Introduction to Reprints of 'Risk and Uncertainty: A Fallacy of Large Numbers' and 'Portfolio of Risky Projects

by John M. Cozzolino

INTRODUCTION TO PAUL A. SAMUELSON'S "RISK AND UNCERTAINTY; A FALLACY OF LARGE NUMBERS"

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The Paper "Risk and Uncertainty: A Fallacy of Large Numbers" by Paul A. Samuelson, was published in Scientia in April-May, 1963. It was later reprinted in the Collected Scientific Papers of Paul A. Samuelson, Volume 1, pp. 153-8, MIT Press, 1966. It had a very distinguished influence on the ideas of risk and portfolio for investment applications.

The paper first got the attention of Pratt, Zeckhauser, and other mathematical economists and thereby spawned several related papers. Unfortunately it apparently did not reach the one group most concerned with property and casualty insurance. We hope that the current republication will rectify this.

Samuelson's Paper proved that risk sharing, which is a fractional participation in one risk, is a more fundamental way to reduce risk than having a replication of identical, independent risks. He proved that if you would not accept one risk, then you would not accept any greater number of identical, independent risks. While people loosely say that the insurance institution exists because of the law of large numbers or because it can insure many risks, this is not the case; the real reason insurance exists is because insurers are risk averse, meaning that their utility curves are concave, and they take fractional participation. He also stated that within a sufficiently small interval of outcomes, the concave function is a linear function. So we are back to the fundamental role of the utility curve. If the expected monetary value is positive then there is a best share greater than zero.

Paul A. Samuelson said this with such clarity that it was obviously true to the reader.

After reading his, I wrote a paper that was published in Decision Sciences in 1974 "Portfolios of Risky Projects". While sharing was always looked at in the perspective of Pareto optimality; Samuelson's paper seemed to suggest that was unnecessary. I realized that the reason there was a best share was because it was assumed that both parties had a concave utility function. Others may have attributed the existence of the best share to the fact that there were two or more parties. In fact, for a partial share to be best for a single party, all you need is that the single party have a concave utility function (be risk averse).

But, you might say, how can you take a share without knowing the party with whom you share? The answer is that when there is an established, deep market for shares, exactly as there is in Lloyds. It is better to assume that you can always find the needed partner or set of needed partners, than to seek the best share for some specific partner who may or may not exist. Finding your preferred share is more to the point. The Paul A. Samuelson paper seems to suggest the framework of the single decision maker trying to find his best decision for himself. Perhaps other observers saw this differently and did not realize the fundamental nature of the "Best Share". From a pure mathematical perspective, share is just a number multiplying the random variable "loss". Therefore the variance of that product must be proportional to that multiplier squared. Cutting the share by one half cuts the variance by one quarter. That is risk reduction. Finance, both real and theoretical, has advanced from the dark ages by inventing markets.

My paper proves the existence of a best share by assuming only a concave utility function and a positive expected monetary value. Whether you speak of size of share retained or of the number of equal partners is up to you. It appears more realistic to solve this "best share" problem, at least in the insurance context. In any case it seems good to expose the casualty actuaries to Samuelson's insightful work so that they can draw their own conclusions.

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