Forecasting Mass Action Losses Using a Hybrid Development Model

by Roger Hayne

FORECASTING MASS ACTION LOSSES USING A HYBRID DEVELOPMENT MODEL

by Roger M. Hayne

Abstract

Mass action losses often emerge differently than other losses for a line of business. Using asbestos as an example, general liability development began to show some unexpected late development in the late 1970's and early 1980's. After some investigation it was concluded that much of this development could be attributed to asbestos related claims. In addition these claims did not seem to exhibit the dependence on accident year age that other general liability losses usually experience. Thus, it could be concluded that normal development methods may not be appropriate for forecasting such losses.

One alternative that has been considered is to assume that future emergence of asbestos losses will depend not on the age of the particular accident year, but on the valuation year of the particular losses. This assumes future development of all losses would be the same, independent of the accident year. In this paper we will propose an alternative, hybrid, of these two models (pure accident year and pure calendar year). In the hybrid model we will allow the data to dictate what mix of the two models best fits the experience emerged to date. The method itself is not very difficult to implement in practice. Given numerical solution methods available in current personal computer spreadsheet software, PC solutions can be generated in a very short amount of time.

We discuss the concept and support the discussion with examples applied to some real-but-disguised data. We then explore an approach that to apply what is learned from this asbestos example to other situations with example hazardous waste data as an example. The concepts could apply to other mass action types of exposure and provide a separate, independent, test of results implied by other forecast methods.

Biography

Roger Hayne is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries and a Consulting Actuary in the Pasadena, California office of Milliman & Robertson, Inc. (M&R). He holds a Ph.D. in mathematics from the University of California and joined M&R in 1977. Roger has been involved in reserve estimation for a wide range of property and liability coverages with emphasis on exposures with longer tails and in situations where full data may not be readily available.

FORECASTING MASS ACTION LOSSES USING A HYBRID DEVELOPMENT MODEL

1. Introduction

Mass action losses, such as those arising from asbestos, DES, or hazardous waste exposure, often emerge differently than other losses for a line of business, and may affect usual actuarial projection methods for that coverage. Insurer experience with these various sources of claims are not all at the same stage of maturity. The industry has been dealing with asbestos related claims for some time, whereas claims from hazardous waste sites, DES, or potentially silicone implants, are not quite as mature. The emergence of asbestos claims may provide some insight into the potential future emergence for other claims from other sources.

For example, general liability development began to show some unexpected late development in the late 1970's and early 1980's. After some investigation, insurers began to conclude that much of this development could be attributed to asbestos related claims. When such unusual events affect development patterns, it is not unusual for the actuary to consider such claims separately when analyzing the experience for reserves. First attempts to deal with such losses may have been to separate asbestos losses from other claims and develop them separately, possibly using development from some other, longer tail, business.

However, the asbestos claims did not seem to exhibit the dependence on accident year age that other general liability losses usually experience. Rather, it seemed that asbestos claims emerged for most accident years, whether relatively old or relatively new, at pretty much the same time. For example, the percentage increase in asbestos related claims coded to 1968 accidents during 1982, might have looked very similar to that for asbestos related claims coded to 1975 accidents during that same year.

There are many characteristics of these claims that could help explain this. One problem is in identifying the "accident date" for a particular claim. Claimants may have been exposed over a span of years, with asbestos related injury not manifesting itself for many more years. Compounded with this are various court decisions regarding coverage triggers and indicating which policies are to respond to what losses. Thus, there may be practical questions as to which accident year or years the losses for a particular claim should be assigned.

Technical elements were not the only influence. With the emergence of asbestos related claims came increased notoriety of the hazards of asbestos exposure, and the likelihood that compensation may be available for injured claimants. Thus, claims may have been reported more because of this notoriety than, because of the time lag from the accident.

Still another complication arises from additional "waves" of asbestos related losses. For example, losses related to asbestos abatement, or containment, have been emerging recently. There are also recent reports of claims being advanced against owners', landlords' and tenants' policies, and liability policies for coverage other than products liability, which were thought to be relatively free of asbestos risk.

Thus, it could be concluded that normal development methods may not be appropriate for forecasting such losses. Compounding this difficulty in the past has been the relative scarcity of data available. Thus actuaries, as in many similar situations, have constructed models of the underlying exposure, latency period, emergence and costs of asbestos claims to estimate reserves for carriers. These models are often very sophisticated and may incorporate both the potential exposure of all workers and an insurer's exposure based on its insureds over time.

One particularly difficult aspect of such models, however, is incorporating them with losses that are emerging to the insurer. Often the insurer's own data base may be too large to ignore and may be exhibiting loss emergence different from what would be predicted by these models. Thus, some alternative may need to be found to incorporate the insurer's experience. This leads us to consider alternative models that incorporate the insurer's own development experience.

One alternative to the traditional accident period loss development method that has been considered is to assume that future emergence of asbestos losses will depend on the valuation year of the particular losses, rather than on the age of the particular accident year. This assumes that the future rate of development of all losses would be the same, independent of the accident year. This assumption potentially ignores latency periods inherent in asbestos claims.

Of course, aggregating all claims of a particular age loses the advantage of the traditional accident year development method of being able to "learn" from the emergence of older accident years. With this approach there is but one "accident" year, composed of all claims. Several alternatives have been advanced to deal with this problem. One is to assume a particular loss runoff curve and fit it to the data. Another is to assume that asbestos claims are somewhat similar to general liability claims, or to some other group of claims with more or less well known emergence characteristics. One could then assume that future asbestos emergence would be similar to the emergence of an appropriately mature accident year for the selected coverage.

There may be some attractiveness to this approach. It could be argued that now the legal climate for asbestos claims may be much more settled and may actually be similar to that for other liability claims. Hence, the argument would proceed, that one could expect future movement of these total asbestos claims that have been known for, say, five years, to be similar to future movement of a five-year-old accident year of liability claims.

In this paper we will propose an alternative, hybrid, of these two development models (pure accident year and pure calendar or valuation year). In the hybrid model, we will allow the data to dictate what mix of the two models best fits the experience emerged to date. The method itself is not very difficult to implement in practice. Given numerical solution methods available in current personal computer spreadsheet software, PC solutions can be generated in a very short amount of time.

Stepping back for a moment, we note that the above discussion indicates that asbestos claims experience has passed through several stages:

1. General liability losses started to experience some late development, though the losses were not separately analyzed.
2. Unusual development continued with the cause identified as asbestos claims, those claims removed from general liability data and developed separately, possibly using some other, longer tail, development.
3. Exposure based models were developed to estimate asbestos losses, often from an all-industry or individual insured basis. These models often required significant amounts of exposure and claims data and are based on the underlying asbestos exposure, health effects, and assumptions regarding costs.
4. Insurers have developed more experience in dealing with asbestos claims, and the legal environment is more certain than in the early stages of asbestos litigation. Insurers are collecting separate asbestos loss data and there may be differences between actual emerged experience and that expected by exposure models.
5. There may be sufficient data to consider emergence models based on those data. These models could be used to augment exposure based model estimates.

It may not be unreasonable to expect that other mass action claims would follow a similar life cycle. If this is the case, we could draw from what we have learned from asbestos movement, to obtain a better understanding of the future development potential for other mass action claims. For example, it appears that hazardous waste claims are in the third stage above, but there is claim experience emerging. Other loss causes, such as DES, or silicone implants are, of course, at other stages of maturity. It is possible, however, to consider the models used to analyze asbestos emergence, to gain additional insight into future emergence for these other loss causes. Rather than proposing these alternatives as replacement for other methods, we believe that they can be used as separate, independent, tests of results implied by other forecast methods.

In the remainder of this paper, we will first discuss the use of development from other coverages as a model of future asbestos experience. At this point we will introduce a hybrid of pure calendar year and pure accident year models. We will then discuss fitting this hybrid model to asbestos loss data, rather than depending on the emergence from other sources. Finally, we will present an example of using this fitted asbestos emergence pattern to estimate future hazardous waste losses.

2. *Notation and Definitions*

We will denote by X_{ij} cumulative losses for accident year i at j years of development and by D_{ij} the development factor for accident year i from year j to year j+1, i.e. $D_{ij} = X_{ij+1}/X_{ij}$. The traditional accident year development model selects factors $d_1, d_2, \ldots, d_\infty$, with the forecast for a particular accident year at age j:

(2.1)
$$X_{i\infty} = X_{ij}\prod_{k=j}^{\infty} d_k = X_{ij}\,f(j)$$

In the traditional methods, the factors $d_1, d_2, \ldots, d_\infty$ are usually selected using the historical factors D_{ij}, with d_j usually selected considering historical factors at age j; D_{ij}.

Implicit in this method is the assumption that the development of losses for each accident year is dependent only on the age of that accident year. So, under these assumptions, the movement of older accident years at a particular age is indicative of movement to be expected for more recent years at that same age. In the usual development triangle format, this assumes that, except for random fluctuations, development factors at a give age are constant.

As discussed in the first section (*Introduction*) above, there are many characteristics of asbestos, hazardous waste, and other mass action losses, that may violate this implicit assumption. Thus we search for alternatives.

