Using the Whole Triangle to Estimate Loss Reserves

by Frank Pierson

INTRODUCTION

This paper will suggest an easy, straightforward way to complement the basic methods currently used by most actuaries to estimate ultimate losses. Most actuaries use some variation of standard loss development or Bornheutter-Ferguson methods. These methods can be applied to a variety of data, e.g., paid, incurred, claim counts or average severities. The last step of most analyses is to apply a development pattern to the latest evaluation of data to estimate ultimate values.

All of these methods rely, to some degree, on analyzing "data triangles" to determine the appropriate development patterns. Most actuarial papers have concentrated on the appropriate adjustments to the underlying data(e.g., Berquist-Sherman) or determining the correct way to calculate these patterns(e.g., Sherman, Weller). There is not much written on how to improve the estimate of the forecasts after the actuary has developed the factors.

In this paper, I propose adding a step to the standard methods by applying the selected development pattern to all values in the data triangle. This step can be used in most methods in use by actuaries today and can be applied to data aggregated by policy year, underwriting year, accident year or report year. This paper uses accident year without loss of generality.

CRITIQUE OF STANDARD METHODS

There are a number of shortcomings associated with the typical actuarial analysis.

Although the historical data is used to select the development pattern, once selected, the development pattern is usually applied to the data as of the latest evaluation date only. This is particularly true when the development pattern is based solely on external data. The analysis ignores the fact that historical data other than those at the latest evaluation date ever existed, however, if the development pattern is correct then it should apply equally to data at evaluation dates other than the latest one.

Given that most projections are a function of the latest diagonal only, they are very sensitive to random movements from year to year in the known losses even if the selected development pattern remains unchanged. The projected ultimates will move up or down from year to year solely due to these random movements. There are times when these movements are substantial and, therefore, result in large movements in the projected ultimate loss. If the actuarial analysis truly measured the underlying losses and their development pattern, then twelve months of additional data should not alter significantly the actuary's view of the ultimate loss. At a minimum, there should be some credibility weighting between the latest indication and prior indications.

In many loss reserve analyses, the projections tend to creep up or down(mainly up) from one evaluation to the next. One standard explanation is that the change in ultimate loss was due to "unexpected adverse development." This explanation is valid once or twice, but is not valid year after year. At some point, continued unexpected development should alert the actuary that the method is not matching the data accurately.

METHODOLOGY

This section of the paper will outline the steps needed to add the proposed procedure to various standard actuarial techniques.

Before we start, let's define a little notation. For each accident year i, i=1,2,...,n, evaluated at the end of year t, t=1,2,...,n-i+1:

Q	=	n-i+1, i.e., the latest evaluation date of each accident year (I will ignore the subscript unless needed in the context),
L(i,t)	=	the cumulative loss for accident year i at evaluation date t,
d(t)	=	factor to develop losses evaluated at year t to ultimate,
U(i,t)	=	projected loss based on d(t) and L(i,t),
Ult(i)	=	selected ultimate for year i,
Pult(i)	=	a priori ultimate for year i,
XL(i,t)	=	expected cumulative loss for accident year i evaluated at the end of year t,
E(i,t)	=	error term, and
BF(i,t)	=	Bornheutter-Ferguson estimate or accident year i evaluated at the end of year t.

The d(t)'s are based on the standard analysis and may be based solely on the company's actual data, i.e., L(i,t), external data or a combination of both.

For clarity during the discussions that follow, I refer to the "standard" method as the one under discussion without the proposed additional steps and the "augmented" method as the one with the additional steps.

Both the standard and augmented methods are highly dependent on accurate estimates of the loss development patterns including the selection of tail factors. The following discussion assumes that the selected pattern is accurate (including the appropriate tail factors) and that variability in projected ultimates is due to random fluctuations.

Loss Development Method

The standard loss development method sets the projected ultimate loss equal to:

$$Ult(i) = L(i,Q)*d(Q).$$

This method is criticized, as outlined above, for being much too sensitive to movements in losses over the latest calendar period. However, if there were multiple projections of ultimate for each accident year based on the selected development pattern at various evaluation points, this method would be much less sensitive to random noise.

Using the augmented method, one can calculate a "triangle" of projected ultimates for each accident year i, t=1,2,...,Q:

$$U(i,t) = L(i,t)*d(t).$$

The selected ultimate, Ult(i), could then be based on some or all of the U(i,t) and not just U(i,Q). Exhibit I shows how the proposed method could be used in analyzing industry-wide general liability paid losses.

Each U(i,t),t<Q, represents the projected ultimates from the standard method at prior evaluations, assuming that the most recently selected development pattern applied at all prior evaluation dates. The change in U(i,t) for a given accident year, say, 1985, approximates the change in the projected ultimate loss for 1985 using the standard method. If we assume, simplistically, that the ultimate loss under the augmented method equals the average of all U(i,j), j=1,...,t, then we can compare the variability of loss projections over time between the two methods. Exhibit II shows graphically the change over time in the projections of ultimate for accident year 1985 based on the standard loss development method versus the projections based on the proposed method. As you can see, the variability in the projected ultimates is less using the augmented method.

Bornheutter-Ferguson Method

The standard Bornheutter-Ferguson method sets the projected ultimate loss equal to:

$$Ult(i) = L(i,Q)+[Pult(i)-XL(i,Q)], \text{ where}$$

$$XL(i,Q)= Pult(i)/d(Q).$$

This method is commonly described as a combination of the loss development method and the expected loss ratio method. The major advantage of this method over the loss development method is that it is less sensitive to random noise in L(i,t). However, I believe that this method loses some of its advantage relative to the loss development method due to the fact that it is usually applied to the latest diagonal only. The assumption underlying adding expected IBNR to L(i,Q) is that future losses are more a function of the Pult(i) and d(t) than they are of L(i,Q) because of the effect of random noise on L(i,Q). Many times, in practice, Ult(i) is significantly different from Pult(i) which may indicate that either, or both of Pult(i) and d(q) are incorrect. If, however, one assumes that Pult(i) and d(t) are valid, then BF(i,t), t<Q, should produce valid estimates of Ult(i) as well.

