Column (9) then shows the resulting 1993 level severities limited to $500,000 per claim, again using the lognormal distribution, the coefficients of variation in column (2) and the unlimited means in column (8).

Finally the various observations of indicated 1993 total limited losses are shown in column (10). Based on these observations we expect $13,054 thousand in losses in 1993 with a variance of 3,082,167 million, assuming the observations are independent. This corresponds to an average of $25,298 per claim limited to $500,000 and an unlimited average of $29,346. This latter amount is the unlimited severity necessary for a lognormal distribution with coefficient of variation 5.0 to have a mean limited to $500,000 equal to $25,298.

These assumptions, including our selected contagion parameter, then result in an expected variance of 4,027,361 million. This in turn results in a negative value for b when we solve equation 6.2. Thus we conclude that our assumptions are sufficient to account for observed variation in these estimates and we will select an overall b parameter equal to zero.

As with calculations without parameter uncertainty, we calculated the aggregate distributions for reserves for each year separately. In this case we used the selected contagion parameter and selected b parameters shown in Exhibit 10. We then convoluted the resulting distributions with a mixing parameter set to zero.

Similar to Exhibit 8, Exhibit 12 shows the estimated distributions of reserves including these estimates of parameter uncertainty. Comparing these two exhibits shows the significant impact of including parameter uncertainty as described here. For example, without parameter uncertainty 97% of the estimated 1991 reserves fall within 30% of the expected value whereas less than 56% fall in this range if parameter uncertainty is included.

A similar observation, though not as dramatic, also holds for the aggregate distributions. Without parameter uncertainty 90% of losses are within 10% of the expected. With parameter uncertainty only 51% of the losses are in that range. Another comparison shows that the 90% probability level is approximately $45 million without parameter uncertainty but is approximately $50 million when parameter uncertainty is considered. Exhibits 12 and 13 graphically show this comparison for the cumulative density functions and probability density functions respectively.

7. Conclusions

Now that our presentation is complete, we once again point out that the methodology we presented does not depend on the choice of the underlying claim size distribution, nor does it require the use of the same distributions for both open and IBNR claims. Of course, calculations of the limited mean and variance would change with different claim size distributions but all concepts and methodology still apply.

We note that this methodology attempts to recognize uncertainty arising from the process, in the selection of parameters, and, to some extent, in the selection of reserve forecasting model. We also recognize that much more work is necessary before we have a comprehensive approach to measure all these sources of uncertainty. However, echoing, Meyers and Schenker, we conclude that parameter uncertainty can be have a significant impact on the distribution of reserves.

EXAMPLE MEDICAL MALPRACTICE DATA

Incurred Losses

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1985	$290	$516	$1,071	$1,461	$1,666	$2,090	$2,289	$2,351
1986	483	1,071	1,691	2,284	2,621	3,197	3,222	
1987	546	1,194	2,073	3,093	4,240	4,838		
1988	873	1,863	3,214	5,720	6,114			
1989	1,123	1,997	5,014	7,373				
1990	871	3,346	6,348					
1991	1,293	4,890						
1992	1,579							

Cumulative Paid Losses

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1985	$13	$41	$144	$299	$447	$818	$1,264	$1,582
1986	4	53	202	364	752	1,430	1,898	
1987	30	115	248	507	1,140	1,771		
1988	5	79	381	977	1,852			
1989	21	83	360	1,129				
1990	17	159	627					
1991	21	157						
1992	21							

NOTE:
1. All dollar amounts are in thousands.

EXAMPLE MEDICAL MALPRACTICE DATA

Reported Claim Count

Accident Year	Months of Development: 12	24	36	48	60	72	84	96	Projected Ultimate
1985	107	168	219	252	256	259	261	263	263
1986	102	185	231	269	275	278	280		282
1987	130	251	314	375	387	392			398
1988	135	273	352	421	446				458
1989	138	283	367	467					497
1990	136	277	362						463
1991	155	279							459
1992	160								500

Cumulative Closed Claim Count

Accident Year	Months of Development: 12	24	36	48	60	72	84	96
1985	32	84	119	137	153	182	208	227
1986	36	89	116	134	165	202	226	
1987	42	118	142	195	244	286		
1988	31	117	169	232	294			
1989	29	144	213	279				
1990	33	135	196					
1991	41	132						
1992	40							

COMPARISON OF AVERAGE PAYMENT AND AVERAGE RESERVE TRENDS

Average Reserve per Open Claim

Accident	Months of Development							
Year	12	24	36	48	60	72	84	96
1985	$3,693	$5,655	$9,270	$10,104	$11,835	$16,519	$19,340	$21,361
1986	7,258	10,604	12,948	14,222	16,991	23,250	24,519	
1987	5,864	8,113	10,610	14,367	21,678	28,934		
1988	8,346	11,436	15,481	25,095	28,039			
1989	10,110	13,770	30,221	33,213				
1990	8,291	22,444	34,464					
1991	11,158	32,197						
1992	12,983							
Indicated Trend	15.6%	29.5%	31.1%	34.3%	32.7%	32.3%	26.8%	

Average Payment per Closed Claim

Accident	Months of Development							
Year	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
1985	$402	$539	$2,971	$8,620	$9,199	$12,669	$17,084	$16,634
1986	110	919	5,487	9,129	12,403	18,452	19,533	
1987	706	1,115	5,644	4,928	12,994	14,948		
1988	161	862	5,782	9,477	14,085			
1989	724	541	4,003	11,709				
1990	518	1,394	7,635					
1991	517	1,494						
1992	525							
Indicated Trend	12.9%	12.0%	11.5%	6.7%	14.2%	8.6%	14.3%	

Exhibit 3

EXAMPLE MEDICAL MALPRACTICE DATA

Ratios of Closed to Projected Ultimate Claims

Accident	Months of Development							
Year	12	24	36	48	60	72	84	96
1985	12.2%	31.9%	45.2%	52.1%	58.2%	69.2%	79.1%	86.3%
1986	12.8%	31.6%	41.1%	47.5%	58.5%	71.6%	80.1%	
1987	10.6%	29.6%	35.7%	49.0%	61.3%	71.9%		
1988	6.8%	25.5%	36.9%	50.7%	64.2%			
1989	5.8%	29.0%	42.9%	56.1%				
1990	7.1%	29.2%	42.3%					
1991	8.9%	28.8%						
1992	8.0%							

Adjusted Cumulative Closed Claim Count

Accident	Months of Development							
Year	12	24	36	48	60	72	84	96
1985	21	76	111	148	169	189	211	227
1986	23	81	119	158	181	203	226	
1987	32	115	168	223	256	286		
1988	37	132	194	257	294			
1989	40	143	210	279				
1990	37	133	196					
1991	37	132						
1992	40							

EXAMPLE MEDICAL MALPRACTICE DATA

Cumulative Paid Losses Adjusted for Closure Rates

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1985	$2	$34	$111	$396	$623	$919	$1,317	$1,582
1986	0	32	210	620	966	1,417	1,898	
1987	7	106	354	817	1,287	1,771		
1988	6	123	554	1,272	1,852			
1989	24	82	337	1,129				
1990	19	153	627					
1991	12	157						
1992	21							

Incurred Losses Adjusted for Closure Rates and Reserve Changes

Accident Year	Months of Development 12	24	36	48	60	72	84	96
1985	$422	$1,315	$1,962	$2,371	$2,227	$2,450	$2,383	$2,351
1986	443	1,697	2,417	3,044	2,959	3,304	3,222	
1987	640	2,610	3,663	4,634	4,481	4,838		
1988	733	3,108	4,671	6,008	6,114			
1989	861	3,490	5,042	7,373				
1990	991	4,185	6,348					
1991	1,344	4,890						
1992	1,579							

NOTE:
1. All dollar amounts are in thousands.

Exhibit 5

EXAMPLE MEDICAL MALPRACTICE DATA

Ultimate Loss Projections

	Unadjusted Methods				Adjusted Methods				
	Development		Severity	Hindsight	Development		Severity	Hindsight	Weighted
Accident Year	Incurred	Paid	Projection	Method	Incurred	Paid	Projection	Method	Average
1985	$2,414	$2,300	$2,300		$2,351	$1,902	$1,901		$2,242
1986	3,399	3,454	3,354		3,180	2,741	2,674		3,075
1987	5,317	4,536	4,865		4,649	3,519	3,714		4,279
1988	7,979	8,149	7,586	$6,797	6,438	5,254	5,249	$5,413	5,806
1989	11,222	9,697	9,818	8,862	7,631	4,878	6,107	6,430	6,783
1990	14,746	13,215	11,247	11,049	8,671	7,326	6,877	6,838	7,999
1991	22,083	12,250	13,372	14,924	9,814	7,591	7,763	8,126	9,263
1992	19,360	10,141	17,740	20,673	12,419	9,964	9,717	10,273	11,335

Selected Weights

	Unadjusted Methods				Adjusted Methods			
	Development		Severity	Hindsight	Development		Severity	Hindsight
Accident Year	Incurred	Paid	Projection	Method	Incurred	Paid	Projection	Method
1985	2	1	1		2	1	1	
1986	2	1	1		8	4	2	
1987	2	1	1		9	6	3	
1988	1	1	1	1	4	4	8	8
1989	1	1	1	1	4	4	8	8
1990	1	1	1	1	4	4	8	8
1991	1	1	1	1	4	4	8	8
1992	1	1	1	1	4	4	8	8

NOTE:
1. All dollar amounts are in thousands.