One such alternative assumes that the accident date assigned to a claim is not particularly relevant to its potential for future development, but rather, it is the valuation date that determines future development. Under this alternative, all claims will experience the same future development. In the case of asbestos and hazardous waste, there may be some attraction to this model. In both cases, the date of the occurrence may have less of an influence on future development than for most other claims. If we were to accept this assumption, we would then model future development, by assuming that all losses are at the same age. In this case for accident year i, currently at age j, the forecast becomes:

(2.2) $$X_{i\infty} = X_{ij}\prod_{k=j+i}^{\infty} d_k = X_{ij}f(j+i)$$

In the usual development triangle, the quantity $i+j$ is constant along the diagonal with $i+j=n+1$, where n is the number of columns (assuming annual development of annual data). In this case, the estimates of the development factors d_k might not follow the traditional approach, but are similar to the problem of estimating a factor to account for development beyond that available in historical data. Such factors are often dubbed "tail" factors.

Neither set of assumptions, however, appear to be completely satisfied. On the one hand, we would probably not expect the future development on accident year 1975 asbestos claims after 1994, to be the same as the development of accident year 1965 asbestos claims after 1984. If this is the case, the pure accident year method may not be appropriate. On the other hand, we may expect that there is more development potential after 1994 for accident year 1985 asbestos claims, than for 1965 claims. If this is case, the pure calendar year method may not be appropriate.

Reviewing formulae (2.1) and (2.2), we note that they can be thought of as two extremes of the more general model:

(2.3) $$X_{i\infty} = X_{ij}f(j+\alpha i),\ 0 \le \alpha \le 1$$

The pure accident year model results from the case $\alpha = 0$ and the pure valuation year model results from the case $\alpha = 1$. The factor estimates in this case are less clear, especially since, at least theoretically, we could require factors at non-integral ages. However, given f, the corresponding development factors can be calculated as:

$$d(j+\alpha i)=\frac{f(j+\alpha i)}{f(j+\alpha i+1)} \tag{2.4}$$

If α is between 0 and 1, the model will fall between the development implied by either the pure accident year or the pure valuation year model. In this case, later accident years will be considered as less mature than earlier accident years, but not at the normal one-for-one rate inherent in the pure accident year model.

For example, in the pure accident year model, the future development for accident year 1972 after 1996 would be the same as that for accident year 1970 after 1994. If α = 0.5 in the hybrid model, the future development for accident year 1972 after 1996 would be the same as that for accident year 1970 after <u>1995</u>.

Hence, if α is between 0 and 1, implicit in this hybrid model is the assumption that each accident year is successively less mature than the prior year, but only by a fraction of a year. Similar to the pure accident year and pure valuation year models, we implicitly assume that, except for this difference in maturity, all accident years will develop the same.

If we assume that the underlying development model is hybrid, as opposed to purely accident year or purely valuation year, then estimating the development factors is not as readily apparent as in the usual development factor method. In the pure accident year case, actuaries often consider the factors for older accident years at a given stage of development, to estimate the development for later years. The hybrid model, however, loses this convenient means of estimation since, without prior assumptions regarding α, we do not know the differences in relative maturity between accident years.

For this reason, in the applications we will present, we will use smoothed development models and allow the data to provide an estimate of α, along with the other parameters of the smooth models. We again caution that this proposed approach is not a substitute for a thorough understanding of the exposures being reserved for. Rather, it is an attempt to provide another check on other methods, incorporating loss experience that has already emerged.

3. Development Models

Lacking sufficient development information, a first approximation actuaries often make is to use development for another, and possibly related, coverage or group of insurers writing similar business. Actuaries often consider the development from peer companies as available from such sources as A.M. Best Company, Inc., the Reinsurance Association of America, published financials or rate filing materials.

It could be argued that asbestos claims have been known for some time and that the legal basis for such claims is relatively well defined. It would follow that general liability development experience (excluding pollution, asbestos, and other mass action claims) may provide a reasonable basis for extrapolating future development. The first column of Exhibit 1 shows some sample general liability development.

These sample factors show some continued movement even far out in the tail. Thus we will not assume that the development is finished, but rather we will fit some sample development curves to smooth the factors and extrapolate future development. We acknowledge that there are a wide variety of models available, so for illustrative purposes, we have confined this discussion to three, fairly simple, models. We emphasize, however, that the methods we will present here are not restricted to these three simple models, but can be adapted to a wide range of assumed future development.

There have been several forms of future development mentioned in the literature. For example, Sherman [1] suggests the use of an inverse power function to model future development and also discusses an exponential variation of this curve among others, Weller [2] in his discussion of generalized Bondy development suggests an exponential decay model for development factors, and Zehnwirth [3] suggests the use of Hoerl curves to model loss runoff and in [4] suggests various regression models. In addition, we have found that a Weibull distribution often provides a reasonable model of loss runoff over time for certain coverages.

3.1. *Exponential Development Model*

In this model we assume that the development factor from age t to t+1 is given by:

$$d^{(e)}(t) = 1 + ae^{-bt} \tag{3.1.1}$$

where a and b are constants. We usually require $b > 0$ to assure that the factors decay over time.

3.2. *Inverse Power Curve Model*

In this model we assume that the development factor from age t to age t+1 is given by:

$$d^{(p)}(t) = 1 + at^{-b} \tag{3.2.1}$$

where a and b are constants. Again we require $b > 0$ to assure that the factors decay over time. It is clear that these two models are related, in fact, d(t) is inverse power, if and only if d(ln t) is exponential.

3.3. Weibull Model

In this case, we note that a Weibull distribution can be parameterized such that the cumulative density function can be written as:

$$\text{(3.3.1)} \qquad F(t) = 1 - e^{-\left(\frac{t}{a}\right)^{b}}$$

If we then assume that the percentage of losses at time *t* equals F(*t*), then we obtain:

$$\text{(3.3.2)} \qquad d^{(w)}(t) = \frac{1 - e^{-\left(\frac{t+1}{a}\right)^{b}}}{1 - e^{-\left(\frac{t}{a}\right)^{b}}}$$

Again, to assure convergence, we require that *a*>0. In addition, to assure that F(*t*) is increasing, we will require that *b*>0.

We will include example calculations with exponential, inverse power and exponential models. Again, we emphasize that these three models are selected here more for convenience, than due to any inherent limitation in the methods we will discuss. The same methods could be used for a wide range of smooth development models.

Exhibit 1 also shows fits of these three models to the sample development data shown in the first column. Rather than linearizing the exponential and power models, as is usually done, we selected parameters that directly minimized the total weighted square errors between the sample and fitted factors. For this we used numerical methods to minimize the appropriate error function. Since our primary interest will be in the "tail" development, we selected the square of the number of years of development as the weights in our fits, thereby giving more weight to fitting of the tail in the various distributions. Also shown in Exhibit 1 are the resulting residuals, the total of the residuals (or bias), and the weighted total square residuals for the three fits. From these fits we conclude that the inverse power curve provides the best fit of those sampled.

As an aside, the following table compares the results of fitting a power and an exponential to these factors, minimizing the simple sum of the squared residuals, sometimes called nonlinear regression, with the results of the "usual" linearized approach, i.e. applying linear regression to the natural logs of the development factors minus 1:

COMPARISON OF LINEARIZED AND NON-LINEAR REGRESSION FITS

	Linearized Fit		Nonlinear Fit	
	Power	Exponential	Power	Exponential
a	1.497	0.314	1.970	4.971
b	1.522	0.153	1.636	0.942
Total Error	0.646	2.041	-0.024	0.719
Square Error	0.245	3.084	0.006	0.067

We note that the nonlinear fit used in the above table gives equal weight to the square of each of the errors, which is the assumption of usual linear regression. Thus, the parameters and error terms do not agree with those shown in Exhibit 1.

3.4 Additional Notation

Our first approximation, then, will assume that future asbestos development patterns will be the same as general liability development, that is, we will use the curves from Exhibit 1 as the basic development model, but we will select the parameters based on actual emerged asbestos losses. Thus we will assume that $f(j+\alpha i)$ will have the form:

(3.4.1)

$$\begin{aligned} f(j+\alpha i) &= d^{(m)}(\beta+j+\alpha i)\times d^{(m)}(\beta+j+\alpha i+1)\times\ldots \\ &= \prod_{k=\beta+j+\alpha i}^{\infty} d^{(m)}(k) \end{aligned}$$

for some possibly negative value of the parameter β. Here β adjusts for any lag that may be inherent in the actual development experience, from that inherent in the un-lagged model. In this section we consider three of many possible representations for the function $d^{(m)}(k)$.

Of course, the actual model selected will significantly influence the ultimate loss projections for this method. This is no different than any other actuarial projection method. In practice we would select the development model that we would expect to most closely follow the expected future development. For example, if we found that general liability development patterns closely paralleled a power curve and we assumed that mass action losses would develop similar to general liability losses, then a power model would be the natural first choice for asbestos development. In addition, if we suspected additional complications in the mass action losses, for example additional "waves" of asbestos claims, we could modify the model accordingly. Thus the nature of the exposure, and the development inherent in the various models, should be considered in selecting the development model to use.

Exhibit 2 shows example asbestos loss development based roughly on some actual emerged experience. Although these are asbestos data, we note that these methods could also be applied to estimate development data for other mass action type of claims.

Though a bit of a digression at this point, Exhibit 3 shows the resulting development factors with selections corresponding to the "column sum" method as described by Stanard [6]. These factors may be similar to those we would select if we use a traditional development factor method to forecast losses.