Under the augmented method, one can calculate a "triangle" of ultimates for each accident year i, (t=1,2,...,Q):

$$BF(i,t) = L(i,t)+[Pult(i)-XL(i,t)].$$

The selected ultimate, Ult(i), is then based on all of the U(i,t) and not just U(i,Q). Exhibit III shows how the augmented method could be used for this method for industry-wide general liability paid losses. For the purposes of Exhibit III, Pult(i) is based on the results of the loss development analysis.

One of the major drawbacks of the Bornheutter-Ferguson method is that the actuary must select both the development pattern and the initial ultimate loss, Pult(i). The Pult(i) is usually calculated by multiplying the ultimate premium for accident year i times an expected loss ratio or is based on the result of the prior reserve study. If Pult(i) and d(Q) are correct, then Ult(i) should equal Pult(i). A significant difference between Ult(i) and Pult(i) would indicate that either Pult(i), d(Q) or both are wrong. Of course, the difference could be due to random noise and Pult(i) and d(Q) were correct. This determination is made easier by reviewing the triangle of BF(i,t) calculated above, e.g., seeing systematic increases or decreases in the projections over time.

ADVANTAGES OF THE AUGMENTED METHODOLOGY

Using the augmented methodology can improve the analysis in the following ways:

The actuary now has more than one estimate of ultimate on which to base his selections. This reduces the sensitivity of the selected ultimate to random fluctuation in L(i,t).

Many actuaries use some form of curve fitting to smooth out fluctuations in the observed data(for example, see Sherman, Weller, Clarke). The tail factor is usually extrapolated from this curve. Unfortunately, most curves in use today do not fit the data equally well over the entire historical period. For example, many curves do not fit well at early maturities (less than 36 months). This can be a significant problem since the largest reserves are usually associated with the most recent accident years which have data only at these early maturities.

To overcome this problem, two adjustments to the augmented method can be made, either individually or together. The first, assuming that data exists at early maturities for older accident years, is to analyze the historical relationship between the projected ultimates at the early maturities to those projected for later maturities, e.g., 12 months versus 36 months and subsequent. This analysis may indicate whether or not there is any significant and systematic bias in the projection at early maturities. The actuary now has information on which to adjust the projections for the less mature accident years based on a straight application of the underlying method.

The second adjustment compares the variation in the L(i,t) to the variation in the projected ultimates. In many cases, there is a relationship between L(i,t) or movements in L(i,t) from one accident year to the next and the distortion in projected ultimates at early maturities compared to those at later maturities. For example, the actuary may believe that for a given data set that even though there does not exist any apparent bias in the projected ultimates at early maturities, historically, the projected ultimates appear to be overstated whenever there is a significant increase in L(i,1) over l(i-1,1). Now the actuary is presented with the case where L(n,1) represents a significant increase over L(n-1,1). How should the projected ultimate for accident year n be adjusted? Exhibit IV shows how these adjustments might be calculated.

In addition to calculating an estimate of the expected reserve, there is a growing interest on the part of companies for a "range of reasonableness" or "confidence interval." This is very difficult to develop using most standard methods. The augmented method may help the actuary get a better feel for the variability in the estimates by analyzing the variance of the historical projections either in absolute dollar or relative "error" terms. The actual mechanics of this are beyond the scope of this paper.

Many times the actuary must deal with changes in either speed of payment/closure or case reserve adequacy. This is usually handled by either adjusting the data or selecting the development pattern based on the latest N diagonals. N is selected to include only the data that is consistent with current conditions. By reviewing the entire triangle of projected ultimates, the actuary can confirm the change by looking for a change in the pattern of ultimates. If there is no significant change in the ultimates, the suspected underlying change may not have had any significant impact on the development pattern. This procedure can also help identify unidentified changes which can lead to the need for further investigation.

For any of the standard methods, one can calculate a triangle of error terms for each accident year at each evaluation point, $t=1,2,...,Q$, i.e.,

$$E(i,t) = [XL(i,t)-L(i,t)]/L(i,t),$$

where $XL(i,t)$ is calculated using the method underlying the projected ultimate losses.

If the selected development pattern truly fits the data, then the error terms should be randomly distributed with a mean near zero. Patterns in the error terms can highlight problems such as auto-correlation and other statistical problems. Since the standard loss development methods are linear estimators, then the assumptions underlying classic linear regression should apply to these methods. As such, if the development pattern is correct, then the error terms should have an expected value of zero, equal variance (for a given development period) and not be correlated with one another. Graphing E(i,t) can help the actuary determine whether there is any bias or auto-correlation in the estimates.

In addition to the standard view of error terms, the actuary can also compare expected to actual calendar year losses. This part of the analysis is rarely performed, but is one that is important to anyone who is concerned with the aggregate cash flow of losses across more than one accident year. Relatively small accident year errors may mask significant calendar year errors. A large calendar year "error" may indicate a significant structural change in the loss process during that year, e.g., a change in the claims handling or a large commutation.

The augmented method focuses attention where it belongs, i.e., on the variation in the estimated ultimates. We have, as a profession, tended to focus on variability in the development patterns and how to best evaluate these patterns. We have not looked at the variability in the resulting ultimates (one significant exception is Stanard).

