

Accounting for Risk Margins

*by Stephen W. Philbrick with an
introduction by Paul G. O'Connell*

Introduction

by Paul G. O'Connell

The CAS Committee on Reserves is pleased to present a funded research paper titled "Accounting for Risk Margins", authored by Stephen W. Philbrick. The committee's charge to Mr. Philbrick was to explore possible ways to adapt statutory and GAAP accounting to reflect formal existence of margins for adverse deviations in loss reserves. The focus was not to be on methods for calculating margins, but rather on proper accounting treatment for the calculated margin. In his paper he has accomplished this and more.

Mr. Philbrick demonstrates the conflict between profit recognition and the true economic reality of the insurance transaction under both current accounting principles and in an environment where losses are discounted at a risk-free rate. Through his research in this area, he has advanced a theoretical framework that addresses the appropriate accounting technique for reflecting and amortizing a risk margin, which when used with discounted loss reserves, results in a more accurate formula for profit recognition.

Mr. Philbrick's paper is a valuable addition to casualty actuarial literature. It is certain to prompt debate among actuaries, accountants and others as well as to inspire additional research on the appropriate method or methods for calculating risk margins.

The current and former members of the CAS Committee on Reserves who assisted on this project are as follows:

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Accounting for Risk Margins

By Stephen W. Philbrick

Introduction

The importance of risk margins is growing rapidly. Not long ago, the subject of risk margins was not considered a burning issue within the actuarial profession, much less the insurance community at large. The recent insolvencies in the industry and the attendant search for causes and solutions, however, have led to heightened interest in risk margins.

Risk margins, whether in pricing or in loss reserves, have always been easy to understand superficially but difficult to pin down precisely. It is well known that case reserves and IBNR reserves are estimates of *unknown* future loss payments. Actual results will differ from estimated amounts, and the concept of risk margin reflects that fact.

A risk-averse individual or corporation would prefer a fixed liability of \$100 to a liability whose expected value is \$100, but whose actual payout amount is uncertain. “The greater the uncertainty, the larger the risk margin.” Beyond that statement, however, there is little universal agreement. Methods vary not only for calculating uncertainty, but also for determining risk margins from uncertainty measures.

Furthermore, assuming a risk margin has been calculated, it is not obvious how that risk margin should be incorporated into statutory accounting, and it is arguable whether it should be incorporated at all.

The purpose of this paper is to explore how such a risk margin should be incorporated in statutory accounting. Rather than researching methods of calculating risk, this paper will assume that a satisfactory method for calculating risk margins will be separately developed.

Nevertheless, a discussion of the accounting treatment of risk margins can hardly proceed without a clear understanding of what we mean by the term “risk margin”. Unfortunately, the actuarial profession not only needs to develop methods to calculate a risk margin, but it also needs to agree on a common definition. There are three common situations where the term “risk margin” is used: undiscounted loss reserves, loss portfolio transfers, and self-insurance trust funds. Each of these situations will be discussed briefly.

Undiscounted Loss Reserves

The term risk margin is commonly used in the observation that stating the loss reserve at nominal (rather than discounted) values provides an implicit risk margin. It is clear that the amount of the risk margin in this circumstance is the difference between undiscounted and discounted reserves. This observation, however, does not

provide much insight into the purpose or definition of a risk margin. Compounding the problem is the fact that there is no general agreement regarding which discount rate should be used in such a calculation; thus, the specific value of the implicit risk margin is not clearly defined.

Loss Portfolio Transfers/Commutations

There is a market (albeit tiny) for loss reserves. A company retiring from business may sell a portfolio of reserves to another company. In theory, the amount of the purchase price less the present value of the expected payments represents a risk margin. Unfortunately, this does not provide a good empirical source for risk margin data because most of these transactions involve other factors, such as tax considerations and stop loss agreements. In concept, however, this provides one measure of a risk margin. Conceptually, a risk margin represents the difference between the expected (discounted) value of the reserves and the certainty equivalent of the reserves, where certainty equivalent means the amount certain one would accept (or pay) now in exchange for a stream of payments in the future whose amount and timing is uncertain. A lump sum payment in exchange for a portfolio of reserves will represent a certainty equivalent, if no extraneous items (such as taxes, expenses, side agreements or default risk) are involved.

A commutation is a special case of a loss portfolio transfer, where the loss reserves are transferred back to the ceding company. In most cases, one of the parties

to the transaction may be in financial difficulty, which will distort the value agreed upon. If both parties are financially strong (and the commutation is not simply the unwinding of a financial reinsurance agreement), the terms of the commutation may provide insight into the value associated with the riskiness of the loss reserves.

Self-Insurance Trust Funds

The use of risk margins is fairly well developed in the case of Self-Insurance Trust Funds. From a financial structure standpoint, one notable difference between a traditional insurance company and a trust fund is that a trust fund typically does not have a capital or surplus account. Rather, it establishes a funding requirement such that the available funds correspond to the p^{th} percentile of the aggregate loss distribution, where p is typically 75% or 90%. The risk margin is the recommended funding requirement less the expected present value of the reserves. It is typical for an actuarial funding study to explicitly show the amount of the risk margin in the report. It is important to note, however, that this risk margin is not intended to represent the difference between the expected value of the reserves and the certainty equivalent.

Thus, while an actuary has a tool kit of methods to arrive at a best estimate of loss reserves, the goal of all such methods is the same. The actuary also has a variety of methods to calculate risk margins, however, the goals of the various methods are not all the same although all are termed risk margins.

Risk Margins in Canada

In developing recommendations for the calculation and accounting of risk margins, it may be helpful to review developments in other countries.

The Canadian Institute of Actuaries (CIA) has adopted a standard of practice which requires a provision for adverse deviations. The mechanics of the standard are quite different from the methodology outlined in this paper. The CIA methodology is heavily judgmental, requiring the actuary to select margins relating to three variables:

- claims development
- reinsurance recovery
- interest rate

Each margin is selected from a range (with options for selecting outside the range) based on a qualitative list of considerations. The claims development margin range is 0-15%, the reinsurance recovery is 0-25% and the interest rate margin is a downward adjustment, (which can vary by line of business) to the interest rate used for discounting. The range is from 50 basis points to 200 basis points. The overall provision for adverse deviations is added to discounted liabilities.¹

¹Memorandum to Fellows and Property/Casualty Actuaries of the Canadian Institute of Actuaries from the Committee on Property and Casualty Financial Reporting May 5, 1993.

Types of Risk Margin

It is critically important to recognize the potential differences in the type of risk margin that might be proposed in any attempt to standardize risk margin calculations. At least four different possibilities could occur.

1. **A risk margin based on a certainty equivalent concept.** Under this concept, a risk margin would be calculated such that, when added to a present-value reserve, it produces an estimate of the certainty equivalent value; that is, the amount of cash immediately payable to transfer the liability. This concept corresponds to loss portfolio transfers.

2. **A risk margin based on a theory of ruin concept.** Under this concept, a risk margin would be calculated such that the probability of insolvency or the expected cost of insolvency is reduced to an acceptable level. This concept corresponds to risk theoretic discussions of insurance enterprises.

3. **A risk margin based on probability intervals.** Under this concept, a probability, such as 75% or 90%, is specified. A risk margin is calculated such that the actual loss amount is less than or equal to the expected loss plus the risk margin in the specified proportion of times. These intervals are sometimes referred to as confidence intervals.² This concept is commonly used in trust fund analyses. A reasonable question is

²Technically, confidence intervals are intervals around parameters, while prediction intervals are intervals around results.

whether the probability intervals should be applied to discounted or undiscounted losses. The use of probability intervals is discussed in Loss Reserving for Solvency (David A. Arata [1983], PCAS LXX, P. 1). Although that article does not explicitly discuss discounting, it appears that the calculations are performed with undiscounted losses. This concept is also discussed in a CLRS presentation by Robin Harbage (1989 CLRS Transcripts, p. 1075). Again, it is not explicitly stated whether discounted values are used, but the context of the discussion implies that undiscounted values are used.

4. **A risk margin intended to simply provide a *relative* measure of risk.** It is conceivable that the actuarial profession may conclude that the calculation of a risk margin satisfying the goals of methods 1 or 2 is beyond current capabilities. Alternatively, the profession may decide that it is possible to design a measure of *relative* risk. For example, some fixed percentage of the aggregate loss variance might be proposed as a risk measure. This value would be higher for companies with more risk, thereby providing a relative measure. The absolute value of the measure, however, might not have a precise meaning. This is analogous to the concept of utility functions, which attempts to *rank* preferences, but not necessarily to ascribe a meaning to the absolute level of the utility function value. The implicit margin in undiscounted loss reserves corresponds to this concept, because the absolute amount of the risk margin does not result from risk theory, but lines of business considered to be riskier (i.e., long-tail) will tend to have relatively larger implicit risk margins.

Relevant Accounting Issues

Accounting for risk margins will be dependent on the concepts underlying the calculation. The implications for balance sheet and income statement are different for the different choices. In particular, a measure following the fourth concept of risk margin may not easily be transferable to a balance sheet and might have to be accounted for in a separate schedule.

Accounting for risk should be largely coincident with accounting for profit. An entrepreneur (or group of entrepreneurs) starts an insurance company with the intention and expectation of earning a profit.³ An entrepreneur wishing to earn a profit without taking risk will find few opportunities. In order to earn a profit substantially in excess of rates available in the United States government securities, an entrepreneur must assume some risk.

³Exceptions arguably exist; there are some nonprofit organizations formed primarily for the purpose of providing difficult-to-obtain insurance. However, mutual insurance companies do operate as profit-maximizing firms, despite the blurred distinction between entrepreneurs and customers.

Types of Risk

An enterprise engaged in the business of insurance faces a wide variety of types of risks. Some of these risks include:

Underwriting risk—This is often defined as the risk that actual losses and expenses will exceed premiums. For the purpose of this paper, underwriting risk represents the possibility that discounted actual losses exceed the loss portion of the premium.

Investment risk—This term encompasses all risks related to the returns on invested assets. This risk is often subdivided into:

Interest rate risk—Possibility that asset values may drop due to a change in market yield rates

Default risk—Possibility of non-payment of interest and/or principal

Reinvestment risk—shortfall in investment income due to lower available yields for reinvested assets

Market risk—Generally referring to stocks or real estate; possible reductions in asset values due to changes market prices

Timing risk—the risk that the actual payout pattern of losses will differ from expected.

Reinsurance risk—The risk that reinsurance placed by an enterprise may not be collectible.

Credit risk—The risk that an insured might not pay all premiums due.

Risks are also classified as process risk or parameter risk. Process risk represents the possibility that actual results differ from expected results while parameter risk represents the possibility that the estimated expected differs from the true expected.

This paper will concentrate on underwriting risk. The assumption will be made that assets are invested in risk-free securities such as T-bills, where the durations are closely matched to the expected. This is not an optimal investment strategy, nor does it eliminate all investment risk. Some timing risk remains. However, there is an important *interrelationship between timing risk and underwriting risk (as defined here)*.

Sometimes a company can settle a claim for a smaller amount by settling it earlier than “expected”. If the reduction in nominal costs equaled the discount associated with the length of time, this would produce no net change in underwriting risk calculated on a discounted basis. Conversely, a settlement for more than the “expected” amount arising from a protracted settlement might also have no effect on underwriting risk. In reality, these amounts will not precisely offset, however, it should be clear that at least some portion of timing risk is mitigated by the possibility that nominal settlements will be dependent on the timing of settlement. Major changes in timing can still have a deleterious effect on investment results when a portfolio has been precisely duration-matched. However, this is beyond the scope of this paper.

This paper will incorporate process risks and some aspects of parameter risks. Parameter risk, by its very nature, cannot be precisely estimated. Some aspects of the estimation process, such as model selection, are considered by many to be parameter

risk, but are beyond the scope of this paper. Other significant issues, such as the possibility of a government takeover of the workers compensation system, might be considered parameter risk, but are almost certainly not incorporated into the pricing decision and are excluded from consideration in this paper.⁴

The largest single source of profit for a typical insurance enterprise is the assumption of underwriting risk.⁵ Some companies earn income for operations that are not strictly labeled as underwriting. For instance, companies that provide fronting services earn profits for the assumption of credit risk. Other companies offer “unbundled services” and may sell services such as claims-handling and loss prevention without incurring underwriting risk.

The fact that “underwriting income” has been negative for the industry as a whole for many years does not mean that companies are not engaging in underwriting risk. The definition of underwriting income (excess of premium over expenses and losses in nominal dollars) is an anachronism, determined when the time value of money was a much smaller component of income (both because interest rates were lower than today and because the length of time between premium receipt and loss payment was shorter).