Exhibit 6

ESTIMATED TOTAL RESERVES

Accident Year	(1) Selected Ultimate Losses	(2) Losses Paid to Date	(3) Indicated Reserves (1) - (2)	(4) Estimated Ultimate Claims	(5) Claims Closed to Date	(6) Indicated Open & IBNR Claims (4) - (5)	(7) Indicated Average Reserve (3)/(6)	(8) Selected Coefficient of Variation	(9) Indicated Lognormal Parameters μ	(10) Indicated Lognormal Parameters σ
1985	$2,242	$1,582	$660	263	227	36	$18,333	3.4	8.5995	1.5908
1986	3,075	1,898	1,177	282	226	56	21,018	3.6	8.7009	1.6236
1987	4,279	1,771	2,508	398	286	112	22,393	3.8	8.7279	1.6544
1988	5,806	1,852	3,954	458	294	164	24,110	4.0	8.7702	1.6832
1989	6,783	1,129	5,654	497	279	218	25,936	4.2	8.8152	1.7104
1990	7,999	627	7,372	463	196	267	27,610	4.4	8.8520	1.7360
1991	9,263	157	9,106	459	132	327	27,847	4.6	8.8294	1.7602
1992	11,335	21	11,314	500	40	460	24,596	4.8	8.6557	1.7832
Total	$50,782	$9,037	$41,745							

NOTE:
1. Amounts in columns (1), (2), and (3) are in thousands of dollars.

Exhibit 7

SELECTED CLAIM SIZE DISTRIBUTIONS

Loss	Accident Year							
Amount	1985	1986	1987	1988	1989	1990	1991	1992
$50	0.00139	0.00138	0.00156	0.00169	0.00181	0.00194	0.00229	0.00346
100	0.00549	0.00535	0.00590	0.00625	0.00653	0.00685	0.00786	0.01125
250	0.02612	0.02476	0.02602	0.02657	0.02688	0.02737	0.03002	0.03951
500	0.06697	0.06290	0.06446	0.06461	0.06435	0.06453	0.06895	0.08595
750	0.10693	0.10021	0.10155	0.10097	0.09990	0.09952	0.10498	0.12719
1,000	0.14405	0.13495	0.13588	0.13452	0.13262	0.13162	0.13775	0.16384
1,250	0.17806	0.16688	0.16730	0.16516	0.16247	0.16085	0.16740	0.19640
1,500	0.21001	0.19694	0.19682	0.19392	0.19047	0.18824	0.19508	0.22644
2,000	0.26564	0.24955	0.24837	0.24413	0.23933	0.23603	0.24310	0.27766
2,500	0.31402	0.29555	0.29337	0.28795	0.28201	0.27775	0.28484	0.32148
3,500	0.39369	0.37186	0.36799	0.36071	0.35297	0.34718	0.35397	0.39280
5,000	0.48100	0.45649	0.45082	0.44171	0.43222	0.42489	0.43097	0.47050
6,000	0.52587	0.50043	0.49391	0.48399	0.47373	0.46570	0.47128	0.51050
7,500	0.58142	0.55520	0.54772	0.53693	0.52585	0.51703	0.52191	0.56031
8,500	0.61120	0.58482	0.57690	0.56573	0.55429	0.54512	0.54959	0.58723
10,000	0.65070	0.62430	0.61584	0.60425	0.59242	0.58283	0.58671	0.62315
12,500	0.70072	0.67482	0.66585	0.65394	0.64182	0.63186	0.63494	0.66931
15,000	0.74028	0.71516	0.70595	0.69396	0.68176	0.67164	0.67406	0.70639
20,000	0.79521	0.77194	0.76270	0.75097	0.73903	0.72896	0.73043	0.75921
25,000	0.83318	0.81180	0.80280	0.79156	0.78009	0.77030	0.77110	0.79686
35,000	0.88178	0.86369	0.85546	0.84533	0.83495	0.82593	0.82590	0.84700
50,000	0.91994	0.90547	0.89839	0.88977	0.88087	0.87299	0.87238	0.88889
60,000	0.93492	0.92224	0.91583	0.90804	0.89996	0.89274	0.89195	0.90632
75,000	0.95108	0.94053	0.93497	0.92822	0.92117	0.91480	0.91385	0.92573
85,000	0.95822	0.94876	0.94366	0.93748	0.93099	0.92509	0.92410	0.93475
100,000	0.96685	0.95877	0.95429	0.94885	0.94311	0.93784	0.93681	0.94589
125,000	0.97598	0.96956	0.96586	0.96136	0.95656	0.95211	0.95109	0.95833
150,000	0.98167	0.97640	0.97328	0.96947	0.96539	0.96155	0.96057	0.96654
175,000	0.98559	0.98119	0.97852	0.97525	0.97171	0.96836	0.96745	0.97246
200,000	0.98837	0.98464	0.98232	0.97947	0.97638	0.97343	0.97256	0.97685
225,000	0.99043	0.98722	0.98519	0.98268	0.97995	0.97732	0.97651	0.98022
250,000	0.99200	0.98921	0.98741	0.98519	0.98275	0.98039	0.97963	0.98288
275,000	0.99321	0.99076	0.98915	0.98716	0.98497	0.98283	0.98213	0.98500
300,000	0.99421	0.99204	0.99061	0.98882	0.98685	0.98491	0.98425	0.98680
350,000	0.99563	0.99390	0.99273	0.99126	0.98962	0.98800	0.98742	0.98948
400,000	0.99658	0.99517	0.99419	0.99295	0.99156	0.99018	0.98966	0.99137
450,000	0.99727	0.99610	0.99526	0.99421	0.99302	0.99182	0.99136	0.99281
500,000	0.99777	0.99677	0.99605	0.99514	0.99410	0.99305	0.99263	0.99388

Exhibit 8

ESTIMATED PROBABILITY LEVELS FOR RESERVES

Without Parameter Uncertainty

	Accident Year								
	1985	1986	1987	1988	1989	1990	1991	1992	Total
	Expected Reserve								
	$660	$1,177	$2,508	$3,954	$5,654	$7,372	$9,106	$11,314	$41,745
Ratio to Expected	Estimated Probability Level								
0.3	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	0.0115	0.0024	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.0519	0.0202	0.0017	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.1322	0.0743	0.0174	0.0051	0.0004	0.0007	0.0003	0.0001	0.0000
0.7	0.2424	0.1710	0.0748	0.0376	0.0095	0.0123	0.0075	0.0031	0.0000
0.8	0.3635	0.2955	0.1918	0.1366	0.0710	0.0792	0.0626	0.0421	0.0006
0.9	0.4794	0.4278	0.3567	0.3134	0.2491	0.2576	0.2378	0.2095	0.0479
1.0	0.5815	0.5541	0.5359	0.5281	0.5200	0.5200	0.5179	0.5162	0.5074
1.1	0.6670	0.6665	0.6960	0.7213	0.7667	0.7596	0.7749	0.7981	0.9452
1.2	0.7375	0.7599	0.8182	0.8579	0.9140	0.9070	0.9230	0.9434	0.9990
1.3	0.7962	0.8330	0.9001	0.9369	0.9757	0.9719	0.9805	0.9892	1.0000
1.4	0.8449	0.8874	0.9492	0.9753	0.9946	0.9932	0.9962	0.9985	1.0000
1.5	0.8842	0.9262	0.9760	0.9914	0.9990	0.9987	0.9994	0.9999	1.0000
1.6	0.9150	0.9530	0.9894	0.9973	0.9999	0.9998	0.9999	1.0000	1.0000
1.7	0.9384	0.9708	0.9956	0.9992	1.0000	1.0000	1.0000	1.0000	1.0000
1.8	0.9558	0.9823	0.9983	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
1.9	0.9685	0.9895	0.9993	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.0	0.9777	0.9939	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.1	0.9844	0.9965	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.2	0.9892	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.3	0.9926	0.9989	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.4	0.9950	0.9994	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.5	0.9967	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.6	0.9978	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.7	0.9985	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.8	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.9	0.9994	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3.0	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3.1	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3.2	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3.3	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

NOTE:
1. Reserve estimates are in thousands.

Exhibit 9

ESTIMATE OF CONTAGION PARAMETER

(1) Accident Year	(2) Estimated Ultimate Claims	(3) Earned Exposures	(4) Indicated Frequency (2)/(3)	(5) Selected On-Level Frequency	(6) Indicated 1993 Claims (5)x8,700
1985	263	5,907	4.45%	5.34%	465
1986	282	4,965	5.68%	6.66%	579
1987	398	7,719	5.16%	5.91%	514
1988	458	7,922	5.78%	6.48%	564
1989	497	11,361	4.37%	4.79%	417
1990	463	7,525	6.15%	6.58%	572
1991	459	8,376	5.48%	5.73%	499
1992	500	8,649	5.78%	5.91%	514

Indicated Trend	2.3%	
Arithmetic Average		516
Variance Estimate		3,158
Indicated c Value		0.0099

Exhibit 10

ESTIMATES OF PARAMETER UNCERTAINTY FOR MEANS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Indicated Lognormal Parameters		Estimated Number of Claims		Expected			Variance		
Accident Year	μ	σ	Open	IBNR	Average Reserve	$E(X^2 \mid L)$	Explained Variance	in Selected	Implied b value	Selected b value
1985	8.5995	1.5908	36	0	$18,333	2,267	69,525	40,192	-0.0581	0.0000
1986	8.7009	1.6236	54	2	21,018	2,920	139,662	71,526	-0.0477	0.0000
1987	8.7279	1.6544	106	6	22,393	3,322	319,139	373,623	0.0091	0.0091
1988	8.7702	1.6832	152	12	24,110	3,821	539,092	746,291	0.0147	0.0147
1989	8.8152	1.7104	188	30	25,936	4,366	831,265	2,277,671	0.0574	0.0574
1990	8.8520	1.7360	166	101	27,610	4,890	1,256,128	4,180,470	0.0974	0.0974
1991	8.8294	1.7602	147	180	27,847	5,044	1,784,293	9,390,867	0.1742	0.1742
1992	8.6557	1.7832	120	340	24,596	4,280	2,588,688	8,436,909	0.0720	0.0720

Selected Contagion Parameter: 0.0099

NOTE:

1. Amounts in columns (6), (7) and (8) are in millions.

Exhibit 11

ESTIMATE OF OVERALL MIXING PARAMETER

Accident Year	(1) Indicated Limited Severity	(2) Selected Coefficient of Variation	(3) Indicated Unlimited Severity	(4) Selected Frequency	(5) Indicated Unlimited Pure Premium (3)x(4)	(6) Unlimited Pure Premium at 1993 Level	(7) Indicated 1993 Unlimited Loss (6)x8,700	(8) Indicated 1993 Unlimited Severity (7)/516	(9) Indicated 1993 Limited Severity	(10) Indicated 1993 Limited Loss (4)x (9)x516
1985	$8,525	5.0	$8,913	4.45%	$397	$1,554	$13,520	$26,202	$22,916	$11,825
1986	10,904	5.0	11,572	5.68%	657	2,168	18,866	36,562	30,547	15,762
1987	10,751	5.0	11,399	5.16%	588	1,636	14,237	27,591	23,976	12,372
1988	12,677	5.0	13,605	5.78%	786	1,844	16,046	31,097	26,599	13,725
1989	13,648	5.0	14,736	4.37%	644	1,274	11,085	21,483	19,219	9,917
1990	17,276	5.0	19,081	6.15%	1,173	1,957	17,024	32,992	27,987	14,441
1991	20,181	5.0	22,692	5.48%	1,244	1,750	15,223	29,502	25,415	13,114
1992	22,670	5.0	25,882	5.78%	1,496	1,774	15,436	29,915	25,723	13,273

	(5)	(10)
Indicated Trend	18.6%	
Average (000)		$13,054
Variance Estimate (000,000)		3,082,167
Average Limited Severity		$25,298
Corresponding Unlimited Severity		29,346
$E(X^2 \mid L)$ (000,000)		4,536
Selected 1993 Claim Counts		516
Explained Variance (000,000)		4,027,361
Implied b value		-0.00542
Selected Overall b value		0

NOTE:

1. Columns (7) and (10) are in thousands.

Exhibit 12

ESTIMATED PROBABILITY LEVELS FOR RESERVES

With Parameter Uncertainty

Ratio to Expected	Accident Year 1985	1986	1987	1988	1989	1990	1991	1992	Total
	Expected Reserve								
	$660	$1,177	$2,508	$3,954	$5,854	$7,372	$9,106	$11,314	$41,745
	Estimated Probability Level								
0.3	0.0008	0.0001	0.0000	0.0000	0.0000	0.0001	0.0008	0.0000	0.0000
0.4	0.0115	0.0024	0.0001	0.0000	0.0006	0.0026	0.0117	0.0008	0.0000
0.5	0.0519	0.0202	0.0037	0.0015	0.0083	0.0211	0.0541	0.0101	0.0000
0.6	0.1322	0.0743	0.0264	0.0151	0.0439	0.0779	0.1382	0.0502	0.0001
0.7	0.2424	0.1710	0.0936	0.0686	0.1284	0.1798	0.2527	0.1400	0.0052
0.8	0.3635	0.2955	0.2152	0.1851	0.2597	0.3120	0.3775	0.2733	0.0638
0.9	0.4794	0.4278	0.3749	0.3549	0.4137	0.4511	0.4965	0.4248	0.2630
1.0	0.5815	0.5541	0.5421	0.5401	0.5630	0.5789	0.6007	0.5688	0.5476
1.1	0.6670	0.6665	0.6899	0.7028	0.6898	0.6861	0.6874	0.6900	0.7769
1.2	0.7375	0.7599	0.8043	0.8239	0.7879	0.7706	0.7570	0.7840	0.9051
1.3	0.7962	0.8330	0.8840	0.9032	0.8589	0.8347	0.8118	0.8527	0.9626
1.4	0.8449	0.8873	0.9350	0.9500	0.9080	0.8818	0.8543	0.9011	0.9856
1.5	0.8842	0.9262	0.9652	0.9755	0.9409	0.9159	0.8870	0.9341	0.9944
1.6	0.9150	0.9530	0.9822	0.9885	0.9623	0.9402	0.9122	0.9564	0.9977
1.7	0.9384	0.9708	0.9912	0.9948	0.9761	0.9575	0.9315	0.9712	0.9990
1.8	0.9558	0.9823	0.9958	0.9977	0.9849	0.9697	0.9464	0.9810	0.9995
1.9	0.9685	0.9895	0.9980	0.9990	0.9904	0.9783	0.9578	0.9874	0.9998
2.0	0.9777	0.9939	0.9991	0.9996	0.9939	0.9845	0.9667	0.9916	0.9999
2.1	0.9844	0.9965	0.9996	0.9998	0.9961	0.9888	0.9735	0.9944	0.9999
2.2	0.9892	0.9981	0.9998	0.9999	0.9975	0.9919	0.9788	0.9962	1.0000
2.3	0.9926	0.9989	0.9999	1.0000	0.9984	0.9941	0.9830	0.9975	1.0000
2.4	0.9950	0.9994	1.0000	1.0000	0.9990	0.9957	0.9863	0.9983	1.0000
2.5	0.9967	0.9997	1.0000	1.0000	0.9993	0.9968	0.9889	0.9988	1.0000
2.6	0.9978	0.9998	1.0000	1.0000	0.9996	0.9976	0.9910	0.9992	1.0000
2.7	0.9985	0.9999	1.0000	1.0000	0.9997	0.9982	0.9926	0.9994	1.0000
2.8	0.9990	1.0000	1.0000	1.0000	0.9998	0.9987	0.9939	0.9996	1.0000
2.9	0.9994	1.0000	1.0000	1.0000	0.9999	0.9990	0.9950	0.9997	1.0000
3.0	0.9996	1.0000	1.0000	1.0000	0.9999	0.9992	0.9959	0.9998	1.0000
3.1	0.9997	1.0000	1.0000	1.0000	0.9999	0.9994	0.9966	0.9999	1.0000
3.2	0.9998	1.0000	1.0000	1.0000	1.0000	0.9996	0.9971	0.9999	1.0000
3.3	0.9999	1.0000	1.0000	1.0000	1.0000	0.9997	0.9976	0.9999	1.0000

NOTE:
1. Reserve estimates are in thousands.

Exhibit 13

Estimated Aggregate Reserve Cumulative Densities

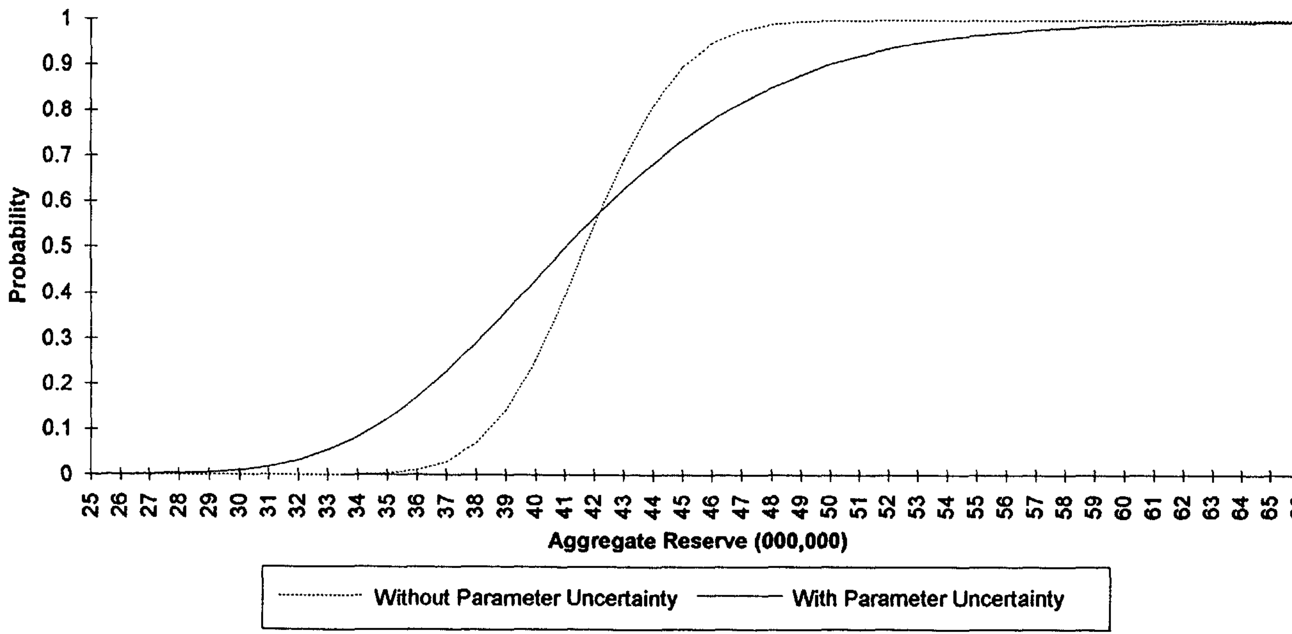
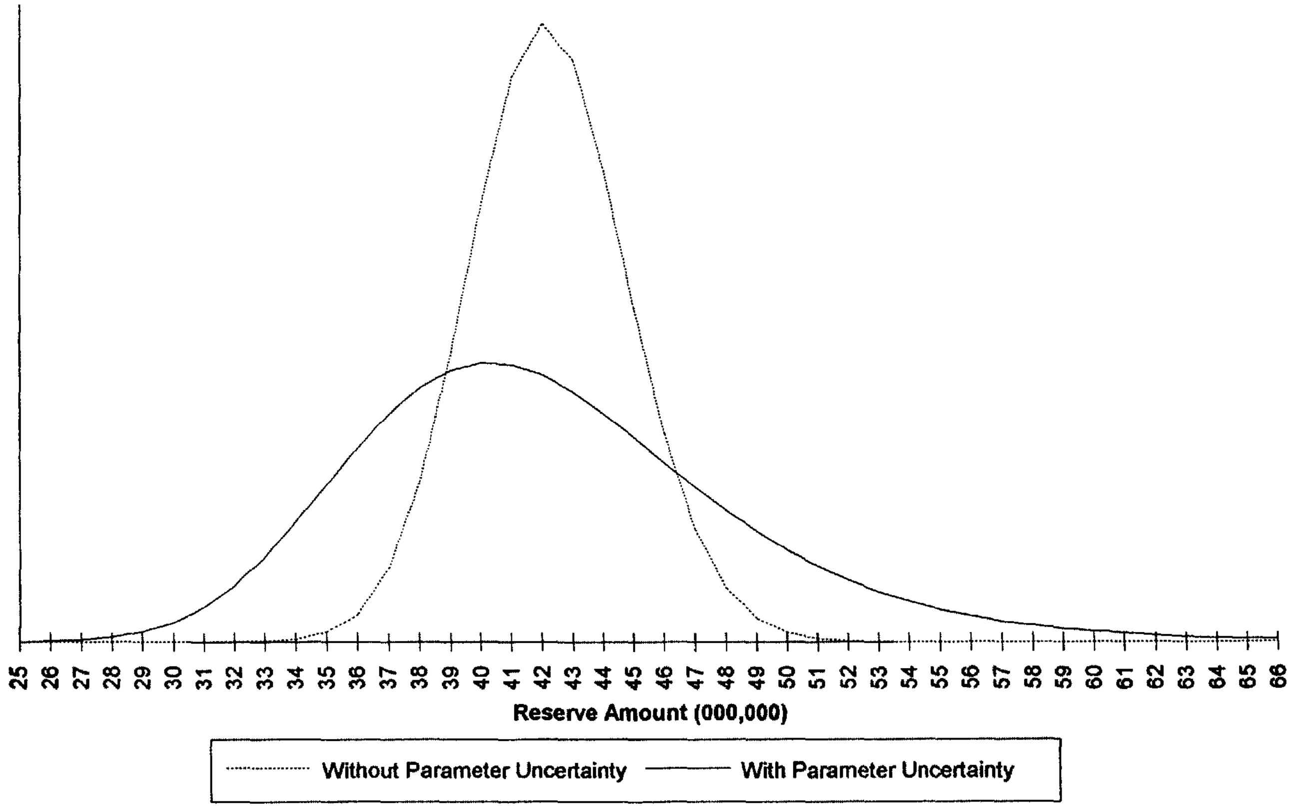