I believe that the augmented method is an improvement over the standard method in meeting the four key attributes of a reserving system as outlined by Steven Lowe. This procedure improves the stability of the indication from year to year; it objectively combines more of the available information in deriving the current indication because it uses the entire historical data; and it is as integrated and interactive as the standard method.

DISADVANTAGES OF THE AUGMENTED METHOD

The augmented method may be less sensitive to changes in the underlying losses because the selected ultimate loss is based on more than data at the latest evaluation.

The augmented method does not, by itself, eliminate changes in Ult(i) from year to year due to changes in the assumed development pattern. If the basic analysis indicates a change in the assumed pattern, it is not clear whether the standard or augmented method would be affected more by such a change. If there is a large difference in the assumed development pattern from one year to the next without a significant change in the underlying business or claims handling practices, one should question the methodology used in selecting the development pattern.

PRACTICAL POINTS

I have used the augmented method for some time and have a few practical observations.

The augmented method can be used for most lines of business, even those that are inherently volatile such as Excess or Surplus Lines, or where data volume is sparse. In these lines, many actuaries apply Bornheutter-Ferguson for stability because other methods such as paid or incurred development methods are too sensitive to volatility in the latest value. Since the augmented method adds stability to most standard development methods, actuaries might be able to use methods other than Bornheutter-Ferguson. More than one approach can be more important for these lines of business than for lines that are very stable.

If the selected development pattern is based on the latest N diagonals because of a perceived change in the data, it is usually appropriate to base the selection of ultimate loss on no more than the latest N+1 projections. It is inconsistent to exclude historical data when calculating the development pattern and then to include projections based on the excluded data in selecting the final estimate of ultimate.

The selection of N is not always easy. Sometimes the correct value of N is apparent from the data or from discussions with management, e.g., the discussions may indicate that a change in claims handling took place three years ago and, therefore, the correct choice for N might be 2. When the choice for N is not so apparent, the augmented method can help the actuary select the optimal N, although the process might be iterative by viewing the effect of different values of N on the triangle of ultimates. Typically, the data fluctuates up and down period to period and, therefore, the standard method may over or understate the ultimate losses depending on whether the losses are at a peak or trough. Using more than one estimate will smooth out this "expected" variability in the estimates.

If d(t) is correct, then U(i,t) should converge to the true ultimate over time. Many times, however, some accident years converge while others do not. For example, the older accident years might converge while the later years do not. This might indicate that some unaccounted for change took place and the actuary should investigate further. It may be necessary to use a different pattern for the two groups of years. In other cases, the years that converge may be spread among years that do not. In addition, some years may trend up and others trend down while some years move up(or down) for a few evaluations and then down(up). The actuary must use his/her judgment to decide for which years the selected pattern is appropriate.

This procedure should be easy to incorporate into most analyses since all the needed elements are already calculated.

Although most of the above discussion deals with the loss development and Bornheutter-Ferguson methods, it can be adapted for other methods as well. For example, the method outlined by Clarke fits a curve to cumulative paid or incurred loss ratios for each accident year or groups of accident years. If we define R(i,t) as the fitted ratio at time t for accident year i, then we can calculate a triangle of U(i,t):

$$U(i,t) = L(i,t)*R(i,u)/R(i,t),$$

where u is when losses reach ultimate. R(i,u)/R(i,t) is equivalent to d(t) in this case. If the curve fit is correct, then the U(i,t)'s should be stable. Exhibit V reproduces the graph for the 1981 year of account contained in Mr. Clarke's paper(p. 30) with the U(i,t)'s superimposed on it. As expected, the U(i,t) begin to converge, but not until after the first 8 quarters.

In his discussion of Mr. Clarke's paper, John Narvell makes a number of observations that are germane to this paper:

1) "The difference between a simple LDF and the more sophisticated approach in this present paper is that the most current observation is not simply multiplied by the appropriate LDF to ultimate. Rather there is some consideration for a random error contained in the endpoint.... Effectively each historical data point is given equal credibility in the estimation of ultimate losses."

2) "...a major difference between the author's approach and the traditional LDF or B-F methods...[is that] the negative exponential considers only the development patterns for the particular year before year t...[and the] traditional LDF or B-F methods [consider] only development data (for other years) after age t...."

3) "... [a] major advantage is that the curve form naturally leads to graphical display and interpretation."

I believe that the augmented methods discussed above compare favorably to Clarke's "sophisticated approach" in that they:

1) consider the random error contained in the end-point,

2) give some credibility to historical data points,

3) consider development from both before and after age t, and

4) they lead naturally to graphical display and interpretation.

With respect to (2), I do not agree with the implication that giving equal credibility to each historical data point is desirable. Given how external and internal changes influence losses, the latest data points should probably, though not automatically, be given greater weight than earlier points. An augmented method would allow the actuary to give the appropriate weight to each data point.

With respect to (3), the augmented methods consider both development after age t in the calculation of the age-to-ultimate development factors and development before age t by selecting the ultimate losses as a function of historical projections.

With respect to (4), graphing the projected ultimates or error terms for an accident year is the quickest way to determine how well the development pattern fits the historical data. In addition, putting the projections from various methods together in a single graph can help the actuary assess the quality of his/her estimate. Exhibit VI shows the projected ultimates for two accident years based on standard paid and incurred development methods. As you can see, the paid and incurred projections for accident year 1985 (sheet B) are converging while for accident year 1983 (sheet A), only the paid projections are converging. The non-convergence of the incurred projections would lead me to dig deeper into the numbers for that year.

Different methods are more stable or they converge more quickly than others. Bornheutter-Ferguson, for example, tends to converge more quickly than paid or incurred development methods. This should be expected given the underlying theory of each method.