The factor for development after 312 months is based on the fit of an exponential curve to the selected development factors less 1. Though not shown, this method results in an ultimate loss estimate of approximately $136 million for all years combined, based on a total of $13.3 million in incurred losses. Had we used the inverse power curve as suggested by Sherman in [1] to estimate the "tail," the resulting factor would have been approximately 11.5 with an ultimate loss estimate of more than $500 million. If the observations in section 1 (*Introduction*) above regarding the emergence of these claims are correct, the emergence of these losses do not satisfy the assumptions of the development factor method; thus, the resulting estimates would not be appropriate.

If, now, we were to use the fitted power curve from Exhibit 1 and the pure valuation year approach described above, we would set $\alpha = 1$. If we assume that since the first losses emerged in 1984, then all losses would develop as would general liability for accident year 1984. Here we would have $\beta = -18$ since accident year 1968 at 1984 is at 17 years of development. If we select the power model, this results in an indicated age-to-ultimate factor of 1.617 and ultimate loss forecast of $21.5 million.

These estimates also ignore information present in the data. We could assume that the emergence will follow the fitted power curve from Exhibit 1 but with α and β values fitted to the development factors in Exhibit 3. We address our approach to estimating these parameters in the next section.

4. Parameter Estimation

Our problem now is to estimate the parameters α and β using historical data. We will use numerical methods to minimize a selected error function that compares actual loss emergence with that expected from the particular model. One error function that suggests itself is the usual square error:

$$Err = \sum (A_i - E_i)^2$$

Of course, least squares regression is based on minimizing this error function. We note, however, that if we would expect different values to have different variances, this particular error function may not be appropriate since all differences will be given equal weight. We thus select an error function that is more akin to a chi-squared test:

(4.1)
$$Err = \sum_{ij} \frac{\left(X_{ij} - P_{ij}\right)^2}{P_{ij}}$$

Here we compare the actual payments for accident year i, age j; X_{ij}, with the (one period) forecast from the model; P_{ij}, using an error term like that used in chi-squared tests.

If we assume that the expected losses at age j can be given by:

(4.2)
$$P_{ij}(\alpha,\beta) = X_{ij-1} d^{(m)}\left(\beta + j - 1 + \alpha i \middle| a, b\right)$$

where m could refer to any of the models described above and we let D_{ij-1} denote the actual development factor from time j-1 to time j for accident year i, then the error function in (4.1) becomes:

(4.3)
$$\begin{aligned} Err &= \sum_{ij} \frac{\left(X_{ij-1} D_{ij-1} - X_{ij-1} d^{(m)}(\beta + j - 1 + \alpha i)\right)^2}{X_{ij-1} d^{(m)}(\beta + j - 1 + \alpha i)} \\ &= \sum_{ij} \frac{X_{ij-1}}{d^{(m)}(\beta + j - 1 + \alpha i)} \left(D_{ij-1} - d^{(m)}(\beta + j - 1 + \alpha i)\right)^2 \end{aligned}$$

Thus, our selected error function weights the square of the difference between observed and fitted development factors proportionate to the size of the prior losses and inversely proportionate to the size of the fitted development factor itself. Given the general expectation that the lower the initial losses or the higher the development factor, the more variation is inherent in that factor, this may indeed be a reasonable weighting of the factors and is probably preferable to the uniform weighting provided in the usual sum-of-squares error function.

We acknowledge that this is simply one approach to weighting the individual errors and that others are possible. Following Klugman [5], we note that practical considerations are often valid reasons for weighting errors differently than what may be "optimal" from purely statistical reasoning. We note that this weighting scheme gives more weight to more mature (larger) data in the development tail. The resulting fitted surfaces will tend to track the tail more closely than a pure regression model and hence may be more useful for extrapolating future development.

5. Example Calculations

Page 1 of Exhibit 4 shows the results of using the power curve parameters a and b from Exhibit 1 and fitting the parameters α and β using the error function discussed in section 4 (*Estimating Parameters*). Page 2 of Exhibit 4 shows the forecast future factors along with the resulting loss forecast of \$20.4 million. Page 3 of Exhibit 4 shows the one-year forecast error for this model; that is, the difference between the actual losses in the cell with the one-year model forecast for that cell. For example, on page 1 we see the fitted factor for accident year 1970 development through 1987 is 1.242. When applied to the losses through 1986 of \$600 thousand, this provides an estimate through 1987 of \$745 thousand, which is \$155 thousand above the actual \$590 thousand for that age.

This example assumes that the fitted power curve from Exhibit 1 is the proper development model to be used to estimate future development. This ignores, however, development data in the data. There is nothing in the foregoing discussion that requires us to use that fitted curve. We will use the data and estimate the three parameters *a*, *b*, and α. Since we are estimating all the parameters, we will take $\beta = 0$. We again minimize the error function from section 4 (*Estimating Parameters*) for each of the three models. Exhibits 5 through 7 parallel Exhibit 4 but use the fitted exponential, power, and Weibull curves respectively.

We see that the power curve again results in the smallest of the error functions; 3,378 compared with 3,404 for the exponential and 3,561 for the Weibull. The forecast accuracy test shown on page 3 of those exhibits show a slightly different picture. In this case the exponential model has the smallest absolute total error, with the Weibull second and power third. Without additional assumptions regarding the underlying distributions for these models, we cannot now say if these differences are statistically significant. We note, however, that the inclusion of the parameter α does affect the fits as summarized by the following table:

COMPARISON OF RESULTS FOR VARIOUS MODELS

	Model		
	Exponential	Power	Weibull
Pure CY Model ($\alpha = 1$)			
Weighed Error	3,701	3,669	3,831
Bias	-$1,650	-$1,621	-$1,848
Forecast	$22,687	$29,050	$15,285
Pure AY Model ($\alpha = 0$)			
Weighed Error	3,722	3,719	3,724
Bias	-$1,738	-$1,917	-$1,907
Forecast	$101,224	$338,523	$88,321
Hybrid Model (α fitted to data)			
Weighed Error	3,404	3,378	3,561
Bias	-$1,539	-$1,580	-$1,577
Forecast	$22,710	$30,866	$16,183

NOTE:
1. Dollar amounts are in thousands.

Since our primary concern is to forecast future development, we note that the one-year forecast error of 1993 losses for the Weibull model is positive, indicating that the model, on the average, underestimated the development during that year. On the other hand, the one-year errors for the exponential and power models are negative, indicating an average overstatement. If these errors hold for future forecasts, they may lead to the conclusion that the exponential and power models may slightly overestimate the tail while the Weibull model may understate it.

Exhibit 8 provides another, "ex-ante," test of the models. In this exhibit we compare the actual calendar year 1993 factors by accident year with the forecasts from the three models. In this case, however, the models were fitted to data through calendar year 1992 only. That is, this exhibit shows the actual forecast accuracy of the three models considered. The total errors are reasonably small with the exponential having the smallest absolute total prediction error, followed by the power model, with the Weibull model third. As above, the power and exponential models tended to overstate losses while the Weibull model tended to underestimate it.

The projections from the three models, as shown on page 2 of Exhibits 5 through 8, are $30.9 million for the power, $22.7 million for the exponential and $16.2 million for the Weibull. The above tests tend to suggest the power and exponential models may be better predictors in this case, with the Weibull generally lacking in all regards. One final test may tend to confirm these observations. If we compare the actual factors at the top of page 2 of these exhibits with the forecast factors shown in the bottom portion, we may conclude that the Weibull model decays more rapidly than we would expect, given the data in the top portions. The same observation could arguably be made regarding the exponential model, though it is not as apparent. Based on these observations, we may thus conclude ultimate losses in this case to be in the neighborhood of $25 to $30 million.

6. Application to Example Pollution Development

Exhibit 9 shows some example pollution development data. As with the asbestos data in Exhibit 2, these data are roughly based on some actual emerged experience. We will assume that each of the fitted asbestos models provide reasonable approximation to the future development of these pollution losses, but that the development is lagged by some unknown amount. As with the general liability development data from Exhibit 1, we will use the fitted curves but solve for the single added lag parameter β using numerical methods to minimize the error function (4.3).

Exhibits 10 through 12 summarize the results for these fits. These exhibits contain the same information as Exhibits 5 through 7; however, since the pollution data have only been available for the past three years, we are able to compress the format. All three models seem to indicate an approximate 3.5 year lag in pollution emergence relative to asbestos (β values near -3.5).

That is, under these very specific assumptions, pollution now is expected to develop as asbestos did three and one-half years ago, even though the actual emergence lag shown is seven years.

In this case the exponential model has the smallest error function, followed by the power and then the Weibull. All of the models had a tendency of underestimating 1992 losses and overestimating 1993 losses. This is due to the relatively mild development experienced during 1993. Overall, the exponential has the lowest absolute total bias of -$797 thousand for the two years, followed by the Weibull with -$813 thousand and the power with -$819 thousand. The forecasts range between $7.2 million and $13.5 million. We did not, however, perform the ex-ante test described above due to the limited data available.

7. Other Applications

These two approaches can also be useful in estimating development of losses from other causes. Just as our first approach used general liability data to extrapolate asbestos losses, if we assume that the development of other loss causes, say DES claims, will generally follow the asbestos model, but with a different lag, we can derive estimates of future development for those other losses as we did with the pollution example above.