⁴While it may appear obvious that our pricing mechanism does not formally incorporate the possibility of a government takeover of some portion of the insurance business, it is not as obvious as it sounds. To the extent that investors are truly worried about such an event, capital will be less likely to flow into the industry and the remaining capital may be able to command a higher rate of return than otherwise.

⁵Operations such as Berkshire Hathaway may be exceptions because of their large investment amounts relative to insurance operations.

To the extent that companies invest in securities with significantly higher risk characteristics than Treasury bills, they are truly earning income from their assumption of investment risks. For the purposes of this discussion, underwriting returns will be defined as the excess of premium over the sum of *discounted* expenses and losses, where expenses and losses are discounted at risk-free rates of return, over the time period between receipt of premium and payment of expenses or losses. Under this definition, the largest single component of insurance companies' returns will arise from underwriting returns.

Accounting rules—whether statutory, tax, or GAAP—do not explicitly state how the accounting for either the profit or risk margin should take place. Rather, accounting rules specify how to account for the various components of the insurance transaction (premium, expenses, losses, investment income) that drive the accounting treatment of the profit component. It should not be inferred, though, that the resulting accounting treatment of profit is of minor relevance. In fact, with respect to GAAP accounting, the determination of rules for accounting of the various elements is guided in part by whether the resulting income statement bears a close resemblance to “reality.”

Accounting is needed whenever the timing of cash flows associated with a particular transaction occur at different times. In a classic goods manufacturing company, the initial cash flow is the capital inflow to the company. This is followed by cash outflow for capital equipment, which is followed by cash outflows for labor and materials. Finally, there is a cash inflow as the customer buys the product. Because

the time frame typically encompasses several years, accounting principles were developed in order to more closely match revenues and expense. For example, the cash outflow for capital equipment is not expensed in the year purchased, but capitalized over some time frame, effectively charging a portion of the total cost to each year it provides service.

Similarly, an insurance company has an original cash inflow of capital, followed by cash inflows of premium and the cash outflows of expenses and losses. Accounting conventions, including such items as loss reserves, exist so that balance sheets and income statement more accurately reflect the economic reality of the corporation.

In any industry, accounting conventions are not expected to perfectly reproduce economic reality. Instead, the goal of closely approximating economic reality is balanced with the desire for reasonable simplicity, consistency and efficiency of performing the accounting. We might argue therefore that the theoretically correct way to depreciate an item of capital equipment is to precisely measure its life time and its yearly contribution to the business. These calculations would be expensive and are subject to dispute and manipulation, so accounting conventions exist to prorate the original cost of capital equipment over some fixed length of time which only approximates the actual useful lifetime. When significant changes occur to the environment (e.g., new classes of equipment), accounting conventions must be devised or revised to reflect the new situation. In any such situation, the goal is to promulgate

accounting conventions such that the resulting accounting statements are a reasonable reflection of economic reality.

In the property-casualty insurance industry, accounting conventions have generally dictated that loss reserves should be established on a nominal basis, that is, without any reduction for the time value of money. These accounting standards were established at a time when:

- reserves were smaller (relative to premiums) than today
- interest rates, and therefore the potential amount of discount, were much lower

The decision to carry reserves on a nominal basis was not justified on theoretical grounds, but rather, pragmatic ones. As additional evidence, it should be noted that life insurance has always formally incorporated the time value of money in its accounting. The length of time associated with life contracts has always been long enough that the simplicity arising from nominal reserves is far overshadowed by the material distortions which would result.

This situation has changed in three significant ways:

- The property-casualty industry has migrated from a predominantly property (i.e., short-tail) to a predominantly casualty (longer tail) book of business
- Individual lines of business have experienced a lengthening of the payment tail
- Interest rates, while lower today than a decade ago, are still well above rates prevalent over the first half of this century.

Each of these three changes has combined to increase the financial impact of the time value of money. Appendix A provides an analysis of the accounting of a single policy, starting with a low-interest scenario on short-tail business, and gradually changing assumptions to a level consistent with today's marketplace. The conclusions of that analysis are:

- if very short-tail business is written in a low interest environment, the timing of profit recognition arising from accounting rules roughly mirrors the pattern that corresponds to economic reality.
- if longer-tail business is written in a higher interest rate environment, accounting conventions significantly delay the recognition of profit. The accounting of a single policy implies that the business loses money in the year it is written, and profits are earned in subsequent years.

Pricing and Risk Margins

Before directly addressing the accounting for loss reserve risk margins it will be helpful to review basic assumptions associated with the pricing of a policy. Risk margins for loss reserves should not be considered separately from premium pricing issues, but rather as a different point on a continuum of a policy from inception to final loss payment.

A block of business is normally priced at a level intended to provide a sufficient profit after paying expected losses and expenses. Premium levels will be affected by many external events, but over long periods of time for the industry as a whole, it is reasonable to assume that the profit margins will be related to the amount of risk assumed by the company.

An insurance enterprise must be financially able to withstand actual loss payments in excess of expected payments. There are two ultimate sources of funds to provide for this contingency:⁶

1. Surplus⁷ from investors, and
2. Profit margin from insureds

The term capital is sometimes used to refer to the original amount of assets provided by the investors (or subsequent infusions) as distinct from retained earnings. This paper will use the term surplus to refer to the entire amount of policyholders surplus, including original contributed capital, subsequent capital or surplus infusions and any retained earnings.

⁶The absence of investment income is deliberate. For the purposes of this paper, we will assume that investment income does accrue to the insurance enterprise but that the amount is not under the control of the insurance company. Thus, if an insurance enterprise determines that it faces higher potential losses than contemplated in its current financial structure, it may raise capital, raise rates (or some combination) but it cannot choose to raise investment income rates.

⁷Theoretically, a third potential source is debt. Because debt is so rarely used in the insurance industry, it will not be treated in this discussion.

Appendix B contains a more in-depth discussion of the nature of the surplus and profit margin components, including a discussion of how the relative amounts are determined.

In the pricing context, there is little confusion about the term risk margin. While there are two sources of funds which pay losses in excess of expected losses, one of these sources, the profit margin, is generally considered to be the risk margin, and the other source, surplus, is not. When we turn to loss reserves, the situation is not as clear.

Loss Reserves and Risk Margins

The actuarial profession has not yet settled on a methodology to determine risk margins for loss reserves. It is possible, however, to consider conceptually what such a provision means without necessarily specifying the calculation procedure. A loss reserve margin is an amount needed over and above the expected (discounted) reserves to reflect the inherent riskiness of the reserves. While this description is obviously imprecise, it is difficult to refine it without specifying, or implying, a calculation methodology. For example, if we define loss reserve margin as that amount, which when added to the reserves, provides a total amount sufficient to pay actual losses with probability $x\%$ (where x might be 99 or 99.5), then we have essentially adopted a ruin theory approach to loss reserve margins.

Despite the vagueness of the definition, there is an important conclusion that can be drawn. Specifically, the normal use of the term “risk margin” in the context of loss reserves does not provide for a distinction between the two ultimate sources of fund—the insured and the investor. Indeed, some actuaries argue that a loss reserve margin should be merely an earmarked surplus item which is equivalent to implying that the source of the amount is the investor.

Risk Margin Calculation

For the purpose of this paper, it will be necessary to divide the total risk margin into two components:

- a loss reserve risk margin arising from the original profit margin
- an earmarked surplus amount

To avoid confusion between two distinct terms with the same name, the first of these two items will be referred to as the “narrow risk margin” (NRM) and the second of the two items will be referred to as the “surplus risk margin” (SRM)⁸. The sum of the two amounts will be referred to as the “broad risk margin” (BRM). The term NRM will be defined to mean that portion of the total risk margin which belongs “above the line”, that is, the portion which theoretically should be considered a liability of the company rather than any part of surplus. The term SRM is defined to be that

⁸This choice of terminology is deliberately based on the convention used to distinguish between two overlapping definitions of IBNR.

portion of the total risk margin which belongs in the surplus section of the balance sheet.

Calculation of these three elements will probably proceed in one of two ways: either the NRM and SRM values will be separately calculated directly, or the total BRM value will be calculated and apportioned into the two components. A methodology to directly calculate the NRM value might start by exploring what it would cost to “sell” the liabilities.

Assume that an efficient secondary market exists for loss reserves. The difference between the sale price and the best estimate reserves on a discounted basis would represent the narrow risk margin. In this case, this amount would identically be the amount that a company should carry as a liability (that is, above the line) in the normal case that it does not sell its reserves.

Several caveats should be noted. First, actual sales of loss reserves typically include aggregate limits on the amount that the assuming company will pay. The appropriate narrow risk margin must be calculated without any such limits.⁹ Second, actual transactions often reflect a different tax situation between the ceding and assuming company. The narrow risk margin should be calculated exclusive of tax considerations. Third, this “thought experiment” doesn't specify how such a sales price should be calculated.

⁹Obviously, existing policy limits and aggregates should be incorporated into the calculations.

Alternatively, industry risk margin calculations may, instead, directly calculate the broad definition, BRM. If the methodology is based on aggregate loss distributions or ruin theory approaches, it is likely that the resulting amount will be either the total amount (including loss reserves) needed as available assets to ensure the viability of the insurance enterprise or the amount needed in excess of the best estimate loss reserves. It is highly likely that in the second case, the amounts will be relative to *discounted* loss reserves. If the resulting margin is to be added to *nominal* loss reserves, an additional calculation will be required.

Once the broad risk margin is calculated, then the two components, NRM and SRM, can be calculated using formulas outlined in a later section.

Another possible approach to risk margin calculations is an adjusted discount rate calculation where the best estimate loss reserves are discounted at a rate less than prevailing market rates.¹⁰ In this case, the difference between the reserves discounted at the adjusted rate and discounted at market rates will represent the risk margin. This calculation normally produces a narrow risk margin.

Many questions still need to be addressed. Subsequent sections will discuss:

- Transition from the current situation to the proposed situation
- Proper handling of the change in risk margin arising from a consistent application of the methodology

¹⁰In the case of property, it is conceivable that the appropriate adjustment to interest rates may produce “discounted” loss reserves slightly larger than the nominal amount.

- Proper handling of the change in risk margin arising from a change in the methodology used to calculate risk margins

However, before we go into those issues, we should discuss specific examples with a proposed methodology for the calculation of the BRM and its components, NRM and SRM.

Assume that an insurance company has the opportunity to write a volume of business with expected nominal losses of \$300.00. For simplicity, we will treat the entire business as if it consists of a single policy. This amount is chosen to keep the numbers in the exhibits manageable. The reader is invited to think of this as a surrogate for a more realistic number such as 30,000,000 or 300,000,000.¹¹

We will examine three different scenarios, starting with an over-simplified example and moving toward a more realistic example.

In the first example, only one loss payment is made (at the middle of the policy period), the premium is paid at the beginning of the policy period, and only a single policy is written. This will be referred to as the SINGLE PERIOD model.

¹¹It would be preferable to examine a single policy added to an existing company. However, exploration of this alternative suggests that proper handling of a single policy requires analysis of the covariance of the individual policy with the remainder of the portfolio. It was felt that this complication would detract from the central theme of this paper, so the decision has been made to analyze the company as a whole. The correct treatment of covariance is important to the issue of pricing an individual contract, but less important to the accounting of the risk margin of a contract which has already been priced.

In the second example, loss payments are made at the middle of three calendar periods following the inception date. The premium is paid at the beginning of the policy year. Only one policy is issued; this will be referred to as the **THREE PERIOD** model.

Finally, we will assume that a policy identical to the one in the second example is written each year for three years. This allows the company to reach a steady state. Expected loss reserve reductions from expected payments exactly offset additions from the new policy. This will be called the **STEADY STATE** model.

Several assumptions will be common to all models:

- Policy period - One year
- Policy inception date - January 1
- Risk-free interest rate - 6%
- Company's desired rate of return on equity - 15%
- Expenses - none
- Taxes (federal and premium) - none

SINGLE PERIOD MODEL

Assume that a policy is issued whose undiscounted expected losses are \$300.00. All losses are paid at the middle of the year, so the present value of expected losses is \$291.39 ($300/1.06^{.5}$). The present value of *actual* losses may turn out to be less than or

greater than this amount. This variability will be quantified by assuming that the aggregate distribution of losses is modeled by a lognormal distribution with mean \$291.39 and coefficient of variation (CV) equal to .128. To put this value in perspective, the probability that actual losses could be 23% higher than expected is approximately 5%. In other words, there is roughly a 95% probability that actual losses will be less than \$359 (291×1.23).