Exhibit 14

Estimated Aggregate Probability Density Functions



APPENDIX

This appendix summarizes the analysis of another data set using the methods presented in this paper. The data used are those provided to the panelists for the Advanced Case Study session of the 1992 Casualty Loss Reserve Seminar, as summarized in Exhibit A-1. The first two pages of that exhibit give a summary background information regarding the data source while the last three pages give summary triangles and exposure information. Included are eighteen years of development for eighteen accident years including data on paid and outstanding losses, claims closed with payment, reported claims, open claims and earned exposures.

Our analysis indicated that there seemed to be changes in the percentage of reported claims that are paid for the various accident years. It appears that the court decision cited in the background material resulted in a higher proportion of reported claims being paid than the levels prior to that decision. We noted other changes in these ratios in the data. We thus selected paid counts, as opposed to reported, as the denominator in calculating severities in our severity and hindsight projection methods.

We used four projection methods to estimate ultimate reported counts. The first two were development factor methods applied to historical paid claims and historical incurred claims (paid claims plus outstanding claims). The third method estimated ultimate paid claims as the product of the number of ultimate reported claims and the forecast percentage of ultimate claims that will be paid. We used development factor methods applied to the historical ratios of paid to closed (defined to be reported minus open) claims. We considered trends in both the resulting reported frequencies and indicated percentages paid to temper the leveraging effect of development factor methods for more immature years.

The fourth method was a hindsight method based on frequencies. This method is similar to what we used to estimate losses, as described in the main portion of this paper.

Exhibit A-2 summarizes these projections and shows our selections and various diagnostics. These projections indicate an increase in estimated ultimate reported frequency in 1987 after a general decrease in prior years, as shown in column (12), and a marked increase in the percentage of reported that are estimated to be paid as shown in column (13).

After an analysis similar to that for the sample medical malpractice data, we noted that there appears to be a change in the rate at which claims are being closed. We thus considered loss projections based on paid loss data adjusted to remove this apparent change. Exhibit A-3 then shows a summary of our ultimate loss projections similar to Exhibit 5.

Exhibit A-4 then summarizes the assumptions we used to estimate the distribution of aggregate reserves before consideration of parameter uncertainty. In this case we assumed that claims closing with payment would have lognormal distributions with unlimited means equal to the average reserve per estimated future paid claim, shown in column (3). We assumed that all claims closing with payment would have a coefficient of variation equal to 1.25 and judgmentally scaled this back as shown in column (7). Though 1.25 may seem arbitrary and possibly low, its selection was based on discussions with the source of these sample data.

We have also elected to combine accident years 1984 and prior. This is due primarily to the relative scarcity of data for those years and the resulting "noise" in estimates for individual accident years.

As with the analysis in the main section of this paper, we assumed that open and IBNR claims both had the same loss distribution. Again, this is more of a convenience than a requirement of this approach. In this case, however, we assumed that the distribution of claims closing with payment would be lognormal and included $0.01 losses in the input distribution with the complement of the probability of a claim closing with payment. We then adjusted the remaining distribution accordingly. Exhibit A-5 shows an example using accident year 1986.

Exhibit A-6 shows the resulting aggregate distributions for the reserves without consideration of parameter uncertainty, similar to Exhibit 8. As can be seen from this exhibit, the rather large number of claims results in relatively little variation in aggregate amounts. Virtually all of the distribution is within 5% of the expected value of $203.2 million.

Exhibit A-7 corresponds to Exhibit 9 and results in an estimate for the overall contagion parameter of 0.0097. As shown in Exhibit A-2, however, due to changes that appeared in the data we used several different forecasting methods to estimate ultimate paid claims with variance among the methods as shown in column (10) of Exhibit A-2 and summarized in column (2) of Exhibit A-8.

Assuming our forecasts of the percentage of ultimate reported claims that will be paid, we can translate these variance estimates for ultimate paid claims to variance estimates for reported claims, as shown in column (4) of Exhibit A-8. We calculated the amount shown for 1984 and Prior as the sum of the corresponding amounts for the individual accident years.

We then solved for the contagion parameter, using the ultimate reported count estimates in column (1) and the variance estimates in column (4) to derive the estimates in column (5). In most accident years, the variance in the estimates is greater than what would be expected from a Poisson distribution. In addition to this variance for individual accident years, there is additional variation from year to year as shown in Exhibit A-7. We thus selected our contagion parameters as the sum of the indicated parameters in column (5) and the overall indicated parameter shown in Exhibit A-7.

Exhibit A-9 shows our estimates of the mixing parameters for the individual years. Since we assume that the losses are unlimited we can easily determine the indicated standard deviation, and hence variance using the unlimited mean and assumed coefficient of variation. Column (10) then shows the variance explained using the selected contagion parameters from Exhibit A-8 and the claim counts and claim size variances. Column (11) shows the variance among methods and shows that, except for accident years 1985 and 1991, the variance in methods exceeds what can be explained by our other assumptions. Column (12) gives the resulting implied values for the mixing parameter b while column (13) shows our selections.

As with Exhibit 11, we also calculated the variation in ultimate losses over the accident years, shown in Exhibit A-10. In this case the observed variance exceeds the amount that can be explained with the overall contagion parameter and our estimates of claim count and claim size distributions. This then implies an additional mixing parameter of 0.00069 shown at the bottom of Exhibit A-10.

We then calculated the individual distributions for each of the accident years separately, using the estimates of contagion and mixing parameters shown in Exhibits A-8 and A-9. We used the overall mixing parameter from Exhibit A-10 to reflect additional uncertainty in our final convolution of the distributions for individual accident years.

Exhibit A-11 then presents a summary of our estimates for the individual years and for the aggregate reserves. As with the analysis in the main section of this paper, the introduction of parameter uncertainty markedly widens the aggregate distribution. Whereas without parameter uncertainty, 90% of the losses were within 2.5% of the expected, with parameter uncertainty this percentage drops to 33%. Without parameter uncertainty 99.9% of the reserves were within 5% of the expected while with parameter uncertainty 60% fall in this range and we would have to widen the range to 20% to capture more than 99% of the indicated values. Exhibits A-12 and A-13 show these comparisons graphically.

BACKGROUND INFORMATION RELATING TO SAMPLE DATA

These data are based on actual bodily injury liability experience for an insurer, though we have randomly disturbed the true data to protect the identity of the insurer. The liability coverage is not particularly long-tailed and does not contain exposure to continuing damage or latent exposure claims such as asbestos or pollution.

For your information, the incremental paid counts and amounts and the incremental reported counts as well as outstanding counts and amounts were all multiplied by values selected randomly from a lognormal distribution. The corresponding normal distribution [that of ln(X)] had a mean of 0 and a standard deviation of 0.05. Thus the data should be close to "real." The exposures shown have also been modified from the actual data, however the underlying frequencies and pure premiums remain unchanged from that which would have arisen from the randomly perturbed data.

We have included five summary triangles:

1. Cumulative Paid Losses. Total loss payments at annual valuations for each accident year.

2. Outstanding Losses. Carried case reserves, without any actuarial or bulk adjustments, valued at successive year-ends.

3. Cumulative Paid Claims. Total claims closed with payment at annual valuations.

4. Outstanding Claims. Total claims open at year-end valuation dates whether or not the claim subsequently closes with payment.

5. Reported Claims. Total claims reported to the insurer, whether or not the claim subsequently closes with payment.

The accident years shown are real. Losses included are total direct losses and the insurer has experienced some drift to higher policy limits over time. This drift has been gradual and somewhat consistent over the time period under consideration. The exposure counts are not inflation-sensitive but do not reflect changes in the mix of exposures between lower and higher risk insureds that may have occurred over time. Similar to the drift in policy limits there has been a general, and gradual, drift to a greater proportion of lower risk insureds in this book.

The exposures are relatively homogeneous over time and contain no claims from outside the United States. There have been no changes in the overall mix of legal jurisdictions affecting these claims. There was, however, a notable legal decision near the end of 1986 affecting claims under this coverage. You can assume that this change made it easier to initiate claims and more difficult for the insurer to settle those claims early as compared to the situation prior to that time.

You may note a decrease in payments and reported claims during calendar year 1991. This is not the result of the random disturbances we introduced in the data but is present in the actual data. The Company is unable to provide a specific explanation as to the reason for this decrease.