These general techniques could also be used with more complex models. For example, if after testing simpler models such as these we find evidence for a "second wave" in the data, we could specify compound models that include such a wave by, for example, adding two simpler models with a lag reflecting the timing of the second wave. Again, these numeric techniques could be used to estimate the parameters for those models.

Again we reiterate that these approaches can provide a different view of potential development for unusual loss causes. They are relatively easy to apply, but rely critically on the choice of underlying development model. It is possible that more detailed models of pollution and asbestos exposure could provide useful insight as to the appropriate model. Given this insight and model choice, these methods can readily be used to derive additional loss estimates that incorporate actual development experienced.

8. *Acknowledgments*

The author wishes to acknowledge the assistance provided by Don Rainey in the preparation of this paper. His input contributed substantially to improving this presentation.

Exhibit 1

SAMPLE FITTED GENERAL LIABILITY DEVELOPMENT

Year of Development	Sample Factors	Fitted Factors			Indicated Error		
		Power	Exponential	Weibull	Power	Exponential	Weibull
1	2.969	2.946	2.413	2.214	0.023	0.556	0.755
2	1.633	1.628	1.794	1.550	0.005	-0.161	0.083
3	1.321	1.324	1.446	1.339	-0.003	-0.125	-0.018
4	1.249	1.203	1.250	1.235	0.046	-0.001	0.014
5	1.156	1.141	1.141	1.173	0.015	0.015	-0.017
6	1.096	1.105	1.079	1.133	-0.009	0.017	-0.037
7	1.060	1.081	1.044	1.104	-0.021	0.016	-0.044
8	1.043	1.066	1.025	1.083	-0.023	0.018	-0.040
9	1.025	1.054	1.014	1.068	-0.029	0.011	-0.043
10	1.027	1.046	1.008	1.055	-0.019	0.019	-0.028
11	1.029	1.039	1.004	1.045	-0.010	0.025	-0.016
12	1.032	1.034	1.002	1.037	-0.002	0.030	-0.005
13	1.018	1.030	1.001	1.031	-0.012	0.017	-0.013
14	1.026	1.026	1.001	1.026	0.000	0.025	0.000
15	1.015	1.023	1.000	1.021	-0.008	0.015	-0.006
16	1.017	1.021	1.000	1.018	-0.004	0.017	-0.001
17	1.035	1.019	1.000	1.015	0.016	0.035	0.020
18	1.018	1.017	1.000	1.012	0.001	0.018	0.006
19	1.014	1.016	1.000	1.010	-0.002	0.014	0.004
20	1.029	1.015	1.000	1.008	0.014	0.029	0.021
21	1.027	1.014	1.000	1.007	0.013	0.027	0.020
22	1.024	1.013	1.000	1.006	0.011	0.024	0.018
23	1.019	1.012	1.000	1.005	0.007	0.019	0.014
24	1.011	1.011	1.000	1.004	0.000	0.011	0.007
25+		1.161	1.000	1.018			
Total					0.009	0.670	0.693
Weighted Square Error					0.584	2.859	1.935
Fitted Parameters:							
a		1.946	2.516	8.013			
b		1.631	0.577	1.221			

EXAMPLE ASBESTOS INCURRED LOSS EMERGENCE

Accident	Months of Development															
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192
1968	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
1969	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	120
1970	0	0	0	0	0	0	0	0	0	0	0	0	0	0	250	360
1971	0	0	0	0	0	0	0	0	0	0	0	0	0	190	280	310
1972	0	0	0	0	0	0	0	0	0	0	0	0	90	180	280	400
1973	0	0	0	0	0	0	0	0	0	0	0	120	190	200	230	250
1974	0	0	0	0	0	0	0	0	0	0	90	180	200	240	270	340
1975	0	0	0	0	0	0	0	0	0	40	90	150	200	230	250	330
1976	0	0	0	0	0	0	0	0	10	110	160	190	230	240	410	480
1977	0	0	0	0	0	0	0	50	150	190	190	340	320	410	460	590
1978	0	0	0	0	0	0	110	310	200	280	420	550	700	910	850	900
1979	0	0	0	0	0	40	120	120	150	280	350	470	460	550	800	
1980	0	0	0	0	10	30	30	100	150	170	240	400	510	600		
1981	0	0	0	0	0	0	70	120	90	200	350	530	700			
1982	0	0	0	0	0	10	20	40	20	180	240	300				
1983	0	0	0	0	0	90	50	40	130	70	100					
1984	0	0	0	0	80	110	100	100	100	100						
1985	0	0	0	0	0	0	0	0	0							
1986	0	0	0	0	0	0	0	0								
1987	0	0	0	0	0	0	0									
1988	0	0	0	0	0	0										
1989	0	0	0	0	0											
1990	0	0	0	0												
1991	0	0	0													
1992	0	0														
1993	0															

Accident	Months of Development									
Year	204	216	228	240	252	264	276	288	300	312
1968	$180	$390	$630	$650	$830	$1,130	$1,330	$1,580	$1,850	$2,000
1969	210	290	330	410	640	760	910	1,000	1,200	
1970	600	590	710	930	1,070	1,180	1,260	1,400		
1971	580	670	820	900	960	1,060	1,200			
1972	430	550	600	650	700	800				
1973	340	450	420	580	600					
1974	400	680	780	900						
1975	480	540	600							
1976	590	500								
1977	600									

NOTE:

1. All amounts are in thousands.

Exhibit 3

EXAMPLE ASBESTOS INCURRED DEVELOPMENT FACTORS

Accident Year	Months of Development 24/12	36/24	48/36	60/48	72/60	84/72	96/84	108/96	120/108	132/120	144/132	156/144	168/156	180/168	192/180	204/192
1968	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
1969	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1.750
1970	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1.440	1.667
1971	–	–	–	–	–	–	–	–	–	–	–	–	–	1.474	1.107	1.871
1972	–	–	–	–	–	–	–	–	–	–	–	–	2.000	1.556	1.429	1.075
1973	–	–	–	–	–	–	–	–	–	–	–	1.583	1.053	1.150	1.087	1.360
1974	–	–	–	–	–	–	–	–	–	–	2.000	1.111	1.200	1.125	1.259	1.176
1975	–	–	–	–	–	–	–	–	–	2.250	1.667	1.333	1.150	1.087	1.320	1.455
1976	–	–	–	–	–	–	–	–	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229
1977	–	–	–	–	–	–	–	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	–	–	–	–	–	–	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059	
1979	–	–	–	–	–	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455		
1980	–	–	–	–	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176			
1981	–	–	–	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321				
1982	–	–	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250					
1983	–	–	–	–	–	0.556	0.800	3.250	0.538	1.429						
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000							
1985	–	–	–	–	–	–	–	–								
1986	–	–	–	–	–	–	–									
1987	–	–	–	–	–	–										
1988	–	–	–	–	–											
1989	–	–	–	–												
1990	–	–	–													
1991	–	–														
1992	–															
Weighted Average Age to Age	–	–	–	–	2.667	1.393	1.660	1.125	1.580	1.408	1.460	1.198	1.228	1.216	1.207	1.330
Cumulative Age to Ultimate					760.450	285.133	204.690	123.307	109.606	69.371	49.269	33.746	28.169	22.939	18.864	15.629

Accident Year	Months of Development 216/204	228/216	240/228	252/240	264/252	276/264	288/276	300/288	312/300	Ultimate/312
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.061	
1969	1.361	1.138	1.242	1.561	1.188	1.197	1.099	1.200		
1970	0.983	1.203	1.310	1.151	1.103	1.068	1.111			
1971	1.155	1.224	1.098	1.067	1.104	1.132				
1972	1.279	1.091	1.083	1.077	1.143					
1973	1.324	0.933	1.381	1.034						
1974	1.700	1.147	1.154							
1975	1.125	1.111								
1976	0.847									
Weighted Average Age to Age	1.223	1.175	1.170	1.165	1.174	1.138	1.137	1.182	1.061	
Cumulative Age to Ultimate	11.751	9.608	8.177	6.969	5.999	5.110	4.490	3.949	3.341	3.091

NOTE:

Exhibit 4
Page 1 of 3

COMPARISON OF ACTUAL AND FITTED DEVELOPMENT

Power Model Using General Liability Fit

Accident	Actual Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident	Fitted Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	1.777	1.372	1.225	1.153	1.112	1.086	1.069	1.057	1.047
1969	1.839	1.391	1.233	1.157	1.115	1.088	1.070	1.057	1.048
1970	1.911	1.411	1.242	1.162	1.117	1.090	1.071	1.058	1.049
1971	1.993	1.434	1.251	1.167	1.120	1.092	1.073	1.059	1.049
1972	2.087	1.458	1.261	1.172	1.123	1.094	1.074	1.060	1.050
1973	2.197	1.484	1.271	1.177	1.126	1.096	1.075	1.061	1.051
1974	2.326	1.513	1.282	1.183	1.130	1.098	1.077	1.062	1.052
1975	2.478	1.545	1.294	1.189	1.133	1.100	1.078	1.063	1.052
1976	2.662	1.579	1.307	1.195	1.136	1.102	1.080	1.064	1.053
1977	2.884	1.618	1.321	1.201	1.140	1.104	1.081	1.065	1.054
1978	3.160	1.661	1.336	1.208	1.144	1.107	1.083	1.066	1.055
1979	3.506	1.710	1.351	1.215	1.148	1.109	1.084	1.067	1.055
1980	3.953	1.764	1.368	1.223	1.152	1.111	1.086	1.069	1.056
1981	4.542	1.825	1.387	1.231	1.156	1.114	1.088	1.070	1.057
1982	5.349	1.894	1.407	1.240	1.161	1.117	1.089	1.071	1.058
1983	6.498	1.974	1.429	1.249	1.166	1.120	1.091	1.072	1.059
1984	8.232	2.065	1.452	1.259	1.171	1.123	1.093	1.074	1.060