The insurance company must provide for an amount of assets sufficiently large so that regulators and policyholders will be satisfied that the company is highly likely to pay any losses which arise under this policy. Obviously, it would be desirable if the company could be *certain* that it could pay losses under any scenario. However, with an unlimited potential for loss, no finite amount of assets can guarantee payment in all circumstances. Therefore, regulators and policyholders must be satisfied that non-performance is reduced to an acceptable level.

Non-performance can be measured in two important ways. The most common way is to measure the probability of non-performance, which is the probability that the available assets of the company are insufficient to pay the actual losses. This approach is often referred to as the “probability of ruin” approach. We could specify the acceptable probability of ruin, for example, 1%, and solve for the amount of assets necessary to cover the 99th percentile of the aggregate loss distribution.

Another way to measure non-performance is to measure the total cost of non-performance, which is the expected losses in excess of available assets. This approach will be referred to as the “expected deficit” approach. We specify the acceptable deficit (as a percentage of expected discounted losses) and solve for the amount of assets. If we specify an expected deficit of 2%, then we need to find the amount of assets such that the area of the aggregate loss distribution above the asset amount is 2% of \$291.39 or \$5.83. This asset amount is \$359.42¹². Thus, the insurance company needs to provide \$359.42 in additional assets in order to write this policy.

The company can get some of these assets from the policyholder and some from the investors in the company.

The policyholder will provide assets by paying a premium. The amount of the premium will be equal to expected losses plus a risk premium. The risk premium will be called the narrow risk margin (NRM). The investor will supply surplus, which will be called the surplus risk margin (SRM). The sum of the NRM and the SRM produces the broad risk margin (BRM). The BRM is that amount of assets needed in addition to

¹²The actual calculation involves solving the equation for expected deficit directly

$\int_A^\infty (z - A)dF = .003$. In this equation, Z is a random variable representing the possible aggregate losses of the company, F is the cumulative distribution of aggregate losses, and A is the desired amount of assets required to satisfy the expected deficit criterion. In words find the value A such that the sum of losses in excess of A is .3% of the total. Alternatively, solve the following equation which represents the proportion of losses which can be covered by a company with assets equal to A.

$$\int_A^\infty z dF + A[1 - F(A)] = 997$$

In both cases, F is the aggregate distribution of losses incorporating both process and parameter risks, and in this example, is a lognormal distribution with CV = .128.

expected losses to satisfy policyholders and regulators that the company is financially sound. Consequently, the BRM is equal to the total asset need (\$359.42), less the expected losses (\$291.39) which produces a BRM value of \$68.03.

We now need to determine the relative contributions of the policyholder and the investor to the BRM. How is the BRM apportioned between NRM and SRM? The answer is that we solve for a value of NRM sufficiently large to provide the required rate of return (ROR). The solution (for the single period model) is:

$$NRM = \frac{(ROR - i) \times BRM}{1 + ROR}$$

Although this is the formula we would use in practice for a one-period model, this formula will be more understandable in a slightly different form. Using the relationship $BRM = SRM + NRM$, the formula for NRM can be written in terms of SRM as follows:

$$NRM = \frac{(ROR - i)SRM}{1 + i}$$

In words, the investor will supply the amount SRM and expects to earn a return on this amount at a rate of ROR. The SRM amount can be invested during the year at rate i , so the policyholder must supply the difference $ROR - i$. This amount is needed at the end of the year, so the amount required at the beginning of the year is discounted by one year's interest $(1 + i)$.

Solving, we find in this specific example:

$$NRM = \$5.32$$

SRM = \$62.71

BRM = \$68.03

Now we will examine how to handle the statutory accounting for this policy.

Exhibit 1 contains the suggested handling of this policy in terms of the effect on existing lines in the balance sheet and income statement pages, as well as required additional lines. Exhibit 2 contains the identical information as Exhibit 1, as well as some supplemental information which will be used to better understand how the process works. This discussion will concentrate on Exhibit 2 for ease of explanation, but the conclusions apply to Exhibit 1.

In Exhibit 2, the relevant lines of the balance sheet and income statement are shown. The shaded lines contain the supplemental, explanatory material not intended to be included in actual financials. (A quick glance at Exhibits 1 and 2 will verify that the only difference between them is the shaded lines.) The lines in bold print are lines not currently in statutory financials that would be required if we incorporate discounting and loss reserve margins into the statement. The annual statement line numbers are shown on the left side of the exhibits. Four-digit numbers corresponding to write-in lines are used to indicate where the newly required information should reside if a current statement format is used. Obviously, if these recommendations are accepted, the organization and line numbering of these exhibits may change.

One-Period Model

Exhibit 1

An St	Balance Sheet	At 12/31/X-1	At 1/1/X	At 12/31/X
Line #'s	Assets			
1.	T-Bills	\$62.71	\$359.42	\$0.00

	Liabilities			
1.	Undiscounted Loss Reserve		\$0.00	\$0.00
9.	UPR	0	\$291.39	\$0.00
2101.	Reserve for Discount		\$0.00	\$0.00
2101a.	Discounted Loss Reserve	0	\$0.00	\$0.00
2102.	Risk Margin	0	\$5.32	\$0.00
22.	Total Liabilities	0	\$296.71	\$0.00
26.	Surplus	\$62.71	\$62.71	\$0.00

	Statement of Income	Year X
1.	Premium	\$296.71
2.	Undiscounted Incurred Loss	(\$300.00)
8.	Investment Income	\$12.70
0501.	Change in Risk Margin	\$0.00
0502.	Change in Reserve for Discount	\$0.00
16.	Net Income	\$9.41
	Capital and Surplus Account	0
17.	Surplus, Dec. 31 previous year	\$62.71
	Gains And (Losses) in Surplus	
18.	Net Income	\$9.41
25(a).	Surplus Adjustments: Paid in	(\$62.71)
27.	Investor Dividend	(\$9.41)
31.	Changes in Surplus for the year	(\$62.71)
32.	Surplus, Dec. 31 current year	\$0.00

NEWEX.XLS

One-Period Model

Exhibit 2

With Supplemental Information

An St	Balance Sheet	At 12/31/X-1	At 1/1/X	At 12/31/X
Line #'s	Assets			
1.	T-Bills	\$62.71	\$359.42	\$0.00

	Liabilities			
1.	Undiscounted Loss Reserve		\$0.00	\$0.00
9.	UPR	0	\$291.39	\$0.00
2101.	Reserve for Discount		\$0.00	\$0.00
2101a.	Discounted Loss Reserve	0	\$0.00	\$0.00
2102.	Risk Margin	0	\$5.32	\$0.00
22.	Total Liabilities	0	\$296.71	\$0.00
26.	Surplus	\$62.71	\$62.71	\$0.00

Statement of Income		Year X
	Loss Portion of Premium	\$291.39
	Risk Margin Portion of Premium	\$5.32
1.	Premium	\$296.71
2.	Undiscounted Incurred Loss	(\$300.00)
	Inv. Inc. (Discounted Loss Reserves)	\$8.61
	Inv. Inc. (Risk Margin and Surplus)	\$4.08
8.	Investment Income	\$12.70
0501.	Change in Risk Margin	\$0.00
0502.	Change in Reserve for Discount	\$0.00
16.	Net Income	\$9.41
Capital and Surplus Account		
17.	Surplus, Dec. 31 previous year	\$62.71
Gains And (Losses) in Surplus		
18.	Net Income	\$9.41
25(a).	Surplus Adjustments: Paid in	(\$62.71)
	Narrow Risk Margin Component	\$5.32
	Inv. Inc. Component	\$4.08
27.	Investor Dividend	(\$9.41)
31.	Changes in Surplus for the year	(\$62.71)
32.	Surplus, Dec. 31 current year	\$0.00
	Investor Return (line 18/line17)	15.00%

NEWEX.XLS

Balance Sheet

We presume that the required surplus must be in the company *before* writing the new policy. With an effective date of 1/1/X, this means we need to include the \$62.71 of required surplus in the company on the previous day, 12/31/X-1. Before the policy is written, the balance sheet will contain only this surplus amount.¹³

Financial statements are typically calculated at year-ends, but we have shown a balance sheet on 1/1/X, immediately following the writing of this policy, to help follow the transactions. The premium of \$296.71 is received by the company on January 1 and invested in Treasury bills. Consequently, the asset side of the balance sheet shows the surplus as well as the total premium, invested in Treasury bills. This total is \$359.42. Technically, this entire amount of the premium should be established as an unearned premium reserve on that date. However, it will be difficult to trace the flow of the narrow risk margin if it is buried in the UPR, so we have placed the expected loss portion of the premium into the UPR and the narrow risk margin of \$5.32 into a risk margin reserve.

¹³The mathematics would be slightly “cleaner” if we could assume that the risk margins are contributed on a discounted basis. However, it is unlikely that regulators would accept such a concept, so a convention is adopted where expected loss amounts are provided on a discounted basis while surplus and risk margins are provided on a nominal basis.

Income Statement

Now let us examine the income statement. The premium line (line 1) contains the entire premium of \$296.71. The shaded lines show that this total is comprised of an expected loss portion and a narrow risk margin. In the expected case, actual losses of \$300.00 are incurred and paid halfway through the year (line 2). The company earns investment income of \$12.70 during the year (line 8). The shaded lines show that this total is comprised of two amounts, 4.08 and 8.61. The first value arises from the 6% rate applied to the narrow risk margin and surplus amounts, which are held for the full year $((62.71 + 5.32) \times .06 = 4.08)$. The second arises from investment income on assets supporting loss reserves.

These assets are only held for six months before the loss is paid, so the asset amount, \$291.39, earns 6% for six months or \$8.61. Note that the \$8.61, added to the \$291.39 precisely provides enough money to pay the \$300.00 loss. The net income (line 16) is the sum of these values, or \$9.41 (line 16).

Capital and Surplus Account

The remaining part of the exhibit shows the reconciliation of the surplus account. It starts at \$62.71 (line 17) at the previous year-end. The net income of \$9.41 (line 18) is an addition. In this example, the policy is now completed so the company can return the surplus to the investor. This is shown as a negative paid-in amount of \$62.71 (line 25(a)). At the end of the year, we would issue a stockholder dividend to the investor in the amount of the net income of \$9.41 (line 27). The shaded lines show that this total is comprised of the \$4.08 investment income earned on the surplus and narrow risk margin during the year, and the \$5.32 provided by the policyholder in the form of a narrow risk margin. If we then calculate the ratio of the net income to the surplus at risk, we find that the investor has earned a 15% return on the investment. This exhibit should help provide an understanding of the rationale behind the formula for determining the narrow risk margin. The investor can earn a “safe” return of 6% in Treasury bills. Instead the investor risks the surplus in an insurance enterprise with the expectation of a higher reward. The required return is 15% on the surplus. The surplus can be invested at rate i , so the remaining requirement (ROR- i) must be provided by the policyholder.

At the end of the year, all losses are paid and all surplus returned to the investor, so that balance sheet contains all zeros.

Of course, actual loss experience might be less than or greater than the expected loss experience. The excess amount, if actual losses are less than expected, will result in a higher rate of return. The short-fall, if actual losses exceed expected, will result in a reduced return or a need to use surplus to pay losses. The risk that the latter may occur is, of course, the reason that the investor can expect a return in excess of a risk-free rate.

While this example illustrates some of the dynamics of the situation, note that the lines for change in risk margin (line 0501) and change in reserve for discount (line 0502) contain zeros. This is correct in this example, because the reserves for these items are zero before the policy is written and zero after the last loss is paid, so the year-end changes are zero. We will next examine a policy with payments over a three-year period so we can better understand the dynamics of these items.

THREE PERIOD MODEL

In this example, assume a single policy is written on 1/1/X. The expected nominal losses are \$600.00. The expected payout of these losses is \$300.00, \$200.00, and \$100.00, with payments taking place at the middle of years X, X+1, and X+2. The expected losses, discounted at 6%, are \$561.09.

Again assume that the actual nominal losses can be described by a lognormal model with mean of \$561.09 and CV of .128. We will make the simplifying

assumption that the overall amount is variable, but the timing of the payments is fixed. As calculated before, the total assets required is found by multiplying the expected losses by 1.233. This produces:

$$\textit{Total Asset Need} = \$561.09 \times 1.233 = \$692.09$$

Arguably, not all of these assets are needed presently because some assets are needed for payments in the future. It is unrealistic, however, to assume that regulators would permit surplus to be promised at some future date. We will assume that all surplus must be supplied prior to the policy inception date.