SIMULATION

Based on my usage of this procedure with a wide range of books of business, I believe that it increases the stability and predictability of the underlying, basic reserve methods. In order to put this hypothesis to a stricter test, I propose the following simulation of the loss development method (I have not had time to model this adequately)..

The reader should note that as I worked on the simulation, I found it extremely difficult to program the proposed procedure to work "correctly" for the latest two accident years in the wide range of outcomes created during the simulation because of the amount of judgment needed. Therefore, the simulation may have to be limited to all but the latest two years. One must keep in mind that the augmented method does not make loss reserving mechanical, particularly for the most immature accident years; it simply gives the actuary more information than the standard methods on which to base his/her judgment. The simulation, therefore, should be viewed as an approximation at best since it was impossible to include "actuarial judgment" for each iteration.

In order to test the augmented method against the standard method, the simulation would have to create a triangle of losses and then calculate loss development factors based on that triangle of data. Using those items, the simulation would then calculate ultimate losses based on the standard and augmented method at various points in time, e.g., t=Q-2, Q-1 and Q. Each method would produce ultimate losses very close to the true ultimates over a large number of iterations since both methods are not significantly biased. However, the variability in the ultimates from one iteration to the next may be significant. The following model would compare the variability in the ultimate losses from one evaluation point to the next for the standard versus the augmented methods.

Step 1.

Select the true ultimate loss for each accident year, L(i,u), and the underlying development pattern, d(t). The simulation should look at many different situations, e.g.:

a. L(i,u) = $100 million for all accident years,
b. L(i,u) increasing at a constant rate,
c. L(i,u) decreasing at a constant rate, and
d. L(i,u) = $100 million +/-10%(uniformly distributed).

Step 2.

Assume random noise around d(t) and calculate a "historical" loss triangle, i.e.,

$$L'(i,t) = L(i,u)/[d(t)+(RAND-.5)*2*RN(t)]$$

where RAND is a random number uniformly distributed over (0,1) and RN(t) is the selected range of random noise allowed at each evaluation point. RN(t) should decrease "over time" to reflect the fact that the random noise apparent in d(t) decreases as losses mature.

Step 3.

Calculate d'(t) based on the weighted average of all years. The tail could be set equal to the true d(n) in order to eliminate any distortion in the results of the simulation due to mis-estimation of the tail factor. This assumption should not affect the conclusions of this analysis because the simulation is designed to compare the relative stability and predictability of the augmented versus traditional loss development method.

Step 4.

Calculate triangle of U(i,t) based on the L'(i,t) simulated in step 2 and d'(t) calculated in step 3. For the standard method, set Ult(i) equal to U(i,Q). For the augmented method, set Ult(i) equal to the average of U(i,t), t=2,3...Q. U(i,1) is not used except when Q-2=1. The most variability is in U(i,1) and may require significant judgment. To minimize the need for judgment in the simulation, U(i,1) is not used unless it is the only projected ultimate available.

For each method, the average and standard deviation of the U(i,t) values are calculated by accident year. Although both methods produce an accurate estimate of the ultimate losses, on average, across all iterations, the hypothesis is that: 1) the standard method has a larger standard deviation at each point in time than that for the augmented method and 2) there is more variability in the ultimates based on the standard method from one evaluation to the next than for the augmented method.

CONCLUSIONS

The purpose of this paper is to outline a simple method to help actuaries do a better job of projecting ultimate losses, whether for pricing or reserving. I hope that it sparks some interest on the part of other actuaries.

I would like to thank John Narvell for the inspiration to write this paper and Carol Rennie for making it much easier to read.

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INDUSTRY COMPOSITE - MEDICAL MALPRACTICE

Exhibit I

Loss Development Method

Historical Paid Loss + ALAE at N months - L(i,t)

AY	12	24	36	48	60	72	84	96	108	120
1982	50	172	383	675	952	1,197	1,385	1,517	1,631	1,706
1983	67	218	487	800	1,121	1,397	1,604	1,772	1,898	
1984	104	298	609	973	1,337	1,612	1,836	2,000		
1985	43	254	602	1,025	1,406	1,736	1,924			
1986	52	261	626	1,006	1,362	1,621				
1987	37	267	635	1,029	1,348					
1988	56	338	733	1,092						
1989	79	396	853							
1990	88	445								
1991	98									
Cum LDFs = d(t)	64.485	11.674	4.747	2.729	1.941	1.569	1.369	1.251	1.178	1.129

Projected Ultimate Loss + ALAE at N months - U(i,t) = L(i,t)*d(t)

AY	12	24	36	48	60	72	84	96	108	120	Average of All	Average of last 4
1982	3,224	2,008	1,818	1,842	1,848	1,878	1,896	1,898	1,921	1,927	2,026	1,910
1983	4,320	2,545	2,312	2,183	2,176	2,192	2,195	2,217	2,235		2,486	2,210
1984	6,706	3,479	2,891	2,655	2,595	2,529	2,513	2,503			3,234	2,535
1985	2,773	2,965	2,858	2,797	2,729	2,723	2,633				2,783	2,721
1986	3,353	3,047	2,972	2,745	2,644	2,543					2,884	2,726
1987	2,386	3,117	3,014	2,808	2,617						2,788	2,889
1988	3,611	3,946	3,479	2,980							3,504	3,504
1989	5,094	4,623	4,049								4,589	4,589
1990	5,675	5,195									5,435	5,435
1991	6,319										6,319	6,319
Cum 1987 Avg	2,386	2,751	2,839	2,831	2,788							