Cumulative Paid Losses

Accident Year	Months of Development: 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	$267	$1,975	$4,587	$7,375	$10,661	$15,232	$17,888	$18,541	$18,937	$19,130	$19,189	$19,209	$19,234	$19,234	$19,246	$19,246	$19,246	$19,246
1975	310	2,809	5,686	9,386	14,884	20,654	22,017	22,529	22,772	22,821	23,042	23,060	23,127	23,127	23,127	23,127	23,159	
1976	370	2,744	7,281	13,287	19,773	23,888	25,174	25,819	26,049	26,180	26,268	26,364	26,371	26,379	26,397	26,397		
1977	577	3,877	9,612	16,962	23,764	26,712	28,393	29,656	29,839	29,944	29,997	29,999	29,999	30,049	30,049			
1978	509	4,518	12,067	21,218	27,194	29,617	30,854	31,240	31,598	31,889	32,002	31,947	31,965	31,986				
1979	630	5,763	16,372	24,105	29,091	32,531	33,878	34,185	34,290	34,420	34,479	34,498	34,524					
1980	1,078	8,066	17,518	26,091	31,807	33,883	34,820	35,482	35,607	35,937	35,957	35,962						
1981	1,646	9,378	18,034	26,652	31,253	33,376	34,287	34,985	35,122	35,161	35,172							
1982	1,754	11,256	20,624	27,857	31,360	33,331	34,061	34,227	34,317	34,378								
1983	1,997	10,628	21,015	29,014	33,788	36,329	37,446	37,571	37,681									
1984	2,164	11,538	21,549	29,167	34,440	36,528	36,950	37,099										
1985	1,922	10,939	21,357	28,488	32,982	35,330	36,059											
1986	1,962	13,053	27,869	38,560	44,461	45,988												
1987	2,329	18,086	38,099	51,953	58,029													
1988	3,343	24,806	52,054	66,203														
1989	3,847	34,171	59,232															
1990	6,090	33,392																
1991	5,451																	

Claims Closed with Payment

Accident Year	Months of Development: 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	268	607	858	1,090	1,333	1,743	2,000	2,076	2,113	2,129	2,137	2,141	2,143	2,143	2,145	2,145	2,145	2,145
1975	294	691	913	1,195	1,620	2,076	2,234	2,293	2,320	2,331	2,339	2,341	2,343	2,343	2,343	2,343	2,344	
1976	283	642	961	1,407	1,994	2,375	2,504	2,549	2,580	2,590	2,596	2,600	2,602	2,603	2,603	2,603		
1977	274	707	1,176	1,688	2,295	2,545	2,689	2,777	2,809	2,817	2,824	2,825	2,825	2,826	2,826			
1978	269	658	1,228	1,819	2,217	2,475	2,613	2,671	2,691	2,706	2,710	2,711	2,714	2,717				
1979	249	771	1,581	2,101	2,528	2,816	2,930	2,961	2,973	2,979	2,986	2,988	2,992					
1980	305	1,107	1,713	2,316	2,748	2,942	3,025	3,049	3,063	3,077	3,079	3,080						
1981	343	1,042	1,608	2,260	2,596	2,734	2,801	2,835	2,854	2,859	2,860							
1982	350	1,242	1,922	2,407	2,661	2,834	2,887	2,902	2,911	2,915								
1983	428	1,257	1,841	2,345	2,683	2,853	2,908	2,920	2,925									
1984	291	1,004	1,577	2,054	2,406	2,583	2,622	2,636										
1985	303	1,001	1,575	2,080	2,444	2,586	2,617											
1986	318	1,055	1,906	2,524	2,874	2,958												
1987	343	1,438	2,384	3,172	3,559													
1988	391	1,671	3,082	3,771														
1989	433	1,941	3,241															
1990	533	1,923																
1991	339																	

Cumulative Reported Claims

Accident Year	Months of Development																	
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,912	2,854	3,350	3,945	4,057	4,104	4,149	4,155	4,164	4,167	4,169	4,169	4,169	4,170	4,170	4,170	4,170	4,170
1975	2,219	3,302	3,915	4,462	4,618	4,673	4,696	4,704	4,708	4,711	4,712	4,716	4,716	4,716	4,716	4,716	4,717	
1976	2,347	3,702	4,278	4,768	4,915	4,983	5,003	5,007	5,012	5,012	5,013	5,014	5,015	5,015	5,015	5,015		
1977	2,983	4,346	5,055	5,696	5,818	5,861	5,884	5,892	5,896	5,897	5,900	5,900	5,900	5,900	5,900			
1978	2,538	3,906	4,633	5,123	5,242	5,275	5,286	5,292	5,298	5,302	5,304	5,304	5,306	5,306				
1979	3,548	5,190	5,779	6,206	6,313	6,329	6,339	6,343	6,347	6,347	6,348	6,348	6,348					
1980	4,583	6,106	6,656	7,032	7,128	7,139	7,147	7,150	7,151	7,153	7,154	7,154						
1981	4,430	5,967	6,510	6,775	6,854	6,873	6,883	6,889	6,892	6,894	6,895							
1982	4,408	5,849	6,264	6,526	6,571	6,589	6,594	6,596	6,600	6,602								
1983	4,861	6,437	6,869	7,134	7,196	7,205	7,211	7,212	7,214									
1984	4,229	5,645	6,053	6,419	6,506	6,523	6,529	6,531										
1985	3,727	4,830	5,321	5,717	5,777	5,798	5,802											
1986	3,561	5,045	5,656	6,040	6,096	6,111												
1987	4,259	6,049	6,767	7,206	7,282													
1988	4,424	6,700	7,548	8,105														
1989	5,005	7,407	8,287															
1990	4,889	7,314																
1991	4,044																	

Outstanding Claims

Accident Year	Months of Development																	
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,381	1,336	1,462	1,660	1,406	772	406	191	98	57	23	13	3	4	0	0	0	0
1975	1,289	1,727	1,730	1,913	1,310	649	358	167	73	30	9	6	4	2	2	1	1	
1976	1,605	1,977	1,947	1,709	1,006	540	268	166	79	48	32	18	14	10	10	7		
1977	2,101	2,159	2,050	1,735	988	582	332	139	66	38	27	21	21	8	3			
1978	1,955	1,943	1,817	1,384	830	460	193	93	56	31	15	9	7	2				
1979	2,259	2,025	1,548	1,273	752	340	150	68	36	24	18	13	4					
1980	2,815	1,991	1,558	1,107	540	228	88	55	28	14	8	6						
1981	2,408	1,973	1,605	954	480	228	115	52	27	15	11							
1982	2,388	1,835	1,280	819	354	163	67	44	21	10								
1983	2,641	1,765	1,082	663	335	134	62	34	18									
1984	2,417	1,654	896	677	284	90	42	15										
1985	1,924	1,202	941	610	268	98	55											
1986	1,810	1,591	956	648	202	94												
1987	2,273	1,792	1,059	626	242													
1988	2,403	1,966	1,166	693														
1989	2,471	2,009	1,142															
1990	2,642	2,007																
1991	2,366																	

Outstanding Losses

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
	Months of Development																	
1974	$5,275	$8,867	$12,476	$11,919	$8,966	$5,367	$3,281	$1,524	$667	$348	$123	$82	$18	$40	$0	$0	$0	$0
1975	6,617	11,306	13,773	14,386	10,593	4,234	2,110	1,051	436	353	93	101	10	5	5	3	3	
1976	7,658	11,064	13,655	13,352	7,592	4,064	1,895	1,003	683	384	216	102	93	57	50	33		
1977	8,735	14,318	14,897	12,978	7,741	4,355	2,132	910	498	323	176	99	101	32	14			
1978	8,722	15,070	15,257	11,189	5,959	3,473	1,531	942	547	286	177	61	67	7				
1979	9,349	16,470	14,320	10,574	6,561	2,864	1,328	784	424	212	146	113	38					
1980	11,145	16,351	14,636	11,273	5,159	2,588	1,290	573	405	134	81	54						
1981	10,933	15,012	14,728	9,067	5,107	2,456	1,400	584	269	120	93							
1982	13,323	16,218	12,676	6,290	3,355	1,407	613	398	192	111								
1983	13,899	16,958	12,414	7,700	4,112	1,637	576	426	331									
1984	14,272	15,806	10,156	8,005	3,604	791	379	159										
1985	13,901	15,384	12,539	7,911	3,809	1,404	827											
1986	15,952	22,799	16,016	8,964	2,929	1,321												
1987	22,772	24,146	18,397	8,376	3,373													
1988	25,216	26,947	17,950	8,610														
1989	24,981	30,574	19,621															
1990	30,389	34,128																
1991	28,194																	

Accident Year	Earned Exposures
1974	11,000
1975	11,000
1976	11,000
1977	12,000
1978	12,000
1979	12,000
1980	12,000
1981	12,000
1982	11,000
1983	11,000
1984	11,000
1985	11,000
1986	12,000
1987	13,000
1988	14,000
1989	14,000
1990	14,000
1991	13,000

SAMPLE BODILY INJURY LIABILITY LOSS DATA

Projections of the Ultimate Number of Claims Closed with Payment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Selected Weights					Indicated Variance
Accident	Development		Percent	Hindsight	Development		Percent	Hindsight	Weighted	in Selected
Year	Paid	Incurred	Paid	Frequency	Paid	Incurred	Paid	Frequency	Average	Methods
1974	2,145	2,145	2,143		1	0	0		2,145	0.0
1975	2,344	2,345	2,345		1	0	0		2,344	0.0
1976	2,603	2,610	2,608		1	0	0		2,603	0.0
1977	2,826	2,827	2,828		1	0	0		2,826	0.0
1978	2,718	2,715	2,716		1	0	0		2,718	0.0
1979	2,994	2,987	2,996		1	0	0		2,994	0.0
1980	3,085	3,075	3,083		1	1	1		3,081	18.7
1981	2,865	2,857	2,864		1	1	1		2,862	12.7
1982	2,924	2,907	2,911		1	1	1		2,915	53.7
1983	2,941	2,919	2,930		1	1	1		2,930	80.7
1984	2,661	2,620	2,640	2,647	1	1	1	1	2,642	218.5
1985	2,660	2,626	2,643	2,639	1	1	1	1	2,642	147.5
1986	3,056	2,978	3,023	3,018	1	1	1	1	3,019	766.8
1987	3,879	3,676	3,813	3,728	1	2	2	3	3,755	4,596.6
1988	4,718	4,279	4,585	4,373	1	2	2	3	4,446	23,048.9
1989	5,233	4,540	5,014	4,641	1	2	2	3	4,783	60,976.5
1990	5,398	4,516	5,137	4,821	1	2	2	3	4,896	84,230.1
1991	3,903	3,990	4,574	4,447	1	2	2	2	4,275	76,972.0