As before, we expect the policyholder to pay a premium consisting of the expected losses \$561.09 plus the NRM, while the investor will supply the SRM. The total asset need less the expected losses produces the sum of the NRM and SRM which we call the BRM:

$$\textit{BRM} = \$692.09 - \$561.09 = \$131.00$$

The calculation of the apportionment between NRM and SRM is now more complicated. The investor must commit surplus, not simply for the upcoming twelve month period, but over the entire life of the policy, which extends until the last loss is paid. Similarly, regulators and policyholders require that assets over and above expected losses need to remain in the company until the last loss is paid. Denote L_t to be the expected unpaid losses at time t . The policy inception date will correspond to $t=0$, but subsequent values of t will correspond to year-end points in time. Make the

simplifying assumption that the same aggregate loss distribution can be used to describe the unpaid losses at any point in time¹⁴. Then the total asset need at any time will be calculated the same way as at the beginning of the policy period:

$$\text{Asset need at time } t = 1.233 \times L_t$$

For simplicity we will assume that the evaluation of asset need does not take place continuously, but at year-end points in time. For each time t , we can also calculate a value we denote BRM_t , which will be the total asset need, less the expected losses:

$$BRM_t = (1.233 \times L_t) - L_t = .233L_t$$

Now we have a sequence of future BRM_t values, each of which can be decomposed into an NRM_t and SRM_t . The formula for calculating the initial, as well as all subsequent NRM_t is:

$$NRM_t = (ROR - i) \sum \frac{BRM_t}{(1 + ROR)^t}$$

In words, the future BRM_t values are discounted at rate ROR. The total of the discounted values is multiplied by $(ROR - i)$, reflecting the fact that the BRM_t will earn investment income at rate i , and the policyholder must supply the remaining amount in order to pay the investor the required rate of return.

¹⁴This assumption will be discussed further in a later section.

Applying these formulas produces the following values:

$$\text{BRM}_0 = .233$$

$$L_0 = \$131.00$$

$$\text{NRM}_0 = \$16.14$$

$$\text{SRM}_0 = \$114.87$$

Now let us examine the financial statements corresponding to this policy. As before, two exhibits are provided. Exhibit 3 contains the proposed accounting. Exhibit 4 contains the identical information, as well as supplemental information in the shaded lines.

Balance Sheet (Year X)

On the day before the policy is issued, (12/31/X-1) the required surplus of \$114.87 is supplied (Exhibit 4 Line 1). On the following day, the total premium is paid, consisting of expected losses of \$561.09 and narrow risk margin of \$16.14, for a total premium of \$577.23. When added to the surplus, this produces total assets on 1/1/X of \$692.09. On that same day, a UPR reserve is established in the amount of \$561.09 (line 9) and a risk margin reserve of \$16.14 is established (line 2102).

During the year, a \$300.00 loss is paid. By year-end, the premium is earned, so the UPR reserve drops to zero. The remaining unpaid losses, on an undiscounted basis are

Three-Period Model

Exhibit 3

An St	Balance Sheet	At 12/31/X-1	At 1/1/X	At 12/31/X	At 12/31/X+1	At 12/31/X+2
Line #'s	Assets					
1.	T-Bills	\$114.87	\$692.09	\$352.64	\$119.81	\$0.00
<hr/>						
	Liabilities					
1.	Undiscounted Loss Reserve		\$0.00	\$300.00	\$100.00	\$0.00
9.	UPR	0	\$561.09	\$0.00	\$0.00	\$0.00
2101.	Reserve for Discount		\$0.00	(\$14.11)	(\$2.87)	\$0.00
2101a.	Discounted Loss Reserve	0	\$0.00	\$285.89	\$97.13	\$0.00
2102.	Risk Margin	0	\$16.14	\$6.77	\$1.77	\$0.00
22.	Total Liabilities	0	\$577.23	\$292.65	\$98.90	\$0.00
26.	Surplus	\$114.87	\$114.87	\$59.98	\$20.90	\$0.00

Statement of Income		Year X	Year X + 1	Year X + 2
1.	Premium	\$577.23	\$0.00	\$0.00
2.	Undiscounted Incurred Loss	(\$800.00)	\$0.00	\$0.00
8.	Investment Income	\$32.66	\$15.25	\$4.23
0501.	Change in Risk Margin	(\$6.77)	\$4.99	\$1.77
0502.	Change in Reserve for Discount	\$14.11	(\$11.24)	(\$2.87)
16.	Net Income	\$17.23	\$9.00	\$3.14
Capital and Surplus Account				
17.	Surplus, Dec. 31 previous year	\$114.87	\$59.98	\$20.90
Gains And (Losses) in Surplus				
18.	Net Income	\$17.23	\$9.00	\$3.14
25(a).	Surplus Adjustments: Paid in	(\$54.88)	(\$39.08)	(\$20.90)
27.	Investor Dividend	(\$17.23)	(\$9.00)	(\$3.14)
31.	Changes in Surplus for the year	(\$54.88)	(\$39.08)	(\$20.90)
32.	Surplus, Dec. 31 current year	\$59.98	\$20.90	\$0.00

NEWEX.XLS

Three-Period Model

Exhibit 4

With Supplemental Information

An St	Balance Sheet	At 12/31/X-1	At 1/1/X	At 12/31/X	At 12/31/X+1	At 12/31/X+2
Line #'s	Assets					
1.	<i>T-Bills</i>	\$114.87	\$692.09	\$352.64	\$119.81	\$0.00
Liabilities						
1.	<i>Undiscounted Loss Reserve</i>		\$0.00	\$300.00	\$100.00	\$0.00
9.	<i>UPR</i>	0	\$561.09	\$0.00	\$0.00	\$0.00
2101.	<i>Reserve for Discount</i>		\$0.00	(\$14.11)	(\$2.87)	\$0.00
2101a.	<i>Discounted Loss Reserve</i>	0	\$0.00	\$285.89	\$97.13	\$0.00
2102.	<i>Risk Margin</i>	0	\$16.14	\$6.77	\$1.77	\$0.00
22.	<i>Total Liabilities</i>	0	\$577.23	\$292.65	\$98.90	\$0.00
26.	<i>Surplus</i>	\$114.87	\$114.87	\$59.98	\$20.90	\$0.00

Statement of Income		Year X	Year X+1	Year X+2
	<i>Loss Portion of Premium</i>	\$661.09		
	<i>Risk Margin Portion of Premium</i>	\$18.14		
1.	<i>Premium</i>	\$577.23	\$0.00	\$0.00
2.	<i>Undiscounted Incurred Loss</i>	(\$600.00)	\$0.00	\$0.00
	<i>Inv. Inc. (Discounted Loss Reserve)</i>	\$24.85	\$11.24	\$2.87
	<i>Inv. Inc. (Risk Margin and Surplus)</i>	\$7.66	\$4.00	\$1.38
8.	<i>Investment Income</i>	\$32.66	\$15.25	\$4.23
0501.	<i>Change in Risk Margin</i>	(\$6.77)	\$4.99	\$1.77
0502.	<i>Change in Reserve for Discount</i>	\$14.11	(\$11.24)	(\$2.87)
16.	<i>Net Income</i>	\$17.23	\$9.00	\$3.14
Capital and Surplus Account				
17.	<i>Surplus, Dec. 31 previous year</i>	\$114.87	\$59.98	\$20.90
Gains And (Losses) in Surplus				
18.	<i>Net Income</i>	\$17.23	\$9.00	\$3.14
25(a).	<i>Surplus Adjustments: Paid in</i>	(\$54.88)	(\$39.08)	(\$20.90)
	<i>Market Risk Margin Component</i>	\$5.37	\$4.99	\$1.77
	<i>Inv. Inc. Component</i>	\$7.56	\$4.00	\$1.38
27.	<i>Investor Dividend</i>	(\$17.23)	(\$9.00)	(\$3.14)
31.	<i>Changes in Surplus for the year</i>	(\$54.88)	(\$39.08)	(\$20.90)
32.	<i>Surplus, Dec. 31 current year</i>	\$59.98	\$20.90	\$0.00
	<i>Investor Return (line 18/line 17)</i>	15.00%	15.00%	15.00%

NEWEX.XLS

\$300.00 (line 1). The present value of these losses at year-end is \$285.89, so the difference between the discounted and undiscounted amounts is established as a reserve for discount of \$ -14.11 (line 2101). The discounted loss reserve is shown as the sum of line 9 and line 2101 in line 2101a.

The required year-end risk margin and surplus are calculated as before—the expected losses are now \$285.89, so the BRM_1 is:

$$BRM_1 = .233 \times \$285.89 = \$66.75$$

$$NRM_1 = \$6.77$$

$$SRM_1 = \$59.98$$

Thus, we need to maintain \$6.77 in the risk margin reserve and \$59.98 in the surplus account.

Income Statement (Year X)

Looking at the income statement (Exhibit 4), we see that income includes a premium of \$577.23 (line 1) comprised of the loss portion of the premium \$561.09, plus the risk margin portion, \$16.14. Incurred losses are shown on an discounted basis, and are equal to the total of payments made during the year, \$300.00 and all expected future payments. This results in a total incurred loss amount of \$600.00 (line 2). The company earns investment income of \$32.66 during the year (line 8). This is comprised of two items, the amount earned on the assets underlying the loss reserves and the

investment income earned on the risk margin and surplus. These two amounts are \$24.80 and \$7.86 (shaded lines).

In our previous example, the investment income associated with loss reserves added to the loss reserve precisely paid off the loss payment. In this example, the situation is slightly more complicated. The total discounted losses at policy inception of \$561.09 is comprised of two components: \$291.39 for the expected payment of \$300.00 in the first year, and \$269.70 for subsequent loss payments. Consequently, \$8.61 of the investment income is combined with the \$291.39 to make the midyear loss payment of \$300.00. The remaining investment income, $\$24.80 - \$8.61 = \$16.19$ is added to the \$269.70 value to produce the needed loss reserve of \$285.89 (line 2101a).

As a check, either confirm that \$285.89 represents the correct expected value of the remaining two payments, or note that the original discounted amount of \$269.70 should be multiplied by 1.06 to bring it forward one year.

The change in risk margin is \$-6.77 (line 0501). This amount may appear puzzling at first. It is important to note the entire effect of the risk margin on the income statement. The entire risk margin of \$16.14 is included in the income because it is a part of the earned premium (see shaded line above line 1). The \$-6.77 ensures that not all of the original risk margin is included in income in this year. The total of these two

items, $\$16.14 - \$6.77 = \$9.37$, represents the portion of the total risk margin that is included in the first year's income.¹⁵

The change in reserve for discount is \$14.11 (line 0502), representing the fact that the prior year-end reserve was zero, and the required reserve of \$-14.11 (line 2101) must be established. The sum of all the income items produces the net income for the year of \$17.23 (line 16).

Capital and Surplus Account (Year X)

The previous year-end surplus \$114.87 (Exhibit 4, line 17) is carried down from line 26 of the balance sheet. To this amount we will add the net income for the year, calculated above to be \$17.23 (line 16 as well as line 18). Now that some of the losses are paid, the remaining obligations require less total assets and less surplus to support possible fluctuation in actual payments. The new surplus requirement (SRM_t) is \$59.98. Thus, the difference between the surplus on hand, and the required surplus, is returned to the investor as a negative paid-in surplus amount¹⁶. This amount, $\$114.87 - \$59.98 = \$54.88$, is shown on line 25(a).

¹⁵If we had assumed that the policy has an effective date of 12/31/X-1, the individual line entries would be different, although the total effect would be the same. This effective date of 1/1 was deliberately chosen to make sure the accounting treatment for this situation would be understood. The subsequent years treatment should be reviewed for understanding.

¹⁶In practice, the investor would leave the surplus in to support a *new* policy written in the second year.

We also paid the entire net income amount out to the investor as a stock dividend¹⁷ (line 27). The shaded lines show that this dividend is comprised of the portion of the narrow risk margin that has been taken into income \$9.37, and the investment income earned during the year on the surplus and risk margin, \$7.86.

The bottom shaded line shows that the investor return, \$17.23, divided by the surplus at risk during the year, \$114.87, produces a return on surplus of 15%.