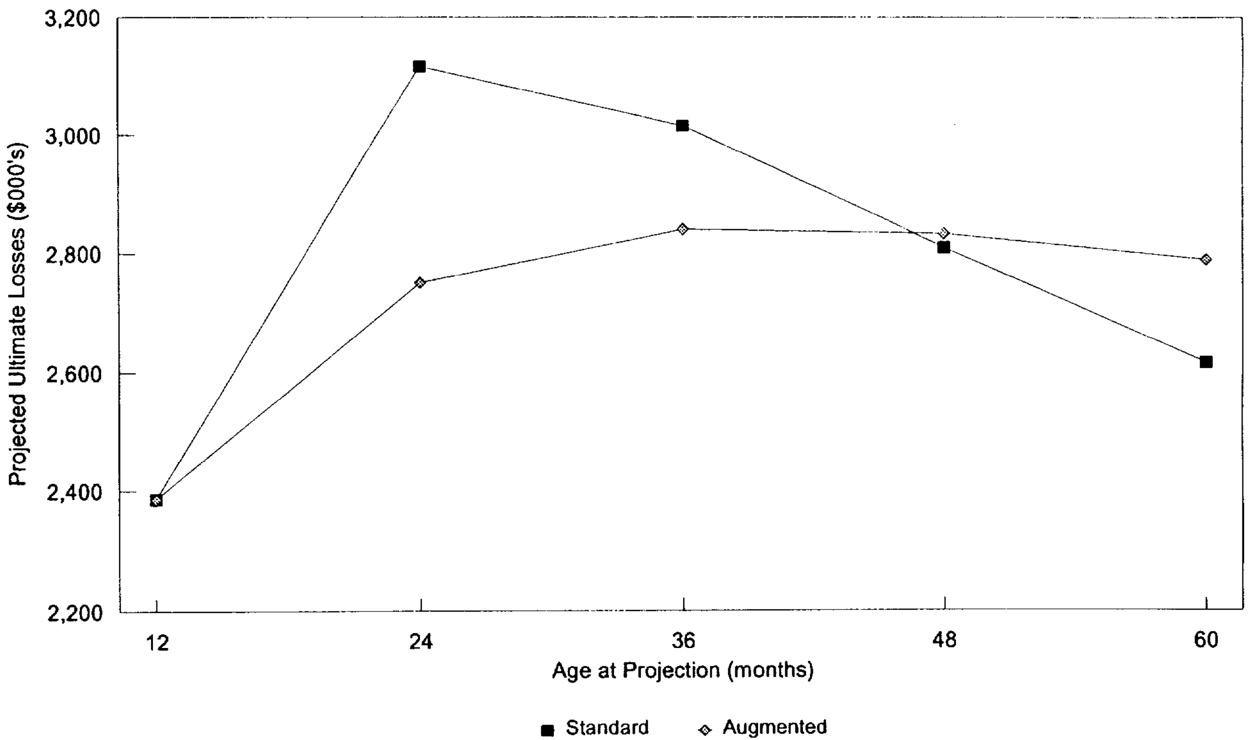
Notes: [1] Accident Year 1982 @12months: 3,224 = 50*64.485
[2] The 1987 AY is used in Exhibit 2 - A graphical comparison of the relative stability of the Standard and the Augmented methods

Data Source: Best's Aggregates & Averages - 1991

Exhibit II

Change in Projections of Ultimate Losses by Age - AY 87

Standard vs. Augmented



Source: Best's Aggregates & Averages - 1991
INDUSTRY COMPOSITE - MEDICAL MALPRACTICE

Exhibit III

INDUSTRY COMPOSITE - MEDICAL MALPRACTICE

Bornheutter-Ferguson Method

Historical Paid Loss + ALAE at N months - L(i,t)

AY	12	24	36	48	60	72	84	96	108	120	'Prior' Ult Pult(i)
1982	50	172	383	675	952	1,197	1,385	1,517	1,631	1,706	1,910
1983	67	218	487	800	1,121	1,397	1,604	1,772	1,898		2,210
1984	104	298	609	973	1,337	1,612	1,836	2,000			2,535
1985	43	254	602	1,025	1,406	1,736	1,924				2,721
1986	52	261	626	1,006	1,362	1,621					2,726
1987	37	267	635	1,029	1,348						2,889
1988	56	338	733	1,092							3,504
1989	79	396	853								4,589
1990	88	445									5,435
1991	98										6,319
Cum LDFs = d(t)	64.485	11.674	4.747	2.729	1.941	1.569	1.369	1.251	1.178	1.129	
% Reported	1.6%	8.6%	21.1%	36.6%	51.5%	63.7%	73.1%	79.9%	84.9%	88.5%	
% Unreported	98.4%	91.4%	78.9%	63.4%	48.5%	36.3%	26.9%	20.1%	15.1%	11.5%	

Projected Ultimate Loss + ALAE at N months - $U(i,t) = L(i,t) + Pult(i)*(1-1/d(t))$

AY	12	24	36	48	60	72	84	96	108	120	Average of All	Average of last 4
1982	1,931	1,919	1,891	1,885	1,878	1,890	1,900	1,901	1,919	1,925	1,904	1,911
1983	2,243	2,239	2,231	2,200	2,192	2,198	2,199	2,216	2,232		2,217	2,211
1984	2,600	2,616	2,610	2,579	2,566	2,531	2,519	2,509			2,566	2,531
1985	2,722	2,742	2,750	2,749	2,725	2,722	2,657				2,724	2,713
1986	2,736	2,753	2,778	2,733	2,684	2,609					2,715	2,701
1987	2,881	2,909	2,915	2,859	2,749						2,863	2,858
1988	3,506	3,542	3,499	3,312							3,465	3,465
1989	4,597	4,592	4,475								4,554	4,554
1990	5,438	5,414									5,426	5,426
1991	6,319										6,319	6,319