	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
Accident Year	Estimated Ultimate Reported	Indicated Reported Frequency	Indicated Percent Paid	Number Reported to Date	Number Open	Number IBNR (11)-(14)	Number Paid to Date	Indicated Future Paid (9)-(17)	Future Percent Paid (18)/ [(15)+(16)]
1974	4,170	0.379	51.4%	4,170	0	0	2,145	0	--
1975	4,719	0.429	49.7%	4,717	1	2	2,344	0	0.0%
1976	5,016	0.456	51.9%	5,015	7	1	2,603	0	0.0%
1977	5,904	0.492	47.9%	5,900	3	4	2,826	0	0.0%
1978	5,306	0.442	51.2%	5,306	2	0	2,717	1	50.0%
1979	6,348	0.529	47.2%	6,348	4	0	2,992	2	50.0%
1980	7,154	0.596	43.1%	7,154	6	0	3,080	1	16.7%
1981	6,900	0.575	41.5%	6,895	11	5	2,860	2	12.5%
1982	6,602	0.600	44.2%	6,602	10	0	2,915	0	0.0%
1983	7,216	0.656	40.6%	7,214	18	2	2,925	5	25.0%
1984	6,534	0.594	40.4%	6,531	15	3	2,636	6	33.3%
1985	5,808	0.528	45.5%	5,802	55	6	2,617	25	41.0%
1986	6,120	0.510	49.3%	6,111	94	9	2,958	61	59.2%
1987	7,319	0.563	51.3%	7,282	242	37	3,559	196	70.3%
1988	8,232	0.588	54.0%	8,105	693	127	3,771	675	82.3%
1989	9,002	0.643	53.1%	8,287	1,142	715	3,241	1,542	83.0%
1990	8,918	0.637	54.9%	7,314	2,007	1,604	1,923	2,973	82.3%
1991	7,982	0.614	53.6%	4,044	2,366	3,938	339	3,936	62.4%

SAMPLE BODILY INJURY LIABILITY LOSS DATA

Projections of the Ultimate Losses

	Unadjusted Paid Methods			Incurred	Paid Methods Adjusted for Claims Closing Changes			
Accident Year	Devel-opment	Severity Method	Hindsight	Devel-opment	Devel-opment	Severity Method	Hindsight	Weighted Average
1974	$19,246	$19,245		$19,246	$19,246	$19,245		$19,246
1975	23,159	23,159		23,162	23,161	23,159		23,160
1976	26,397	26,397		26,430	26,400	26,397		26,406
1977	30,049	30,049		30,054	30,061	30,063		30,057
1978	31,996	31,994		31,971	32,021	32,023		32,003
1979	34,559	34,563		34,510	34,572	34,572		34,554
1980	36,012	36,023		35,955	36,012	36,011		35,999
1981	35,221	35,231		35,131	35,221	35,217		35,199
1982	34,478	34,464		34,344	34,426	34,423		34,416
1983	37,941	37,864		37,811	37,768	37,765		37,812
1984	37,474	37,371		36,979	37,214	37,205		37,205
1985	36,715	36,505	$36,409	36,543	36,394	36,407	$36,429	36,463
1986	47,818	47,338	47,044	46,916	47,083	47,054	47,055	47,117
1987	63,861	62,577	62,799	60,585	61,685	61,571	62,844	62,173
1988	83,555	80,717	79,763	74,708	78,748	78,001	79,268	78,809
1989	99,338	94,900	90,936	84,444	91,348	89,375	91,514	90,845
1990	110,157	105,279	94,068	92,617	102,640	95,849	96,509	98,101
1991	127,250	104,212	94,090	87,770	312,670	91,947	96,203	94,044

Selected Weights

	Unadjusted Paid Methods			Incurred	Paid Methods Adjusted for Claims Closing Changes			Indicated Variance
Accident Year	Devel-opment	Severity Method	Hindsight	Devel-opment	Devel-opment	Severity Method	Hindsight	in Selected Methods
1974	1	1		2	2	2		0
1975	1	1		2	2	2		2
1976	1	1		2	2	2		194
1977	1	1		2	2	2		31
1978	1	1		2	2	2		453
1979	1	1		2	2	2		659
1980	1	1		2	2	2		655
1981	1	1		2	2	2		1,547
1982	1	1		2	2	2		2,102
1983	1	1		2	2	2		3,455
1984	1	1		2	2	2		25,279
1985	1	1	2	2	2	2	3	7,936
1986	1	1	2	2	2	2	3	50,268
1987	1	1	2	2	2	2	3	876,278
1988	1	1	2	2	2	2	3	4,889,756
1989	1	1	2	2	2	2	3	13,592,826
1990	1	1	2	2	2	2	3	26,807,766
1991	0	1	2	2	0	2	3	20,489,727

NOTES:
1. Dollar amounts are in thousands.
2. Variance amounts are in millions.

Exhibit A-4

SAMPLE BODILY INJURY DATA

Summary Reserve and Claim Indications

Accident Year	(1) Indicated Reserves	(2) Indicated Future Paid Claims	(3) Indicated Average Claim to be Paid (1)/(2)	(4) Total Number Open	(5) Total Number IBNR	(6) Selected Percent to be Paid (2)/ [(4)+(5)]	(7) Selected Coefficient of Variation
1984 & Prior	$404	17	$23,765	77	17	18.1%	1.050
1985	404	25	16,160	55	6	41.0%	1.075
1986	1,129	61	18,508	94	9	59.2%	1.100
1987	4,144	196	21,143	242	37	70.3%	1.125
1988	12,606	675	18,676	693	127	82.3%	1.150
1989	31,613	1,542	20,501	1,142	715	83.0%	1.175
1990	64,709	2,973	21,766	2,007	1,604	82.3%	1.200
1991	88,593	3,936	22,508	2,366	3,938	62.4%	1.225
Total	$203,198	9,408	$21,598	6,599	6,436	72.2%	

NOTE:
1. Amounts in column (1) are in thousands.

Exhibit A-5

SAMPLE BODILY INJURY DATA

Severity Input for Accident Year 1986

Loss Amount	Step Function Approximation for Lognormal	Selected Input Distribution .408 + .592 x (1)
$0.01	–	0.40800
950	0.00007	0.40804
2,316	0.02575	0.42324
4,358	0.11754	0.47758
7,117	0.26685	0.56598
10,625	0.43335	0.66454
14,909	0.58465	0.75411
19,994	0.70653	0.82627
25,902	0.79769	0.88023
32,651	0.86274	0.91874
40,259	0.90778	0.94541
48,743	0.93837	0.96352
58,118	0.95890	0.97567
68,399	0.97260	0.98378
79,598	0.98170	0.98917
91,728	0.98774	0.99274
104,801	0.99176	0.99512
118,829	0.99444	0.99671
133,822	0.99623	0.99777
149,791	0.99743	0.99848
166,746	0.99824	0.99896
184,696	0.99879	0.99928
203,651	0.99916	0.99950
223,619	0.99942	0.99966
244,608	0.99959	0.99976
266,629	0.99971	0.99983
289,687	0.99980	0.99988
313,791	0.99986	0.99992
338,949	0.99990	0.99994
365,168	0.99993	0.99996
392,455	0.99995	0.99997
420,817	0.99996	0.99998
450,261	0.99997	0.99998
480,793	0.99998	0.99999
512,420	0.99999	0.99999
545,148	0.99999	0.99999
578,984	0.99999	0.99999
613,932	0.99999	0.99999
650,000	1.00000	1.00000

NOTE:

1. The amounts in column (1) are based on a lognormal distribution with mean 18,508 and coefficient of variation 1.100.

Exhibit A-6

SAMPLE BODILY INJURY DATA

Estimated Probability Levels for Reseves Without Parameter Uncertainty

	Accident Year								
	1984 & Prior	1985	1986	1987	1988	1989	1990	1991	Total
	Expected Reserve								
	$404	$404	$1,129	$4,144	$12,806	$31,613	$64,709	$88,593	$203,198
Ratio to Expected	Estimated Probability Level								
0.300	0.0030	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.500	0.0433	0.0117	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.600	0.0996	0.0445	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.700	0.1864	0.1169	0.0208	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.750	0.2396	0.1703	0.0502	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000
0.800	0.2978	0.2342	0.1024	0.0088	0.0000	0.0000	0.0000	0.0000	0.0000
0.825	0.3281	0.2693	0.1383	0.0205	0.0000	0.0000	0.0000	0.0000	0.0000
0.850	0.3590	0.3062	0.1813	0.0424	0.0005	0.0000	0.0000	0.0000	0.0000
0.875	0.3903	0.3443	0.2305	0.0792	0.0034	0.0000	0.0000	0.0000	0.0000
0.900	0.4217	0.3833	0.2849	0.1346	0.0166	0.0013	0.0000	0.0000	0.0000
0.925	0.4530	0.4228	0.3433	0.2098	0.0584	0.0132	0.0013	0.0013	0.0000
0.950	0.4842	0.4622	0.4046	0.3026	0.1533	0.0727	0.0242	0.0242	0.0009
0.975	0.5149	0.5014	0.4668	0.4079	0.3116	0.2388	0.1665	0.1665	0.0458
1.000	0.5451	0.5398	0.5285	0.5177	0.5104	0.5069	0.5050	0.5050	0.4951
1.025	0.5744	0.5770	0.5883	0.6236	0.7019	0.7663	0.8340	0.8340	0.9488
1.050	0.6028	0.6130	0.6449	0.7187	0.8462	0.9222	0.9715	0.9715	0.9994
1.075	0.6303	0.6475	0.6973	0.7985	0.9329	0.9819	0.9975	0.9975	1.0000
1.100	0.6567	0.6802	0.7449	0.8618	0.9752	0.9970	0.9999	0.9999	1.0000
1.125	0.6820	0.7110	0.7876	0.9090	0.9921	0.9997	1.0000	1.0000	1.0000
1.150	0.7063	0.7397	0.8251	0.9422	0.9979	1.0000	1.0000	1.0000	1.0000
1.175	0.7291	0.7665	0.8572	0.9646	0.9995	1.0000	1.0000	1.0000	1.0000
1.200	0.7507	0.7913	0.8846	0.9792	0.9999	1.0000	1.0000	1.0000	1.0000
1.225	0.7710	0.8140	0.9077	0.9880	1.0000	1.0000	1.0000	1.0000	1.0000
1.250	0.7902	0.8348	0.9268	0.9933	1.0000	1.0000	1.0000	1.0000	1.0000
1.275	0.8082	0.8537	0.9423	0.9964	1.0000	1.0000	1.0000	1.0000	1.0000
1.300	0.8250	0.8709	0.9549	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000
1.350	0.8548	0.9002	0.9730	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000
1.400	0.8803	0.9236	0.9842	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
1.500	0.9202	0.9563	0.9948	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.600	0.9478	0.9757	0.9984	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.800	0.9786	0.9928	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.000	0.9915	0.9978	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.500	0.9991	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3.000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

NOTE:
1. Dollar amounts are in thousands.

Exhibit A-7

SAMPLE BODILY INJURY DATA

Selection of Overall Contagion Parameter

Accident Year	Indicated Ultimate Reported Frequency	Selected On-Level Frequency	Indicated 1992 Claims (2)x13,000
1974	0.379	0.571	7,423
1975	0.429	0.631	8,203
1976	0.456	0.656	8,528
1977	0.492	0.692	8,996
1978	0.442	0.608	7,904
1979	0.529	0.711	9,243
1980	0.596	0.783	10,179
1981	0.575	0.738	9,594
1982	0.600	0.753	9,789
1983	0.656	0.805	10,465
1984	0.594	0.713	9,269
1985	0.528	0.619	8,047
1986	0.510	0.585	7,605
1987	0.563	0.631	8,203
1988	0.588	0.644	8,372
1989	0.643	0.688	8,944
1990	0.637	0.667	8,671
1991	0.614	0.628	8,164
Indicated Trend	2.3%		

Arithmetic Average	8,756
Estimate of Variance	753,367
Indicated Overall Contagion Parameter	0.0097

Exhibit A-8

SAMPLE BODILY INJURY DATA

Selected Contagion Parameters

Accident Year	(1) Estimated Ultimate Reported	(2) Indicated Variance in Selected Methods	(3) Estimated Proportion Paid	(4) Estimated Variance in Reported (2)/[(3)x(3)]	(5) Indicated Individual Contagion Parameter	(6) Selected Contagion Parameter
1984 & Prior	65,869	384.2	--	1,338.7	0.0000	0.0097
1985	5,808	147.5	45.5%	712.5	-0.0002	0.0096
1986	6,120	766.8	49.3%	3,154.7	-0.0001	0.0096
1987	7,319	4,596.6	51.3%	17,466.4	0.0002	0.0099
1988	8,232	23,048.9	54.0%	79,042.8	0.0010	0.0108
1989	9,002	60,976.5	53.1%	216,258.6	0.0026	0.0123
1990	8,918	84,230.1	54.9%	279,462.0	0.0034	0.0131
1991	7,982	76,972.0	53.6%	267,918.8	0.0041	0.0138

Exhibit A-9

SAMPLE BODILY INJURY DATA

Estimates of Mixing Parameters

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Estimates Based on Claims With Payment						$E(x^2)$
Accident Year	Estimated Average Reserve	Selected Coefficient of Variation	Indicated Standard Deviation (1)x(2)	Indicated Variance (3)x(3)	$E(x^2)$ (4)+(1)x(1)	Indicated Percent Paid	Based on Reported Claims (5)x(6)
1984 & Prior	$23,765	1.050	$24,953	622.665	1,187.440	18.1%	214.927
1985	16,160	1.075	17,372	301.786	562.932	41.0%	230.802
1986	18,508	1.100	20,359	414.481	757.027	59.2%	448.160
1987	21,143	1.125	23,786	565.768	1,012.794	70.3%	711.994
1988	18,676	1.150	21,477	461.279	810.072	82.3%	666.689
1989	20,501	1.175	24,089	580.264	1,000.555	83.0%	830.461
1990	21,766	1.200	26,119	682.213	1,155.971	82.3%	951.364
1991	22,508	1.225	27,572	760.232	1,266.842	62.4%	790.509

	(8)	(9)	(10)	(11)	(12)	(13)
	Estimated Number of Claims					
Accident Year	Open	IBNR	Explained Variance	Variance in Selected	Implied b Value	Selected b Value
1984 & Prior	77	17	18,830	34,377	0.1161	0.1161
1985	55	6	11,680	7,936	-0.0256	0.0000
1986	94	9	34,969	50,268	0.0138	0.0138
1987	242	37	148,177	876,278	0.0544	0.0544
1988	693	127	423,955	4,889,756	0.0379	0.0379
1989	1,142	715	3,027,679	13,592,826	0.0200	0.0200
1990	2,007	1,604	13,618,136	26,807,766	0.0062	0.0062
1991	2,366	3,938	46,708,007	20,489,727	-0.0062	0.0000

NOTE:
1. Amounts in columns (4), (5), (7), (10), and (11) are in millions.

Exhibit A-10

SAMPLE BODILY INJURY DATA

Estimate of Overall Mixing Parameter

Accident Year	(1) Estimated Ultimate Losses	(2) Earned Exposures	(3) Indicated Pure Premium (1)/(2)	(4) Estimated Pure Premium at 1992 Level	(5) Indicated 1992 Loss (4)x13,000
1974	$19,246	11,000	$1,750	$6,764	$87,932
1975	23,160	11,000	2,105	7,547	98,111
1976	26,406	11,000	2,401	7,985	103,805
1977	30,057	12,000	2,505	7,728	100,464
1978	32,003	12,000	2,667	7,633	99,229
1979	34,554	12,000	2,880	7,646	99,398
1980	35,999	12,000	3,000	7,388	96,044
1981	35,199	12,000	2,933	6,701	87,113
1982	34,416	11,000	3,129	6,631	86,203
1983	37,812	11,000	3,437	6,757	87,841
1984	37,205	11,000	3,382	6,168	80,184
1985	36,463	11,000	3,315	5,608	72,904
1986	47,117	12,000	3,926	6,161	80,093
1987	62,173	13,000	4,783	6,963	90,519
1988	78,809	14,000	5,629	7,602	98,826
1989	90,845	14,000	6,489	8,129	105,677
1990	98,101	14,000	7,007	8,143	105,859
1991	94,044	13,000	7,234	7,798	101,374

A. Indicated Trend	7.8%	
B. Average (000)		$93,421
C. Variance Estimate (000,000)		93,442,417
D. Estimated 1992 Claims Reported		8,756
E. Indicated Severity (000) (A/C)		$10.669
F. Selected Coefficient of Variation		1.250
G. Indicated Standard Deviation (000) (ExF)		$13.336
H. Indicated Variance (000,000) (GxG)		177.849
I. Indicated $E(x^2)$(000,000) (H+ExE)		291.677
J. Selected Overall Contagion Parameter		0.0097
K. Explained Variance (000,000)		87,317,281
L. Indicated Overall Mixing Parameter		0.00069
M. Selected Overall Mixing Parameter		0.00069

NOTE:
1. Amounts in columns (1) and (5) are in thousands of dollars.