Balance Sheet (Year X+1)

In the second year, the loss payment of \$200.00 is made, leaving an undiscounted loss reserve equal to the single remaining payment of \$100.00 (Liabilities, line 1). The present value of this payment of \$97.13 (line 2101a) leaving a reserve for discount of \$-2.87 (line 2101). The total required assets are calculated as before:

$$\text{Required Assets} = 1.233 \times \$97.13 = \$119.81$$

The BRM_1 is calculated as before:

$$BRM_1 = .233 \times \$97.13 = \$22.68$$

which is apportioned into its components:

$$NRM_1 = \$1.77$$

$$SRM_1 = \$20.90$$

Thus, the risk margin on the liability side becomes \$1.77 (line 2101) and the surplus amount becomes \$20.90 (line 26).

¹⁷In practice, this amount, or a portion of this amount would be left in the company as retained earnings, to support growth in premium writings.

Income Statement (Year X+1)

No premium is received in this year, and our estimate of ultimate losses for prior years does not change, so the incurred losses are zero. Our assets earn a total of \$15.25 during the year (line 8). This is comprised of \$11.24 earned on assets supporting loss reserves and \$4.00 earned on the surplus and risk margin (shaded lines). Once the losses are paid, we will need a smaller broad risk margin, hence a smaller narrow risk margin and a smaller surplus risk margin. Consequently, we can release \$4.99 of the risk margin amount into income (line 0501). This is calculated as the difference between the required risk margin at the prior year-end, \$6.66 and the amount needed at the current year-end, \$1.77.

Similarly, we have to charge against income the need to increase the reserve for discount from \$-14.11 to \$-2.87. This increase in reserves of \$11.24 is a charge to income of \$-11.24 (line 0502). The net income for the year is the total of these items, or \$9.00 (line 16)

Finally, we reconcile the surplus account. It started with \$59.98 (line 17). We earned \$9.00 in net income (line 18), returned the surplus not needed for the next year's operations, \$-39.08 (line 25(a)), and paid an investor dividend equal to total net income of \$9.00 (line 27). This amount is comprised of the \$4.99 of the original narrow risk margin released into earnings this year plus the investment income on the remaining risk margin and surplus (shaded lines above line 27). The sum of these

changes is the total change in surplus for the year, \$-39.08 (line 31), yielding the year-end surplus of \$20.90 (line 37), which is precisely the amount needed to support the remaining loss reserves. The investor dividend of \$9.00 divided by the surplus throughout the year of \$59.98, provides the investor with a return on invested surplus of 15%.

Balance Sheet (Year X+2)

In the third year, the final loss is paid, so at the end of the year, there is no need for reserves or surplus, and therefore no need for assets. Consequently, all values in the balance sheet are zero.

Income Statement (Year X+2)

There is no premium in this year. Loss payments exactly equal as expected, so incurred losses are zero. Investment income of \$4.23 is earned during the year (line 8). This is comprised of the \$2.87 earned on the assets supporting loss reserves of \$97.13, and the \$1.36 earned on the risk margin and surplus (Exhibit 4, shaded lines above line 8). Note that the sum of the investment income on loss reserve assets plus the reserve of \$97.13 provides enough money to pay the loss of \$100.00.

The required risk margin at the end of the year is zero, so the change in the risk margin of \$1.77 flows into income (line 0501). The \$-2.87 corresponds to the fact that

the reserve for discount becomes zero (line 0502). The total of the income items is the net income for the year of \$3.14 (line 16).

Finally, we reconcile the surplus account. The beginning surplus is \$20.90 (line 17). Net income of \$3.14 is added (line 18). All remaining surplus can be returned to the investor, \$-20.90 (line 25(a)), and we pay a stock holder dividend to the investor equal to the net income (line 27). This dividend is comprised of the release of the remaining portion of the narrow risk margin \$1.77, and the investment income earned during the year on the risk margin and surplus, \$1.36 (shaded lines above line 27). The total dividend of \$3.14, divided by the surplus invested throughout the year \$20.90, provides the investor with a 15% return.

STEADY STATE

In this example, a policy is written every year. Each policy will be identical to the policy in the three period model. The nominal losses will be \$600.00, paid out over three calendar years with \$300.00 in the current year, \$200.00 in the second year, and 100.00 in the third year. Because there are no losses outstanding after the third year, any point in time in the steady state model needs only to consider the current year, the prior year and the second prior year.

The results are summarized in Exhibit 5, (This exhibit is laid out in the same format as Exhibits 2 and 4.)

Three-Period Model

Exhibit 5

Steady State

An St		Balance Sheet		At 12/31/X-1	At 1/1/X	At 12/31/X	At 12/31/X+1	At 12/31/X+2
Line #'s		Assets						
1.		<i>T-Bills</i>		\$603.45	\$1,164.54	\$603.45	\$603.45	\$603.45
Liabilities								
1.		<i>Undiscounted Loss Reserve</i>		\$400.00	\$400.00	\$400.00	\$400.00	\$400.00
9.		<i>UPR</i>		0	\$561.09	\$0.00	\$0.00	\$0.00
2101.		<i>Reserve for Discount</i>		(\$16.98)	(\$16.98)	(\$16.98)	(\$16.98)	(\$16.98)
2101a.		<i>Discounted Loss Reserve</i>		\$383.02	\$383.02	\$383.02	\$383.02	\$383.02
2102.		<i>Risk Margin</i>		\$24.68	\$24.68	\$24.68	\$24.68	\$24.68
22.		<i>Total Liabilities</i>		\$407.70	\$968.79	\$407.70	\$407.70	\$407.70
26.		<i>Surplus</i>		\$195.75	\$195.75	\$195.75	\$195.75	\$195.75

		Statement of Income		Year X	Year X + 1	Year X + 2
		<i>Loss Portion of Premium</i>		\$581.09	\$581.09	\$581.09
		<i>Risk Margin Portion of Premium</i>		\$16.14	\$16.14	\$16.14
1.		<i>Premium</i>		\$577.23	\$577.23	\$577.23
2.		<i>Undiscounted Incurred Loss</i>		(\$600.00)	(\$600.00)	(\$600.00)
		<i>Inv. Inc. (Discounted Loss Reserves)</i>		\$38.91	\$38.91	\$38.91
		<i>Inv. Inc. (Risk Margin and Surplus)</i>		\$13.23	\$13.23	\$13.23
8.		<i>Investment Income</i>		\$52.13	\$52.13	\$52.13
0501.		<i>Change in Risk Margin</i>		\$0.00	\$0.00	\$0.00
0502.		<i>Change in Reserve for Discount</i>		\$0.00	\$0.00	\$0.00
16.		<i>Net Income</i>		\$29.36	\$29.36	\$29.36
		Capital and Surplus Account				
17.		<i>Surplus, Dec. 31 previous year</i>		\$195.75	\$195.75	\$195.75
		Gains And (Losses) in Surplus				
18.		<i>Net Income</i>		\$29.36	\$29.36	\$29.36
25(a).		<i>Surplus Adjustments: Paid in</i>		\$0.00	\$0.00	\$0.00
		<i>Narrow Risk Margin Component</i>		\$16.14	\$16.14	\$16.14
		<i>Inv. Inc. Component</i>		\$13.23	\$13.23	\$13.23
27.		<i>Investor Dividend</i>		(\$29.36)	(\$29.36)	(\$29.36)
31.		<i>Changes in Surplus for the year</i>		\$0.00	\$0.00	\$0.00
32.		<i>Surplus, Dec. 31 current year</i>		\$195.75	\$195.75	\$195.75
		<i>Investor Return: (line 18/line17)</i>		15.00%	15.00%	15.00%

NEWEX.XLS

Most of the values in the exhibits can be calculated as the sum of the appropriate amounts in Exhibit 4 associated with each of the three years. For example, the undiscounted loss reserve is \$400.00 at each year-end. This is the sum of the unpaid amount of \$300.00 on the previous year's policy, and the remaining unpaid of \$100.00 on the second prior year's policy. (The current policy does not contribute to loss reserves, but rather to the UPR account.) The narrow risk margin is \$24.68 at each year end, corresponding to the sum of the three narrow risk margins in Exhibit 4.

Note that surplus adjustments (line 25(a)) are always zero. On December 31, there is a potential for a return of surplus, arising from the fact that losses have been paid during the year and therefore do not require surplus to support the loss reserves. However, a new policy will be written the following day, which will require surplus. In the steady state model, this return of surplus and the required additional surplus exactly offset. Of course, a company which is growing would require either additional surplus paid in, or (more likely) an increase in surplus by paying out to the investor less than the profit earned in the year (i.e., retained surplus).

ADVERSE RESULTS

To this point, the assumption has been made that actual loss emergence follows expected loss emergence. If actual experience always matched expected, there would be no need for risk margins (or, for that matter, insurance). It will be helpful to analyze what happens if actual experience deviated from the expected. This paper will only analyze adverse departures from expected. Positive deviations can, and do, occur but the proper accounting should be obvious from the following discussion. The discussion will focus on two types of deviation:

1. Actual payments not equal to expected payments
2. A change in the estimated outstanding reserve

Of course, these two types of changes often occur simultaneously, but they will be separated for the purposes of discussion.

Actual paid not equal to expected paid

This is the easiest type of deviation to handle. In the three-period model (Exhibit 4), expected payments are \$300, \$200, and \$100 for the three calendar periods. If

actual paid losses differ from that amount, but *this does not affect the estimate of outstanding*, the only adjustments are that:

- Actual incurred (line 2) will be different, which will flow dollar for dollar through to
- Net Income (line 16), which will flow through to
- Investor Dividend (line 27) and will change the
- Investor Return (bottom line)

If the adverse results are large enough, i.e., greater than income, then it would require a surplus contribution (line 25(a)).

Risk margins are unaffected whenever the loss reserves are unchanged. It might seem reasonable that investment income should be affected, but note that assets are not reduced because investor dividends are reduced by the excess payments dollar for dollar. (Technically, there is a small impact, if loss payments are made mid-year and investor dividends and surplus replenishments are made at year-end.)

Change in estimated outstanding reserves

The more complicated situation occurs when there is a change in the estimate of outstanding reserves¹⁸. As noted above, the expected payments for the three-period model are \$300, \$200, and \$100. Let us suppose that, sometime during the second year, we determine that the best estimate of third year payments is \$200, rather than \$100. Exhibit 6 summarizes how this change flows through accounting statements. A comparison of Exhibits 4 and 6 will reveal that some items are different. Each entry in Exhibit 6 which differs from its counterpart in Exhibit 4 has a box around it.

Not surprisingly, there are no changes in any of the values up through 12/31/X. During year X + 1, the determination is made that nominal loss reserves should be set at \$200. This creates a change in the Undiscounted Loss Reserve (line 1). The Reserve for Discount and Discounted Loss reserves (line 2101 and 2101a) are similarly affected. The formula for risk margin is applied to the revised amounts. In this simplified situation (only one remaining payment), the required risk margin (line 2102) increases directly in proportion to the loss reserve change. The required surplus (line 26) is also increased. Changes in the loss reserve and required risk margin also change the total liability entry (line 22)).

¹⁸ Of course, this does not include reductions in reserves due to the payment of losses originally as expected, or reserve changes arising from a change in discount. It only covers changes due to a change in future expected payments.