Notes: [1] Accident Year 1982 @12months: 1,931 = 50 + 1,910*(1-1/64.485)
[2] 'Prior' Ultimate equals the Average of the last 4 ultimates as projected by the LDF method

Data Source: Best's Aggregates & Averages - 1991

Adjustment of Indicated Ultimates
Based on Actual Company Data

Exhibit IVa

Historical Paid Loss +ALAE at N months - L(i,t)

AY	12	24	36	48	60	72	84	96	108	120
1	553	7,034	14,473	22,365	27,140	35,561	39,822	42,418	46,121	48,164
2	631	8,281	14,590	22,431	28,727	34,241	41,279	43,457	47,044	
3	682	6,431	17,260	26,945	34,464	40,194	45,640	50,271		
4	933	10,942	22,880	37,076	42,430	51,883	58,648			
5	999	11,208	21,225	30,108	38,568	43,636				
6	1,221	12,050	24,735	35,563	45,488					
7	1,369	14,689	29,190	40,431						
8	1,169	9,580	22,461							
9	878	7,819								
10	672									

Age-to-Age Development Factors

	12	24	36	48	60	72	84	96	108	120
1	12.722	2.058	1.545	1.214	1.310	1.120	1.065	1.087	1.044	
2	13.114	1.762	1.537	1.281	1.192	1.206	1.053	1.083		
3	9.433	2.684	1.561	1.279	1.166	1.135	1.101			
4	11.727	2.091	1.620	1.144	1.223	1.130				
5	11.219	1.894	1.419	1.281	1.131					
6	9.869	2.053	1.438	1.279						
7	10.730	1.987	1.385							
8	8.195	2.345								
9	8.903									
Selected	10.436	2.080	1.489	1.243	1.200	1.145	1.074	1.085	1.044	
Cumulative = d(t)	72.028	6.902	3.319	2.229	1.794	1.496	1.306	1.216	1.121	1.073

Indicated Ultimates - U(i,t) = L(i,t)*d(t)

AY	12	24	36	48	60	72	84	96	108	120	Indicated @12 mths	Indicated @24 mths	Ult(i) Avg Indic 36 & Subseq
1	39,826	48,546	48,034	49,854	48,687	53,182	52,002	51,565	51,680	51,680	39,826	48,546	50,836
2	45,483	57,154	48,420	50,001	51,534	51,208	53,905	52,829	52,715		45,483	57,154	51,516
3	49,103	44,383	57,282	60,064	61,826	60,111	59,600	61,113			49,103	44,383	59,999
4	67,206	75,519	75,935	82,646	76,115	77,592	76,586				67,206	75,519	77,775
5	71,958	77,357	70,440	67,114	69,189	65,258					71,958	77,357	68,000
6	87,945	83,167	82,092	79,275	81,601						87,945	83,167	80,989
7	98,604	101,378	96,874	90,125							98,604	101,378	93,499
8	84,203	66,116	74,544								84,203	66,116	74,544
9	63,258	53,962											
10	48,433												

Adjustment of Indicated Ultimates

Exhibit IVb

Calculation of Adjustment Factors

AY	U(i,12)	U(i,24)	Avg of U(i) 36 & Subs
1	39,826	48,546	50,836
2	45,483	57,154	51,516
3	49,103	44,383	59,999
4	67,206	75,519	77,775
5	71,958	77,357	68,000
6	87,945	83,167	80,989
7	98,604	101,378	93,499
8	84,203	66,116	74,544
9	63,258	53,962	
10	48,433		

Adjustment Factors @ 12 mths

AY	Ult(i) / U(i,12)
1	1.28
2	1.13
3	1.22
4	1.16
5	0.95
6	0.92
7	0.95
8	0.89

Regression Output:	
Constant	1.3168
Std Err of Y Est	6.79%
R Squared	83.09%
No. of Observations	8
Degrees of Freedom	6
X Coefficient(s)	-0.0569
Std Err of Coef.	1.05%

Adjustment Factors @ 24 mths

AY	Ult(i) / U(i,24)
1	1.05
2	0.90
3	1.35
4	1.03
5	0.88
6	0.97
7	0.92
8	1.13

Regression Output:	
Constant	1.0622
Std Err of Y Est	16.59%
R Squared	1.36%
No. of Observations	8
Degrees of Freedom	6
X Coefficient(s)	-0.0074
Std Err of Coef.	2.56%

Application of Adjustment Factors

AY	U(i,12)	U(i,24)	Avg of U(i,t) t=36 & Subs	Selected Adjustment Factors @12 mths	@24 mths	Ult(i)	
1	39,826	48,546	50,836			50,836	
2	45,483	57,154	51,516			51,516	
3	49,103	44,383	59,999			59,999	
4	67,206	75,519	77,775			77,775	
5	71,958	77,357	68,000			68,000	
6	87,945	83,167	80,989			80,989	
7	98,604	101,378	93,499			93,499	
8	84,203	66,116	74,544			74,544	
9	63,258	53,962		0.81	1.00	52,447	= Avg of 63,258*.81 & 53,962*1.00
10	48,433			0.75		36,242	= 48,433*.75

Notes: [1] The selected Adjustment Factors @ 12 months equal the regression predicted factors
[2] The selected Adjustment Factor @ 24 months equals 1.00. The regression fit has a very low R-Squared value (1.36%), and is, therefore, not used. The average of the prior factors equals 1.03 and the average of the prior factors excluding the high and the low equals 1.00. Hence, there does not appear to be sufficient evidence to justify an adjustment to Indicated ultimates @ 24 months.