Exhibit A-11

SAMPLE BODILY INJURY DATA

Estimated Probability Levels for Reseves With Parameter Uncertainty

	Accident Year								
	1984 & Prior	1985	1986	1987	1988	1989	1990	1991	Total
	Expected Reserve								
	$404	$404	$1,129	$4,144	$12,606	$31,613	$64,709	$88,593	$203,198
Ratio to Expected	Estimated Probability Level								
0.300	0.0128	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.500	0.1059	0.0118	0.0005	0.0016	0.0001	0.0000	0.0000	0.0000	0.0000
0.600	0.1925	0.0446	0.0081	0.0192	0.0039	0.0002	0.0000	0.0000	0.0000
0.700	0.2940	0.1170	0.0486	0.0848	0.0375	0.0084	0.0001	0.0000	0.0000
0.750	0.3468	0.1705	0.0930	0.1425	0.0817	0.0292	0.0014	0.0001	0.0000
0.800	0.3993	0.2344	0.1573	0.2152	0.1489	0.0758	0.0111	0.0021	0.0000
0.825	0.4252	0.2695	0.1963	0.2559	0.1904	0.1116	0.0250	0.0071	0.0009
0.850	0.4508	0.3064	0.2397	0.2988	0.2367	0.1561	0.0502	0.0194	0.0033
0.875	0.4754	0.3445	0.2864	0.3428	0.2866	0.2089	0.0908	0.0455	0.0127
0.900	0.5001	0.3834	0.3357	0.3874	0.3388	0.2686	0.1494	0.0925	0.0394
0.925	0.5235	0.4228	0.3865	0.4321	0.3924	0.3335	0.2259	0.1658	0.0982
0.950	0.5467	0.4623	0.4380	0.4761	0.4462	0.4013	0.3175	0.2657	0.2007
0.975	0.5690	0.5014	0.4893	0.5190	0.4993	0.4700	0.4187	0.3859	0.3449
1.000	0.5904	0.5398	0.5395	0.5603	0.5506	0.5374	0.5225	0.5150	0.5113
1.025	0.6115	0.5770	0.5877	0.5998	0.5995	0.6017	0.6219	0.6395	0.6715
1.050	0.6310	0.6130	0.6338	0.6371	0.6454	0.6616	0.7113	0.7484	0.8018
1.075	0.6505	0.6475	0.6768	0.6722	0.6880	0.7160	0.7875	0.8351	0.8927
1.100	0.6685	0.6801	0.7169	0.7049	0.7273	0.7646	0.8489	0.8985	0.9478
1.125	0.6862	0.7109	0.7533	0.7352	0.7626	0.8070	0.8961	0.9413	0.9770
1.150	0.7030	0.7396	0.7866	0.7630	0.7946	0.8434	0.9308	0.9673	0.9907
1.175	0.7188	0.7664	0.8164	0.7885	0.8231	0.8742	0.9552	0.9834	0.9966
1.200	0.7344	0.7911	0.8429	0.8117	0.8481	0.8998	0.9718	0.9915	0.9988
1.225	0.7486	0.8139	0.8662	0.8326	0.8703	0.9210	0.9828	0.9960	0.9996
1.250	0.7627	0.8347	0.8867	0.8515	0.8896	0.9381	0.9898	0.9980	0.9999
1.275	0.7755	0.8536	0.9045	0.8686	0.9063	0.9519	0.9940	0.9990	1.0000
1.300	0.7880	0.8708	0.9198	0.8839	0.9208	0.9629	0.9966	0.9990	1.0000
1.350	0.8109	0.9001	0.9442	0.9098	0.9437	0.9783	0.9990	1.0000	1.0000
1.400	0.8315	0.9235	0.9617	0.9303	0.9604	0.9875	0.9997	1.0000	1.0000
1.500	0.8665	0.9562	0.9826	0.9588	0.9808	0.9961	1.0000	1.0000	1.0000
1.600	0.8942	0.9756	0.9924	0.9758	0.9909	0.9988	1.0000	1.0000	1.0000
1.800	0.9334	0.9927	0.9986	0.9918	0.9980	0.9999	1.0000	1.0000	1.0000
2.000	0.9578	0.9978	0.9998	0.9972	0.9996	1.0000	1.0000	1.0000	1.0000
2.500	0.9859	0.9999	1.0000	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
3.000	0.9949	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

NOTE:
1. Dollar amounts are in thousands.

Exhibit A-12

Estimated Aggregate Reserve Cumulative Densities

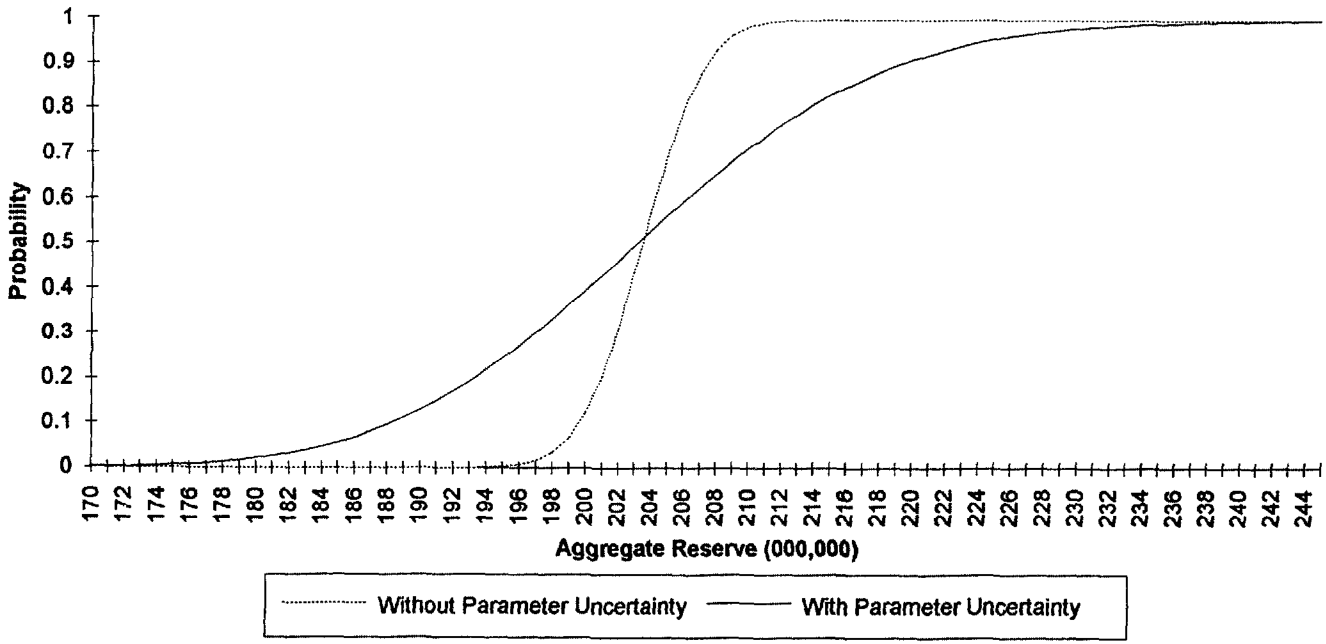
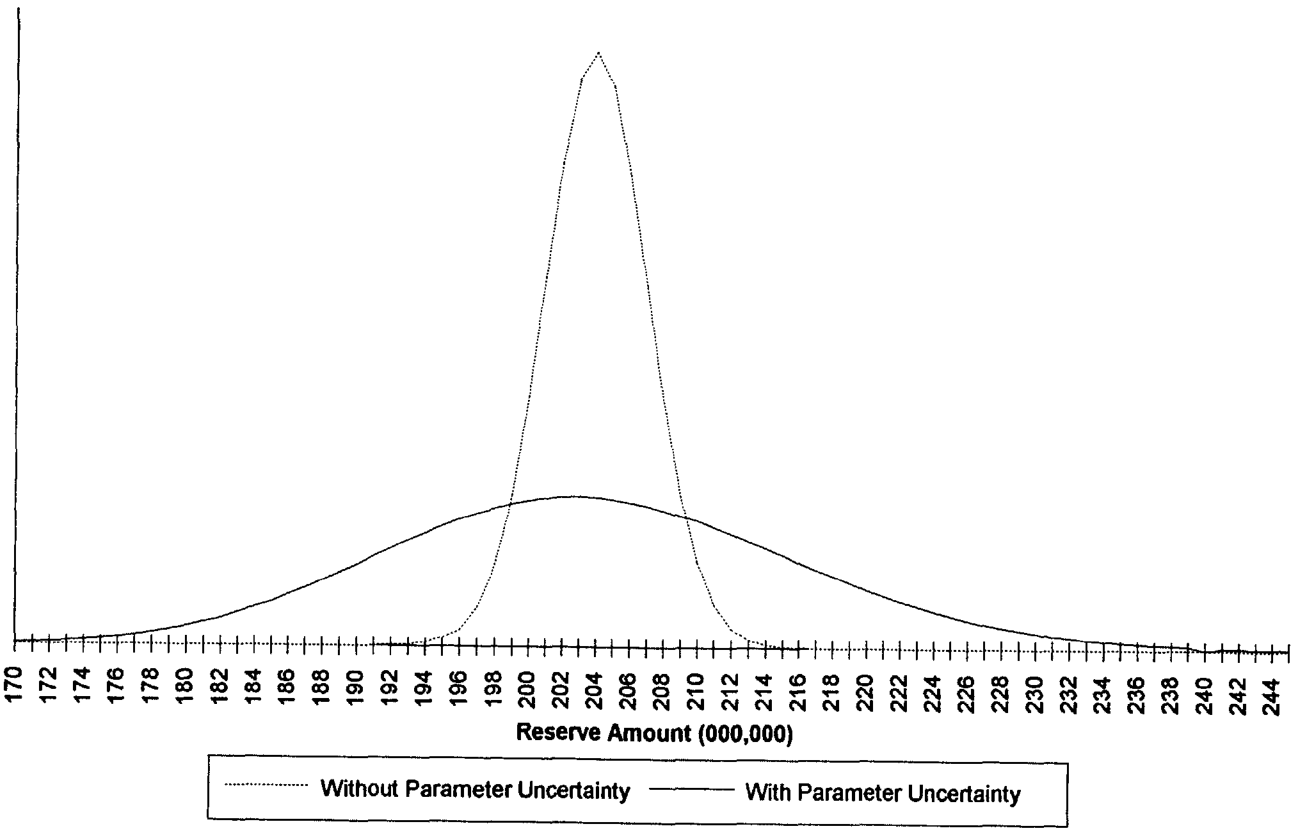


Exhibit A-13

Estimated Aggregate Probability Density Functions



References

[1] R.E. Beard, T. Pentikäinen, E. Pesonen, *Risk Theory, The Stochastic Basis of Insurance*, Third Edition. Chapman and Hall, 1984.

[2] G.G. Meyers and N. Schenker, "Parameter Uncertainty in the Collective Risk Model," *PCAS* LXX, 1983, p. 11.

[3] R.M. Hayne, "Application of Collective Risk Theory to Estimate Variability in Loss Reserves," *PCAS* LXXVI, 1989, p. 77.

[4] P.E. Heckman and G.G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, p. 22.

[5] J.R. Berquist and R.E. Sherman, "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *PCAS* LXIV, 1977, p. 123.

[6] R.S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, p. 27

References

[1] R.E. Beard, T. Pentikäinen, E. Pesonen, *Risk Theory, The Stochastic Basis of Insurance*, Third Edition. Chapman and Hall, 1984.

[2] G.G. Meyers and N. Schenker, "Parameter Uncertainty in the Collective Risk Model," *PCAS* LXX, 1983, p. 11.

[3] R.M. Hayne, "Application of Collective Risk Theory to Estimate Variability in Loss Reserves," *PCAS* LXXVI, 1989, p. 77.

[4] P.E. Heckman and G.G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, p. 22.

[5] J.R. Berquist and R.E. Sherman, "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *PCAS* LXIV, 1977, p. 123.

[6] R.S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, p. 27