Three-Period Adverse Model

Exhibit 6

With Supplemental Information

An St	Balance Sheet	At 12/31/X-1	At 1/1/X	At 12/31/X	At 12/31/X+1	At 12/31/X+2
Line #'s	Assets					
1.	<i>T-Bills</i>	\$114.87	\$692.09	\$352.64	\$239.61	\$0.00
Liabilities						
1.	<i>Undiscounted Loss Reserve</i>		\$0.00	\$300.00	\$200.00	\$0.00
9.	<i>UPR</i>	0	\$561.09	\$0.00	\$0.00	\$0.00
2101.	<i>Reserve for Discount</i>	\$0.00	\$0.00	(\$14.11)	(\$6.74)	\$0.00
2101a.	<i>Discounted Loss Reserve</i>	0	\$0.00	\$285.89	\$194.26	\$0.00
2102.	<i>Risk Margin</i>	0	\$16.14	\$6.77	\$3.55	\$0.00
22.	<i>Total Liabilities</i>	0	\$577.23	\$292.65	\$197.81	\$0.00
26.	<i>Surplus</i>	\$114.87	\$114.87	\$59.98	\$41.81	\$0.00

Statement of Income		Year X	Year X+1	Year X+2
	<i>Loss Portion of Premium</i>	\$861.09		
	<i>Risk Margin Portion of Premium</i>	\$16.14		
1.	<i>Premium</i>	\$577.23	\$0.00	\$0.00
2.	<i>Undiscounted Incurred Loss</i>	(\$600.00)	(\$100.00)	\$0.00
	<i>Inv. Inc. (Discounted Loss Reserves)</i>	\$24.80	\$11.24	\$6.76
	<i>Inv. Inc. (Risk Margin and Surplus)</i>	\$7.86	\$4.00	\$2.72
8.	<i>Investment Income</i>	\$32.66	\$15.25	\$8.48
0501.	<i>Change in Risk Margin</i>	(\$6.77)	\$3.22	\$3.55
0502.	<i>Change in Reserve for Discount</i>	\$14.11	(\$8.37)	(\$5.74)
16.	<i>Net Income</i>	\$17.23	(\$89.91)	\$6.27
Capital and Surplus Account				
17.	<i>Surplus, Dec. 31 previous year</i>	\$114.87	\$59.98	\$41.81
Gains And (Losses) in Surplus				
18.	<i>Net Income</i>	\$17.23	(\$89.91)	\$6.27
25(a).	<i>Surplus Adjustments: Paid in</i>	(\$54.88)	\$71.73	(\$41.81)
	<i>Narrow Risk Margin Component</i>	\$9.37	\$3.22	\$3.55
	<i>Inv. Inc. Component</i>	\$7.86	\$4.00	\$2.72
27.	<i>Investor Dividend</i>	(\$17.23)	\$0.00	(\$6.27)
31.	<i>Changes in Surplus for the year</i>	(\$54.88)	(\$18.18)	(\$41.81)
32.	<i>Surplus, Dec. 31 current year</i>	\$59.98	\$41.81	\$0.00
<i>Investor Return (line 18/line 17)</i>		15.00%	-149.89%	15.00%

NEWEX.XLS

Note carefully that we can simply “change” our liabilities by calculating new values. However, simply calculating a new required surplus amount, which also affects the required asset amount (line 1) does not, by itself, change the surplus or assets. The other required changes will be discussed after the income statement changes.

The undiscounted incurred loss (line 2) is changed by the \$100 change in reserve estimate. Investment income *during this year* is unaffected, because we won't change any cash items prior to year end. A change in risk margin still flows into income, but the amount is less than in Exhibit 4. (We only get credit for a reduction in the nominal reserve from \$300 to \$200, rather than the anticipated change from \$300 to \$100). Similarly, the reserve for discount account is increased, but by a smaller amount.

Net income is now a loss of \$89.91, compared to a gain of \$9.00 in Exhibit 4. Now we inform our investor that we will be unable to pay a dividend this year (line 27). In fact, we do not have enough surplus to support our liabilities, so we need a surplus infusions of \$71.73 (line 25(a)).

The investor has earned a negative return in this year of almost -150% (bottom line). Note that, in the subsequent year, investment income amounts are higher, because the assets are higher. The investor earns exactly 15% on surplus in the final year, although this policy represents an overall loss.

Practical Considerations

Although the methodology for the calculation of risk margins outlined in this paper is straight forward, there are several practical reasons for choosing computationally simple algorithms. The most complex step of the process involves the determination of the aggregate distribution of losses and the solution of an equation which produces the BRM value. It is likely that a company would not determine an aggregate distribution for its entire portfolio of business directly, but would determine distributions for individual blocks or lines of business. It is reasonable to assume that aggregate distributions for a particular line of business do not change materially in a short period of time. Thus, it may be reasonable to determine an aggregate distribution for a particular line of business as a one time project and use the resulting factors for some period of years. Moreover, it may also be reasonable to assume that aggregate distributions for a particular line of business do not vary materially among companies writing similar business. Thus it may be possible for an industry wide effort to generate aggregate distributions for lines of business with individual companies either using these results or making adjustments based on their own specific characteristics.

It might be necessary to calculate different factors for different ages. However, I believe that we will find that the ratio of BRM values to expected losses does not vary materially by age. If so, then the same factor can be used for any year of outstanding, and in particular, for the entire block of outstanding losses combined. Alternatively, we may find that the factors vary over the first one or two years and then stabilize. In

which case, we would have separate factors for the first year or two, and then an all-subsequent factor. A bit more problematic is the fact that aggregate distributions are likely to vary considerably based on the retentions kept by the company. It may be possible to specify an aggregate distribution applying to unlimited losses and an algorithm to adjust this distribution to various underlying retention and limit profiles. It will take some additional research to determine how robust these factors are with respect to the characteristics of the various lines of business, but it may be possible to generate a table of factors which could be applied to the outstanding reserves of a particular company.

Not only would this approach simplify the calculations of risk margins of individual companies, it would also allow/facilitate a comparison of risk margins among companies. If Company A has approximately the same amount of business as Company B, but has a higher risk margin on its financial statements, this may be due to the fact that Company A writes lines of business that require a higher risk margin. A better comparison would be to look at the specific factor that Company A uses for its auto liability outstanding reserves vs. the comparable factor for Company B.

GAAP ACCOUNTING

The discussion to this point has concentrated on the appropriate statutory accounting for risk margins. An obvious question is how GAAP accounting should incorporate risk margins.

With minor exceptions, the proposals for the statutory treatment of risk margins should apply equally to the appropriate GAAP treatment. This conclusion arises from the fact that the derivation of the proposed statutory accounting treatment was motivated by the GAAP accounting principle of matching revenues and expenses. GAAP theory suggests that the value of uncertainty, i.e. risk margins, should be incorporated into financial statements.

The fact that GAAP does not currently incorporate risk margins or discounting into property-casualty financial statements does not arise from theoretical but, rather, practical considerations. There are a number of practical issues, including calculation methodologies, which need to be resolved before the accounting profession decides to incorporate both elements into GAAP accounting.

The main difference between GAAP and statutory accounting with respect to risk margins is likely to be related to the handling of discounting. This paper suggests that statutory accounting should continue to deal with loss reserves on an undiscounted basis with a contra liability line reflecting the amount of discount. The reason for this is to preserve the existence of undiscounted liabilities in order to best track runoff of liabilities on a basis consistent with history. This amount will also allow continued calculation of traditional formulas and ratios such as are incorporated in the IRIS tests. GAAP accountants may conclude that it would be preferable to show the discounted liabilities as a single line item, with undiscounted liabilities disclosed in the notes to the financial statements, if these amounts are needed.

Alternative Treatment of the Risk Margin

The proposed treatment of the loss reserve risk margin is based upon several assumptions believed to be reasonable. However, it is possible to put forth arguments for a different set of assumptions, which might lead to different recommended accounting treatments. Alternatively, it is possible to accept the overall assumptions, yet reach a different conclusion regarding accounting treatment based upon criteria such as simplicity or ease of presentation. This section will explore some of these alternatives.

Earmarked Surplus

A key assumption is that an insured purchases an insurance policy to receive a service—the specific service being the elimination (or at least reduction) of the risk associated with the unknown losses covered by the policy. This paper argues that the performance of that service is not complete until the final loss is paid. Consequently, the earnings should be recognized over the same period. However, some may argue that the performance of the service is complete when the policy period has ended. Indeed, the concept of earned premium is consistent with that theory. Under this theory, the entire NRM should be included in income by the time the policy is fully earned.

But a different opinion on the time frame associated with the service provided does not change the fact that uncertainty in loss reserves exists after the end of the policy period. The NRM amount is still a necessary component of an adequate financial structure. Under this scenario, it would be a required component of surplus, rather than a liability item.

We have already determined that the SRM values belong in the surplus account. (In fact, we have presumed that they are identical to the surplus account. However, this paper does not treat other sources of risk such as asset risk. Inclusion of asset risk would require surplus amounts other than those supporting loss reserves.) If only the risk associated with loss reserves is included, then the NRM value and the SRM value can be recombined into the total BRM. This amount could be shown as an earmarked surplus item.

In summary, if one subscribes to the theory that the service under an insurance policy is complete by the end of the policy year, and agrees that discounting of reserves is appropriate, the following accounting treatment would follow:

- Start with the accounting treatment suggested in this paper, including the establishment of an undiscounted loss reserve and a contra-liability for discount (calculated at risk-free rates of return)

- Earn the NRM, along with the remainder of the premium, over the policy period

- Earmark the entire amount BRM_t within surplus, not just SRM_t , which recognizes that the asset needs are not reduced under this scenario. The NRM amounts are still required for financial adequacy, but they will be part of surplus rather than as a liability item.

Identification of an earmarked surplus amount may make sense in a scenario where risk-based capital concepts are incorporated into the financial statements. Such a treatment might include an earmarked surplus item for each category of risk-based surplus. In theory, the surplus account for an insurer would contain an earmarked amount for reserve risk and asset risk, with the remainder in a “free” surplus account. Presumably, regulators could put restrictions on the ability to pay dividends if the amounts would exceed the “free” surplus amounts.

Footnote or Alternative Schedule

The accounting treatment proposed in this paper would require only two items in a balance sheet - the total NRM value shown in the liability section and the SRM amount contained in the surplus account (optionally, this does not even need to be shown as a separate item). The alternative treatment discussed above only requires one item, an optional earmarked surplus account. It may be that regulators and other

users of financial statements may wish to have additional information. This additional information should be included as a footnote or as a separate schedule, depending on the complexity of the additional information desired. For example it may be desirable to show NRM and SRM values for each accident year. Such a schedule might show the current totals of the NRM and SRM values as the sum of individual components, as well as an historical registry of values at prior valuation periods. This schedule would be especially useful if the methodology used to calculate the values changed over time, or if critical parameters such as ROR changed over time.

Other categorizations may also be desirable:

- Ongoing versus runoff business
- Gross, net and ceded business
- Special situations, such as environmental coverage

Transition Issues

This paper has outlined a methodology to account for loss reserve risk margins. An obvious question is how to get from here to there. Two methods of conversion are possible. One method is a complete restatement of the annual statement at a particular year-end. It may be necessary to restate the beginning year balances so that income for the year is stated correctly. The difference between the prior year end balance sheet and the restated beginning year would be reported as an extraordinary gain (or loss) due to an accounting change.

Another method would be to phase in the new accounting rules prospectively. This could be done on an accident year or policy year basis, although accident year would be preferred. Presumably, after some period of time (five to ten years), the remaining prior years would also be restated to avoid carrying multiple accounting conventions forever.

There are advantages and disadvantages to both methods. The first method, complete conversion, is “cleaner”, but to the extent that some companies have understated their undiscounted liabilities by implicitly recognizing some time value of money, they would be forced to either book inadequate discounted reserves or admit openly that the undiscounted reserves were deficient. The phase-in approach would mitigate this problem, but would mean that annual statement accounting would be a

hybrid for years to come. More research into the advantages and disadvantages of these alternatives (as well as possible other approaches) is needed.

In addition to transition issues, there are other reasons why the actual implementation of the recommended procedures does not have to precisely follow the theory. This paper has outlined an accounting treatment consistent with a theoretically correct calculation of NRM and SRM values at each point in time. It is important to keep the calculation of these values in context. Once the initial (NRM) value is calculated, the subsequent calculations of NRM_t values dictates how the original amount is released into earnings. A different method for calculating $NRM_t, t \geq 1$ would only affect the timing of the release into earnings, not the absolute amount. As an analogy, consider the purchase of capital equipment. In general, the entire purchase price is not taken as an expense immediately. Instead, a depreciation schedule is established. The purpose of this schedule is not the depreciation amounts per se, but the time frame over which the expense is recognized. In theory, we might wish to create a schedule which shows how the equipment contributes to the corporation over its life time, and charge expenses on a proportional basis. Instead, an arbitrary lifetime is ascribed to the equipment, and the original purchase price is spread over this time period according to specified rules (e.g. pro-rata, double declining balance). For NRM values, we might decide to apply similar procedures. This would mean assigning periods of time to lines of business, and an associated schedule which might approximate a payout schedule. For example, we might assign a 10 year life to

general liability, and select an exponential pattern. This schedule would be applied to the original NRM values at all future points in time.

Alternatively, a study might be completed which calculates total NRM values under a variety of circumstances. It might be possible to analyze the results and conclude that selected factors applied to discounted reserves will approximate the theoretically correct amounts.

Non-constant Variability of Loss Reserves

One assumption used in this paper is the assumption that an aggregate loss distribution appropriate at the time of pricing is also appropriate for the unpaid losses at future points in time. Mathematically, let L_0 be a random variable which represents unpaid losses at time zero (i.e. the entire loss amount). Now consider the unpaid losses at some future time, L_t . Of course $E(L_t) < E(L_0)$, so the distributions cannot be identical. But if we assume that the unpaid loss amounts are scaled up to the same level as the original loss distribution, how would we expect the two distributions to differ? First, we might anticipate that some of the smaller, more predictable losses have been paid, leaving larger, potentially more variable O/S claim. With additional information we should anticipate a “tighter”, less variable distribution. In total, the unpaid loss distribution will probably differ from the original, but it may not be obvious that the riskiness is materially different.