Adjustment of Indicated Ultimates

Exhibit IVb

Calculation of Adjustment Factors

AY	U(i,12)	U(i,24)	Avg of U(i) 36 & Subs
1	39,826	48,546	50,836
2	45,483	57,154	51,516
3	49,103	44,383	59,999
4	67,206	75,519	77,775
5	71,958	77,357	68,000
6	87,945	83,167	80,989
7	98,604	101,378	93,499
8	84,203	66,116	74,544
9	63,258	53,962	
10	48,433		

Adjustment Factors @ 12 mths

AY	Ult(i) / U(i,12)
1	1.28
2	1.13
3	1.22
4	1.16
5	0.95
6	0.92
7	0.95
8	0.89

Regression Output:	
Constant	1.3168
Std Err of Y Est	6.79%
R Squared	83.09%
No. of Observations	8
Degrees of Freedom	6
X Coefficient(s)	-0.0569
Std Err of Coef.	1.05%

Adjustment Factors @ 24 mths

AY	Ult(i) / U(i,24)
1	1.05
2	0.90
3	1.35
4	1.03
5	0.88
6	0.97
7	0.92
8	1.13

Regression Output:	
Constant	1.0622
Std Err of Y Est	16.59%
R Squared	1.36%
No. of Observations	8
Degrees of Freedom	6
X Coefficient(s)	-0.0074
Std Err of Coef.	2.56%

Application of Adjustment Factors

AY	U(i,12)	U(i,24)	Avg of U(i,t) t=36 & Subs	Selected Adjustment Factors @12 mths	Selected Adjustment Factors @24 mths	Ult(i)	
1	39,826	48,546	50,836			50,836	
2	45,483	57,154	51,516			51,516	
3	49,103	44,383	59,999			59,999	
4	67,206	75,519	77,775			77,775	
5	71,958	77,357	68,000			68,000	
6	87,945	83,167	80,989			80,989	
7	98,604	101,378	93,499			93,499	
8	84,203	66,116	74,544			74,544	
9	63,258	53,962		0.81	1.00	52,447	= Avg of 63,258*.81 & 53,962*1.00
10	48,433			0.75		36,242	= 48,433*.75

Notes: [1] The selected Adjustment Factors @ 12 months equal the regression predicted factors
[2] The selected Adjustment Factor @ 24 months equals 1.00. The regression fit has a very low R-Squared value (1.36%), and is, therefore, not used. The average of the prior factors equals 1.03 and the average of the prior factors excluding the high and the low equals 1.00. Hence, there does not appear to be sufficient evidence to justify an adjustment to Indicated ultimates @ 24 months.