Selected Model Parameters:

$a = 1.946$ $b = 1.631$ $\alpha = 0.918$ $\beta = -17.998$ Error = 3,924

Exhibit 4
Page 2 of 3

COMPARISON OF ACTUAL AND FORECAST DEVELOPMENT

Power Model Using General Liability Fit

Accident Year	Actual Annual Development Through Year Ending 12/31/ 1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident Year	Forecast Annual Development Through Year Ending 12/31/ 1994	1995	1996	1997	1998	1999	2000+	1994+	Forecast Ultimate
1968	1.040	1.035	1.031	1.027	1.024	1.022	1.226	1.462	$2,924
1969	1.041	1.035	1.031	1.027	1.024	1.022	1.235	1.474	1,769
1970	1.041	1.036	1.031	1.028	1.025	1.022	1.243	1.488	2,083
1971	1.042	1.036	1.032	1.028	1.025	1.022	1.251	1.501	1,801
1972	1.043	1.037	1.032	1.028	1.025	1.022	1.259	1.513	1,210
1973	1.043	1.037	1.032	1.028	1.025	1.023	1.267	1.524	914
1974	1.044	1.037	1.033	1.029	1.026	1.023	1.275	1.540	1,386
1975	1.044	1.038	1.033	1.029	1.026	1.023	1.282	1.550	930
1976	1.045	1.038	1.033	1.029	1.026	1.023	1.289	1.560	780
1977	1.045	1.039	1.034	1.030	1.026	1.023	1.297	1.574	944
1978	1.046	1.039	1.034	1.030	1.026	1.024	1.304	1.586	1,427
1979	1.047	1.040	1.034	1.030	1.027	1.024	1.311	1.599	1,279
1980	1.047	1.040	1.035	1.031	1.027	1.024	1.318	1.611	967
1981	1.048	1.041	1.035	1.031	1.027	1.024	1.325	1.622	1,135
1982	1.049	1.041	1.036	1.031	1.028	1.025	1.332	1.637	491
1983	1.049	1.042	1.036	1.032	1.028	1.025	1.339	1.649	165
1984	1.050	1.042	1.037	1.032	1.028	1.025	1.345	1.659	166
									$20,371

Selected Model Parameters:
$a = 1.946$ $b = 1.631$ $\alpha = 0.918$ $\beta = -17.998$ Error = 3,924

NOTE:
1. The forecast ultimate losses are in thousands of dollars.

Exhibit 4
Page 3 of 3

ACTUAL LOSSES MINUS ONE YEAR FITTED DEVELOPMENT FORECAST

Power Model Using General Liability Fit

Accident	Comparison for Year Ending 12/31/									
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	Total
1968	$70	$95	-$122	$81	$207	$103	$158	$180	$63	$835
1969	-11	-2	-28	28	183	64	97	38	152	521
1970	-118	92	-155	24	137	57	34	12	78	161
1971	-99	-92	192	-7	70	5	-5	43	88	195
1972	-8	18	47	-39	67	-2	5	11	65	164
1973	-74	-82	-24	-21	59	78	-64	134	-10	-4
1974	-29	-72	-16	-14	35	27	249	58	80	318
1975	-9	11	6	-8	-11	55	124	30	32	230
1976	83	-14	-19	3	-21	145	37	79	-122	171
1977	6	-53	-61	112	-68	57	17	100	-22	88
1978	-38	-315	13	82	70	91	152	-120	3	-62
1979	-20	-85	-12	98	29	82	-49	59	220	322
1980	-10	-23	59	28	-3	51	139	82	61	384
1981	—	—	—	34	-49	100	132	155	140	512
1982	—	—	—	8	17	-25	158	47	46	251
1983	—	—	—	—	-55	-16	86	-69	26	-28
1984	—	—	—	—	16	-24	-9	-7	-6	-30
Total	-$257	-$522	-$120	$409	$683	$848	$1,261	$832	$894	$4,028
Percent	-19.8%	-20.1%	-3.6%	9.7%	12.4%	12.3%	15.0%	8.1%	7.6%	

NOTE:

1. Dollar amounts are in thousands.

Exhibit 5
Page 1 of 3

COMPARISON OF ACTUAL AND FITTED DEVELOPMENT

Exponential Model

Accident	Actual Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident	Fitted Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	1.461	1.370	1.297	1.239	1.192	1.154	1.124	1.099	1.080
1969	1.498	1.400	1.321	1.258	1.207	1.166	1.134	1.107	1.086
1970	1.538	1.432	1.347	1.279	1.224	1.180	1.144	1.116	1.093
1971	1.581	1.467	1.375	1.301	1.242	1.194	1.156	1.125	1.101
1972	1.628	1.504	1.405	1.325	1.261	1.210	1.169	1.135	1.109
1973	1.678	1.545	1.438	1.351	1.282	1.227	1.182	1.146	1.118
1974	1.733	1.589	1.473	1.380	1.305	1.245	1.197	1.158	1.127
1975	1.792	1.636	1.511	1.410	1.330	1.265	1.213	1.171	1.137
1976	1.855	1.687	1.552	1.443	1.356	1.286	1.230	1.185	1.148
1977	1.924	1.742	1.596	1.479	1.385	1.309	1.248	1.199	1.160
1978	1.999	1.802	1.644	1.517	1.416	1.334	1.268	1.215	1.173
1979	2.079	1.867	1.696	1.559	1.449	1.361	1.290	1.233	1.187
1980	2.166	1.936	1.752	1.604	1.485	1.390	1.313	1.251	1.202
1981	2.259	2.012	1.813	1.653	1.524	1.421	1.338	1.272	1.218
1982	2.361	2.093	1.878	1.705	1.566	1.455	1.365	1.294	1.236
1983	2.470	2.181	1.949	1.762	1.612	1.492	1.395	1.317	1.255
1984	2.589	2.276	2.025	1.823	1.661	1.531	1.427	1.343	1.275

Selected Model Parameters:

$a = 29.233$ $b = 0.219$ $\alpha = 0.647$ Error = 3,404

Exhibit 5
Page 2 of 3

COMPARISON OF ACTUAL AND FORECAST DEVELOPMENT

Exponential Model

Accident	Actual Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.168	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident	Forecast Annual Development Through Year Ending 12/31/								Forecast
Year	1994	1995	1996	1997	1998	1999	2000+	1994+	Ultimate
1968	1.064	1.051	1.041	1.033	1.027	1.021	1.089	1.373	$2,746
1969	1.069	1.056	1.045	1.036	1.029	1.023	1.097	1.411	1,693
1970	1.075	1.060	1.048	1.039	1.031	1.025	1.106	1.450	2,030
1971	1.081	1.065	1.052	1.042	1.034	1.027	1.115	1.494	1,793
1972	1.087	1.070	1.056	1.045	1.036	1.029	1.125	1.539	1,231
1973	1.094	1.076	1.061	1.049	1.039	1.032	1.136	1.596	958
1974	1.102	1.082	1.066	1.053	1.042	1.034	1.147	1.654	1,489
1975	1.110	1.089	1.071	1.057	1.046	1.037	1.160	1.722	1,033
1976	1.119	1.096	1.077	1.062	1.050	1.040	1.174	1.798	899
1977	1.129	1.103	1.083	1.067	1.054	1.043	1.189	1.881	1,129
1978	1.139	1.112	1.090	1.072	1.058	1.046	1.206	1.975	1,778
1979	1.150	1.121	1.097	1.078	1.062	1.050	1.224	2.081	1,665
1980	1.162	1.130	1.105	1.084	1.068	1.054	1.244	2.202	1,321
1981	1.175	1.141	1.113	1.091	1.073	1.059	1.266	2.342	1,639
1982	1.189	1.152	1.122	1.098	1.079	1.063	1.290	2.497	749
1983	1.205	1.164	1.132	1.106	1.085	1.068	1.316	2.678	268
1984	1.221	1.178	1.143	1.115	1.092	1.074	1.345	2.892	289
									$22,710

Selected Model Parameters:

a = 29.233 b = 0.219 α = 0.647 Error = 3,404

NOTE:

1. The forecast ultimate losses are in thousands of dollars.

Exhibit 5
Page 3 of 3

ACTUAL LOSSES MINUS ONE YEAR FITTED DEVELOPMENT FORECAST

Exponential Model

Accident Year	Comparison for Year Ending 12/31/ 1985	1986	1987	1988	1989	1990	1991	1992	1993	Total
1968	$127	$96	-$167	$25	$140	$26	$85	$114	$2	$448
1969	30	-4	-53	-5	145	14	48	-7	114	282
1970	-25	85	-218	-45	61	-27	-44	-57	23	-247
1971	-20	-101	154	-85	-12	-79	-80	-20	33	-210
1972	33	9	7	-100	8	-65	-52	-38	24	-174
1973	-11	-93	-58	-61	20	33	-112	99	-49	-232
1974	24	-86	-55	-61	-12	-23	201	-7	21	2
1975	18	3	-27	-52	-56	14	80	-22	-14	-56
1976	91	-26	-58	-44	-72	101	-24	21	-178	-189
1977	54	-71	-113	59	-151	-9	-52	39	-84	-328
1978	90	-359	-49	-5	-45	-34	22	-256	-97	-733
1979	37	-104	-54	46	-56	-6	-146	-17	147	-153
1980	8	-28	47	-10	-53	4	85	10	-13	50
1981	–	–	–	4	-93	72	82	85	55	205
1982	–	–	–	3	9	-38	153	7	3	137
1983	–	–	–	–	-95	-35	74	-101	12	-145
1984	–	–	–	–	-23	-68	-43	-34	-28	-196
Total	$456	-$679	-$644	-$331	-$285	-$120	$277	-$184	-$29	-$1,539
Percent	35.1%	-26.1%	-19.2%	-7.9%	-5.2%	-1.7%	3.3%	-1.8%	-0.2%	

NOTE:
1. Dollar amounts are in thousands.