More important, while we might find that the shape of the unpaid distribution is significantly different than the original distribution, the important question is whether the expected deficit, as a ratio to the expected, has changed materially.

If the answer to these questions is that the expected deficits of the unpaid loss distributions are not materially different than the original, then it may be acceptable to use a simplifying assumption that the same distribution, or at least the same critical ratio values can be used for all ages. Keeping in mind that the goal is simply to determine the timing of the release of the original NRM value, we might conclude that expected deficit values as a ratio to expected amounts could vary as much as the age.

For example, the critical value used to calculate BRM_0 is 1.233. Suppose the corresponding value for BRM_t is a sequence of values each greater than 1.233. The formula for calculating NRM_0 still works. It will produce an NRM value somewhat higher than the original problem. In words, higher critical ratios arising from more variability of loss reserves means that more surplus is needed at future points in time to support the loss reserves. In order to produce our target ROR, we will need a higher NRM_0 value. In other words, if the line of business is significantly more volatile over its entire life span than the original example, the company is subject to more risk and can command a larger price in the marketplace.

Note carefully that insureds with more risk do not automatically imply a higher required ROR. Instead, the higher risk may require a higher surplus commitment with

the same ROR. The target ROR will be related to the expected deficit, rather than to the risk of the individual insured.

SUMMARY

Current statutory and GAAP accounting rules performed adequately when investment income was modest and the length of time until loss payment was reasonably short. However, with high interest rates or longer payout patterns, current accounting rules provide a poor match between recognition of profit and the true economic reality of the insurance transaction. Specifically, current accounting rules artificially defer the recognition of profit.

A change in accounting rules to reflect loss reserves discounted at risk-free rates of return would over-correct. Specifically, this change would accelerate the recognition of profit (in the expected case) entirely into the initial policy year, despite the fact that risk exists with respect to loss reserves.

This paper outlined a procedure which provides a more appropriate balance between these two incorrect alternatives. It specifies a formula to determine how much of an overall asset requirement consistent with regulatory goals should be established as a narrow risk margin as a liability on the balance sheet. This formula provides for the proper release of the original profit margin into earnings over time.

This paper discussed specific methodologies for the calculation of margins (e.g., based on expected deficit). Other methodologies may exist or may be developed. More research is necessary to determine the proper choice of a methodology for the calculation of margins. Once that research is complete, an acceptable method for *calculating and accounting for risk margins* will exist.

It will be instructive to review the accounting treatment of the profit margin for a variety of assumptions. This discussion will trace how the timing of profit recognition differs under various assumptions, which will roughly correspond to changes in the insurance environment over time. This discussion will form a background and framework for a discussion of how accounting for loss reserve margins ought to be handled.

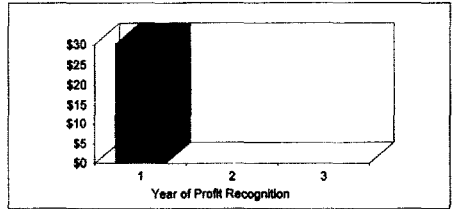
The first example considers a situation where investment income is earned at a rate of 3%. This forms a benchmark starting point. The following examples will also ignore expenses. While the treatment of expenses plays some part in the evolution of accounting rules, it is not particularly relevant to the issue of risk margins and therefore will not be discussed.

In Case 1, we assume that a policy is written with expected losses of \$600 and a premium of \$611.65. Interest is assumed to be 3%. Losses are paid at the end of the policy period.

In the first example, the premiums shown were calculated by selecting a paid amount of \$600.00, an expected profit of 5% (or \$30.00), and calculating the required premium. A \$30.00 profit at the end of the year represents a 4.9% return on premium.

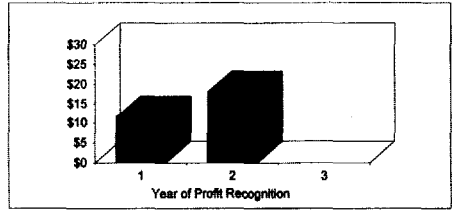
Case 1

Premium	\$811.65
Inv Inc.	\$18.35
Paid	\$600.00
Δ O/S	0.00
Incurred	\$800.00
Profit	\$30.00



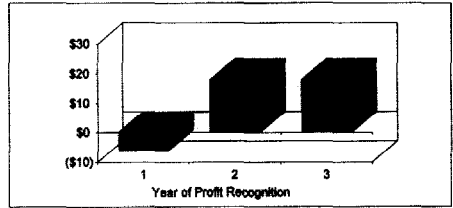
Case 2

Premium	\$593.84	\$0.00
Inv Inc.	\$17.82	\$18.00
Paid	\$0.00	\$600.00
Δ O/S	600.00	(600.00)
Incurred	\$800.00	\$0.00
Profit	\$11.65	\$18.00



Case 3

Premium	\$576.54	\$0.00	
Inv Inc.	\$17.30	\$18.00	\$18.00
Paid	\$0.00	\$0.00	\$600.00
Δ O/S	600.00	0.00	(800.00)
Incurred	\$800.00	\$0.00	\$0.00
Profit	(\$6.16)	\$18.00	\$18.00



All other examples were constructed such that the expected profit (discounted to the end of Year 1 where necessary) is 4.9% of premium.

The policy is written on January 1, premium is earned evenly over the next twelve months, and losses are paid at the end of the year. As a consequence, the net income (i.e., profit) is earned proportionately throughout the policy period. The company has entered into a risk-taking enterprise. The result of the insurance policy could be a no-claim situation or a situation with a large claim. This first example will assume that, once the loss occurs, the amount is known with certainty. The risk is taken during the policy year and no risk exists at the end of the policy period. In this example, the incidence of risk undertaken by the insurance company and the emergence of profit precisely track each other.

In case 2, we continue the 3% interest rate, but we now assume that the losses are paid out over a longer time frame. Specifically, losses are paid 24 months after they are incurred. This represents the evolution of the industry from predominately property coverage to an increased proportion of longer tailed liability coverage.

When we inspect the graphs that identify when the profit is recognized, we see that a large proportion of the profit is recognized during the policy year, but now a fairly significant amount of the profit is recognized subsequent to the end of the policy period. This fact should not be troubling. The reserves could be too high or too low at any point up to the final payment period. Thus, the insurance company which has

taken assumed risk from the insured for a fee finds itself with risk on its books until the last day of payment. It earns a profit over this period of time and statutory accounting has recognized profit over this entire period of time. It is possible to argue whether the exact heights of the bars in the graph precisely conform to the exact amount of profit that should be earned in each period, but we should all be in agreement that the general shape of the profit recognition curve conforms to what we think is the true economic reality of this company.

However, the last two or three decades have not conformed to assumptions such as this. Interest rates are higher and the average length of time to loss payout is longer. Let's relax these assumptions one at a time. In Case 3, we will assume that the loss payment period is lengthened to thirty-six months on average, but the interest rate remains at 3%. Under this scenario, the premium necessary to generate a reasonable profit will be set at \$576.54.

Now when we examine the graphs that show profit recognition, they begin to depart from our assumptions about the reality of the insurance transaction. Statutory accounting would generate a loss during the policy period followed by gains in the three subsequent years. While the overall profit is finally recognized, statutory accounting rules cause the profit to be earned much later than would conform to economic reality. We would believe that an insurance policy whose final loss payment does not occur for thirty-six months earns at least some of its profit in the two years after the policy period ends, but we would be hard pressed to argue that no profit,

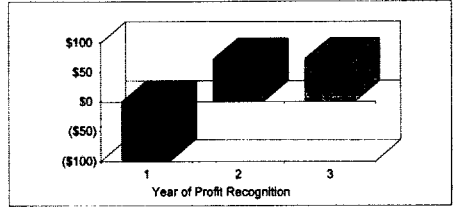
indeed even a loss, is incurred during the policy period. Obviously, losses can be incurred on particular policies that end up with losses for the entire period, but this example is intended to represent a situation that is profitable for its lifetime.

The situation becomes even more extreme if we increase the interest rate to 12%. Case 4 summarizes the situation where loss payments occur thirty-six months after occurrence, but interest is assumed at a 12% annual rate. This rate obviously exceeds rates available today, but is consistent with rates available in the early 80's. The policy generates significant losses during the policy period, which are offset by significant gains in the subsequent years. Of course, it should be recognized that overall calendar year results for an entire company would not look like this. Any particular calendar year would contain a mixture of new policies with a negative contribution to profit and older policies whose artificially high profits are running off. Eventually, a steady state situation might be reached, but in a situation where interest rates were growing and loss payment lags were increasing and the overall business volume was increasing, statutory accounting produced an understatement of income. Even if a steady state situation were reached, statutory accounting produces a poor match between reality and the accounting for any particular set of policies.

The insurance industry had a mixed reaction to this situation. On the one hand, there is a desire to report as high an earnings result as possible to satisfy the stockholders (in the case of stock companies) and to portray the financial strength of their companies to the policyholders, (in the case of mutual companies). On the other

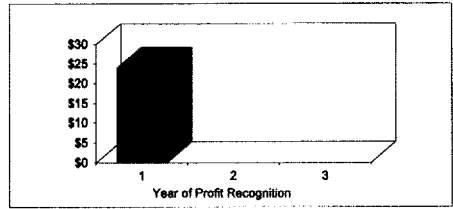
Case 4

Premium	\$448.42	\$0.00	
Inv Inc.	\$53.81	\$72.00	\$72.00
Paid	\$0.00	\$0.00	\$800.00
Δ O/S	600.00	0.00	(600.00)
Incurred	\$600.00	\$0.00	\$0.00
Profit	(\$97.77)	\$72.00	\$72.00



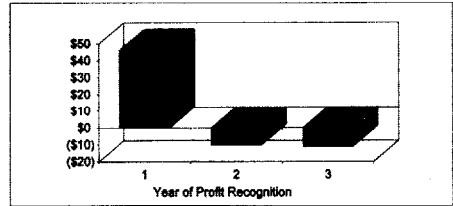
Case 6

Premium	\$448.42	\$0.00	
Inv Inc.	\$53.81	\$57.40	\$64.29
Paid	\$0.00	\$0.00	\$800.00
Δ O/S	478.32	57.40	(535.71)
Incurred	\$478.32	\$57.40	\$64.29
Profit	\$23.92	\$0.00	\$0.00



Case 6

Premium	\$528.96	\$0.00	
Inv Inc.	\$31.74	\$30.86	\$33.33
Paid	\$0.00	\$0.00	\$800.00
Δ O/S	514.40	41.15	(555.58)
Incurred	\$514.40	\$41.15	\$44.44
Profit	\$46.29	(\$10.29)	(\$11.11)



hand, a delayed recognition of profit meant a reduced, or at least deferred, tax bill to the extent that the Internal Revenue Service accepted statutory income as tax income. (Income for tax purposes was not identical to statutory income, but the magnitude of the other adjustments is rather small compared to the discounting issue.)

Not surprisingly, the Internal Revenue Service did not accept this situation quietly. For many years, the definition of income for tax purposes was virtually the same as statutory income. However, as the length of time to loss payment increased and interest rates increased, the use of nominal reserves artificially depressed statutory income to such a degree that the Internal Revenue Service decided to make changes. They decreed that reserves should be established on a discounted basis. (The precise formula for the discount is beyond the scope of this paper, but it was roughly based on T-bill rates.) The imposition of discounting dramatically changed the profit incidence. As Case 5 shows, under discounting almost all of the income for a policy is recognized in the first twelve months and only modest amounts of income are recognized in the following three years. While some might argue that the premium has been fully earned by the end of the policy period, it is certainly a fact that much uncertainty still remains at the end of the policy period and prior to the loss payments. If any one questions the existence of uncertainty, the following thought experiment should suffice.

Consider a company issuing a policy for \$448.47 with expected losses of \$600.00. This premium is set at a level which will generate a profit over its lifetime. If the

company wished to sell this policy, that is, sell off the loss reserves at the end of the premium earning period, would they be able to sell it at a rate such that they would recoup the entire profits? I submit that they would only be able to earn a modest amount of profit by that period and the premium for transferring the remaining uncertain loss liabilities would be sufficiently high that a significant amount of the original profit would need to be transferred to the reinsuring company.