Exhibit V

Fitted Loss Ratios
1981 Year of Account

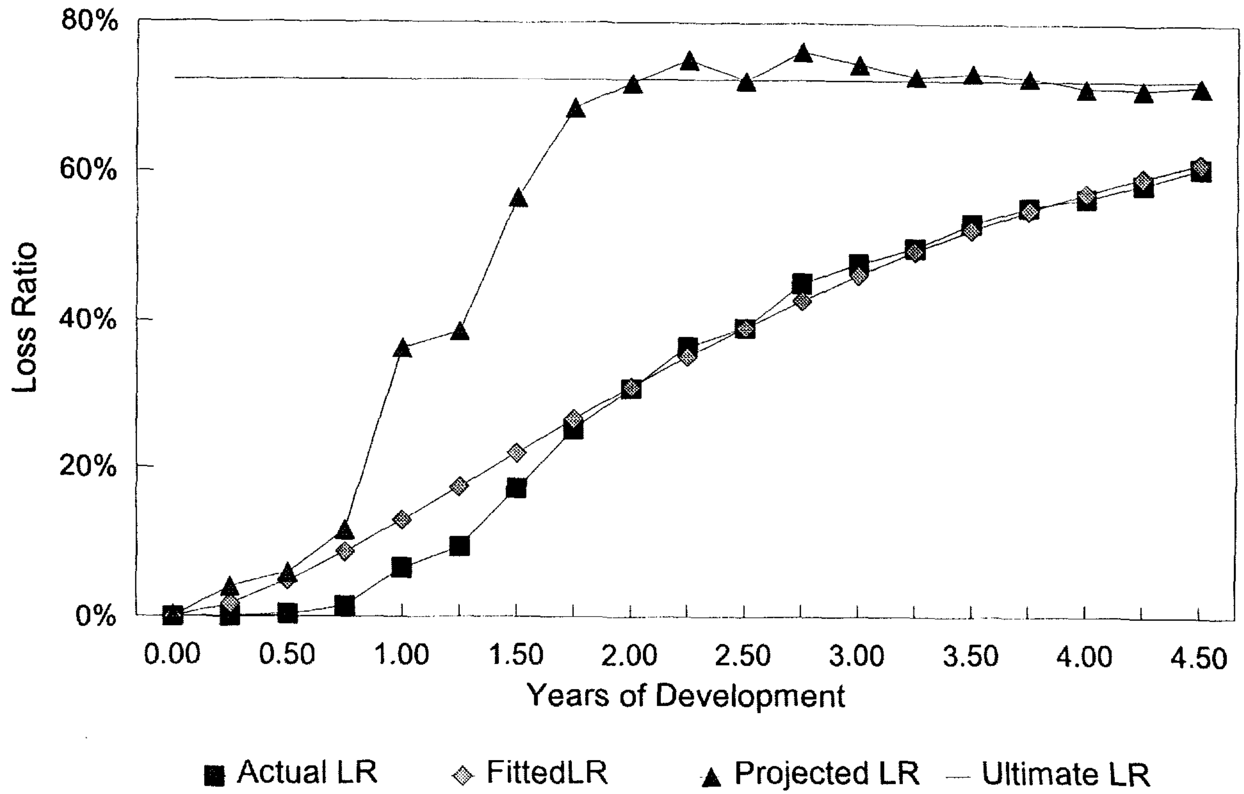


Exhibit VI
Sheet A

XYZ RE Casualty Loss Ratios

Comparison Paid vs Incd Indic

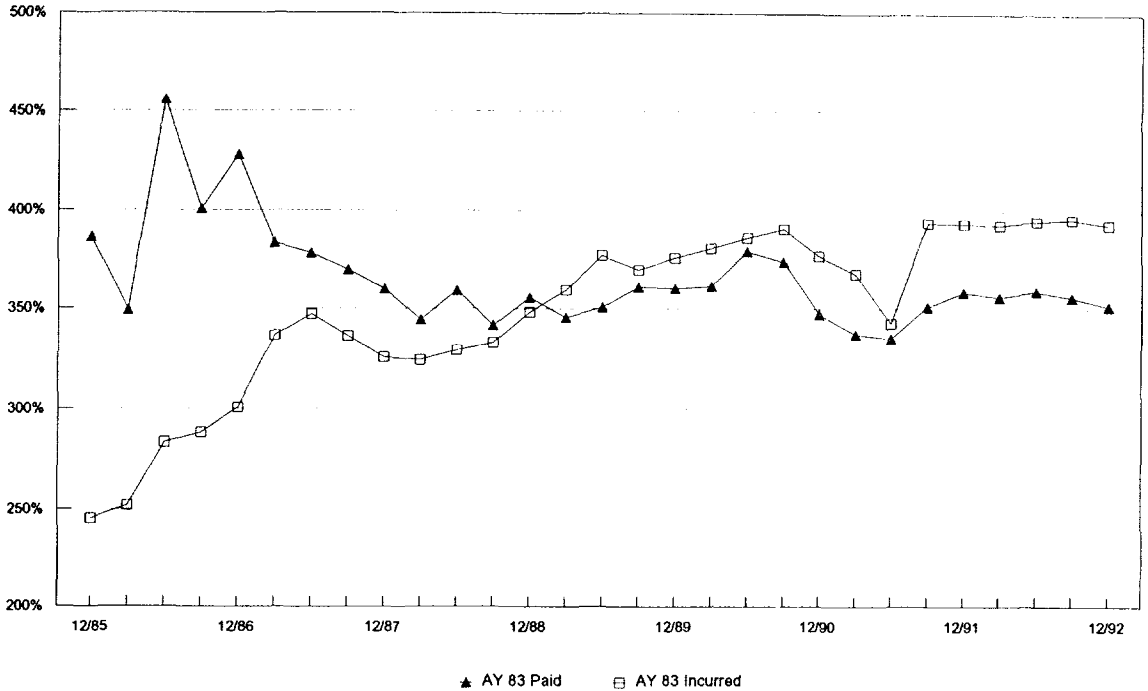
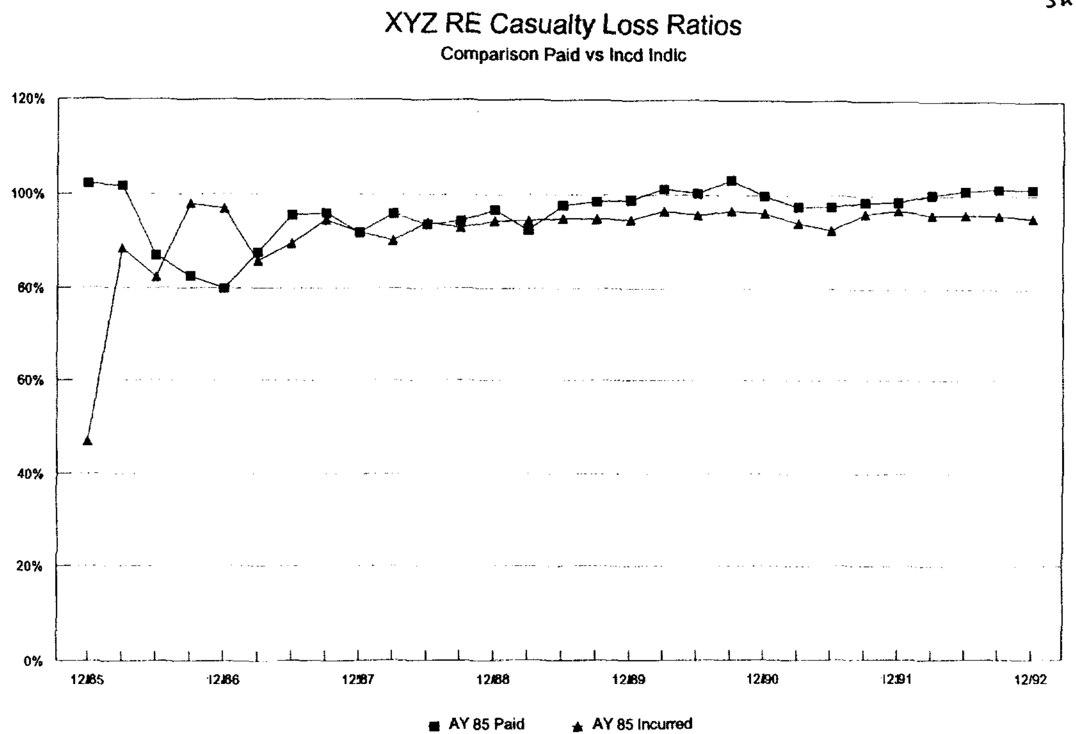


Exhibit VI
Sheet 6



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