Exhibit 6
Page 1 of 3

COMPARISON OF ACTUAL AND FITTED DEVELOPMENT

Power Model

Accident Year	Actual Annual Development Through Year Ending 12/31/								
	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident Year	Fitted Annual Development Through Year Ending 12/31/								
	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	1.456	1.362	1.291	1.236	1.193	1.160	1.133	1.111	1.094
1969	1.495	1.391	1.313	1.253	1.207	1.170	1.141	1.118	1.100
1970	1.538	1.424	1.338	1.272	1.221	1.182	1.151	1.126	1.106
1971	1.586	1.459	1.365	1.293	1.237	1.194	1.160	1.134	1.112
1972	1.639	1.499	1.394	1.315	1.255	1.208	1.171	1.142	1.119
1973	1.698	1.542	1.427	1.340	1.274	1.223	1.183	1.151	1.126
1974	1.764	1.590	1.463	1.367	1.295	1.239	1.195	1.161	1.134
1975	1.837	1.644	1.502	1.397	1.317	1.256	1.209	1.172	1.143
1976	1.920	1.703	1.546	1.430	1.342	1.276	1.224	1.184	1.152
1977	2.013	1.770	1.595	1.466	1.369	1.296	1.240	1.197	1.162
1978	2.117	1.844	1.649	1.506	1.400	1.319	1.258	1.210	1.173
1979	2.235	1.928	1.709	1.550	1.433	1.344	1.277	1.225	1.185
1980	2.368	2.021	1.776	1.599	1.469	1.372	1.298	1.242	1.198
1981	2.520	2.127	1.851	1.654	1.509	1.402	1.321	1.260	1.212
1982	2.692	2.246	1.935	1.714	1.554	1.436	1.347	1.279	1.227
1983	2.889	2.380	2.030	1.782	1.604	1.472	1.374	1.300	1.243
1984	3.114	2.534	2.136	1.858	1.659	1.513	1.405	1.323	1.261

Selected Model Parameters:

a = 248,731 b = 4.489 $\alpha = 0.657$ Error = 3,378

Exhibit 6
Page 2 of 3

COMPARISON OF ACTUAL AND FORECAST DEVELOPMENT

Power Model

Accident Year	Actual Annual Development Through Year Ending 12/31/ 1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.558	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.845	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident Year	Forecast Annual Development Through Year Ending 12/31/ 1994	1995	1996	1997	1998	1999	2000+	1994+	Forecast Ultimate
1968	1.080	1.068	1.059	1.051	1.044	1.038	1.308	1.820	$3,640
1969	1.084	1.072	1.062	1.053	1.046	1.040	1.328	1.877	2,252
1970	1.089	1.076	1.065	1.056	1.048	1.042	1.349	1.941	2,717
1971	1.094	1.080	1.068	1.059	1.051	1.044	1.371	2.010	2,412
1972	1.100	1.085	1.072	1.062	1.053	1.046	1.394	2.086	1,669
1973	1.106	1.090	1.076	1.065	1.056	1.049	1.418	2.170	1,302
1974	1.113	1.095	1.081	1.069	1.059	1.051	1.443	2.262	2,036
1975	1.119	1.101	1.085	1.073	1.062	1.054	1.470	2.360	1,416
1976	1.127	1.107	1.090	1.077	1.066	1.056	1.499	2.471	1,236
1977	1.135	1.113	1.095	1.081	1.069	1.059	1.529	2.588	1,553
1978	1.144	1.120	1.101	1.086	1.073	1.062	1.561	2.725	2,453
1979	1.153	1.128	1.107	1.091	1.077	1.066	1.595	2.876	2,301
1980	1.163	1.136	1.114	1.096	1.081	1.069	1.631	3.040	1,824
1981	1.174	1.144	1.121	1.102	1.086	1.073	1.670	3.229	2,260
1982	1.186	1.154	1.128	1.108	1.091	1.077	1.712	3.441	1,032
1983	1.199	1.164	1.136	1.114	1.096	1.082	1.757	3.680	368
1984	1.213	1.175	1.145	1.121	1.102	1.086	1.805	3.952	395
									$30,866

Selected Model Parameters:

$a = 248{,}731$ $b = 4.489$ $\alpha = 0.657$ Error = 3,378

NOTE:

1. The forecast ultimate losses are in thousands of dollars.

Exhibit 6
Page 3 of 3

ACTUAL LOSSES MINUS ONE YEAR FITTED DEVELOPMENT FORECAST

Power Model

Accident	Comparison for Year Ending 12/31/									
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	Total
1968	$128	$99	-$163	$27	$139	$19	$73	$95	-$24	$393
1969	31	-2	-51	-4	145	12	43	-17	100	257
1970	-25	87	-213	-41	63	-29	-51	-68	6	-271
1971	-21	-99	157	-80	-9	-79	-84	-29	21	-223
1972	32	10	10	-96	10	-64	-53	-42	17	-176
1973	-14	-93	-55	-58	22	34	-113	97	-53	-233
1974	21	-86	-53	-58	-10	-21	202	-10	16	1
1975	17	2	-25	-49	-53	16	81	-23	-17	-51
1976	91	-27	-57	-42	-69	104	-22	22	-180	-180
1977	49	-75	-113	61	-146	-5	-48	40	-86	-323
1978	77	-372	-50	-2	-38	-25	29	-251	-97	-729
1979	31	-111	-55	48	-51	0	-140	-13	149	-142
1980	6	-31	47	-10	-50	7	89	13	-11	60
1981	–	–	–	4	-91	74	86	89	58	220
1982	–	–	–	3	9	-37	153	10	6	144
1983	–	–	–	–	-94	-34	75	-99	13	-139
1984	–	–	–	–	-23	-66	-41	-32	-26	-188
Total	$423	-$698	-$621	-$297	-$246	-$94	$279	-$218	-$108	-$1,580
Percent	32.5%	-26.8%	-18.5%	-7.1%	-4.4%	-1.4%	3.3%	-2.1%	-0.9%	

NOTE:

1. Dollar amounts are in thousands.

Exhibit 7
Page 1 of 3

COMPARISON OF ACTUAL AND FITTED DEVELOPMENT

Weibull Model

Accident Year	Actual Annual Development Through Year Ending 12/31/ 1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident Year	Fitted Annual Development Through Year Ending 12/31/ 1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	1.437	1.383	1.328	1.270	1.212	1.155	1.103	1.060	1.029
1969	1.457	1.404	1.349	1.292	1.234	1.177	1.122	1.075	1.039
1970	1.478	1.425	1.371	1.315	1.257	1.199	1.142	1.092	1.052
1971	1.498	1.445	1.392	1.337	1.279	1.221	1.164	1.111	1.066
1972	1.519	1.466	1.413	1.358	1.302	1.244	1.186	1.130	1.082
1973	1.539	1.486	1.433	1.379	1.324	1.266	1.208	1.151	1.100
1974	1.560	1.507	1.454	1.400	1.345	1.289	1.230	1.173	1.119
1975	1.582	1.527	1.474	1.421	1.367	1.311	1.253	1.195	1.139
1976	1.604	1.548	1.495	1.442	1.388	1.333	1.275	1.217	1.160
1977	1.627	1.569	1.515	1.462	1.409	1.354	1.298	1.240	1.182
1978	1.650	1.591	1.536	1.482	1.430	1.376	1.320	1.262	1.204
1979	1.675	1.613	1.557	1.503	1.450	1.397	1.342	1.285	1.226
1980	1.700	1.636	1.578	1.523	1.470	1.417	1.363	1.307	1.249
1981	1.727	1.660	1.600	1.544	1.491	1.438	1.384	1.329	1.271
1982	1.756	1.685	1.622	1.565	1.511	1.458	1.405	1.350	1.294
1983	1.786	1.711	1.646	1.587	1.532	1.479	1.426	1.372	1.316
1984	1.818	1.739	1.670	1.609	1.553	1.499	1.446	1.393	1.338

Selected Model Parameters:

$a = 23.214$ $b = 7.909$ $\alpha = 0.614$ Error = 3,561

Exhibit 7
Page 2 of 3

COMPARISON OF ACTUAL AND FORECAST DEVELOPMENT

Weibull Model

Accident	Actual Annual Development Through Year Ending 12/31/								
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993
1968	2.167	1.615	1.032	1.277	1.361	1.177	1.188	1.171	1.081
1969	1.750	1.381	1.138	1.242	1.561	1.188	1.197	1.099	1.200
1970	1.440	1.667	0.983	1.203	1.310	1.151	1.103	1.068	1.111
1971	1.474	1.107	1.871	1.155	1.224	1.098	1.067	1.104	1.132
1972	2.000	1.556	1.429	1.075	1.279	1.091	1.083	1.077	1.143
1973	1.583	1.053	1.150	1.087	1.360	1.324	0.933	1.381	1.034
1974	2.000	1.111	1.200	1.125	1.259	1.176	1.700	1.147	1.154
1975	2.250	1.667	1.333	1.150	1.087	1.320	1.455	1.125	1.111
1976	11.000	1.455	1.188	1.211	1.043	1.708	1.171	1.229	0.847
1977	3.000	1.267	1.000	1.789	0.941	1.281	1.122	1.283	1.017
1978	2.818	0.645	1.400	1.500	1.310	1.273	1.300	0.934	1.059
1979	3.000	1.000	1.250	1.867	1.250	1.343	0.979	1.196	1.455
1980	3.000	1.000	3.333	1.500	1.133	1.412	1.667	1.275	1.176
1981	–	–	–	1.714	0.750	2.222	1.750	1.514	1.321
1982	–	–	–	2.000	2.000	0.500	9.000	1.333	1.250
1983	–	–	–	–	0.556	0.800	3.250	0.538	1.429
1984	–	–	–	–	1.375	0.909	1.000	1.000	1.000

Accident	Forecast Annual Development Through Year Ending 12/31/								Forecast
Year	1994	1995	1996	1997	1998	1999	2000+	1994+	Ultimate
1968	1.011	1.003	1.001	1.000	1.000	1.000	1.000	1.015	$2,030
1969	1.017	1.005	1.001	1.000	1.000	1.000	1.000	1.023	1,228
1970	1.024	1.009	1.002	1.000	1.000	1.000	1.000	1.035	1,449
1971	1.033	1.013	1.004	1.001	1.000	1.000	1.000	1.052	1,262
1972	1.044	1.019	1.007	1.002	1.000	1.000	1.000	1.073	858
1973	1.057	1.028	1.010	1.003	1.001	1.000	1.000	1.102	661
1974	1.072	1.037	1.016	1.005	1.001	1.000	1.000	1.136	1,022
1975	1.089	1.049	1.023	1.008	1.002	1.000	1.000	1.180	708
1976	1.107	1.063	1.031	1.012	1.004	1.001	1.000	1.234	617
1977	1.127	1.079	1.042	1.018	1.006	1.001	1.000	1.299	779
1978	1.147	1.096	1.055	1.026	1.010	1.003	1.001	1.380	1,242
1979	1.169	1.115	1.070	1.036	1.015	1.004	1.001	1.474	1,179
1980	1.191	1.135	1.086	1.047	1.021	1.007	1.002	1.583	950
1981	1.213	1.156	1.104	1.061	1.030	1.011	1.004	1.717	1,202
1982	1.236	1.178	1.123	1.076	1.040	1.017	1.007	1.874	562
1983	1.258	1.200	1.144	1.093	1.052	1.024	1.012	2.058	206
1984	1.281	1.222	1.165	1.112	1.067	1.034	1.019	2.280	228
									$16,183

Selected Model Parameters:

a = 23.214 b = 7.909 α = 0.614 Error = 3,561

NOTE:

1. The forecast ultimate losses are in thousands of dollars.

Exhibit 7
Page 3 of 3

ACTUAL LOSSES MINUS ONE YEAR FITTED DEVELOPMENT FORECAST

Weibull Model

Accident	Comparison for Year Ending 12/31/									
Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	Total
1968	$131	$90	-$186	$5	$124	$25	$113	$175	$96	$573
1969	35	-5	-61	-17	134	7	57	22	161	333
1970	-10	87	-233	-66	38	-45	-42	-28	74	-225
1971	-5	-95	148	-106	-37	-101	-87	-7	70	-220
1972	43	16	4	-113	-10	-84	-62	-34	43	-197
1973	5	-82	-57	-67	9	20	-124	97	-38	-237
1974	40	-71	-51	-66	-23	-38	188	-18	27	-12
1975	27	13	-21	-54	-64	2	67	-34	-15	-79
1976	94	-10	-49	-44	-79	90	-43	6	-185	-220
1977	69	-45	-98	62	-159	-23	-72	20	-97	-343
1978	128	-293	-27	5	-50	-57	-14	-298	-123	-729
1979	53	-74	-37	55	-56	-19	-171	-41	126	-164
1980	13	-19	53	-2	-51	-1	73	-13	-37	16
1981	–	–	–	12	-89	71	73	65	27	159
1982	–	–	–	4	10	-38	152	-3	-11	114
1983	–	–	–	–	-88	-34	73	-108	8	-149
1984	–	–	–	–	-14	-65	-45	-39	-34	-197
Total	$623	-$488	-$615	-$392	-$405	-$290	$136	-$238	$92	-$1,577
Percent	47.9%	-18.8%	-18.3%	-9.3%	-7.3%	-4.2%	1.6%	-2.3%	0.8%	

NOTE:
1. Dollar amounts are in thousands.

Exhibit 8

COMPARISON OF ACTUAL 1993 DEVELOPMENT WITH FORECASTS FITTED THROUGH 1992

Accident	Actual	Fitted Factors			1993 Loss Forecast Error		
Year	Factor	Exponential	Power	Weibull	Exponential	Power	Weibull
1968	1.081	1.079	1.095	1.120	$4	-$26	-$72
1969	1.200	1.086	1.101	1.120	114	99	80
1970	1.111	1.093	1.107	1.120	23	5	-11
1971	1.132	1.100	1.113	1.120	34	20	13
1972	1.143	1.108	1.120	1.120	24	16	16
1973	1.034	1.117	1.128	1.120	-48	-54	-50
1974	1.154	1.127	1.136	1.120	21	14	26
1975	1.111	1.137	1.144	1.120	-14	-18	-5
1976	0.847	1.148	1.154	1.120	-177	-181	-161
1977	1.017	1.160	1.164	1.120	-84	-87	-61
1978	1.059	1.173	1.175	1.120	-97	-99	-52
1979	1.455	1.187	1.187	1.120	147	147	184
1980	1.176	1.202	1.200	1.120	-13	-12	29
1981	1.321	1.219	1.214	1.120	54	57	106
1982	1.250	1.237	1.229	1.120	3	5	31
1983	1.429	1.256	1.245	1.120	12	13	22
1984	1.000	1.276	1.263	1.120	-28	-26	-12
Total Error					-$25	-$127	$83
Percentage Error					-0.2%	-1.0%	0.6%

NOTE:

1. Dollar amounts are in thousands

Exhibit 9

SAMPLE POLLUTION DEVELOPMENT DATA

Accident	As of 12/31/			Development Factors	
Year	1991	1992	1993	92/91	93/92
1968	$320	$460	$530	1.4375	1.1522
1969	120	240	300	2.0000	1.2500
1970	320	530	620	1.6563	1.1698
1971	240	330	430	1.3750	1.3030
1972	100	110	110	1.1000	1.0000
1973	80	120	110	1.5000	0.9167
1974	110	150	110	1.3636	0.7333
1975	100	110	90	1.1000	0.8182
1976	50	50	40	1.0000	0.8000
1977	90	60	60	0.6667	1.0000
1978	10	20	30	2.0000	1.5000
1979	110	110	120	1.0000	1.0909
1980	0	0	0	–	–
1981	40	70	40	1.7500	0.5714
1982	40	50	70	1.2500	1.4000
1983	50	150	120	3.0000	0.8000
1984	160	320	170	2.0000	0.5313
1985	170	170	150	1.0000	0.8824

NOTE:

1. Dollar amounts are in thousands.

Exhibit 10

COMPARISON OF ACTUAL AND FITTED SAMPLE POLLUTION DEVELOPMENT

Exponential Model

Accident	Actual Through 12/		Fitted Annual Development Through Year Ending 12/31/										Forecast	One-Year Error	
Year	1992	1993	1992	1993	1994	1995	1996	1997	1998	1999	2000+	1994+	Ultimate	1992	1993
1968	1.438	1.152	1.215	1.172	1.138	1.111	1.089	1.072	1.058	1.046	1.202	1.963	$1,040	$71	-$9
1969	2.000	1.250	1.232	1.186	1.150	1.120	1.097	1.078	1.062	1.050	1.221	2.074	622	92	15
1970	1.656	1.170	1.251	1.201	1.162	1.130	1.104	1.084	1.067	1.054	1.241	2.193	1,360	130	-17
1971	1.375	1.303	1.271	1.217	1.175	1.140	1.113	1.091	1.073	1.058	1.263	2.332	1,003	25	28
1972	1.100	1.000	1.293	1.235	1.189	1.152	1.122	1.098	1.079	1.063	1.287	2.491	274	-19	-26
1973	1.500	0.917	1.316	1.254	1.204	1.164	1.132	1.106	1.085	1.068	1.314	2.672	294	15	-40
1974	1.364	0.733	1.341	1.274	1.220	1.177	1.142	1.114	1.092	1.074	1.343	2.877	316	2	-81
1975	1.100	0.818	1.369	1.296	1.238	1.191	1.154	1.123	1.099	1.080	1.375	3.118	281	-27	-53
1976	1.000	0.800	1.399	1.320	1.257	1.207	1.166	1.133	1.107	1.086	1.410	3.398	136	-20	-26
1977	0.667	1.000	1.431	1.346	1.278	1.223	1.179	1.144	1.116	1.093	1.449	3.726	224	-69	-21
1978	2.000	1.500	1.465	1.374	1.300	1.241	1.194	1.156	1.125	1.100	1.492	4.111	123	5	3
1979	1.000	1.091	1.503	1.404	1.324	1.261	1.209	1.168	1.135	1.108	1.540	4.566	548	-55	-34
1980	–	–	1.543	1.436	1.350	1.282	1.226	1.182	1.146	1.117	1.593	5.114	0	–	–
1981	1.750	0.571	1.587	1.471	1.379	1.304	1.244	1.196	1.158	1.127	1.653	5.772	231	7	-63
1982	1.250	1.400	1.634	1.509	1.409	1.329	1.264	1.212	1.170	1.137	1.719	6.560	459	-15	-5
1983	3.000	0.800	1.685	1.550	1.442	1.355	1.285	1.229	1.184	1.148	1.793	7.520	902	66	-113
1984	2.000	0.531	1.740	1.595	1.478	1.384	1.308	1.247	1.199	1.160	1.877	8.710	1,481	42	-340
1985	1.000	0.882	1.800	1.642	1.516	1.414	1.333	1.267	1.215	1.173	1.971	10.170	1,526	-136	-129
Total													$9,294	$114	-$911
Percent														5.9%	-31.6%

Selected Model Parameters:

$a = 29.233$ $b = 0.219$ $\alpha = 0.647$ $\beta = -3.515$ Error = 2,357

NOTE:

1. Dollar amounts are in thousands.

Exhibit 11

COMPARISON OF ACTUAL AND FITTED SAMPLE POLLUTION DEVELOPMENT

Power Model

Accident	Actual Through 12/		Fitted Annual Development Through Year Ending 12/31/										Forecast	One-Year Error	
Year	1992	1993	1992	1993	1994	1995	1996	1997	1998	1999	2000+	1994+	Ultimate	1992	1993
1968	1.438	1.152	1.200	1.166	1.139	1.117	1.099	1.085	1.073	1.063	1.528	2.644	$1,401	$76	-$6
1969	2.000	1.250	1.215	1.178	1.149	1.125	1.106	1.090	1.077	1.067	1.570	2.811	843	94	17
1970	1.656	1.170	1.233	1.192	1.160	1.134	1.113	1.096	1.082	1.071	1.616	3.005	1,863	135	-12
1971	1.375	1.303	1.252	1.207	1.172	1.144	1.121	1.103	1.087	1.075	1.665	3.225	1,387	30	32
1972	1.100	1.000	1.273	1.224	1.185	1.154	1.129	1.109	1.093	1.080	1.718	3.472	382	-17	-25
1973	1.500	0.917	1.296	1.242	1.199	1.166	1.139	1.117	1.099	1.085	1.775	3.765	414	16	-39
1974	1.364	0.733	1.322	1.262	1.215	1.178	1.149	1.125	1.106	1.090	1.837	4.097	451	5	-79
1975	1.100	0.818	1.351	1.284	1.232	1.192	1.160	1.134	1.113	1.096	1.904	4.487	404	-25	-51
1976	1.000	0.800	1.383	1.309	1.251	1.207	1.172	1.143	1.121	1.102	1.978	4.943	198	-19	-25
1977	0.667	1.000	1.418	1.336	1.272	1.223	1.185	1.154	1.129	1.109	2.058	5.482	329	-68	-20
1978	2.000	1.500	1.458	1.366	1.296	1.241	1.199	1.165	1.138	1.117	2.147	6.131	184	5	3
1979	1.000	1.091	1.502	1.399	1.321	1.261	1.215	1.178	1.148	1.125	2.244	6.910	829	-55	-34
1980	–	–	1.552	1.437	1.350	1.283	1.232	1.191	1.159	1.134	2.352	7.856	0	–	–
1981	1.750	0.571	1.608	1.479	1.382	1.308	1.251	1.206	1.171	1.143	2.473	9.027	361	6	-64
1982	1.250	1.400	1.672	1.526	1.417	1.335	1.272	1.223	1.184	1.154	2.607	10.482	734	-17	-6
1983	3.000	0.800	1.744	1.579	1.457	1.365	1.295	1.241	1.199	1.165	2.758	12.313	1,478	63	-117
1984	2.000	0.531	1.825	1.638	1.501	1.399	1.321	1.261	1.214	1.177	2.928	14.635	2,488	28	-354
1985	1.000	0.882	1.918	1.706	1.551	1.436	1.349	1.283	1.231	1.191	3.121	17.639	2,646	-156	-140
Total													$13,746	$101	-$920
Percent														5.2%	-31.9%

Selected Model Parameters:

a = 108,782 b = 4.247 α = 0.602 β = -3.382 Error = 2,378

NOTE:

1. Dollar amounts are in thousands.

Exhibit 12

COMPARISON OF ACTUAL AND FITTED SAMPLE POLLUTION DEVELOPMENT

Weibull Model

Accident	Actual Through 12/		Fitted Annual Development Through Year Ending 12/31/										Forecast	One-Year Error	
Year	1992	1993	1992	1993	1994	1995	1996	1997	1998	1999	2000+	1994+	Ultimate	1992	1993
1968	1.438	1.152	1.246	1.188	1.133	1.084	1.046	1.020	1.007	1.002	1.000	1.322	$701	$61	-$16
1969	2.000	1.250	1.269	1.211	1.154	1.102	1.059	1.029	1.011	1.003	1.001	1.407	422	88	9
1970	1.656	1.170	1.291	1.233	1.175	1.121	1.074	1.039	1.016	1.005	1.001	1.502	931	117	-33
1971	1.375	1.303	1.313	1.256	1.197	1.141	1.091	1.051	1.024	1.008	1.003	1.621	697	15	16
1972	1.100	1.000	1.335	1.278	1.220	1.163	1.110	1.065	1.033	1.013	1.005	1.764	194	-24	-31
1973	1.500	0.917	1.357	1.300	1.242	1.184	1.129	1.081	1.044	1.019	1.008	1.925	212	11	-46
1974	1.364	0.733	1.378	1.322	1.265	1.207	1.150	1.099	1.057	1.027	1.013	2.122	233	-2	-88
1975	1.100	0.818	1.399	1.344	1.287	1.229	1.171	1.118	1.071	1.037	1.021	2.348	211	-30	-58
1976	1.000	0.800	1.420	1.366	1.310	1.252	1.194	1.138	1.088	1.049	1.032	2.625	105	-21	-28
1977	0.667	1.000	1.441	1.387	1.331	1.274	1.216	1.159	1.106	1.062	1.048	2.942	177	-70	-23
1978	2.000	1.500	1.461	1.408	1.353	1.296	1.238	1.180	1.126	1.078	1.068	3.321	100	5	2
1979	1.000	1.091	1.481	1.428	1.374	1.319	1.261	1.203	1.146	1.095	1.094	3.774	453	-53	-37
1980	–	–	1.502	1.449	1.396	1.340	1.283	1.225	1.168	1.114	1.128	4.315	0	–	–
1981	1.750	0.571	1.522	1.469	1.416	1.362	1.306	1.248	1.190	1.134	1.170	4.963	199	9	-63
1982	1.250	1.400	1.543	1.490	1.437	1.383	1.328	1.270	1.212	1.155	1.221	5.729	401	-12	-5
1983	3.000	0.800	1.564	1.510	1.457	1.404	1.349	1.293	1.234	1.177	1.284	6.654	798	72	-107
1984	2.000	0.531	1.586	1.531	1.478	1.425	1.371	1.315	1.257	1.199	1.359	7.777	1,322	66	-320
1985	1.000	0.882	1.608	1.552	1.498	1.445	1.392	1.337	1.279	1.221	1.449	9.116	1,367	-103	-114
Total													$7,156	$129	-$942
Percent														6.6%	-32.7%

Selected Model Parameters:

a = 23.214 b = 7.909 α = 0.614 β = -3.593 Error = 2,391

NOTE:

1. Dollar amounts are in thousands.

Bibliography

[1] Sherman, R., "Extrapolating, Smoothing, and Interpolating Development Factors," Proceedings of the Casualty Actuarial Society, Volume LXXI (1984), pp. 122-155.

[2] Weller, A., "Generalized Bondy Development," Presented at the 1990 Casualty Loss Reserve Seminar, Sessions 2G & 7G, September 9-11, 1990, Transcripts pp. 541-598.

[3] Zehnwirth, B., "Stochastic Regression Models with Applications to Loss Reserving," Presented at the 1989 Casualty Loss Reserve Seminar, Session 4G, September 18-19, 1989, Transcripts pp. 825-918.

[4] Zehnwirth, B., "Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals and Risk Based Capital," Casualty Actuarial Society Forum, Spring 1994, pp. 447-606.

[5] Klugman, S. & Parsa, R., "Minimum Distance Estimation of Loss Distributions," To appear, Proceedings of the Casualty Actuarial Society, Volume LXXX (1993)

[6] Stanard, J., "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques," Proceedings of the Casualty Actuarial Society, Volume LXXII (1985), pp. 124-148.