Finally, we look at Case 6. This also follows the tax accounting rules, except it approximates the situation in place right now where the discount rate for tax purposes exceeds the amount of interest a company can earn on T-bills. (This arises because the discount rate used for IRS calculations is based on 60-month average rates, so when rates are dropping rapidly the average rate exceeds the current rate.) In this situation, we see that, while the overall profit of the contract is unchanged, the tax accounting rules actually “recognize” more than all of the profit by the end of the policy period and then recognize losses in subsequent periods. An insurance company is in the unenviable position of having an artificially high income calculated for the purposes of determining income tax, but artificially low statutory accounting income to report to stockholders.

At low interest rates with short-tail lines of business, the true incidence of profit and the incidence of profit arising under statutory accounting are approximately the same. As the length of time between occurrence of the claim and payment of the claim increases, the incidence of reported profit and the incidence of true profit (that is, the

price paid to transfer risk) remain approximately in sync. However, as the length of time gets fairly long, the comparison starts to get out of sync and at modestly high interest rates, statutory accounting rules generate a profit recognition picture which bears almost no resemblance to reality.

Tax accounting attempted to correct this distortion, but it has over-corrected. Discounting essentially forces the profit to be fully recognized during the policy period, despite the fact that an insurance company remains at risk substantially after the last dollar of premium is earned.

In summary, the failure to formally incorporate risk margins and recognition of discount produced only modest distortions between statutory recognition of profit and the true economic earning of profit when interest rates were low and most business was short-tailed. However, the situation is very different today. The recognition of profit under statutory and GAAP accounting is artificially deferred, and under tax accounting is artificially accelerated relative to the true economic incidence of profit. Simultaneous recognition of discounting and risk margins could correct these problems.

The theoretical justification for a loss reserve margin arises because the loss liabilities are uncertain. Economic values of assets are reduced when uncertainty is introduced; the economic value (or cost) of a liability is increased because of uncertainty. Before analyzing the uncertainty associated with loss reserves, it will be helpful to examine the treatment of uncertainty in the pricing situation. Then, by analogy, the reserving situation can be analyzed.

Consider an entrepreneur in the process of establishing a brand-new insurance company. Assume that the entrepreneur has identified a portfolio of insureds, whose losses can be described by a random variable z , with distribution $F(z)$. In order to obtain regulatory approval to start the company, as well as convince potential insureds to become customers, the company will have to show evidence of its ability to meet its financial obligations. This is normally done by starting the company with a certain amount of capital, C .

A ruin theory approach to solvency would require that the company have sufficient assets such that the probability of being unable to satisfy its obligation, i.e., probability of ruin is q , where q might be .01 or .005. Mathematically, we need to solve the following equation for A where A represents the total assets of the company (which, of course, equals the expected liabilities plus surplus):

$$\int_A^{\infty} dF(z) = q$$

Alternatively, we might specify an expected deficit requirement. Rather than simply being interested in the *probability* that a company is unable to meet some of its obligations, we may be more interested in the *cost* of insolvency. In this case, we want the sum of all losses in excess of total assets. An expected deficit requirement consistent with actual industry insolvency costs would be approximately .5% of premium or .3% of expected losses. Let d equal .003 times all expected unpaid losses.

Mathematically, we need to solve the following equation for A

$$\int_A^{\infty} (z - A) dF(z) = d$$

(For convenience, we will ignore operating expenses of the company and include LAE with loss.)

The company will charge a premium, P . At start-up, the company will have assets equal to the premium plus the start-up capital:

$$A = P + C$$

P is supplied by the policyholder and C is supplied by the stockholder. To determine these values, we need another equation. Note that the premium can be decomposed into two components — the expected losses, $E(z)$, and a risk margin, R (ignoring expenses). Thus:

$$P = E(z) + R$$

Substituting:

$$A = E(z) + R + C$$

In words, the total assets of the company include an amount necessary to cover the expected losses, plus an amount necessary to cover the possibility that losses exceed the expected amount. On average (or over the long term), the company will pay $E(z)$, leaving profits of R on capital of C . Thus:

$$\frac{R}{C} = \textit{return on capital}$$

We can presume that the market will determine the acceptable level of profit for a company with this level of risk. Call this amount ROE . In summary, the value ROE is fixed by the market, and the value A is fixed by the characteristics of the portfolio of risks. The two equations

$$A = E(Z) + R + C$$

$$ROE = \frac{R}{C}$$

simultaneously determine the total amount needed to start up the company, and the relative contribution needed from policyholder and stockholder. (It should be noted that ROE in the above discussion is a total return on capital, not an annual rate of return, unless the company expects to pay all its losses by year-end. The appropriate calculations reflecting annualized rates of return are handled in the main text of the paper.)

In the pricing context, there is no confusion regarding the term “margin.” Although both R and C are needed to meet the solvency requirements, only R is considered the pricing margin.

To examine the reserving situation, we need to add some notation. Define L_t as the random variable representing losses unpaid at age t . For example, L_{24} represents losses unpaid 24 months after the inception of the policy period. By definition, L_o represents the unpaid losses at the beginning of the period, so $L_o = L$. If we let u be the date the last loss is paid (ultimate), then $L_u = 0$. For every value of t , we have a random variable L_t , and a distribution associated with that variable. We are typically interested in the inverse distribution at the value q ; that is, the dollars of loss unpaid at time t corresponding to the q^{th} percentile of the aggregate loss distribution. For notational convenience, we will define $G(t)$ to be the inverse of the distribution function associated with the random variable L_t , and cumulative probability q . In most cases, we will not need to distinguish between various values of q . If needed, we can extend the notation to $G(t,q)$ to allow for varying values of q .

Assume the new company starts with capital C and premium P . Assume no new business is written and we examine our company at time t . Some of the losses have been paid — the remaining unpaid are represented by the random variable L_t . Assuming that our criteria for solvency is unchanged, we would require total assets of $A_t = G(t)$ which, as before, can be written as:

$$A_t = E(L_t) + R_t + C_t$$

In words, the assets required for an insurance company at time t are the expected value of the liabilities plus an amount $(R_t + C_t)$ such that the probability of insolvency is sufficiently low. This latter amount is comprised of two quantities. The first, R_t is the portion of the original profit margin which must still remain in the company. The second quantity, C_t is the proportion of original capital required to remain in the company. We can presume that the amount $R - R_t$ has been returned to the stockholder in the form of a dividend and the amount $C - C_t$ has been returned as a return of capital. If we assume that the market rate of return is unchanged, then it is reasonable to assume that

$$\frac{R_t}{C_t} = ROE$$

which means that

$$\frac{R_t}{R} = \frac{C_t}{C}$$

or that the release of profit has been proportional to the reduction in the capital requirements.

It is important to note what assumptions are being made and what assumptions are not being made, at least at this time. If a block of business is priced at inception, use the following relationship to determine the overall required assets and the relative contributions from owner and policyholder:

$$A_o = E(L_o) + R_t + C_t$$

At time t , use the following relationship to determine the required assets which must remain in the insurer if the outstanding liabilities are retained or the amount of assets needed by another company if the liabilities are fully reinsured:

$$A_t = E(L_t) + R_t + C_t$$

Suppose that, at time t , the expected remaining losses are some proportion of the original expected. For concreteness, assume that this value is 25%. That is:

$$\frac{E(L_t)}{E(L_o)} = .25$$

We are not making the assumption that $G(t)$ is 25% of $G(0)$. Nor are we making the assumption that A_t is 25% of A_o . More importantly, we are not making the assumption that $(R_t + C_t)$ is 25% of $(R_o + C_o)$.

We might examine the characteristics of the business being written and conclude that the outstanding and IBNR and time t are more volatile than the overall block of business. In this case, we would conclude that, while

$$\frac{E(L_t)}{E(L_o)} = .25$$

the ratio of the required assets is greater than 25%:

$$\frac{A_t}{A_o} + \frac{G(t)}{G(0)} > .25$$

In this case, the amounts in excess of expected losses would also, in total, have a ratio greater than 25%:

$$\frac{R_t + C_t}{R_o + C_o} > .25$$

Additionally, from the formula for assets, we note that:

$$\frac{A_t}{A_o} = \frac{G(t)}{G_o} > \frac{R_t + C_t}{R_o + C_o}$$

Alternatively, if we conclude that the additional information provided by the paid data and outstanding case reserve information sufficiently improves our ability to estimate the ultimate cost of the unpaid liabilities, such that:

$$\frac{A_t}{A_o} = \frac{G_t}{G_o} > .25$$

The relationship between $R_t + C_t$ and $R_o + C_o$ will be determined by the characteristics of the business written. As we have seen, this ratio might be greater than or less than the ratio of the expected unpaid losses to the original expected losses.

On the other hand, the determination of the individual components of $R_t + C_t$ will depend on the market rate of return for runoff business of this type. Unfortunately, this market is very thinly traded, so empirical evidence may not be available to determine the appropriate conclusions. Instead, we will have to use more theoretical approaches.

It is tempting to conclude that, if the unpaid liabilities are viewed as more “risky” than the original business, the required **ROE** is higher. Equivalently, it is tempting to conclude that, if solid information on outstanding claims reduces the “riskiness” of the unpaid liabilities, the required **ROE** would be lower. This does not necessarily follow. If the outstanding liabilities are “risky,” then the value of $G(t)$ will be relatively high, which means a relatively large amount of assets is required, but this can be satisfied by requiring more capital and leaving the **ROE** the same. That is, riskier business either requires a higher return on a fixed amount of capital, or higher capital. If we conclude that we establish our company (either the original company writing the business or a company formed to reinsure the runoff) using a ruin criteria concept, that is, we fix the required probability of solvency q , then, to a high degree of

approximation, we are assuming that riskier business requires more capital rather than a higher return on capital. Thus, it will be reasonable to assume that the ROE for a runoff situation will be the same as the original business and:

$$\frac{R_o}{C_o} = \frac{R_t}{C_t} = m$$

I suggest that we should adopt the convention that the term “loss reserve margin” should refer to R_o , not to the total of R_t and C_t . One reason for this suggestion is that we should not combine two very dissimilar concepts (a portion of the original premium provided by the policyholder, and a portion of the original capital supplied by the owner). Unfortunately, there is some precedence which is inconsistent with this convention. In many hospital trust funds, it is typical to establish a funding requirement consistent with the formula

$$A_t = E(L_t) + R_t + C_t$$

This formula specifically refers to the funding requirement for one particular historical accident year (specifically, the one at age t). Conceptually, this formula is calculated for all open years plus possibly an amount for the upcoming year:

$$A_o = E(L_o) + R_o + C_o$$

In practice, all years other than the upcoming year may be handled as a group. More importantly, there are two related issues that distinguish this situation from the classic insurance company example.

First, the value of q is typically set at .75 or .90 rather than a value very close to one. This decision arises from the second distinction, which is that the policyholder and the owner are often identical. Because of this identity, a value of q can reasonably be chosen lower than that of a stand-alone insurance company. Also, because of the identity, the distinction between funds provided as a policyholder (R_t) and funds provided as an owner (C_t) is often not made. As a consequence, the fact that the term “loss reserve margin” is used to refer to the entire amount $R_t + C_t$ is not surprising.

The appropriate theoretical accounting for loss reserve margins is quite straightforward, given these assumptions. An insurance company would write premiums equal to P in year zero. This premium amount contains R_0 of expected profit. The company would need to have capital C_0 to support this business. At time $t = 1$, the end of the year, the company should book actual incurred losses (including an appropriate amount for IBNR). The company conceptually can return $C_1 - C_0$ to the owners, although in practice, this amount will be “rolled over” to support new writings in the next year. The company can “release” $R_1 - R_0$ into earnings. If actual loss experience exactly matches expected loss experience, then booked incurred losses will exactly equal $E(L_0)$, and a profit of $R_1 - R_0$ will be reported. To the extent that actual experience is better or worse than expected, so will the reported results. The company will maintain a “reserve” at year-end of R_1 , which it will label the loss reserve margin. It will be carried above the line as a liability, not below the line as a part of surplus.

The introduction of this paper stated that the term “risk margin” requires careful definition because the term is used to refer to fundamentally different concepts. This appendix has laid a theoretical framework for terminology. As a result, issues in the introduction can now be stated in precise terms: When we refer to risk margins, it is important to clarify whether we are discussing only R_I , or the larger quantity $R_I + C_I$ (or perhaps some other quantity). In the terminology of the main text, R_I corresponds to the narrow risk margin and $R_I + C_I$ corresponds to the broad risk margin.

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